扫地机定位方案（初稿）

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1. VXO 框架

目前的扫地机拥有、可用于定位的传感器有：双目、轮式里程计、IMU、光流；根据前期对传感器的试验、文献阅读，笔者选定使用双目、轮式里程计、IMU 的陀螺仪构建定位方案，各传感器数据的时序关系如图 1 所示。

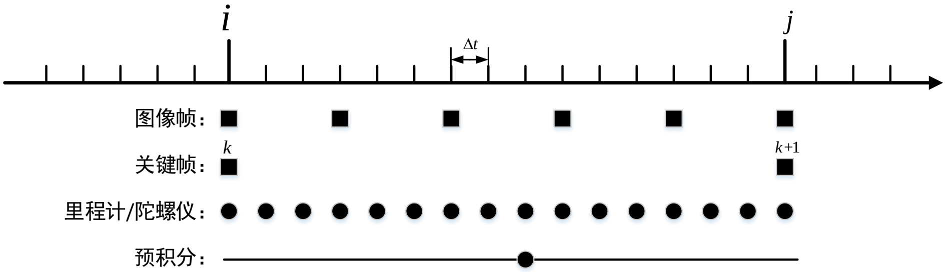

图 1 各传感数据的时序关系

该方案对应的因子图如图 2 所示。另外，光流具有与轮式里程计相同属性的测量值，因此光流预期可用于：辅助判断轮子是否打滑、与轮式里程计融合后传入速度信息（待定）。该方案具有如下特点：

a) 基于关键帧的滑动窗口优化框架；
b) 系统建模为三维运动；
c) 使用二维平面运动约束；
d) 轮式里程计与陀螺仪预积分融合；
e) 在线更新相机与轮式里程计外参；
f) 回环检测、重定位、建图作为后端，额外的线程，主要服务于定位、建图的精度提升（图 2 中未画出）；
g) 基于 EKF 航位推算主要服务于运动控制（已初步验证，图 2 中未画出），VXO 融合后的定位信息会反馈更新 EKF 相关状态；

