Artificial Intelligence 1

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Problem Solving: Uninformed Search

Introduction

- **Simple reflex agent** „only" transfers the current percept to one action.
- **Goal-based agent** can plan action sequences that achieve agent's goals.
  - We will now explore one kind of goal-based agent – a **problem-solving agent**.
  - What is a **problem** and what is its **solution**?
  - How to **find a solution**?
  - **Search** algorithms
    BFS, DFS, DLS, IDS, BiS

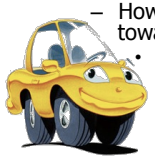
Note:
First, we will explore "uninformed" search algorithms, that is, algorithms that do not exploit any problem specific information (and use an atomic representation).

Example

- Intelligent agents are supposed to maximize their performance measure. This can be simplified if the agent can adopt a **goal** and aim at satisfying it.

A model problem and a possible way to solve it

- an agent is in the city of Arad (Romania)
- the performance measure contains many factors (improve its suntan, take in the sights, enjoy the nightlife, …), which make decisions hard
- now, suppose the agent has a non-refundable ticket to fly out of Bucharest the following day
- **goal formulation** – getting to Bucharest – greatly simplifies the agent's decision problem
- a goal is a set of world states, where the goal is satisfied, but we also need to assume other world states and actions for moving between – **problem formulation**
  - states could correspond to being in major towns (the exact location is too fine)
  - actions could correspond to driving from one major town to another (acceleration and breaking are too fine)
- How to find a path to Bucharest, if three roads lead out from Arad, one toward Sibiu, one to Timisoara, and one to Zerind?
  - Suppose the agent has a map of Romania and can explore possible journeys (**search**), select the best one, and then finally **execute** the actions.



Problem solving

Problem solving consists of four steps:

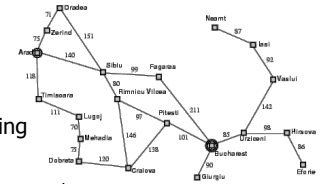
- goal formulation
  - What are the desired states of the world?
- problem formulation
  - What are the states and actions assumed to reach the goal?
- **problem solving**
  - **How to find the best sequence of states reaching the goal?**
- solution execution
  - If we know the actions, how to execute them?

```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
  static: seq, an action sequence, initially empty
          state, some description of the current world state
          goal, a goal, initially null
          problem, a problem formulation
  state ← UPDATE-STATE(state, percept)
  if seq is empty then do
      goal ← FORMULATE-GOAL(state)
      problem ← FORMULATE-PROBLEM(state, goal)
      seq ← SEARCH(problem)
  action ← FIRST(seq)
  seq ← REST(seq)
  return action
```

Problem formulation

Well-defined problems consist of:

- **the initial state**
  - ***in(Arad)***
- a description of possible **actions**
  can be done via a **transition model** describing for each state possible actions and states to which the action leads
  - ***SUCCESSOR-FN(in(Arad)) = ⟨ go(Sibiu), in(Sibiu) ⟩***
  - implicitly defines the **state space** (the set of all states reachable from the initial state by any sequence of actions)
  - a **path** is a sequence of states connected by actions.
- **the goal test**
  a function determining whether a given state is a goal state or not
  - ***{ in(Bucharest) }***
- **a path cost**
  a function that assigns a numeric cost to each path (reflects the performance measure)
  - We assume that the cost of a path can be described as a sum of the costs of individual actions along the path.



A **solution to a problem** is an action sequence that leads from the initial state to a goal state.

An optimal solution is a solution that has the lowest path cost among all solutions.

Abstraction

- In the problem formulation, we used
  - **abstraction of world states**
    - we ignored weather, traffic conditions, …
  - **abstraction of actions**
    - we ignored turning on the radio, looking out of the window, …
- **Abstraction** is the process of removing detail from representation.
- What is the **appropriate level of abstraction**?
  - the abstraction is **valid**
    - we can expand any abstract solution into a solution in the more detailed world
    - we can find a way from any place in Arad to any place in Sibiu
  - the abstraction is **useful**
    - carrying out each of the actions in the solution is easier than the original problem
    - the path from Arad to Sibiu can executed by average driving agent

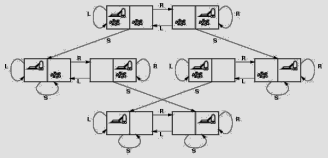
Intelligent agents would be completely swamped by the real world without using abstractions.

