Exam Notes

Made by Leon :) *Note: Any reference numbers are to the lecture notes*

1 Revisiting FPM

Definition 1.1: Nested Sequences

A sequence $(I_n)_{n\in\mathbb{N}}$ of sets is said to be **nested** if

$$I_1 \subset I_2 \subset I_3 \subset \cdots$$

Theorem 1.1: Nested Interval Property

If (I_n) is a nested sequence of nonempty closed bounded intervals then

$$E = \bigcap_{n\in\mathbb{N}} I_n = \{x \in \mathbb{R} : x \in I_n, \forall n \in \mathbb{N}\}$$

is nonempty (i.e. it contains at least one number). Moreover if $\lambda(I_n) \to 0$, where $\lambda(I_n)$ denotes the length of interval I_n, then E contains exactly one number

Theorem 1.2: Covers

Let E be a subset of $\mathbb{R}^n$

- A **cover** of E is a collection of sets $\{I_\alpha\}_{\alpha\in A}$ such that
$$E \subseteq \bigcup_{\alpha\in A} I_\alpha$$
- An **open covering** of E is a cover such that each I_α is open, i.e.(a, b) compared to $[a, b]$
- A **finite subcover** of E is a collection of sets $(I_\alpha)_{\alpha\in A_0}$ where there exists a subset $A_0 = \{\alpha_1, \alpha_2, \dots, a_N\}$ of A such that $(I_\alpha)_{\alpha\in A_0}$ is a finite subset of $(I_\alpha)_{\alpha\in A}$ that is also a cover
- The set E is said to be **compact** iff every open covering of E has a **finite subcovering**; that is
$$E \subseteq \bigcup_{j=1}^{N} I_{aj} \quad \text{or} \quad E \subseteq I_{\alpha_1} \cup I_{a_2} \cup \cdots \cup I_{a_N}$$

Definition 1.2: Epsilon-N Convergence of Sequence

A sequence of real numbers (x_n) is said to **converge** to a real number $a \in \mathbb{R}$ iff for every $\epsilon > 0$ there is an $N \in \mathbb{N}$ such that

$$n \geq N \text{ implies } |x_n - a| < \epsilon$$

If (x_n) converges to a, we will write $\lim\limits_{n\to\infty} x_n = a$, or $x_n \to a$. The number a is called the limit of the sequence (x_n). A sequence that does not converge to some real number is said to *diverge

Definition 1.3: Cauchy Sequence

A sequence (x_n) of numbers $x_n \in \mathbb{R}$ is said to be **Cauchy** if for every $\epsilon > 0$ there is $N \in \mathbb{N}$ such that

$$|x_n - x_m| < \epsilon \quad \forall n, m \geq N$$

Theorem 1.3: Convergent Sequences are Cauchy

Let (x_n) be a sequence of real numbers. Then (x_n) is a Cauchy sequence if and only if (x_n) is a convergent sequence.
Note: This works both ways ((x_n) is a convergent seq $\implies$ Cauchy)

Definition 1.4: Subsequences

Suppose $(x_n)_{n\in\mathbb{N}}$ is a sequence. A subsequence of this sequence is a sequence of the form $(x_{n_k})_{k\in\mathbb{N}}$ where for each k there is a positive integer n_k such that

$$n_1 < n_2 < \cdots < n_k < n_{k+1} < \cdots$$

Thus, $(x_n)_{n\in\mathbb{N}}$ is just a selection of some (possibly all) of the x_n's taken in order

Theorem 1.5: Bolzano-Weierstrass

Every bounded sequence of real numbers has a convergent subsequence

Definition 1.5: Limit Superior and Inferior

If (x_n) is a bounded sequence of real numbers we denote by

$$\limsup_{n\to\infty} x_n = \lim_{n\to\infty}\left(\sup_{k\geq n} x_k\right), \qquad \liminf_{n\to\infty} x_n = \lim_{n\to\infty}\left(\inf_{k\geq n} x_k\right)$$

