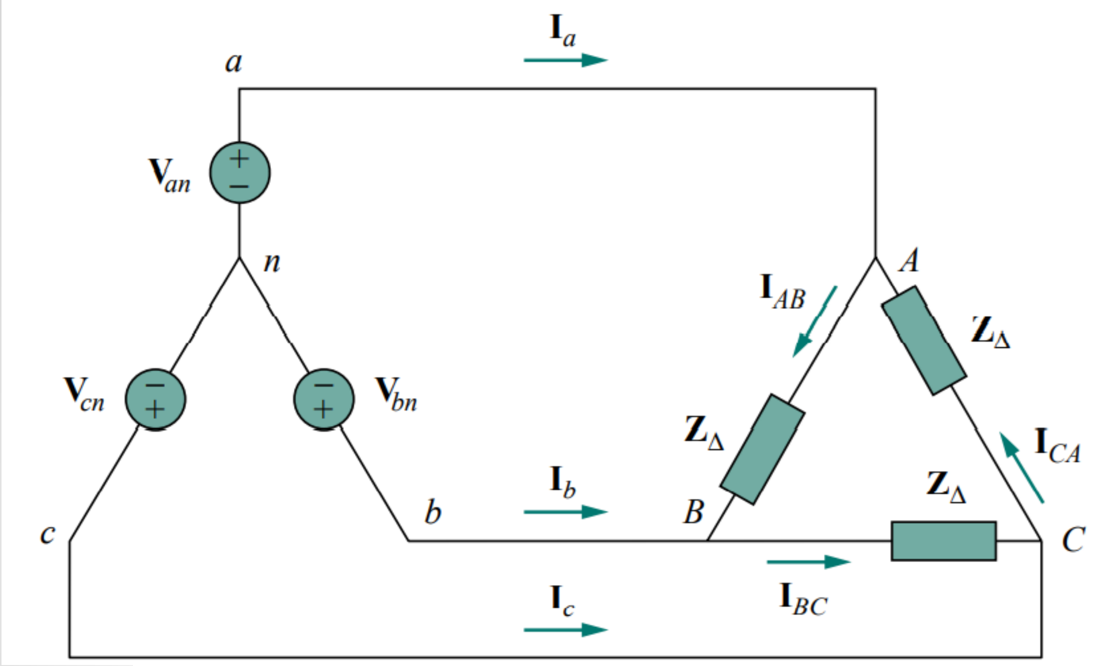
Introduction to Three Phase System

Title and Content Layout with List

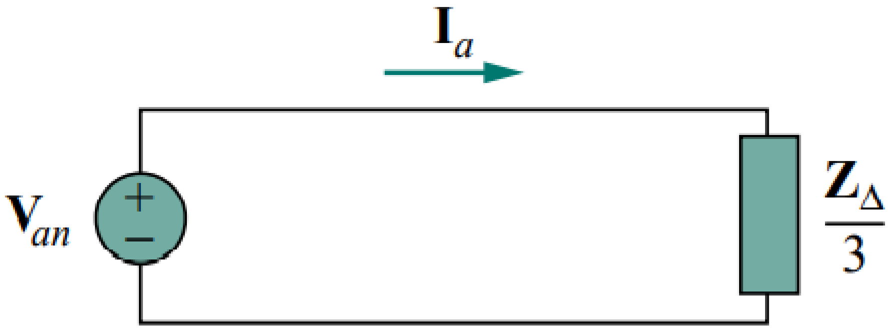
- After the ending of this lesson, students are able to know about
  - Wye Delta Configuration with mathematical analysis
    - Delta Delta configuration with mathematical analysis
      - Wattmeter Connection

BALANCED WYE-DELTA CONNECTION



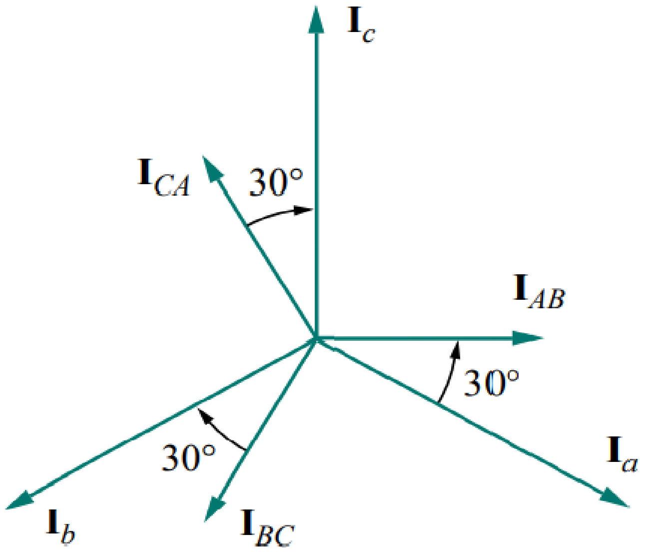
$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$



