Assignment 4-6: Modeling With Stan

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2018/12/06

```
#Importing relevant packages
import pandas as pd
import numpy as np
import pystan
import matplotlib.pyplot as plt
```

1 Call Center Modeling

1.1 Load Data

```
'''
Call Center Modeling
'''

#Load the data locally
data = np.loadtxt('/Users/ash/Downloads/call-center.csv')

# Split the data into 24 separate series, one for each hour of the day
# Code from Scheffler's gist
# (https://gist.github.com/cscheffler/6a03c9473297f21b78363ec7301d19d8#file-cs146-2-2-pre-class-work-ipynb)
current_time = 0
waiting_times_per_hour = [[] for _ in range(24)]  # Make 24 empty lists, one per hour
for t in data:
    current_hour = int(current_time // 60)
    current_time += t
    waiting_times_per_hour[current_hour].append(t)

print('Number of datapoints:', len(waiting_times_per_hour[11]))
data = waiting_times_per_hour[11]

-->Output:
Number of datapoints: 892
```

1.2 Stan Model

```
#Stan model
model_cc = """

data {
//length is the size of the data, waitime is the individual wait time
    between calls
    int<lower=1> length;
    real<lower=0> waitime[length];
//alpha and beta are the gamma prior parameters
    real<lower=0> alpha;
    real<lower=0> beta;
}

parameters {
//lamba is the exponetial likelihood parameter
    real<lower=0> lambda;
}

model {
    lambda ~ gamma(alpha, beta);
    for(i in 1:length) {
        waitime[i] ~ exponential(lambda);
    }
}

"""
#Compiling the model
stan_model_cc = pystan.StanModel(model_code=model_cc)
```

1.3 Results

```
#Specifying call center data
call_center_data = {
    'length': len(data),
    'waitime': data,
    'alpha': 1,
    'beta': 0.25
}

#Sampling using Stan and displaying the sampling results
stan_cc_results = stan_model_cc.sampling(data=call_center_data)
cc_results_lambda = stan_cc_results.extract()['lambda']
print(stan_cc_results)

#Computing the confidence interval for lambda
print('95% confidence interval for lambda is:',(np.percentile(
    cc_results_lambda, 2.5),np.percentile(cc_results_lambda,97.5)))

#Plotting the histogram for lambda
plt.hist(cc_results_lambda, density=True, bins=50)
plt.title('Histogram of $\lambda$')
plt.show()
```

```
-->Output:
```

```
Inference for Stan model: anon_model_134b0d50066e1a97cfd72f284bc79e99.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

          mean se_mean     sd    2.5%     25%     50%     75%   97.5%   n_eff    Rhat
lambda   14.84    0.01   0.51   13.85   14.49   14.85   15.18   15.81    1460     1.0
lp__    1516.4    0.02   0.73  1514.3  1516.2  1516.6  1516.8  1516.9    1707     1.0

Samples were drawn using NUTS at Wed Oct 17 12:12:02 2018.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).

-----

95% confidence interval for lambda is: (13.850487346728022, 15.803830966391658)

-----

Figure 1
```

