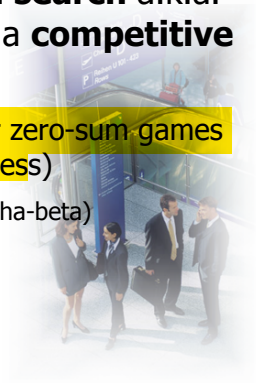
Artificial Intelligence 1

Roman Barták
Department of Theoretical Computer Science and Mathematical Logic

Adversarial Search: Games

Introduction

- So far we assumed a single-agent environment, but what if there are more agents and some of them „playing" against us?
- Today we will discuss **adversarial search** a.k.a. **game playing**, as an example of a **competitive multi-agent environment**.
  - deterministic, turn-taking, two-player zero-sum games of perfect information (tic-tac-toe, chess)
    - optimal (perfect) decisions (minimax, alpha-beta)
    - imperfect decisions (cutting off search)
  - stochastic games (backgammon)



Games

- **Mathematical game theory** (a branch of economics) views any multi-agent environment as a game, provided that the impact of each agent on others is significant.
  - environments with many agents are called economies (rather than games)
- AI deals mainly with **turn-taking**, **two-player zero-sum** games (one player wins, the other one loses).
  - **deterministic** games vs. stochastic games
  - **perfect** information vs. imperfect information
  - Why games in AI? Because games are:
    - hard to play
    - easy to model (not that many actions)
    - funny

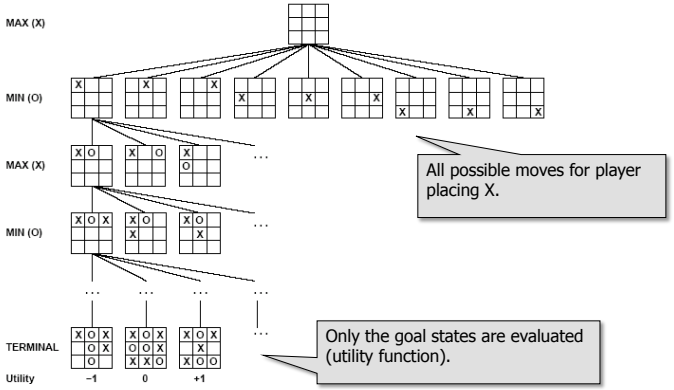
	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information		bridge, poker, scrabble nuclear war

Problem setting

- We consider two players **MAX** and **MIN**
  - MAX moves first, and then the players take turns moving until the game is over
  - we are looking for the strategy of MAX
- Again, we shall see game playing as a search problem:
  - **initial state**: specifies how the game is set up at the start
  - **successor function**: results of the moves (move, state)
    - the initial state and the successor function define the **game tree**
  - **terminal test**: true, when the game is over (a goal state)
  - **utility function**: final numeric value for a game that ends in terminal state (win, loss, draw with values +1, 0, -1)
    - higher values are better for MAX, while lower values are better for MIN

Game tree – tic-tac-toe

- Two players place X and O in an empty square until a line of three identical symbols is reached or all squares are full.



Optimal strategy

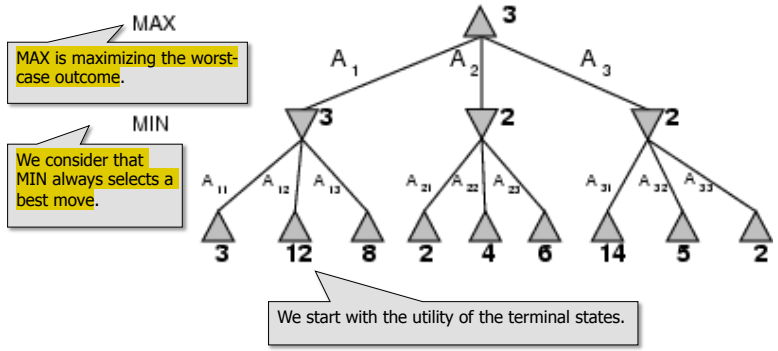
- Classical search is looking a **(shortest) path to a goal state**.
- Search for games is looking for a **path to the terminal state with the highest utility**, but MIN has something to say about it.
- MAX is looking for a contingent strategy, which specifies
  - MAX's move in the initial state
  - MAX's moves in the states resulting from every possible response by MIN
  - an **optimal strategy** leads to outcomes at least as good as any other strategy when one is playing an infallible opponent

Minimax value

- The optimal strategy can be determined from the **minimax value** of each node computed as follows:

