HOM project instructions 2017/18

School bus routing problem with bus stop selection

Problem definition

School bus routing problem, described in these instructions, is a variation of the vehicle routing problem. Unlike vehicle routing problem, where all the stops are known, and routes should be determined, in school bus routing problem only *potential* stops are given, and selection of stops and determining bus routes depends on students' locations and capacity of each bus.

The goal of solving this problem is:

1. to determine the set of stops to visit,
2. determine for each student which stop (s)he should walk to, and
3. determine routes that lie along the chosen stops, so that the total travelled distance is minimized.

At the same time, following **constraints** and **assumptions** must be taken into account:

1. Each student should be picked up on a bus stop (s)he can reach. (Stop's distance from the student location is shorter than maximum allowed walking distance.)
2. All of the buses have the same capacity. Capacity of the bus must not be exceeded.
3. Each bus stop is visited by only one bus. This implies that the students that go to a bus stop may not be divided into groups that may then each take a different bus.

Figure 1 illustrates this type of a problem. In the figure, dots represent the students, small squares potential stops, and a big square represents school. Dotted lines show which stations are within reach for each student. If, for example, capacity of the bus equals 8, then a possible solution is depicted with Figure 2. In the solution example,

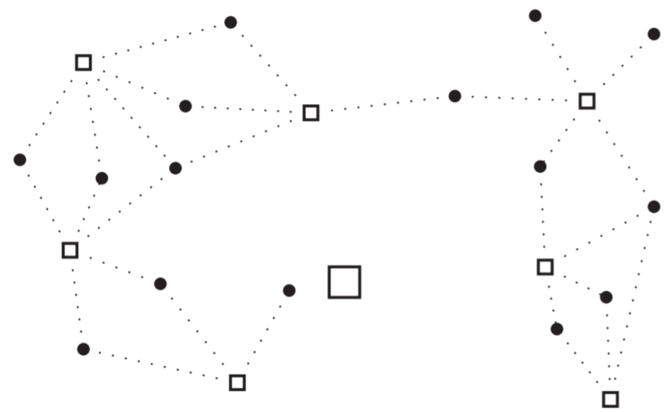

Figure 1: Problem instance example.

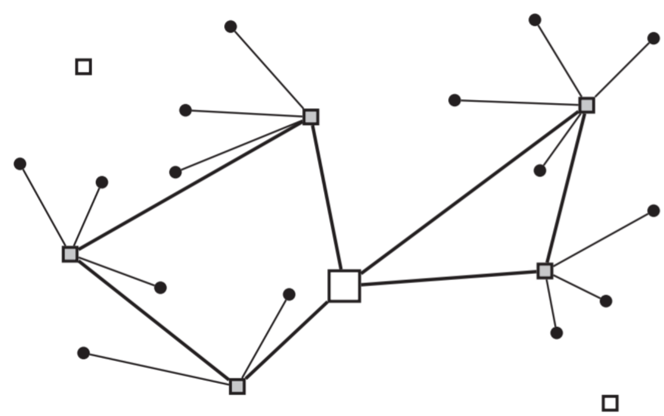

Figure 2: Problem solution example.

two bus routes are defined, and each must start and finish at the school, thus creating a cycle.

The following text describes the format of a problem instance (input file), gives a mathematical formulation of the problem and describes your project task related to this problem.

Problem instance

Problem instance defines the location of the school, locations of potential bus stops, and locations of students given as coordinates in euclidean plane, together with the problem constraints.