Toy problems

- **Toy problems** are used to illustrate and comparison of solving techniques.

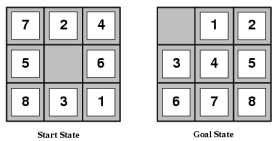
The vacuum world

states (robot × dirt)
start, goal
actions (L, R, S)



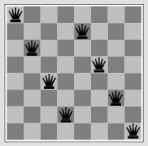
The 8-puzzle

states (block position)
start, goal
actions (L, R, U, D)



The 8-queens

place queens on chessboard without attacks
- incremental formulation (adding queens)
- complete-state formulation (moving queens)

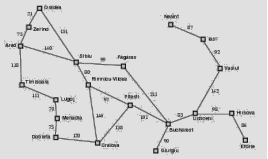


Real problems

- That is what matters!

Route-finding problems

states (places)
start (here and now), goal (there, on-time)
successor
cost function (time, cost,…)

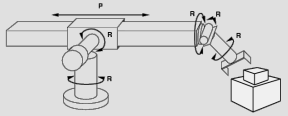


Touring problems

goal (to visit each place at least once)
states (current and visited places)
TSP (Travelling Salesman Problem), NP-hard

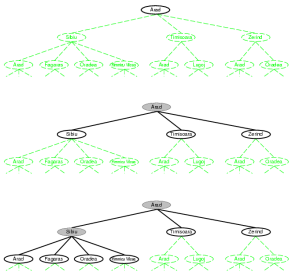
Product assembly problems

state (hand location × components)
goal (assembled product)
successor (movement of „hinges")
cost (total assembly time)
protein design from a sequence of amino acids



Searching for solutions

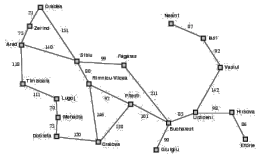
- Having formulated the problem, how do we solve it?
- **State space search**
  - start in the **initial state** (root node)
  - **check** whether **the initial state** is **the goal state**
  - if the state is not a goal state then **expand the state**, it generates a set of new states
  - **select a next** state using a **search strategy**