MINIMAX-VALUE(*n*)=

UTILITY(*n*)	if *n* is a terminal state
$\max_{s \in successors(n)}$ MINIMAX-VALUE(*s*)	if MAX plays in *n*
$\min_{s \in successors(n)}$ MINIMAX-VALUE(*s*)	if MIN plays in *n*



Algorithm minimax

```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

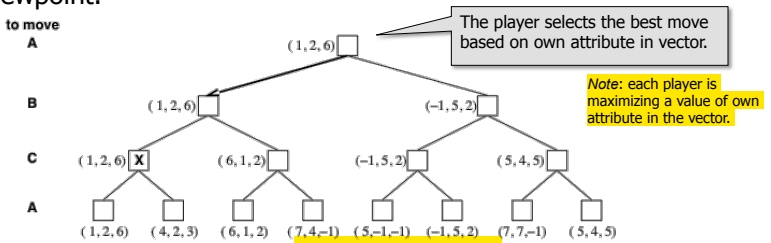
The algorithm assumes that the player plays optimally. Otherwise, the utility is even higher

- **Time complexity $O(b^m)$**
- **Space complexity O(bm)**

(b - #actions in states, m - #moves)

Minimax for more players

- For multiplayer games we can use a **vector of utility values** – this vector gives the utility of the state from each player's viewpoint.

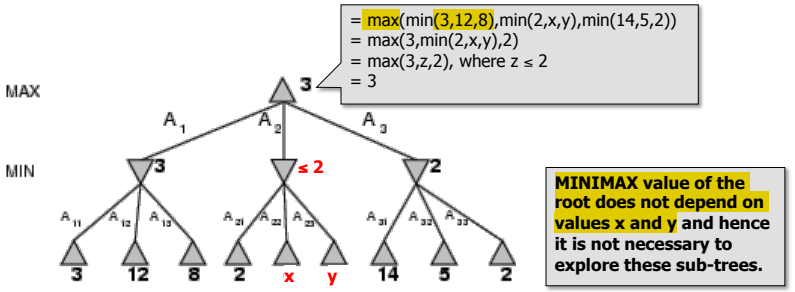


- Multiplayer games usually involve **alliances**, whether formal or informal, among the players.
  - Alliances seems to be a natural consequence of optimal strategies for each player.
  - For example, suppose A and B are in weak positions and C is in a stronger position. Then it is often optimal for both A and B to attack C rather than each other.

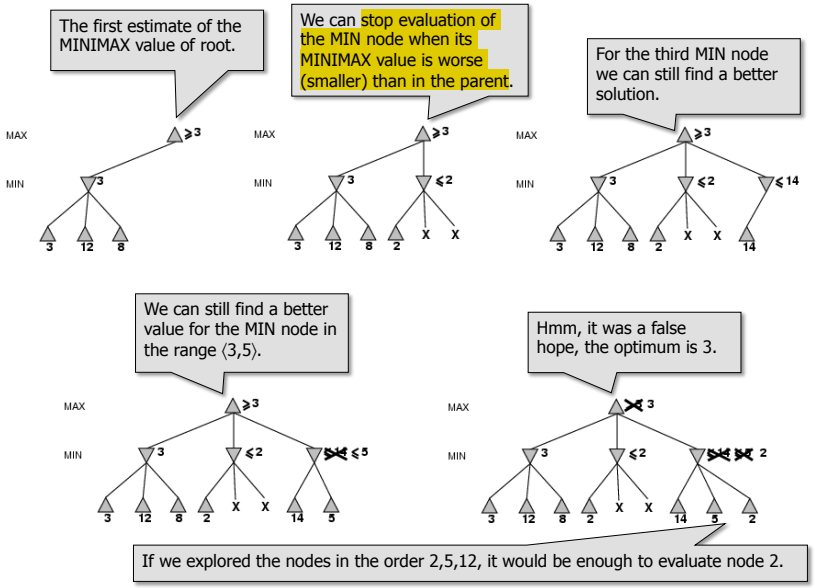
    Of course, as soon as C weakens under the joint onslaught, the alliance loses its value.

Improving minimax

- The minimax algorithm always finds an optimal strategy, but it has to explore a complete game tree.
- Can we speed-up the algorithm?
  - YES!
    We do not need to explore all states, if the are "very bad".
  - **α-β pruning** eliminates branches that cannot possibly influence the final decisions.



α-β pruning - example



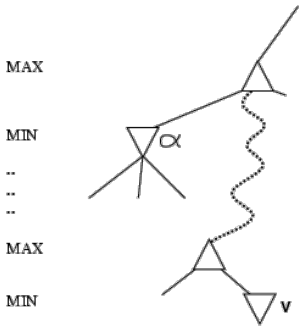
Algorithm α-β

function ALPHA-BETA-SEARCH(*state*) **returns** *an action*
  inputs: *state*, current state in game
  $v \leftarrow$ MAX-VALUE(*state*, $-\infty, +\infty$)
  return the *action* in SUCCESSORS(*state*) with value v

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(s, α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Why α-β?



- α is the value of best (i.e. the highest-value) choice we have found so far at any choice point along the path for MAX
  - if α is not worse (smaller) than v, MAX will never play in the direction to v and hence the sub-tree below v does not need to be explored
- β is the value of best (i.e. the lowest-value) choice we have found so far at any choice point along the path for MIN
  - we can similarly prune the sub-trees for MIN

Properties:

- By cutting off the sub-trees we do not miss optimum.
- By „perfect ordering" we can decrease time complexity to $O(b^{m/2})$, which gives a branching factor $\sqrt{b}$ (b for minimax), so we can solve a tree roughly twice as deep as minimax in the same amount of time.

„Imperfect" strategies

- Both minimax and α-β have to **search all the way to terminal states**.
  - This is not practical for bigger depths (depth = #moves to reach a terminal state).
- We can **cut off search** earlier and apply a heuristic evaluation function to states in the search.
  - does not guarantee finding an optimal solution, but
  - can finish search in a given time
- **Realisation:**
  - terminal test → cutoff test
  - utility function → heuristic evaluation function EVAL

Evaluation function

- Returns an estimate of the expected utility of the game from a given position (similar to the heuristic function h).
- Obviously, quality of the algorithm depends on the quality of evaluation function.

Properties:

- terminal states must be ordered in the same way as if ordered by the true utility function
