University of Colombo, Sri Lanka

UCSC *University of Colombo School of Computing*

BIT

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2023 — 3rd Year Examination — Semester 5

IT5506 — Mathematics for Computing II

Structured Question Paper

(2 Hours)

To be completed by the candidate

Index Number							

Important Instructions

- The duration of the paper is **2 hours.**
- The medium of instructions and questions is English. Students should answer in the medium of English language only.
- This paper has **4 questions** on **18** pages. Each question carries 25 marks. Answer **all** questions.
- Write your answers **only on the space provided** on this question paper.
- Do not tear off any part of this question paper. Under no circumstances may this paper (or any part of this paper), used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper. If a page or part of a page is not printed, please inform the supervisor/invigilator immediately.
- Any electronic device capable of storing and retrieving text, including electronic dictionaries, smartwatches, and mobile phones, is not allowed.
- Calculators are **not allowed.**
-

To be completed by the examiners

1	
2	
3	
4	
Total	

Index Number | | | | | | | |

1. George loves Food X and Food Y. He has decided to go on an exclusive diet consisting primarily of Food X and Food Y for all his meals. Recognizing the potential health risks associated with this choice of diet, George is keen on ensuring that he consumes the optimal quantities of both X and Y to meet essential daily nutritional needs. He has obtained the following nutritional and cost information.

Ingredient	**Grams of Ingredient per Serving**		**Daily Requirement (Grams)**
	Food X	**Food Y**	
Carbohydrates	5	20	≥ 60
Protein	20	5	≥ 40
Fat	10	5	≤ 50

The daily cost per serving of Food X and Food Y are $8 and $4 respectively. George's goal is to determine the minimum-cost daily servings of X and Y to fulfil these nutritional requirements. Assume that all assumptions of a linear programming problem are held.

(a). Formulate a linear programming problem to **find the minimum daily requirement of X and Y to minimize the overall cost.**

[5 marks]

① (1)

X - Servings of Food x in diet
Y - Servings of Food Y in diet

Minimize $\$8X + \$4Y$

ⓐ $5x + 20y \geq 60 \Rightarrow x=0, y=3 / y=0, x=12$

$20x + 5y \geq 40 \Rightarrow x=0, y=8 / y=0, x=2$

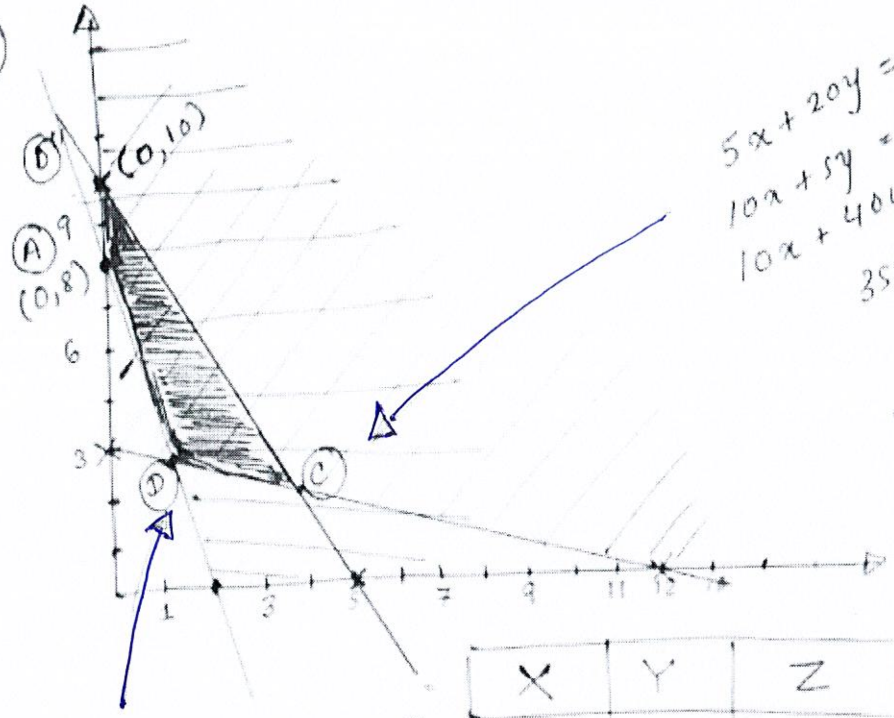
$10x + 5y \leq 50 \Rightarrow x=0, y=10 / y=0, x=5$

$x \geq 0, \ y \geq 0.$

Index Number

(b) Use the graphical method to solve the problem and find the minimum amount of daily servings of X and Y and the minimum cost incurred. [10 marks]

(b)



