McGill University
Department of Electrical and
Computer Engineering
Patrick Diez, U4 Undergraduate
Sean Lawlor, U4 Undergraduate

Navigation

ECSE 211: Design Principles and Methods

McGill University
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Patrick Diez, U4 Undergraduate
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Introduction

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Patrick Diez, U4 Undergraduate
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- With an accurate odometer, it is now possible to position the robot in the field
- However, no means has been established for moving the robot from one point to another in the field
- Using the same principles as those used to derive the algorithm for odometry, a means of driving the robot to a specific position on the field will be developed

Derivation

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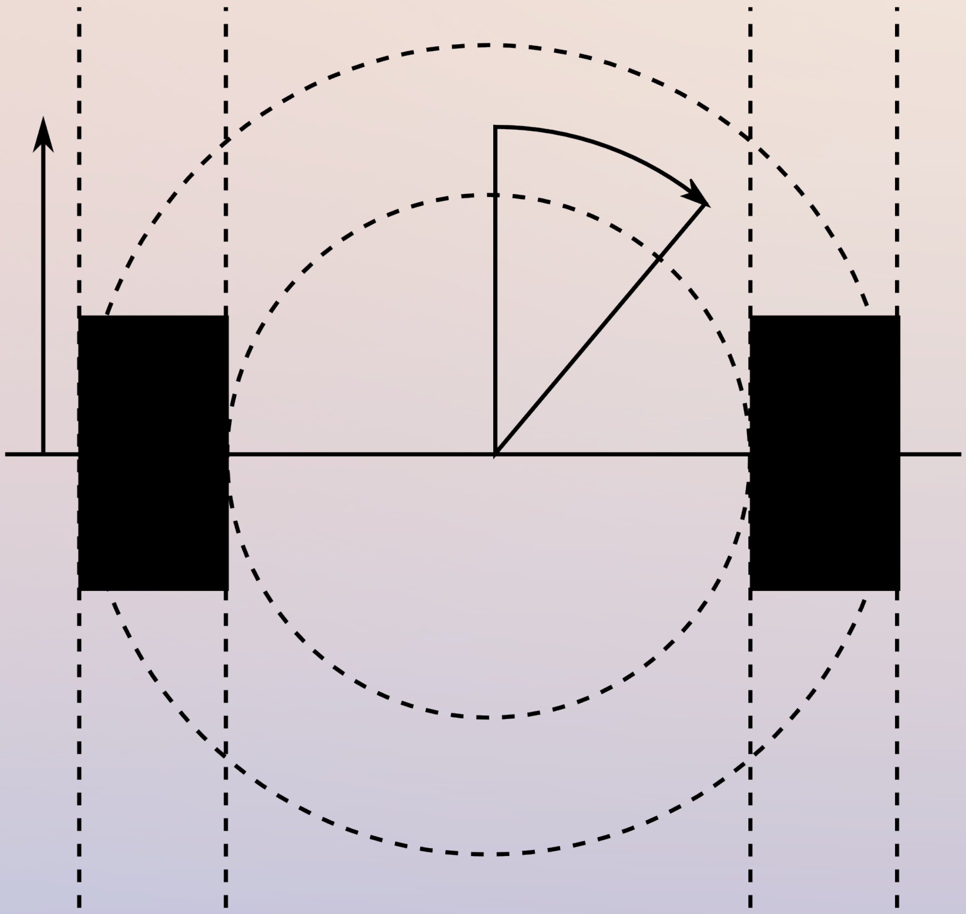
Sean Lawlor, U4 Undergraduate

- The robot's motion is broken into two motions, forward and rotary, that are assumed to be independent



