STATISTICAL DATA MINING 1

Homework 3

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Class Number 3

1) **Boston Data Set :**

```
     crim                zn             indus            chas        
 Min.   : 0.00632   Min.   :  0.00   Min.   : 0.46   Min.   :0.00000  
 1st Qu.: 0.08204   1st Qu.:  0.00   1st Qu.: 5.19   1st Qu.:0.00000  
 Median : 0.25651   Median :  0.00   Median : 9.69   Median :0.00000  
 Mean   : 3.61352   Mean   : 11.36   Mean   :11.14   Mean   :0.06917  
 3rd Qu.: 3.67708   3rd Qu.: 12.50   3rd Qu.:18.10   3rd Qu.:0.00000  
 Max.   :88.97620   Max.   :100.00   Max.   :27.74   Max.   :1.00000  

      nox               rm             age              dis              rad        
 Min.   :0.3850   Min.   :3.561   Min.   :  2.90   Min.   : 1.130   Min.   : 1.000  
 1st Qu.:0.4490   1st Qu.:5.886   1st Qu.: 45.02   1st Qu.: 2.100   1st Qu.: 4.000  
 Median :0.5380   Median :6.208   Median : 77.50   Median : 3.207   Median : 5.000  
 Mean   :0.5547   Mean   :6.285   Mean   : 68.57   Mean   : 3.795   Mean   : 9.549  
 3rd Qu.:0.6240   3rd Qu.:6.623   3rd Qu.: 94.08   3rd Qu.: 5.188   3rd Qu.:24.000  
 Max.   :0.8710   Max.   :8.780   Max.   :100.00   Max.   :12.127   Max.   :24.000  

      tax           ptratio          black            lstat            medv      
 Min.   :187.0   Min.   :12.60   Min.   :  0.32   Min.   : 1.73   Min.   : 5.00  
 1st Qu.:279.0   1st Qu.:17.40   1st Qu.:375.38   1st Qu.: 6.95   1st Qu.:17.02  
 Median :330.0   Median :19.05   Median :391.44   Median :11.36   Median :21.20  
 Mean   :408.2   Mean   :18.46   Mean   :356.67   Mean   :12.65   Mean   :22.53  
 3rd Qu.:666.0   3rd Qu.:20.20   3rd Qu.:396.23   3rd Qu.:16.95   3rd Qu.:25.00  
 Max.   :711.0   Max.   :22.00   Max.   :396.90   Max.   :37.97   Max.   :50.00  

    crim_med  
 Min.   :0.0  
 1st Qu.:0.0  
 Median :0.5  
 Mean   :0.5  
 3rd Qu.:1.0  
 Max.   :1.0  
```

The following analysis is for the Boston data set , where we try to build a model to predict whether a given suburb has a high or low crime rate (above or below median) . We explore Logistic regression , Linear Discriminant Analysis and KNN for the same .

First we split the data into training and test set . We then perform the training of the model on the train set and ultimately calculate the prediction (test) error by running the model against our test set .

a) Logistic Regression :

By fitting a logistic regression model over the training set and predicting the test error on the predicted values for the test set , we obtain the logistic regression test error as :

```
lr.test.error
[1] 0.09448819
```

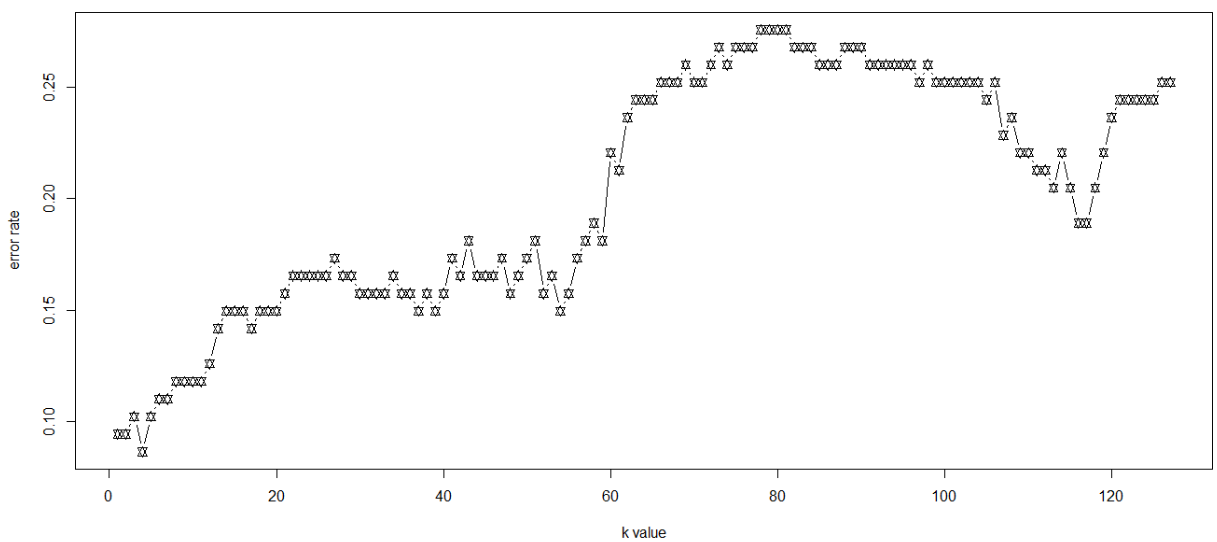
b) LDA :

