

Artificial Intelligence¹

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Automated Planning

Today we will explore techniques for **action planning** – how to find a sequence of actions to reach a given goal.

- **problem representation**

- situation calculus (pure logical representation)
- using sets of predicates (instead of formulas)
- planning domain vs. planning problem

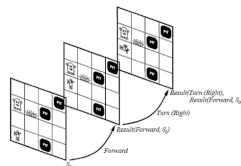
- **planning techniques**

- state-space planning
 - forward and backward
- plan-space planning
 - partially ordered plans



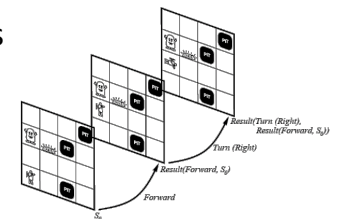
Actions and situations

- So far we modelled a static world only.
- **How to reason about actions and their effects in time?**
- **In propositional logic** we need a **copy of each action for each time (situation)**:
 - $L_{x,y}^t \wedge \text{FacingRight}^t \wedge \text{Forward}^t \Rightarrow L_{x+1,y}^{t+1}$
 - We need an upper bound for the number of steps to reach a goal but this will lead to a huge number of formulas.
- Can we do it better in **first order logic**?
 - We do not need copies of axioms describing state changes; this can be implemented using a universal quantifier for time (situation)
 - $\forall t$ P is the result of action A in time t+1



Situation calculus

- **actions** are represented by terms
 - $\text{Go}(x,y)$
 - $\text{Grab}(g)$
 - $\text{Release}(g)$
- **situation** is also a term
 - initial situation: S_0
 - situation after applying action a to state s : $\text{Result}(a,s)$
- **fluent** is a predicates changing with time
 - the situation is in the last argument of that term
 - $\text{Holding}(G, S_0)$
- **rigid (eternal) predicates**
 - $\text{Gold}(G)$
 - $\text{Adjacent}(x,y)$

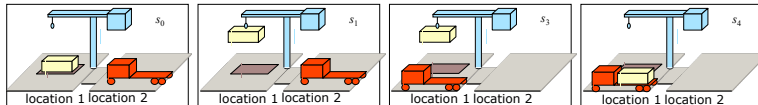


- We need to **reason about sequences of actions** – about **plans**.

- $\text{Result}([], s) = s$
- $\text{Result}([a | \text{seq}], s) = \text{Result}(\text{seq}, \text{Result}(a, s))$

- What are the typical tasks related to plans?

- **projection task** – what is the state/situation after applying a given sequence of actions?
 - $\text{At}(\text{Agent}, [1,1], S_0) \wedge \text{At}(G, [1,2], S_0) \wedge \neg \text{Holding}(o, S_0)$
 - $\text{At}(G, [1,1], \text{Result}([\text{Go}([1,1], [1,2]), \text{Grab}(G), \text{Go}([1,2], [1,1])], S_0))$
- **planning task** – which sequence of actions reaches a given state/situation?
 - $\exists \text{seq } \text{At}(G, [1,1], \text{Result}(\text{seq}, S_0))$



- Each **action** can be described using two axioms:

- **possibility axiom:** $\text{Preconditions} \Rightarrow \text{Poss}(a, s)$
 - $\text{At}(\text{Agent}, x, s) \wedge \text{Adjacent}(x, y) \Rightarrow \text{Poss}(\text{Go}(x, y), s)$
 - $\text{Gold}(g) \wedge \text{At}(\text{Agent}, x, s) \wedge \text{At}(g, x, s) \Rightarrow \text{Poss}(\text{Grab}(g), s)$
 - $\text{Holding}(g, s) \Rightarrow \text{Poss}(\text{Release}(g), s)$
- **effect axiom:** $\text{Poss}(a, s) \Rightarrow \text{Changes}$
 - $\text{Poss}(\text{Go}(x, y), s) \Rightarrow \text{At}(\text{Agent}, y, \text{Result}(\text{Go}(x, y), s))$
 - $\text{Poss}(\text{Grab}(g), s) \Rightarrow \text{Holding}(g, \text{Result}(\text{Grab}(g), s))$
 - $\text{Poss}(\text{Release}(g), s) \Rightarrow \neg \text{Holding}(g, \text{Result}(\text{Release}(g), s))$

- Beware! This is not enough to deduce that a plan reaches a given goal.

