5/3/25

Assignment - 2 : Unit 4

"Laplace Transform"

Q-1) Find Laplace Transform of $t^2 - e^{-2t}$.

$\rightarrow L\{t^2 - e^{-2t}\} = L\{t^2\} - L\{e^{-2t}\}$

$= 2!/s^{2+1} - 1/s+2$

$= \dfrac{2}{s^3} - \dfrac{1}{s+2}$

Q-2) Find Laplace Transform of $2 + e^{-3t} + \sin 2t$

$\rightarrow L\{2 + e^{-3t} + \sin 2t\} = 2L\{1\} + L\{e^{-3t}\} + L\{\sin 2t\}$

$= \dfrac{2}{s} + \dfrac{1}{s+3} + \dfrac{2}{s^2+2^2}$

$= \dfrac{2}{s} + \dfrac{1}{s+3} + \dfrac{2}{s^2+4}$

Q-3) Find $L[2e^{3t} + 5t^2 - 4\sin 2t + 2\cos 2t]$

$\rightarrow 2L\{e^{3t}\} + 5L\{t^2\} - 4L\{\sin 2t\} + 2L\{\cos 2t\}$

$\rightarrow \dfrac{2 \times 1}{s-3} + \dfrac{5 \cdot 2!}{s^{2+1}} - \dfrac{4 \cdot 2}{s^2+2^2} + \dfrac{2 \cdot s}{s^2+2^2}$

$\rightarrow \frac{2}{s-3} + \frac{10}{s^3} + \frac{8}{s^2+4} + \frac{2s}{s^2+4}$

$\rightarrow \frac{2}{s-3} + \frac{10}{s^3} - \frac{8+2s}{s^2+4}$

Q-4) Find $L\{e^{2t} + 4t^3 + 3\cos 2t - 2\sin 3t\}$

$\rightarrow L\{e^{2t}\} + 4L\{t^3\} + 3L\{\cos 2t\} - 2L\{\sin 3t\}$

$\rightarrow \frac{1}{s-2} + \frac{4 \times 3!}{s^{3+1}} + \frac{3 \cdot s}{s^2+2^2} - \frac{2 \times 3}{s^2+3^2}$

$\rightarrow \frac{1}{s-2} + \frac{24}{s^4} + \frac{3s}{s^2+4} - \frac{6}{s^2+9}$

Q-5) Find $L\{\sin^2 3t\}$

$\rightarrow$ We know that : $\sin^2\theta = \frac{1-\cos 2\theta}{2}$, $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\rightarrow \sin^2 3t = \frac{1-\cos 2(3t)}{2}$

$\rightarrow \sin^2 3t = \frac{1-\cos 6t}{2}$

$\longrightarrow$

$\rightarrow L\{\sin^2 3t\} = L\{\frac{1-\cos 6t}{2}\}$

$= L\{\frac{1}{2} - \frac{\cos 6t}{2}\}$

$= L\{\frac{1}{2}\} - \frac{1}{2} L\{\cos 6t\}$

$= \frac{1}{2} L\{1\} - \frac{1}{2} \cdot \frac{s}{s^2+6^2}$

$= \frac{1}{2} \cdot \frac{1}{s} - \frac{1}{2} \cdot \frac{s}{s^2+36}$

$= \frac{1}{2s} - \frac{s}{2(s^2+36)}$

Q-6) Find $L\{4\sin 3t - 3\cos 2t\}$

$\rightarrow 4L\{\sin 3t\} - 3L\{\cos 2t\}$

$\rightarrow \frac{4 \cdot 3}{s^2+3^2} - \frac{3 \cdot s}{s^2+2^2}$

$\rightarrow \frac{12}{s^2+9} - \frac{3s}{s^2+4}$

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Q-7) Find $L\{3e^{2t} - 2\sin 3t\}$