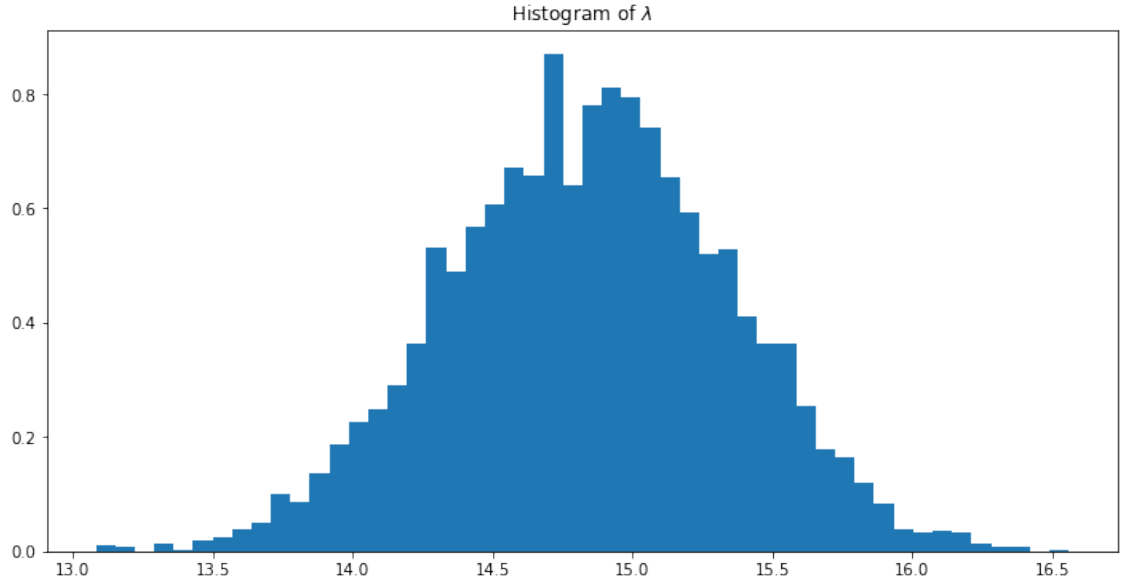


Figure 1: Histogram of sampled λ by Stan

2 Normal Inverse Gamma Prior

2.1 Load Data

```
'''
Normal Inverse Gamma Prior
'''

#Load the data
data_ = np.array([3.54551763569501, 4.23799861761927, 4.72138425951628,
    -0.692265320368236, 3.04473513808788, 3.10721270732507,
    3.42982225852764, 3.12153903971176, 3.60532628639808, 2.46561737557325,
```

```
 1.64059465916131, 2.4621623937158, 2.76744495617481, 2.11580054750407,
 5.14077208608354, 4.90288499104252, 1.43357579078348,
4.78997817363558, 1.93633438207439, 2.43698838097178, 3.95389148701877,
 2.4242295507716, 2.90256268679023, 2.90931728045901,
0.658072819386888, 3.05946763895983, 3.42615331539605,
2.68842833004417, 2.35850130765166, 2.20014998540933, 4.73846511350084,
 4.19839721414451, 2.11805510171691, -0.572742936038015,
0.389413982010623, 3.87846130744249, 1.34057656890858, 0.7235748351719,
 5.11042369840174, 4.00747556696571, 3.18080956726965,
3.24677964069676, 5.1154659863626, 1.80276616697155, 0.305877679021404,
 -0.449168307882718, 4.63705561194774, 1.37783714058301,
4.9608149859515, 6.7764195802069, 1.75515522922399, 7.04457337435215,
0.625185284955128, 2.25130734369064, 2.19770178119255,
2.16858257249432, 6.25367644481438, 0.116081323476489,
2.06315857864341, 1.82409781471718, 5.15226741230987, 2.03408231293173,
 -1.12450854337596, 5.03511270642234, 2.03841989653263,
5.80911741751597, 2.31718128783245, 4.97575010580997, 3.34262752222776,
 -0.786983904253601, 0.777362359850013, 0.975825009321195,
3.76354577515958, 7.27215002907876, 1.35404089480189, 3.76567940257157,
 3.48573993343334, 1.85976988586156, 1.93567061960716,
5.31071812003942, 2.96832987672751, 3.32378908637275, 2.61631960054551,
 5.80897964052825, 4.95215217171488, 1.32036772796131,
3.79932542233371, 3.08108492766309, 2.6734110081666, -0.14251851138521,
 2.48744375588965, 3.98463042123415, 6.32781680028, 4.0029172024315,
4.23210369459457, 1.71412938967325, 5.16492114963802, 2.53409673107906,
 4.77346963973334, 3.34088878725551, 4.77681472750664,
3.81135755590976, 1.14054269983137, 1.42057452397702,
0.132142311125433, 7.12577254064672, 4.85422012781764,
4.15745720676399, 4.48763147363348, 1.56060322283629, 2.64821761542887,
 1.26655351354548, 4.48497722937931, 4.3286302403783, 4.26157679512625,
 4.0597558651364, 5.14051109132496, 2.5660348362221, 1.10764013818617,
0.386889523012303, 3.54150473246237, 3.57480214382351,
1.95150869584847, 2.70688970563118, 2.47971849820016, 6.50838037000679,
 4.01511556826974, 1.11562740835344, 5.02637639472439,
4.38184491686864, 5.60423144047386, 2.40067408379298, 5.7849941378344,
2.37225791084559, 6.86031465910273, 4.09214858239736, 6.85994063692621,
 3.62202415158781, -1.11220646958158, 3.73920971696866,
3.24533871512216, 1.28724203643002, 0.291152541773164,
0.368630935755111, 6.71607270510525, 5.42278455200833,
5.35188416119281, 2.305874586163, -1.85878097203032, 2.69877382351447,
4.84121860550417, 4.40973060799391, 5.04399320650774, 2.68632252661298,
 6.06531610659912, 3.11881325011993, 3.45532087005125,
3.08442259840346, 4.43564424136733, 2.84252623135804, 1.50536798885106,
 1.48868622407603, 2.07322837615663, 2.5476910210998, 5.66941808257884,
 2.16731067416426, 2.49843958833905, 3.94586413879977,
0.316433764679541, -0.608937441815983, 2.5943436558557,
1.05516869528337, 2.1447601332725, 6.65846634141906, 2.1771555267834,
5.23953812029442, 3.53629759842647, 6.03263538017003, 3.85739159396599,
 5.95093453004638, 1.12856987160476, 3.5559912886093, 2.21974864244489,
 3.38471394882135, -1.90805399279409, 3.5113699258973,
4.49319955412346, 5.10507952638867, 1.08277895384184, 4.58403638422759,
 1.37304994426824, 4.17566975753523, 3.36454182510378,
0.177136582644021, 2.91337423388405, 3.22796455457526,
2.80124198378441, 1.95189718582788, 3.37659263896246,
```

```
    -1.6463045238231])
print(len(data_), "data")

--->Output:
200 data
```