$/_p\angle -90^\circ = \mathbf{V}_{BC}$

CA



$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

Calculation of line currents

- The line currents are obtained from the phase currents by applying KCL at nodes A, B and C

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \qquad \textbf{(12.23)}$$

Since $\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/-240^\circ}$,

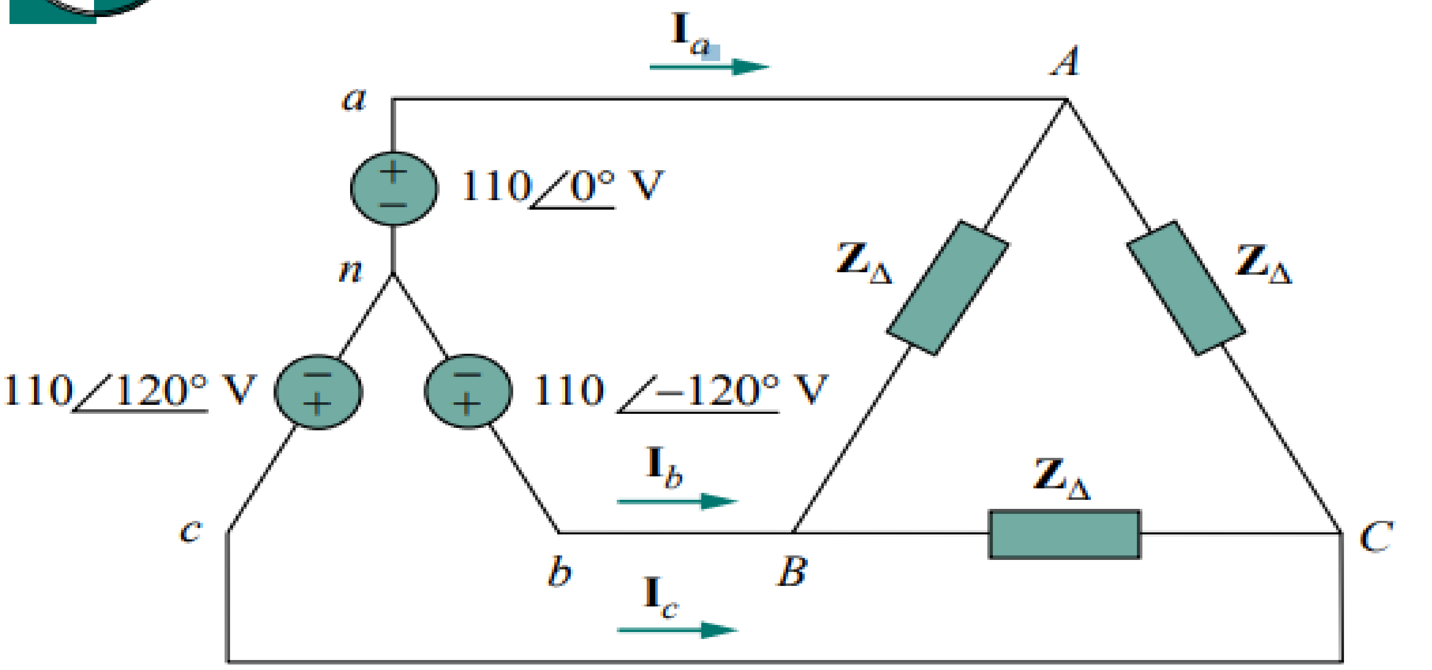
$$\begin{aligned} \mathbf{I}_a &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\underline{/-240^\circ}) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} \end{aligned} \qquad \textbf{(12.24)}$$