$\rightarrow 3L\{e^{2t}\} - 2L\{\sin 3t\}$

$\rightarrow 3 \cdot \frac{1}{s-2} - 2 \cdot \frac{3}{s^2+3^2}$

$\rightarrow \frac{3}{s-2} - \frac{6}{s^2+9}$

Q-8) Find $L\{2\sin 4t - 5\cos 2t\}$

$\rightarrow 2L\{\sin 4t\} - 5L\{\cos 2t\}$

$\rightarrow \frac{2 \cdot 4}{s^2+4^2} - \frac{5 \cdot s}{s^2+2^2}$

$\rightarrow \frac{8}{s^2+16} - \frac{5s}{s^2+4}$

Q-9) Find $L\{(e^t - 1)^2\}$

$\rightarrow L\{(e^t-1)^2\} = L\{e^{2t} - 2 \cdot e^t \cdot 1 + 1^2\}$

$= L\{e^{2t}\} - 2L\{e^t\} + L\{1\}$

$\rightarrow \frac{1}{s-2} - \frac{2}{s-1} + \frac{1}{s}$

$\rightarrow \frac{1}{s-2} - \frac{2}{s-1} + \frac{1}{s}$

Q-10) Find $L\{\sin 2t \cdot \sin 3t\}$

$\rightarrow$ We know that,

$\sin A \cdot \sin B = \frac{1}{2}[\cos(A+B) - \cos(A-B)]$

$\rightarrow A = 2t, B = 3t$

$\rightarrow \sin 2t \cdot \sin 3t = \frac{1}{2}[\cos(2t+3t) - \cos(2t-3t)]$

$= \frac{1}{2}[\cos(5t) - \cos(-t)]$

$\rightarrow L\{\sin 2t \cdot \sin 3t\} = L\{\frac{1}{2}[\cos 5t - \cos t]\}$

$= \frac{1}{2} L\{\cos 5t\} - \frac{1}{2} L\{\cos t\}$

$= \frac{1}{2} \cdot \frac{s}{s^2+5^2} - \frac{1}{2} \cdot \frac{s}{s^2+1^2}$

$$= \frac{s}{2(s^2+25)} - \frac{s}{2(s^2+1)}$$

Q-11) Find $L\{3\sinh 7t - \cos 3t\}$

$\rightarrow 3L\{\sinh 7t\} - L\{\cos 3t\}$

$\rightarrow \frac{3 \cdot 7}{s^2 - 7^2} - \frac{s}{s^2+3^2}$

$\rightarrow \frac{21}{s^2-49} - \frac{s}{s^2+9}$

Q-12) Find $L\{e^{4t} + 5t^2 - 3\sin 4t + 2\cosh 2t\}$.

$\rightarrow L\{e^{4t}\} + 5L\{t^2\} - 3L\{\sin 4t\} + 2L\{\cosh 2t\}$

$\rightarrow \frac{1}{s-4} + \frac{5 \cdot 2!}{s^{2+1}} - \frac{3 \cdot 4}{s^2+4^2} + \frac{2 \cdot s}{s^2-2^2}$

$\rightarrow \frac{1}{s-4} + \frac{10}{s^3} - \frac{12}{s^2+16} + \frac{2s}{s^2-4}$

Q-13) Find $L\{2t^2 + \cos 4t + e^{-2t}\}$

$\rightarrow 2L\{t^2\} + L\{\cos 4t\} + L\{e^{-2t}\}$

$\rightarrow \frac{2.2!}{s^{2+1}} + \frac{s}{s^2+4^2} + \frac{1}{s+2}$

$\rightarrow \frac{4}{s^3} + \frac{s}{s^2+16} + \frac{1}{s+2}$

Q-14) Find $L\{4e^{5t} - 3\sin 4t + 5\}$

$\rightarrow 4L\{e^{5t}\} - 3L\{\sin 4t + L\{5\}$

$\rightarrow \frac{4.1}{s-5} - \frac{3.4}{s^2+4^2} + 5L\{1\}$

$\rightarrow \frac{4}{s-5} - \frac{12}{s^2+16} + \frac{5}{s}$

Based on First Shifting Theorem:

Q-1) Find $L\{e^{-t}.\sin 3t\}$

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Q-1) $\to L\{f(t)\} = L\{\sin 3t\} = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

$\to$ Replacing s by s+1

$\to L\{e^{-t}.\sin 3t\} = \frac{3}{(s+1)^2+9} = \frac{3}{s^2+2s+1+9}$

$= \frac{3}{s^2+2s+10}$

Q-2) Find $L\{t^2.e^{-3t}\}$

$\to L\{f(t)\} = L\{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3} = F(s)$

