Single Source Shortest Path

Dijkstra’s Algorithm

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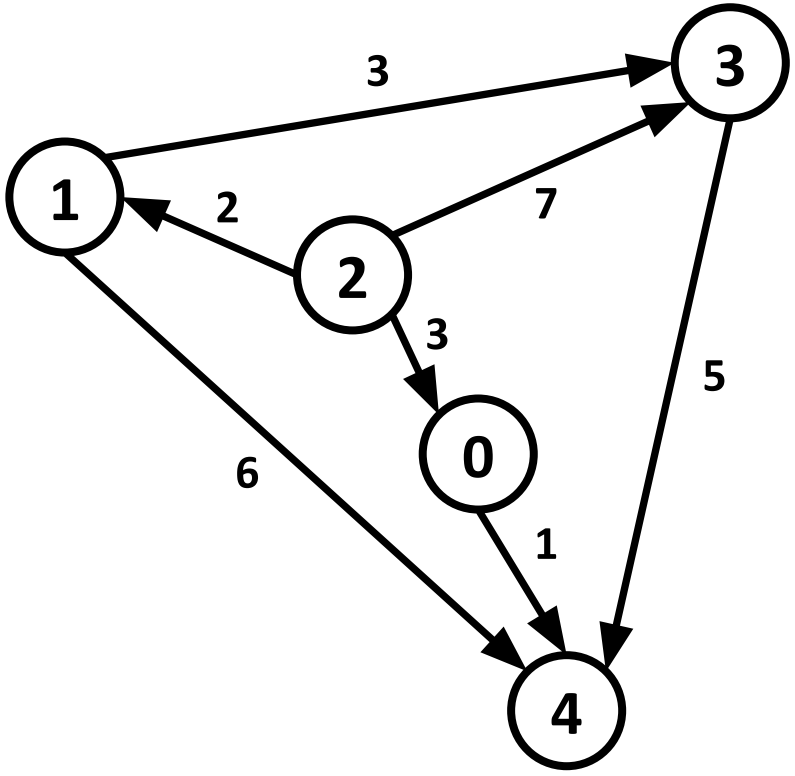
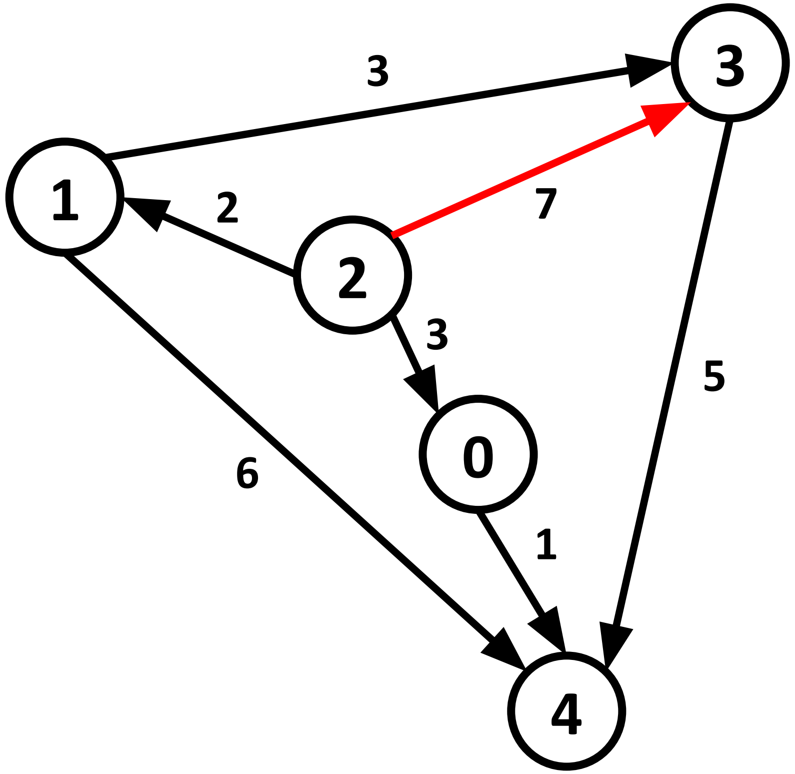
What is Shortest Path?



Reach node 3 from node 2 with minimum cost

How??????

What is Shortest Path?



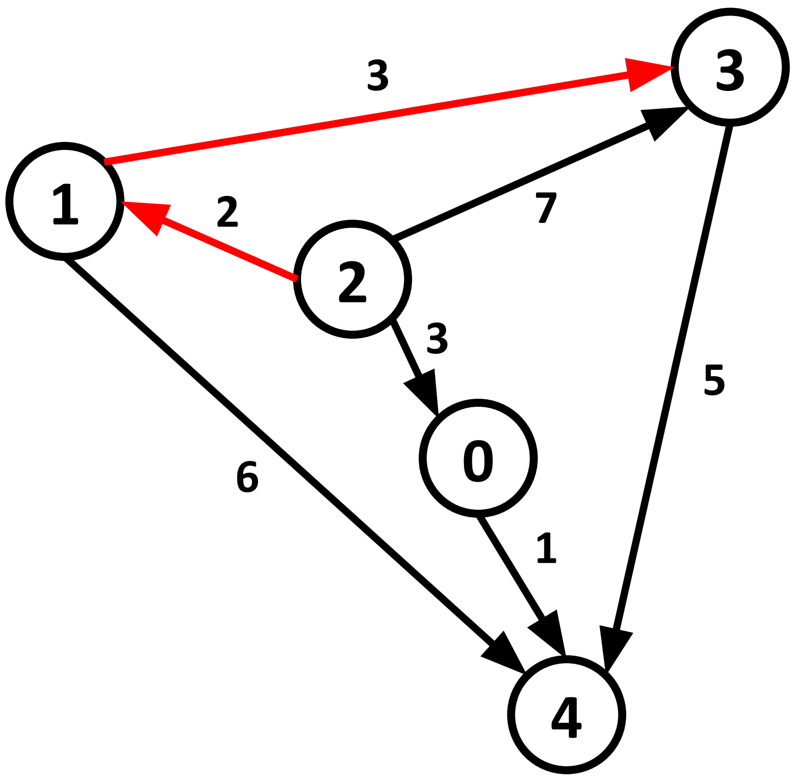
Reach node 3 from node 2 with minimum cost

How??????

One possible way

2 -> 3

What is Shortest Path?



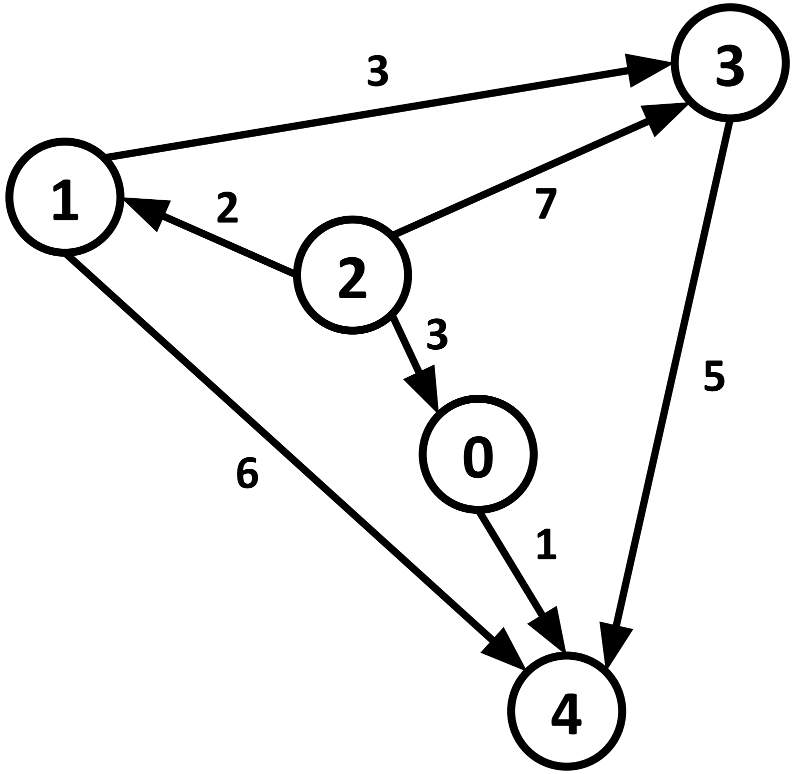
Reach node 3 from node 2 with minimum cost

How??????

Another possible way

2 -> 1 -> 3

What is Shortest Path?



Reach node 3 from node 2 with minimum cost

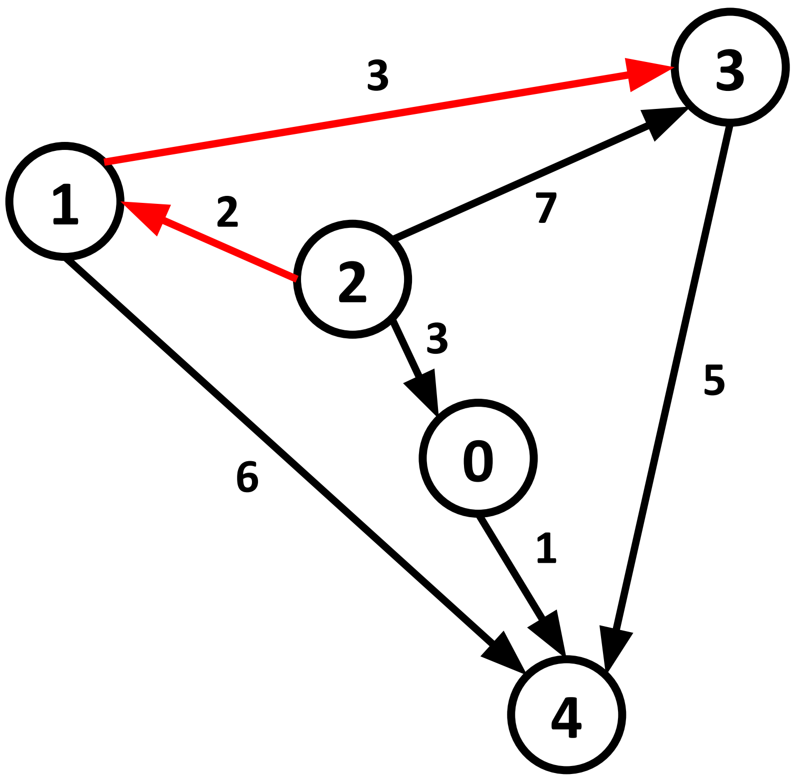
How??????

Two possible way

2-> 3 = Cost(7)

2 -> 1 -> 3 = Cost(2+3=5)

What is Shortest Path?



Reach node 3 from node 2 with minimum cost

How??????

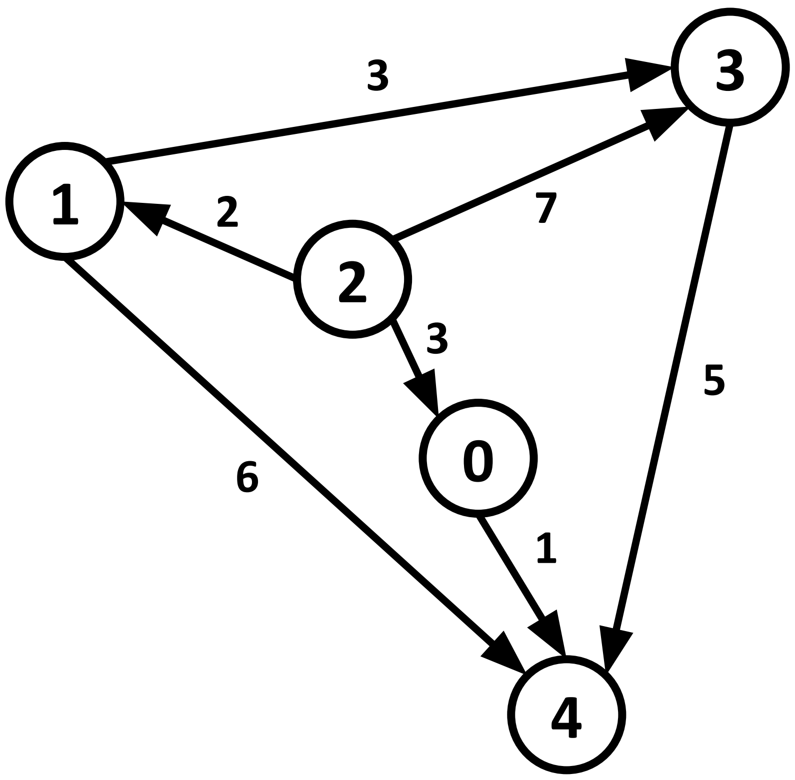
Two possible way

2-> 3 = Cost(7)

2 -> 1 -> 3 = Cost(2+3=5)

Shortest Path from node 2 to node 3

What is Single Source Shortest Path?

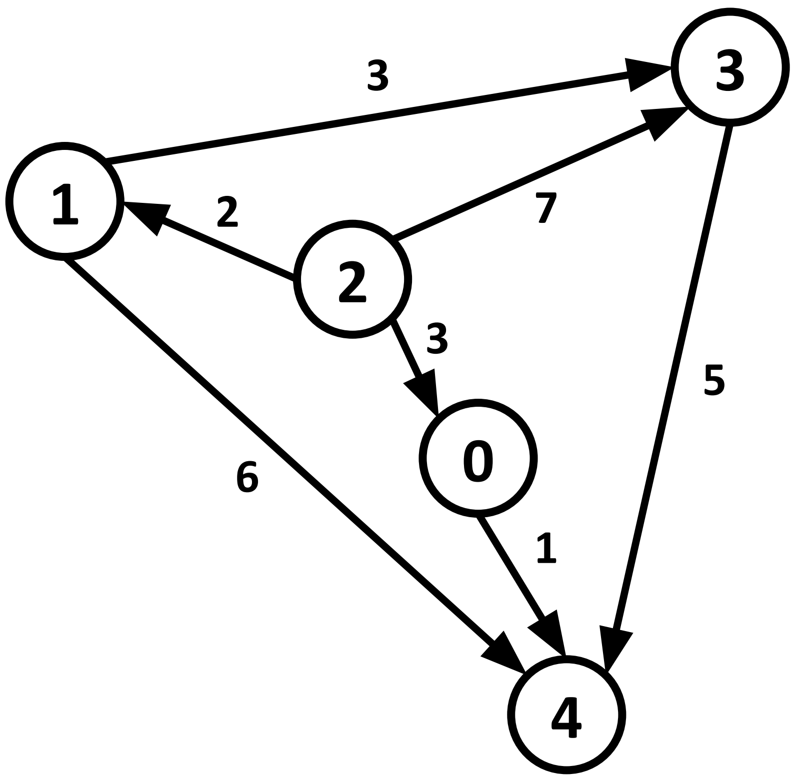


Shortest path for all reaching nodes from a predefine source node

If source node(start node) is 2, then calculate shortest path of other nodes from 2

For single source shortest path calculation, source node is predefined

What is Single Source Shortest Path?



