COM2109

Automata

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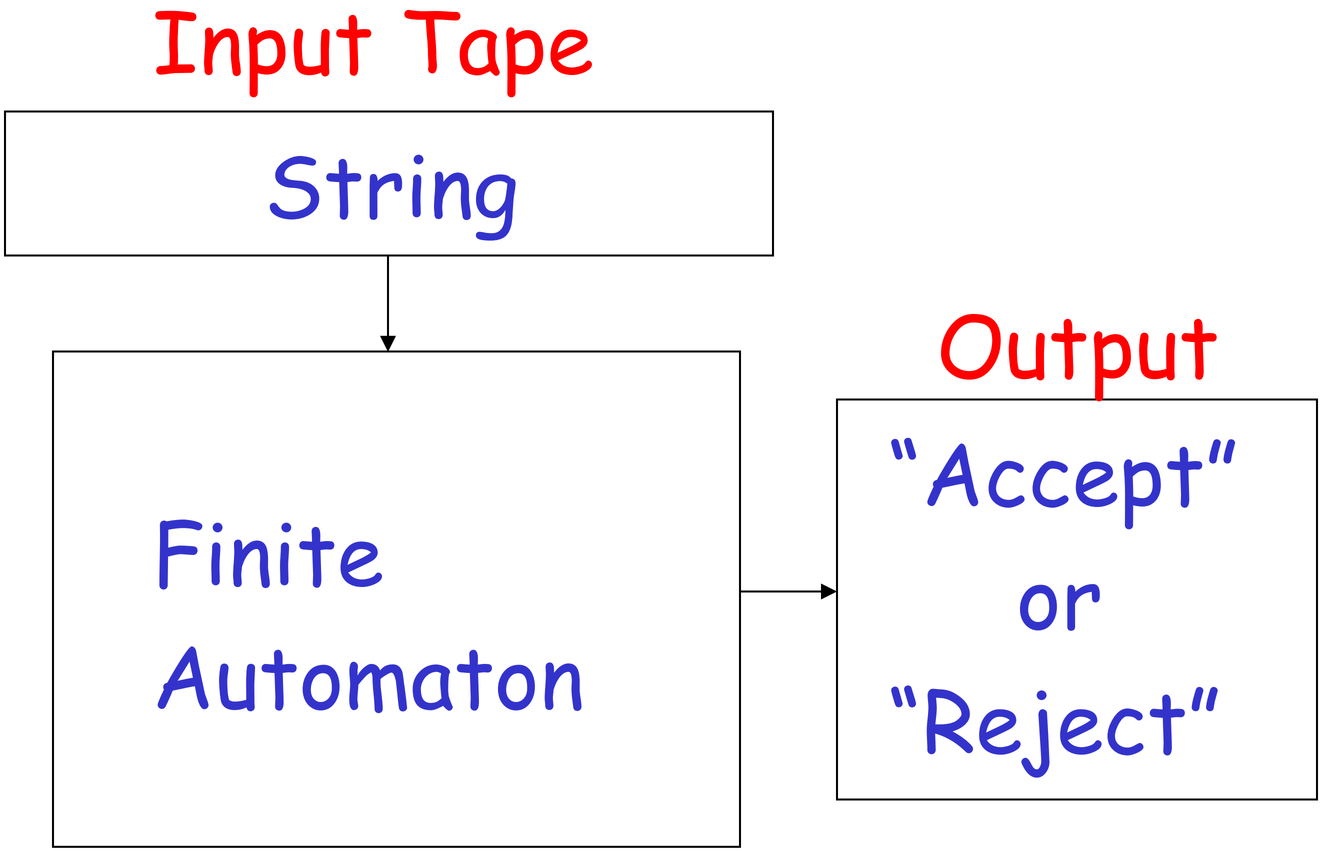
Based on the work of Lucia Specia,
M.S. Moorthy and Costas Busch