2.2 Stan Model

```
#Stan model

model_nig = """

data {
//Data and its length
    int<lower=1> length;
    real data_[length];
//alpha, beta, mu, nu of the NIG prior
    real<lower=0> alpha;
    real<lower=0> beta;
    real<lower=0> nu;
    real mu;
}

parameters {
//mean and var are the mean and variance of the normal likelihood (with
    unknown mean and variance)
    real mean_;
    real<lower=0> var_;
}

model {
    var_ ~ inv_gamma(alpha, beta);
    mean_ ~ normal(mu, sqrt(var_/nu));
    for(i in 1:length) {
        data_[i] ~ normal(mean_, sqrt(var_));
    }
}

"""
stan_model_nig = pystan.StanModel(model_code=model_nig)
```

2.3 Results

```
#Specifying data for Stan
nig_data = {
    'length': len(data_),
    'data_': data_,
    'alpha': 1.12,
    'beta': 0.4,
    'nu':0.054,
    'mu':0
}

#Sampling using Stan and displaying the sampling results
```

```
stan_nig_results = stan_model_nig.sampling(data=nig_data)
nig_results_mean = stan_nig_results.extract()['mean_']
nig_results_var = stan_nig_results.extract()['var_']
print(stan_nig_results)

#Computing the confidence interval for mean and variance
print('95% confidence interval for mean is:', (np.percentile(
    nig_results_mean, 2.5),np.percentile(nig_results_mean,97.5)))
print('95% confidence interval for variance is:', (np.percentile(
    nig_results_var, 2.5),np.percentile(nig_results_var,97.5)))

#Plotting 10 samples from Stan results
from scipy.stats import norm
x = np.linspace(-15,15,400)
choices = np.random.choice(4000,size=10,replace=False)
plt.figure(figsize=(12, 6))
for _ in choices:
    plt.plot(x, norm.pdf(x,loc=nig_results_mean[_],scale=np.sqrt(
    nig_result_var[_])))

plt.title('10 Sampled Normal Distribution By Stan')
plt.show()
```

```
-->Output:

Inference for Stan model: anon_model_f3d02ba78fd42d0a638632520a6867e6.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

         mean se_mean     sd    2.5%     25%     50%     75%   97.5%   n_eff    Rhat
mean_    3.07  2.3e-3   0.14     2.8    2.97    3.07    3.16    3.33    3614     1.0
var_     3.62  6.6e-3   0.36    2.98    3.37     3.6    3.85     4.4    3039     1.0
lp__   -233.2    0.02   1.04  -236.1  -233.6  -232.9  -232.5  -232.2    1737     1.0

Samples were drawn using NUTS at Wed Oct 17 13:39:51 2018.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).

-----

95% confidence interval for mean is: (2.798198119655621, 3.3294936656934633)
95% confidence interval for variance is: (2.97808915862992, 4.403567082267464)

-----

Figure 2
```

