

obtained by directly applying KCL and KVL to the appropriate three-phase circuits.

A balanced Y-connected load with a phase impedance of $40 + j25 \Omega$ is supplied by a balanced, positive sequence Δ -connected source with a line voltage of 210 V. Calculate the phase currents. Use \mathbf{V}_{ab} as a reference.

Example 12.5

Solution:

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17 \angle 32^\circ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210 \angle 0^\circ \text{ V}$$

When the Δ -connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}} \angle -30^\circ = 121.2 \angle -30^\circ \text{ V}$$

The line currents are

$$\begin{aligned}\mathbf{I}_a &= \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2 \angle -30^\circ}{47.12 \angle 32^\circ} = 2.57 \angle -62^\circ \text{ A} \\ \mathbf{I}_b &= \mathbf{I}_a \angle -120^\circ = 2.57 \angle -178^\circ \text{ A} \\ \mathbf{I}_c &= \mathbf{I}_a \angle 120^\circ = 2.57 \angle 58^\circ \text{ A}\end{aligned}$$

which are the same as the phase currents.

In a balanced Δ -Y circuit, $\mathbf{V}_{ab} = 240 \angle 15^\circ$ and $\mathbf{Z}_Y = (12 + j15) \Omega$. Calculate the line currents.

Practice Problem 12.5

Answer: $7.21 \angle -66.34^\circ \text{ A}, 7.21 \angle +173.66^\circ \text{ A}, 7.21 \angle 53.66^\circ \text{ A}$.

12.7 Power in a Balanced System

Let us now consider the power in a balanced three-phase system. We begin by examining the instantaneous power absorbed by the load. This requires that the analysis be done in the time domain. For a Y-connected load, the phase voltages are

$$\begin{aligned}v_{AN} &= \sqrt{2}V_p \cos \omega t, & v_{BN} &= \sqrt{2}V_p \cos(\omega t - 120^\circ) \\ v_{CN} &= \sqrt{2}V_p \cos(\omega t + 120^\circ)\end{aligned}\quad (12.41)$$

where the factor $\sqrt{2}$ is necessary because V_p has been defined as the rms value of the phase voltage. If $\mathbf{Z}_Y = Z \angle \theta$, the phase currents lag behind their corresponding phase voltages by θ . Thus,

$$\begin{aligned}i_a &= \sqrt{2}I_p \cos(\omega t - \theta), & i_b &= \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ) \\ i_c &= \sqrt{2}I_p \cos(\omega t - \theta + 120^\circ)\end{aligned}\quad (12.42)$$

where I_p is the rms value of the phase current. The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$\begin{aligned} p &= p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c \\ &= 2V_p I_p [\cos \omega t \cos(\omega t - \theta) \\ &\quad + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) \\ &\quad + \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ)] \end{aligned} \quad (12.43)$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)] \quad (12.44)$$

gives

$$\begin{aligned} p &= V_p I_p [3 \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\ &\quad + \cos(2\omega t - \theta + 240^\circ)] \\ &= V_p I_p [3 \cos \theta + \cos \alpha + \cos \alpha \cos 240^\circ + \sin \alpha \sin 240^\circ \\ &\quad + \cos \alpha \cos 240^\circ - \sin \alpha \sin 240^\circ] \\ &\quad \text{where } \alpha = 2\omega t - \theta \\ &= V_p I_p \left[3 \cos \theta + \cos \alpha + 2 \left(-\frac{1}{2} \right) \cos \alpha \right] = 3V_p I_p \cos \theta \end{aligned} \quad (12.45)$$

Thus the total instantaneous power in a balanced three-phase system is constant—it does not change with time as the instantaneous power of each phase does. This result is true whether the load is Y- or Δ -connected. This is one important reason for using a three-phase system to generate and distribute power. We will look into another reason a little later.