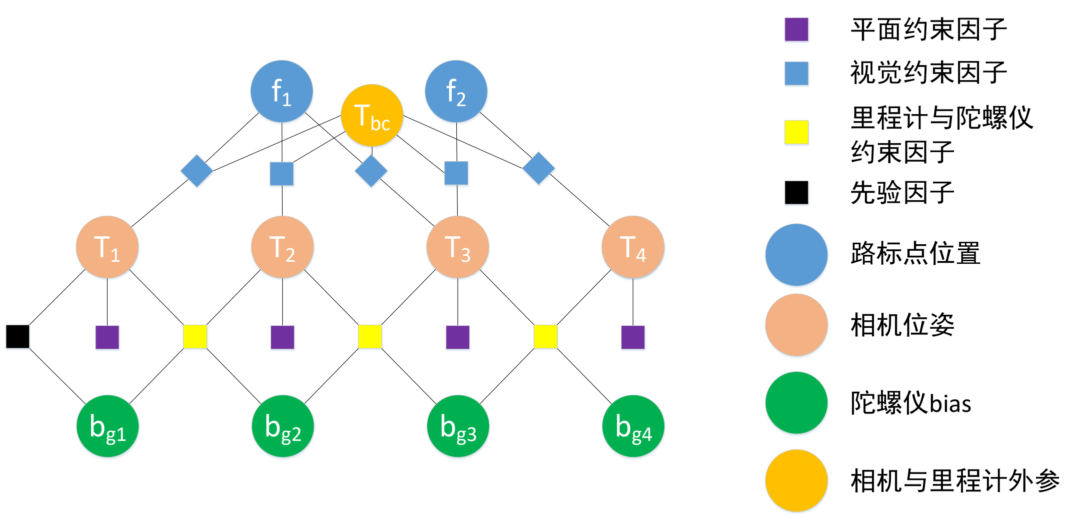

图 2 VXO 框架因子图

2. 符号说明

一些约定：

误差 = 测量 – 估计

李群流形采用右乘更新

$\mathbf{q}_{\mathrm{AB}}$：表征坐标系 B 到坐标系 A 的四元数，采用 Hamilton 形式

$\mathbf{R}_{\mathrm{AB}}$：表征坐标系 B 到坐标系 A 的旋转矩阵

${}^{A}\mathbf{p}_{B}$：表征点 B 在坐标系 A 的位置

坐标系：

系统使用的坐标系包括：世界坐标系 W、体坐标系 B、相机坐标系 C、里程计坐标系 O、陀螺仪坐标系 I；其中，选择里程计坐标系选择为体坐标系。

状态向量：

$$\chi=[\mathbf{x}_0,\mathbf{x}_1,\cdots\mathbf{x}_{\mathrm{n}},\mathbf{x}_{\mathrm{bc}},\lambda_0,\lambda_1,\cdots\lambda_m]$$
$$\mathbf{x}_{\mathrm{k}}=[{}^{w}\mathbf{p}_{b_k},\mathbf{q}_{wb_k},\mathbf{b}_{g_k}],k\in[0,n]$$
$$\mathbf{x}_{\mathrm{bc}}=[{}^{b}\mathbf{p}_{c},\mathbf{q}_{bc}]$$

最小化目标函数：

$$\chi^{*}=\arg\min_{\chi}\left\|\mathbf{b}_p-\mathbf{H}_p\chi\right\|^2+\sum_{k\in\kappa}\rho\left(\left\|\mathbf{r}_{o_{k,k+1}}\right\|^2_{\mathrm{I}_{o_{k,k+1}}}\right)+\sum_{k\in\kappa}\sum_{l\in L}\rho\left(\left\|\mathbf{r}_{C_{kl}}\right\|^2_{\mathrm{I}C_{kl}}\right)+\sum_{k\in\kappa}\rho\left(\left\|\mathbf{r}_{p_k}\right\|^2_{\mathrm{I}_{p_k}}\right)$$

3. 视觉因子

1) 测量值

路标点 l 在第 k 时刻归一化相机坐标系的坐标：

$$^{c_k}\bar{\mathbf{P}}_l = \pi_c^{-1}\left(\begin{bmatrix} ^{c_k}u_l \\ ^{c_k}v_l \end{bmatrix}\right)$$

2) 估计值

路标点 l 在 **k 时刻相机坐标系**的坐标(i 表示第一次看到路标点 l 的时刻)：

左相机：

$$^{c_k}\mathbf{P}_l = \mathbf{R}_{bc}{}^T(\mathbf{R}_{wb_k}{}^T(\mathbf{R}_{wb_i}(\mathbf{R}_{bc}\frac{1}{\lambda_l}\pi_c^{-1}\left(\begin{bmatrix} ^{c_i}u_l \\ ^{c_i}v_l \end{bmatrix}\right) + {}^b\mathbf{p}_c) + {}^w\mathbf{p}_{b_i} - {}^w\mathbf{p}_{b_k}) - {}^b\mathbf{p}_c)$$

右相机：

$\mathbf{R}_{LR}$、${}^L\mathbf{p}_R$ 表征左右相机外参，有

$$\mathbf{R}_{bc_r} = \mathbf{R}_{bc}\mathbf{R}_{LR}$$
$$^b\mathbf{p}_{c_r} = \mathbf{R}_{bc}{}^L\mathbf{p}_R + {}^b\mathbf{p}_c$$

代入有：

$$^{c_k}\mathbf{P}_l = \mathbf{R}_{LR}{}^T\mathbf{R}_{bc}{}^T(\mathbf{R}_{wb_k}{}^T(\mathbf{R}_{wb_i}(\mathbf{R}_{bc}\mathbf{R}_{LR}\frac{1}{\lambda_l}\pi_c^{-1}\left(\begin{bmatrix} ^{c_i}u_l \\ ^{c_i}v_l \end{bmatrix}\right) + \mathbf{R}_{bc}{}^L\mathbf{p}_R + {}^b\mathbf{p}_c) + {}^w\mathbf{p}_{b_i} - {}^w\mathbf{p}_{b_k}) - \mathbf{R}_{bc}{}^L\mathbf{p}_R - {}^b\mathbf{p}_c)$$

3) 误差值

路标点在 k 时刻归一化相机坐标系的坐标差值；

$$\mathbf{r}_{c_{kl}} = \begin{bmatrix}\mathbf{e}_1 & \mathbf{e}_2\end{bmatrix}^T\left(^{c_k}\bar{\mathbf{P}}_l - \frac{^{c_k}\mathbf{P}_l}{\left\|^{c_k}\mathbf{P}_l\right\|}\right)$$

