Video Compression Assignment 2

Prerequisites

- Programming langauge: Python 3.10+ (IPython)
- Framework: Jupyter

Install dependencies from PyPI:

Note: you may need to restart the kernel to use updated packages.

Task 1

Motion Estimation

Please apply the full-search block-matching algorithm to the luma component of $foreman_qcif_1_rgb.bmp$ with the reference frame $foreman_qcif_0_rgb.bmp$.

The collocated position in the reference frame of the top-left pixel of each MB is the center of the search window.

The search range is set to [-16, 15]. The similarity metric is SAD. Please show MVs for all the MBs in the video frame.

Solution of Task 1

The marcoblock size is 16x16, and the search window size is 32x32:

```
In [3]: block_size = (16, 16)
    search_size = (32, 32)
```

The top-left pixel of a macroblock is the block index as well as the center of the search window:

```
plaintext
o: block index
+: other pixel

0 + + +
+ + + +
+ + + +
```

For a search window locates at the center , the range of block index to search is [center - window_size // 2, center]:

Wrap these concepts into functions:

```
In [5]: from typing import Iterable
from PIL.Image import Image

def get_grid_size(frame: Image) -> tuple[int, int]:
```

```
return (frame.width, frame.height)
def get_block_indexs(
    grid_size: tuple[int, int],
    block_size: tuple[int, int],
) -> tuple[Iterable[int], Iterable[int]]:
    assert grid_size[0] >= block_size[0], (grid_size[0], block_size[0])
assert grid_size[1] >= block_size[1], (grid_size[1], block_size[1])
        range(0, grid_size[0] - block_size[0] + 1, block_size[0]),
        range(0, grid_size[1] - block_size[1] + 1, block_size[1]),
def get_block_index_bound(
    frame: Image,
) -> tuple[tuple[int, int], tuple[int, int]]:
    return ((0, max(frame.width - 1, 0)), (0, max(frame.height - 1, 0)))
def get_search_block_indexs(
    center_block_index: tuple[int, int],
    search_block_index_range: tuple[tuple[int, int], tuple[int, int]],
    block_index_bound: tuple[tuple[int, int], tuple[int, int]],
) -> tuple[Iterable[int], Iterable[int]]:
    def minmax(value: int, min_value: int, max_value: int) -> int:
        return max(min(value, max_value), min_value)
    return (
        range(
            minmax(
                 center_block_index[0] + search_block_index_range[0][0],
                 block_index_bound[0][0],
                 block_index_bound[0][1],
            minmax(
                 center_block_index[0] + search_block_index_range[0][1],
                 block_index_bound[0][0],
                 block_index_bound[0][1],
            + 1,
        ).
        range(
                 center_block_index[1] + search_block_index_range[1][0],
                 block_index_bound[1][0],
                 block_index_bound[1][1],
            minmax(
                 center_block_index[1] + search_block_index_range[1][1],
                 block_index_bound[1][0],
                 block_index_bound[1][1],
            + 1.
        ),
```

The similarity metric is SAD, which is the sum of absolute differences between two blocks:

```
In [6]: from numpy.typing import NDArray

def get_score_sad(
    block_pair: tuple[NDArray, NDArray],
) -> int:
    from numpy import abs
    return abs(block_pair[0] - block_pair[1]).sum(dtype=int)
```

Load the images and extract their luma component:

```
In [7]: import PIL.Image as Image
    from numpy import asarray

    reference_frame = Image.open("../resources/foreman_qcif_0_rgb.bmp").convert("L")
    current_frame = Image.open("../resources/foreman_qcif_1_rgb.bmp").convert("L")

    reference_buffer = asarray(reference_frame)
    current_buffer = asarray(current_frame)

In [8]: grid_size = get_grid_size(current_frame)
    block_index_bound = get_block_index_bound(current_frame)

display(
    dict(
        grid_size=grid_size,
        block_index_bound=block_index_bound,
    )
    )
)
```