Different Algorithms for SSSP

-> Dijkstra's Algorithm

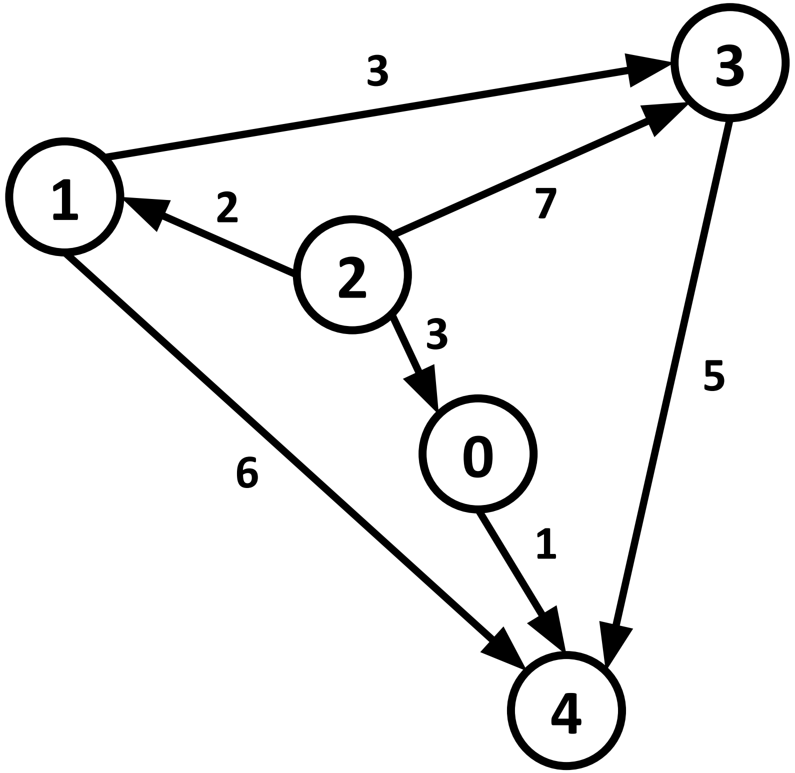
Example

Parent

0	1	2	3	4
-1	-1	-1	-1	-1

Distance

0	1	2	3	4
∞	∞	∞	∞	∞



Source = 2

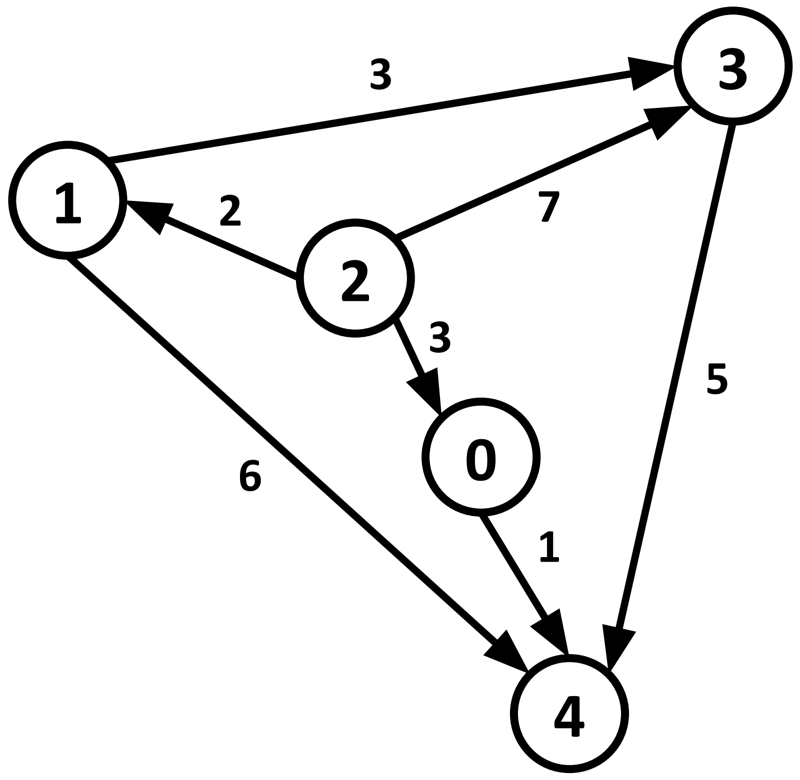
Example

Parent

0	1	2	3	4
-1	-1	-1	-1	-1

Distance

0	1	2	3	4
∞	∞	0	∞	∞



Source = 2

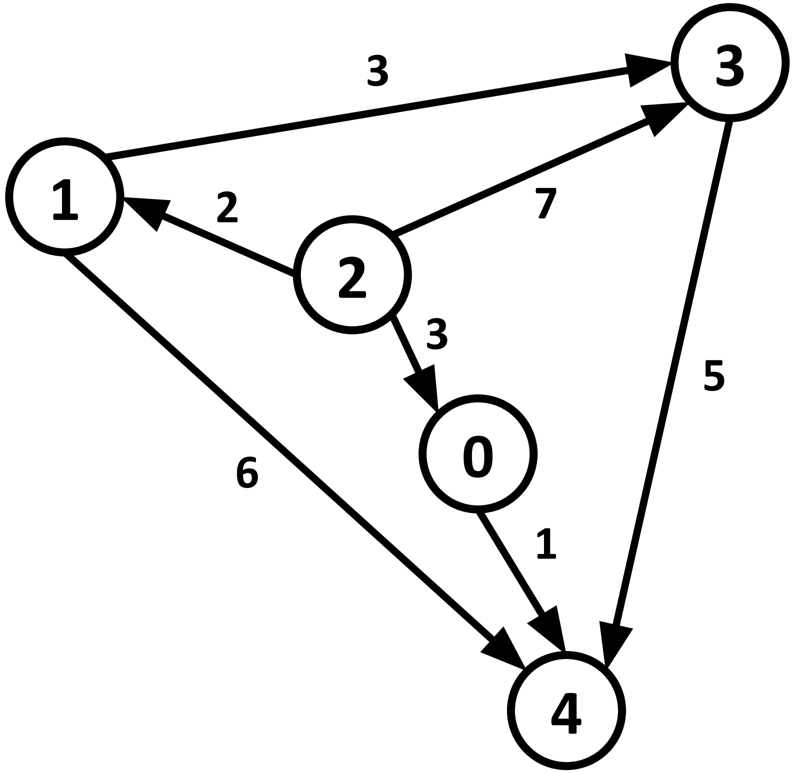
Example

Parent

0	1	2	3	4
-1	-1	-1	-1	-1

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞



Source = 2

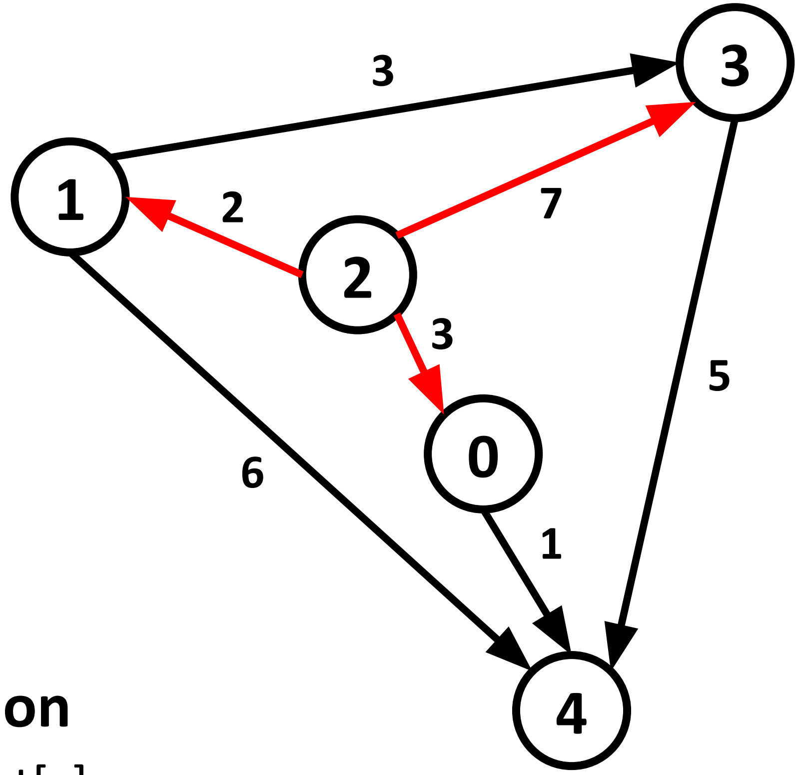
Example

Parent

0	1	2	3	4
-1	-1	-1	-1	-1

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞



Relaxation Condition

If Dist[u] + Dist[u,v] < Dist[v]

Then

Dist[v] = Dist[u] + Dist[u,v]

Source = 2

Example

Parent

0	1	2	3	4
~~-1~~ 2	-1	-1	-1	-1

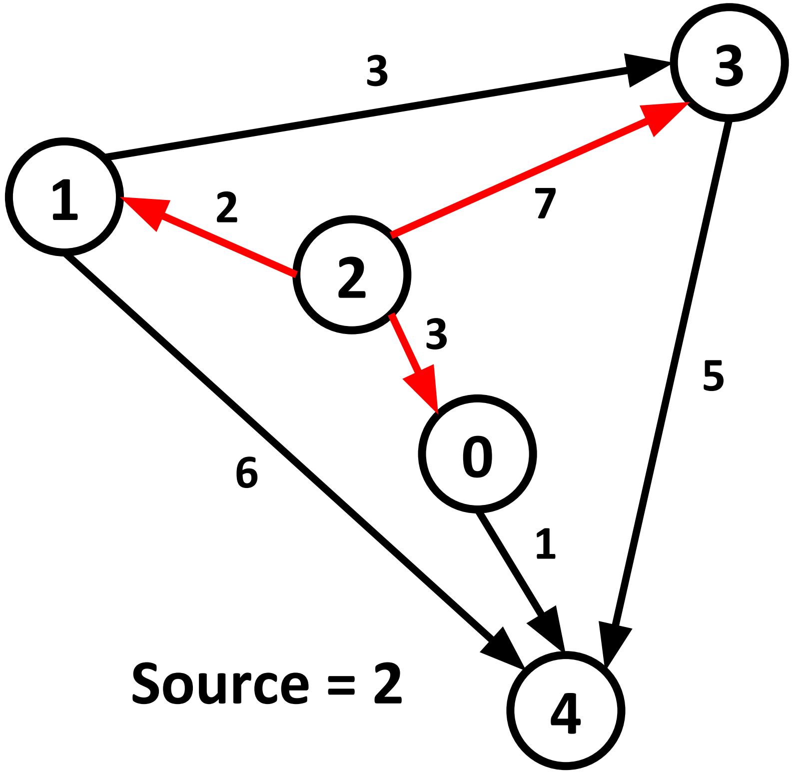
Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
	Min(∞,0+3) 3				

Dist[2] + Dist[2,0] < Dist[0]
0+3 < ∞
True

Relaxation Condition

If Dist[u] + Dist[u,v] < Dist[v]
Then
Dist[v] = Dist[u] + Dist[u,v]



Source = 2

Example

Parent

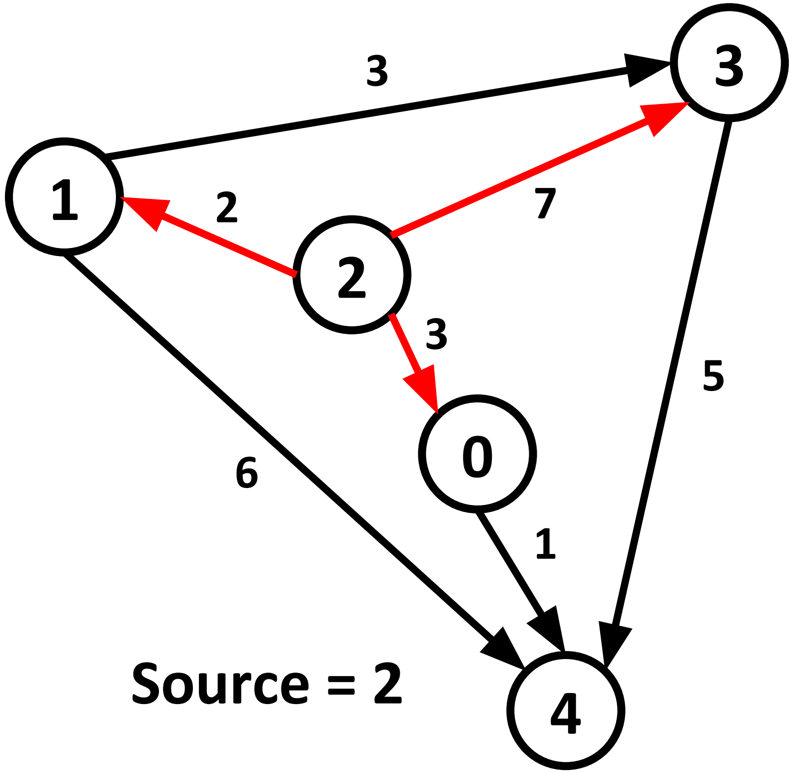
0	1	2	3	4
~~-1~~ 2	~~-1~~ 2	-1	-1	-1

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
	Min(∞,0+3) 3	Min(∞,0+2) 2			

Relaxation Condition

If Dist[u] + Dist[u,v] < Dist[v]
Then
Dist[v] = Dist[u] + Dist[u,v]



Source = 2

Dist[2] + Dist[2,1] < Dist[1]
0+2 < ∞
True

Example

Parent

0	1	2	3	4
~~-1~~ 2	~~-1~~ 2	-1	~~-1~~ 2	-1

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
	Min(∞,0+3) 3	Min(∞,0+2) 2	$\boxed{0}$	Min(∞,0+7) 7	∞

Dist[2] + Dist[2,3] < Dist[3]

0+7 < ∞

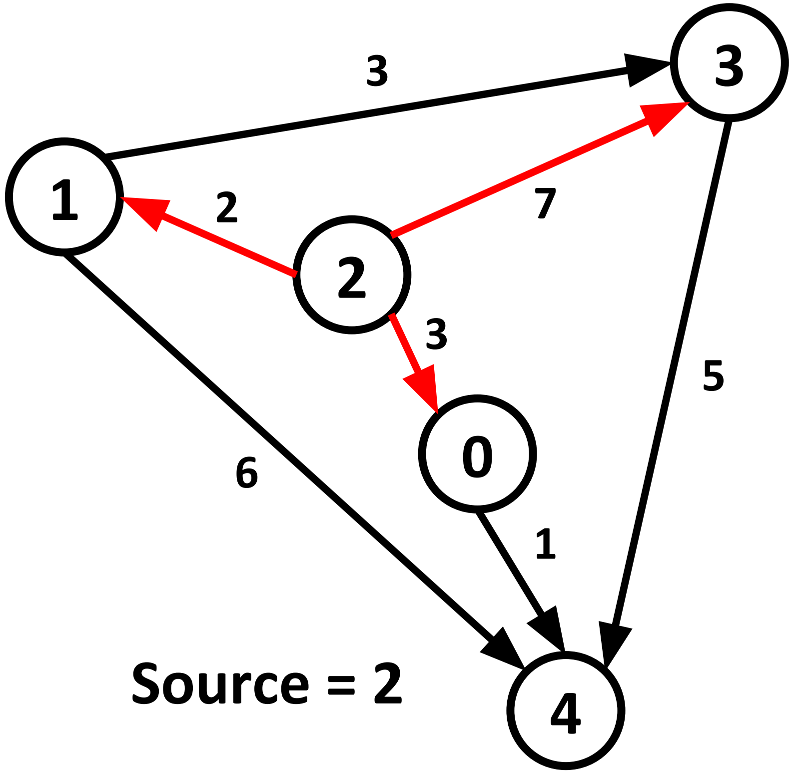
True

Relaxation Condition

If Dist[u] + Dist[u,v] < Dist[v]

Then

Dist[v] = Dist[u] + Dist[u,v]