4) 信息矩阵

$$\mathrm{I}_{c_{kl}} = \begin{bmatrix} (fx/\sigma_u)^2 & 0 \\ 0 & (fy/\sigma_v)^2 \end{bmatrix}$$

σ_u、σ_v 分别为特征点提取误差，可假定二者为 1.5 像素

5) 雅克比矩阵

$$\mathbf{J}_{c_{kl}}=\frac{\partial \mathbf{r}_{c_{kl}}}{\partial \boldsymbol{\chi}}=-\frac{1}{\left\|{}^{c_k}\mathbf{P}_l\right\|}\begin{bmatrix}\mathbf{e}_1 & \mathbf{e}_2\end{bmatrix}^T\frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial \boldsymbol{\chi}}$$

$$=-\frac{1}{\left\|{}^{c_k}\mathbf{P}_l\right\|}\begin{bmatrix}\mathbf{e}_1 & \mathbf{e}_2\end{bmatrix}^T\left[\mathbf{0}\quad \frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial \mathbf{x}_i}\quad \mathbf{0}\quad \frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial \mathbf{x}_k}\quad \mathbf{0}\quad \frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial \mathbf{x}_{bc}}\quad \mathbf{0}\quad \frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial \lambda_l}\quad \mathbf{0}\right]$$

a) 关于第 i 时刻相机位姿的雅克比

右相机使用$\mathbf{R}_{bc_r}$、${}^b\mathbf{p}_{c_r}$ 替换$\mathbf{R}_{bc}$、${}^b\mathbf{p}_c$

假定：${}^{b_i}\mathbf{P}_l=\mathbf{R}_{bc}\frac{1}{\lambda_l}\pi_c^{-1}\left(\begin{bmatrix}{}^{c_i}u_l\\ {}^{c_i}v_l\end{bmatrix}\right)+{}^b\mathbf{p}_c$

$$\frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial \mathbf{R}_{wb_i}}=-\mathbf{R}_{bc}{}^T\mathbf{R}_{wb_k}{}^T\mathbf{R}_{wb_i}\left[{}^{b_i}\mathbf{P}_l\right]_\times$$

$$\frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial\,{}^w\mathbf{p}_{b_i}}=\mathbf{R}_{bc}{}^T\mathbf{R}_{wb_k}{}^T$$

b) 关于第 k 时刻相机位姿的雅克比

右相机使用$\mathbf{R}_{bc_r}$、${}^b\mathbf{p}_{c_r}$ 替换$\mathbf{R}_{bc}$、${}^b\mathbf{p}_c$

$$\frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial \mathbf{R}_{wb_k}}=\mathbf{R}_{bc}{}^T\left[\mathbf{R}_{wb_k}{}^T(\mathbf{R}_{wb_i}{}^{b_i}\mathbf{P}_l+{}^w\mathbf{p}_{b_i}-{}^w\mathbf{p}_{b_k})\right]_\times$$

$$\frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial\,{}^w\mathbf{p}_{b_k}}=-\mathbf{R}_{bc}{}^T\mathbf{R}_{wb_k}{}^T$$

c) 关于相机与里程计外参的雅克比

左相机：

$$\frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial \mathbf{R}_{bc}}=\left[\mathbf{R}_{bc}{}^T\mathbf{R}_{wb_k}{}^T\mathbf{R}_{wb_i}\mathbf{R}_{bc}\frac{1}{\lambda_l}\pi_c^{-1}\left(\begin{bmatrix}{}^{c_i}u_l\\ {}^{c_i}v_l\end{bmatrix}\right)\right]_\times-\mathbf{R}_{bc}{}^T\mathbf{R}_{wb_k}{}^T\mathbf{R}_{wb_i}\mathbf{R}_{bc}\left[\frac{1}{\lambda_l}\pi_c^{-1}\left(\begin{bmatrix}{}^{c_i}u_l\\ {}^{c_i}v_l\end{bmatrix}\right)\right]_\times$$

$$+\left[\mathbf{R}_{bc}{}^T\left(\mathbf{R}_{wb_k}{}^T\left(\mathbf{R}_{wb_i}{}^b\mathbf{p}_c+{}^w\mathbf{p}_{b_i}-{}^w\mathbf{p}_{b_k}\right)-{}^b\mathbf{p}_c\right)\right]_\times$$

$$\frac{\partial\,{}^{c_k}\mathbf{P}_l}{\partial\,{}^b\mathbf{p}_c}=\mathbf{R}_{bc}{}^T\mathbf{R}_{wb_k}{}^T\mathbf{R}_{wb_i}-\mathbf{R}_{bc}{}^T$$