- the computation must not take too long
- for nonterminal states, the evaluation function should be strongly correlated with the actual chances of winning
  - given the limited amount of computation, the best the algorithm can do is make a guess about the final outcome

- How to construct such a function?

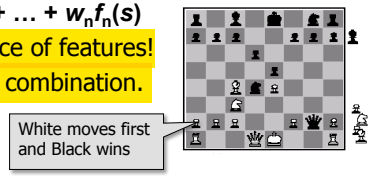
Evaluation function - examples

Expected value

- based on selected features of states, we can define various *categories* (equivalence classes) of states
- each category is evaluated based on the proportion of winning and losing states
  - EVAL = (0.72 × +1) + (0.20 × -1) + (0.08 × 0) = 0.52

„Material" value

- estimate the numerical contribution of each feature
  - chess: pawn = 1, knight = bishop= 3, rook = 5, queen = 9
- combine the contributions (e.g. weighted sum)
  - **EVAL(*s*) = $w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$**
  - The sum assumes independence of features!
  - It is possible to use non-linear combination.



Problems with cut off

- The situation may change dramatically by assuming one more move after the cut-off limit.

Identical material value (better for Black) for both states,
but **White wins the right position** by capturing the queen.

  - **quiescent**: if the opponent can capture a chess-man then the estimate is not stable and it is better to explore a few more moves (for example only selected moves)
- **horizon effect**
  - the unavoidable bad situation can be delayed after the cut-off limit (horizon) and hence it is not recognized as a bad state

Black has a better material value, but if White changes a pawn to a queen, then **White wins**.
Black may consider checking the white king so the situation does not look so bad.

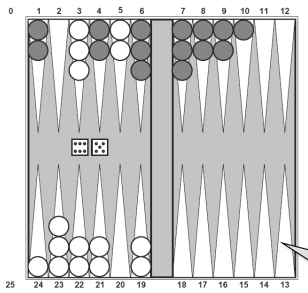
Possible improvements

- **Singular extension**
  - explore the sequence of moves that are "clearly better" than all other moves
  - a fast way to explore the area after the depth limit (quiescent is a special case)
- **Forward pruning**
  - some moves at a given state are not assumed at all (a human approach)
  - dangerous as it can miss the optimal strategy
  - safe, if symmetric moves are pruned
- **Transposition tables**
  - similarly to classical search, we can remember already evaluated states for the case when they are reached again by a different sequence of moves

Stochastic games

- In real life, many unpredictable external events can put us into unforeseen situations.
- Games mirror **unpredictability** by including a random element, such as throwing of dice.

Backgammon



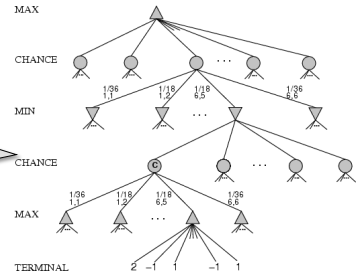
  - the goal is to move all one's pieces off the board (clockwise)
  - who finishes first, wins
  - dice are rolled to determine the legal moves
    - the total travelled distance

There are four legal moves for White:
(5-10,5-11), (5-11,19-24), (5-10,10-16), (5-11,11-16)

Playing stochastic games

- Game tree is extended with **chance nodes** (in addition to MAX and MIN nodes) describing all rolls of dice.
  - 36 results for two dice, 21 without symmetries (5-6 and 6-5)
  - chance for double is 1/36, other results 1/18

Chance nodes are added to each layer, where the move is influenced by randomness.
MAX rolls the dice here.



- Instead of the MINIMAX value, we use **expected MINIMAX value** (based on probability of chance actions):

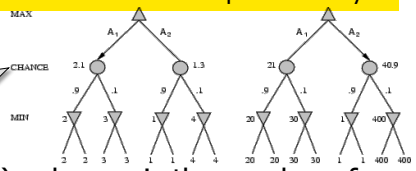
EXPECTIMINIMAX-VALUE(n)=

UTILITY(n)	if n is a terminal node