Source = 2

Example

Parent

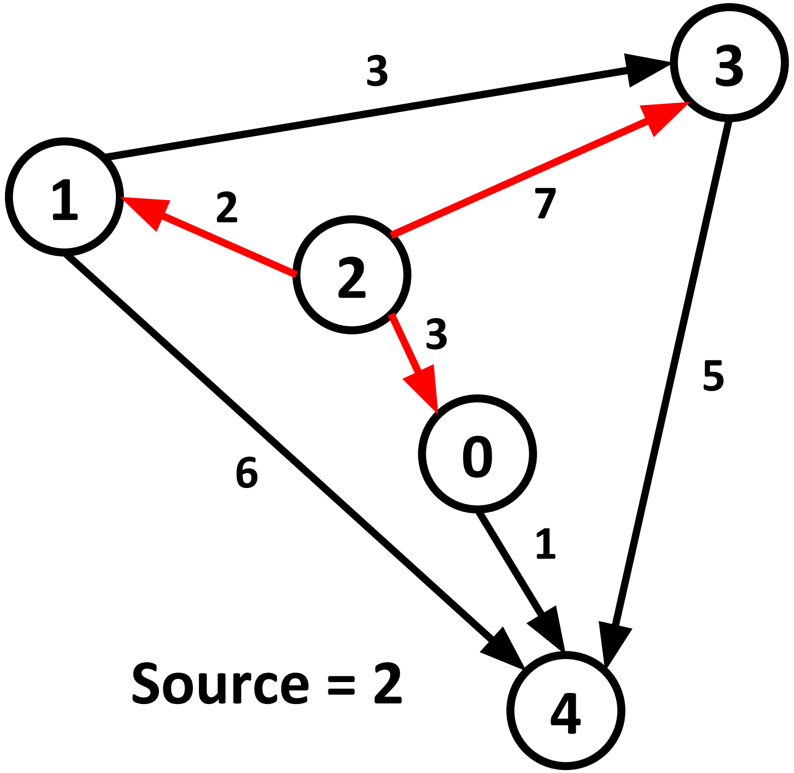
0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ 2	-1

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
	Min(∞,0+3) 3	Min(∞,0+2) 2	$\boxed{0}$	Min(∞,0+7) 7	∞

Relaxation Condition

If Dist[u] + Dist[u,v] < Dist[v]
Then
Dist[v] = Dist[u] + Dist[u,v]



Source = 2

Example

Parent

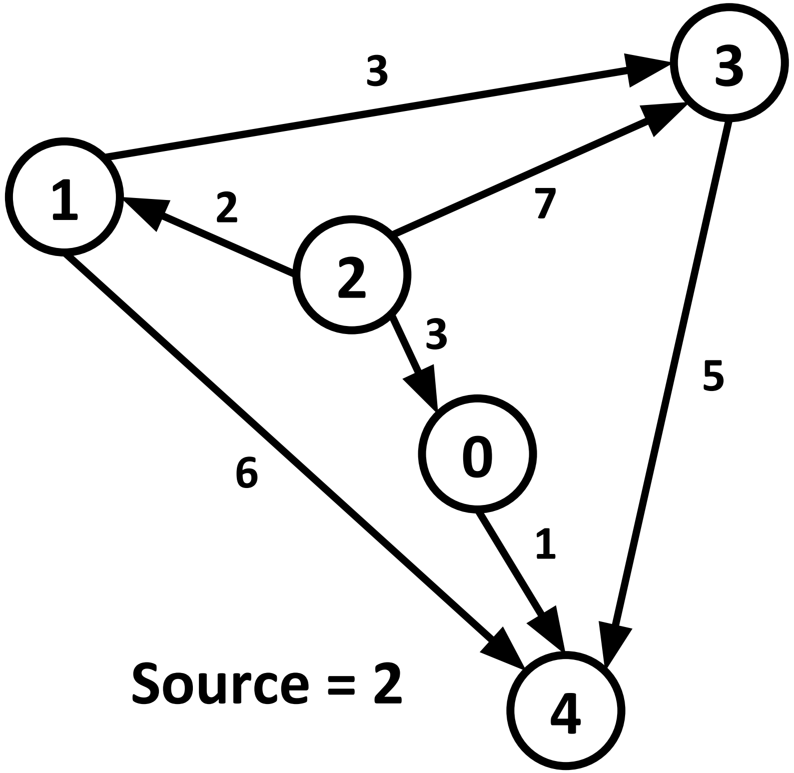
0	1	2	3	4
~~-1~~ 2	~~-1~~ 2	-1	~~-1~~ 2	-1

Distance

	0	1	2	3	4
2	∞	∞	0	∞	∞
	Min(∞,0+3) 3	Min(∞,0+2) 2	0	Min(∞,0+7) 7	∞

Relaxation Condition

If Dist[u] + Dist[u,v] < Dist[v]
Then
Dist[v] = Dist[u] + Dist[u,v]



Source = 2

Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ 2	-1

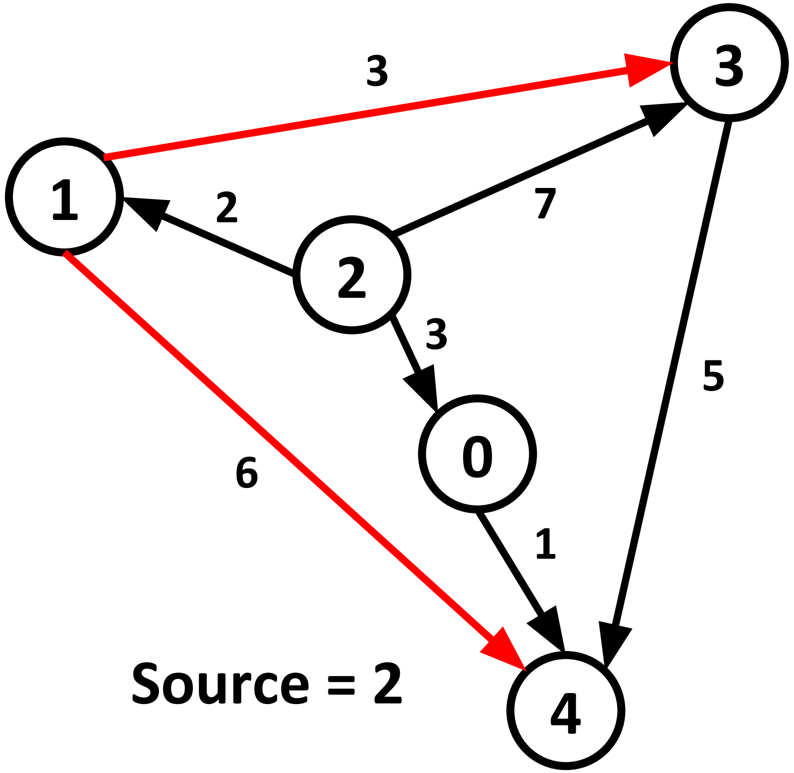
Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞

Dist[2] + Dist[2,3] < Dist[3]

0+7 < ∞

True



Source = 2

Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-1

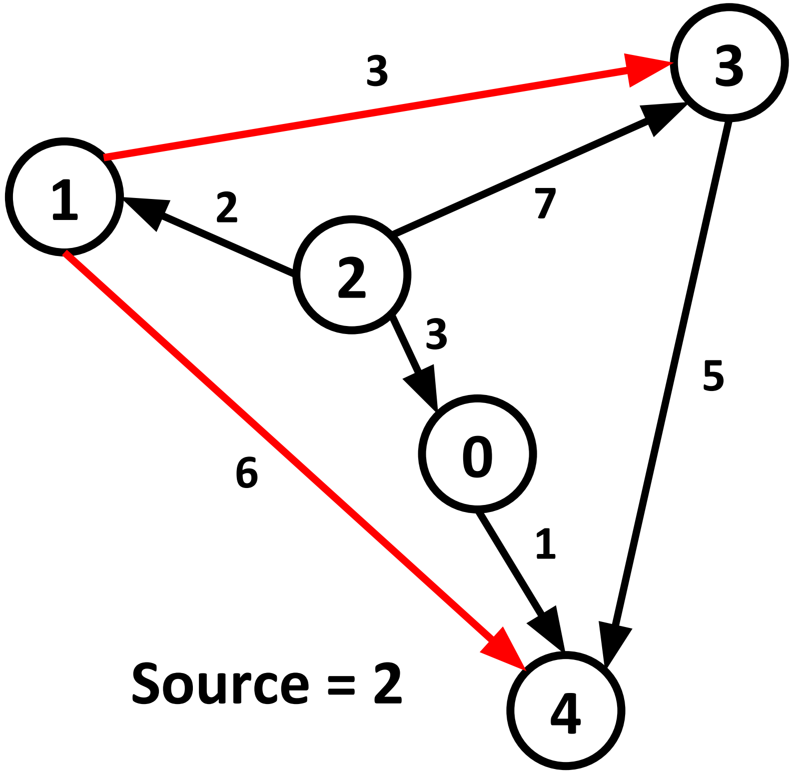
Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
	3	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	

Dist[1] + Dist[1,3] < Dist[3]

2+3 < 7

True



Source = 2

Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ 1

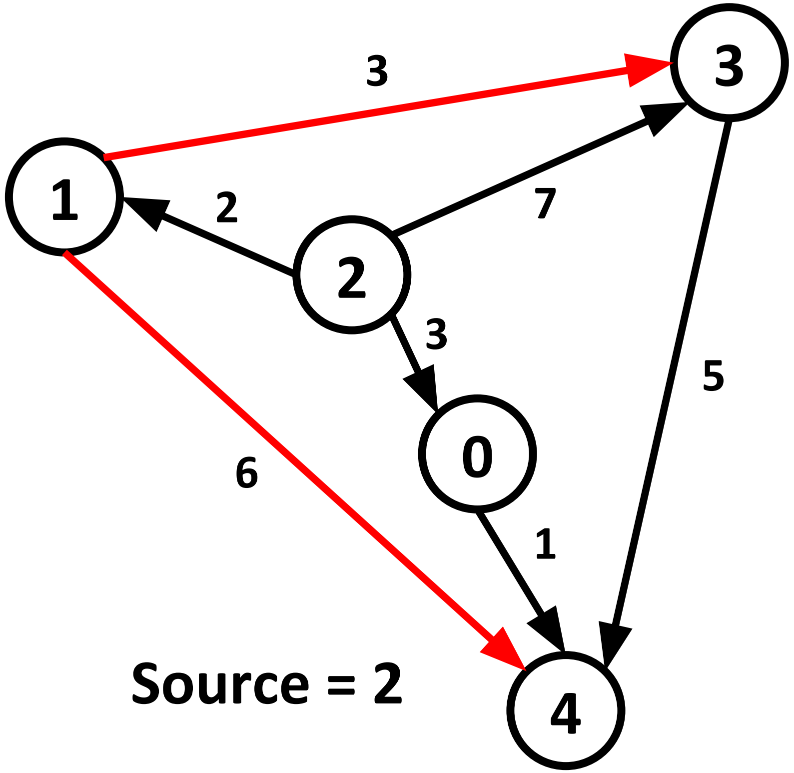
Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
	3	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(∞,2+6) 8

Dist[1] + Dist[1,4] < Dist[4]

2+6 < ∞

True



Source = 2

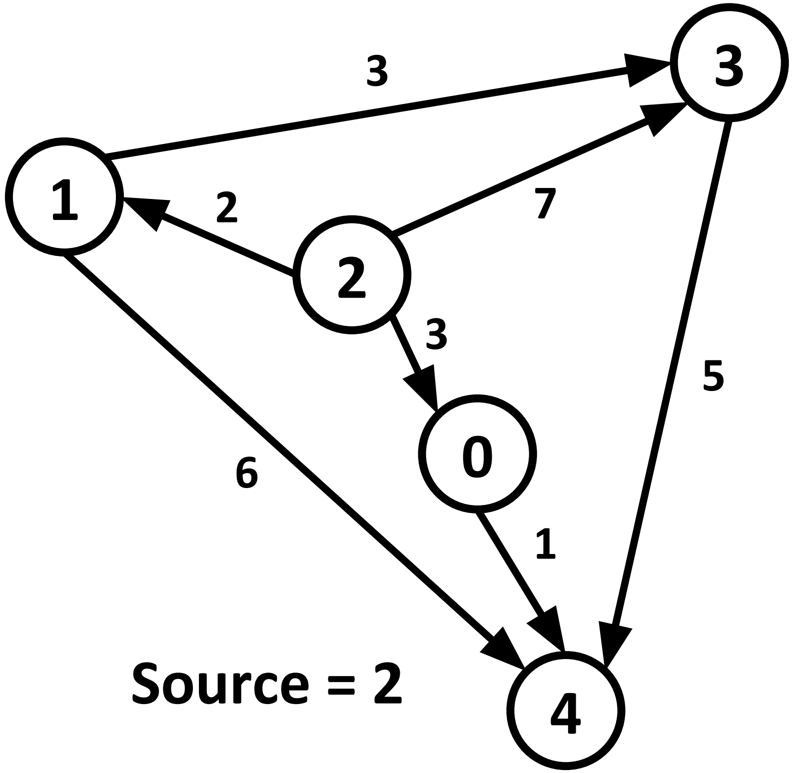
Example

Parent

0	1	2	3	4
~~-1~~ 2	~~-1~~ 2	-1	~~-1~~ ~~2~~ 1	~~-1~~ 1

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
	3	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(∞,2+6) 8



Source = 2

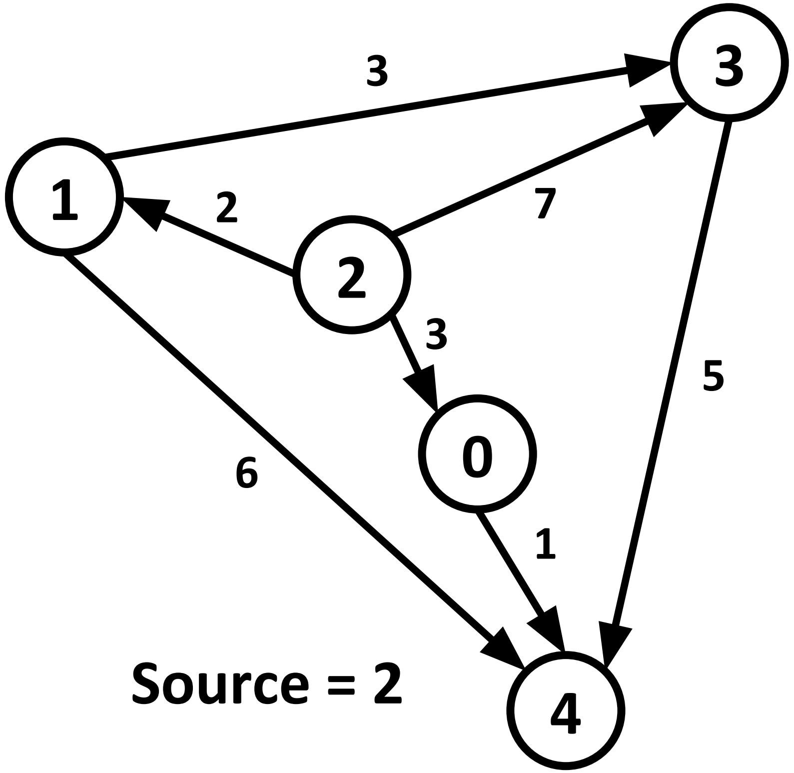
Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ 1