By fitting a linear discriminant analysis model over the training set and predicting the test error on the predicted values for the test set , we obtain the LDA test error as :

```
lda.test.error
[1] 0.1653543
```

c) KNN :

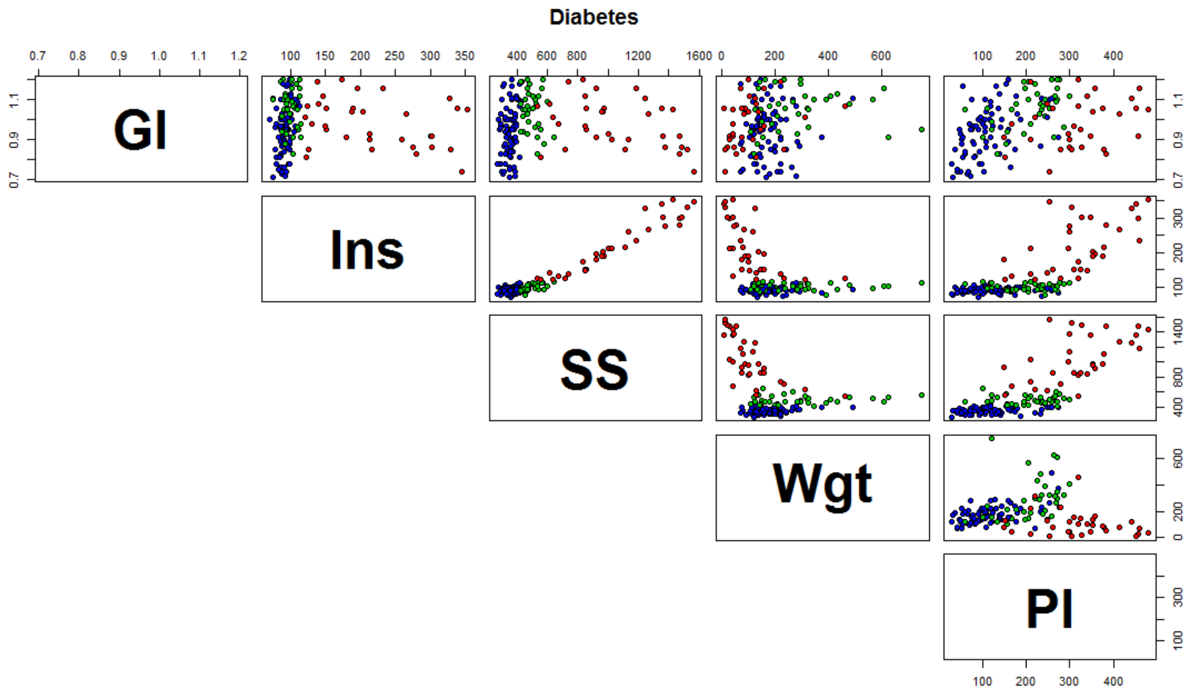
To find the test error on a Knn model , we first train our model over the training set for all values of K starting from 1 to Number of test samples and evaluate it over the test set .The following plot shows how the error values vary for each selection of K .