Since the total instantaneous power is independent of time, the average power per phase P_p for either the Δ -connected load or the Y-connected load is $p/3$, or

$$P_p = V_p I_p \cos \theta \quad (12.46)$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin \theta \quad (12.47)$$

The apparent power per phase is

$$S_p = V_p I_p \quad (12.48)$$

The complex power per phase is

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^* \quad (12.49)$$

where \mathbf{V}_p and \mathbf{I}_p are the phase voltage and phase current with magnitudes V_p and I_p , respectively. The total average power is the sum of the average powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos \theta = \sqrt{3}V_L I_L \cos \theta \quad (12.50)$$

For a Y-connected load, $I_L = I_p$ but $V_L = \sqrt{3}V_p$, whereas for a Δ -connected load, $I_L = \sqrt{3}I_p$ but $V_L = V_p$. Thus, Eq. (12.50) applies for both Y-connected and Δ -connected loads. Similarly, the total reactive power is

$$Q = 3V_p I_p \sin \theta = 3Q_p = \sqrt{3}V_L I_L \sin \theta \quad (12.51)$$

and the total complex power is

$$\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p \mathbf{I}_p^* = 3I_p^2 \mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*} \quad (12.52)$$

where $\mathbf{Z}_p = Z_p/\theta$ is the load impedance per phase. (\mathbf{Z}_p could be \mathbf{Z}_Y or \mathbf{Z}_Δ .) Alternatively, we may write Eq. (12.52) as

$$\mathbf{S} = P + jQ = \sqrt{3}V_L I_L/\theta \quad (12.53)$$

Remember that V_p , I_p , V_L , and I_L are all rms values and that θ is the angle of the load impedance or the angle between the phase voltage and the phase current.

A second major advantage of three-phase systems for power distribution is that the three-phase system uses a lesser amount of wire than the single-phase system for the same line voltage V_L and the same absorbed power P_L . We will compare these cases and assume in both that the wires are of the same material (e.g., copper with resistivity ρ), of the same length ℓ , and that the loads are resistive (i.e., unity power factor). For the two-wire single-phase system in Fig. 12.21(a), $I_L = P_L/V_L$, so the power loss in the two wires is

$$P_{\text{loss}} = 2I_L^2 R = 2R \frac{P_L^2}{V_L^2} \quad (12.54)$$

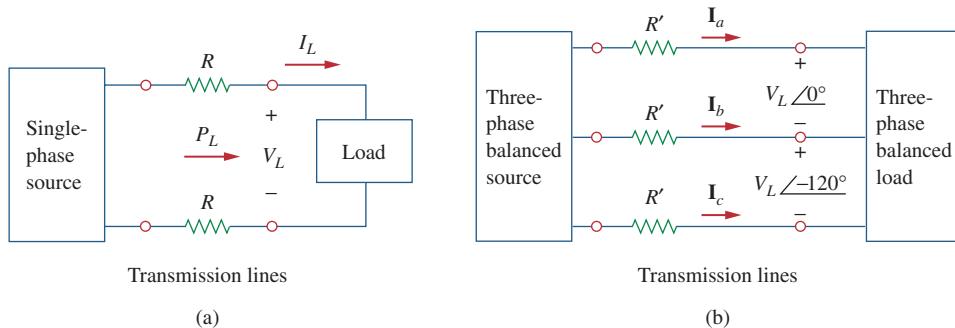


Figure 12.21
Comparing the power loss in (a) a single-phase system, and (b) a three-phase system.

For the three-wire three-phase system in Fig. 12.21(b), $I'_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| = P_L/\sqrt{3}V_L$ from Eq. (12.50). The power loss in the three wires is

$$P'_{\text{loss}} = 3(I'_L)^2 R' = 3R' \frac{P_L^2}{3V_L^2} = R' \frac{P_L^2}{V_L^2} \quad (12.55)$$

Equations (12.54) and (12.55) show that for the same total power delivered P_L and same line voltage V_L ,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2R}{R'} \quad (12.56)$$

But from Chapter 2, $R = \rho\ell/\pi r^2$ and $R' = \rho\ell/\pi r'^2$, where r and r' are the radii of the wires. Thus,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2r'^2}{r^2} \quad (12.57)$$

If the same power loss is tolerated in both systems, then $r^2 = 2r'^2$. The ratio of material required is determined by the number of wires and their volumes, so

$$\begin{aligned} \frac{\text{Material for single-phase}}{\text{Material for three-phase}} &= \frac{2(\pi r^2 \ell)}{3(\pi r'^2 \ell)} = \frac{2r^2}{3r'^2} \\ &= \frac{2}{3}(2) = 1.333 \end{aligned} \quad (12.58)$$

since $r^2 = 2r'^2$. Equation (12.58) shows that the single-phase system uses 33 percent more material than the three-phase system or that the three-phase system uses only 75 percent of the material used in the equivalent single-phase system. In other words, considerably less material is needed to deliver the same power with a three-phase system than is required for a single-phase system.