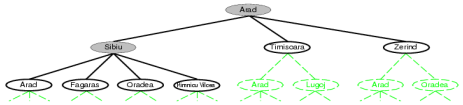


```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

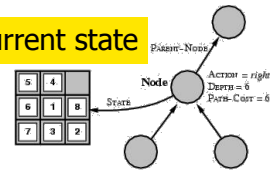
Node vs. state

- **State space is different from search tree – world state is different from search node.**





- **Node** of the search tree consists of:
  - a current **state**
  - a link to its **parent**
  - **action** leading from parent to the current state
  - **cost** of path from the root g(n)
  - **depth** (the number of steps from the root)



Fringe/frontier

- **Fringe** (**frontier**) is a set of nodes not yet expanded.

- Nodes from the fringe are called **leafs**.
- The algorithms can be described using the following operations over the fringe represented as a **queue**:
  - MAKE-QUEUE(element,…)
  - EMPTY?(queue)
  - FIRST(queue)
  - REMOVE-FIRST(queue)
  - INSERT(element, queue)
  - INSERT-ALL(elements, queue)

Tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        fringe ← INSERTALL(EXPAND(node, problem), fringe)

function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

Measuring the perfomance

We can evaluate an **algorithm's performance** in four ways:

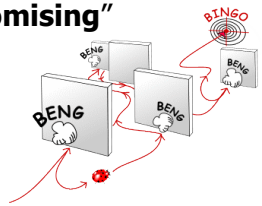
- **completeness**
  - Is the algorithm to guaranteed to find a solution when there is one?
- **optimality**
  - Does the strategy find the optimal solution?
- **time complexity**
  - How long does it take to find a solution?
- **space complexity**
  - How much memory is needed to perform search?
- Time and space complexity are always considered with respect to some measure of problem difficulty.
  - **branching factor b** (maximum number of successors of any node) – useful for implicit representation of world states (the initial state and successors)
  - **depth d** (the path length from the root to a shallowest goal node)
  - **path length m** (the maximum length of any path in the state space)

- **search cost** (how much time and space we need to find a solution)
- **total cost** (combines the search cost and the path cost of the solution found)

Information in search

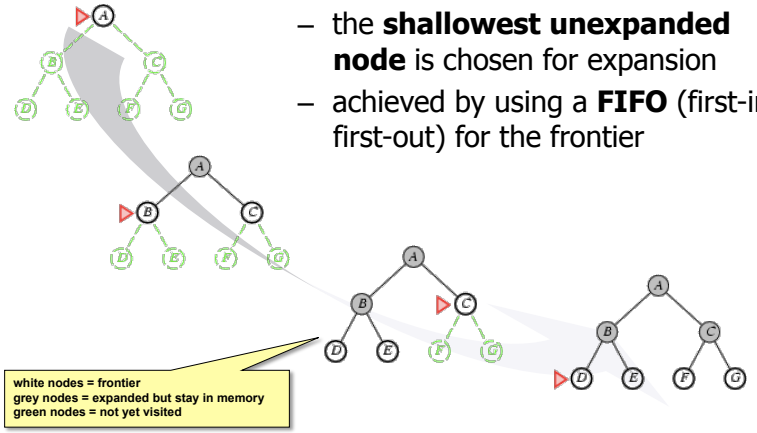
- Today we will cover **uninformed (blind) search**
  - no additional information about states beyond that provided in the problem definition
  - it can generate successors and distinguish a goal state from a non-goal state

- Next lesson will cover **informed** (**heuristic**) **search**
  - information to distinguish "**more promising**" non-goal states from other states
  - for example by estimating distance to some goal state



Breadth-first search

- **All nodes are expanded at a given depth** in the search tree before any nodes at the next level are expanded.
  - the **shallowest unexpanded node** is chosen for expansion
  - achieved by using a **FIFO** (first-in-first-out) for the frontier



Breadth-first search (analysis)

- It is a **complete method** (provided the branching factor is finite).
- The shallowest goal state is not necessarily the optimal one!
  - BFS is optimal if the path cost is a non-decreasing function of the depth of the node
- **Time complexity** (the number of visited nodes for the goal at depth d)
  - $1+b+b^2+b^3+\ldots+b^d + b(b^d-1) =$ **$O(b^{d+1})$**
- **Space complexity**
  - store every expanded node in the explored set
  - **$O(b^{d+1})$**

The memory requirements are a bigger problem than is the execution time.

depth	nodes	time	memory
2	1100	0.11 sec.	1 megabyte
4	111100	11 seconds	106 megabytes
6	10^7	19 minutes	10 gigabytes
8	10^9	31 hours	1 terabyte
10	10^{11}	129 days	101 terabytes
12	10^{13}	35 years	10 petabytes
14	10^{15}	3523 years	1 exabyte

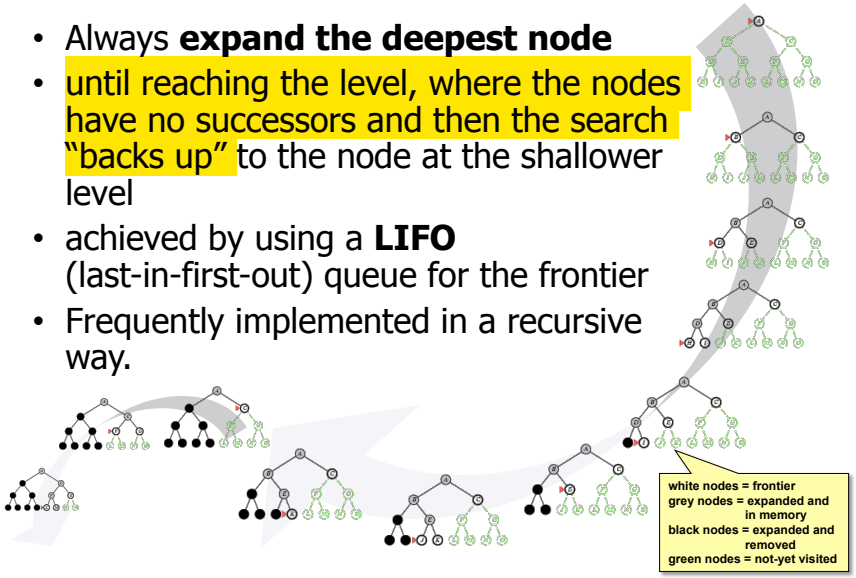
b = 10
10.000 nodes/sec.
1000 bytes/node

Uniform-cost search

- Modifying BFS for **finding an optimal solution**.
- Expand the node n with the **lowest path cost** g(n).
- Beware of zero-cost steps – they can cause cycling!
- **Completeness** (and optimality) can be guaranteed if the cost of each step is lower-bounded by some ε.
- **Time (and space) complexity**
  - depends on the path cost rather than on the depth
  - **$O(b^{1+\lfloor C^*/\varepsilon \rfloor})$**, where C* is the cost of the optimal solution
  - **Could be much worse than for BFS**, because the algorithm prefers long paths with "cheap" steps over shorter paths with more expensive steps.

Depth-first search

- Always **expand the deepest node**
- until reaching the level, where the nodes have no successors and then the search "backs up" to the node at the shallower level
- achieved by using a **LIFO** (last-in-first-out) queue for the frontier
- Frequently implemented in a recursive way.



Depth-first search (analysis)

- If the algorithm selects a wrong path, it may not find a goal during tree search (it is **not complete**) and of course it may **not find optimum**.
- **Time complexity**
  - **$O(b^m)$**, where m is the maximum depth
  - it may happen that d << m, where d is the depth of the optimum
- **Space complexity**
  - needs to store only a single path from the root to a leaf node, along with the remaining unexpanded sibling nodes
  - expanded node can be removed as soon as all its descendants have been fully explored
  - **O(bm)**, where m is the maximum visited depth
  - Space complexity can be decreased via **backtracking**!
    - generate one successor (instead of all) → **O(m)** states
    - modify the current state rather than copying it (we must be able to undo each modification when going back to generate the next successor → **O(1)** states and **O(m)** actions

Depth-limited search

- The failure of DFS in infinite state spaces can be alleviated by supplying DFS with a **predetermined depth limit l**.
  - nodes at depth l are treated as if they have no successors
  - terminates with a solution, with a failure, or with cut-off
  - **time** complexity **$O(b^l)$**, **space** complexity **O(bl)**
  - if l < d, then the algorithm is not complete (d is a depth of solution)
  - if d << l, then the algorithm explores many useless nodes

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
  RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)
function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
  cutoff-occurred? ← false
  if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
  else if DEPTH[node] = limit then return cutoff
  else for each successor in EXPAND(node, problem) do
      result ← RECURSIVE-DLS(successor, problem, limit)
      if result = cutoff then cutoff-occurred? ← true
      else if result ≠ failure then return result
  if cutoff-occurred? then return cutoff else return failure
```

