

GaleShapelyDrill4

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1 Drill

1.1 Rankings

List of Rankings for Men:

	Choice 1	Choice 2	Choice 3
1	A	B	C
2	B	A	C
3	A	B	C

List of Rankings for Women:

	Choice 1	Choice 2	Choice 3
A	2	1	3
B	1	3	2
C	1	2	3

1.2 GaleShapely

	Day 1	Day 2	Day 3	Day 4	Day 5
A	1, 3	1	1, 2	2	2
B	2	2, 3	3	1, 3	1
C					3

2 Proofs

2.1 Statement 1: In any execution of the algorithm, if a woman receives a proposal on day(i), then she receives some proposal on every subsequent day until the algorithm terminates

Let's try a direct Proof:

Let's assume that a woman W was proposed to on some day D by some guy G . That means that on day $D+1$ since G wasn't rejected he will propose again and will continue to propose until he is rejected. When G gets rejected, by definition of the Algorithm it must be that W accepted a higher ranked man on her list. Then that man will continue to propose to her until she will reject him for a different higher ranked man.

Once someone proposes to a woman she has 2 possible choices and 2 possible choices only. She can either continue to accept the man she already has, or option 2 she can reject him and take a higher ranked man. (Option 1 will always be there; option 2 might be there) In both possible outcomes she always has at least 1 man proposing to her.

QED

2.2 Statement 2: In any execution of the algorithm, if a woman receives no proposal on day(i), then she receives no proposal on any previous day(j), $1 \leq j < i$

Let's try a proof by contrapositive :

In Statement 1 we have already proven that if a woman receives a proposal on a day D , then she will always have at least 1 proposal on every following day. Therefore if a woman was proposed to she must always have at least 1 proposal. Then the inverse must be true, if she has no proposals on a given day then she also has to have no proposals before. If she had any proposals before then Statement 1 would be incorrect and we already proved it to be correct so Statement 2 must also be correct

QED

2.3 Statement 3: In any execution of the algorithm, there is at least one woman who only receives a single proposal.

Let's try a direct proof:

The reason there is a woman who only receives one proposal is because the program terminates as soon as everyone has a match. If every woman has at least 1 proposal it means that every woman has a match and the program will terminate. It is impossible for every single woman to have more than 1 proposal because if every single woman has at least 1 proposal the program terminates so therefore it is impossible for every woman to have more than 1 proposal

QED