CS5691 Assignment 1 Report

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Motivation

- To understand the effects of dimensionality reduction on reconstructing a decomposed image
- To compare the efficiencies of decomposing an image using Eigen Value Decomposition and Singular Value Decomposition

EVD

$$A = PDP^{-1}$$

where,

A is the original matrix
P is the matrix made from the eigen vectors of A
D is the diagonal matrix with the elements as the eigen values of A

SVD

$$A = U\Sigma V^T$$

where,

A is the original matrix
U is the right singluar matrix of A
V is the left singular matrix of A
D is the diagonal matrix with elements as the singular values of A

Experimental Results

- Figure 1 displays the reconstructed images besides the error images(original image - reconstructed image) for certain values of k, where k represents the number of top eigen/singular values used to reconstruct the image
- Figure 2 displays the Frobenius norm of the error matrix vs k using EVD(blue) and SVD(orange), for values of k from 1 to 256.

Inferences

- As seen from figure 2, the values of k for which the norm of the error image is equal to 1000, when using SVD is around 50, whereas when using EVD is around 180. The clarity of these two reconstructed images are almost indistinguishable from the original image. From this, we can conclude that using SVD for decomposing an image is more effective than EVD since less data, $(i.e)$ less number of eigen value-vector/ singular value-vectors pairs are required to recreate the image of equal quality.
- Figure 2 depicts high instability in the norm of the error image produced using EVD. This adds to the reasons why SVD would be preferred over EVD wherever applicable, besides the fact SVD can be used on non square matrices as well.
- We can also conclude that SVD is able to efficiently utilise the fact that most of the data of the image is contained/represented in the eigen/singluar values with higher magnitude than compared to those values with lower magnitude that holds a much smaller fraction of the data of the image, since the norm of the error matrix decreases drastically in the start when using SVD.

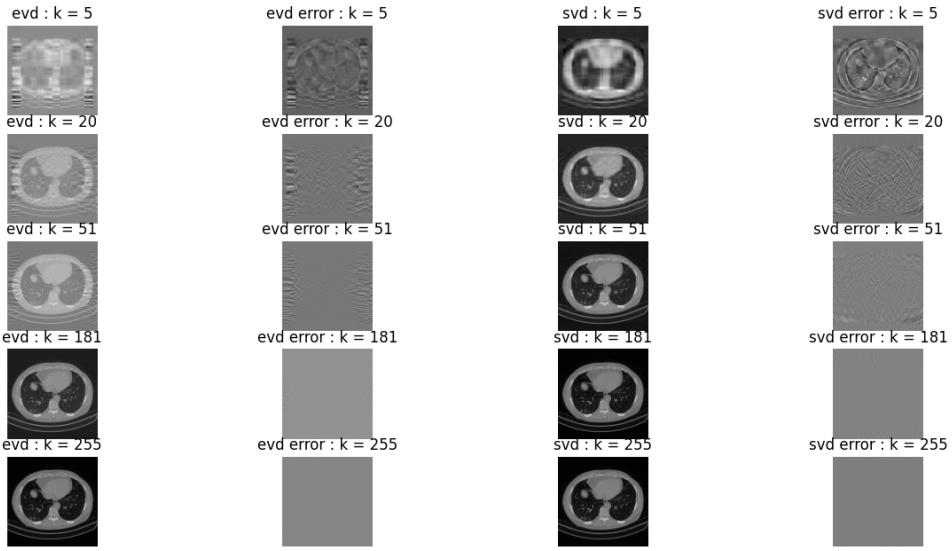

Fig 1

- It is easier to use SVD compared to EVD, since we don't have to take care of using both complex conjugates in SVD, since all the singular values are always positive(taken by convention instead of using negative values as well) compared to the eigen values that come as complex conjugate pairs or real numbers.

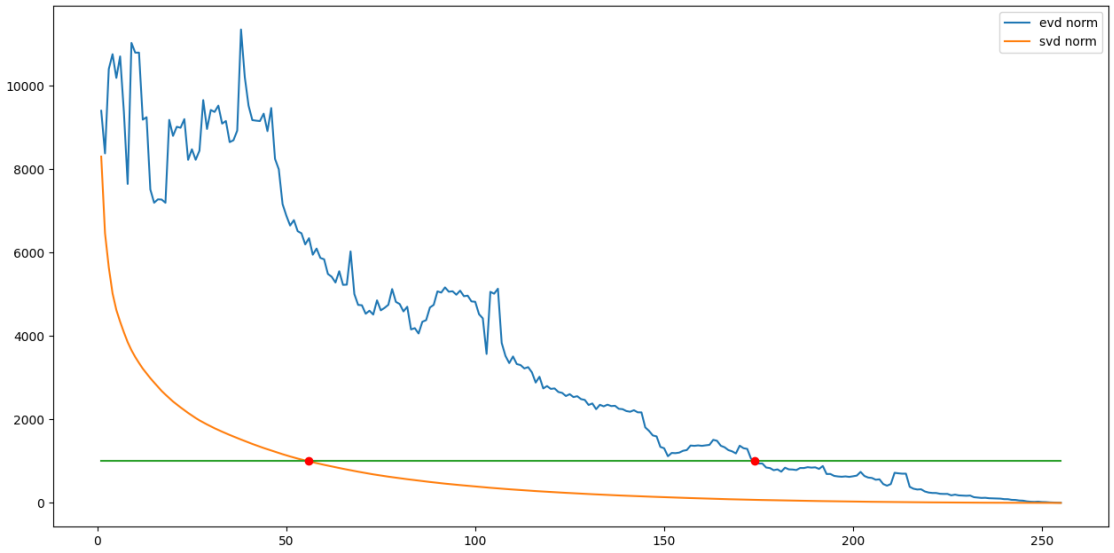

Fig 2