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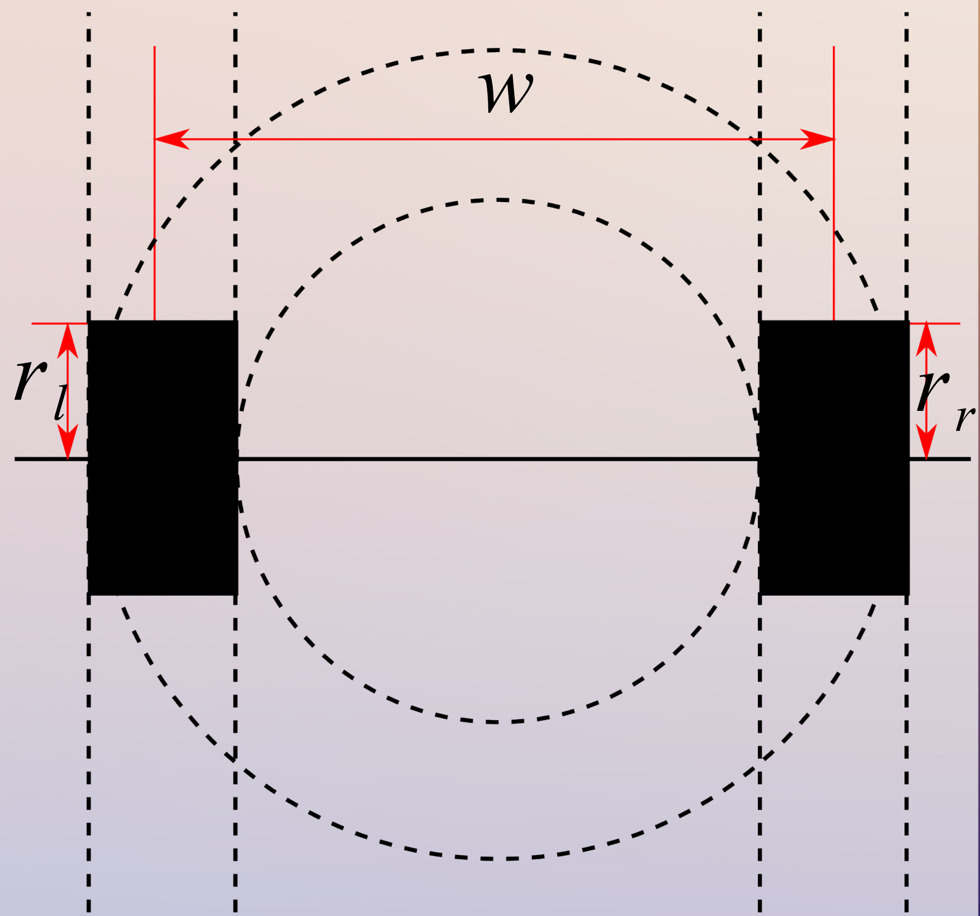
Derivation

- As with odometry, the robot's wheel radii and its width are defined
- If the robot is moving forward at a velocity v, then it holds that:

$$\omega_l = \frac{v}{r_l}, \quad \omega_r = \frac{v}{r_r}$$

where ω_l and ω_r are the angular velocities of the left and right wheels, respectively.



Derivation

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- Similarly, if the robot is only rotating clockwise on point at a speed ω, then it holds that:

$$\omega_l = \frac{w}{2r_l}\omega, \quad \omega_r = \frac{-w}{2r_r}\omega$$

- Since the value $r_l\omega_l + r_r\omega_r$ is 0 when the robot is rotating, and $r_l\omega_l - r_r\omega_r$ is 0 when the robot is moving forward, we solve for v and ω in terms of $r_l\omega_l + r_r\omega_r$ and $r_l\omega_l - r_r\omega_r$:

$$v = \frac{r_l\omega_l + r_r\omega_r}{2}, \quad \omega = \frac{r_l\omega_l - r_r\omega_r}{w}$$

Example

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- The robot's right wheel is travelling in an arc of radius 1 m. If the robot is 15 cm wide (wheel center to wheel center), and its forward velocity is 10 cm/s, determine its angular velocity, assuming the left wheel is rotating faster than the right wheel.

  Solution: The robot is travelling in a circle of radius 1.075 m. Since it is travelling at a forward velocity of 0.1 m / s, it will complete 1 rad of this circle in 10.75 s. Its angular velocity is thus:

$$\omega = \frac{1}{10.75} \approx 0.093\ \mathrm{rad/s}$$

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Example

- If we assume wheel radii of 2.8 cm, we can then compute the speed of the robot's wheels:

$$\omega_l = \frac{v}{r_l} + w\frac{\omega}{2r_l} \approx 3.82\,\mathrm{rad/s}$$

$$\omega_r = \frac{v}{r_r} - w\frac{\omega}{2r_r} \approx 3.32\,\mathrm{rad/s}$$

or 219 and 190 degrees per second, respectively.