We find that a KNN model with k=4 has the least testing error (0.866) . Hence the prediction used by this model would be accurate in determining whether a suburb belongs to a region with high crime rate or low crime rate . We obtain the Knn test error as :

```
min.knn.error
[1] 0.08661417
```

```
Therefore , we see that the KNN model performs the best (Has a minimum error)
compared to Logistic regression model and Linear Discriminant Model .
```



2)

a) When the data is a multivariate normal distribution and the covariance matrix is common, then based on the assumptions of the LDA model , prediction may be accurate (Provided the model is able to fit the data with a low error) . But if the assumption that the covariance matrix is same throughout does not hold , then the QDA may be a better model to be used as a predictor (Again , provided the model is able to fit the data with a low error) . In this case , we see that for each class , the covariance between each pair of predictor variables are not the same . Only certain predictor pairs , such as (Glucose,SSPG) , (Insulin,SSPG) , (SSPG, Weight) show common correlations for atmost 2 classes . But other pairs of predictors such as (Weight, Fasting Plasma Glucose) (Glucose , Insulin) and (Glucose and Fasting Plasma Glucose) have data points belonging to different classes interspersed with each other , hence the assumption of a common covariance matrix may not lead to an optimal decision boundary in our LDA model. Also from the plots, its easy to see that most pairs of predictor variables are normally distributed , although in some of the plots not all classes seem to be normally distributed . Overall its safe to assume normal distribution of priors .

Covariance Matrix :

	V1	V2	V3	V4	V5
V1	1.000000000	-0.008813193	0.0239843	0.222237813	0.384319804
V2	-0.008813193	1.000000000	0.9646281	-0.396234858	0.715480192
V3	0.023984304	0.964628091	1.0000000	-0.337020435	0.770942459
V4	0.222237813	-0.396234858	-0.3370204	1.000000000	0.007914263
V5	0.384319804	0.715480192	0.7709425	0.007914263	1.000000000

b)

```
LDA_train_error
[1] 0.0862069
LDA_test_error
[1] 0.1724138

QDA_train_error
[1] 0.02586207
QDA_test_error
[1] 0.1724138
```

The LDA training error as expected is less than the LDA test error . Similarly , the QDA training error is lesser than the QDA test error , when tested on the same data split . Also , the Overall training error in the QDA case seems to be lesser than the test error , but overall test error in both the cases are the same . This indicates that ,provided the assumptions that each model takes for granted holds , they both can be used for similar predictions . But we'll see later why QDA would be a better selection.

c) The LDA model predicts the new data to be in class 3 whereas the QDA model predicts it to be in class 2 . But since the assumption needed for an Optimal LDA classifier may not withhold here , the QDA offers a better classification accuracy . In other words , the data [0.98,122,544,186,184] , assuming our assumptions hold , belongs to class 2 .

3) Under the logistic regression model , $p(X) = \exp(\beta 0 + \beta 1X)/ (1+ \exp(\beta 0 + \beta 1X))$

a) **Posterior probability for k=K is given by** :

P1 = P(C=k/X=x) = $1/(1+\sum_{[1 \text{ to } K-1]} \exp(\beta_{l0}+\beta^T_l x))$

Posterior probability for k=1…K-1 is given by :

P2= P(C = [1..K-1] / X=x) = $\exp(\beta_{k0}+\beta^T_k x)/(1+\sum_{[1 \text{ to } K-1]}\exp(\beta_{l0}+\beta^T_l x))$

P3 = Sum(P2) over all k from 1 to K-1

$= \sum_{[1 \text{ to } K-1]} \exp(\beta_{l0}+\beta^T_l x) / (1+\sum_{[1 \text{ to } K-1]} \exp(\beta_{l0}+\beta^T_l x))$

We see that P1 + P3 = $(1 + \sum_{[1 \text{ to } K-1]} \exp(\beta_{l0}+\beta^T_l x)) / (1+\sum_{[1 \text{ to } K-1]} \exp(\beta_{l0}+\beta^T_l x))$

$= 1$.

b)The logistic function is given by exp(β0 + β1X)/ (1+ exp(β0 + β1X)) and the Logit is given by the log of odds ration , i.e , log(p(X)/ (1-p(X))) .