- we can deduce $\text{At}(\text{Agent}, [1,2], \text{Result}(\text{Go}([1,1], [1,2]), S_0))$
- but we **cannot deduce** $\text{At}(G, [1,2], \text{Result}(\text{Go}([1,1], [1,2]), S_0))$
- Effect axioms describe what has been changed in the world but say **nothing about the property that everything else is not changed!**
- This is a so called **frame problem**.

- We need to represent properties that are not changed by actions.
- A **simple frame axiom** says what is not changed:

$$\text{At}(o, x, s) \wedge o \neq \text{Agent} \wedge \neg \text{Holding}(o, s) \Rightarrow \text{At}(o, x, \text{Result}(\text{Go}(y, z), s))$$

- for F fluents and A actions we need $O(FA)$ frame axioms
- This is a lot especially taking in account that most predicates are not changed.



- Can we use less axioms to model the frame problem?

- successor-state axiom**

$$\text{Poss}(a, s) \Rightarrow (\text{fluent holds in } \text{Result}(a, s) \Leftrightarrow (\text{fluent is effect of } a \vee (\text{fluent holds in } s \wedge a \text{ does not change fluent})))$$

- We get F axioms (F is the number of fluents) with $O(AE)$ literals in total (A is the number of actions, E is the number of effects).

Examples:

$$\begin{aligned} \text{Poss}(a, s) &\Rightarrow (\text{At}(\text{Agent}, y, \text{Result}(a, s)) \Leftrightarrow a = \text{Go}(x, y) \vee (\text{At}(\text{Agent}, y, s) \wedge a \neq \text{Go}(y, z))) \\ \text{Poss}(a, s) &\Rightarrow (\text{Holding}(g, \text{Result}(a, s)) \Leftrightarrow a = \text{Grab}(g) \vee (\text{Holding}(g, s) \wedge a \neq \text{Release}(g))) \end{aligned}$$

- **Beware of implicit effects!**

- If an agent holds some object and the agent moves then also the object moves.
- This is called a **ramification problem**.

$$\begin{aligned} \text{Poss}(a, s) &\Rightarrow (\text{At}(o, y, \text{Result}(a, s)) \Leftrightarrow (a = \text{Go}(x, y) \wedge (o = \text{Agent} \vee \text{Holding}(o, s))) \vee (\text{At}(o, y, s) \wedge \neg \exists z (y \neq z \wedge a = \text{Go}(y, z) \wedge (o = \text{Agent} \vee \text{Holding}(o, s))))) \end{aligned}$$



Frame problem: even better axioms

- Successor-state axiom is still too big with $O(AE/F)$ literals in average.
 - To solve the projection task with t actions, the time complexity depends on the total number of actions – $O(AEt)$ – rather than on the actions in plan.
 - If we know each action, cannot we do it better say $O(Et)$?

classical successor-state axiom:

$$\text{Poss}(a,s) \Rightarrow (F_i(\text{Result}(a,s)) \Leftrightarrow (a=A_1 \vee a=A_2 \vee \dots) \vee (F_i(s) \wedge a \neq A_3 \wedge a \neq A_4 \dots))$$

actions having F_i among effects

actions having $\neg F_i$ among effects

- We can introduce **positive** and **negative effects** of actions:
 - **PosEffect(a, F_i)** action a causes F_i to become true
 - **NegEffect(a, F_i)** action a causes F_i to become false

modified successor state axiom:

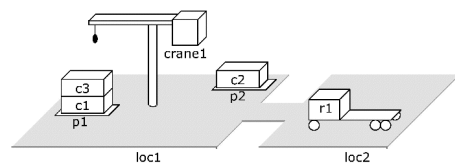
$$\begin{aligned} \text{Poss}(a,s) \Rightarrow & (F_i(\text{Result}(a,s)) \Leftrightarrow \text{PosEffect}(a, F_i) \vee (F_i(s) \wedge \neg \text{NegEffect}(a, F_i))) \\ & \text{PosEffect}(A_1, F_i) \\ & \text{PosEffect}(A_2, F_i) \\ & \text{NegEffect}(A_3, F_i) \\ & \text{NegEffect}(A_4, F_i) \end{aligned}$$



Classical representation: states

We can simplify the full FOL model into a so called **classical representation of planning problems**.

