

Assignment No: 1

Calculate the mean, median, mode and standard deviation for the problem statements 1 & 2.

Pbm I

The marks awarded for an assignment set for a year 8 class of 20 students were as follows:

6, 7, 5, 7, 7, 8, 7, 6, 9, 7, 4, 10, 6, 8, 8, 9, 5, 6, 4, 8.

$$\text{Mean} = \frac{137}{20} = \underline{\underline{6.85}}$$

Median

4, 4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10.

$$\frac{7+7}{2} = \underline{\underline{7}}$$

$$\text{Mode} = \underline{\underline{7}}$$
$$\text{Standard deviation}(\sigma) = \sqrt{\frac{\sum (x_i - \mu)^2}{N}} = \underline{\underline{1.589}}.$$

Pbm 2

The number of calls from motorists per day for roadside service was recorded for a particular month:

28, 122, 217, 130, 120, 86, 80, 90, 140, 120, 70, 40, 145,
113, 90, 68, 174, 194, 170, 100, 75, 104, 97, 75, 123, 100,
75, 104, 97, 75, 123, 100, 89, 120, 109

$$\text{Mean} = \frac{3763}{37} = \underline{\underline{101.7}}$$

$$\text{Median} = \underline{\underline{100.}}$$

$$\text{Mode} = \underline{\underline{75}}$$

$$\text{Standard deviation} = \underline{\underline{38.77}}$$

Pbm-3

The number of times I go to the gym in weekdays, are given below along with its associated probability

$$x = 0, 1, 2, 3, 4, 5$$

$$f(x) = 0.09, 0.15, 0.40, 0.25, 0.10, 0.01$$

Calculate the mean no. of workouts in a week. Also evaluate the variance involved in it

$$\text{Mean } (\mu) = \sum xp$$

$$\begin{aligned}\mu &= 0 \times 0.09 + 1 \times 0.15 + 2 \times 0.4 \\ &\quad + 3 \times 0.25 + 4 \times 0.1 + 5 \times 0.01 \\ &= \underline{\underline{2.15}}\end{aligned}$$

$\frac{x}{}$

0

1

2

3

4

5

$P(x)$

0.09

0.15

0.40

0.25

0.10

0.01

$$\text{Var}(x) = \sum (x_i - \mu)^2 P_i$$

$$= 14.622 = \underline{\underline{14.188}}$$

$$= \underline{\underline{1.5217}}$$

Pbm-5

A company manufactures LED bulbs with faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it

$$P(r/N) = {}^N C_r P^r q^{N-r}$$

$$P = 0.3$$

$$q = 1 - 0.3$$

$$= 0.7$$

$$P(2/6) = {}^6C_2 (0.3)^2 (0.7)^4$$

$$= 15 \times 0.09 \times 0.2401$$

$$= 0.324$$

Mean (μ_x) = $n \times P$

Standard deviation $\sigma_x = \sqrt{n \times P \times (1-P)}$

$$\mu_x = 2 \times 0.3 = 0.6$$

$$\sigma_x = \sqrt{(2 \times 0.3) \times 0.7} = 0.648$$

Pbm 6

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give

Gaurav:

$$P(4/8) = \frac{8!}{4!4!} \times 0.75^4 \times 0.25^4$$

$$= 0.0865$$

$$P(n/N) = {}^NC_n P^n q^{N-n}$$

$$P = 0.75$$

$$q = 0.25$$

$$N = 8$$

$$n = 4$$

$$P(5/8) = \frac{8!}{5!3!} 0.75^5 \times 0.25^3$$

$$= 0.207$$

$$P(6/8) = \frac{8!}{6!2!} (0.75)^6 \times (0.25)^2$$

$$= 0.3114$$

Barakaha

$$P(4/12) = \frac{12!}{4!8!} 0.45^4 \times 0.55^8$$

$$= 0.1699$$

$$P = 0.45$$

$$q = 0.55$$

$$N = 12$$

$$P(5/12) = \frac{12!}{5!7!} 0.45^5 \times 0.55^7$$

$$= 0.222$$

$$P(6/12) = \frac{12!}{6!6!} (0.45)^6 \times (0.55)^6$$

$$= 0.212$$

Pbm 7

Customers arrive at a rate of 72 per hour to my shop. What is the probability of 4 customers arriving at 4 minutes? a) 5 customers b) not more than 3 customers. c) more than 3 customers.

$$\text{Avg}(\mu) = 04.8$$

a) 5 customers

$$P = \frac{e^{-\mu} \mu^x}{x!}$$

$$= \frac{e^{-4.8} \times (4.8)^5}{5!} = \underline{\underline{0.174}}$$

b) not more than 3.

$$P(0) + P(1) + P(2) + P(3) = \underline{\underline{0.292}}$$

$$P(1) = \frac{e^{-4.8} \times (4.8)^1}{1!} = \underline{\underline{0.03936}}$$

$$P(2) = \frac{e^{-4.8} \times (4.8)^2}{2!} = \underline{\underline{0.0944}}$$

$$P(3) = \frac{e^{-4.8} \times (4.8)^3}{3!} = \underline{\underline{0.1511}}$$

$$P(0) = \frac{e^{-4.8} \times (4.8)^0}{0!} = 0.0082$$

c) More than 3.

$$1 - (P(0) + P(1) + P(2) + P(3))$$

$$= 1 - 0.292$$

$$= \underline{\underline{0.708}}$$

Pbm-8

I work as a data analyst in Acorn Learning Pvt Ltd. After analyzing data, I make reports, where I have the efficiency of entering 77 words per minutes with 6 errors per hour. What is the probability that I will commit 2 errors in a 455 word financial report?