How to decide the depth limit?

- using knowledge of the problem
- for example, for path finding for 20 cities, we can use the depth limit 19
- if we study the map carefully, we can discovered that any city can be reached from any other city in at most 9 steps (the **diameter** of the state space)

Iterative deepening

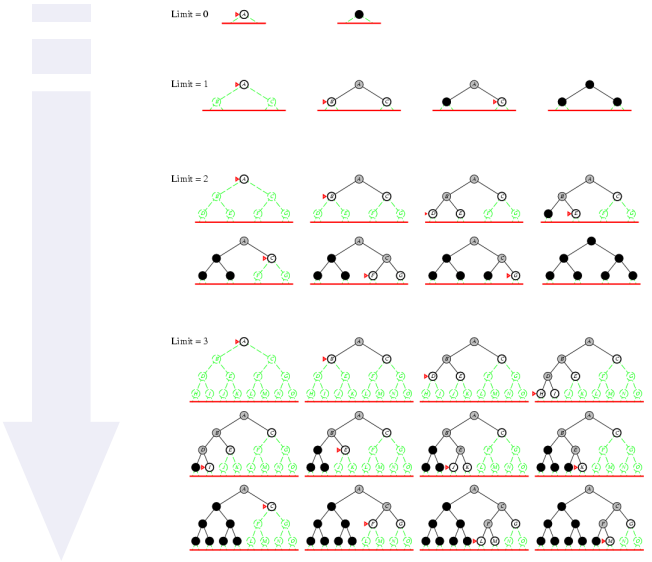
How to make the depth limited search complete?

- by gradually increasing the limit
- combines the benefits of BFS and DFS
  - **completeness** (when the branching factor is finite)
  - **optimal** (when the path cost is a non-decreasing function of the depth of the node)
  - low **memory** consumption **O(bd)**
  - What about **time complexity**?
    - d b^1 + (d-1)b^2 + … + $3b^{d-2}$ +$2b^{d-1}$ + $1b^d$ = **$O(b^d)$**
    - in fact, this is even better than BFS, which explores one more level
      - Ex. (b= 10, d = 5): BFS = 1.111.100, DLS = 111.110, IDS = 123.450

Iterative deepening is the preferred uninformed search method when the search space is large and the depth of the solution is not known.

