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1. Find the equation of parabola $y = A + Bx + Cx^2$ that passes through 3 points $(1,1)$, $(2,-1)$ & $(3,1)$ using gaussian elimination

$$y = A + Bx + Cx^2$$

Substituting the points $(1,1)$, $(2,1)$ and $(3,1)$

$$1 = A + B(1) + C(1)^2$$
$$A + B + C = 1$$

$$-1 = A + (2)B + C(2)^2$$
$$A + 2B + 4C = -1$$

$$1 = A + B(3) + C(3^2)$$
$$A + 3B + 9C = 1$$

Gaussial elimination

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad x = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$[A\ b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - R_1$

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$$[A \ b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

$2C = 4$ $\qquad$ $B + 3C = -2$

$C = 2$ $\qquad$ $B = -8$

$A + B + C = 1$

$A = 7$

$$y = 7 + (-8)x + 2x^2$$

2) $$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

Find the LU decomposition for the matrix

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$$E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad E_{31} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_{41} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -5 & 0 & 0 & 1 \end{bmatrix}$$

$$A' = E_{41} E_{31} E_{21} A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad E_{42} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

$$A'' = E_{42} E_{32} A' = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

$$E_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{bmatrix}$$

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$$U = E_{43} A'' = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

$$E_{43} E_{42} E_{32} E_{41} E_{31} E_{21} A = U$$

$$A = E_{21}^{-1} E_{31}^{-1} E_{41}^{-1} E_{32}^{-1} E_{42}^{-1} E_{43}^{-1} U.$$

$$= L U$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

3 $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$

i) $T(1,0,0) = (1,0,1)$

$T(0,1,0) = (2,1,1)$

$T(0,0,1) = (-1,1,-2)$

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

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ii) Column space $C(T)$

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$R3 \rightarrow R3 - R1$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$U = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Corresponding columns in T are
$(1,0,1)$ $(2,1,1)$

Basis of $C(T) = \{(1,0,1), (2,1,1)\}$

Basis of $C(T^T) = \{(1,2,-1)\ (2,1,1)\}$

Null space

$Ux = 0$ $\quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$y + z = 0$$
$$y = -z$$

$$x + 2y - z = 0$$
$$x - 2z - z = 0$$
$$x = 3z$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3z \\ -z \\ z \end{bmatrix} = z \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

Based on pivot columns z is free var.

Basis of $N(T) = \{(3, -1, 0)\}$

def Nullspace of $N(T^T)$

$$T^T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

Find U $\quad R_3 \to R_3 + R_1$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$.

$$U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y - z = 0$$
$$y = z$$
$$x + z = 0$$
$$x = -z$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -z \\ z \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Basis of $N(T^T) = \{(-1, 1, 1)\}$.

iii) $$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$Tx = \lambda x$$

$$(T - I\lambda)x = 0$$
$$|T - I\lambda| = 0 \quad \text{for eigen values}$$

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix} = 0$$

$$\lambda(\lambda^2 - 3) = 0$$
$$\lambda = 0, \sqrt{3}, -\sqrt{3}$$

$\lambda = 0$

$$(T - I\lambda)x = 0$$

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$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

x is $N(T)$

$x = 3, -1, 1$

$\lambda = \sqrt{3}$

$$\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(1-\sqrt{3})y + z = 0$$

$$y = \frac{-z}{1-\sqrt{3}} = \frac{1+\sqrt{3}}{2} z$$

$$(1-\sqrt{3})x + z + \sqrt{3}z - z = 0$$

$$x = \frac{-\sqrt{3}z}{1-\sqrt{3}} = \frac{\sqrt{3}(1+\sqrt{3})z}{2}$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = z \begin{bmatrix} \frac{\sqrt{3}(1+\sqrt{3})}{2} \\ 1+\sqrt{3}/3 \\ 1 \end{bmatrix}$$

Eigen Vector for $\lambda = \sqrt{3}$ $= \left(\frac{\sqrt{3}(1+\sqrt{3})}{2}, \frac{1+\sqrt{3}}{2}, 1 \right)$