右相机：

$$\frac{\partial\, {}^{c_k}\mathbf{P}_l}{\partial \mathbf{R}_{bc}} = \left(\begin{array}{l} \left[\mathbf{R}_{bc_r}{}^{T}\mathbf{R}_{wb_k}{}^{T}\mathbf{R}_{wb_i}\mathbf{R}_{bc_r}\frac{1}{\lambda_l}\pi_c{}^{-1}\left(\begin{bmatrix} {}^{c_i}u_l \\ {}^{c_i}v_l \end{bmatrix}\right)\right]_{\times} - \mathbf{R}_{bc_r}{}^{T}\mathbf{R}_{wb_k}{}^{T}\mathbf{R}_{wb_i}\mathbf{R}_{bc_r}\left[\frac{1}{\lambda_l}\pi_c{}^{-1}\left(\begin{bmatrix} {}^{c_i}u_l \\ {}^{c_i}v_l \end{bmatrix}\right)\right]_{\times} \\ + \left[\mathbf{R}_{bc_r}{}^{T}\left(\mathbf{R}_{wb_k}{}^{T}\left(\mathbf{R}_{wb_i}{}^{b}\mathbf{p}_{c_r} + {}^{w}\mathbf{p}_{b_i} - {}^{w}\mathbf{p}_{b_k}\right) - {}^{b}\mathbf{p}_{c_r}\right)\right]_{\times} \end{array} \right)\mathbf{R}_{LR}{}^{T}$$

$$\frac{\partial\, {}^{c_k}\mathbf{P}_l}{\partial\, {}^{b}\mathbf{p}_c} = \mathbf{R}_{bc_r}{}^{T}\mathbf{R}_{wb_k}{}^{T}\mathbf{R}_{wb_i} - \mathbf{R}_{bc_r}{}^{T}$$

d) 关于路标点逆深度的雅克比

右相机使用$\mathbf{R}_{bc_r}$、${}^{b}\mathbf{p}_{c_r}$替换$\mathbf{R}_{bc}$、${}^{b}\mathbf{p}_c$

$$\frac{\partial\, {}^{c_k}\mathbf{P}_l}{\partial \lambda_l} = -\frac{1}{\lambda_l{}^2}\mathbf{R}_{bc}{}^{T}\mathbf{R}_{wb_k}{}^{T}\mathbf{R}_{wb_i}\mathbf{R}_{bc}\pi_c{}^{-1}\left(\begin{bmatrix} {}^{c_i}u_l \\ {}^{c_i}v_l \end{bmatrix}\right)$$

4. 平面约束因子

一些约定运算：

$$\mathrm{SE2} \rightarrow SE3: \boldsymbol{\xi}(\mathbf{v}) = \begin{bmatrix} v_1 & v_2 & 0 & 0 & 0 & \phi \end{bmatrix}$$

$$SE3 \rightarrow SE2: \mathbf{v}(\boldsymbol{\xi}) = \begin{bmatrix} r_1 & r_2 & \omega_3 \end{bmatrix}$$

$$\mathbf{T}_{wb_k} = \begin{bmatrix} \mathbf{R}_{wb_k} & {}^{w}\mathbf{p}_{b_k} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{T}_{b_k w} = \mathbf{T}_{wb_k}{}^{-1} = \begin{bmatrix} \mathbf{R}_{wb_k}{}^{T} & -\mathbf{R}_{wb_k}{}^{T}\,{}^{w}\mathbf{p}_{b_k} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{T}_{bc} = \begin{bmatrix} \mathbf{R}_{bc} & {}^{b}\mathbf{p}_c \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{T}_{cb} = \mathbf{T}_{bc}{}^{-1} = \begin{bmatrix} \mathbf{R}_{bc}{}^{T} & -\mathbf{R}_{bc}{}^{T}\,{}^{b}\mathbf{p}_c \\ \mathbf{0} & 1 \end{bmatrix}$$