$\to$ Replace s by s+3

$\to L\{t^2.e^{-3t}\} = \frac{2}{(s+3)^3} = \frac{2}{(s+3)^3}$

Q-3) Find $L\{e^{-t}.\cos 2t\}$

$\to L\{f(t)\} = L\{\cos 2t\} = \frac{s}{s^2+2^2} = \frac{s}{s^2+4} = F(s)$

→ Replace s by s + 1

→ $L\{e^{-t} \cdot \cos 2t\} = \frac{(s+1)}{(s+1)^2+4} = \frac{s+1}{s+2s+5}$

Q-4) Find $L\{t^3 \cdot e^{2t} + t^2 \cdot e^{3t}\}$

→ $L\{e^{2t} \cdot t^3 + e^{3t} \cdot t^2\}$

→ $L\{t^3\} = \frac{3!}{s^{3+1}} = \frac{6}{s^4}$

→ $L\{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

→ Replacing s by s-2 and s-3 respectively

→ $L\{t^3 \cdot e^{2t} + t^2 \cdot e^{3t}\} = L\{e^{2t} \cdot t^3\} + L\{e^{3t} \cdot t^2\}$

$= \frac{6}{(s-2)^4} + \frac{2}{(s-3)^3}$

Q-5) Find $L\{e^{6t}(\cosh 2t + \cosh 4t)\}$

$\rightarrow L\{e^{6t}.\cosh 2t + e^{6t}.\cosh 4t\}$

$\rightarrow L\{\cosh 2t\} = \frac{s}{s^2-2^2} = \frac{s}{s^2-4}$

$\rightarrow L\{\cosh 4t\} = \frac{s}{s^2-4^2} = \frac{s}{s^2-16}$

Replacing s by s-6 respectively,

$\rightarrow L\{e^{6t}.\cosh 2t + e^{6t}.\cosh 4t\}$

$= L\{e^{6t}.\cosh 2t\} + L\{e^{6t}-\cosh 4t\}$

$= \frac{(s-6)}{(s-6)^2-4} + \frac{(s-6)}{(s-6)^2-16}$

$= \frac{(s-6)}{s^2-12s+36-4} + \frac{(s-6)}{s^2-12s+36-16}$

$= \frac{s-6}{s^2-12s+32} + \frac{s-6}{s^2-12s+20}$

Q-6) Find $L\{e^{-2t}(\sin 2t + \sinh 4t)\}$

$\rightarrow L\{e^{-2t} \cdot \sinh 2t + e^{-2t} \cdot \sinh 4t\}$

$\rightarrow L\{\sinh 2t\} = \frac{2}{s^2-2^2} = \frac{2}{s^2-4}$

$\rightarrow L\{\sinh 4t\} = \frac{4}{s^2-4^2} = \frac{4}{s^2-16}$

$\rightarrow$ Replace s by $s+2$,

$\rightarrow L\{e^{-2t} \cdot \sinh 2t + e^{-2t} - \sinh 4t\}$

$= L\{e^{-2t} \cdot \sinh 2t\} + L\{e^{-2t} \sinh 4t\}$

$= \frac{2}{(s+2)^2-4} + \frac{4}{(s+2)^2-16}$

$= \frac{2}{s^2+4s+4-4} + \frac{4}{s^2+4s+4-16}$

$= \frac{2}{s^2+4s} + \frac{4}{s^2+4s-12}$

Q-7) Find $L\{e^{3t} \cdot t^2\}$

$\rightarrow L\{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

$\rightarrow L\{e^{3t} \cdot t^2\} = \frac{2}{(s-3)^3}$ .... replace s by $s-3$

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Q-8) Find $L\{t.\cos at\}$

→ Here $f(t) = \cos at$

$\therefore L(f(t)) = L(\cos at)$

$= s/s^2+a^2$

→ Using second shifting property,

if $L\{t^n.f(t)\} = (-1)^n.\frac{d^n}{ds^n}L\{f(t)\}$

→ $L\{t.\cos at\} = -1\frac{d}{ds}\left(\frac{s}{s^2+a^2}\right)$

→ $\frac{d}{ds}\left(\frac{s}{s^2+a^2}\right) = \frac{(s^2+a^2)\frac{d}{ds}(s) - s\frac{d}{ds}(s^2+a^2)}{(s^2+a^2)^2}$