$\lambda = -\sqrt{3}$

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$$\begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad Ux = 0$$

$$(1+\sqrt{3})y + z = 0 \qquad y = \frac{-z}{1+\sqrt{3}} = \frac{1-\sqrt{3}}{2} z$$

$$(1+\sqrt{3})x + 2y - z = 0$$
$$(1+\sqrt{3})x + z - \sqrt{3}z - z = 0$$

$$x = \frac{\sqrt{3}z}{1+\sqrt{3}} = \frac{\sqrt{3}(\sqrt{3}-1)}{2} z$$

Eigen vector for $(\lambda = -\sqrt{3})$ $\left(\frac{\sqrt{3}(\sqrt{3}-1)}{2}, \frac{1-\sqrt{3}}{2}, 1\right)$

iv) $$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$q_1 = \frac{a}{\|a\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$q = b - q_1 q_1^T b / \| b - q_1 q_1^T b$$

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$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \left(\sqrt{2} + \frac{1}{\sqrt{2}}\right) \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 3/2 \\ 0 \\ 3/2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix} \Big/ \sqrt{1/4+1+1/4}.$$

$$= \begin{bmatrix} 1/\sqrt{6} \\ \sqrt{2/3} \\ -1/\sqrt{6} \end{bmatrix} \qquad q_3 = \frac{c}{\|c\|}$$

$$c = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} - \left(\frac{3}{\sqrt{2}}\right) \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} - \sqrt{3}/2 \begin{bmatrix} 1/\sqrt{6} \\ \sqrt{2/3} \\ -1/\sqrt{6} \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 3/2 \\ 0 \\ 3/2 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$q_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ Because T is singular.

$$Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & \sqrt{2/3} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix}$$

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$$R = \begin{bmatrix} q_1^T a & q_1^T b & q_1^T c \\ 0 & q_2^T b & -q_2^T c \\ 0 & 0 & q_3^T c \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & \sqrt{3}/2 & \sqrt{3}/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & \sqrt{2}/3 & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & \sqrt{3}/2 & \sqrt{3}/2 \\ 0 & 0 & 0 \end{bmatrix}$$

4)

x	-4	1	2	3
y	4	6	10	8

$$y = c + dx$$

$$A = \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \qquad x = \begin{bmatrix} c \\ d \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

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$$(A^T A)^{-1} = \frac{1}{116}\begin{bmatrix} 30 & -2 \\ -2 & 4 \end{bmatrix}$$

$$\hat{x} = \frac{1}{116}\begin{bmatrix} 30 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 28 \\ 34 \end{bmatrix} = \begin{bmatrix} 193/29 \\ 20/29 \end{bmatrix}$$

$$c = \frac{193}{29} \qquad d = \frac{20}{29}$$

$$y = \frac{193}{29} + \frac{20}{29}x \;//$$

4 $x_1 + x_2 + 3x_3 + 4x_5 = 0$

Since there is x_5 the vector space $\in R^5$

A general vector

let $x_1 = s$ $\quad x_3 = t$, $x_5 = u$, $x_4 = v$

$x_2 = -s - 3t - 4u$

vector $(s, -s - 3t - 4u, t, u, v)$

$$= s\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + u\begin{bmatrix} 0 \\ -4 \\ 0 \\ 0 \\ 1 \end{bmatrix} + v\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & -3 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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projection onto plane = projection onto column space

= P.

projection onto orthogonal complement = projection onto left nullspace.

= Q.

$P + Q = I$

$P = I - Q$

$N(A^T)$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -3 & 1 & 0 & 0 \\ 0 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 4/3\, R_2$

$$U = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -3 & 1 & 0 & 0 \\ 0 & 0 & -4/3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$U_x = 0$

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$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -3 & 1 & 0 & 0 \\ 0 & 0 & -4/3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$s = 0$$

$$-4/3\, r + t = 0$$
$$r = 3/4\, t$$

$$-3q + r = 0$$
$$-3q + 3/4\, t = 0$$

$$q = 1/4\, t$$

$$p - q = 0$$
$$p = q = 1/4\, t$$

$$x = \begin{bmatrix} p \\ q \\ r \\ s \\ t \end{bmatrix} = \begin{bmatrix} 1/4\, t \\ 1/4\, t \\ 3/4\, t \\ 0 \\ t \end{bmatrix} = t \begin{bmatrix} 1/4 \\ 1/4 \\ 3/4 \\ 0 \\ 1 \end{bmatrix}$$

Basis of $N(A^T) = \{(1/4, 1/4, 3/4, 0, 1)\} = q$.