1) 测量值

$$\mathbf{v}_{wb_k} = \mathbf{v}(Log(\mathbf{T}_{wb_k}))$$

$$\overline{\mathbf{T}}_{wb_k} = Exp(\boldsymbol{\xi}(\mathbf{v}_{wb_k}))$$

2) 估计值

$$\mathbf{T}_{wb_k} = \mathbf{T}_{wb_k}$$

3) 误差值

$$\mathbf{r}_{p_k} = \overline{\mathbf{T}}_{wb_k} \ominus \mathbf{T}_{wb_k} = Log(\mathbf{T}_{wb_k}{}^{-1}\overline{\mathbf{T}}_{wb_k}) = Log\left(\mathbf{T}_{wb_k}{}^{-1}Exp\left(\boldsymbol{\xi}\left(\mathbf{v}\left(Log\left(\mathbf{T}_{wb_k}\right)\right)\right)\right)\right)$$

4) 信息矩阵

$$
\begin{aligned}
&\Delta\boldsymbol{\xi}_{wb_k} = \bar{\boldsymbol{\xi}}_{wb_k} - \hat{\boldsymbol{\xi}}_{wb_k} \\
&= Log(\bar{\mathbf{T}}_{wb_k}) - Log(\mathbf{T}_{wb_k}) \\
&= Log(\mathbf{T}_{wb_k} Exp(\mathbf{r}_{p_k})) - Log(\mathbf{T}_{wb_k}) \\
&= \mathbf{J}_r(\hat{\boldsymbol{\xi}}_{wb_k})^{-1}\mathbf{r}_{p_k}
\end{aligned}
$$

令 $\boldsymbol{\Omega}_{\mathrm{p_k}} = diag(0 \quad 0 \quad \frac{1}{\sigma_{r3}{}^2} \quad \frac{1}{\sigma_{\omega 1}{}^2} \quad \frac{1}{\sigma_{\omega 2}{}^2} \quad 0)$，则有

$$
\mathrm{I}_{p_k} = \mathbf{J}_r(\hat{\boldsymbol{\xi}}_{wb_k})^{-T}\boldsymbol{\Omega}_{\mathrm{p_k}}\mathbf{J}_r(\hat{\boldsymbol{\xi}}_{wb_k})^{-1}
$$

5) 雅克比矩阵

$$
\mathbf{J}_{p_k} = \frac{\partial \mathbf{r}_{p_k}}{\partial \boldsymbol{\chi}} = \begin{bmatrix} \mathbf{0} & \frac{\partial \mathbf{r}_{p_k}}{\partial \mathbf{x}_k} & \mathbf{0} \end{bmatrix}
$$

关于 k 时刻机体位姿的雅克比

$$
\begin{aligned}
&\frac{\partial \mathbf{r}_{p_k}}{\partial \delta\boldsymbol{\xi}_{wb_k}} \\
&= \frac{\partial Log((\mathbf{T}_{wb_k} Exp(\delta\boldsymbol{\xi}_{wb_k}))^{-1}\bar{\mathbf{T}}_{wb_k})}{\partial \delta\boldsymbol{\xi}_{wb_k}} \\
&= \frac{\partial Log(Exp(-\delta\boldsymbol{\xi}_{wb_k})\mathbf{T}_{wb_k}{}^{-1}\bar{\mathbf{T}}_{wb_k})}{\partial \delta\boldsymbol{\xi}_{wb_k}} \\
&= \mathbf{J}_l(\mathbf{r}_{p_k})^{-1}
\end{aligned}
$$

5. 里程计与陀螺仪约束因子

1) 测量值

解析形式：

$$
\Theta_h = \begin{bmatrix} \frac{\Delta l_h + \Delta r_h}{2} & 0 & 0 \end{bmatrix}^T
$$

$$
{}^{b_k}\mathbf{p}_{b_{k+1}} = \sum_{h=i}^{j} \mathbf{R}_{b_i b_h}\Theta_h \qquad o_{k,k+1}: i \to j
$$

$$
\mathbf{R}_{b_k b_{k+1}} = \prod_{h=i}^{j} Exp\left(\mathbf{R}_{BI}\left(\boldsymbol{\omega}_h - \mathbf{b}_{g_k}\right)\Delta t\right) \qquad o_{k,k+1}: i \to j
$$