Note: These are only defined for bounded sequences

- If (x_n) is not bounded from above then we write $\limsup_{n\to\infty} x_n = +\infty$
- If (x_n) is not bounded from below then we write $\liminf_{n\to\infty} x_n = +\infty$

Theorem 1.6: Convergence from Limsup and Liminf

A sequence (x_n) of real numbers is convergent if and only if $\limsup_{n\to\infty} x_n$ and $\liminf_{n\to\infty} x_n$ are real numbers and

$$\limsup_{n\to\infty} x_n = \liminf_{n\to\infty} x_n$$

Definition 1.6: Convergent Infinite Series

Let $S = \sum_{k=1}^{\infty} a_k$ be an infinite series a_k. For each $n \in \mathbb{N}$, the partial sum of S of order n is defined by

$$s_n = \sum_{k=1}^{n} a_k$$

S is said to **converge** iff its sequence of partial sums (s_n) converges to some $s \in \mathbb{R}$ as $n \to \infty$; that is, iff for every $\epsilon > 0$ there is an $N \in \mathbb{N}$ such that for all $n \geq N$ we have $|s_n - s| < \epsilon$. In this case we shall write

$$\sum_{k=1}^{\infty} a_k = s$$

and call s the **sum** or **value** of the series $\sum_{k=1}^{\infty} a_k$

A series $S = \sum_{k=1}^{\infty} a_k$ is said to be **absolutely convergent** if the series $\sum_{k=1}^{\infty} |a_k|$ is convergent. A series is called **conditionally convergent** if it is convergent but not absolutely convergent.

Theorem 1.7: Cauchy Criteron

Let $S = \sum_{k=1}^{\infty} a_k$ be a series. Then the series S is convergent iff for any $\epsilon > 0$ there exists N such that for all $m \geq n \geq N$ we have that

$$\left|\sum_{k=n+1}^{m} a_k\right| < \epsilon$$

Theorem 1.8: Rearrangements of Abs. Convergent Series

Let $S = \sum_{k=1}^{\infty} a_k$ be an absolutely convergent series. Then

- The series S is convergent
- Let $z : \mathbb{N} \to \mathbb{N}$ be a bijection. Then the series $\sum_{k=1}^{\infty} a_{z(k)}$ is convergent and
$$\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} a_{z(k)}$$

The series $\sum_{k=1}^{\infty} a_{z(k)}$ is called a **rearrangement** of the series $\sum_{k=1}^{\infty} a_k$. What we do here is add the terms of the sum in a different order to the original one, for example

$$a_3 + a_7 + a_1 + a_{100} + a_2 + \dots$$

Since $z : \mathbb{N} \to \mathbb{N}$ is a bijection, we will miss no terms.

Theorem 1.9: Rearrangements of Cond. Convergent Series

Let $S = \sum_{k=1}^{\infty} a_k$ be any conditionally convergent series. Then there exists rearrangements $z : \mathbb{N} \to \mathbb{N}$ (where z is a bijection) such that

- For any $r \in \mathbb{R}$ the series $\sum_{k=1}^{\infty} a_{z(k)}$ is conditionally convergent and its sum is r
- The series $\sum_{k=1}^{\infty} a_{z(k)}$ diverges to $+\infty$
- The series $\sum_{k=1}^{\infty} a_{z(k)}$ diverges to $-\infty$
- The partial sums of the series $\sum_{k=1}^{\infty} a_{z(k)}$ oscillate between any two real numbers

Definition 1.7: Continuity

Let f be a function $f : \mathrm{dom}(f) \to \mathbb{R}$ where $\mathrm{dom}(f) \subset \mathbb{R}$. We say that f is continuous at some $a \in \mathrm{dom}(f)$ if for any sequence (x_n) whose terms lie in $\mathrm{dom}(f)$ and which converges to a, we have $\lim_{n\to\infty} f(x_n) = f(a)$. If f is continuous at each $a \in S \subset \mathrm{dom}(f)$ then we say f is continuous on S. If f is continuous of $\mathrm{dom}(f)$ then we say f is continuous

