Polyphonic Sound Generation

ECE 362
https://engineering.purdue.edu/ece362/

Generation of a wave table

- To generate a single cycle of a sine wave, use a program like this:

```
// genwave.c
#include <stdio.h>
#include <math.h>
#define N 1024
int main(void) {
        int x;
        printf("const short int wavetable[%d] = {\n", N);
        for(x=0; x<N; x++) {
                int value = 32767 * sin(2 * M_PI * x / N);
                printf("%d, ", value);
                if ((x % 8) == 7) printf("\n");
        }
        printf("};\n");
}
```

Running genwave.c

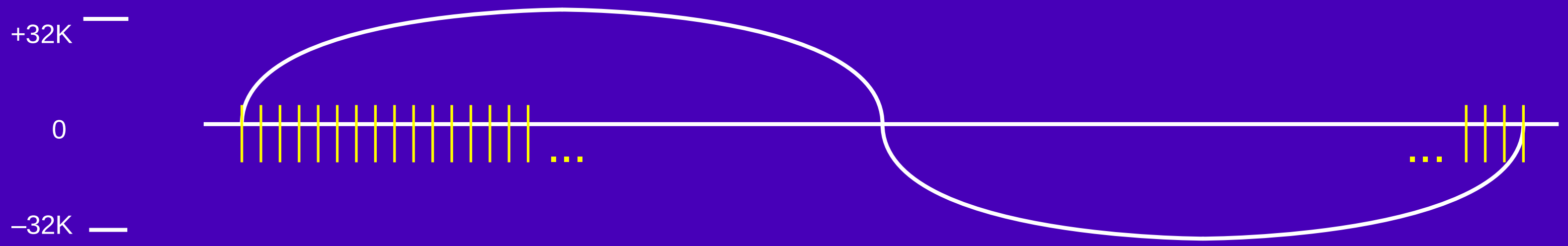
- Compile and run the genwave program:

```
gcc -o genwave genwave.c -lm
./genwave > wave.c
```

The wave table

- The wave table will be a single array
  - Values range from -32767 to +32767: nearly the maximum range that can be specified with a short int (AKA int16_t if you #include <stdint.h>).
  - The array is declared with “const” so that the array is placed in the Flash ROM rather than in RAM.

```
const short int wavetable[1024] = {
0, 201, 402, 603, 804, 1005, 1206, 1406,
1607, 1808, 2009, 2209, 2410, 2610, 2811, 3011,
3211, 3411, 3611, 3811, 4011, 4210, 4409, 4608,
4807, 5006, 5205, 5403, 5601, 5799, 5997, 6195,
6392, 6589, 6786, 6982, 7179, 7375, 7571, 7766,
7961, 8156, 8351, 8545, 8739, 8932, 9126, 9319,

. . . lots more entries . . .

-14009, -13827, -13645, -13462, -13278, -13094, -12909, -12724,
-12539, -12353, -12166, -11980, -11792, -11604, -11416, -11227,
-11038, -10849, -10659, -10469, -10278, -10087, -9895, -9703,
-9511, -9319, -9126, -8932, -8739, -8545, -8351, -8156,
-7961, -7766, -7571, -7375, -7179, -6982, -6786, -6589,
-6392, -6195, -5997, -5799, -5601, -5403, -5205, -5006,
-4807, -4608, -4409, -4210, -4011, -3811, -3611, -3411,
-3211, -3011, -2811, -2610, -2410, -2209, -2009, -1808,
-1607, -1406, -1206, -1005, -804, -603, -402, -201,
};
```