迭代形式：

$$
\begin{aligned}
{}^{b_k}\mathbf{p}_{b_{h+1}} &= {}^{b_k}\mathbf{p}_{b_h} + \mathbf{R}_{b_k b_h}\Theta_h \\
\mathbf{R}_{b_k b_{h+1}} &= \mathbf{R}_{b_k b_h} Exp\left(\mathbf{R}_{BI}\left(\boldsymbol{\omega}_{h+1} - \mathbf{b}_{g_k}\right)\Delta t\right)
\end{aligned}
$$

迭代求解

2) 估计值

$$ {}^{b_k}\mathbf{p}_{b_{k+1}} = \mathbf{R}_{wb_k}{}^{T}\left({}^{w}\mathbf{p}_{b_{k+1}} - {}^{w}\mathbf{p}_{b_k}\right) $$

$$ \mathbf{R}_{b_k b_{k+1}} = \mathbf{R}_{wb_k}{}^{T}\mathbf{R}_{wb_{k+1}} $$

3) 误差值

$$ r_{{}^{b_k}\mathbf{p}_{b_{k+1}}} = \mathbf{R}_{b_k b_{k+1}} - {}^{b_k}\mathbf{p}_{b_{k+1}} = {}^{b_k}\mathbf{p}_{b_{k+1}} - \mathbf{R}_{wb_k}{}^{T}\left({}^{w}\mathbf{p}_{b_{k+1}} - {}^{w}\mathbf{p}_{b_k}\right) $$

$$ r_{\mathbf{R}_{b_k b_{k+1}}} = \mathbf{R}_{b_k b_{k+1}} \ominus \mathbf{R}_{b_k b_{k+1}} = \mathrm{Log}\left(\mathbf{R}_{b_k b_{k+1}}{}^{-1}\mathbf{R}_{b_k b_{k+1}}\right) = Log\left(\mathbf{R}_{wb_{k+1}}{}^{T}\mathbf{R}_{wb_k}\mathbf{R}_{b_k b_{k+1}}\right) $$

$$ \mathbf{r}_{b_g} = \mathbf{b}_{g_{k+1}} - \mathbf{b}_{g_k} $$

4) 信息矩阵

$$
\begin{bmatrix} \delta^{b_k}\mathbf{p}_{b_{h+1}} \\ \delta\boldsymbol{\theta}_{b_k b_{h+1}} \\ \delta\mathbf{b}_{g_k,h+1} \end{bmatrix}
= \begin{bmatrix} \mathbf{I}_{3\times3} & -\mathbf{R}_{b_k b_h}\left[\Theta_h\right]_\times & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & Exp\left(-\mathbf{R}_{BI}\left(\boldsymbol{\omega}_{h+1} - \mathbf{b}_{g_k}\right)\Delta t\right) & -\mathbf{J}_r\left(\mathbf{R}_{BI}\left(\boldsymbol{\omega}_{h+1} - \mathbf{b}_{g_k}\right)\Delta t\right)\mathbf{R}_{BI}\Delta t \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix}
\begin{bmatrix} \delta^{b_k}\mathbf{p}_{b_h} \\ \delta\boldsymbol{\theta}_{b_k b_h} \\ \delta\mathbf{b}_{g_k,h} \end{bmatrix}
$$

$$
+ \begin{bmatrix} \mathbf{R}_{b_k b_h} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & -\mathbf{J}_r\left(\mathbf{R}_{BI}\left(\boldsymbol{\omega}_{h+1} - \mathbf{b}_{g_k}\right)\Delta t\right)\mathbf{R}_{BI}\Delta t & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \Delta t\mathbf{I}_{3\times3} \end{bmatrix}
\begin{bmatrix} \mathbf{n}_o \\ \mathbf{n}_\omega \\ \mathbf{n}_{b_g} \end{bmatrix}
$$

$$
= \mathbf{F}\begin{bmatrix} \delta^{b_k}\mathbf{p}_{b_h} \\ \delta\boldsymbol{\theta}_{b_k b_h} \\ \delta\mathbf{b}_{g_k,h} \end{bmatrix} + \mathbf{G}\begin{bmatrix} \mathbf{n}_o \\ \mathbf{n}_\omega \\ \mathbf{n}_{b_g} \end{bmatrix}
$$

$$ \boldsymbol{\Sigma}_o = diag\left(\sigma_o{}^2 \quad \sigma_g{}^2 \quad \sigma_{b_g}{}^2\right) $$