Theorem 1.10: Properties of Continuity

Let $f, g : D \to \mathbb{R}$ be continuous on D, and let $\alpha \in \mathbb{R}$. Then the following functions are continuous on D :

1. α f
2. $f + g$
3. fg

Definition 1.8: Composition

Let $A, B \subseteq \mathbb{R}$ be nonempty, let $f : A \to \mathbb{R}$, $g : B \to \mathbb{R}$ and $f(A) \subseteq B$. The composition of g with f is the function $g \circ f : A \to \mathbb{R}$ defined by

$$(g \circ f)(x) = g(f(x)), \quad \text{for all } x \in A$$

Theorem 1.11: Continuity of Composition

If f is continuous at $a \in \mathbb{R}$ and g is continuous at $f(a)$ then the composition $g \circ f$ is continuous at a

Theorem 1.12: $\epsilon - \delta$ definition of continuity

Let f be a function $f : \mathrm{dom}(f) \to \mathbb{R}$ where $\mathrm{dom}(f) \subset \mathbb{R}$. Then f is continuous at $a \in \mathrm{dom}(f)$ iff for any $\epsilon > 0$ there exists $\delta > 0$ s.t. whenever $x \in \mathrm{dom}(f)$ and $|x - a| < \delta$ we have $|f(x) - f(a)| < \epsilon$

Definition 1.13: Intermediate Value Theorem

Let $a < b$ real numbers and $f : [a, b] \to \mathbb{R}$ be continuous on $[a, b]$. If $f(a)f(b) < 0$ then there exists at least one $c \in (a, b)$ s.t. $f(c) = 0$

Definition 1.14: Extreme Value Theorem

Let $a < b$ real numbers and $f : [a, b] \to \mathbb{R}$ be continuous on $[a, b]$. Then there exists points $c, d \in [a, b]$ s.t.

$$f(c) = \inf\{f(x) : x \in [a, b]\}, \quad f(d) = \sup\{f(x) : x \in [a, b]\}$$

That is, the function f on the interval $[a, b]$ is bounded and attains its minimal value at some point $c \in [a, b]$. Similarly, the maximal value of f is also attained at some point $d \in [a, b]$

2 Uniform convergence

Definition 2.1: Pointwise Convergence

Let E be a nonempty subset of $\mathbb{R}$. A sequence of functions $f_n : E \to \mathbb{R}$ is said to **converge pointwise** on E, written $f_n \to f$ pointwise on E as $n \to \infty$, iff $f(x) = \lim_{n\to\infty} f_{n(x)}$ exists for each $x \in E$

Let E be a nonempty subset of $\mathbb{R}$. Then a sequence of functions f_n converges pointwise on E, as $n \to \infty$, iff for every $\epsilon > 0$ and $x \in E$ there is an $N \in \mathbb{N}$ (which may depend on x as well we ϵ) such that

$$n \geq N \quad \text{implies} \quad |f_n(x) - f(x)| < \epsilon$$

Remarks:

- The pointwise limit of continuous (respectively, differentiable) functions is not necessarily continuous (respectively, differentiable).
- The pointwise limit of integrable functions is not necessarily integrable.
- There exist continuous functions f_n and f such that $f_n \to f$ pointwise on $[0, 1]$ but

$$\lim_{n\to\infty} \int_0^1 f_n(x)\,dx \neq \int_0^1 \left(\lim_{n\to\infty} f_n(x)\right) dx$$

Definition 2.2: Uniform Convergence

Let E be a nonempty subset of $\mathbb{R}$. A sequence of functions $f_n : E \to \mathbb{R}$ is said to **converge uniformly** on E to a function f (notation: $f_n \to f$ uniformly on E as $n \to \infty$) if and only if for every $\epsilon > 0$ there is an $N \in \mathbb{N}$ such that for all $x \in E$

$$n \geq N \quad \text{implies} \quad |f_n(x) - f(x)| < \epsilon$$

Remark 2.2: Differences between Pointwise and Uniform

Let E be a nonempty subset of $\mathbb{R}$.