Projection matrix Q onto $N(A^T) = \dfrac{q q^T}{q^T q}$

$$= \begin{bmatrix} 1/4 \\ 1/4 \\ 3/4 \\ 0 \end{bmatrix} \begin{bmatrix} 1/4 & 1/4 & 3/4 & 0 & 1 \end{bmatrix} = \times \frac{1}{\frac{25}{16}}$$

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$$\begin{bmatrix} 1/27 & 1/27 & 1/9 & 0 & 4/27 \\ 1/27 & 1/27 & 1/9 & 0 & 4/27 \\ 1/9 & 1/9 & 1/3 & 0 & 4/9 \\ 0 & 0 & 0 & 0 & 0 \\ 4/27 & 4/27 & 4/9 & 0 & 16/27 \end{bmatrix}$$

$P = I - Q$

$$\begin{bmatrix} 26/27 & -1/27 & -1/9 & 0 & -4/27 \\ -1/27 & 26/27 & -1/9 & 0 & -4/27 \\ -1/9 & -1/9 & 2/3 & 0 & -4/9 \\ 0 & 0 & 0 & 1 & 0 \\ -4/27 & -4/27 & -4/9 & 0 & 11/27 \end{bmatrix}$$

6

$$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$$

i) $|a| > 0$

$a > 0$

$$\begin{bmatrix} a & 2 \\ 2 & 0 \end{bmatrix} > 0 \qquad a^2 - 4 > 0$$

$a^2 - 4 > 0 \qquad a < -2 \qquad a > -2$

$$\begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix} > 0$$

$a(a^2-4) - 2(2a-4) + 2(4-2a) > 0$

$a > -4$ //

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From above ranges

$$a \in (2, \infty)$$

$$= a(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3).$$

$$\det B = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \sum_{i=1}^{2} \sum_{j=1}^{3} a_{ij} x_i x_j$$

$a_{11} = 2$, $a_{22} = 2$ $a_{33} = 2$

$a_{12} = a_{21} = -2/2 = -1$

$a_{13} = a_{31} = 0/2 = 0$

$a_{23} = a_{32} = -2/2 = 1$

$$\therefore B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

7) $$A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$$

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$$AA^T = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix}$$

Eigenvectors

$$\begin{bmatrix} 10-\lambda & -20 & -20 \\ -20 & 40-\lambda & 40 \\ -20 & 40 & 40-\lambda \end{bmatrix} = 0$$

$$\lambda^2 (90-\lambda) = 0$$

$$\lambda = 90, \lambda = 0$$

$\lambda = 90$

$$A - 90I = \begin{bmatrix} -80 & -20 & -20 \\ -20 & -50 & 40 \\ -20 & 40 & -50 \end{bmatrix}$$

Echelon form $\begin{bmatrix} -80 & -20 & -20 \\ 0 & -45 & 45 \\ 0 & 0 & 0 \end{bmatrix}$

$Ex = 0$

$$\begin{bmatrix} -80 & -20 & -30 \\ 0 & -45 & 45 \\ 0 & 0 & 0 \end{bmatrix}$$

$Ex = 0$

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$$\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$$

$$AA^T - 0I = \begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix}$$

$$E = \begin{bmatrix} 10 & -20 & -20 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/\sqrt{5} \\ 2/3 & 1/\sqrt{5} & 0 \\ 2/3 & 0 & 1/\sqrt{5} \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sqrt{10} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Eigen Vectors

$$\begin{vmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{vmatrix} = 0$$

$$\lambda(\lambda - 90) = 0$$

$$\lambda = 90 \quad \lambda = 0$$

$$A^T A - 90E = \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix} \quad E = \begin{bmatrix} -9 & -27 \\ 0 & 0 \end{bmatrix}$$

$$Ex = 0.$$

$$\begin{bmatrix} -9 & -27 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$-9x - 27y = 0 \qquad x = -3y$$

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$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3y \\ y \end{bmatrix} = y\begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = y\begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$$

$$\text{Eigen Vector} = \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} \xrightarrow{\text{Norm}} \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

$$V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$

$$V^T = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} -1/3 & 2/\sqrt{5} & 2/\sqrt{5} \\ 2/3 & 1/\sqrt{5} & 0 \\ 2/3 & 0 & 1/\sqrt{5} \end{bmatrix} \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$

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