$5x + 20y = 60$
$10x + 5y = 50$
$10x + 40y = 120$
$35y = 70$
$y = 2$
$x = 4$

$5x + 20y = 60$
$20x + 5y = 40$
$20x + 80y = 240$
$75y = 200$
$y = \frac{200}{75} = \frac{8}{3}$
$5x = 60 - 20 \times \frac{8}{3}$
$x = 4/3$
$x = \frac{4}{5}$

	X	Y	Z
A	0	8	32
B	0	10	40
C	4	2	40
D	4/3	8/3	$\frac{64}{3}$ $21\frac{1}{3}$ ← min

$8x + 4y$

$\frac{32}{3} + \frac{32}{3} = \frac{64}{3}$

Index Number

Index Number

(c) How does the optimal solution in (b) change if the daily requirement of **Fat is made greater than or equal to 50 grams**. (10 Marks)

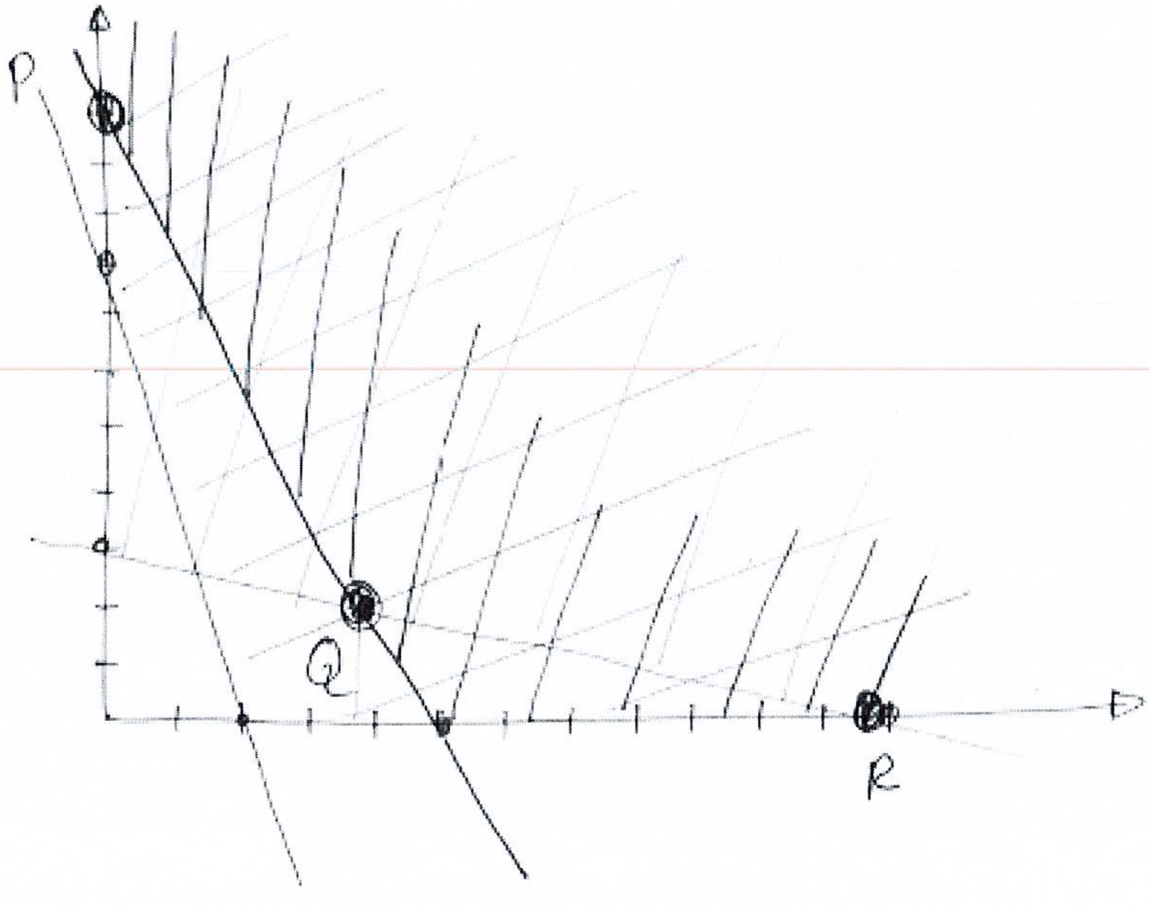
(c)

$5x + 20y \geq 60$ —①

$20x + 5y \geq 40$

$10x + 5y \geq 50$ $x = 0$ $y = 10$

$x \geq 0, y \geq 0.$ $y = 0$ $x = 5$



Index Number | | | | | | | |

	X	Y	Z
P	0	10	40
Q	4	2	40
R	12	0	96

$Z = 8x + 4y$

Either 10 grams of Food Y or

4 g of X & 2 g of Y required daily.