- A sequence of functions f_n **converges pointwise** on E, as $n \to \infty$, if and only if for every $\epsilon > 0$ and $x \in E$ there is an $N \in \mathbb{N}$ (which may depend on x as well we ϵ) such that

$$n \geq N \quad \text{implies} \quad |f_n(x) - f(x)| < \epsilon$$

- A sequence of functions $f_n : E \to \mathbb{R}$ **converges uniformly** on E iff for every $\epsilon > 0$ there is an $N \in \mathbb{N}$ such that for all $x \in E$

$$n \geq N \quad \text{implies} \quad |f_n(x) - f(x)| < \epsilon$$

For a sequence of functions to be pointwise convergent, it is enough to have an N_n for every x_n, but for it to be uniformly convergent, it has to have **the same** N for every x in the sequence

Theorem 2.1: Equivalence of Uniform Convergence

The following are equivalent concerning a sequence of functions $f_n : E \to \mathbb{R}$ and $f : E \to \mathbb{R}$:

- $f_n \to f$ uniformly on E
- $\sup_{x\in E} |f_n(x) - f(x)| \to 0$ as $n \to \infty$
- there exists a sequence $a_n \to 0$ s.t. $|f_n(x) - f(x)| \leq a_n, \forall x \in E$

Theorem 2.1

Let E be a nonempty subset of $\mathbb{R}$ and suppose that $f_n \to f$ uniformly on E as $n \to \infty$. If each f_n is continuous at some $x_0 \in E$, then f is continuous at $x_0 \to E$

Definition 2.2: Uniformly Bounded Sequences

A sequence of functions f_n is said to be **uniformly bounded** on a set E if there is a $M > 0$ such that $|f_n(x)| \leq M$ for all $x \in E$ and all $n \in N$

Theorem 2.2

Suppose that $f_n \to f$ uniformly on a closed interval $[a, b]$. If each f_n is integrable on $[a, b]$, then so is f and

$$\lim_{n\to\infty} \int_a^b f_n(x)\,dx = \int_a^b \left(\lim_{n\to\infty} f_n(x)\right) dx$$

Theorem 2.3

Let (a, b) be a bounded interval and suppose that f_n is a sequence of functions which converges at some $x_0 \in (a, b)$. If each f_n is differentiable on (a, b), and f_n' converges uniformly on (a, b) as $n \to \infty$, then f_n converges uniformly on (a, b) and

$$\lim_{n\to\infty} f_n'(x) = \left(\lim_{n\to\infty} f_n(x)\right)'$$

Definition 2.3: Convergence of series

Let f_k be a sequence of a real functions defined on some set E and set

$$s_n(x) = \sum_{k=1}^{n} f_k(x), \quad x \in E,\ n \in \mathbb{N}$$

- The series $\sum_{k=1}^{\infty} f_k$ is said to **converge pointwise** on E if and only if the sequence $s_n(x)$ converges pointwise on E as $n \to \infty$
- The series $\sum_{k=1}^{\infty} f_k$ is said to **converge uniformly** on E if and only if the sequence $s_n(x)$ converges uniformly on E as $n \to \infty$
- The series $\sum_{k=1}^{\infty} f_k$ is said to **converge absolutely** (pointwise) on E if and only if $\sum_{k=1}^{\infty} |f_k(x)|$ converges for each $x \in E$

Theorem 2.4: Results of Convergent Series

Let E be a nonempty subset of $\mathbb{R}$ and let (f_k) be a sequence of real functions defined on E.