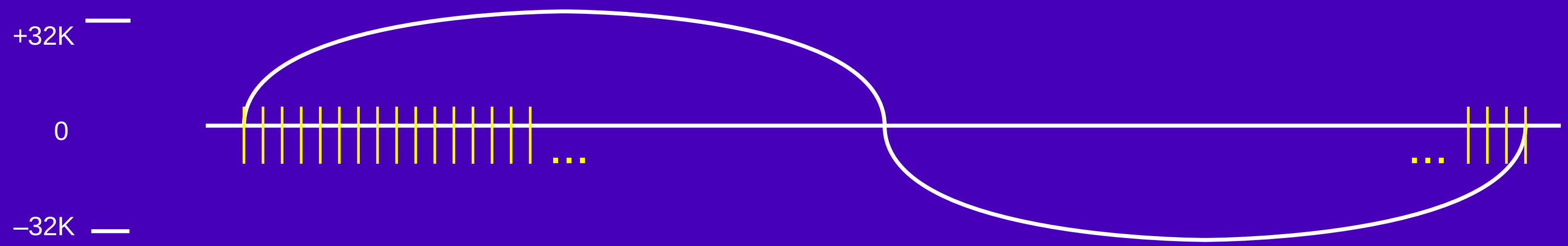


Sine wave output

- Try using the wave table to generate a signal using the DAC output.
  - The wave table contains values between -32767 and +32767, but the DAC can, at best, output values between 0 and 4095.
    - It is important to leave the sine wave in a range centered on zero so that we can later *add* multiple samples together.
    - To output to the DAC take each sample, divide it by 16, and add 2048.
  - A single traversal of the entire wave table will result in a single cycle of a sine wave.
  - Repeated traversal will show a continuous sine wave.

Determining frequency: simple

- Consider a single cycle of a sine wave with 100K samples, used with a DAC sample frequency of 100K/sec.



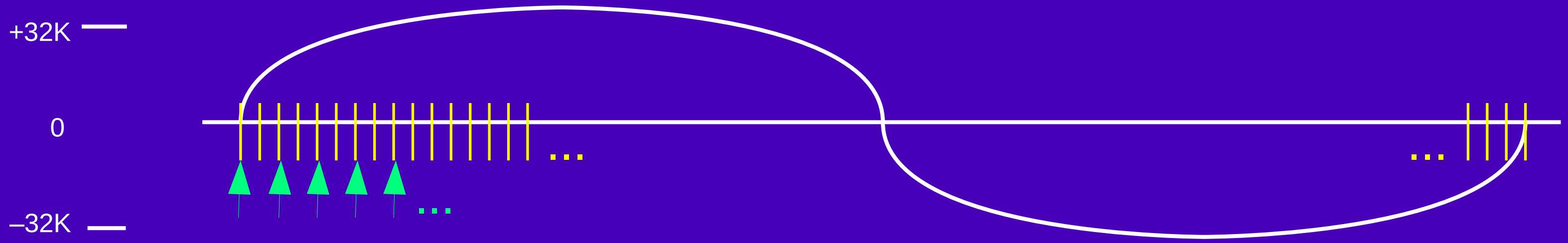
- This would produce a 1Hz output.
  - Setting the DAC rate faster would produce a higher frequency.
  - Setting the DAC rate slower would produce a lower frequency.

Limitations

- Adjusting the DAC rate allows very crude refinements to the output frequency.
  - To get a 2Hz wave, set the DAC rate to 200Ksamples/sec.
  - To get a 3Hz wave, set the DAC rate to 300Ksamples/sec.
  - There is no way to use this scheme to produce a 400Hz output signal.
    - Because there’s a limit to how fast we can drive the DAC.

Determining frequency: step size

- What if, instead of taking every sample, we take every other sample of the wave table with a 100K-entry wavetable and a 100Ksample/sec DAC rate?



- This would still produce a sine wave, but
  - Now the frequency would be 2Hz.
  - Taking every S^{th} step produces a wave of S Hz.

Generalized frequency calculation

- With a DAC rate of F_{DAC}, a single-cycle wavetable of N samples, and a step size of S, a general formula for the frequency, f, of the signal output by the DAC is:

$$f = S * F_{DAC} / N$$

Example for a DAC ISR

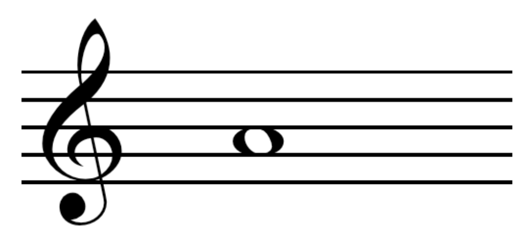
- Construct an ISR to output a signal using a step size like this. The ‘offset’ variable keeps track of the last sample output from the wavetable.

```
const short int wavetable[N] = { … };

int offset = 0;
int step = 440;

void ISR(void) {
    offset += step;
    if (offset >= N)  // If we go past the end of the array,
        offset -= N;  // wrap around with same offset as overshoot.
    int sample = wavetable[offset];    // get a sample
    sample = sample / 16 + 2048;      // adjust for the DAC range
    DAC->DHR12R1 = sample;
    DAC->SWTRIGR |= DAC_SWTRIGR_SWTRIG1; // trigger DAC
}
```