Deterministic

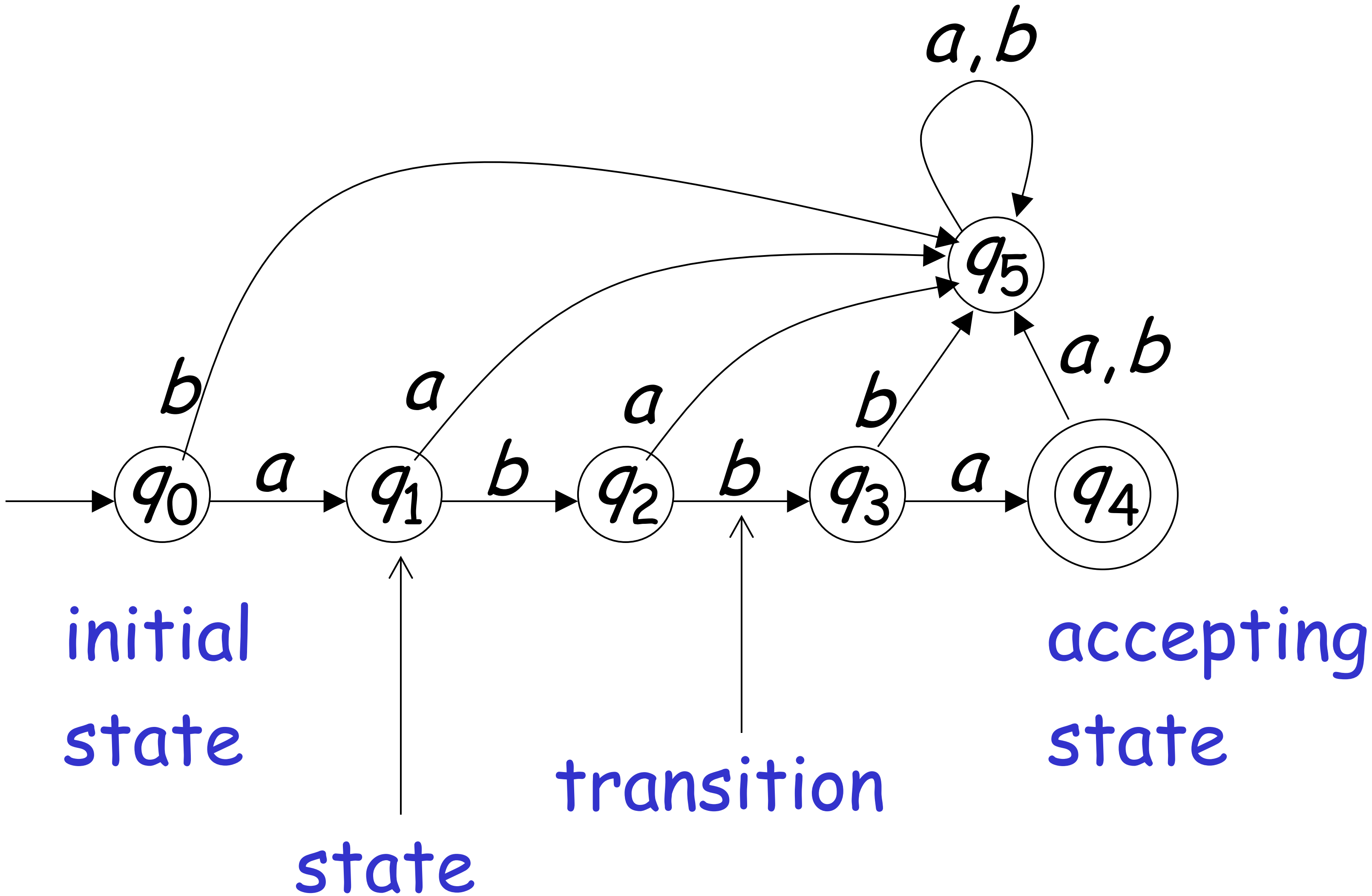
Finite Automata

And Regular Languages

