

# PHYS 512 #1

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$$\textcircled{1} a) f(x \pm \delta) = f(x) \pm \delta f'(x) + \delta^2 \frac{f''(x)}{2} \pm \delta^3 \frac{f'''(x)}{6} + \delta^4 \frac{f^{(4)}(x)}{24} \pm \frac{\delta^5}{120} f^{(5)}(x) \dots$$

$$\textcircled{1} f(x+\delta) - f(x-\delta) = 2\delta f'(x) + \delta^3 \frac{f'''(x)}{3} + 4\delta^5 \frac{f^{(5)}(x)}{120}$$

Similarly

$$\textcircled{2} f(x+2\delta) - f(x-2\delta) = 4\delta f'(x) + \delta^3 \frac{8}{3} f'''(x) + \delta^5 \frac{64}{15} f^{(5)}(x)$$

8 ① - ②

$$8f(x+\delta) - f(x+2\delta) - f(x-2\delta) + 8f(x-\delta) = 12\delta f'(x) + \frac{12}{30}\delta^5 f^{(5)}(x)$$

$$f'(x) = \frac{8f(x+\delta) - f(x+2\delta) + f(x-2\delta) - 8f(x-\delta)}{12\delta} - \frac{f^{(5)}(x)}{30}\delta^4$$

$$b) \bar{f} \rightarrow (1 + g_i \epsilon) f_{i, \text{true}}$$

$\epsilon$  - machine error

$g_i$  order unity random number

$$f_{\text{true}} - \bar{f}$$

$$\epsilon \sim 10^{-16}$$

$$= f' - \frac{8\bar{f}(x+\delta) - \bar{f}(x+2\delta) + \bar{f}(x-2\delta) - 8\bar{f}(x-\delta)}{12\delta}$$

$$= f' - \frac{8f(x+\delta) - f(x+2\delta) + f(x-2\delta) - 8f(x-\delta)}{12\delta} - \epsilon \frac{8g_1 f(x+\delta) + g_2 f(x+2\delta) - g_3 f(x-2\delta) - 8g_4 f(x-\delta)}{12\delta}$$

$$= \frac{f^{(5)}}{30}\delta^4 - \epsilon \frac{8g_1 f(x+\delta) - g_2 f(x+2\delta) + g_3 f(x-2\delta) - 8g_4 f(x-\delta)}{12\delta}$$

to leading order.

$$= \frac{f^{(5)}}{30}\delta^4 - \epsilon f(x) \frac{8g_1 + g_2 + g_3 - 8g_4}{12\delta}$$

$$= \frac{f^{(5)}}{30}\delta^4 - \frac{\epsilon}{\delta} \frac{18}{12} f(x)$$

$$\epsilon = \left| \frac{f^{(5)}}{30}\delta^4 \right| + \frac{3}{2} \frac{\epsilon}{\delta} |f(x)|$$

$$\frac{\partial \epsilon}{\partial \delta} = \frac{2f^{(5)}}{15}\delta^3 - \frac{3}{2} \frac{\epsilon}{\delta^2} f(x) = 0$$

$$\frac{2f^{(5)}(x)}{15}\delta^5 = \frac{3}{2} \epsilon f(x)$$

$$\delta = \left( \frac{45}{4} \frac{\epsilon f(x)}{f^{(5)}(x)} \right)^{1/5}$$