- Suppose that $x_0 \in E$ and that each f_k is continuous at $x_0 \in E$. If $f = \sum_{k=1}^{\infty} f_k$ converges uniformly on E, then f is continuous at $x_0 \in E$.
- Term-by-term integration: Suppose that $E = [a, b]$ and that each f_k is integrable on $[a, b]$. If $f = \sum_{k=1}^{\infty} f_k$ converges uniformly on $[a, b]$, then f is integrable on $[a, b]$ and

$$\int_a^b \sum_{k=1}^{\infty} f_k(x)\, dx = \sum_{k=1}^{\infty} \int_a^b f_k(x)\, dx$$

- Term-by-term differentiation: Suppose that E is a bounded, open interval and that each f_k is differentiable on E. If $\sum_{k=1}^{\infty} f_k(x_0)$ converges at some $x_0 \in E$, and $g = \sum_{k=1}^{\infty} f'(k)$ converges uniformly on E, then $f = \sum_{k=1}^{\infty} f_k$ converges uniformly on E, is differentiable on E, and

$$f'(x) = \left(\sum_{k=1}^{\infty} f_k(x) \right)' = \sum_{k=1}^{\infty} f_k'(x) = g(x)$$

for $x \in E$

Theorem 2.5: Weierstrass M-test

Let E be a nonempty subset of $\mathbb{R}$, let $f_k : E \to \mathbb{R}$, $k \in \mathbb{N}$, and suppose that $M_k > 0$ satisfies $\sum_{k=1}^{\infty} M_k < \infty$. If $|f_k(x)| \leq M_k$ for $k \in \mathbb{N}$ and $x \in E$, then $f = \sum_{k=1}^{\infty} f_k$ converges absolutely and uniformly on E.

Definition 3.0: Power Series

Let (a_n) be a sequence of real numbers, and $c \in \mathbb{R}$. A **power series** is a series of the form

$$\sum_{n=0}^{\infty} a_n (x - c)^n$$

The numbers a_n are called the **coefficients** of the power series, and c is its **centre**. In many cases it suffices to set $c = 0$. Note that the series will always converge at the point $x = c$ as all terms beyond the first are 0.

Definition 3.1: Radius of Convergence

The **radius of convergence** R of the power series

$$\sum_{n=0}^{\infty} a_n (x - c)^n \qquad (*)$$

is defined by

$$R = \sup\{r \geq 0 : (a_n r^n) \text{ is bounded}\}$$

unless $(a_n r^n)$ is bounded for all $r \geq 0$, in which case we say that $R = \infty$

Thm 3.1: Suppose the radius of convergence R of $*$ satisfies $0 < R < \infty$. If $|x - c| < R$, the power series $*$ converges absolutely. If $|x - c| > R$, the power series $*$ diverges

Theorem 3.2: Continuity of Power Series

Assume that $R > 0$. Suppose that $0 < r < R$. Then a power series converges uniformly and absolutely on $|x - c| \leq r$ to a continuous function f. Hence

$$f(x) = \sum_{n=0}^{\infty} a_n (x - c)^n$$

defines a continuous function $f : (c - R, c + R) \to \mathbb{R}$

Lemma 3.1: The two power series

$$\sum_{n=1}^{\infty} a_n (x - c)^n \text{ and } \sum_{n=1}^{\infty} n a_n (x - c)^{n-1}$$

have the same radius of convergence

Theorem 3.3: Differentiation of Power Series

Suppose the radius of convergence of a power series is R. Then the function

$$f(x) = \sum_{n=0}^{\infty} a_n (x - c)^n$$

is infinitely differentiable on $|x - c| < R$, and for such x,

$$f'(x) = \sum_{n=0}^{\infty} n a_n (x - c)^{n-1}$$

and the series converges absolutely, and also uniformly on $[c - r, c + r]$ for any $r < R$. Moreover,

$$a_n = \frac{f^{(n)}(c)}{n!}$$

3 Lebesgue Integration

Definition 4.0: Characteristic Function

Let E be a subset of $\mathbb{R}$. We define its **characteristic function** $\chi_E : \mathbb{R} \to \mathbb{R}$ by $\chi_E(x) = 1$ if $x \in E$ and $\chi_E(x) = 0$ if $x \notin E$. In other words,

$$\chi_E = \begin{cases} 1 & x \in E \\ 0 & x \notin E \end{cases}$$

In other words, this is a function that is 1 at all points of a bounded interval, and 0 elsewhere