3 HRTEM

3.1 Load Data

```
'''
HRTEM
'''
#Loading data locally and changing to log-scale
hrtem = np.loadtxt('/Users/ash/Downloads/hrtem.csv')
print('%i data, min: %f, max: %f' % (len(hrtem), min(hrtem), max(hrtem)))
log_data = np.log(hrtem)

--->Output:
500 data, min: 1.051827, max: 28.942578
```

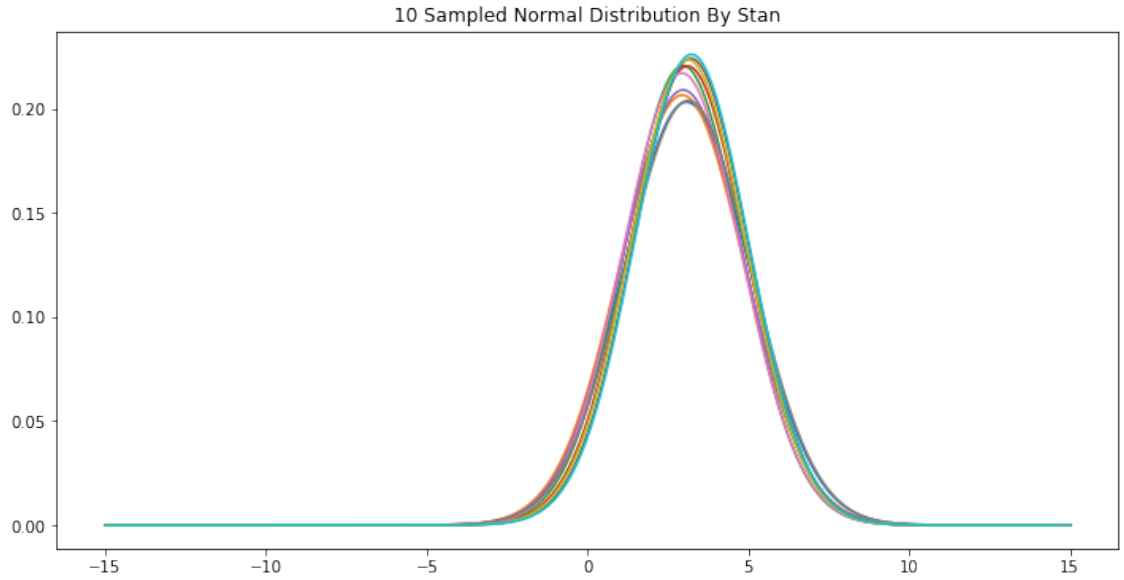


Figure 2: Normal distributions from posterior NIG sampled by Stan

3.2 Stan Model

Since we have transformed the data into log scale, it is now normally distributed and we will reuse the NIG model specified in section 2.2.

3.3 Results

```
#Specifying data for Stan
hrtem_data = {
    'length': len(log_data),
    'data_': log_data,
    'alpha': 2,
    'beta': 5,
    'nu':0.1,
    'mu':2.3
}

#Sampling from Stan and printing out results
stan_hrtem_results = stan_model_nig.sampling(data=hrtem_data)
hrtem_results_mean = stan_hrtem_results.extract()['mean_']
hrtem_results_var = stan_hrtem_results.extract()['var_']
print(stan_hrtem_results)

from scipy.stats import lognorm

choices = np.random.choice(4000,size=10,replace=False)
plt.figure(figsize=(12,6))
x = np.linspace(0, 30, 400)

```

```
for _ in choices:
    plt.plot(x, lognorm.pdf(x, np.sqrt(hrtem_results_var[_]),scale=np.exp(
    hrtem_results_mean[_])))
plt.hist(hrtem, bins=20, density=True, alpha=0.7)
plt.title('10 Log-normal Distributions Sampled By Stan')
plt.show()
```

```
-->Output:

Inference for Stan model: anon_model_f3d02ba78fd42d0a638632520a6867e6.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

        mean se_mean     sd    2.5%     25%     50%     75%   97.5%   n_eff    Rhat
mean_   1.89  5.4e-4   0.03    1.83    1.87    1.89    1.91    1.95    3454     1.0
var_    0.49  6.3e-4   0.03    0.44    0.47    0.49    0.52    0.56    2581     1.0
lp__   -76.05    0.02   1.03  -78.84  -76.42  -75.73  -75.31  -75.04    1897     1.0

Samples were drawn using NUTS at Wed Oct 17 13:58:33 2018.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).

-----

95% confidence interval for mean is: (1.8305745551258448, 1.9526028346420938)
95% confidence interval for variance is: (0.4354322787100587, 0.5588026676368617)

-----

Figure 3
```