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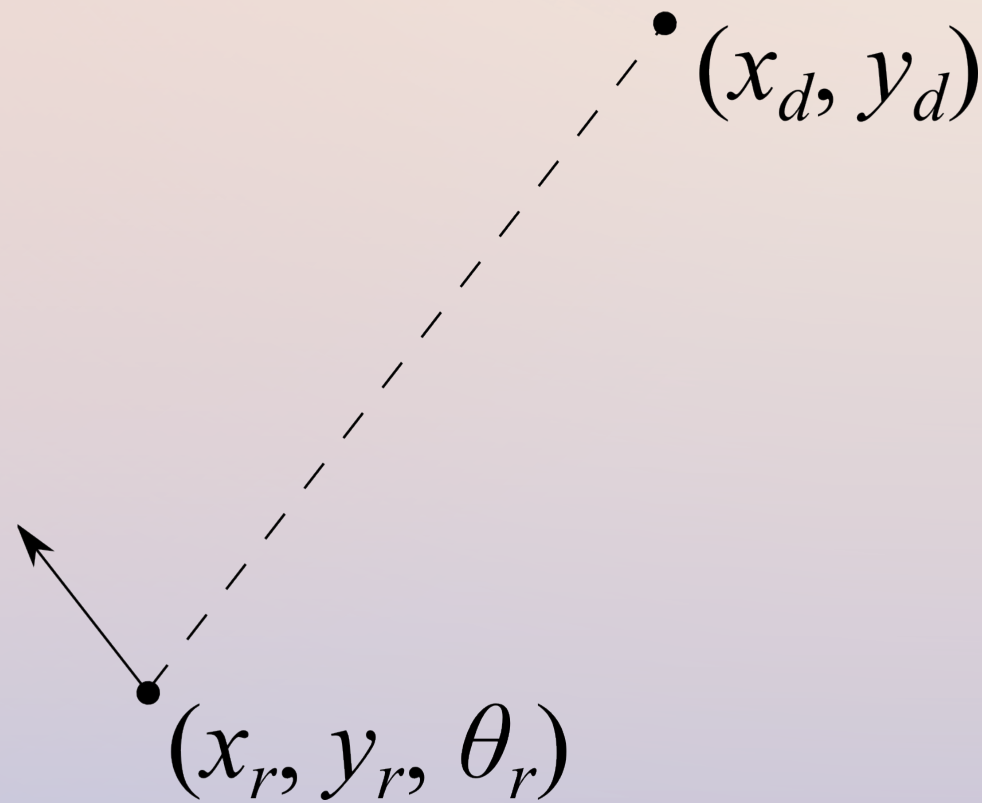
Derivation

- Now let us assume that we want to move the robot from its current position, (x_r, y_r, θ), to a new position (x_d, y_d)
- The robot clearly must rotate to face its destination, which is at a heading of:

$$\theta_d = \tan^{-1}(y_d - y_r, x_d - x_r)$$



Derivation

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- The $\tan^{-1}(y, x)$ function is a binary form of the unary arctan function, which, unlike its unary cousin, provides the correct result when x is negative:

$$\tan^{-1}(y,x)=\begin{cases}\tan^{-1}\left(\dfrac{y}{x}\right) & x>0\\ \tan^{-1}\left(\dfrac{y}{x}\right)+\pi & x<0,\ y>0\\ \tan^{-1}\left(\dfrac{y}{x}\right)-\pi & x<0,\ y<0\end{cases}$$

Derivation

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- The *error* in the robot's heading is $\theta_d - \theta_r$, however this value may not represent the minimal rotation of the robot (e.g. the robot may rotate 359º clockwise instead of 1º counter-clockwise)
- To correct this flaw, consider possible values of $\theta_d - \theta_r$
  - Assume θ_d and θ_r are each angles between 0 and 359
  - Then $\theta_d - \theta_r$ cannot be smaller than -359, nor larger than +359
  - Since the interval (-180, 180) will result in minimal rotation, only the intervals (-359, -180) and (180, 359) need consideration

Derivation

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- When $\theta_d - \theta_r$ is less than -180, the robot will attempt to rotate counter-clockwise by more than 180º, when it should instead rotate clockwise by $360^\circ + (\theta_d - \theta_r)$
- When $\theta_d - \theta_r$ is greater than 180, the robot will attempt to rotate clockwise by more than 180º, when it should instead rotate counter-clockwise by $(\theta_d - \theta_r) - 360^\circ$
- Thus we will define (see next slide) a function $f(x)$ such that:

$$f(\theta_d - \theta_r) \in (-180, 180)$$

Derivation

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- As is seen below, a simple piecewise function will correctly round the error in the heading of the robot to the interval (-180º, 180º)

$$f(\theta_d-\theta_r)=\begin{cases}\theta_d-\theta_r & (\theta_d-\theta_r)\in(-180,180)\\(\theta_d-\theta_r)+360 & (\theta_d-\theta_r)<-180\\(\theta_d-\theta_r)-360 & (\theta_d-\theta_r)>180\end{cases}$$