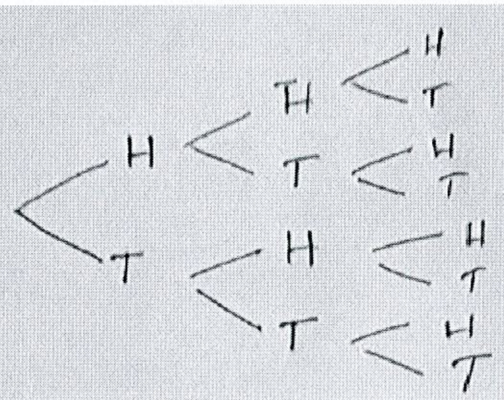
Index Number

2.

a) A fair coin is tossed three times. If X is a random variable that indicates the number of heads that are observed,

(i) Construct the probability distribution table of X. (2 Marks)
(ii) Find the probability that at least two heads are observed in three tossings. (2 Marks)

(2) (a)



H - Getting Head

H	0	1	2	3
P(H)	$1/8$	$3/8$	$3/8$	$1/8$

(ii) $p(2) + p(3) = \frac{3}{8} + \frac{1}{8} = \underline{\underline{0.5}}$

Index Number

b) According to the local health department, the average number of flu cases per year in a specific region is three (3) per 100,000 population, following a poisson distribution. Find the probability that in a town of population 200,000, there will be two (2) flu cases per year.

(Note that $e^{-3} = 0.004979$, $e^{-4} = 0.01831$, $e^{-6} = 0.00248$, $e^{-8} = 0.00034$)

(6 Marks)

(b) $\lambda = 2 \times 3 = 6$

$$P(x=2) = \frac{e^{-6} \times 6^2}{2!} = \frac{0.00248 \times 36}{\cancel{2}}$$

$$= 0.04462$$

Index Number

c) At a toy production facility, the probability of a defective toy is 0.2. Assuming and then justifying a suitable probability distribution for defects, find the **(a) Mean** and, **(b) Standard Deviation** for the number of defective toys in a total of 500 toys. (6 Marks)

c $p = 0.2$

$q = 1 - p = 0.8$

$E(x) = np = 500 \times 0.2 = 100$ //

$sd = \sqrt{npq} = \sqrt{500 \times 0.2 \times 0.8}$

$= \sqrt{80}$

$\simeq 8.95$

Index Number

d) If the weights of 300 individuals are normally distributed with a mean of 68 kilograms and a standard deviation of 3 kilograms, how many individuals have weights greater than 72 kilograms.

(The standard normal distribution table can be found on the last page of the paper) (9 Marks)

$$P(x > 72)$$

$$= 1 - P(x < 72)$$

$$= 1 - P\left(\frac{x-\mu}{\sigma} < \frac{72-68}{3}\right)$$

$$= 1 - P(z < 1.33)$$

$$= 1 - 0.9032$$

$$= \underline{\underline{0.0968}}$$

Individuals weight greater than 72 KGs = 0.0968 x 300 = 29 individuals

Index Number

3.

(a) Given that square matrix A is denoted as $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$

(i) Show that A^{-1} exists.

(03 Marks)

Determinat of $A = 2(12+10) - 4(28-5) - 6(-14-3)$

$= 59, \neq 0$

Determinant of $A \neq 0$

Inverse of A exists.

(ii) Calculate all the Minors of A and find the *Minor Matrix* A.

(06 Marks)

$\begin{bmatrix} \cancel{4} & \cancel{4} & \cancel{6} \\ \cancel{7} & 3 & 5 \\ \cancel{1} & -2 & 4 \end{bmatrix}$ $M_{11} = \det \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix} = 22$.

Similarly calculate all 9 minors of A.

$\begin{bmatrix} 22 & 23 & -17 \\ 4 & 14 & -8 \\ 38 & 52 & -22 \end{bmatrix}$

Index Number

$C_{ij} = M_{ij}(-1)^{i+j}$

(iii) Calculate the cofactors of *A* and find the *Cofactor Matrix A*

(02 Marks)

$C_{ij} = M_{ij}(-1)^{i+j}$

use above relationship and calculate.
all C_{ij} cofactors of matrix A.

Index Number | | | | | | | |

Cofactor Matrix.

$$\begin{bmatrix} 22 & -23 & -17 \\ -4 & 14 & 8 \\ 38 & -52 & -22 \end{bmatrix}$$

(iv) Calculate the *Adjoint* of the Matrix *A*.

