# Proofs0

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## 1 Proof 1:

If x is an odd integer, then x + 1 is even. Formally, you must prove that $\forall x \in Z$ (x is odd $\rightarrow x + 1$ is even).

Proof:

Step 1: By definition an even number is divisible by 2.

Step 2: 1 is half of 2; Since one is half of 2 if you add one to an even number twice($x + 1$ two times) then you will still be at a number that is divisible by 2.

Step 3: But, if you add 1 to an even it is no longer divisible by 2 and therefore every time you add 1 it cycles between being divisible by 2 and not being divisible by 2.

Step 4: Therefore, if x is odd then x+1 is even.

QED

## 2 Proof 2:

First some notation and a definition for divisibility. We say that integer a divides b (or b is divisible by a), if and only if for some integer q, $b = aq$. We write this as $a|b$.

Theorem: $\forall n \in N, 3|(n^3 - n)$ Prove the theorem using induction.

Proof:

Step 1: Assume that $n = 1$; then $1^3 - 1 = 0$ and $0/3 = 0$ so there is a $q$ that exists and that $q$ is 0. Therefore our base case $P(1)$ holds true.

Step 2: Assume that this holds true for some integer $k$. That means that $k^3 - k$ is divisible by 3.

Step 3: Now lets try to prove $(k+1)^3 - (k+1)$ is divisible by 3.

Step 4: $(k+1)^3$ can be simplified to $k^3 + 3k^2 + 3k + 1$. So we now have $k^3 + 3k^2 + 3k + 1 - (k+1)$; and this expression can be simplified even further to $k^3 + 3k^2 + 2k$ by combining like terms.

Step 5: We already know that $k^3 - k$ is divisible by 3(from our assumption in step 2). $k^3 + 3k^2 + 2k$ is equivalent to $k^3 - k + 3k^2 + 3k$ and since $k^3 - k$ is divisible by 3 there exists some integer $a$ so that $k^3 - k = 3a$. (Since they are equal we can just write $k^3 - k$ as $3a$)

Step 6: We have $3a+3k^2+3k$. We can then factor out a 3 so we have $3(a+k^2+k)$. Since you are multiplying by a 3 in the front if you divide by 3 you are left with $a + k^2 + k$. So therefore $3(a + k^2 + k)$ is divisible by 3

QED

# 3 Proof 3:

Theorem: $\forall n \in N$ , for $n > 1$ we have $n! < n^n$.

Step 1: Assume that n = 2. Then $2! < 2^2$ which is the same as $2 < 4$. So therefore our base case $P(2)$ holds.

Step 2: Assume this holds true for some integer $k$. so $k! < k^k$

Step 3: Now let's prove $(k+1)! < (k+1)^{(k+1)}$

Step 4: By the definition of factorials we know $k! * (k+1) = (k+1)!$ We also know that $(k+1)^{k+1} = (k+1)^k * (k+1)$ Since we know all of this we can simply rewrite them together as $k! * (k+1) < (k+1)^k * (k+1)$

Step 5: We can start by canceling out our $(k+1)$ which is on both sides so we can simply divide them out and be left with $k! < (k+1)^k$

Step 6: Since we have already said that this holds true for $k! < k^k$ and since its obvious that $k^k < (k+1)^k$ when dealing with whole positive Integers. Then $k! < (k+1)^k$ and so to $(k+1)! < (k+1)^{k+1}$

QED