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(∞,2+6) 8



Source = 2

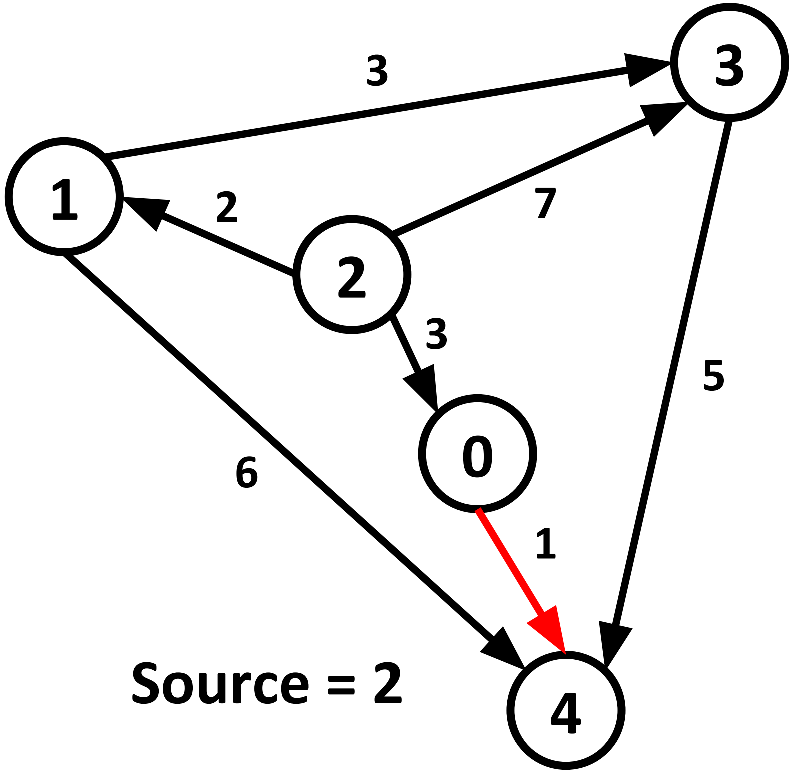
Example

Parent

0	1	2	3	4
~~-1~~ 2	~~-1~~ 2	-1	~~-1~~ ~~2~~ 1	~~-1~~ 1

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(∞,2+6) 8



Source = 2

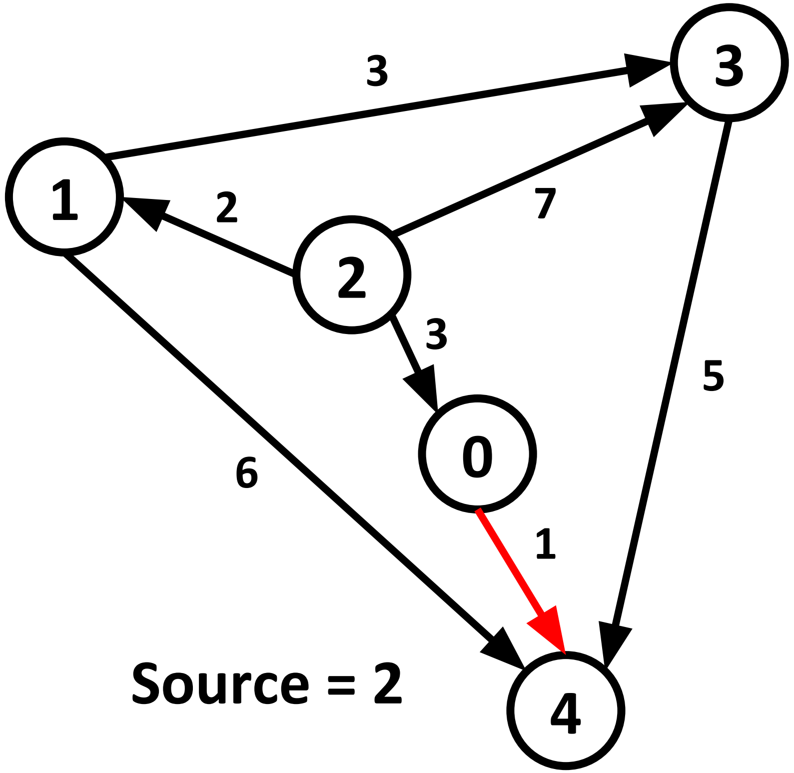
Example

Parent

0	1	2	3	4
$-\cancel{1}$ 2	$-\cancel{1}$ 2	-1	$-\cancel{1}$ $\cancel{2}$ 1	$-\cancel{1}$ 1

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(∞,2+6) 8
	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$		



Source = 2

Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ ~~1~~ 0

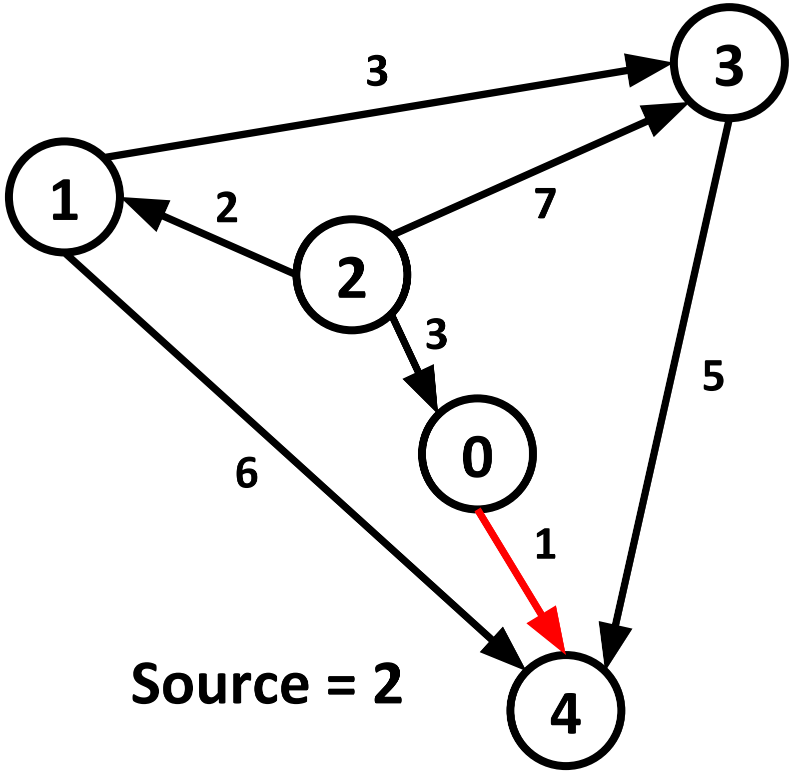
Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(7,2+3) 8
	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	5	Min(8,3+1) 4

Dist[0] + Dist[0,4] < Dist[4]

3+1 < 8

True



Source = 2

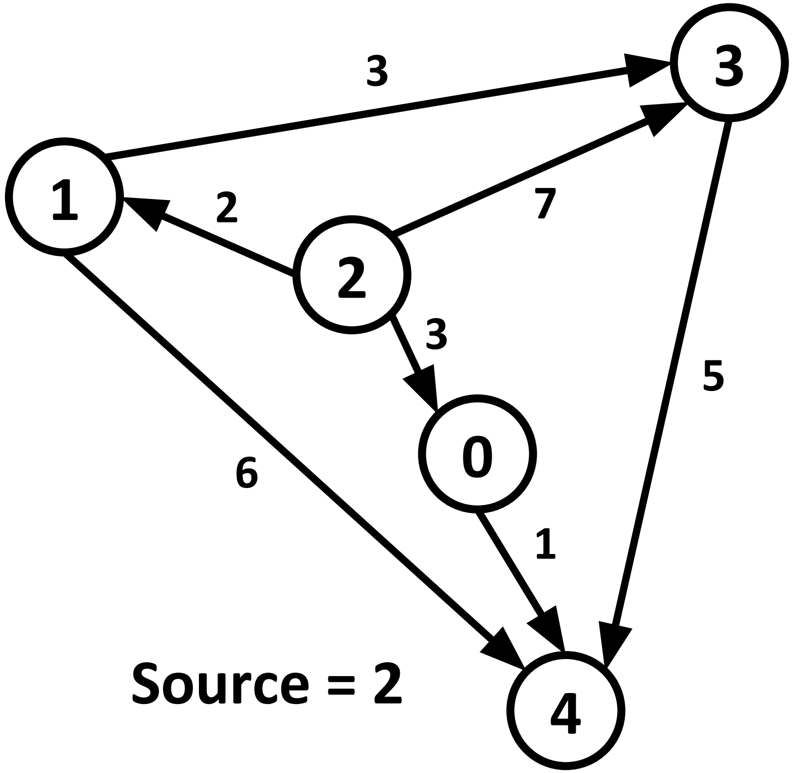
Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ ~~1~~ 0

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(7,2+3) 8
	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	5	Min(8,3+1) 4



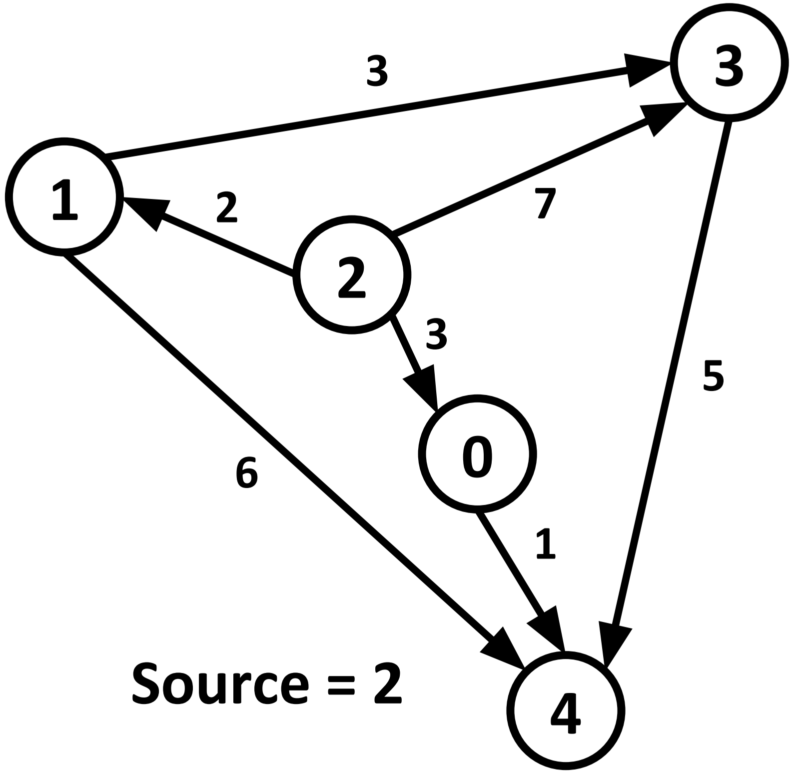
Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ ~~1~~ 0

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(7,2+3) 8
	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	5	Min(8,3+1) $\boxed{4}$



Source = 2

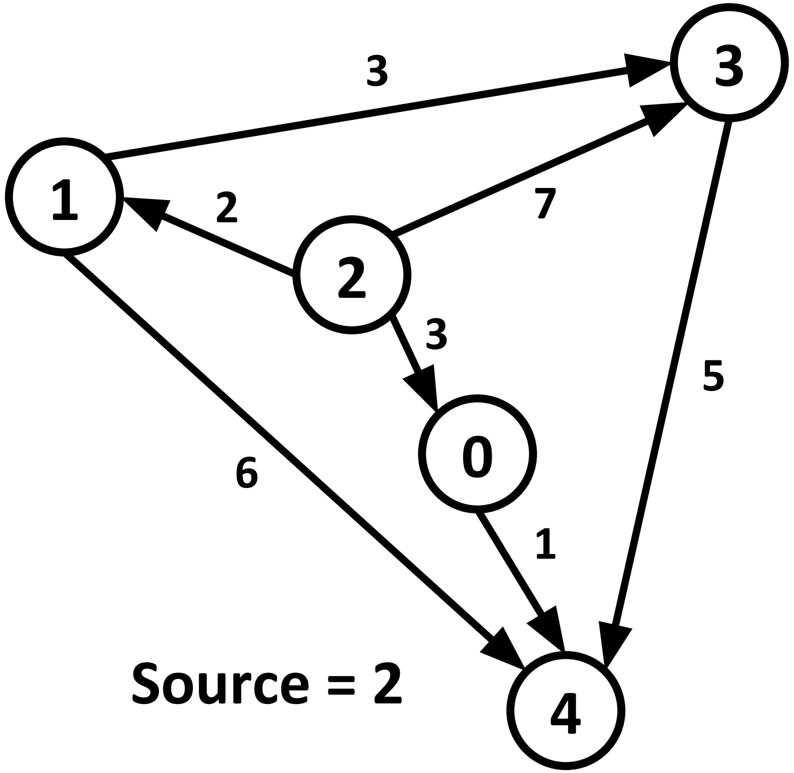
Example

Parent

0	1	2	3	4
~~-1~~ 2	~~-1~~ 2	-1	~~-1~~ ~~2~~ 1	~~-1~~ ~~1~~ 0

Distance

	0	1	2	3	4
2	∞	∞	0	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) 2	0	Min(∞,0+7) 7	∞
0	3	2	0	Min(7,2+3) 5	Min(7,2+3) 8
4	3	2	0	5	Min(8,3+1) 4



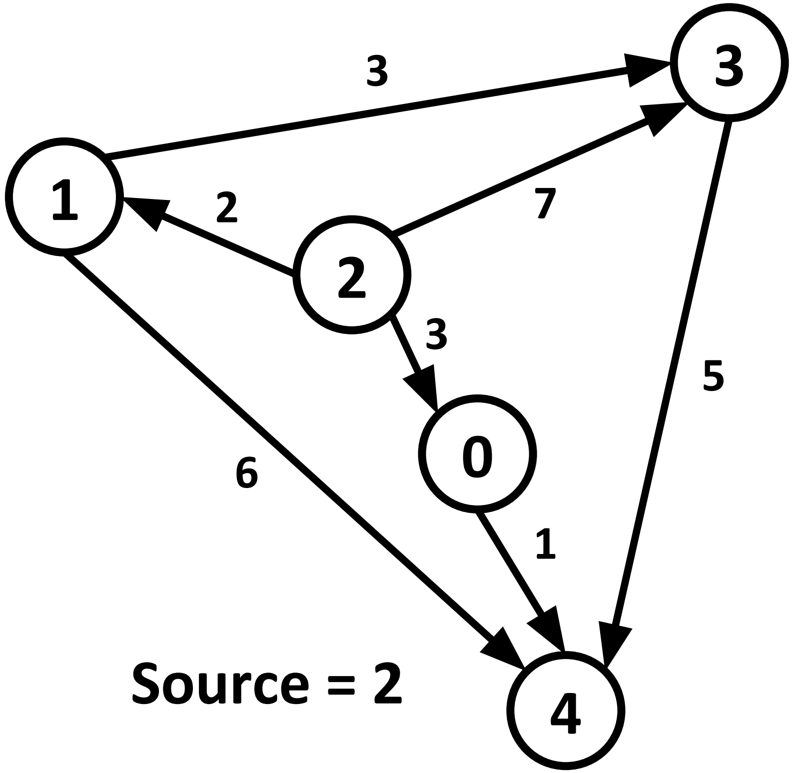
Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ ~~1~~ 0

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(7,2+3) 8
4	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	5	Min(8,3+1) $\boxed{4}$
	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	5	$\boxed{4}$