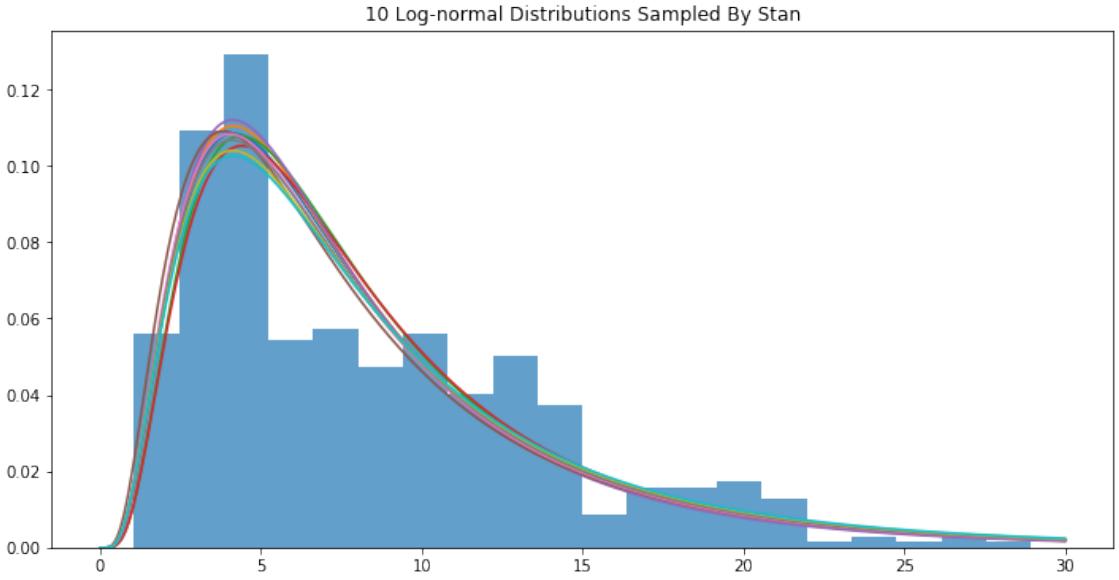


Figure 3: Log-normal distributions from posterior NIG sampled by Stan

4 Mixed Gaussians

Two Mixed Gaussians Sampled From A Normal (μ) and A Log-normal (σ^2)

To specify a mixed model we used a simplex parameter p of size 2 to indicate the mixing probability of each Gaussian, and sampled the Gaussians' means μ_i and variance σ_i^2 from a slightly informative normal and a slightly informative log-normal distribution respectively, which means:

$$\mu_i \sim \mathcal{N}(\mu = 0, \sigma = 10)$$
$$\sigma_i^2 \sim Log - normal(\mu = 0, \sigma = 2)$$

Using this specification, we can add the product of the likelihood and prior probability by transforming them to log scale and add them to the log-scaled posterior probability, using a syntax of Stan ("target" being the log probability of the posterior distribution). With this, we can sample from the posterior distribution using MCMC as usual.

The Stan model is specified as followed:

```
'''
Mixture of Gaussians --1--
'''

model_mix = """

data {
//Data and its length
    int<lower=1> length;
    real<lower=0> data_[length];
}

parameters {
//mean and var are the mean and variance of the normal likelihood (with
    unknown mean and variance)
    simplex[2] p;
    vector[2] mean_;
    vector<lower=0>[2] var_;
}

model {
    mean_ ~ normal(0,10);
    var_ ~ lognormal(0, 2);
    for(i in 1:length) {
        real gau_1;
        real gau_2;
        gau_1 = normal_lpdf(data_[i] | mean_[1], var_[1]) + log(p[1]);
        gau_2 = normal_lpdf(data_[i] | mean_[2], var_[2]) + log(p[2]);
        target += log_sum_exp(gau_1, gau_2);
    }
}
```