Therefore , P(X) = exp(β0 + β1X)/ (1+ exp(β0 + β1X))

And 1 – P(X) = 1 - exp(β0 + β1X)/ (1+ exp(β0 + β1X))

= (1+ exp(β0 + β1X) - exp(β0 + β1X))/(1+ exp(β0 + β1X)

= (1)/(1+ exp(β0 + β1X))

Which means P(X)/(1-P(X)) = exp(β0 + β1X)*(1+ exp(β0 + β1X))/ (1+ exp(β0 + β1X))

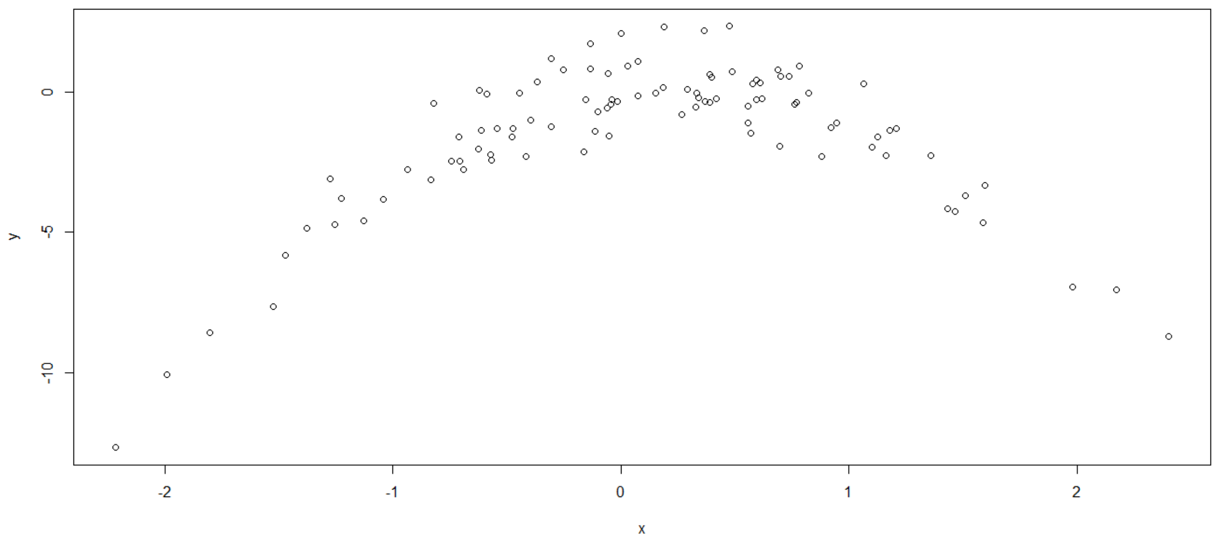
= exp(B0 + B1X)

Which is equivalent to the Logit function

Therefore the logistic function and the logit function representations are equivalent .

4)

We generate an arbitrary data set using a random seed value .



The following analysis is for studying the effect LOOCV has on different models generated arbitrarily from random samples .

We use the following 4 models

Y = β0 + β1X + ε ---------------------1

Y = β0 + β1X + β2X2 + ε ------------------------2

Y = β0 + β1X + β2X2 + β3X3 + ε ----------------------------3

Y = β0 + β1X + β2X2 + β3X3 + β4X4 + ε---------------------------4

Using LOOCV , the following are the errors that we obtained :

```
Model 1 :    [1] 7.288162
Model 2 :    [1] 0.9374236
Model 3:     [1] 0.9566218
Model 4:     [1] 0.9539049
```

b) The second model is the one with the lowest LOOCV error . This is expected because the relation between x and y is quadratic in nature . This can be seen from the figure that the relation between y and x is parabolic or quadratic with degree = 2 , similar to the second model .

c)

This is the summary of our 4 models .

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -1.6254     0.2619  -6.205 1.31e-08 ***
 x            0.6925     0.2909   2.380   0.0192 *
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -1.5500     0.0958  -16.18  < 2e-16 ***
poly(x, 2)1   6.1888     0.9580    6.46 4.18e-09 ***
poly(x, 2)2 -23.9483     0.9580  -25.00  < 2e-16 ***
                         ---
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -1.55002    0.09626 -16.102  < 2e-16 ***
poly(x, 3)1   6.18883    0.96263   6.429 4.97e-09 ***
poly(x, 3)2 -23.94830    0.96263 -24.878  < 2e-16 ***
  poly(x, 3)3   0.26411    0.96263   0.274    0.784
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -1.55002    0.09591 -16.162  < 2e-16 ***
poly(x, 4)1   6.18883    0.95905   6.453 4.59e-09 ***
poly(x, 4)2 -23.94830    0.95905 -24.971  < 2e-16 ***
  poly(x, 4)3   0.26411    0.95905   0.275    0.784
  poly(x, 4)4   1.25710    0.95905   1.311    0.193
```

As expected , according to the summary of the fits of each model , based on the p values and other coefficients , the quadratic model appears to be more statistically significant than the linear , cubic and polynomial with degree 4 .