$= \frac{s^2+a^2-2s^2}{(s^2+a^2)^2}$

now adding the (-1) to the answer,

→ $(-1)\frac{s^2+a^2-2s^2}{(s^2+a^2)^2}$

Q-9) Find $L\{t.\cos 3t\}$

→ Here $f(t) = \cos 3t$

$L\{f(t)\} = L(\cos 3t)$

$= \frac{s}{s^2+3^2} = \frac{s}{s^2+9}$

→ We know using second shifting property →

$(-1)^n . \frac{d^n}{ds} L\{f(t)\}$

→ $\frac{d}{ds}\left(\frac{s}{s^2+9}\right) = \frac{(s^2+9)\frac{d}{ds}(s) - s.\frac{d}{ds}(s^2+9)}{(s^2+9)^2}$

$= \frac{s^2+9-2s^2}{(s^2+9)^2}$

adding the (-1) to the answer,

→ $(-1)\left(\frac{s^2+9-2s^2}{(s^2+9)^2}\right)$

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Q-10) Find $L\{t \cdot \sin 5t\}$

$\rightarrow L\{\sin 5t\} = 5/s^2+5^2 = 5/s^2+25 = F(s)$

$\rightarrow L\{t^n \cdot f(t)\} = (-1)^n \cdot \frac{d^n}{ds^n} \cdot F(s)$

$\rightarrow L\{t^1 \cdot \sin 5t\} = (-1)^1 \cdot \frac{d}{ds} \cdot \left[\frac{5}{s^2+25}\right]$

$\rightarrow -5 \cdot \frac{d}{ds}\left[\frac{1}{s^2+25}\right]$

$\rightarrow -5 \cdot \left[\frac{-1}{(s^2+25)^2}\right] \cdot \frac{d}{ds}(s^2+25)$

$\rightarrow \frac{5}{(s^2+25)^2} \cdot 2s$

$\rightarrow \frac{10s}{(s^2+25)^2}$

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Q-11) Find $L\{e^{3t}(t^2+t)\}$

$\rightarrow L\{e^{3t}.t^2+e^{3t}.t\}$

$\rightarrow L\{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

$\rightarrow L\{t\} = \frac{1}{s^2}$

$\rightarrow$ Replace s with $s-3$, we get-

$\rightarrow L\{e^{3t}.t^2+e^{3t}.t\} = L\{e^{3t}.t^2\} + L\{e^{3t}.t\}$

$\rightarrow \frac{2}{(s-3)^3} + \frac{1}{(s-3)^2}$

Q-12) Find $L\{e^{-3t}.\cos 4t\}$

$\rightarrow L\{\cos 4t\} = \frac{s}{s^2+4^2} = \frac{s}{s^2+16} = F(s)$

$\rightarrow$ Replace s by $s+3$

$\rightarrow L\{e^{-3t}.\cos 4t\} = \frac{(s+3)}{(s+3)^2+16}$

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$\rightarrow \dfrac{(s+3)}{s^2+6s+25}$

Q-13) Find $L\{e^{-t}.\sin 5t\}$

$\rightarrow L\{\sin 5t\} = \dfrac{5}{s^2+5^2} = \dfrac{5}{s^2+25} = F(s)$

$\rightarrow$ Replacing s by s+1

$\rightarrow L\{e^{-t}.\sin 5t\} = \dfrac{5}{(s+1)^2+25}$

$= \dfrac{5}{s^2+2s+26}$

Q-14) Find $L\{e^{-3t}.\cos 7t\}$

$\rightarrow L\{\cos 7t\} = \dfrac{s}{s^2+7^2} = \dfrac{s}{s^2+49} = F(s)$

$\rightarrow$ Replace s by s+3

$\rightarrow L\{e^{-3t}.\cos 7t\} = \dfrac{(s+3)}{(s+3)^2+49} = \dfrac{(s+3)}{s^2+6s+58}$