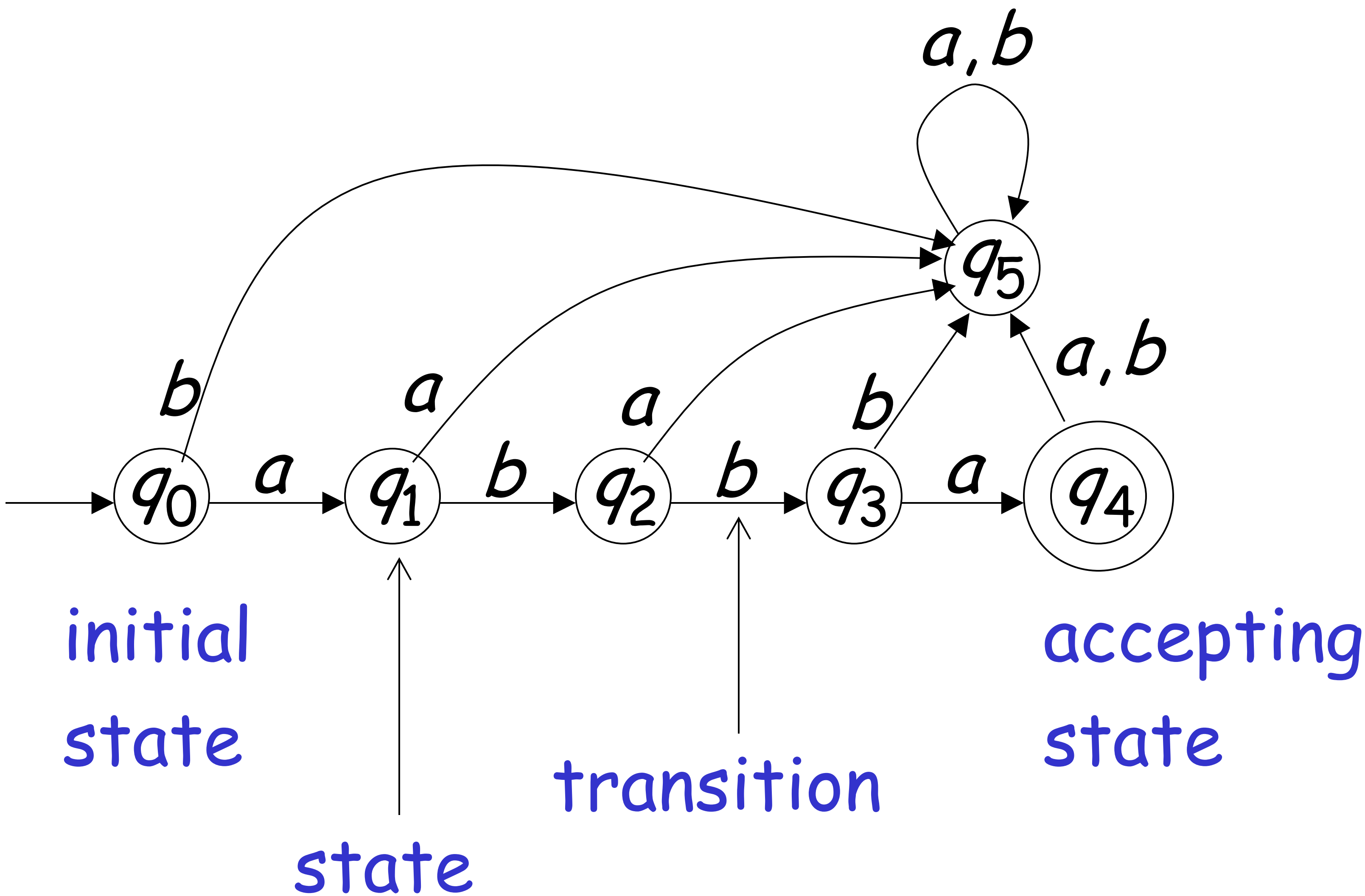
Deterministic Finite Automaton (DFA)