{'grid_size': (176, 144), 'block_index_bound': ((0, 175), (0, 143))}

Perform full-search block-matching algorithm and calculate the motion vectors (reference - current):

```
In [9]: from sys import maxsize
        motion_vectors_by_full_search: list[list[tuple[int, int]]] = []
        center_block_x_indexs, center_block_y_indexs = get_block_indexs(
            grid_size, block_size
        for center_block_y_index in center_block_y_indexs:
            motion_vectors_x: list[tuple[int, int]] = []
            for center_block_x_index in center_block_x_indexs:
                center_block = current_buffer[
                    center_block_y_index : center_block_y_index + block_size[1],
                    center_block_x_index : center_block_x_index + block_size[0],
                search_block_x_indexs, search_block_y_indexs = get_search_block_indexs(
                    (center_block_x_index, center_block_y_index),
                    search_block_index_range,
                    block_index_bound,
                best_score = maxsize
                motion_vector: tuple[int, int] = None
                for search_block_y_index in search_block_y_indexs:
                    for search_block_x_index in search_block_x_indexs:
                        search_block = reference_buffer[
                            search_block_y_index : search_block_y_index + block_size[1],
                            search_block_x_index : search_block_x_index + block_size[0],
                        score = get_score_sad((search_block, center_block))
                        if best_score > score:
                            best_score = score
                            motion_vector = (
                                {\tt search\_block\_x\_index - center\_block\_x\_index,}
                                search_block_y_index - center_block_y_index,
                motion_vectors_x.append(motion_vector)
            motion_vectors_by_full_search.append(motion_vectors_x)
```

Render the output:

```
In [10]: from IPython.display import display, Markdown
        table_markdown_str = "| y \\ x |"
        for center_block_x_index in center_block_x_indexs:
           table_markdown_str += f" {center_block_x_index} |"
        table_markdown_str += "\n"
        table_markdown_str += (
            for center_block_y_index, motion_vectors_x in zip(
            center_block_y_indexs, motion_vectors_by_full_search
        ):
            table_markdown_str += f"| {center_block_y_index} |"
            for motion_vector in motion_vectors_x:
               table_markdown_str += f" {motion_vector} |"
            table_markdown_str += "\n"
        display(
               f"Display the motion vectors `(mx, my)` in the table below:\n{table_markdown_str}"
```

Display the motion vectors (mx, my) in the table below:

y \ x	0	16	32	48	64	80	96	112	128	144	160
0	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(-12, 0)	(-6, 0)	(-4, 0)	(-3, 0)	(-16, 0)
16	(0, -16)	0, -16) (0, 0) (0, 0)		(0, 0)	(0, -2)	(0, -2)	(-6, 0)	(-14, -7)	(-15, -14)	(-16, -13)	(0, 0)
32	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, -16)	(0, 0)	(-5, 0)	(-11, -3)	(-14, -4)	(0, -16)	(-10, -7)
48	(0, 0)	(0, 0)	(-13, -1)	(-3, -4)	(-13, 0)	(-2, -8)	(-6, 0)	(0, 0)	(0, 0)	(-5, -9)	(-1, -16)
64	(0, -7)	(-6, -2)) (0, 0)	(-4, -11)	(-16, -9)	(-16, -12)	(-7, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
80	(0, 0)	(-7, 0)	(-6, -3)	(0, -6)	(-2, -2)	(-16, 0)	(-6, 0)	(-2, 0)	(-3, -6)	(0, -6)	(-6, -7)
96	(0, -2)	(-14, -5)	(-6, -2)	(-7, 0)	(-3, -15)	(-15, -10)	(-16, -12)	(-3, 0)	(-14, -10)	(-12, -14)	(-1, -7)
112	(0, -15)	(-14, -16)	(-13, 0)	(-5, 0)	(0, -16)	(-1, 0)	(-4, 0)	(-3, -5)	(-16, -16)	(-16, -5)	(-6, -13)
128	(0, -5)	(-13, -8)	(-15, 0)	(-16, 0)	(-14, -13)	(-15, -1)	(0, 0)	(-15, -6)	(-16, -14)	(0, 0)	(0, 0)

Motion Estimation

Following the previous question, please implement the diamond-search algorithm.

