Metric Spaces Notes

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1 Introduction to Metric Spaces

Metric is another name for distance. A **Metric Space** is a set equipped with a metric. A standard example is $\mathbb{R}$ with the standard metric

$$d(x, y) = |x - y|$$

We will now formally define what it means to have a metric

Definition 1.1: Definition of a Metric

Let X be a non-empty set. A function $d : X \times X \to \mathbb{R}$ is called a **metric** iff for all x, y, $z \in X$,

- $d(x, y) \geq 0$ and $d(x, y) = 0 \iff x = y$
- $d(x, y) = d(y, x)$
- $d(x, y) \leq d(x, z) + d(z, y)$ (Triangle Inequality)