Transition Graph

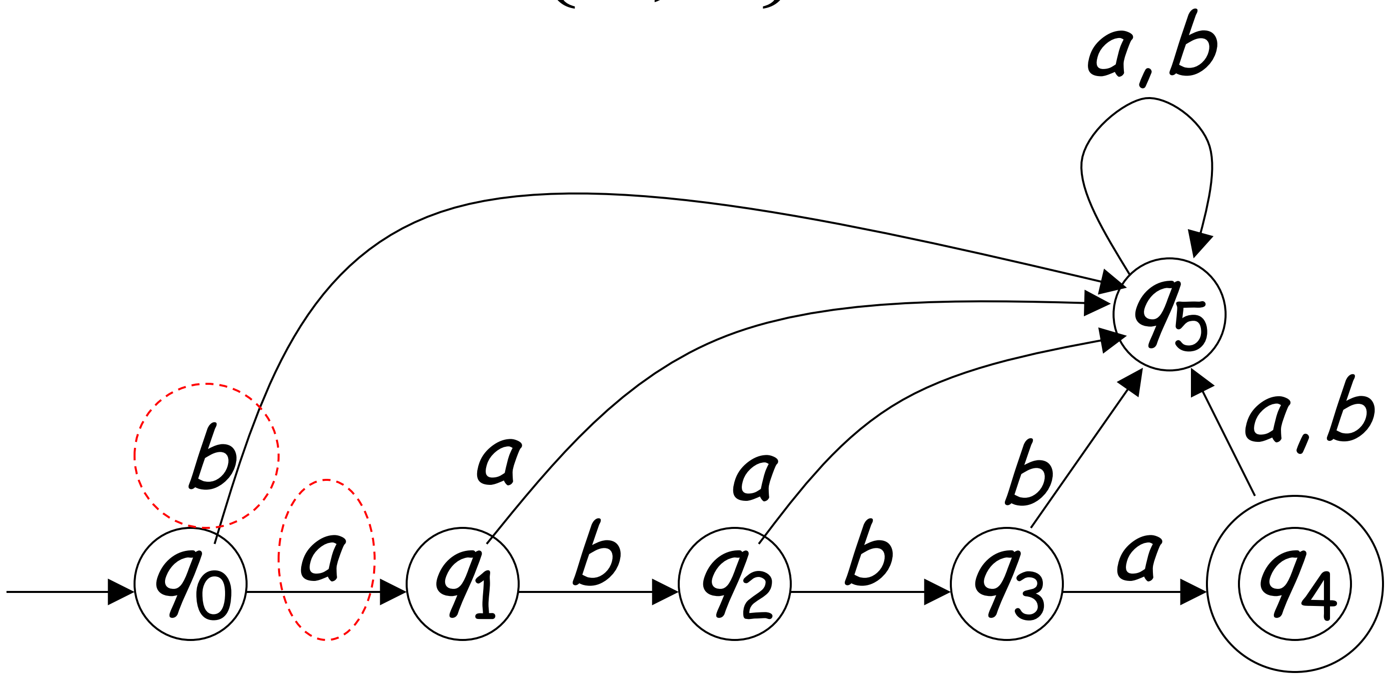


Transition Graph



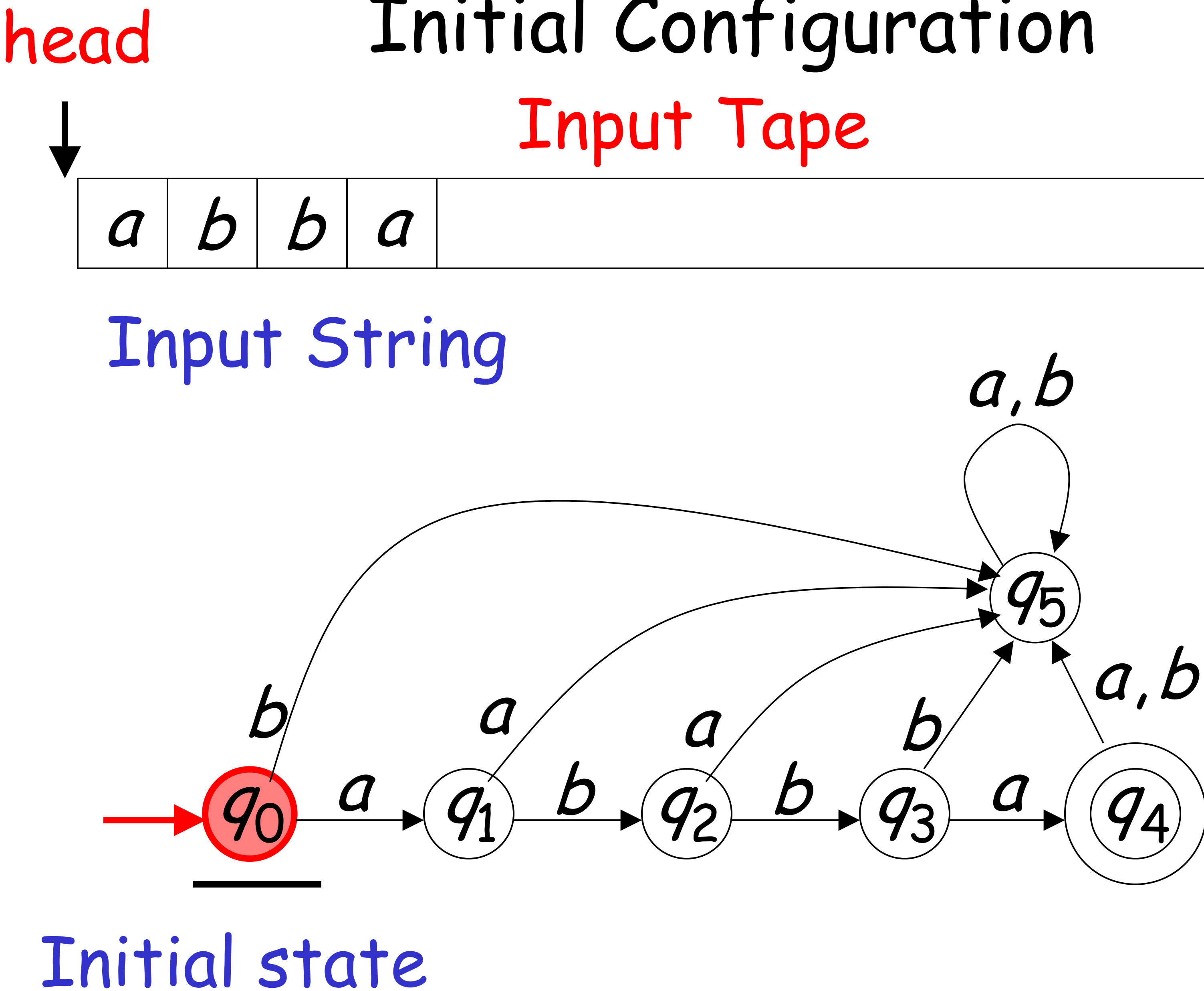