Example 12.6

Refer to the circuit in Fig. 12.13 (in Example 12.2). Determine the total average power, reactive power, and complex power at the source and at the load.

Solution:

It is sufficient to consider one phase, as the system is balanced. For phase a ,

$$\mathbf{V}_p = 110/0^\circ \text{ V} \quad \text{and} \quad \mathbf{I}_p = 6.81/-21.8^\circ \text{ A}$$

Thus, at the source, the complex power absorbed is

$$\begin{aligned} \mathbf{S}_s &= -3\mathbf{V}_p \mathbf{I}_p^* = -3(110/0^\circ)(6.81/21.8^\circ) \\ &= -2247/21.8^\circ = -(2087 + j834.6) \text{ VA} \end{aligned}$$

The real or average power absorbed is -2087 W and the reactive power is -834.6 VAR .

At the load, the complex power absorbed is

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2 \mathbf{Z}_p$$

where $\mathbf{Z}_p = 10 + j8 = 12.81/38.66^\circ$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81/-21.8^\circ$. Hence,

$$\begin{aligned} \mathbf{S}_L &= 3(6.81)^2 12.81/38.66^\circ = 1782/38.66^\circ \\ &= (1392 + j1113) \text{ VA} \end{aligned}$$

The real power absorbed is 1391.7 W and the reactive power absorbed is 1113.3 VAR . The difference between the two complex powers is absorbed by the line impedance $(5 - j2) \Omega$. To show that this is the case, we find the complex power absorbed by the line as

$$\mathbf{S}_\ell = 3|\mathbf{I}_p|^2 \mathbf{Z}_\ell = 3(6.81)^2 (5 - j2) = 695.6 - j278.3 \text{ VA}$$

which is the difference between \mathbf{S}_s and \mathbf{S}_L ; that is, $\mathbf{S}_s + \mathbf{S}_\ell + \mathbf{S}_L = 0$, as expected.

For the Y-Y circuit in Practice Prob. 12.2, calculate the complex power at the source and at the load.

Practice Problem 12.6

Answer: $-(1054.2 + j843.3)$ VA, $(1012 + j801.6)$ VA.

A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

Example 12.7

Solution:
The apparent power is

$$S = \sqrt{3}V_L I_L = \sqrt{3}(220)(18.2) = 6935.13 \text{ VA}$$

Since the real power is

$$P = S \cos\theta = 5600 \text{ W}$$

the power factor is

$$\text{pf} = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.8075$$

Calculate the line current required for a 30-kW three-phase motor having a power factor of 0.85 lagging if it is connected to a balanced source with a line voltage of 440 V.

Practice Problem 12.7

Answer: 46.31 A.

Two balanced loads are connected to a 240-kV rms 60-Hz line, as shown in Fig. 12.22(a). Load 1 draws 30 kW at a power factor of 0.6 lagging, while load 2 draws 45 kVAR at a power factor of 0.8 lagging. Assuming the *abc* sequence, determine: (a) the complex, real, and reactive powers absorbed by the combined load, (b) the line currents, and (c) the kVAR rating of the three capacitors Δ -connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.

Example 12.8

Solution:

(a) For load 1, given that $P_1 = 30$ kW and $\cos\theta_1 = 0.6$, then $\sin\theta_1 = 0.8$. Hence,

$$S_1 = \frac{P_1}{\cos\theta_1} = \frac{30 \text{ kW}}{0.6} = 50 \text{ kVA}$$

and $Q_1 = S_1 \sin\theta_1 = 50(0.8) = 40$ kVAR. Thus, the complex power due to load 1 is

$$S_1 = P_1 + jQ_1 = 30 + j40 \text{ kVA} \quad (12.8.1)$$