State is a set of instantiated atoms (no variables). There is a finite number of states!



{attached(p1,loc1), in(c1,p1), in(c3,p1), top(c3,p1), on(c3,c1), on(c1,pallet), attached(p2,loc1), in(c2,p2), top(c2,p2), on(c2,pallet), belong(crane1,loc1), empty(crane1), adjacent(loc1,loc2), adjacent(loc2,loc1), at(r1,loc2), occupied(loc2), unloaded(r1)}.

- The truth value of some atoms is changing in states:

- **fluents**
- *example:* $at(r1,loc2)$

- The truth value of some state is the same in all states

- **rigid atoms**
- *example:* $adjacent(loc1,loc2)$

We will use a classical **closed world assumption**.

An atom that is not included in the state does not hold at that state!

Hidden assumptions

Example:

- Assume the following claim:
 - „In summer we will teach courses CS101, CS102, CS106, and EE101“
 - so in FOL we have the facts
 - $\text{Course}(\text{CS},101), \text{Course}(\text{CS},102), \text{Course}(\text{CS},106), \text{Course}(\text{EE},101)$
- How many courses will we teach in summer?
 - Something between one and infinity!!

Why?

- We usually assume having a complete information about the world, i.e., what is not explicitly said does not hold – this is called a **closed world assumption (CWA)**
- There is no such assumption in FOL, so we need to complete the knowledge base:
 - $\text{Course}(d,n) \Leftrightarrow [d,n] = [\text{CS},101] \vee [d,n] = [\text{CS},102] \vee [d,n] = [\text{CS},106] \vee [d,n] = [\text{EE},101]$
- We also assumed that different names (constants) denote different objects – this is called a **unique name assumption (UNA)**
- Again, we need to explicitly describe that objects are different:
 - $[\text{CS},101] \neq [\text{CS},102], \dots$

Classical representation: operators

operator o is a triple (name(o), precondition(o), effects(o))

- **name(o):** name of the operator in the form $n(x_1, \dots, x_k)$
 - n : a symbol of the operator (a unique name for each operator)
 - x_1, \dots, x_k : symbols for variables (operator parameters)
 - Must contain all variables appearing in the operator definition!
- **precond(o):**
 - literals that must hold in the state so the operator is applicable on it
- **effects(o):**
 - literals that will become true after operator application (only fluents can be there!)

$\text{take}(k, l, c, d, p)$

;; crane k at location l takes c off of d in pile p

precond: $\text{belong}(k,l), \text{attached}(p,l), \text{empty}(k), \text{top}(c,p), \text{on}(c,d)$

effects: $\text{holding}(k,c), \neg \text{empty}(k), \neg \text{in}(c,p), \neg \text{top}(c,p), \neg \text{on}(c,d), \text{top}(d,p)$

An action is a fully instantiated operator

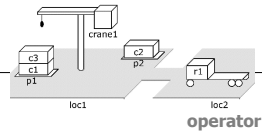
- substitute constants to variables

$take(k, l, c, d, p)$

;; crane k at location l takes c off of d in pile p

precond: $belong(k, l), attached(p, l), empty(k), top(c, p), on(c, d)$

effects: $holding(k, c), \neg empty(k), \neg in(c, p), \neg top(c, p), \neg on(c, d), top(d, p)$