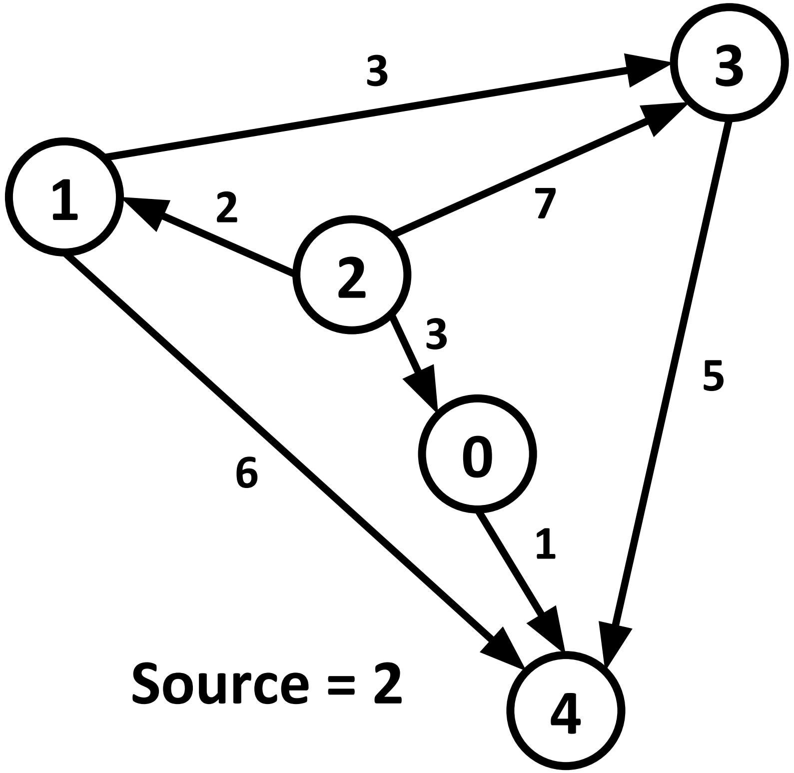
Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ ~~1~~ 0

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(7,2+3) 8
4	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	5	Min(8,3+1) $\boxed{4}$
	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	$\boxed{5}$	$\boxed{4}$



Source = 2

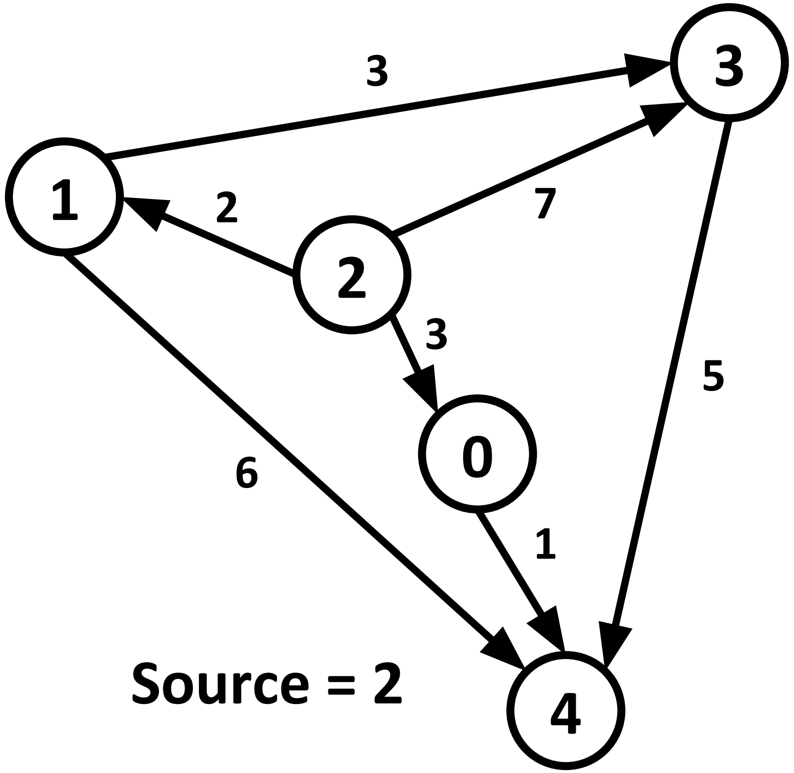
Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ ~~1~~ 0

Distance

	0	1	2	3	4
2	∞	∞	0	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) 2	0	Min(∞,0+7) 7	∞
0	3	2	0	Min(7,2+3) 5	Min(7,2+3) 8
4	3	2	0	5	Min(8,3+1) 4
3	3	2	0	5	4



Source = 2

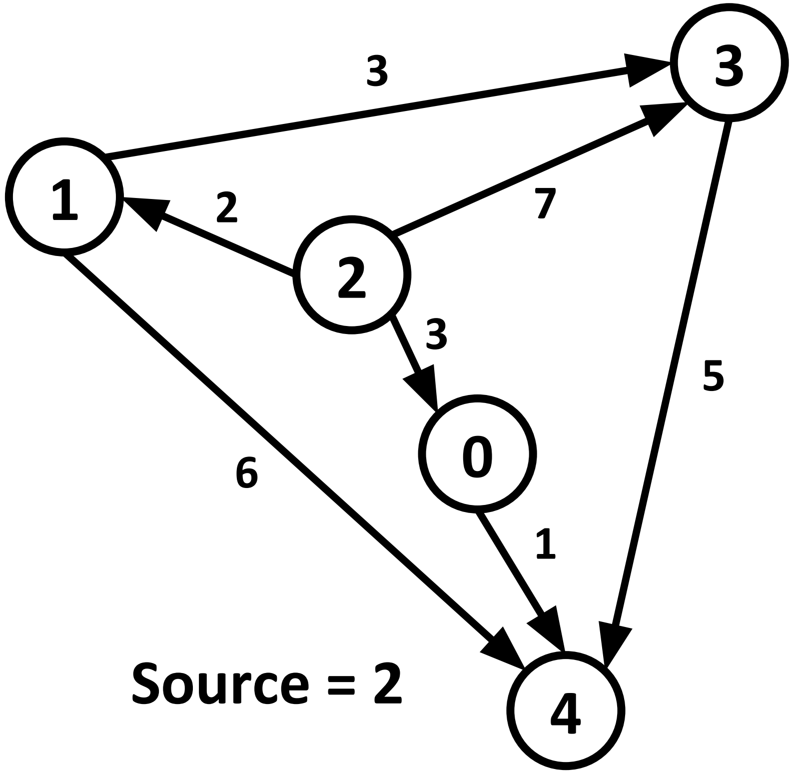
Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ ~~1~~ 0

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(7,2+3) 8
4	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	5	Min(8,3+1) $\boxed{4}$
3	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	$\boxed{5}$	$\boxed{4}$



Example

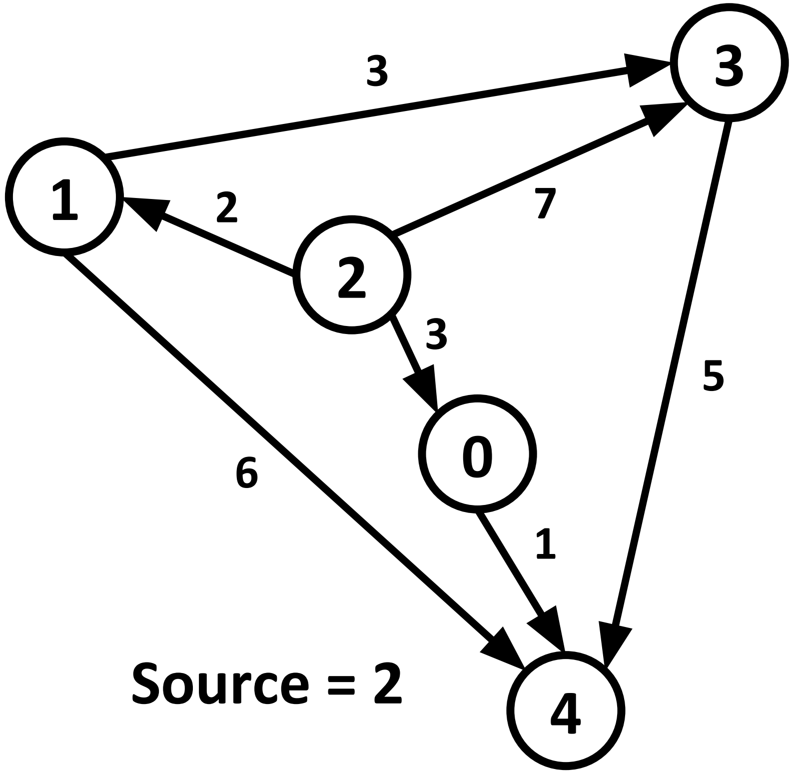
Parent

0	1	2	3	4
~~-1~~ 2	~~-1~~ 2	-1	~~-1~~ ~~2~~ 1	~~-1~~ ~~1~~ 0

Distance

	0	1	2	3	4
2	∞	∞	0	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) 2	0	Min(∞,0+7) 7	∞
0	3	2	0	Min(7,2+3) 5	Min(7,2+3) 8
4	3	2	0	5	Min(8,3+1) 4
3	3	2	0	5	4

Now destination = 3



Source = 2

Example

Parent

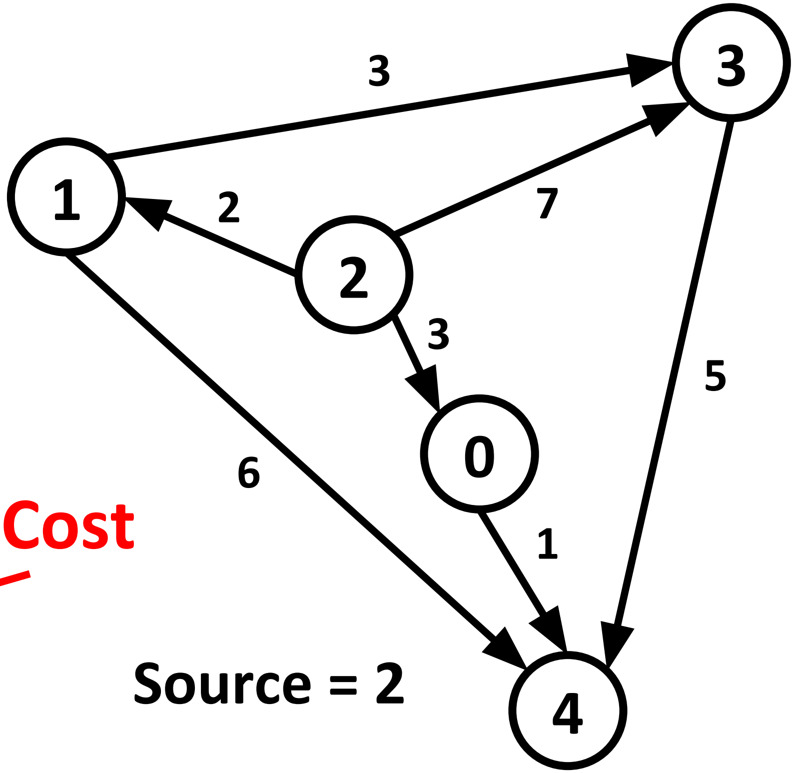
0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ ~~1~~ 0

Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(7,2+3) 8
4	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	5	Min(8,3+1) $\boxed{4}$
3	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	$\boxed{5}$	$\boxed{4}$

Now destination = 3

Cost



Source = 2

Example

Parent

0	1	2	3	4
-~~1~~ 2	-~~1~~ 2	-1	-~~1~~ ~~2~~ 1	-~~1~~ ~~1~~ 0

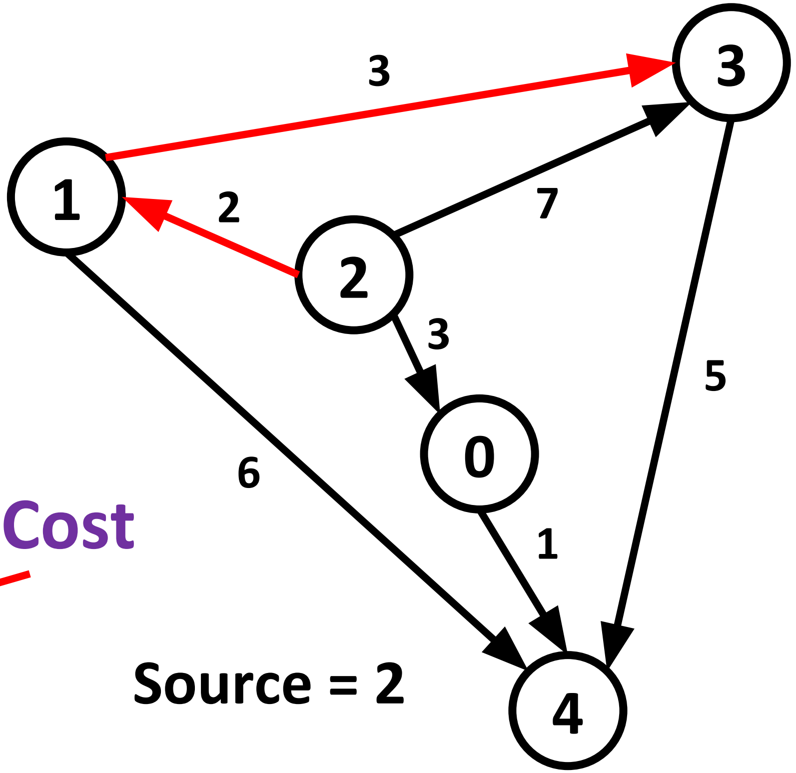
Distance

	0	1	2	3	4
2	∞	∞	$\boxed{0}$	∞	∞
1	Min(∞,0+3) 3	Min(∞,0+2) $\boxed{2}$	$\boxed{0}$	Min(∞,0+7) 7	∞
0	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	Min(7,2+3) 5	Min(7,2+3) 8
4	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	5	Min(8,3+1) $\boxed{4}$
3	$\boxed{3}$	$\boxed{2}$	$\boxed{0}$	$\boxed{5}$	$\boxed{4}$

Path : 2 -> 1-> 3

Now destination = 3

Cost



Source = 2

Implementation

- Using Priority-queue
- Maintain two vector(Distance & Parent)

Implementation

Parent

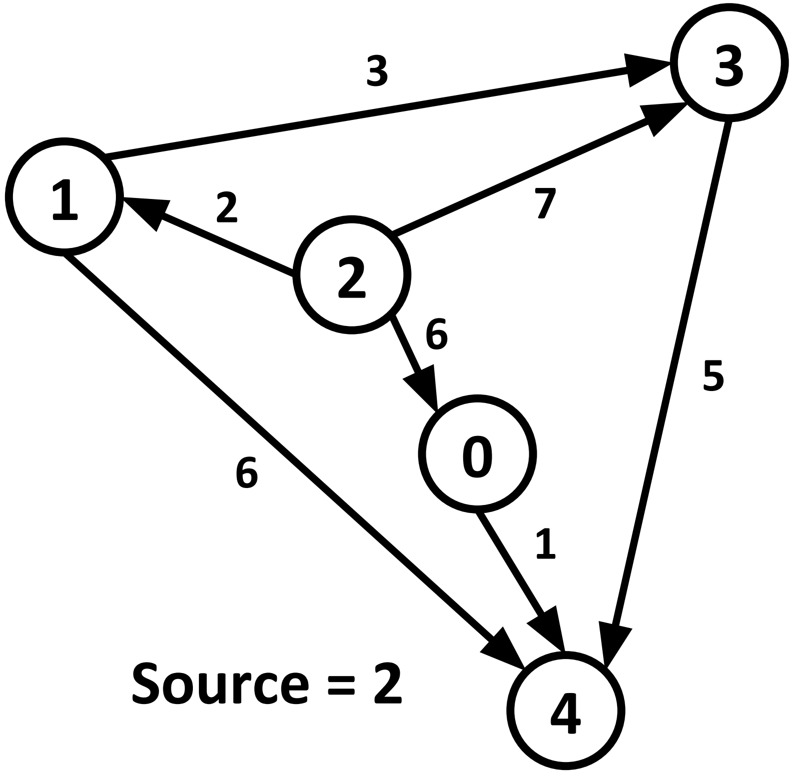
0	-1
1	-1
2	-1
3	-1
4	-1

Distance

0	∞
1	∞
2	0
3	∞
4	∞

Priority-Queue

(-w , -v)



