Process	$gg \to hh$	$gg \to hhg$	$gq \to hhq$	$g\bar{q} \to hh\bar{q}$	$q\bar{q} \to hhg$
σ(14 TeV) [fb]	26.2(1)	9.5(1)	1.80(2)	0.411(6)	0.062(1)
σ(33 TeV) [fb]	145(3)	70.2(9)	10.0(1)	3.39(5)	0.206(3)
σ(100 TeV) [fb]	883(5)	555(7)	60.6(9)	27.1(4)	0.79(1)

Table 1. Cross sections for the partonic $pp \to hh + X$ and for the sub-processes contributing to $pp \to hhj + X$ at 14, 33 and 100 TeV. For the case of real emission, a cut of $p_\perp > 60$ GeV was placed on the associated parton. The factorisation/renormalisation scales were both fixed to $\mu = m_h + p_{\perp,j}$, where $p_{\perp,j}$ is the transverse momentum of the associated parton in the centre of mass frame.

work by partitioning phase space, by means of a jet algorithm, such that the distribution of jets corresponds to that of the partons in the matrix elements, while the distribution of radiation inside the jets is appropriately developed by the shower. In addition, both the MLM and CKKW algorithms augment the distribution of radiation in the matrix element region with Sudakov suppression effects, not present in the matrix elements themselves, thus smoothing the transition from one radiation pattern to another at the phase space partition.[3,4]

HERWIG++ [55–58] includes an implementation of the MLM merging scheme. The current version of the merging algorithm has been validated against its FORTRAN HERWIG [59] counterpart for several processes. For the purposes of this project, the implementation has undergone minor modifications, to accommodate the use of internally-generated matrix elements. We use this algorithm in conjunction with the parton shower in order to merge the two. We fix the factorisation and renormalization scales to be equal, $\mu_F = \mu_R = \mu = \nu(m_h + p_\perp^{hh})$, where ν is a parameter which we vary, m_h and $p_\perp^{hh}$ are the Higgs boson mass and the transverse momentum (as defined in the centre-of-mass frame of the hard process) of the Higgs boson pair respectively. Note that for the LO hh process, $p_\perp^{hh} = 0$ and hence this implies that $\mu = \nu m_h$ for all, even showered, LO samples. We call the merging scale $E_{T\mathrm{clus}}$, inspired by the way the MLM method is implemented in the HERWIG++ generator. We call the lowest-order sample '0-jet' and the sample including one real emission '1-jet'. Broadly speaking, after showering is performed in HERWIG++, the MLM method will effectively veto all events in the '0-jet' sample that contain a jet with transverse momentum larger than $E_{T\mathrm{clus}}$. This will result in what we will call the '0-jet exclusive' sample. In the showered '1-jet' sample the MLM algorithm will effectively veto any events with jets that have not 'matched'[5] the given extra parton produced in association with the Higgs boson pair, as well as events that contain jets harder than the 'matched' jet. The resulting sample is called '1-jet inclusive', meaning it contains no 0-jet contributions but contains jets coming from

[3]For a full, comparative description of the available schemes, see Ref. [46].

[4]It is also conceivable, at least in the case of one extra associated parton, to perform a simulation with the MC@NLO or POWHEG matching prescriptions, with an arbitrary virtual contribution which can be set to zero [51–54].

[5]The term 'matched' in the MLM prescription refers to whether a jet is found to be within a certain distance ΔR, from a given hard parton that appears in the pre-showered event. By default this is taken to be $1.5 \times R_{clus}$, where the R_{clus} is the clustering cone size used in the merging.