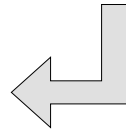
$take(crane1, loc1, c3, c1, p1)$

action

;; crane crane1 at location loc1 takes c3 off c1 in pile p1

precond: $belong(crane1, loc1), attached(p1, loc1),$
 $empty(crane1), top(c3, p1), on(c3, c1)$

effects: $holding(crane1, c3), \neg empty(crane1), \neg in(c3, p1),$
 $\neg top(c3, p1), \neg on(c3, c1), top(c1, p1)$



Notation:

- $S^+ = \{\text{positive atoms in } S\}$
- $S^- = \{\text{atoms, whose negation is in } S\}$

Action a is **applicable** to state s if any only
 $precond^+(a) \subseteq s \wedge precond^-(a) \cap s = \emptyset$

The result of application of action a to s is

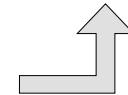
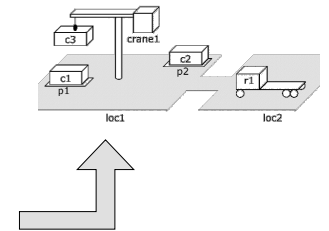
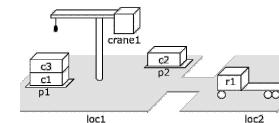
$$\gamma(s, a) = (s - effects^-(a)) \cup effects^+(a)$$

$take(crane1, loc1, c3, c1, p1)$

;; crane crane1 at location loc1 takes c3 off c1 in pile p1

precond: $belong(crane1, loc1), attached(p1, loc1),$
 $empty(crane1), top(c3, p1), on(c3, c1)$

effects: $holding(crane1, c3), \neg empty(crane1), \neg in(c3, p1),$
 $\neg top(c3, p1), \neg on(c3, c1), top(c1, p1)$



Let L be a language and O be a set of operators.

Planning domain Σ over language L with operators O is a triple (S, A, γ) :

- **states** $S \subseteq P(\{\text{all instantiated atoms from } L\})$
- **actions** $A = \{\text{all instantiated operators from } O \text{ over } L\}$
 - action a is **applicable** to state s if
 $precond^+(a) \subseteq s \wedge precond^-(a) \cap s = \emptyset$
- **transition function γ :**
 - $\gamma(s, a) = (s - effects^-(a)) \cup effects^+(a)$, if a is applicable on s
 - S is closed with respect to γ (if $s \in S$, then for every action a applicable to s it holds $\gamma(s, a) \in S$)

• **Planning problem P is a triple (Σ, s_0, g) :**

- $\Sigma = (S, A, \gamma)$ is a planning domain
- s_0 is an initial state, $s_0 \in S$
- g is a set of instantiated literals
 - state s satisfies the goal condition g if and only if
 $g^+ \subseteq s \wedge g^- \cap s = \emptyset$
 - $S_g = \{s \in S \mid s \text{ satisfies } g\}$ – a set of goal states

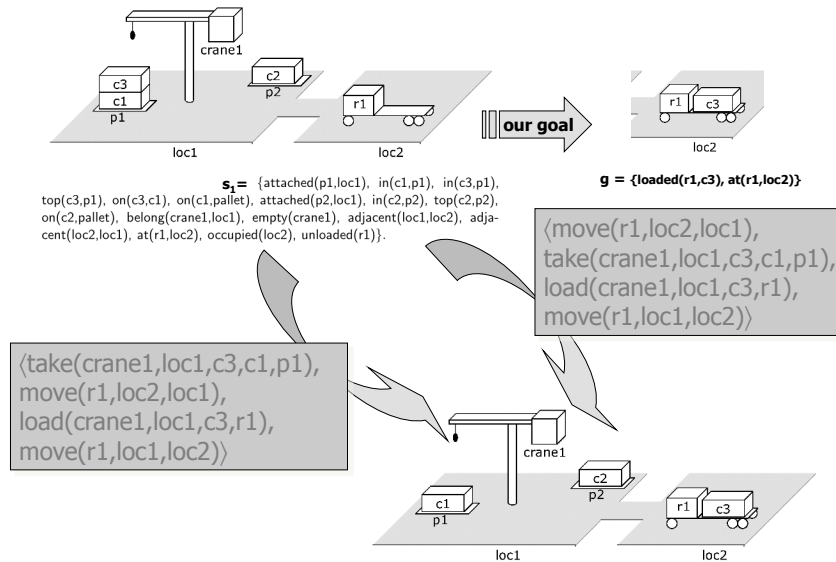
• **Plan** is a sequence of actions $\langle a_1, a_2, \dots, a_k \rangle$.