What happens when the no. of words increase/decrease
(in case of 1000 words, 255 words) ?

$$455 W - 5.9 M - 0.59 (\mu)$$

$$255 W - 3.3 M - 0.33 (\mu)$$

$$1000 W - 13 M - 1.3 (\mu)$$

$$P(2) = \frac{e^{-0.59} \times 0.59^2}{2!} = \underline{\underline{0.096}} \quad \text{on 455 words.}$$

$$P(2) = \frac{e^{-0.33} \times (0.33)^2}{2!} = \underline{\underline{0.0391}} \quad \text{on 255 words}$$

$$P(2) = \frac{e^{-1.3} \times (1.3)^2}{2!} = \underline{\underline{0.230}} \quad \text{on 1000 words.}$$

Pbm 10

Please compute the following

a) $P(Z > 1.26)$, $P(Z < -0.86)$, $P(Z > -1.37)$,

$P(-1.25 < Z < 0.37)$, $P(Z \leq -4.6)$

b) Find the value z such that $P(Z > z) = 0.05$

c) Find the value of z such that $P(-z < Z < z) = 0.99$

a) (i) $P(Z > 1.26)$

$$= 1 - P(Z \leq 1.26)$$

$$= 1 - 0.8962 = \underline{\underline{0.1038}}$$

(ii) $P(Z < -0.86) = \underline{\underline{0.1949}}$

(iii) $P(Z > -1.37) = 1 - P(Z \leq -1.37)$

$$= 1 - 0.0853 = \underline{\underline{0.9147}}$$

$$(iv) P(-1.25 < Z < 0.37)$$

$$= P(Z < 0.37) - P(Z < -1.25)$$

$$= 0.6443 - 0.1056$$

$$= \underline{\underline{0.5387}}$$

$$(v) P(Z \leq -4.6) = \underline{\underline{0.00002}}$$

(b) Find the value z such that $P(Z > z) = 0.05$.

$$1 - P(Z \leq z) = 0.05$$

$$\text{Ans: } = \underline{\underline{1.65}}$$

(c) Find the value of z such that $P(-z < Z < z) = 0.99$.

$$P(Z \leq z) - P(Z \leq -z) = 0.99$$

$$z = \underline{\underline{\pm 2.33}}$$

Pbm 11

The current flow in a copper wire follow a normal distribution with a mean of 10 mA and a variance of 4 (mA)^2 .

What is the probability that a current measurement will exceed 13 mA? What is the probability that a current measurement is between 9 and 11 mA? Determine the current measurement which has a probability of 0.98

$$\mu = 10$$

$$\sigma^2 = 4$$

$$a) P(x > 13)$$

$$Z(13) = \frac{x - \mu}{\sigma} = \frac{13 - 10}{2} = \underline{\underline{1.5}}$$

$$P(x > 1.5) = 1 - P(x \leq 1.5)$$

$$= 1 - 0.9332$$

$$= \underline{\underline{0.0668}}$$

$$b) P(9 < x < 11)$$

$$= P(x \leq Z(11)) - P(x \leq Z(9))$$

$$= P(x \leq 0.5) - P(x \leq -0.5)$$

$$= 0.6915 - 0.3085$$

$$= \underline{\underline{0.383}}$$

$$(c) P_Z(x) = 0.98$$

$$Z(x) = 2.06$$

$$\frac{x - 10}{2} = 2.06$$

$$x - 10 = 4.12$$

$$x = \underline{\underline{14.12}}$$

if $Z = 2$

Prob-12

The shaft in a piston has its diameter normally distributed with a mean of 0.2508 inch and a standard deviation of 0.0005 inch. The specifications of the shaft are 0.2500 ± 0.0015 inch. What proportion of shafts are in syne with the specifications? of the process is centered so that the mean is equal to the

target value of 0.2500, what proportion of shafts conform to the new specifications? what is your conclusion from this experiment?

$$\text{specifications} = 0.2500 \pm 0.0015$$

$$0.2500 + 0.0015 = 0.2515$$

$$0.2500 - 0.0015 = 0.2485$$

$$0.2485 \leq X \leq 0.2515$$

$$Z = \frac{X - \mu}{\sigma} = \frac{0.2485 - 0.2508}{0.0005} = \frac{-0.0023}{0.0005} = -4.6$$

$$Z = \frac{0.2515 - 0.2508}{0.0005} = 1.4$$

$$\begin{aligned} P(-4.6 \leq Z \leq 1.4) &= 0.0002 \leq Z \leq 0.91924 \\ &= 0.91924 - 0.0002 \\ &= 0.91904 \\ &= \underline{\underline{91.904\%}} \end{aligned}$$

$$\text{of } \mu = 0.2500$$

$$Z_1 = \frac{0.2485 - 0.2500}{0.0005} = \frac{-0.0015}{0.0005} = -3$$

$$Z_2 = \frac{0.2515 - 0.2500}{0.0005} = \frac{0.0015}{0.0005} = 3$$

$$\begin{aligned} P(-3 \leq Z \leq 3) &= 0.99865 - 0.00135 \\ &= 0.9973 \\ &= \underline{\underline{99.73\%}} \end{aligned}$$