

obtained by directly applying KCL and KVL to the appropriate three-phase circuits.

A balanced Y-connected load with a phase impedance of  $40 + j25 \Omega$  is supplied by a balanced, positive sequence  $\Delta$ -connected source with a line voltage of 210 V. Calculate the phase currents. Use  $\mathbf{V}_{ab}$  as a reference.

### Example 12.5

#### Solution:

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17 \angle 32^\circ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210 \angle 0^\circ \text{ V}$$

When the  $\Delta$ -connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}} \angle -30^\circ = 121.2 \angle -30^\circ \text{ V}$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2 \angle -30^\circ}{47.17 \angle 32^\circ} = 2.57 \angle -62^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = 2.57 \angle -178^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120^\circ = 2.57 \angle 58^\circ \text{ A}$$

which are the same as the phase currents.

In a balanced  $\Delta$ -Y circuit,  $\mathbf{V}_{ab} = 240 \angle 15^\circ$  and  $\mathbf{Z}_Y = (12 + j15) \Omega$ . Calculate the line currents.

### Practice Problem 12.5

**Answer:**  $7.21 \angle -66.34^\circ \text{ A}$ ,  $7.21 \angle +173.66^\circ \text{ A}$ ,  $7.21 \angle 53.66^\circ \text{ A}$ .

## 12.7 Power in a Balanced System

Let us now consider the power in a balanced three-phase system. We begin by examining the instantaneous power absorbed by the load. This requires that the analysis be done in the time domain. For a Y-connected load, the phase voltages are

$$\begin{aligned} v_{AN} &= \sqrt{2}V_p \cos \omega t, & v_{BN} &= \sqrt{2}V_p \cos(\omega t - 120^\circ) \\ v_{CN} &= \sqrt{2}V_p \cos(\omega t + 120^\circ) \end{aligned} \quad (12.41)$$

where the factor  $\sqrt{2}$  is necessary because  $V_p$  has been defined as the rms value of the phase voltage. If  $\mathbf{Z}_Y = Z \angle \theta$ , the phase currents lag behind their corresponding phase voltages by  $\theta$ . Thus,

$$\begin{aligned} i_a &= \sqrt{2}I_p \cos(\omega t - \theta), & i_b &= \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ) \\ i_c &= \sqrt{2}I_p \cos(\omega t - \theta + 120^\circ) \end{aligned} \quad (12.42)$$

where  $I_p$  is the rms value of the phase current. The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$\begin{aligned} p &= p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c \\ &= 2V_p I_p [\cos \omega t \cos(\omega t - \theta) \\ &\quad + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) \\ &\quad + \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ)] \end{aligned} \quad (12.43)$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)] \quad (12.44)$$

gives

$$\begin{aligned} p &= V_p I_p [3 \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\ &\quad + \cos(2\omega t - \theta + 240^\circ)] \\ &= V_p I_p [3 \cos \theta + \cos \alpha + \cos \alpha \cos 240^\circ + \sin \alpha \sin 240^\circ \\ &\quad + \cos \alpha \cos 240^\circ - \sin \alpha \sin 240^\circ] \\ &\quad \text{where } \alpha = 2\omega t - \theta \\ &= V_p I_p \left[ 3 \cos \theta + \cos \alpha + 2 \left( -\frac{1}{2} \right) \cos \alpha \right] = 3V_p I_p \cos \theta \end{aligned} \quad (12.45)$$

Thus the total instantaneous power in a balanced three-phase system is constant—it does not change with time as the instantaneous power of each phase does. This result is true whether the load is Y- or  $\Delta$ -connected. This is one important reason for using a three-phase system to generate and distribute power. We will look into another reason a little later.

Since the total instantaneous power is independent of time, the average power per phase  $P_p$  for either the  $\Delta$ -connected load or the Y-connected load is  $p/3$ , or

$$P_p = V_p I_p \cos \theta \quad (12.46)$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin \theta \quad (12.47)$$

The apparent power per phase is

$$S_p = V_p I_p \quad (12.48)$$

The complex power per phase is

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^* \quad (12.49)$$

where  $\mathbf{V}_p$  and  $\mathbf{I}_p$  are the phase voltage and phase current with magnitudes  $V_p$  and  $I_p$ , respectively. The total average power is the sum of the average powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos \theta = \sqrt{3}V_L I_L \cos \theta \quad (12.50)$$