• Plan $\langle a_1, a_2, \dots, a_k \rangle$ is a **solution plan** for problem P iff
 $\gamma^*(s_0, \pi)$ satisfies the goal condition g .

• Usually the **planning problem is given by a triple (O, s_0, g) .**

- O defines the the operators and predicates used
- s_0 provides the particular constants (objects)

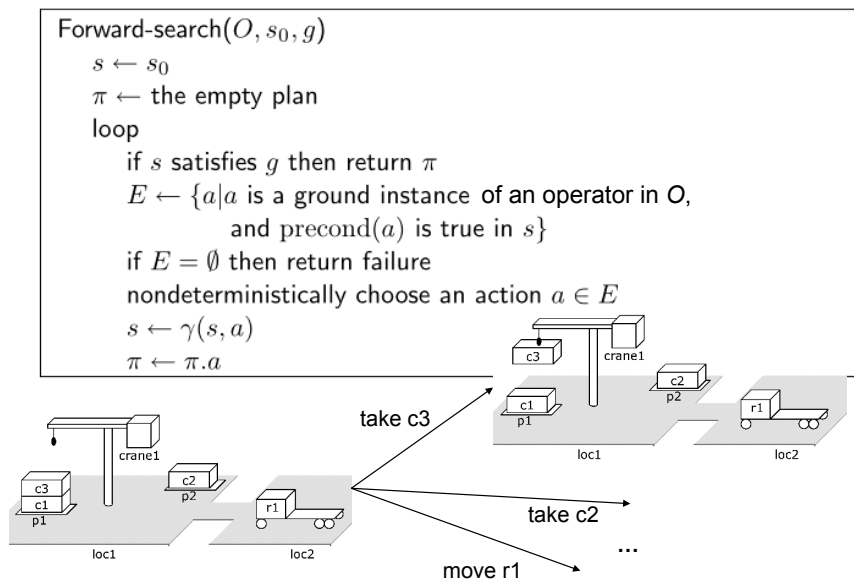
Classical representation: example plan



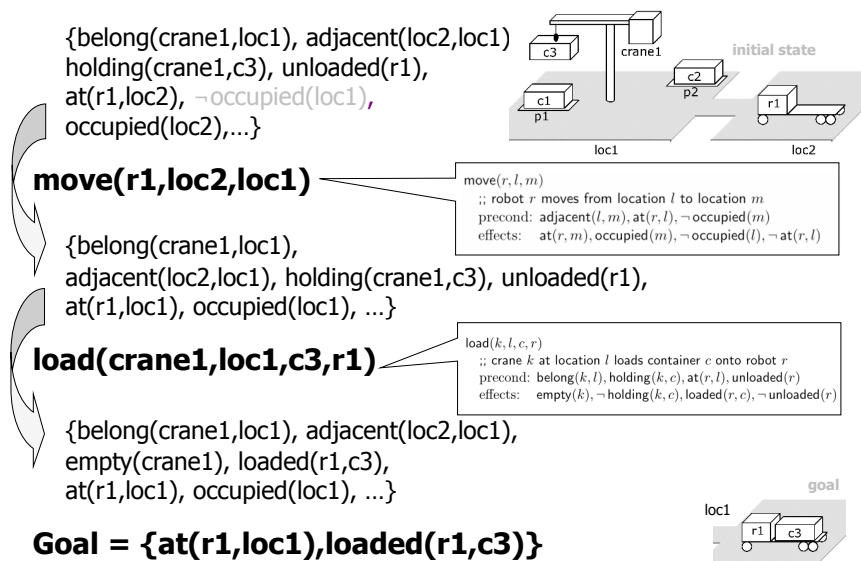
State-space planning

- The search space corresponds to the state space of the planning problem.
 - search nodes correspond to world states
 - arcs correspond to state transitions by means of actions
 - the task is to find a path from the initial state to some goal state
- Basic approaches
 - forward search (progression)
 - start in the initial state and apply actions until reaching a goal state
 - backward search (regression)
 - start with the goal and apply actions in the reverse order until a subgoal satisfying the initial state is reached
 - lifting (actions are only partially instantiated)

Forward planning: algorithm



Forward planning: an example



Backward planning

Start with a goal (not a goal state as there might be more goal states) and through sub-goals try to reach the initial state.