Each instance file first states the number of potential bus stops (including school), number of students, maximum walking distance, and bus capacity. The following lines in the file indicate the coordinates of each bus stop, where the bus stop with index 0 represents the school. After the bus stop coordinates, coordinates of student locations are given.

```
6 stops, 25 students, 20.000 maximum walk, 25 capacity

0 50.000 50.000
1 38.390 30.261
2 21.710 34.625
3 22.467 21.108
4 38.726 79.167
5 33.491 66.206


1 26.080 36.624
2 42.858 22.531
3 36.392 32.102
...
(shortened)
...
24 52.590 62.867
25 30.624 58.687
```

Mathematical programming formulation

Variables used in the mathematical formulation are defined as follows:

Input parameter	Description
C	capacity of a bus
V	set of potential stops
E	set of arcs between stops
S	set of students
c_{ij}	cost of traversing the arc from stop i to stop j
s_{il}	1 if student l can reach stop i, otherwise 0
$i = 0$	index for the school
Decision variables	**Description**
x_{ijk}	1 if bus k traverses the arch from i to j, otherwise 0
y_{ik}	1 if bus k visits stop i, otherwise 0
z_{ilk}	1 if student l is picked up by bus k at stop i, otherwise 0

The objective function of the school bus routing problem models the requirement to minimise the total distance travelled by all the buses. This function can be written mathematically as:

$$min \sum_{i \in V} \sum_{j \in V} c_{ij} \sum_{k=1}^{n} x_{ijk}$$

Constraints of this problem can be defined as follows:

Expression	Description
$\sum_{j \in V} x_{ijk} = \sum_{j \in V} x_{jik} = y_{ik}$ $\forall i \in V, k = 1, ..., n$	If stop i is visited by bus k, then an arc should be traversed by bus k entering stop i and leaving stop i.
$\sum_{i,j \in Q} x_{ijk} \leq \lvert Q \rvert - 1$ $\forall Q \subseteq V \setminus \{v_o\}, \forall k$	For each bus k and each subset of potential stops Q, excluding school, the number of arcs connecting the stops in the route of bus k must be less or equal to the number of elements in set Q minus 1. This constraint imposes the connectivity of the route performed by bus k.
$\sum_{k=1}^{n} y_{ik} \leq 1 \quad \forall i \in V \setminus \{0\}$	Each stop, except from school, is visited at most once.
$\sum_{k=1}^{n} z_{ilk} \leq s_{il} \quad \forall l \in S, \forall i \in V$	Each student is picked up at a stop (s)he can reach.
$\sum_{i \in V} \sum_{l \in S} z_{ilk} \leq C \quad k = 1, ..., n$	The number of students in each bus must be less or equal to the capacity of the bus.

$z_{ilk} \leq y_{ik} \quad \forall i, l, k$	Student l is not picked up at stop i by bus k, if bus k does not visit stop i.
$\sum_{i \in V} \sum_{k=1}^{n} z_{ilk} = 1 \quad \forall l \in S$	Each student is picked up once.

Project task

1. Design and implement a heuristic algorithm to solve the given problem.
2. Execute your algorithm for given instances of the problem.
3. Save **3 solutions for each instance**: solution obtained after 1 minute of algorithm execution, 5 minutes of algorithm execution and without time constraints.
4. Create a report that describes your implemented heuristic algorithm. The report should include the following:
   - A description of the problem.
   - A description of the implemented heuristic algorithm (solution representation, objective/fitness function, way of construction of an initial solution, iteration size and termination criterion, heuristic specific design elements, etc.).
   - Pseudocode of the implemented algorithm.
   - Analysis of results and discussion (e.g., influence of certain parameters of the heuristic algorithm on the quality of solutions, execution time of the algorithm)
   - Conclusion (e.g., discussion of potential further improvements of the implemented algorithm).

Files with obtained solutions must:

- be named "res-time-instance.txt", where "time" equals "1m", "5m" or "un" (unlimited), and "instance" equals "sbr1", "sbr2", etc.

- contain information about bus routes and bus stops assigned to each student, in the following format:

```
6 18 33
25 35 2 14 3
...
32 8 12 1

1 22
2 1
3 18
4 30
...
```

In the given example, the first row indicates that one of the bus lines is $0 \to 6 \to 18 \to 33 \to 0$, where bus stop IDs correspond to ones given in the instance file. Each route is written in a new line. After the bus routes, divided by an empty line, the solution should describe how the students are assigned to bus stops. In the given example, student 1 is picked up at stop 22, student 2 at stop 1 etc. Student IDs and bus stop IDs correspond to the IDs in the instance file.