Limitations of stepping

- It's still easy to produce an output frequency with an integer multiple of 1Hz.
  - Easy to produce 440 Hz output.
  - Still no way to get 466.164 Hz with this scheme.
    - Why would we want 466.164 Hz?
  - We'd like to be able to step by *fractional* amounts.

Musical notes



- The “standard” frequency for the ‘A’ above middle C is 440 Hz.
- Each octave represents a doubling of frequency, so:
  - The ‘A’ below middle C has a frequency 220 Hz.
  - The ‘A’ two octaves above middle C has a frequency 880 Hz.
- What about the notes in between?
  - There are 12 notes in an octave.
  - Each note is then $2^{1/12}$ (about 1.05946) times higher than the previous one.
  - The A# above middle C is then about 440 * 1.05946 = 466.164 Hz.
  - The B above middle C is then about 440 * 1.05946 ^ 2 = 493.883 Hz.
  - Middle C is 220 * 1.05946 ^ 3 = 261.626 Hz (three steps above 220).
  - See page 520 of your text for a handy table.

More limitations of stepping

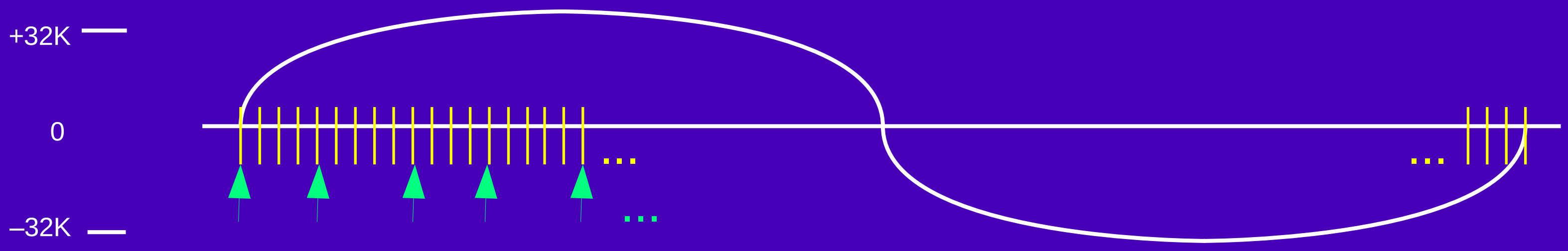
- Difficult to use a table of 100K samples in 256K of Flash ROM.
  - You can do this by storing only ¼ of the wavetable and then flipping it around to produce the other ¾ portions.
    - Your textbook has an example of this on pp 512 – 515.
- Using a step size near the sample size is not going to work.
  - If S == N/2, you would always output zeros.
  - If S == N/4, you would output a triangle wave.
  - S should probably be less than N/20 to produce anything reasonable.
    - Only possible when we have a large N.

Stepping with fixed-point math

- It's tempting to use a floating-point step size and offset to refer to wavetable samples.
  - Our microcontroller has no floating-point math hardware, so this can be very slow. Avoid!
- We can use fixed-point arithmetic to do it (almost) as quickly as integers.

Steps still work with fractions

- If we make fractional steps, we just round down to the next lower integer to get the offset into the wavetable array. E.g., a step of 4.5 would look like:



- Practically, that means taking steps of 4, 5, 4, 5, and so on.
  - This might not be quite as clean of a sine wave, but you won't notice.
  - Other fractional amounts, like 466.164, would work just as well.