$$ \boldsymbol{\Sigma}_{o_{k,t+\Delta t}} = \mathbf{F}\boldsymbol{\Sigma}_{o_{k,t}}\mathbf{F}^T + \mathbf{G}\boldsymbol{\Sigma}_o\mathbf{G}^T \qquad t \in [k, k+1] $$ 迭代求解

$$ \mathbf{I}_{o_{k,k+1}} = \boldsymbol{\Sigma}_{o_{k,k+1}}{}^{-1} $$

5) 雅克比矩阵

$$ \mathbf{J}_{o_{k,t+\Delta t}} = \mathbf{F}\mathbf{J}_{o_{k,t}} \qquad t \in [k, k+1] $$

$$
\mathbf{J}_{o_{k,k+1}} = \begin{bmatrix} \dfrac{\partial^{b_k}\bar{\mathbf{p}}_{b_{k+1}}}{\partial^{b_k}\mathbf{P}_{b_{t+\Delta t}}} & \dfrac{\partial^{b_k}\bar{\mathbf{p}}_{b_{k+1}}}{\partial^{b_k}\mathbf{R}_{b_{t+\Delta t}}} & \dfrac{\partial^{b_k}\bar{\mathbf{p}}_{b_{k+1}}}{\partial\mathbf{b}_{g_k}} \\ \mathbf{0}_{3\times3} & \dfrac{\partial^{b_k}\bar{\mathbf{R}}_{b_{k+1}}}{\partial^{b_k}\mathbf{R}_{b_{t+\Delta t}}} & \dfrac{\partial\bar{\mathbf{R}}_{b_k b_{k+1}}}{\partial\mathbf{b}_{g_k}} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix}
$$ 迭代求解

Bias 改变后更新预积分：

$$ {}^{b_k}\overline{\mathbf{p}}_{b_{k+1}} = {}^{b_k}\mathbf{p}_{b_{k+1}} + \frac{\partial\, {}^{b_k}\overline{\mathbf{p}}_{b_{k+1}}}{\partial \mathbf{b}_g}\delta\mathbf{b}_g $$

$$ \Leftarrow \frac{\partial\, {}^{b_k}\overline{\mathbf{p}}_{b_{k+1}}}{\partial \mathbf{b}_g} = -\sum_{h=i+1}^{j}\mathbf{R}_{b_i b_{h-1}}\left[\Theta_h\right]_\times \frac{\partial \overline{\mathbf{R}}_{b_i b_{h-1}}}{\partial \mathbf{b}_g} $$

$$ \overline{\mathbf{R}}_{b_k b_{k+1}} = \mathbf{R}_{b_k b_{k+1}} Exp\left(\frac{\partial \overline{\mathbf{R}}_{b_k b_{k+1}}}{\partial \mathbf{b}_g}\delta\mathbf{b}_g\right) $$

$$ \Leftarrow \frac{\partial \overline{\mathbf{R}}_{b_k b_{k+1}}}{\partial \mathbf{b}_g} = -\sum_{h=i}^{j-1}\mathbf{R}_{b_{h+1} b_j}{}^T \mathbf{J}_{r_h}\mathbf{R}_{BI}\Delta t \qquad o_{k,k+1}: i \to j $$

a) 姿态残差的雅克比

$$ \frac{\partial r_{\mathbf{R}_{b_k b_{k+1}}}}{\partial \boldsymbol{\chi}} = \begin{bmatrix} \mathbf{0} & \frac{\partial r_{\mathbf{R}_{b_k b_{k+1}}}}{\partial \mathbf{x}_k} & \frac{\partial r_{\mathbf{R}_{b_k b_{k+1}}}}{\partial \mathbf{x}_{k+1}} & \mathbf{0} \end{bmatrix} $$

$$ \frac{\partial r_{\mathbf{R}_{b_k b_{k+1}}}}{\partial \mathbf{R}_{wb_k}} = \mathbf{J}_r\left(Log\left(\mathbf{R}_{wb_{k+1}}{}^T\mathbf{R}_{wb_k}\mathbf{R}_{b_k b_{k+1}}\right)\right)^{-1}\mathbf{R}_{b_k b_{k+1}}{}^T = \mathbf{J}_r\left(r_{\mathbf{R}_{b_k b_{k+1}}}\right)^{-1}\mathbf{R}_{b_k b_{k+1}}{}^T $$