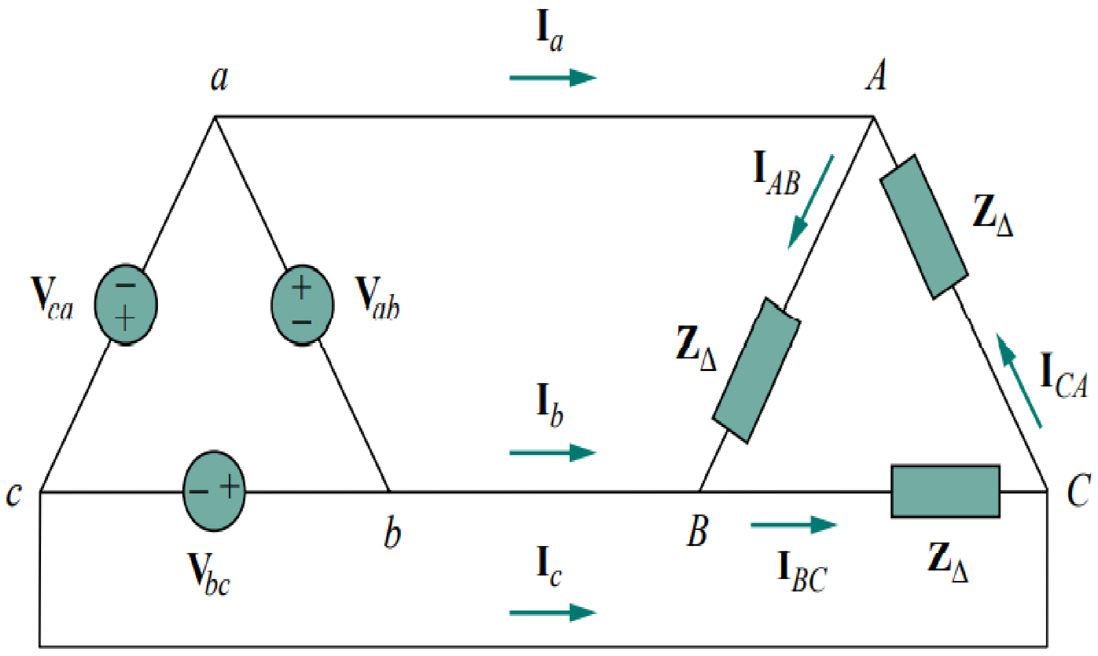
showing that the magnitude I_L of the line current is $\sqrt{3}$ times the magnitude I_p of the phase current, or

$$\boxed{I_L = \sqrt{3}I_p} \qquad \textbf{(12.25)}$$

Problem

12.12 Solve for the line currents in the Y-Δ circuit of Fig. 12.45. Take $\mathbf{Z}_\Delta = 60\angle 45^\circ\ \Omega$.





$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

Hence, the phase currents are

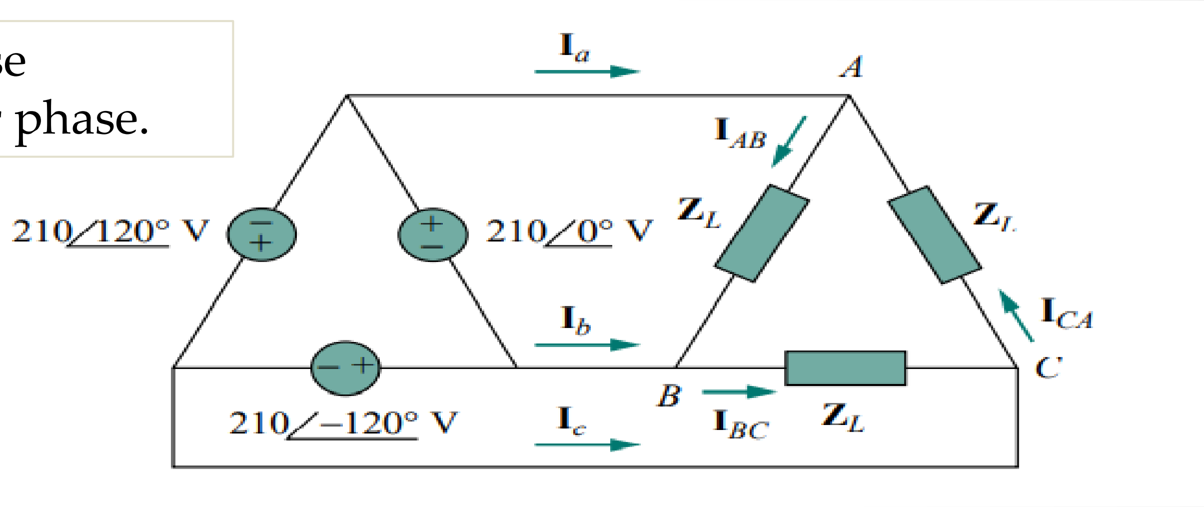
$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_\Delta}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_\Delta}$$

How can you calculate Ia, Ib and Ic ?