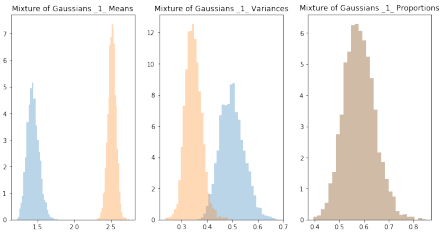
```

"""
stan_model_mix = pystan.StanModel(model_code=model_mix)

#Data for Stan model
mix_data = {
    'length': len(log_data),
    'data_': log_data
}

#Sampling from Stan and printing out results
mix_results = stan_model_mix.sampling(data=mix_data)
print(mix_results)
```

```
--->Output:
Inference for Stan model: anon_model_9d65cfc3fae879b2537b695905afbe5a.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

            mean se_mean     sd    2.5%     25%     50%     75%   97.5%   n_eff    Rhat
p[1]        0.58   1.8e-3   0.06    0.46    0.54    0.58    0.62    0.71    1183     1.0
p[2]        0.42   1.8e-3   0.06    0.29    0.38    0.42    0.46    0.54    1183     1.0
mean_[1]    1.44   2.4e-3   0.08    1.29    1.38    1.43    1.49    1.62    1193     1.0
mean_[2]    2.52   1.5e-3   0.06     2.4    2.48    2.52    2.55    2.62    1304     1.0
var_[1]      0.5   1.3e-3   0.05    0.41    0.46     0.5    0.53     0.6    1317     1.0
var_[2]     0.35   8.5e-4   0.03    0.29    0.32    0.34    0.37    0.41    1471     1.0
lp__      -506.4     0.05   1.69  -510.7  -507.3  -506.1  -505.2  -504.3    1332     1.0

Samples were drawn using NUTS at Wed Oct 17 14:48:40 2018.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

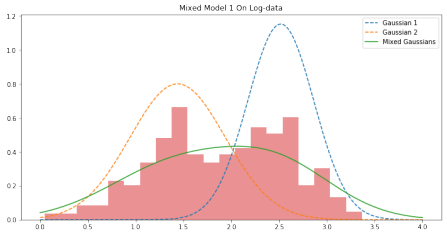
By the convergent results we can see that the mixed proportions in this model is around 50% for each Gaussian, one centered around 1.44 and one centered around 2.52. Plotting the sampling results in figure 4.1:

```
mixed1_mean1 =  mix_results.extract()['mean_'][:,0]
mixed1_mean2 =  mix_results.extract()['mean_'][:,1]
mixed1_var1 = mix_results.extract()['var_'][:,0]
mixed1_var2 = mix_results.extract()['var_'][:,1]
mixed1_p1 = mix_results.extract()['p'][:,0]
mixed1_p2 = mix_results.extract()['p'][:,1]

plt.figure(figsize=(12,6))
plt.subplot(1,3,1)
plt.hist(mixed1_mean1, alpha=0.3, bins=30, density=True)
plt.hist(mixed1_mean2, alpha=0.3, bins=30, density=True)
plt.title('Mixture of Gaussians _1_ Means')
plt.subplot(1,3,2)
plt.hist(mixed1_var1, alpha=0.3, bins=30, density=True)
plt.hist(mixed1_var2, alpha=0.3, bins=30, density=True)
plt.title('Mixture of Gaussians _1_ Variances')
plt.subplot(1,3,3)
plt.hist(mixed1_p1, alpha=0.3, bins=30, density=True)
plt.hist(mixed1_p1, alpha=0.3, bins=30, density=True)
plt.title('Mixture of Gaussians _1_ Proportions')
plt.show()
```

Plotting the resulted mixed model on the log-data, we can see that the model fit not too well with the data (figure 4.2).



(a) Summary histograms of parameters



(b) Log-data with Stan best fit mixed model

Figure 4: Mixed Gaussians 1