Also called a finite state machine

Alphabet $\Sigma = \{a, b\}$

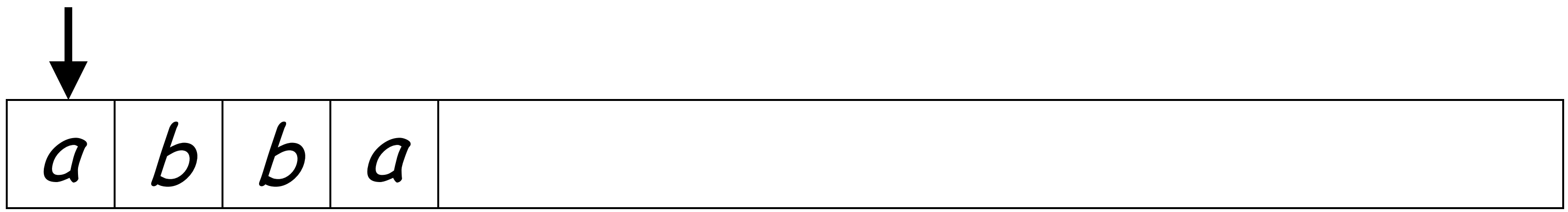


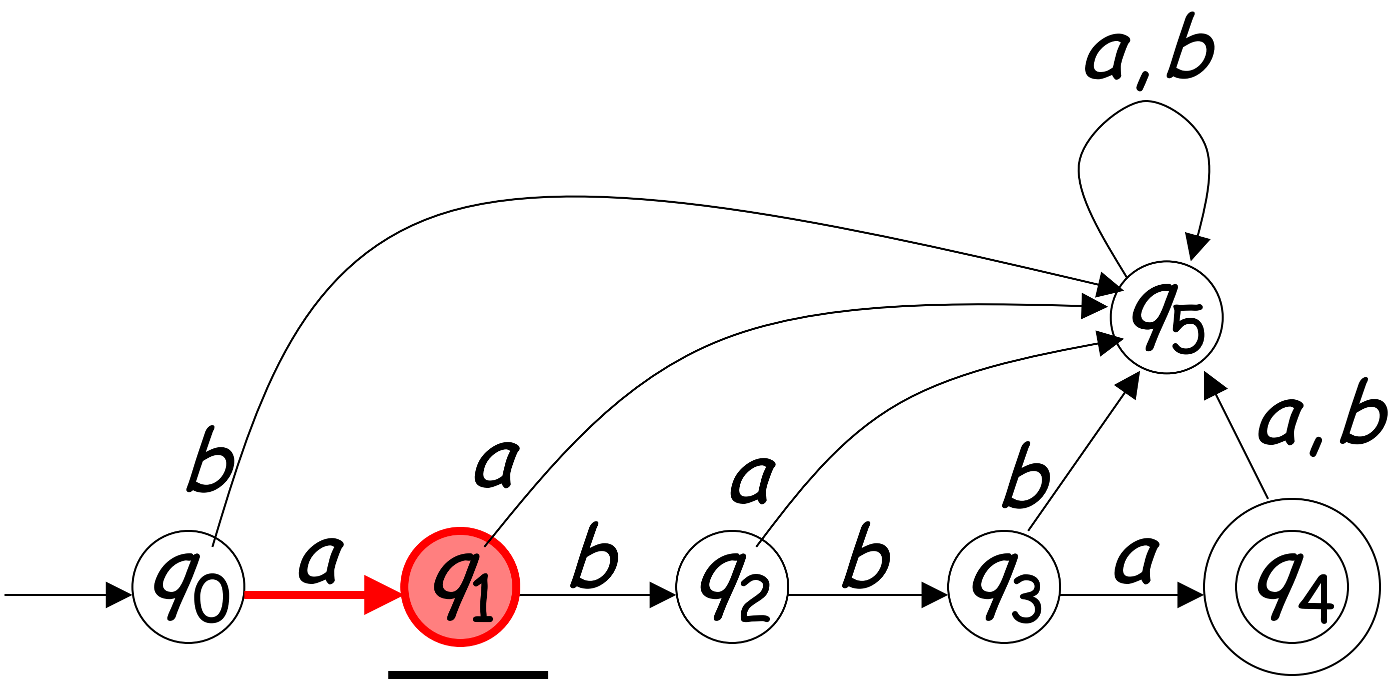
For every state, there is a transition for every symbol in the alphabet