Action **a** is relevant for a goal **g** if and only if:

- action **a** contributes to goal **g**: $g \cap \text{effects}(\mathbf{a}) \neq \emptyset$
- effects of action **a** are not conflicting goal **g**:
 - $g^- \cap \text{effects}^+(\mathbf{a}) = \emptyset$
 - $g^+ \cap \text{effects}^-(\mathbf{a}) = \emptyset$

A **regression set** of the goal **g** for (relevant) action **a** is
 $\gamma^{-1}(g, a) = (g - \text{effects}(\mathbf{a})) \cup \text{precond}(\mathbf{a})$

Example:

goal: **{on(a,b), on(b,c)}**

action **stack(a,b)** is relevant

by backward application of the action we get a new goal:
{holding(a), clear(b), on(b,c)}

stack(x,y)
 Precond: holding(x), clear(y)
 Effects: ~holding(x), ~clear(y),
 on(x,y), clear(x), handempty

Backward planning: algorithm

Backward-search(O, s_0, g)

$\pi \leftarrow$ the empty plan

loop

if s_0 satisfies g then return π

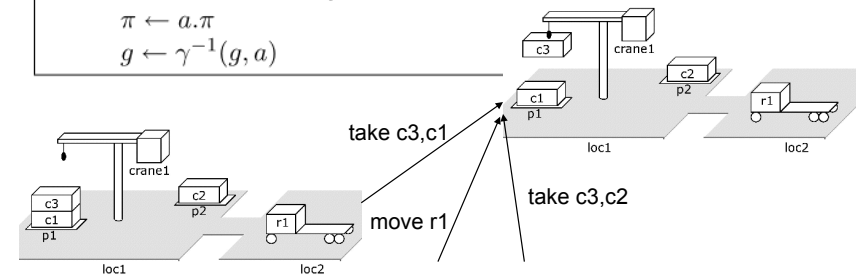
$A \leftarrow \{a \mid a \text{ is a ground instance of an operator in } O$
 and $\gamma^{-1}(g, a)$ is defined}

if $A = \emptyset$ then return failure

nondeterministically choose an action $a \in A$

$\pi \leftarrow a.\pi$

$g \leftarrow \gamma^{-1}(g, a)$



Backward planning: an example

Goal = {at(r1,loc1),loaded(r1,c3)}

load(crane1,loc1,c3,r1)

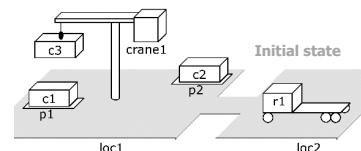
load(k,l,c,r)
 ;; crane k at location l loads container c onto robot r
 precondition: belong(k,l), holding(k,c), at(r,l), unloaded(r)
 effects: empty(k), ~holding(k,c), loaded(r,c), ~unloaded(r)

{at(r1,loc1), belong(crane1,loc1),
 holding(crane1,c3), unloaded(r1)}

move(r1,loc2,loc1)

move(r,l,m)
 ;; robot r moves from location l to location m
 precondition: adjacent(l,m), at(r,l), ~occupied(m)
 effects: at(r,m), occupied(m), ~occupied(l), ~at(r,l)

{belong(crane1,loc1), holding(crane1,c3),
 unloaded(r1),
 adjacent(loc2,loc1),
 at(r1,loc2),
 ~occupied(loc1)}



Backward planning: lifting

Lifted-backward-search(O, s_0, g)