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Let I be a bounded interval with endpoints a, b and $a \leq b$. We call the number $b - a$ the **length of the interval** I and we denote it by $\lambda(I)$. This might also be referred to as $|I|$. That is,

$$\lambda((a,b)) = \lambda([a,b]) = \lambda((a,b]) = \lambda([a,b)) = b - a$$

From our definition of a characteristic function and the length of an interval, we have that the area of the characteristic function is a rectangle with width $\lambda(I)$ and height 1, therefore

$$\int \chi_I = 1 \cdot \lambda(I) = \lambda(I)$$

Definition 4.1: Step function

We say that $\phi : \mathbb{R} \to \mathbb{R}$ is a **step function** if there exist real numbers $x_0 < x_1 < x_2 < \cdots < x_n$ (for some $n \in \mathbb{N}$) such that

1. $\phi(x) = 0$ for $x < x_0$ and $x > x_n$
2. ϕ is constant on (x_{j-1}, x_j) for $1 \leq j \leq n$

We shall use the phrase "ϕ is a step function with respect to $\{x_0, x_1, \ldots, x_n\}$" to describe this situation

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Properties of Step Functions

1. The class of step functions is a vector space - i.e. if ϕ and ψ are step functions and α and β are real numbers, then $\alpha\phi + \beta\psi$ is a step function, and that if ϕ and ψ are step functions, then $\max\{\phi, \psi\}$, $\min\{\phi\psi\}$, $|\phi|$ and $\phi\psi$ are also step functions
2. If ϕ and ψ are step functions, then $\phi + \psi$ is a step function
3. ϕ is a step function if and only if it is of the form

$$\phi = \sum_{j=1}^{n} c_j \chi_{J_j}$$

for some n, c_j, and bounded intervals J_j

Def 4.2: Integral of a Step Function
If ϕ is a step function with respect to $\{x_0, x_1, \ldots, x_n\}$ which takes the value c_j on (x_{j-1}, x_j), then

$$\int \phi := \sum_{j=1}^{n} c_j (x_j - x_{j-1})$$

Therefore, using the characteristic definition of a step function, the integral is

$$\int \phi = \int \sum_{j=1}^{n} c_j \chi_{J_j} = \sum_{j=1}^{n} c_j \int \chi_{I J_j} = \sum_{j=1}^{n} c_j \lambda(J_j)$$

Definition 4.3: Lebesgue Integrals

A function $f : I \to \mathbb{R}$ is said to be **integrable** or more precisely **Lebesgue integrable** on an interval I if there exist numbers c_j and bounded intervals $J_j \subset I$, $j = 1, 2, 3, \ldots$ such that

$$\sum_{j=1}^{\infty} |c_j| \lambda(J_j) < \infty$$

and the equality

$$f(x) = \sum_{j=1}^{\infty} c_j \chi_{J_j}(x)$$

holds for all $x \in I$ at which

$$\sum_{j=1}^{\infty} |c_j| \chi_{J_j}(x) < \infty$$

We denote by $\int_I f$ the number

$$\int_I f = \sum_{j=1}^{\infty} c_j \lambda(J_j)$$

and call it the integral of f over the interval I. If the function f is not integrable on the interval I then we say that the integral of f on I does not exist. Hence if we say that the integral of f on I exists it just means that f is (Lebesgue) integrable on I.

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