For a Y-connected load,  $I_L = I_p$  but  $V_L = \sqrt{3}V_p$ , whereas for a  $\Delta$ -connected load,  $I_L = \sqrt{3}I_p$  but  $V_L = V_p$ . Thus, Eq. (12.50) applies for both Y-connected and  $\Delta$ -connected loads. Similarly, the total reactive power is

$$Q = 3V_p I_p \sin \theta = 3Q_p = \sqrt{3}V_L I_L \sin \theta \quad (12.51)$$

and the total complex power is

$$\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p\mathbf{I}_p^* = 3I_p^2\mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*} \quad (12.52)$$

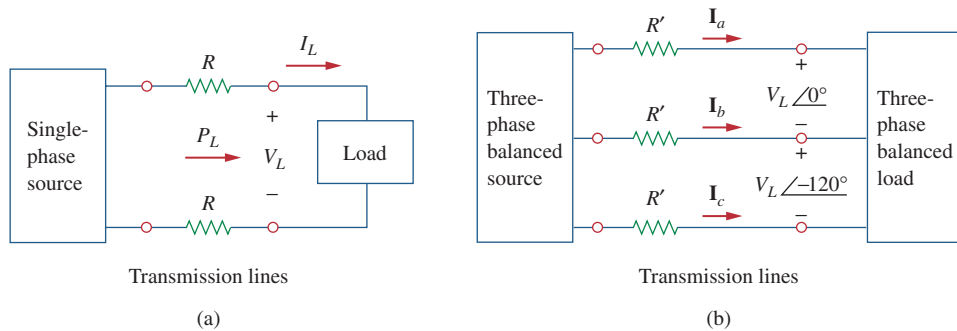
where  $\mathbf{Z}_p = Z_p \angle \theta$  is the load impedance per phase. ( $\mathbf{Z}_p$  could be  $\mathbf{Z}_Y$  or  $\mathbf{Z}_\Delta$ .) Alternatively, we may write Eq. (12.52) as

$$\mathbf{S} = P + jQ = \sqrt{3}V_L I_L \angle \theta \quad (12.53)$$

Remember that  $V_p$ ,  $I_p$ ,  $V_L$ , and  $I_L$  are all rms values and that  $\theta$  is the angle of the load impedance or the angle between the phase voltage and the phase current.

A second major advantage of three-phase systems for power distribution is that the three-phase system uses a lesser amount of wire than the single-phase system for the same line voltage  $V_L$  and the same absorbed power  $P_L$ . We will compare these cases and assume in both that the wires are of the same material (e.g., copper with resistivity  $\rho$ ), of the same length  $\ell$ , and that the loads are resistive (i.e., unity power factor). For the two-wire single-phase system in Fig. 12.21(a),  $I_L = P_L/V_L$ , so the power loss in the two wires is

$$P_{\text{loss}} = 2I_L^2 R = 2R \frac{P_L^2}{V_L^2} \quad (12.54)$$



**Figure 12.21**

Comparing the power loss in (a) a single-phase system, and (b) a three-phase system.

For the three-wire three-phase system in Fig. 12.21(b),  $I'_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| = P_L/\sqrt{3}V_L$  from Eq. (12.50). The power loss in the three wires is

$$P'_{\text{loss}} = 3(I'_L)^2 R' = 3R' \frac{P_L^2}{3V_L^2} = R' \frac{P_L^2}{V_L^2} \quad (12.55)$$

Equations (12.54) and (12.55) show that for the same total power delivered  $P_L$  and same line voltage  $V_L$ ,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2R}{R'} \quad (12.56)$$

But from Chapter 2,  $R = \rho\ell/\pi r^2$  and  $R' = \rho\ell/\pi r'^2$ , where  $r$  and  $r'$  are the radii of the wires. Thus,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2r'^2}{r^2} \quad (12.57)$$

If the same power loss is tolerated in both systems, then  $r^2 = 2r'^2$ . The ratio of material required is determined by the number of wires and their volumes, so

$$\begin{aligned} \frac{\text{Material for single-phase}}{\text{Material for three-phase}} &= \frac{2(\pi r^2 \ell)}{3(\pi r'^2 \ell)} = \frac{2r^2}{3r'^2} \\ &= \frac{2}{3}(2) = 1.333 \end{aligned} \quad (12.58)$$

since  $r^2 = 2r'^2$ . Equation (12.58) shows that the single-phase system uses 33 percent more material than the three-phase system or that the three-phase system uses only 75 percent of the material used in the equivalent single-phase system. In other words, considerably less material is needed to deliver the same power with a three-phase system than is required for a single-phase system.