Source = 2

Implementation

Parent

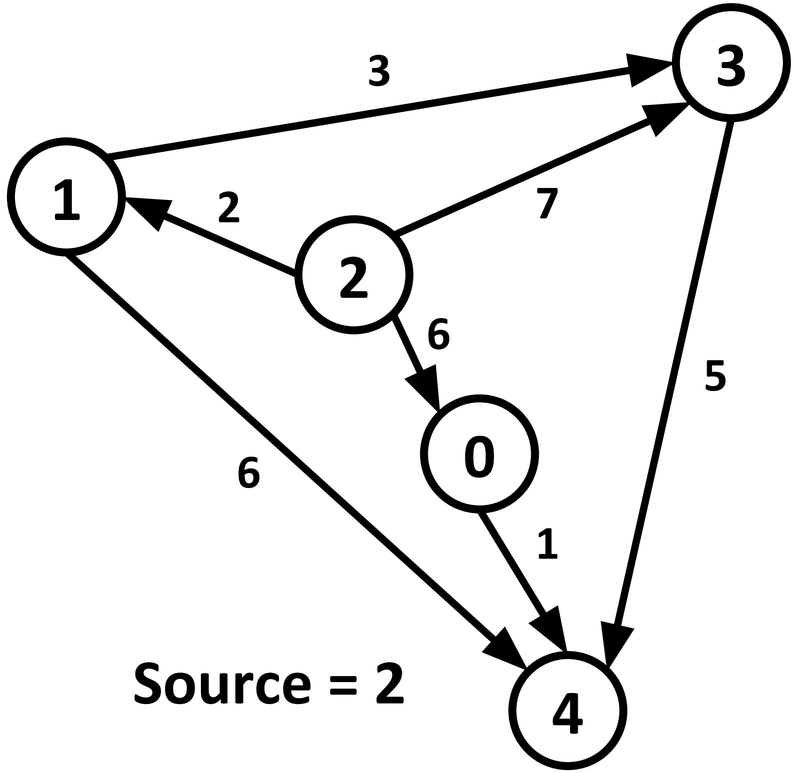
0	-1
1	-1
2	-1
3	-1
4	-1

Distance

0	∞
1	∞
2	0
3	∞
4	∞

Priority-Queue

(0, 2)



Source = 2

Implementation

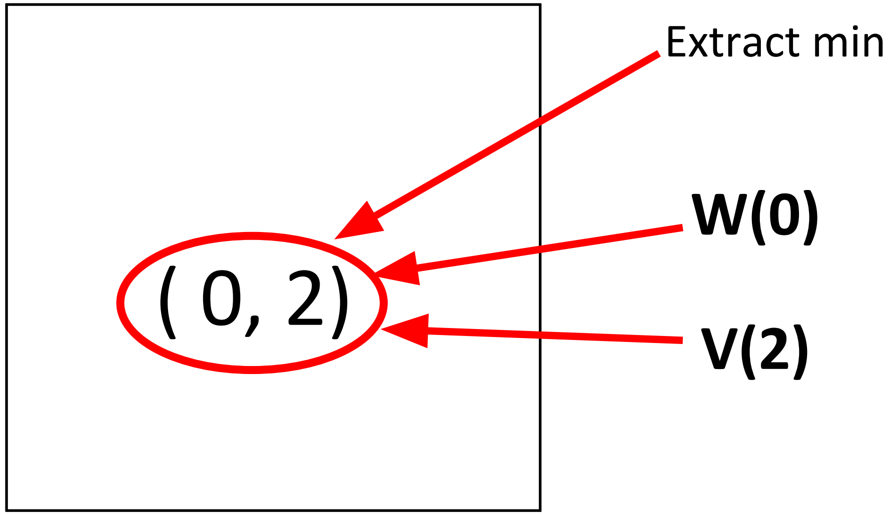
Parent

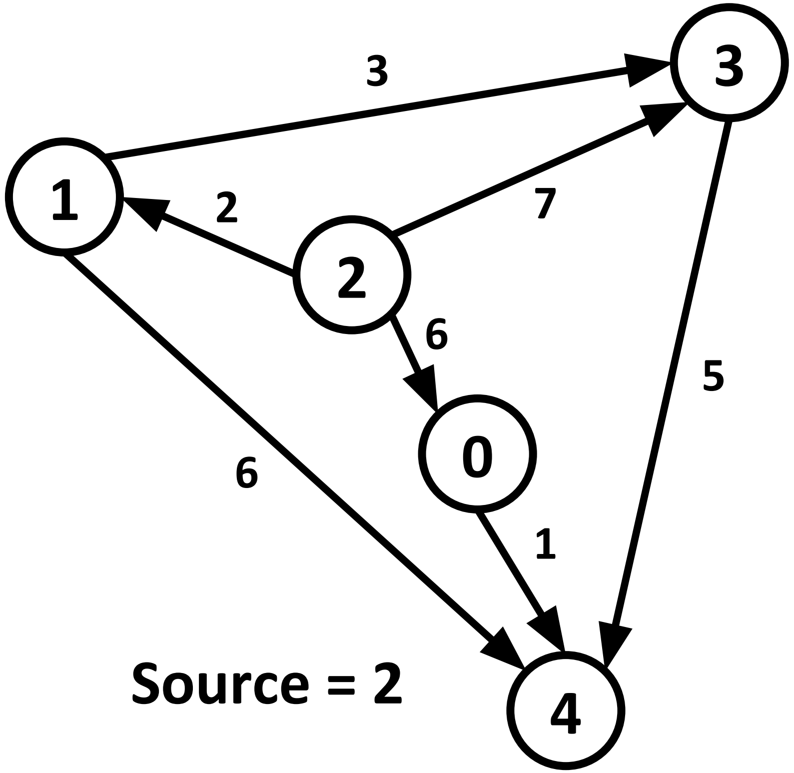
0	-1
1	-1
2	-1
3	-1
4	-1

Distance

0	∞
1	∞
2	0
3	∞
4	∞

Priority-Queue





Implementation

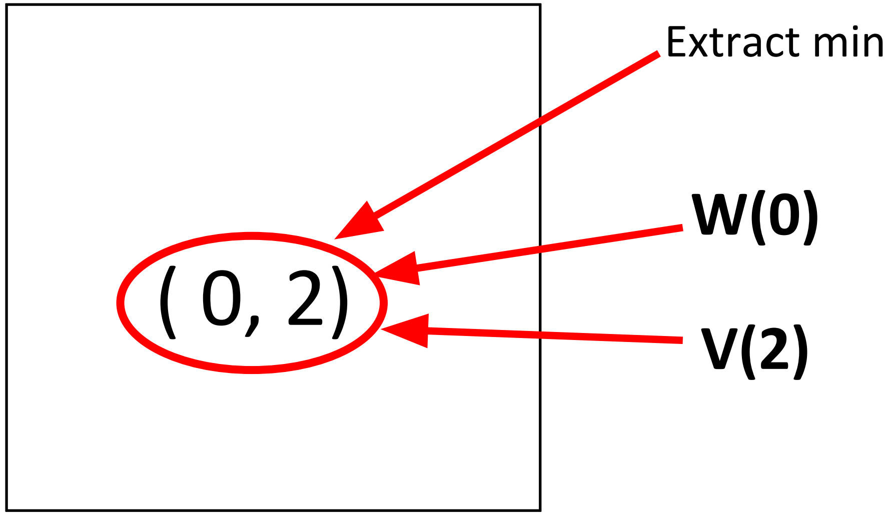
Parent

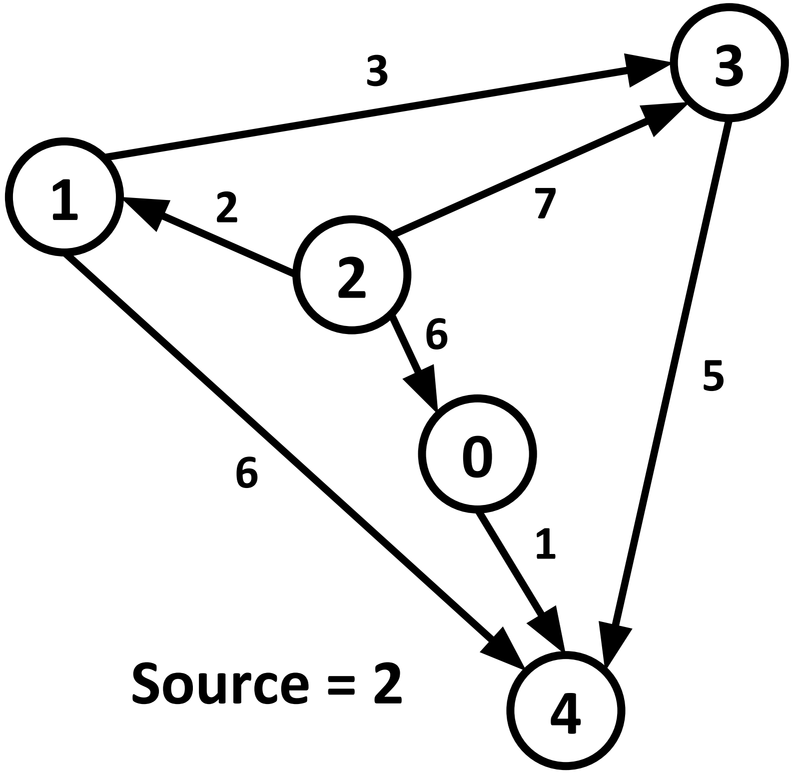
0	-1
1	-1
2	-1
3	-1
4	-1

Distance

0	∞
1	∞
2	0
3	∞
4	∞

Priority-Queue





Implementation

Parent

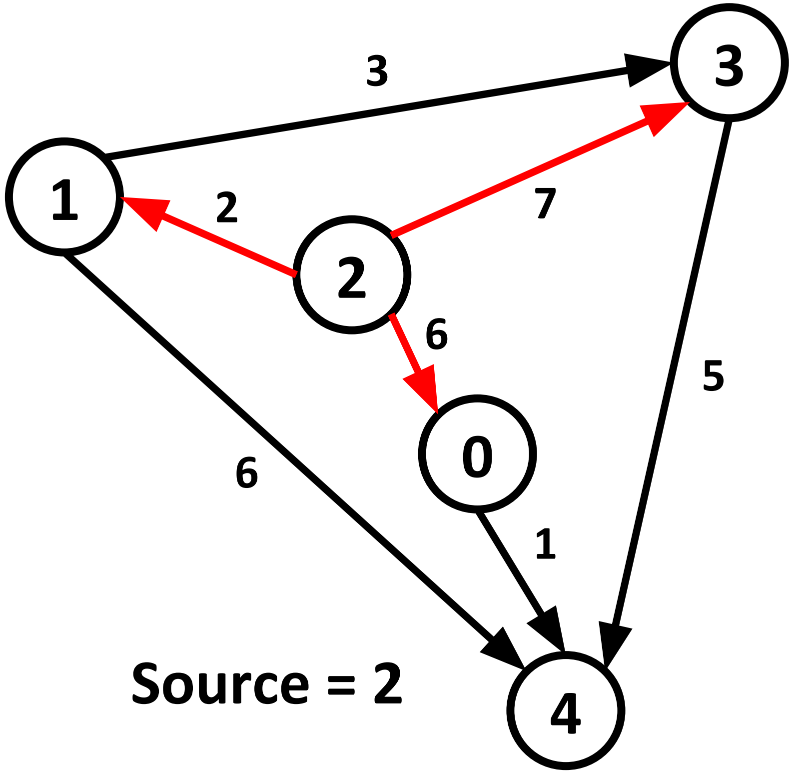
0	2
1	2
2	-1
3	2
4	-1

Distance

0	6
1	2
2	0
3	7
4	∞

Priority-Queue

(2, 1)
(6,0)
(7,3)



Source = 2

Implementation

Parent

0	2
1	2
2	-1
3	2
4	-1

Distance

0	6
1	2
2	0
3	7
4	∞

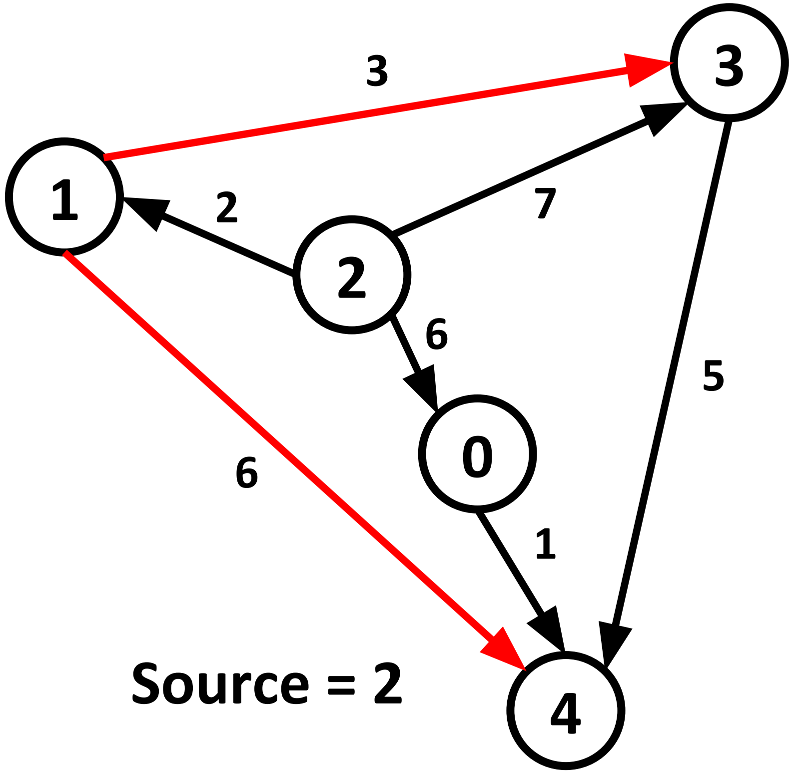
Priority-Queue

(2, 1)
(6,0)
(7,3)

Extract min

W(2)

V(1)



