University of Colombo, Sri Lanka

University of Colombo School of Computing

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2024 — 3^{rd} Year Examination — Semester 5

IT5506 — Mathematics for Computing II

Structured Question Paper
(2 Hours)

To be completed by the candidate							
Index Number							

Important Instructions

- The duration of the paper is **2 hours**.
- The medium of instructions and questions is English. Students should answer in the medium of English language only.
- This paper has **4 questions** on **14 pages**. Answer **all** questions.
- All questions carry **equal** marks.
- Write your answers **only on the space provided** on this question paper.
- Do not tear off any part of this question paper. Under no circumstances may this paper (or any part of this paper), used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper. If a page or part of a page is not printed, please inform the supervisor/invigilator immediately.
- Any electronic device capable of storing and retrieving text, including electronic dictionaries, smartwatches, and mobile phones, is not allowed.
- Calculators are **not allowed**.
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To be completed by the examiners

1	
2	
3	
4	
Total	

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1.

(a) Given that square matrix S is denoted as $S = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

(i) Show that S^{-1} exists.

(03 Marks)

(ii) Find the inverse of the matrix S by using the Gauss-Jordan method.

(07 Marks)

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(b) Matrix multiplication is not commutative in general, but there are instances where it satisfies the commutative property

(i) Show that for two matrices A and B, it is not always true that AB=BA

(03 Marks)

(ii) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ then show that $AB = BA = 8I_3$

(04 Marks)

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(iii) If $AB = BA = 8I_3$ then show that B^{-1} =(1/8)A

(02 Marks)

(iv) Represent the following system of linear equations in matrix form $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$. Using the results from part (a)(ii) and (a)(iii), solve the system of equations.

(06 Marks)

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2.

(a). Let V be a vector space over the field F, and let $S \subseteq V$ be a subset of V

(i) For S to be a subspace over V, what conditions must be satisfied?

(06 Marks)

(ii) Any line through the origin is given by $S = \{ \begin{bmatrix} x \\ y \end{bmatrix} : ax + by = 0 , a, b \in \mathbb{R}^2 \}$. Is S a subspace of $\mathbb{R}^2$? Justify your answer.

(06 Marks)

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(b). Determine whether the following function T is a *linear transformation*.

$$T: \mathrm{R}^3 \rightarrow \mathrm{R}^3, \text{ with } T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x^3 \\ y^2 \\ \sin z \end{bmatrix}$$

(06 Marks)

(c). Consider given tow vector s $\vec{A} = [2 \quad 3 \quad 4]$ and $\vec{B} = [1 \quad 0 \quad -1]$ in R^3

(i) Find the unit vectors along $\vec{A}$ and $\vec{B}$

(02 Marks)

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(ii) Calculate the magnitudes (lengths) of $\vec{A}$ and $\vec{B}$

(02 Marks)

(iii) Compute the angle between vectors $\vec{A}$ and $\vec{B}$

(03 Marks)

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3)

(a) Write down two (2) key differences between integer programming and linear programming.

(04 Marks)

(b) Explain basic components of a linear programming model, providing examples for each component.

(06 Marks)

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(c) A company, XYZ, manufactures two (2) types of cars: Sedans and SUVs. Assembling a Sedan requires three (3) hours, while an SUV requires four (4) hours. The total time available for assembling cars is limited to 60 hours. Additionally, the company aims to produce at least twice as many Sedans as SUVs to meet sales goals. Each Sedan generates a profit of \$200, and each SUV generates a profit of \$100.

(i) Formulate a linear programming problem to determine the number of each type of car the company should produce to maximize its profit. (05 Marks)

(ii) Use the graphical method or any other method to solve the problem in (i) above and find the number of each type of car the company should produce to maximize its profit. (10 Marks)

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4)

(a) Determine the total number of possible outcomes when a die is rolled and then a coin is tossed. List all the outcomes in the form of ordered pairs.

(03 Marks)

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(b) How many different license plates can be created if a license plate consists of two letters followed by four digits, and the second letter cannot be "O"? (04 Marks)

(c) A six-sided die is rolled twice. Let X represent the sum of the numbers obtained on the two rolls.

Find: (i) $P(X=7)$ (ii) $P(X>8)$

(06 Marks)

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(d) A factory produces a batch of 5 electronic devices, and each device has a 0.8 probability of passing quality control. What is the probability that at least 3 devices pass quality control?

(04 Marks)

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(e) A theme park manager observes that guests arrive at the entrance gate at an average rate of 2 guests per minute during the peak hours of 10.00 a.m. to 11.00 a.m. Using this information, determine the following:

(i) Exactly 2 guests arrive between 10:57 a.m. and 11:00 a.m.

(ii) Fewer than 4 guests arrive between 10:57 a.m. and 11:00 a.m.

(08 Marks)

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