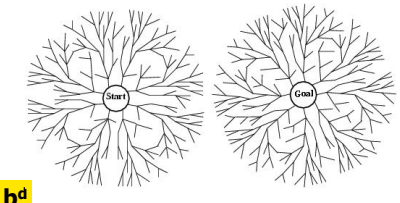
```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or failure
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
```

Iterative deepening (example)



Bidirectional search

We can run two simultaneous searches – **one forward from the initial state and the other backward from the goal** (hoping that the two searches meet in the middle).



Why?

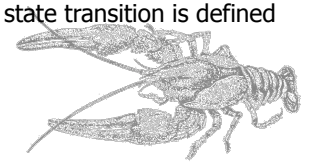
- **$b^{d/2} + b^{d/2} << b^d$**
- Ex. (b= 10, d = 6): BFS = 11.111.100, BiS = 22.200

How?

- Before expanding a node, verify that it is not present in the frontier of the other search tree.
- One of the frontiers must be kept in memory **$O(b^{d/2})$** so the intersection check can be done. With a hash table it will take constant time.
- If using breadth-first search, we obtain a **complete algorithm** (still the first solution found may not be optimal and some additional search is necessary to make sure there isn't any shortcut).

Backward search?

- Forward search explores the set of states, but **what is explored during backward search and how?**
  - If the goal is a single state, we can go through the states (path Arad → Bucharest).
  - If the goal is a set of goal states given explicitly (vacuum world), we can use a **dummy state** such that all the goal states are its predecessors or we can use a **meta-state** to describe a set of states.
  - The worst case is when there is an **abstract description of the goal state** (8-queens).
- Another problem is **definition of predecessors** of a state.
  - The predecessors of n are all those states that have n as a successor.
  - Again, the most complex case is when the state transition is defined using a general function.



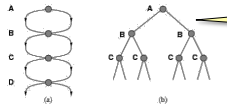
Summary

Criterion	Breadth-First	Uniform-cost	Depth-First	Depth-limited	Iterative deepening	Bidirectional search
Complete?	YES*	YES*	NO	YES, if $l \geq d$	YES*	YES*
Time	b^{d+1}	$b^{C^*/\varepsilon}$	b^m	b^l	b^d	$b^{d/2}$
Space	b^{d+1}	$b^{C^*/\varepsilon}$	bm	bl	bd	$b^{d/2}$
Optimal?	YES*	YES*	NO	NO	YES*	YES*

b – the branching factor
d – the depth of the shallowest solution
C* – the cost of optimal solution
ε – the minimal step cost (the minimal action cost)
m – the maximum depth of the search tree
l – the depth limit
* complete if b is finite (BFS)
optimal if step costs are all identical

Final notes: Graph-search

- So far we ignored one of typical problems of tree search – **expansion of already visited states**.
  - sometimes, it is not a problem (a good problem formulation, ex. 8-queens)
  - sometimes, the search tree is much bigger than the search space



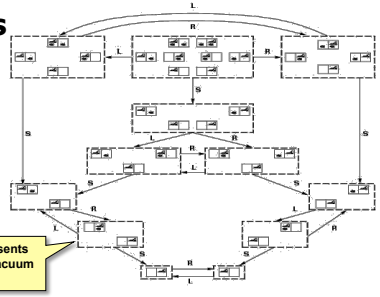
d+1 states (a) give a search tree with 2^d leaves (b)

- **How to resolve it?**
  - remember already expanded states – **closed states**
  - if the closed state is visited again, it is treated as it has no successors

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
  closed ← an empty set
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
    if STATE[node] is not in closed then
      add STATE[node] to closed
      fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Final notes: Partial information

- So far we assumed static, fully observable, discrete and deterministic environment.
- What if some **information is missing**?
  - **no sensors** (the agent does not know the current state)
    - work with **belief states** – sets of real states – looking for a belief state containing only goal states (*conformant problems*)
  - **non-deterministic actions**
    - plans with alternatives (*contingency problems*)
  - **unknown actions**
    - solved by exploration (*exploration problems*)



The belief state represents a set of real states (vacuum world).

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