Solution of Task 2

The diamond search pattern of radius 2:

Write this concept into a function:

```
In [11]: def get_diamond_search_block_indexs(
              center_block_index: tuple[int, int],
              search_block_radius: int,
              block_index_bound: tuple[tuple[int, int], tuple[int, int]],
              block_size: tuple[int, int],
          ) -> Iterable[tuple[int, int]]:
              def get_diamond_search_block_index_relative(search_block_radius: int):
                   search_block_radius = max(search_block_radius, 0)
                  if search_block_radius > 0:
                      yield (-search_block_radius, 0)
                  for rx in range(-search_block_radius + 1, 0):
                       ry = rx + search_block_radius
                       yield (rx, -ry)
                       yield (rx, +ry)
                  if search_block_radius > 0:
                      yield (0, -search_block_radius)
                  yield (0, 0)
                  if search_block_radius > 0:
                      yield (0, +search_block_radius)
                   for rx in range(1, search_block_radius):
                       ry = rx - search_block_radius
                       yield (rx, +ry)
                      yield (rx, -ry)
                   if search_block_radius > 0:
                       yield (+search_block_radius, 0)
              for search_block_index_relative in get_diamond_search_block_index_relative(
                   search_block_radius
                   search_block_index = (
                       center_block_index[0] + search_block_index_relative[0],
center_block_index[1] + search_block_index_relative[1],
                       block_index_bound[0][0]
                       <= search_block_index[0]</pre>
                       <= block_index_bound[0][1]</pre>
                       and block_index_bound[1][0]
                       <= search block index[1]</pre>
                        block_index_bound[1][1]
                       and block_index_bound[0][0]
                       <= search_block_index[0] + block_size[0] - 1</pre>
                       <= block_index_bound[0][1]</pre>
                      and block_index_bound[1][0]
<= search_block_index[1] + block_size[1] - 1</pre>
                        block_index_bound[1][1]
                       yield search_block_index
```

Take this as an example:

```
plaintext
o: search block index
x: center block index (also search block index)
+: other pixel
+ 0 + 0 + +
0 + + + 0 +
+ + x + + 0
```

```
0 + + + 0 +
             + 0 + 0 + +
             + + 0 + + +
In [12]: display(
             list(
                 get_diamond_search_block_indexs(
                     center_block_index=(2, 2),
                     search_block_radius=3,
                     block_index_bound=block_index_bound,
                     block_size=block_size,
             )
         )
        [(0, 1),
         (0, 3),
         (1, 0),
         (1, 4),
(2, 2),
         (2, 5),
         (3, 0),
         (3, 4),
         (4, 1),
         (4, 3),
         (5, 2)
         Perform diamond-search block-matching algorithm and calculate the motion vectors ( reference - current ):
In [13]: search_block_radius_init = block_size[0] // 2
         display(search_block_radius_init)
        8
In [14]: from sys import maxsize
         motion_vectors_by_diamond_search: list[list[tuple[int, int]]] = []
         center_block_x_indexs, center_block_y_indexs = get_block_indexs(
             grid_size, block_size
         for center_block_y_index in center_block_y_indexs:
             motion_vectors_x: list[tuple[int, int]] = []
             for center_block_x_index in center_block_x_indexs:
                 current_center_block_x_index = reference_center_block_x_index = (
                     center_block_x_index
                 current_center_block_y_index = reference_center_block_y_index = (
                     center_block_y_index
                 center_block = current_buffer[
                     current_center_block_y_index : current_center_block_y_index
                     + block_size[1],
                     current_center_block_x_index : current_center_block_x_index
                     + block_size[0],
                 search_block_radius = search_block_radius_init
                 while search_block_radius > 0:
                     search_block_indexs = get_diamond_search_block_indexs(
                              reference_center_block_x_index,
                              reference_center_block_y_index,
                         search_block_radius,
                         block_index_bound,
                         block_size,
                     best_score = maxsize
                     best_reference_center_block_x_index: int = None
                     best_reference_center_block_y_index: int = None
                     for (
                         search_block_x_index,
                         search_block_y_index,
                     ) in search_block_indexs:
                         search_block = reference_buffer[
                             search_block_y_index : search_block_y_index + block_size[1],
                              search_block_x_index : search_block_x_index + block_size[0],
                         score = get_score_sad((search_block, center_block))
                         if best_score > score:
                             best_score = score
                              best_reference_center_block_x_index = search_block_x_index
                              best_reference_center_block_y_index = search_block_y_index
                     if (
                         reference_center_block_x_index
                          == best_reference_center_block_x_index
                         and reference_center_block_y_index
                          == best_reference_center_block_y_index
                      ):
                         search_block_radius -= 1
```

```
reference_center_block_x_index = best_reference_center_block_x_index
reference_center_block_y_index = best_reference_center_block_y_index

motion_vector = (
    reference_center_block_x_index - current_center_block_x_index,
    reference_center_block_y_index - current_center_block_y_index,
)
motion_vectors_x.append(motion_vector)

motion_vectors_by_diamond_search.append(motion_vectors_x)
```