(02 Marks)

Adjoint of matrix A = Transpose of Cofactor matrix ~~A~~.

$$\begin{bmatrix} 22 & -4 & 38 \\ -23 & -14 & -52 \\ -17 & 8 & 22 \end{bmatrix}$$

(v) Find the inverse of A, A^{-1}.

(02 Marks)

$$A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{54}\begin{bmatrix} 22 & -4 & 38 \\ -23 & -14 & -52 \\ -17 & 8 & -22 \end{bmatrix}$$

Index Number | | | | | | | |

(b) Represent the following system of linear equations in matrix form and use elementary row operations on matrices to solve the given system of linear equations.

$$x + y + z = 2$$
$$2x - y + 3z = -7$$
$$x + y - z = 6$$

(10 Marks)

$x + y + z = 2$
$2x - y + 3z = -7$
$x + y - z = 6$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 6 \end{bmatrix}$$

$\downarrow$ Elementary Row operations.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

Index Number | | | | | | | |

4.

(a) Let V be a vector space over R (Real numbers) and $x, y, z \in V$. Focus on the axioms of a vector space.

(i) State relevant axioms and prove the following property:
If $x + y = x + z$ then $y = z$.

(04 Marks)

$x \in V$
$-x$ is exists and it is the additive. invers of x.

$x + y = x + z$

$-x + (x+y) = -x + (x+z)$ — additive invers

$\underbrace{(-x+x)}_{0} + y = \underbrace{(-x+x)}_{0} + z$ — associativity of addition

$0 + y = 0 + z$

$y = z$

(ii) State relevant axioms and prove the following property:
If $\forall\, x \in V$ the additive inverse $-x$ is unique.

(04 Marks)

$x \in V$
$-x$ is the additive invers of x.

$x + (-x) = 0$ ——— (1)

Lets assume. there is anoter aditive. invers. of x. Suppose it is w.

$w \in V$

$x + w = 0$ ——— (2) w also an additive invers

(1), (2)

$x + (-x) = x + w$

$(-x + x) - (x) = (-x + x) + w$

$0 - x = ~~\text{then}~~ 0 + w$

$w = -x \Rightarrow$ additive invers is uniqu.

Index Number | | | | | | | |

(iii) If V is given by $V = \left\{\begin{bmatrix} x \\ y \end{bmatrix} : \ x \geq 0, y \ \geq 0\right\}$, is V a vector space? Prove.

(04 Marks)

If V is a vetor space $\underline{u}, \underline{v} \in V$

$\underline{u} + \underline{v} \in V$ $c\underline{u} \in V$, c is a scalar.

$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $c = -1$

$c\underline{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \notin V$

$\therefore$ V is not a vector space.

(iv) If V is given by $V = \left\{\begin{bmatrix} x \\ y \end{bmatrix} : \ xy \geq 0\right\}$, is V a vector space? Prove.

(04 Marks)

If V is a Vetor space $\underline{u}, \underline{v} \in V$

$\underline{u} + \underline{v} \in V$ $c\underline{u} \in V$, c is a scalar.

$\underline{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$\underline{u} + \underline{v} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \in V$ $xy \ngtr 0$

V is not a vector space.

Index Number | | | | | | | |

(b) Let X and Y be two vector spaces over the same field A. What are the conditions that need to be satisfied by the function $T: X \longrightarrow Y$ to become a *linear transformation*?

(04 Marks)

$T(x+y) = Tx + Ty \quad \forall x \in X \; y \in X$

$T(\alpha x) = \alpha Tx \quad \forall \alpha \in A, \; x \in X$

(c) Determine whether the following function T is a *linear transformation*.

$$T: \mathrm{R}^2 \rightarrow \mathrm{R}^2, \text{ with } T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$$

(05 Marks)

Suppose. $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad T(x) = T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$2T(x) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ — (1)

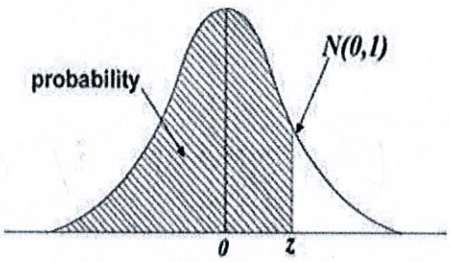
$T(2x) = T\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ — (2)

$2T(x) \neq T(2x)$

$\therefore$ T is not a linear transformation.

Index Number

The Standard Normal Distribution Table



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of Z, the standardized normal deviate, (the proportion P, of the distribution less than Z) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value X is obtained by calculating $Z = (X - \mu)/\sigma$ and reading the proportion corresponding to this value of Z.

		Z	P	Z	P
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997

****