Derivation

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- For the forward position error, several options exist
  - Euclidean distance from the robot to the destination: Easily calculated, but will only work for small angular error.
  - Dot product of vector connecting robot to destination and robot's unit direction vector: Harder to calculate, but will work for large angular error
- Once the forward and angular errors are computed, a *controller* uses these values to determine the speeds of the motors (see block diagram on next slide)

Block Diagram

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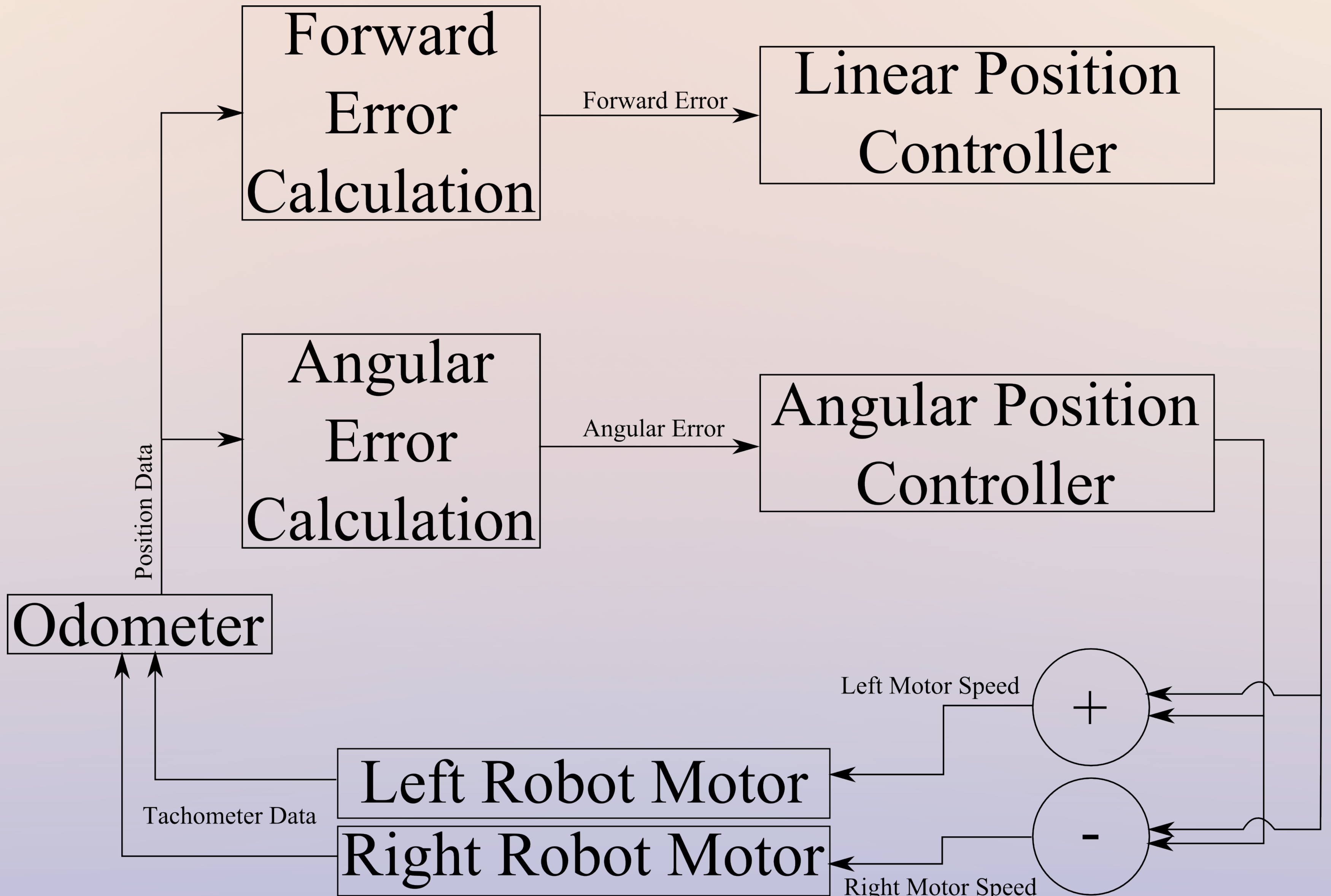

Summary

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- The robot chooses a (possibly hard-coded) destination point on the field
- This point's coordinates are used in the forward and angular error calculations to determine how much the robot needs to advance or turn
- The controllers then compute the linear and angular *velocities* which are added and subtracted for the left and right wheel motors respectively
- This in turn causes the motors to spin, changing the robot's coordinates, and causing new values to be calculated when this process start all over