Render the output:

Display the motion vectors (mx, my) in the table below:

y \ x	0	16	32	48	64	80	96	112	128	144	160
0	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(8, 14)	(6, 21)	(-4, 19)	(-6, 0)	(-4, 0)	(14, 16)	(-4, 23)
16	(0, 0)	(0, 0)	(0, 0)	(9, 12)	(19, -3)	(8, 3)	(-5, 4)	(-6, 5)	(-15, -14)	(0, 3)	(0, 0)
32	(0, 0)	(0, 0)	(0, 0)	(9, -14)	(24, -4)	(0, 0)	(-5, 0)	(-11, -3)	(11, 10)	(-15, 11)	(-10, -7)
48	(17, -6)	(0, 0)	(-2, -5)	(7, -8)	(-15, 3)	(-4, 1)	(-5, 1)	(2, 1)	(3, -6)	(0, 0)	(0, 0)
64	(17, -13)	(-6, -2)	(0, 0)	(7, -16)	(-1, 12)	(2, 3)	(-4, 9)	(2, -2)	(0, 0)	(0, 0)	(0, 0)
80	(0, 9)	(0, 16)	(-6, -3)	(9, 8)	(-2, -2)	(-15, 3)	(7, 5)	(-2, 1)	(17, 4)	(1, -6)	(0, 0)
96	(0, -2)	(-14, -5)	(0, 7)	(-1, 4)	(1, 0)	(5, 11)	(-7, 11)	(-9, 10)	(-14, -10)	(3, -4)	(-1, -7)
112	(0, -23)	(-14, 4)	(-16, 8)	(-1, 4)	(10, -32)	(0, 1)	(-4, 0)	(-3, -5)	(0, 0)	(-19, -5)	(-6, -13)
128	(0, -5)	(-13, -8)	(-29, -4)	(-16, 0)	(-14, -13)	(19, -1)	(1, -2)	(0, 0)	(15, -4)	(9, 0)	(0, 0)

Intra-frame Prediction

Please apply intra-prediction to the luma component of foreman_qcif_0_rgb.bmp .

You only need to implement the four modes (Mode 0, 1, 2, and 4) for the 16x16 luma MB. You do not make an intra-prediction to the top-left MB since there is no predictor for it.

The similarity metric is also SAD. If any pixels are unavailable for a mode, you will not choose it. Please plot the mode for each MB using the example figure below.

plaintext | x | 1 | ... | 2 | | 4 | 1 | ... | 2 | | ... | ... | ... | 4 | | 0 | 0 | ... | 4 |

Solution of Task 3

There are four modes for 16x16 macroblocks in intra-frame prediction:

- Mode 0: Vertical
- Mode 1: Horizontal
- Mode 2: DC
- Mode 4: Diagonal down-right

To simplify illustrations, the block size downgrades to 4x4.

The encoded and prediction partitions are illustrated as follows:

```
plaintext
e: encoded pixel (source)
p: predicted pixel (target)
```

M	ode	e (9			М	ode	e :	1			М	ode	2	2			Мс	ode	2	1		
					- -						- -						- -						-
	е	е	е	е									е	е	е	е		е	е	е	е		
	p	р	р	р		е	р	р	p	р		е	р	p	р	p		е	р	р	р	р	
	p	р	р	р		е	p	р	p	р		е	р	p	р	p		е	р	р	р	р	
	р	р	р	р		е	р	р	р	р		е	р	р	р	р		е	р	р	р	р	
	p	р	р	р		е	p	р	p	р		е	р	p	р	р			р	р	р	р	

The predicted data is calculated as the following illustrations:

plaintext
Before:

Mode 0	Mode 1	Mode 2	Mode 4
abcd		abcd	mabc
	i	1	i
	j	j	j
	k	k	k
	1	1	

After:

```
* u = the average of a, b, c, d, i, j, k, l
```

Wrap these concepts into functions:

```
In [16]: from numpy.typing import NDArray
from typing import Optional

def has_top_predictor(
    block_index: tuple[int, int],
    block_index_bound: tuple[tuple[int, int]],
) -> bool:
```

```
return (
        block_index_bound[1][0] \leftarrow block_index[1] - 1 \leftarrow block_index_bound[1][1]
def has_left_predictor(
    block_index: tuple[int, int],
    block_index_bound: tuple[tuple[int, int], tuple[int, int]],
) -> bool:
    return (
        block_index_bound[0][0] \leftarrow block_index[0] - 1 \leftarrow block_index_bound[0][1]
def get_predicted_block_in_vertical_mode(
    frame_buffer: NDArray,
    block_index: tuple[int, int],
    block_size: tuple[int, int],
    block_index_bound: tuple[tuple[int, int], tuple[int, int]],
) -> Optional[NDArray]:
    from numpy import full
    if not has_top_predictor(block_index, block_index_bound):
    top_predictor_buffer = frame_buffer[
        block_index[1] - 1,
        block_index[0] : block_index[0] + block_size[0],
    predicted_block = full(
        (block_size[1], block_size[0]),
        top_predictor_buffer,
    return predicted_block
def get_predicted_block_in_horizontal_mode(
    frame_buffer: NDArray,
    block_index: tuple[int, int],
    block_size: tuple[int, int],
    block_index_bound: tuple[tuple[int, int], tuple[int, int]],
) -> Optional[NDArray]:
    from numpy import full
    if not has_left_predictor(block_index, block_index_bound):
        return
    left_predictor_buffer = frame_buffer[
        block_index[1] : block_index[1] + block_size[1],
        block_index[0] - 1,
    predicted_block = full(
        (block_size[1], block_size[0]),
        left_predictor_buffer.reshape(-1, 1),
    return predicted_block
def get_predicted_block_in_dc_mode(
    frame_buffer: NDArray,
    block_index: tuple[int, int],
    block_size: tuple[int, int],
    block_index_bound: tuple[tuple[int, int], tuple[int, int]],
) -> Optional[NDArray]:
    from numpy import full
    if not has_left_predictor(
        block_index,
        block_index_bound,
    ) or not has_top_predictor(
        block_index,
        block_index_bound,
    ):
        return
    top_predictor_buffer = frame_buffer[
        block_index[1] - 1,
block_index[0] : block_index[0] + block_size[0],
    left_predictor_buffer = frame_buffer[
        block_index[1] : block_index[1] + block_size[1],
        block_index[0] - 1,
    average_value = (
        top_predictor_buffer.sum(dtype=int)
         + left_predictor_buffer.sum(dtype=int)
    ) // (top_predictor_buffer.size + left_predictor_buffer.size)
    predicted_block = full((block_size[1], block_size[0]), average_value)
    return predicted_block
```

```
def get_predicted_block_in_diagonal_downright_mode(
    frame_buffer: NDArray,
    block_index: tuple[int, int],
   block_size: tuple[int, int],
   block_index_bound: tuple[tuple[int, int], tuple[int, int]],
) -> Optional[NDArray]:
   from numpy import empty
   if not has_left_predictor(
        block_index,
        block_index_bound,
    ) or not has_top_predictor(
        block_index,
        block_index_bound,
        return
   predicted_block = empty(
        (block\_size[1],\ block\_size[0]),\ dtype=frame\_buffer.dtype
   diag_size = min(block_size[0], block_size[1])
   for predict_count, predictor_y_index_relative in zip(
        range(1, diag_size + 1), range(diag_size - 2, -2, -1)
        predictor_index = (
           block_index[0] - 1,
            block_index[1] + predictor_y_index_relative,
        for predicted_index_relative in range(predict_count):
            predicted_index = (
                predicted_index_relative,
                predicted_index_relative + predictor_y_index_relative + 1,
           predicted_block[predicted_index[1], predicted_index[0]] = (
                frame buffer[
                    predictor_index[1],
                    predictor_index[0],
   for predict_count, predictor_x_index_relative in zip(
        range(diag\_size - 1, 0, -1), range(0, diag\_size - 1)
        predictor_index = (
            block_index[0] + predictor_x_index_relative,
           block_index[1] - 1,
        for predicted_index_relative in range(predict_count):
           predicted index = (
                predicted_index_relative + predictor_x_index_relative + 1,
                predicted_index_relative,
           predicted_block[predicted_index[1], predicted_index[0]] = (
                frame buffer
                    predictor_index[1],
                    predictor_index[0],
   return predicted_block
```