$\pi \leftarrow$ the empty plan

loop

if s_0 satisfies g then return π

$A \leftarrow \{(o, \theta) \mid o \text{ is a standardization of an operator in } O,$
 $\theta \text{ is an mgu for an atom of } g \text{ and an atom of effects } (o),$
 and $\gamma^{-1}(\theta(g), \theta(o)) \text{ is defined}\}$

if $A = \emptyset$ then return failure

nondeterministically choose a pair $(o, \theta) \in A$

$\pi \leftarrow$ the concatenation of $\theta(o)$ and $\theta(\pi)$

$g \leftarrow \gamma^{-1}(\theta(g), \theta(o))$

Notes:

- standardization = a copy with fresh variables
- mgu = most general unifier
- by using the variables we can decrease the branching factor but the trade off is more complicated loop check

Plan-space planning: a core idea

- The principle of plan space planning is similar to backward planning:
 - start from an „empty” plan containing just the description of initial state and goal
 - add other actions to satisfy not yet covered (open) goals
 - if necessary add other relations between actions in the plan
- Planning is realised as repairing flaws in a partial plan
 - go from one partial plan to another partial plan until a complete plan is found

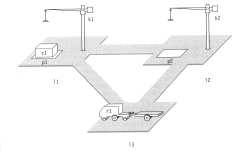
Plan space planning: the initial plan

- The initial state and the goal are encoded using two special actions in the initial partial plan:
 - Action a_0 represents the initial state in such a way that atoms from the initial state define effects of the action and there are no preconditions. This action will be before all other actions in the partial plan.
 - Action a_∞ represents the goal in a similar way – atoms from the goal define the precondition of that action and there is no effect. This action will be after all other actions.
- Planning is realised by repairing flaws in the partial plan.

Plan space planning: an example

- Assume a partial plan with the following two actions:

- take(k1,c1,p1,l1)
- load(k1,c1,r1,l1)



- Possible modifications of the plan:

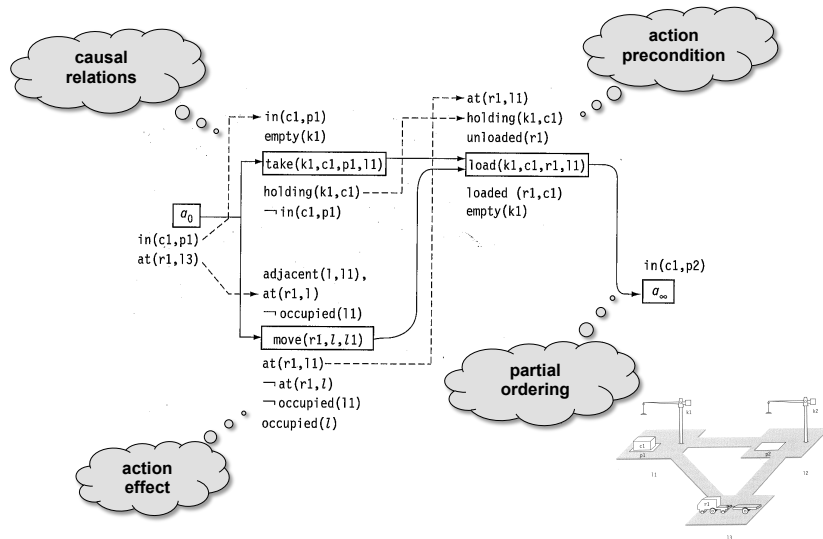
- adding a new action
 - to apply action **load**, robot r1 must be at location l1
 - action **move**(r1,l,l1) moves robot r1 to location l1 from some location l
- binding the variables
 - action **move** is used for the right robot and the right location
- ordering some actions
 - the robot must move to the location before the action **load** can be used
 - the order with respect to action **take** is not relevant
- adding a causal relation
 - new action is added to move the robot to a given location that is a precondition of another action
 - the causal relation between **move** and **load** ensures that no other action between them moves the robot to another location

Search nodes and partial plans

The search nodes correspond to partial plans.