Problem

Refer to the - circuit in Fig. 12.49. Find the line and phase currents. Assume that the load impedance is 12 + j9 per phase.



Solution

The load impedance per phase is

$$\mathbf{Z}_\Delta = 20 - j15 = 25\angle{-36.87^\circ}\ \Omega$$

Since $\mathbf{V}_{AB} = \mathbf{V}_{ab}$, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\angle{0^\circ}}{25\angle{-36.87}} = 13.2\angle{36.87^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle{-120^\circ} = 13.2\angle{-83.13^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle{+120^\circ} = 13.2\angle{156.87^\circ}\ \text{A}$$

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ} = (13.2\angle{36.87^\circ})(\sqrt{3}\angle{-30^\circ})$$

$$= 22.86\angle{6.87^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle{-120^\circ} = 22.86\angle{-113.13^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle{+120^\circ} = 22.86\angle{126.87^\circ}\ \text{A}$$

Balanced Delta-Wye Connection

- A balanced Delta-Wye system consists of a balanced Delta-connected source feeding a balanced Wye-connected load
- Phase voltages, as well as line voltages

$$\mathbf{V}_{ab} = V_p\angle 0^\circ, \qquad \mathbf{V}_{bc} = V_p\angle -120^\circ$$

$$\mathbf{V}_{ca} = V_p\angle +120^\circ$$

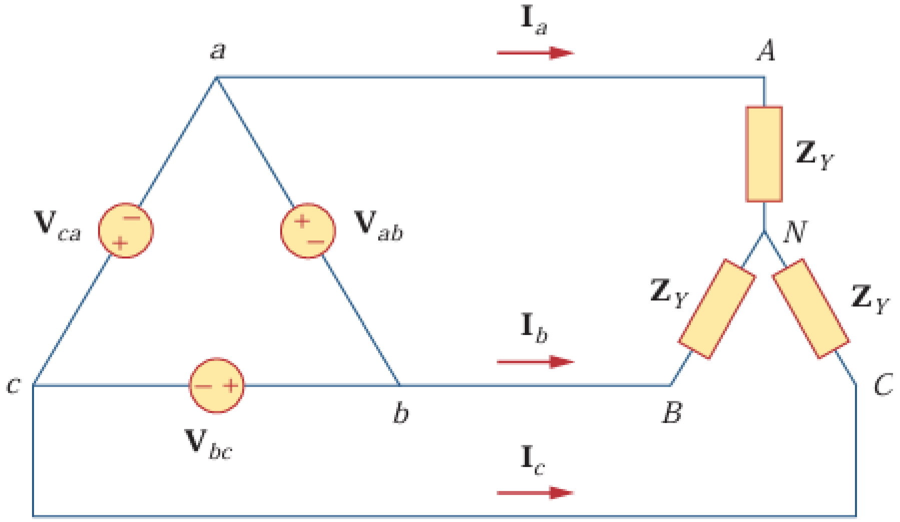


Figure 12.18
A balanced Δ-Y connection.

Line currents

We can obtain the line currents in many ways. One way is to apply KVL to loop *aANBba* in Fig. 12.18, writing

$$-\mathbf{V}_{ab} + \mathbf{Z}_Y\mathbf{I}_a - \mathbf{Z}_Y\mathbf{I}_b = 0$$

or

$$\mathbf{Z}_Y(\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = V_p\underline{/0^\circ}$$

Thus,

$$\mathbf{I}_a - \mathbf{I}_b = \frac{V_p\underline{/0^\circ}}{\mathbf{Z}_Y}$$

But $\mathbf{I}_b$ lags $\mathbf{I}_a$ by 120°, since we assumed the *abc* sequence; that is, $\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$. Hence,

$$\begin{aligned}\mathbf{I}_a - \mathbf{I}_b &= \mathbf{I}_a(1 - 1\underline{/-120^\circ}) \\ &= \mathbf{I}_a\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \mathbf{I}_a\sqrt{3}\underline{/30^\circ}\end{aligned} \tag{12.36}$$

Substituting Eq. (12.36) into Eq. (12.35) gives

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\underline{/-30^\circ}}{\mathbf{Z}_Y} \tag{12.37}$$

Problem (Delta-Wye System)

- A balanced Y-connected load with a phase impedance of 40+j25 Ohms is supplied by a balanced, positive sequence Delta-connected source with a line voltage of 210 V. Calculate the phase currents. Use Vab as reference.