Load the image and extract the luma component:

predicted_buffer = zeros_like(source_buffer)

In [19]: from numpy import zeros_like
 from sys import maxsize
 modes: list[list[str]] = []

prediction_functions = {

```
In [17]: import PIL.Image as Image
    from numpy import asarray

    source_frame = Image.open("../resources/foreman_qcif_0_rgb.bmp").convert("L")
    source_buffer = asarray(source_frame)

In [18]: grid_size = get_grid_size(source_frame)
    block_index_bound = get_block_index_bound(source_frame)

display(
    dict(
        grid_size=grid_size,
        block_index_bound=block_index_bound,
    )
)

{'grid_size': (176, 144), 'block_index_bound': ((0, 175), (0, 143))}

Perform intra-frame prediction and calculate the predicted mode and frame:
```

```
0: get_predicted_block_in_vertical_mode,
   1: get_predicted_block_in_horizontal_mode,
   2: get_predicted_block_in_dc_mode,
   4: get_predicted_block_in_diagonal_downright_mode,
block_x_indexs, block_y_indexs = get_block_indexs(grid_size, block_size)
for block_y_index in block_y_indexs:
   modes_x: list[str] = []
   for block_x_index in block_x_indexs:
        source_block = source_buffer[
            block_y_index : block_y_index + block_size[1],
block_x_index : block_x_index + block_size[0],
        best_score = maxsize
        best_mode = None
        best_predicted_block = None
        for mode, prediction_function \underline{\textbf{in}} prediction_functions.items():
            predicted_block = prediction_function(
                source_buffer,
                (block_x_index, block_y_index),
                block_size,
                block_index_bound,
            if predicted_block is not None:
                score = get_score_sad((predicted_block, source_block))
                if best_score > score:
                     best_score = score
                     best_mode = mode
                     best_predicted_block = predicted_block
        modes_x.append(best_mode)
        if best_predicted_block is not None:
            predicted_buffer[
                block_y_index : block_y_index + block_size[1],
                block_x_index : block_x_index + block_size[0],
            ] = best_predicted_block
   modes.append(modes_x)
```

Render the output:

```
In [23]: from IPython.display import display, Markdown

table_markdown_str = "|"
    for block_x_index in block_x_indexs:
        table_markdown_str += f" |"
    table_markdown_str += "\n"

table_markdown_str += "|" + "---|" * len(block_x_indexs) + "\n"

for modes_x in modes:
    table_markdown_str += "|"
    for mode in modes_x:
        if mode is None:
            mode = "x"
        table_markdown_str += f" {mode} |"
        table_markdown_str += f" {mode} |"
        table_markdown_str += "\n"

display(
        Markdown(
            f"Display the intra prediction modes in the table below:\n{table_markdown_str}"
        )
    )
}
```

Display the intra prediction modes in the table below:

```
1 | 1
              1
                1 | 1
0 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2
         2 1
              4
                2 2
0 2 2 2 2 2 2 2 2 2 2 2 2 2 2
0 2 2 2 2 2 2 2 2 2 2 2 2
0 2 2 2 2 2 2 2
                2 2 2 2
0 2 2
      2 2 2
              2
                2 2
                     2 0
0 2 2 2 2 2 2
                2 2
              2
                     2 2
0 2 2 2 2 2 2 2 2 2 2 2 2 2
```

```
In [21]: display(Markdown("The source frame (Luma component):"))
    display(source_frame)
    display(Markdown("The predicted frame (Luma component):"))
    display(Image.fromarray(predicted_buffer, mode="L"))
```

The source frame (Luma component):



The predicted frame (Luma component):