### Example 12.6

Refer to the circuit in Fig. 12.13 (in Example 12.2). Determine the total average power, reactive power, and complex power at the source and at the load.

#### Solution:

It is sufficient to consider one phase, as the system is balanced. For phase  $a$ ,

$$\mathbf{V}_p = 110\angle 0^\circ \text{ V} \quad \text{and} \quad \mathbf{I}_p = 6.81\angle -21.8^\circ \text{ A}$$

Thus, at the source, the complex power absorbed is

$$\begin{aligned} \mathbf{S}_s &= -3\mathbf{V}_p \mathbf{I}_p^* = -3(110\angle 0^\circ)(6.81\angle 21.8^\circ) \\ &= -2247\angle 21.8^\circ = -(2087 + j834.6) \text{ VA} \end{aligned}$$

The real or average power absorbed is  $-2087 \text{ W}$  and the reactive power is  $-834.6 \text{ VAR}$ .

At the load, the complex power absorbed is

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2 \mathbf{Z}_p$$

where  $\mathbf{Z}_p = 10 + j8 = 12.81\angle 38.66^\circ$  and  $\mathbf{I}_p = \mathbf{I}_a = 6.81\angle -21.8^\circ$ . Hence,

$$\begin{aligned} \mathbf{S}_L &= 3(6.81)^2 12.81\angle 38.66^\circ = 1782\angle 38.66^\circ \\ &= (1392 + j1113) \text{ VA} \end{aligned}$$

The real power absorbed is  $1391.7 \text{ W}$  and the reactive power absorbed is  $1113.3 \text{ VAR}$ . The difference between the two complex powers is absorbed by the line impedance  $(5 - j2) \Omega$ . To show that this is the case, we find the complex power absorbed by the line as

$$\mathbf{S}_\ell = 3|\mathbf{I}_p|^2 \mathbf{Z}_\ell = 3(6.81)^2(5 - j2) = 695.6 - j278.3 \text{ VA}$$

which is the difference between  $\mathbf{S}_s$  and  $\mathbf{S}_L$ ; that is,  $\mathbf{S}_s + \mathbf{S}_\ell + \mathbf{S}_L = 0$ , as expected.

For the Y-Y circuit in Practice Prob. 12.2, calculate the complex power at the source and at the load.

### Practice Problem 12.6

**Answer:**  $-(1054.2 + j843.3)$  VA,  $(1012 + j801.6)$  VA.

A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

### Example 12.7

**Solution:**

The apparent power is

$$S = \sqrt{3}V_L I_L = \sqrt{3}(220)(18.2) = 6935.13 \text{ VA}$$

Since the real power is

$$P = S \cos \theta = 5600 \text{ W}$$

the power factor is

$$\text{pf} = \cos \theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.8075$$

Calculate the line current required for a 30-kW three-phase motor having a power factor of 0.85 lagging if it is connected to a balanced source with a line voltage of 440 V.

### Practice Problem 12.7

**Answer:** 46.31 A.

Two balanced loads are connected to a 240-kV rms 60-Hz line, as shown in Fig. 12.22(a). Load 1 draws 30 kW at a power factor of 0.6 lagging, while load 2 draws 45 kVAR at a power factor of 0.8 lagging. Assuming the *abc* sequence, determine: (a) the complex, real, and reactive powers absorbed by the combined load, (b) the line currents, and (c) the kVAR rating of the three capacitors  $\Delta$ -connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.

### Example 12.8

**Solution:**

(a) For load 1, given that  $P_1 = 30$  kW and  $\cos \theta_1 = 0.6$ , then  $\sin \theta_1 = 0.8$ . Hence,

$$S_1 = \frac{P_1}{\cos \theta_1} = \frac{30 \text{ kW}}{0.6} = 50 \text{ kVA}$$

and  $Q_1 = S_1 \sin \theta_1 = 50(0.8) = 40$  kVAR. Thus, the complex power due to load 1 is

$$S_1 = P_1 + jQ_1 = 30 + j40 \text{ kVA} \quad (12.8.1)$$