Benefit of fractional stepping

- If we can have fractional stepping, we can also use a smaller wave table size. The formula still works. E.g., if N = 1000, F_{DAC} = 100 k/sec, S=4.66164

  $f = S * F_{DAC} / N = 4.66164 * 100000 / 1000 = 466.164$ Hz
- To get the step size for a particular frequency:

  $S = f * N / F_{DAC}$

  $S = 466.164 * 1000 / 100000 = 4.66164$

Implementing fractional stepping

- When using fixed-point arithmetic, you generally choose what part of an integer you want to be the whole number and use the other part as a fraction.
  - Let’s use a 32-bit integer to represent a 16-bit whole number and a 16-bit fraction. (The nomenclature for this fixed-point format is “Q16.16”.)
  - To encode a fixed-point number, we specify it with a floating-point number multiplied by the amount we shift for the fractional part. E.g., to encode 1.5 in Q16.16, we say:
    - int step = 1.5 * (1<<16); // same as 1.5 * 65536 = 65536 + 32768
    - // in binary, this is 0000 0000 0000 0001 . 1000 0000 0000 0000

Q16.16 examples

- We can imagine there's a "binary point" between the whole part of the number and the fractional part of the 32-bit number:

```
2.5 =     0000000000000010.1000000000000000
4.75 =    0000000000000100.1100000000000000
466.164 = 0000000110111110.0010100111111011
```

- To get the whole part, we just shift it right by 16.

Example ISR with fractional steps

```
// Assume N = 1000 and the DAC rate is 100K/sec...

const short int wavetable[N] = { … };

int offset = 0;
int step = 493.883 * N / RATE * (1<<16); // B above middle C (493.883 Hz)

void ISR(void) {
    offset += step;
    if ((offset>>16) >= N) // If we go past the end of the array,
        offset -= N<<16;  // wrap around with same offset as overshoot.
    int sample = wavetable[offset>>16]; // get a sample
    sample = sample / 16 + 2048;        // adjust for the DAC range
    DAC->DHR12R1 = sample;
    DAC->SWTRIGR |= DAC_SWTRIGR_SWTRIG1; // trigger DAC
}
```

Mixing notes

- Everything our ears hear is the result of constructive and destructive interference between multiple frequency sources.
  - Our ears are insensitive to phase of the frequencies, therefore it's easy to mix multiple frequencies together.
  - We can take wavetable samples and <u>ADD</u> them.
    - Do this by maintaining separate offsets and steps for each note.
    - By stepping through the same wavetable at different rates, you effectively produce two different notes.
  - This works as long as wavetable samples are centered on zero.

Clipping

- When adding two different audio sources, sometimes the highs or lows of samples will occur at the same time.
  - This will be a larger value than we can produce with the DAC.
  - We don’t want to write a value too large (or negative) to the DAC holding register because it will not represent an audio level that makes sense. e.g., 4099 == (4096 + 3), when written to the DAC, would be output as a 3.
  - We cut our output amplitude in half by dividing by 32 instead of 16.
  - And we ‘clip’ the values that are too high or too low:

```
if (sample > 4095)
    sample = 4095;
else if (sample < 0)
    sample = 0;
```

ISR with multiple notes

```
// Assume N = 1000 and the DAC rate is 100K/sec...
const short int wavetable[N] = { … };

int offset1 = 0;
int offset2 = 0;
int step1 = 261.626 * N / RATE * (1<<16); // Middle ‘C’ (261.626 Hz)
int step2 = 329.628 * N / RATE * (1<<16); // The ‘E’ above middle ‘C’ (329.628 Hz)

void ISR(void) {
     offset1 += step1;
     if ((offset1>>16) >= N) // If we go past the end of the array,
          offset1 -= N<<16;  // wrap around with same offset as overshoot.
     offset2 += step2;
     if ((offset2>>16) >= N) // If we go past the end of the array,
          offset2 -= N<<16;  // wrap around with same offset as overshoot.
     int sample = 0;
     sample += wavetable[offset1>>16]; // get sample for tone #1
     sample += wavetable[offset2>>16]; // get sample for tone #2
     sample = sample / 32 + 2048;      // adjust for the DAC range
     if (sample > 4095) sample = 4095; // clip
     else if (sample < 0) sample = 0;  // clip
     DAC->DHR12R1 = sample;
     DAC->SWTRIGR |= DAC_SWTRIGR_SWTRIG1; // trigger DAC
}
```

Playing many notes

- Use an array of steps for all possible notes.
- Use an array of offsets for all possible notes.
- Still divide the cumulative amplitude by 32, and still clip it.
  - Probably will not have three or more notes coinciding with a high amplitude, but watch out to make sure.
- When a note starts, set its offset to zero and mix it into the sample on each ISR.
- When a note ends, mark it so it is not mixed in with the sample.