```
from scipy.stats import norm
plt.figure(figsize=(12,6))
x = np.linspace(0, 4, 400)

mixed_gaussians = np.mean(mixed1_p1)*norm.pdf(x, loc=np.mean(mixed1_mean1),
    scale=np.sqrt(np.mean(mixed1_var1))) + (1-np.mean(mixed1_p1))*norm.pdf(
    x, loc=np.mean(mixed1_mean2),scale=np.sqrt(np.mean(mixed1_var2)))
plt.plot(x, norm.pdf(x, loc=np.mean(mixed1_mean1),scale=np.sqrt(np.mean(
    mixed1_var1))), linestyle='--', label='Gaussian 1')
plt.plot(x, norm.pdf(x, loc=np.mean(mixed1_mean2),scale=np.sqrt(np.mean(
    mixed1_var2))), linestyle='--', label='Gaussian 2')
plt.plot(x,mixed_gaussians, label='Mixed Gaussians')
plt.hist(log_data, bins=20, density=True, alpha=0.5)
plt.title('Mixed Model 1 On Log-data')
plt.legend()
plt.show()
```

Two Mixed Gaussians Sampled From A Normal (μ) and An Inverse-Gamma (σ^2)

This time instead of sampling σ^2 from a log-normal prior we will sample it from an inverse-gamma with $\alpha = 1$ and $\beta = 1$. Other specifications are similar to the previous scenario.

```
'''
Mixture of Gaussians __2__
'''

model_mix_2 = """

data {
//Data and its length
    int<lower=1> length;
    real<lower=0> data_[length];
}

parameters {
```

```
//mean and var are the mean and variance of the normal likelihood (with unknown mean and variance)
    simplex[2] p;
    vector[2] mean_;
    vector<lower=0>[2] var_;
}

model {
    var_ ~ inv_gamma(1, 1);
    mean_ ~ normal(0,10);
    for(i in 1:length) {
        real gau_1;
        real gau_2;
        gau_1 = normal_lpdf(data_[i] | mean_[1], sqrt(var_[1])) + log(p[1]);
        gau_2 = normal_lpdf(data_[i] | mean_[2], sqrt(var_[2])) + log(p[2]);
        target += log_sum_exp(gau_1, gau_2);
    }
}

"""
stan_model_mix_2 = pystan.StanModel(model_code=model_mix_2)

mix_data_2 = {
    'length': len(log_data),
    'data_': log_data
}
#Sampling from Stan and printing out results
mix_results_3 = stan_model_mix_2.sampling(data=mix_data_2)
print(mix_results_2)
```

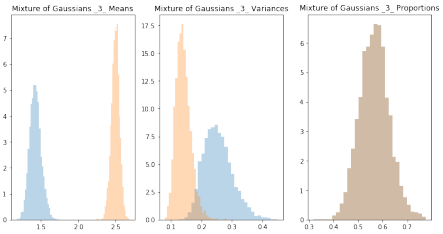
The model successfully converged, and summaries of the results are in figure 5.

```
--->Output:
Inference for Stan model: anon_model_080355cd049d7e529040e1ad4bd50372.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

           mean se_mean     sd    2.5%     25%     50%     75%   97.5%   n_eff    Rhat
p[1]       0.57   1.7e-3   0.06    0.45    0.52    0.57     0.6    0.69    1244     1.0
p[2]       0.43   1.7e-3   0.06    0.31     0.4    0.43    0.48    0.55    1244     1.0
mean_[1]   1.43   2.1e-3   0.08    1.28    1.37    1.42    1.47     1.6    1339     1.0
mean_[2]    2.5   1.4e-3   0.05    2.38    2.46     2.5    2.53     2.6    1436     1.0
var_[1]    0.25   1.2e-3   0.05    0.18    0.22    0.25    0.28    0.36    1499     1.0
var_[2]    0.14   5.9e-4   0.02     0.1    0.12    0.14    0.16     0.2    1720     1.0
lp__      -514.9     0.04   1.68  -519.1  -515.7  -514.5  -513.7  -512.7    1545     1.0

Samples were drawn using NUTS at Thu Oct 18 12:43:06 2018.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

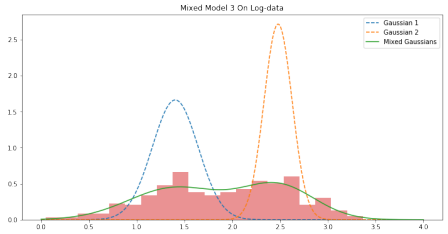
Two Mixed Gaussians Sampled From A Normal Inverse Gamma (σ^2)

This time we sample the two Gaussians from the same Normal Inverse Gamma Distribution Prior. Other specifications are similar to the previous scenario.

```
'''
```