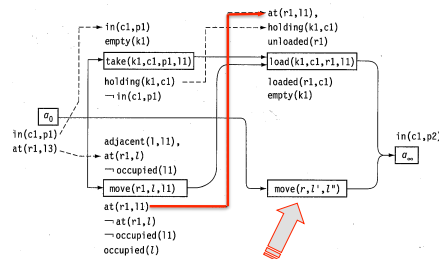
A partial plan Π is a tuple $(A, <, B, L)$, where

- A is a set of partially instantiated planning operators $\{a_1, \dots, a_k\}$
- $<$ is a partial order on A ($a_i < a_j$)
- B is set of constraints in the form $x=y$, $x \neq y$ or $x \in D_i$
- L is a set of causal relations ($a_i \rightarrow^p a_j$)
 - a_i, a_j are ordered actions $a_i < a_j$
 - p is a literal that is effect of a_i and precondition of a_j
 - B contains relations that bind the corresponding variables in p



- **Open goal** is an example of a **flaw**.
- This is a precondition **p** of some operator **b** in the partial plan such that no action was decided to satisfy this precondition (there is no causal relation $a_i \rightarrow^p b$).
- The **open goal p** of action **b** can be resolved by:
 - finding an operator **a** (either present in the partial plan or a new one) that can give **p** (**p** is among the effects of **a** and **a** can be before **b**)
 - binding the variables from **p**
 - adding a causal relation $a \rightarrow^p b$

- **Threat** is another example of **flaw**.
- This is action that can influence existing causal relation.
 - Let $a_i \rightarrow^p a_j$ be a causal relation and action **b** has among its effects a literal unifiable with the negation of **p** and action **b** can be between actions a_i and a_j . Then **b** is threat for that causal relation.
- We can **remove the threat** by one of the ways:
 - ordering **b** before a_i
 - ordering **b** after a_j
 - binding variables in **b** in such a way that **p** does not bind with the negation of **p**



- Partial plan $\Pi = (A, <, B, L)$ is a **solution plan** for the problem $P = (\Sigma, s_0, g)$ if:
 - partial ordering $<$ and constraints **B** are globally consistent
 - there are no cycles in the partial ordering
 - we can assign variables in such a way that constraints from **B** hold
 - Any linearly ordered sequence of fully instantiated actions from **A** satisfying $<$ and **B** goes from s_0 to a state satisfying **g**.
- Hmm, but this definition **does not say how** to verify that a partial plan is a solution plan!

Claim: Partial plan $\Pi = (A, <, B, L)$ is a solution plan if:

- there are no flaws (no open goals and no threats)
- partial ordering $<$ and constraints **B** are globally consistent

- **PSP = Plan-Space Planning**

```

PSP( $\pi$ )
   $flaws \leftarrow \text{OpenGoals}(\pi) \cup \text{Threats}(\pi)$ 
  if  $flaws = \emptyset$  then return( $\pi$ )
  select any  $\phi \in flaws$ 
   $resolvers \leftarrow \text{Resolve}(\phi, \pi)$ 
  if  $resolvers = \emptyset$  then return(failure)
  nondeterministically choose a resolver  $\rho \in resolvers$ 
   $\pi' \leftarrow \text{Refine}(\rho, \pi)$ 
  return(PSP( $\pi'$ ))
end

```

Notes:

- The selection of flaw is deterministic (all flaws must be resolved).
- The resolver is selected non-deterministically (search in case of failure).

- **Course Planning and scheduling**

– <http://ktiml.mff.cuni.cz/~bartak/planovani/>



Course summary

- An **agent view** of Artificial Intelligence
 - an agent is an entity perceiving environment and acting upon it
 - a **rational agent** maximizes expected performance
- **Problem solving** with simple state space
 - **search** techniques
 - exploiting extra information \rightarrow heuristic search **A***
 - structured states \rightarrow **constraint satisfaction**
 - more agents \rightarrow **adversarial search** (games)
- **Knowledge representation**
 - propositional and first-order **logic**
 - **inference** procedures
- **Automated planning**
 - situation calculus
 - state-space and plan-space planning



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