Implementation

Parent

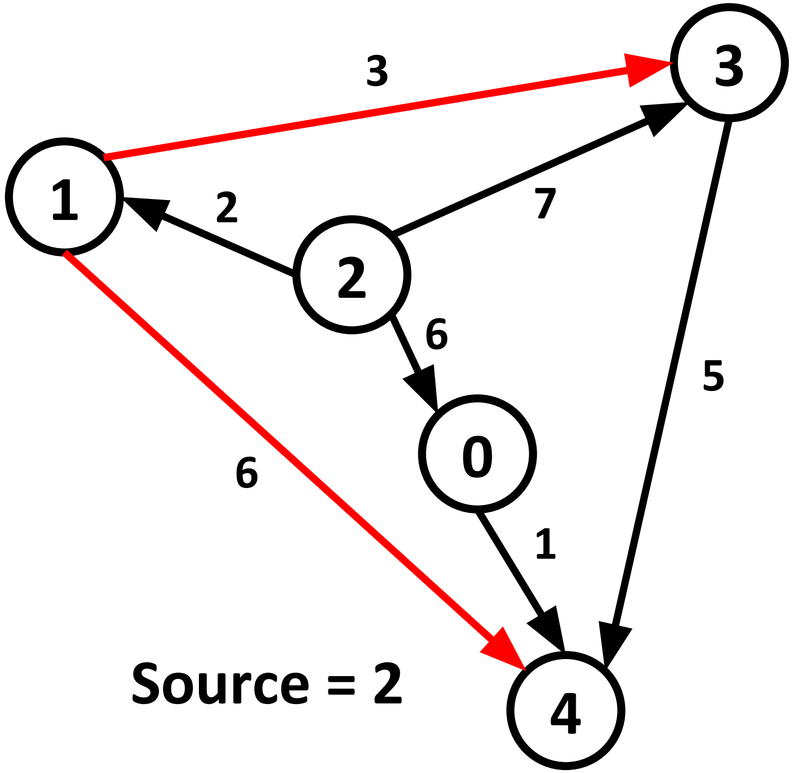
0	2
1	2
2	-1
3	1
4	1

Distance

0	6
1	2
2	0
3	5
4	8

Priority-Queue

(5,3)
(6,0)
(7,3)
(8,4)



Source = 2

Implementation

Parent

0	2
1	2
2	-1
3	1
4	1

Distance

0	6
1	2
2	0
3	5
4	8

Priority-Queue

(5,3)

(6,0)

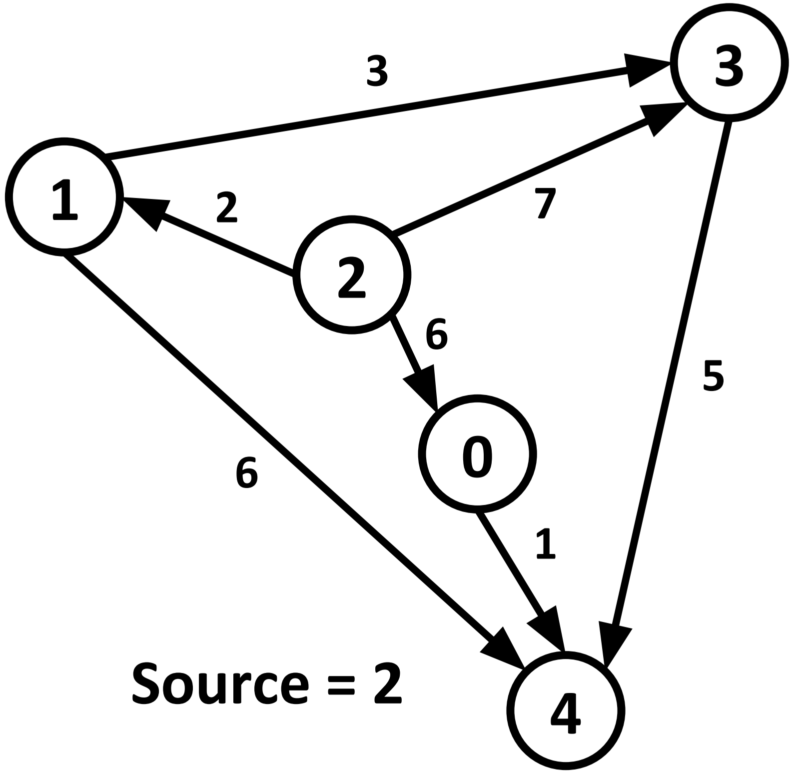
(7,3)

(8,4)

Extract min

W(5)

V(3)



Source = 2

Implementation

Parent

0	2
1	2
2	-1
3	1
4	1

Distance

0	6
1	2
2	0
3	5
4	8

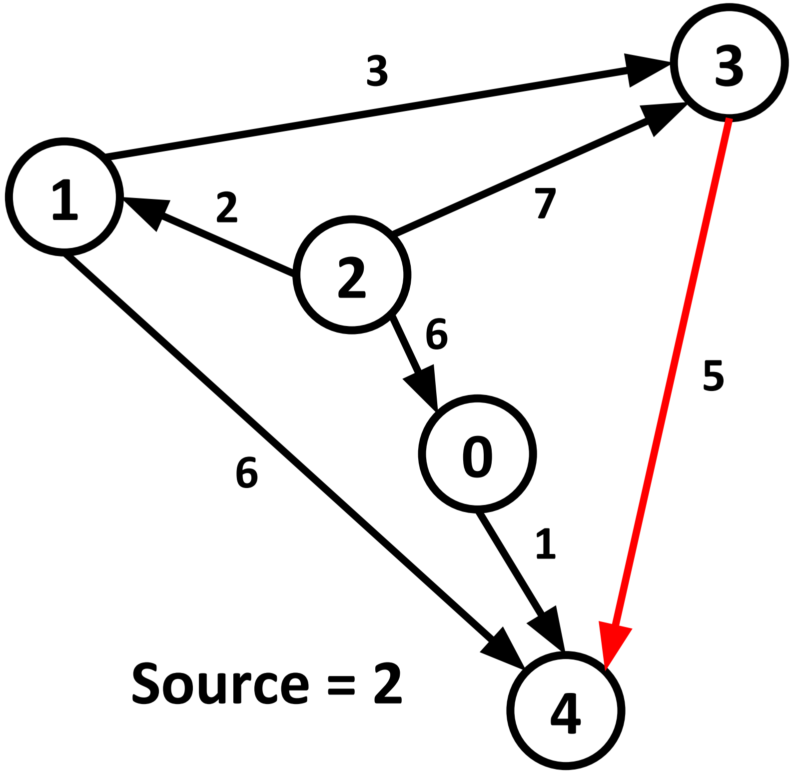
Priority-Queue

(5,3)
(6,0)
(7,3)
(8,4)

Extract min

W(5)

V(3)



Source = 2

Implementation

Parent

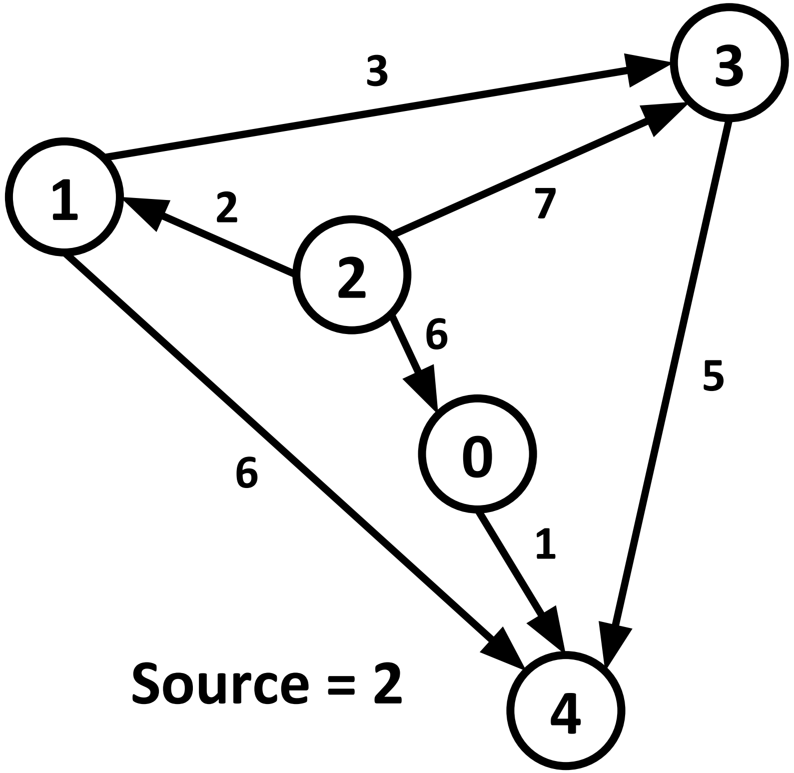
0	2
1	2
2	-1
3	1
4	1

Distance

0	6
1	2
2	0
3	5
4	8

Priority-Queue

(6,0)
(7,3)
(8,4)



Source = 2

Implementation

Parent

0	2
1	2
2	-1
3	1
4	1

Distance

0	6
1	2
2	0
3	5
4	8

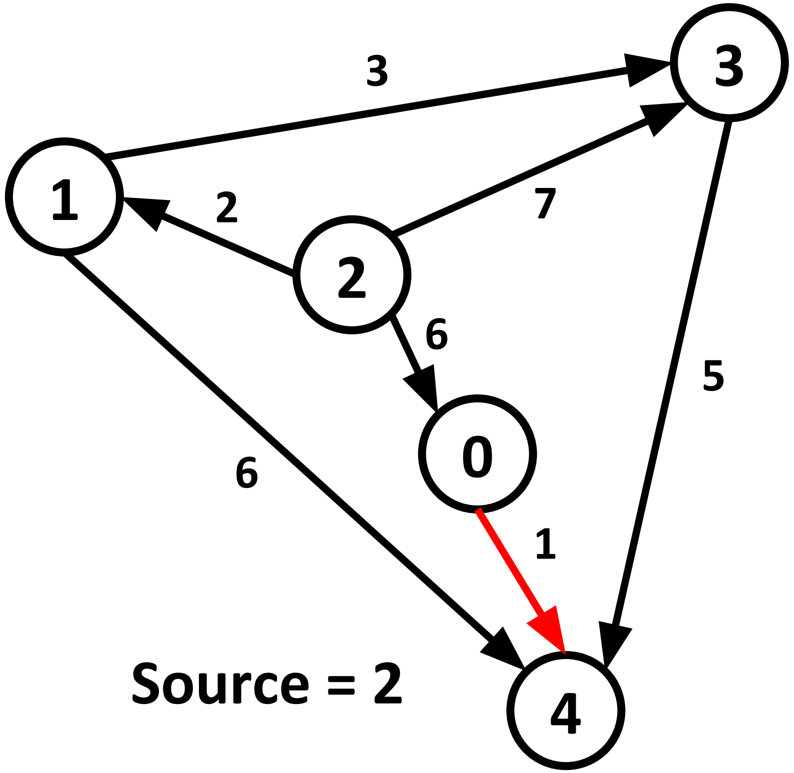
Priority-Queue

(6,0)
(7,3)
(8,4)

Extract min

W(6)

V(0)



Source = 2

Implementation

Parent

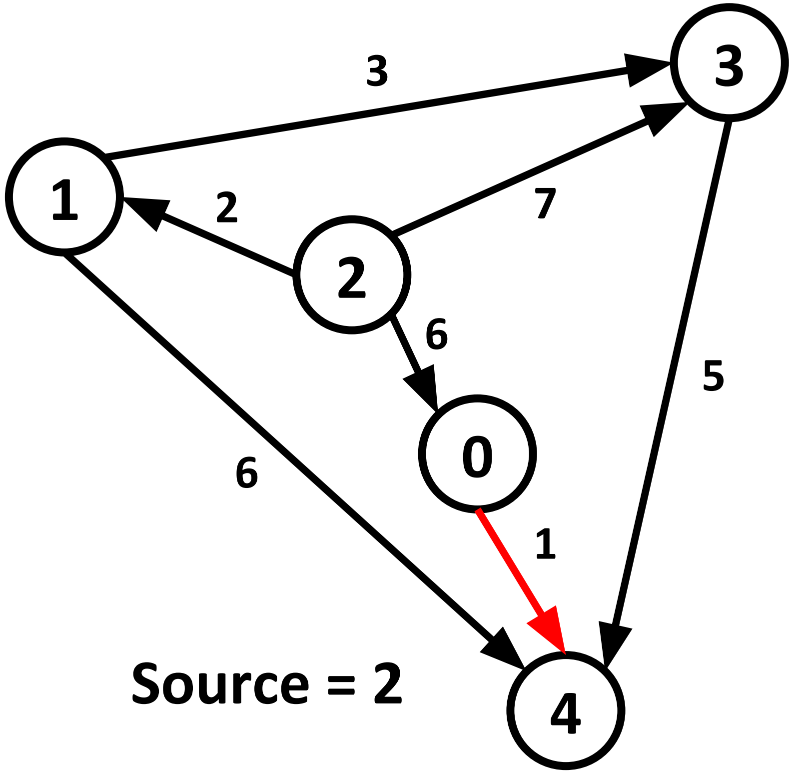
0	2
1	2
2	-1
3	1
4	0

Distance

0	6
1	2
2	0
3	5
4	7

Priority-Queue

(7,3)
(7,4)
(8,4)



Source = 2

Implementation

Parent

0	2
1	2
2	-1
3	1
4	0

Distance

0	6
1	2
2	0
3	5
4	7

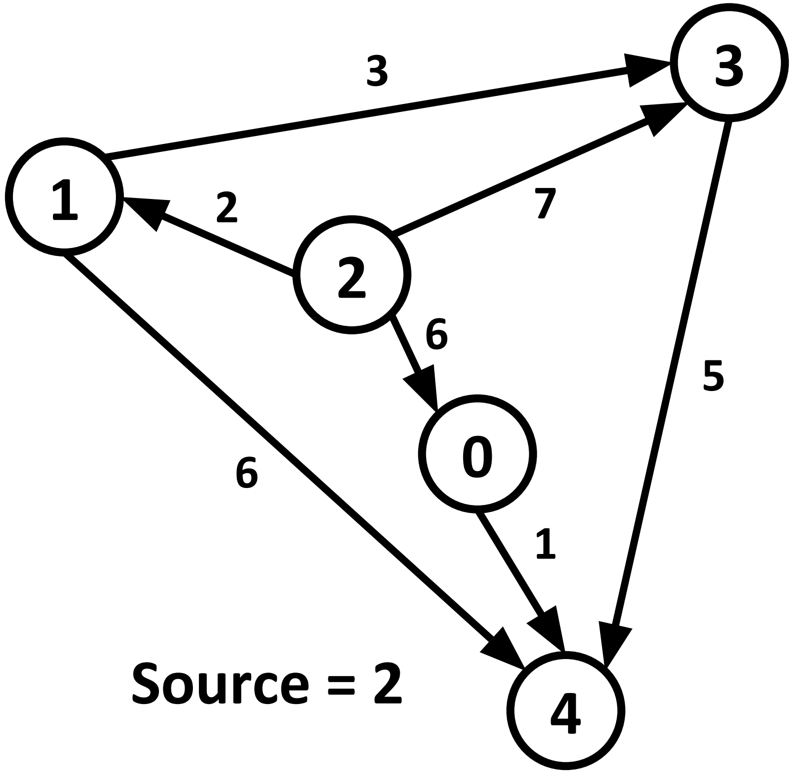
Priority-Queue

(7,3)
(7,4)
(8,4)

Extract min

W(7)

V(3)