$\max_{s \in successors(n)}$ EXPECTMINIMAX-VALUE(s)	if MAX plays in n
$\min_{s \in successors(n)}$ EXPECTMINIMAX-VALUE(s)	if MIN plays in n
$\sum_{s \in successors(n)} P(s)$. EXPECTMINIMAX(s)	if n is a chance node

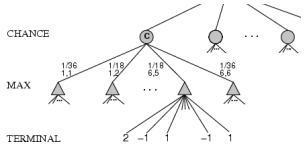
Stochastic games - discussion

- **Beware of the evaluation function** (for cut-off)
  - the absolute value of nodes may play a role
  - the values should be a linear transformation of expected utility in the node

The left tree is better for A_1 while the right tree is better for A_2, though the order of nodes is identical.



- **Time complexity** $O(b^m n^m)$, where n is the number of random moves
  - it is not realistic to reach a bigger depth especially for larger random branching
- **Using cut-off à la α-β**
  - we can cut-off the chance nodes if the evaluation function is bounded
  - the expected value can be bounded when the value is not yet computed



Card Games

- Card games may look like the stochastic games, but the dice are rolled just once at the beginning!
- Card games are an example of games with **partial observability** (we do not see opponent's cards).

Example: card game "higher takes" with open cards

Situation 1: MAX: ♥6 ♦6 ♣9 8 MIN: ♥4 ♠2 ♣10 5

1. MAX gives ♣9, MIN confirms colour ♣10 MIN wins
2. MIN gives ♠2, MAX gives ♦6 MIN wins
3. MAX gives ♥6, MIN confirms colour ♥4 MAX wins
4. MIN gives♣5, MAX confirms colour ♣8 MAX wins

– ♣9 is the optimal first move for MAX

Situation 2: MAX: ♥6 ♦6 ♣9 8 MIN: ♦4 ♠2 ♣10 5

– a symmetric case, ♣9 is again the optimal first move for MAX

Situation 3: MIN hides the first card (♥4 or ♦4), what is the optimal first move for MAX now?

– Independently of ♥4 and ♦4 the optimal first move was ♣9, so it is the first optimal move now too.
– **Really?**

Incomplete information

Example: how to become rich (a different view of cards)

- **Situation 1**: Trail A leads to a gold pile while trail B leads to a road-fork. Go left and there is a mound of diamonds, but go right and a bus will kill you (diamonds are more valuable than gold). Where to go?
  - the best choice is B and left
- **Situation 2**: Trail A leads to a gold pile while trail B leads to a road-fork. Go right and there is a mound of diamonds, but go left and a bus will kill you. Where to go?
  - B a right
- **Situation 3**: Trail A leads to a gold pile while trail B leads to a road-fork. Select the correct side and you will reach a mound of diamonds, but select a wrong side and a bus will kill you. Where to go?
  - a reasonable agent (not risking the death;-) goes A
- This is the same case as in the previous slide. We do not know what happens at the road-fork B. In the card game, we do not know which card (♥4 or ♦4) the opponent has, 50% chance of failure.
- **Lesson learnt:** We need to assume information that we will have at a given state (the problem of using ♣9 is that MAX plays differently when all cards are visible).

Computer games– the state of the art

- **Chees**
  - 1997 Deep Blue wins over Kasparov 3.5 – 2.5
  - 2006 „regular" PC (DEEP FRITZ) beats Kramnik 4 – 2
- **Checkers**
  - 1994 Chinook became the official world champion
  - 29. 4. 2007 solved – optimal policy leads to draw
- **Go**
  - branching factor 361 makes it challenging
  - today computers play at a master level (using Monte Carlo methods based on the UCT scheme)
- **Bridge**
  - 2000 GIB was twelve at world championship
  - Jack and Wbridge5 play at the level of best players