$$ \frac{\partial r_{\mathbf{R}_{b_k b_{k+1}}}}{\partial \mathbf{b}_{g_k}} = \mathbf{J}_r\left(r_{\mathbf{R}_{b_k b_{k+1}}}\right)^{-1}\frac{\partial \overline{\mathbf{R}}_{b_k b_{k+1}}}{\partial \mathbf{b}_{g_k}} $$

$$ \frac{\partial r_{\mathbf{R}_{b_k b_{k+1}}}}{\partial \mathbf{R}_{wb_{k+1}}} = -\mathbf{J}_r\left(-Log\left(\mathbf{R}_{wb_{k+1}}{}^T\mathbf{R}_{wb_k}\mathbf{R}_{b_k b_{k+1}}\right)\right)^{-1} = -\mathbf{J}_r\left(-r_{\mathbf{R}_{b_k b_{k+1}}}\right)^{-1} $$

b) 位置残差的雅克比

$$ \frac{\partial r_{{}^{b_k}\mathbf{p}_{b_{k+1}}}}{\partial \boldsymbol{\chi}} = \begin{bmatrix} \mathbf{0} & \frac{\partial r_{{}^{b_k}\mathbf{p}_{b_{k+1}}}}{\partial \mathbf{x}_k} & \frac{\partial r_{{}^{b_k}\mathbf{p}_{b_{k+1}}}}{\partial \mathbf{x}_{k+1}} & \mathbf{0} \end{bmatrix} $$

$$ \frac{\partial r_{{}^{b_k}\mathbf{p}_{b_{k+1}}}}{\partial \mathbf{R}_{wb_k}} = -\left[\mathbf{R}_{wb_k}{}^T\left({}^w\mathbf{p}_{b_{k+1}} - {}^w\mathbf{p}_{b_k}\right)\right]_\times $$

$$ \frac{\partial r_{{}^{b_k}\mathbf{p}_{b_{k+1}}}}{\partial\, {}^w\mathbf{p}_{b_k}} = \mathbf{R}_{wb_k}{}^T $$

$$ \frac{\partial r_{{}^{b_k}\mathbf{p}_{b_{k+1}}}}{\partial \mathbf{b}_{g_k}} = \frac{\partial\, {}^{b_k}\overline{\mathbf{p}}_{b_{k+1}}}{\partial \mathbf{b}_{g_k}} $$

$$ \frac{\partial r_{{}^{b_k}\mathbf{p}_{b_{k+1}}}}{\partial\, {}^w\mathbf{p}_{b_{k+1}}} = -\mathbf{R}_{wb_k}{}^T $$

c) 陀螺仪 bias 的雅克比

$$\frac{\partial \mathbf{r}_{b_g}}{\partial \boldsymbol{\chi}} = \begin{bmatrix} \mathbf{0} & \frac{\partial \mathbf{r}_{b_g}}{\partial \mathbf{x}_k} & \frac{\partial \mathbf{r}_{b_g}}{\partial \mathbf{x}_{k+1}} & \mathbf{0} \end{bmatrix}$$

$$\frac{\partial \mathbf{r}_{b_g}}{\partial \mathbf{b}_{g_k}} = -\mathbf{I}_{3\times 3}$$

$$\frac{\partial \mathbf{r}_{b_g}}{\partial \mathbf{b}_{g_{k+1}}} = \mathbf{I}_{3\times 3}$$

6. 先验因子

假定优化求解过程中，正规方程为

$$\mathbf{H}\boldsymbol{\chi} = \mathbf{b}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{mm} & \mathbf{H}_{mr} \\ \mathbf{H}_{rm} & \mathbf{H}_{rr} \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} \mathbf{b}_m \\ \mathbf{b}_r \end{bmatrix}$$

边缘化后，有

$$\mathbf{b}_{p'} = \mathbf{b}_r - \mathbf{H}_{rm}\mathbf{H}_{mm}^{-1}\mathbf{b}_m$$

$$\mathbf{H}_{p'} = \mathbf{H}_{rr} - \mathbf{H}_{rm}\mathbf{H}_{mm}^{-1}\mathbf{H}_{mr}$$

状态增广后，有

$$\boldsymbol{\Xi} = \begin{bmatrix} \mathbf{I}_{\dim \boldsymbol{\chi}_r} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{b}_p = \boldsymbol{\Xi}^T \mathbf{b}_{p'}$$

$$\mathbf{H}_p = \boldsymbol{\Xi}^T \mathbf{H}_{p'} \boldsymbol{\Xi}$$