Answer:

Ia = 2.57<-62

Ib = 2.57 <-178

Ic = 2.57<58

Book solution

Solution:

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17\angle 32^\circ\ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210\angle 0^\circ\ \text{V}$$

When the Δ-connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30^\circ = 121.2\angle -30^\circ\ \text{V}$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\angle -30^\circ}{47.12\angle 32^\circ} = 2.57\angle -62^\circ\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 2.57\angle -178^\circ\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = 2.57\angle 58^\circ\ \text{A}$$

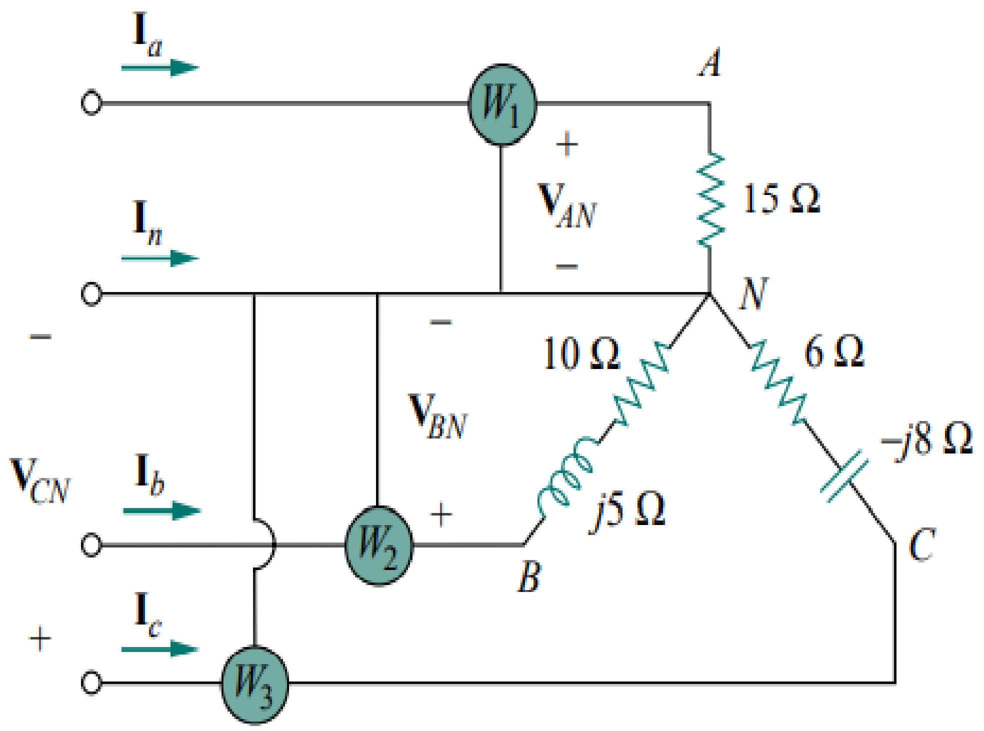
which are the same as the phase currents.

- Please carry a printed copy of this page while working on problems related to three-phase systems
- Memorise this part, very very important

Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0°$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$
	$\mathbf{V}_{bn} = V_p\angle -120°$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120°$
	$\mathbf{V}_{cn} = V_p\angle +120°$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120°$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0°$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30°$
	$\mathbf{V}_{bn} = V_p\angle -120°$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120°$
	$\mathbf{V}_{cn} = V_p\angle +120°$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120°$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0°$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120°$	
	$\mathbf{V}_{ca} = V_p\angle +120°$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0°$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120°$	
	$\mathbf{V}_{ca} = V_p\angle +120°$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30°}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120°$

Wattmeter Related Math



$\mathbf{V}_{AN} = 100\underline{/0^\circ}, \qquad \mathbf{V}_{BN} = 100\underline{/120^\circ}, \qquad \mathbf{V}_{CN} = 100\underline{/-120^\circ}\text{ V}$

while

$\mathbf{I}_a = 6.67\underline{/0^\circ}, \qquad \mathbf{I}_b = 8.94\underline{/93.44^\circ}, \qquad \mathbf{I}_c = 10\underline{/-66.87^\circ}\text{ A}$

Predict the wattmeter readings.


FINISH