ISR pseudocode

```
char pressed[90] = { 0 }; // which piano keys are pressed?
int offset[90] = { 0 };
int step[90] = { 16.352 * N / RATE * (1<<16),  // Low ‘C’ (16.352 Hz)
                 …
                 7902.133 * N / RATE * (1<<16), // High ‘B’ (7902.133 Hz)
               }

void ISR(void) {
     int x;
     int sample = 0;
     for(x=0; x<90; x++) {
          if (pressed[x]) {
               offset[x] += step[x];
               if (offset[x] >= N<<16)
                    offset[x] -= N<<16;
               sample += wavetable[offset[x]>>16];
          }
     }
     sample = sample / 32 + 2048;      // adjust for the DAC range
     if (sample > 4095) sample = 4095; // clip
     else if (sample < 0) sample = 0;  // clip
     DAC->DHR12R1 = sample;
     DAC->SWTRIGR |= DAC_SWTRIGR_SWTRIG1; // trigger DAC
}
```

You probably cannot do all this at a 100kHz DAC rate.

Small suggestion

- When producing multiple notes, the time taken to run the ISR will be different depending on:
  - how many notes are generated
  - other calculations you might want to add
- Since the DAC trigger happens at the end of the ISR, it could be slightly delayed sometimes which will lead to an inconsistent sample speed.
- Solution: Put the DAC trigger at the very beginning of the ISR, and store the result for the NEXT trigger at the end of the ISR.

ISR pseudocode (trigger at start)

```
char pressed[90] = { 0 }; // which piano keys are pressed?
int offset[90] = { 0 };
int step[90] = { 16.352 * N / RATE * (1<<16),  // Low ‘C’ (16.352 Hz)
                 …
                 7902.133 * N / RATE * (1<<16), // High ‘B’ (7902.133 Hz)
               }

void ISR(void) {
     int x;
     int sample = 0;
     DAC->SWTRIGR |= DAC_SWTRIGR_SWTRIG1; // You should trigger the DAC here this time...
     for(x=0; x<90; x++) {                // ...because we have no idea how long this will take.
          if (pressed[x]) {
               offset[x] += step[x];
               if (offset[x] >= N<<16)
                    offset[x] -= N<<16;
               sample += wavetable[offset[x]>>16];
          }
     }
     sample = sample / 32 + 2048;       // adjust for the DAC range
     if (sample > 4095) sample = 4095; // clip
     else if (sample < 0) sample = 0;  // clip
     DAC->DHR12R1 = sample; // only store to the DAC here this time.
}
```

Multiple voices

- If you can store a wave table in 1K samples (2K bytes), you will have room for multiple wave tables.
  - Not all wave tables need to have a sine wave:
    - Make pointier wave shapes for sharper sounds.
      - Like a sawtooth wave: wavetable[x] = x * 65535.0 / N - 32768
    - Add small ‘harmonics’ to a sine wave for a richer sound.
      - e.g., Let each sample be sin(x) + sin(2*x) / 4 + sin(3*x) / 8
    - Record your own sound by capturing it with the ADC.
  - This lets you assign multiple ‘voices’ per note if you want to.
  - By adding and averaging the samples of multiple wave tables, you can produce many types of sounds. (e.g., what is the average of a sine wave and a square wave if they have the same frequency? How about a sine wave and a sawtooth wave?)
  - Maybe you want to have some wave tables with more samples than other wave tables?

ADSR

- Your textbook describes an ADSR (attack / decay / sustain / release) model for a synthesizer. (pages 521 – 523)
  - If you keep track of the time at which you ‘press’ a note, you can modulate its contribution to a combined sample with its ADSR curve.
- Also easy to create vibrato and echo effects this way.

Pitch Bending

- One of the most interesting things you can do with sound synthesis is pitch ‘bending’.
  - Dynamically change the step size of each note to correspond to a different frequency.
  - By changing each frequency to that of the frequency below it, you can shift your sound output by a half step.
  - By *<u>multiplying</u>* each step size by $(1 + \Delta)$, you can shift your sound output by a variable offset.
    - Determine that multiplier by an ADC sample of a sliding potentiometer will let you have trombone-like glissando effect.