Source = 2

Implementation

Parent

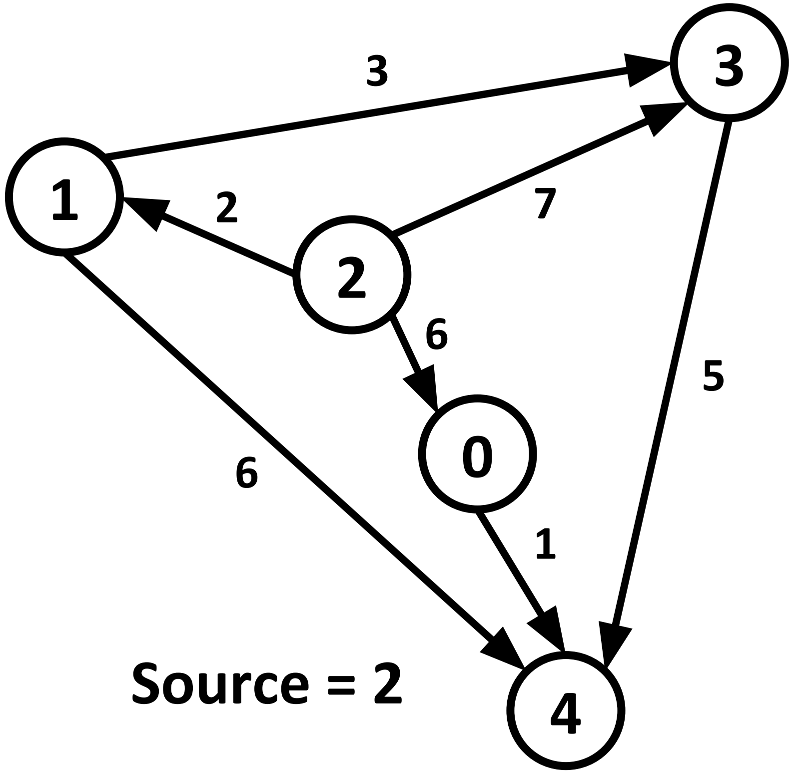
0	2
1	2
2	-1
3	1
4	0

Distance

0	6
1	2
2	0
3	5
4	7

Priority-Queue

(7,4)
(8,4)



Source = 2

Implementation

Parent

0	2
1	2
2	-1
3	1
4	0

Distance

0	6
1	2
2	0
3	5
4	7

Priority-Queue

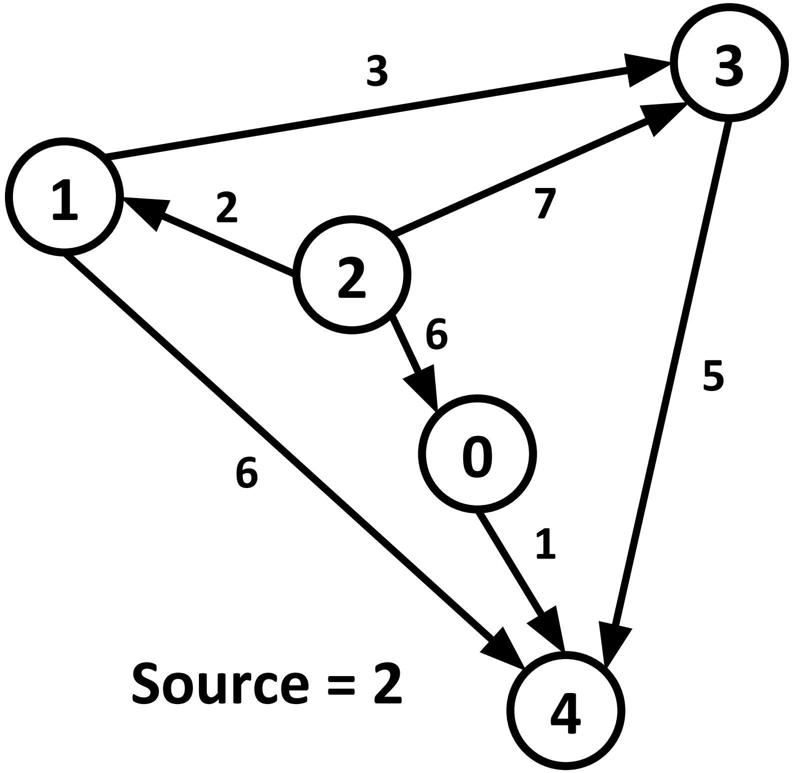
(7,4)

(8,4)

Extract min

W(7)

V(4)



Implementation

Parent

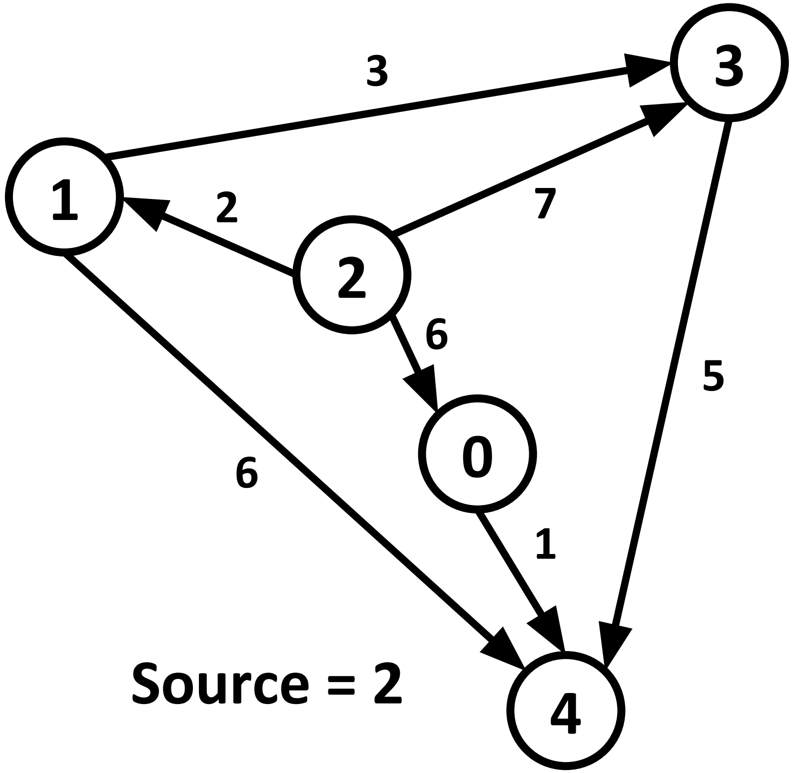
0	2
1	2
2	-1
3	1
4	0

Distance

0	6
1	2
2	0
3	5
4	7

Priority-Queue

(8,4)



Source = 2

Implementation

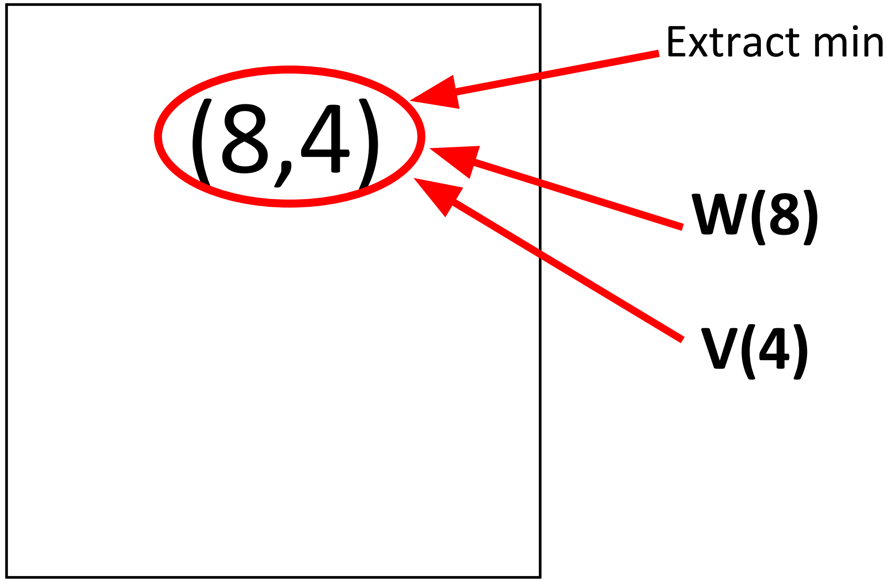
Parent

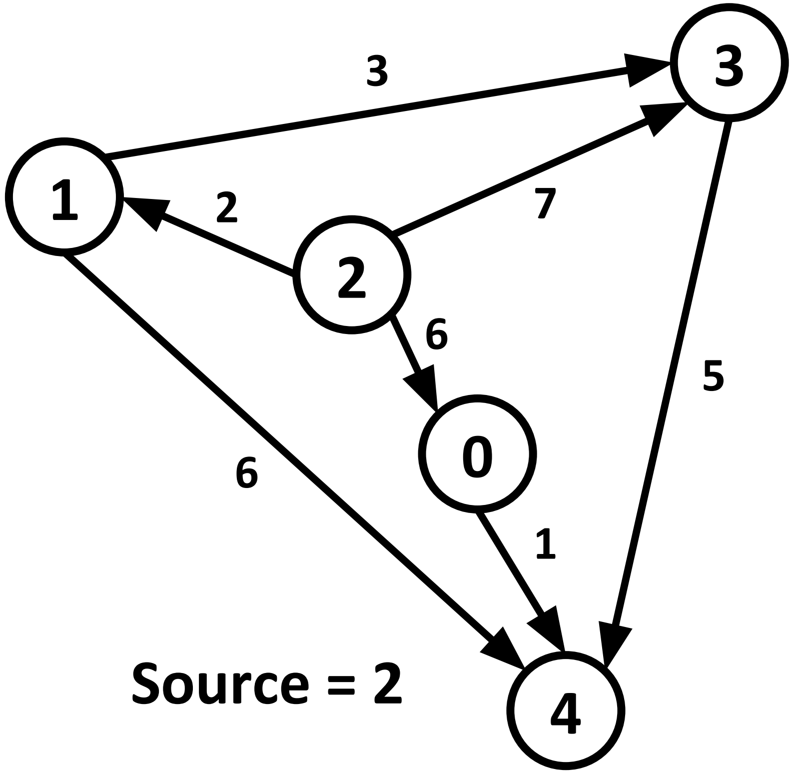
0	2
1	2
2	-1
3	1
4	0

Distance

0	6
1	2
2	0
3	5
4	7

Priority-Queue





Implementation

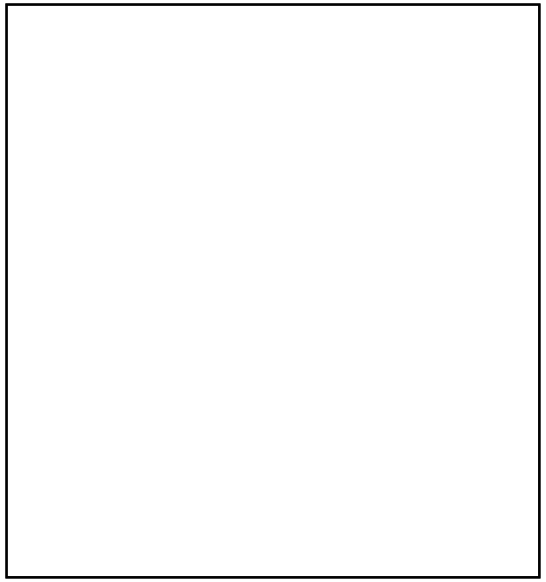
Parent

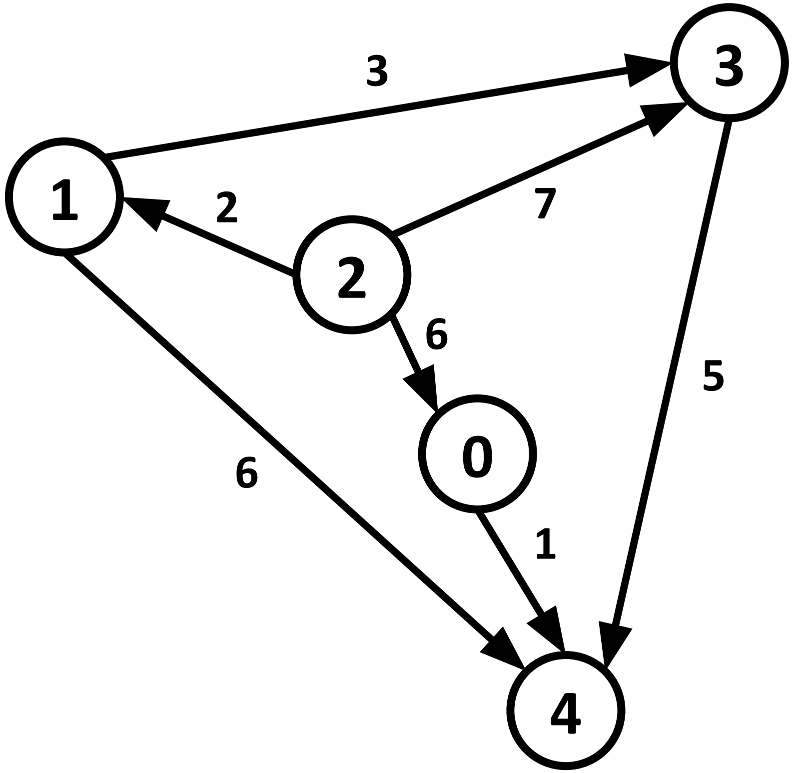
0	2
1	2
2	-1
3	1
4	0

Distance

0	6
1	2
2	0
3	5
4	7

Priority-Queue





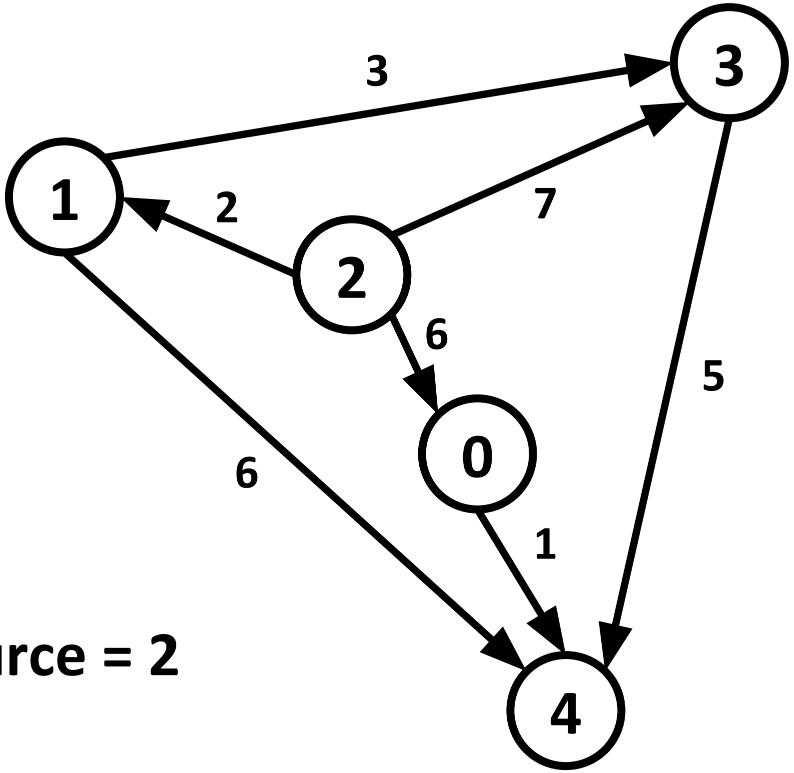
Source = 2

Complexity

- Build Max Heap: O(log E)
- Extract Min: O(1)
- Edge excess total complexity: O(E log E) = O(E log V) **)** **[E<=|V|^2]**
- Vertex excess complexity: O(V)
- Total : O(V+E log V)

Implementation

0	(4, 1)		
1	(3, 3)	(4, 6)	
2	(0, 6)	(1, 2)	(3, 7)
3	(4, 5)		
4			



Source = 2