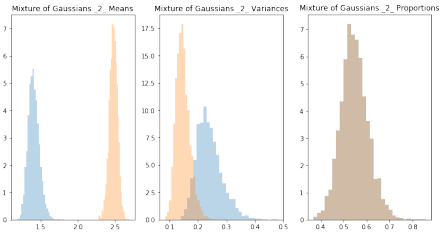
(a) Summary histograms of parameters



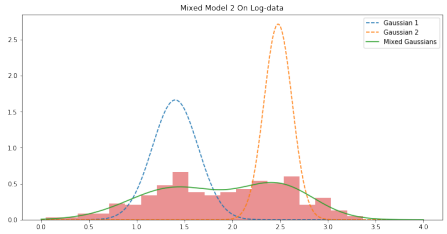
(b) Log-data with Stan best fit mixed model

Figure 5: Mixed Gaussians 2

```
Mixture of Gaussians __3__
'''

model_mix_3 = """

data {
//Data and its length
    int<lower=1> length;
    real<lower=0> data_[length];
    real<lower=0> alpha;
    real<lower=0> beta;
    real<lower=0> nu;
    real mu;
}

parameters {
//mean and var are the mean and variance of the normal likelihood (with unknown mean and variance)
    simplex[2] p;
    vector[2] mean_;
    vector<lower=0>[2] var_;
}

model {
    var_ ~ inv_gamma(alpha, beta);
    mean_ ~ normal(mu, sqrt(var_/nu));
    for(i in 1:length) {
        real gau_1;
        real gau_2;
        gau_1 = normal_lpdf(data_[i] | mean_[1], sqrt(var_[1])) + log(p[1]);
        gau_2 = normal_lpdf(data_[i] | mean_[2], sqrt(var_[2])) + log(p[2]);
        target += log_sum_exp(gau_1, gau_2);
    }
}

"""
stan_model_mix_3 = pystan.StanModel(model_code=model_mix_3)

```



(a) Summary histograms of parameters



(b) Log-data with Stan best fit mixed model

Figure 6: Mixed Gaussians 3

```
mix_data_3 = {
    'length': len(log_data),
    'data_': log_data
}
#Sampling from Stan and printing out results
mix_results_3 = stan_model_mix_2.sampling(data=mix_data_3)
print(mix_results_3)
```

The model successfully converged, and summaries of the results are in figure 6.

```
--->Output:
Inference for Stan model: anon_model_cc255d0eaad30c6dc5ca0a069ab29537.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

          mean se_mean     sd    2.5%     25%     50%     75%   97.5%  n_eff   Rhat
p[1]      0.55  1.8e-3   0.06    0.43    0.51    0.54    0.58    0.68   1147    1.0
p[2]      0.45  1.8e-3   0.06    0.32    0.42    0.46    0.49    0.57   1147    1.0
mean_[1]   1.4  2.2e-3   0.08    1.27    1.35     1.4    1.45    1.57   1193    1.0
mean_[2]  2.48  1.6e-3   0.06    2.36    2.44    2.48    2.52    2.58   1253    1.0
var_[1]   0.24  1.2e-3   0.05    0.17    0.21    0.23    0.27    0.34   1374    1.0
var_[2]   0.15  6.5e-4   0.02    0.11    0.13    0.14    0.16     0.2   1476    1.0
lp__      -514.6    0.05   1.67  -518.6  -515.5  -514.3  -513.4  -512.4   1291    1.0

Samples were drawn using NUTS at Thu Oct 18 14:09:19 2018.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

Discussion

With three different specifications of priors the mixing proportions came out roughly the same $(0.55 - 0.45)$ and two modes around 2.5 and 1.4, which is quite good if we eye-ball them in the histograms of the log-data. Variances seem to differ quite a lot if we are sampling the variances from the inverse-gamma instead of log-normal, which is reasonable since the two distributions signify different prior knowledge. Since the variance for the first mixture model is quite large for both Gaussians, the mixture model becomes more of a skew Gaussians (figure 4) rather than a bimodal distribution, but when the variance is smaller like with the second and the third case the bimodal distribution shows a little better (in figure 5 and 6). Overall the second and third mixture models seem to fit better with the bimodality of the log-data, which probably happened because the inverse-gamma and normal inverse gamma distribution has a more suitable shape for drawing σ^2 from.