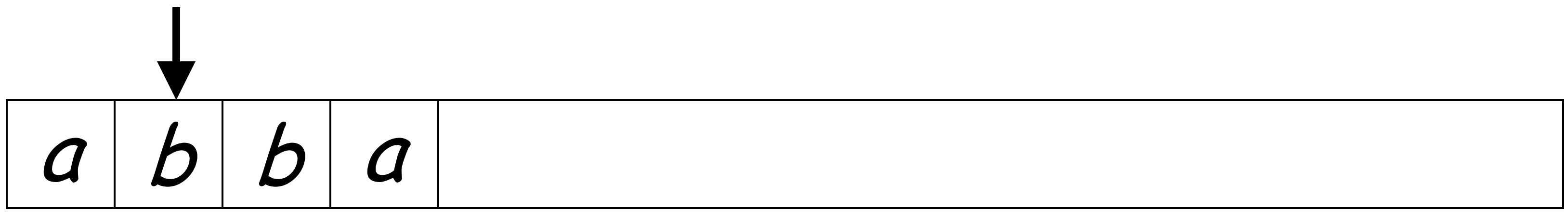
Initial Configuration

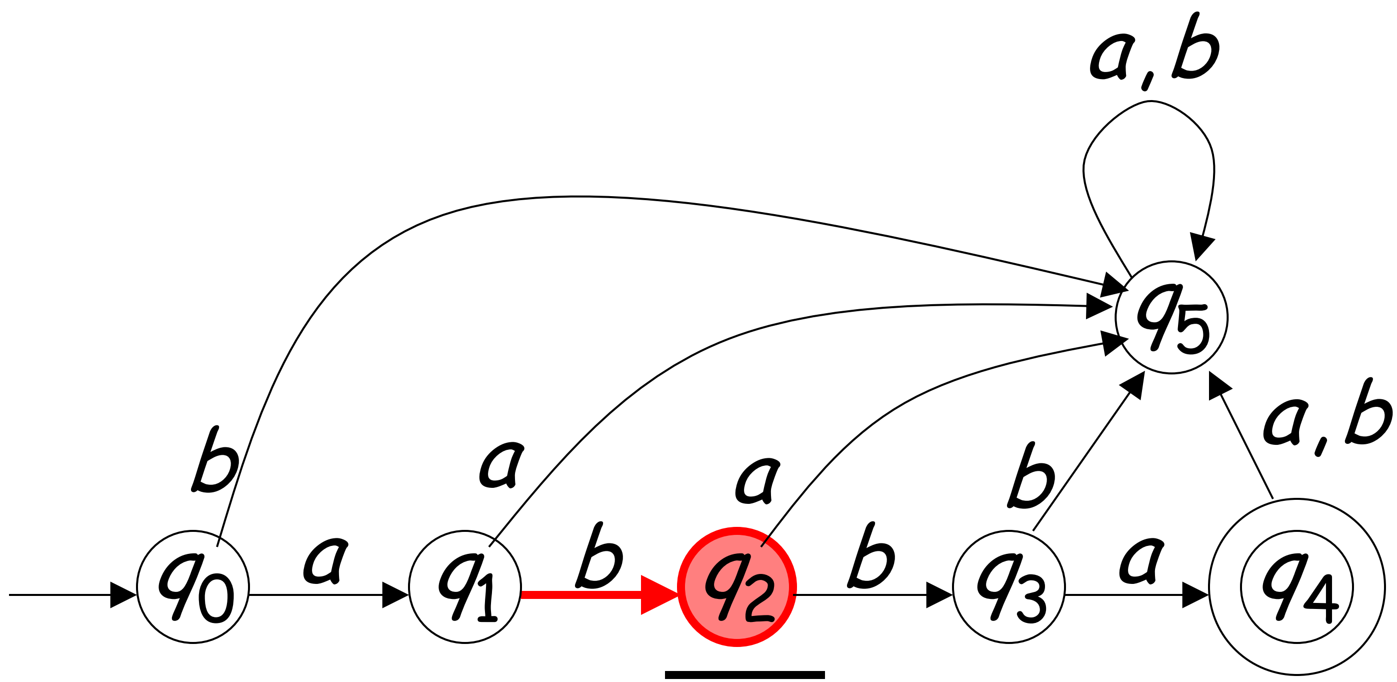


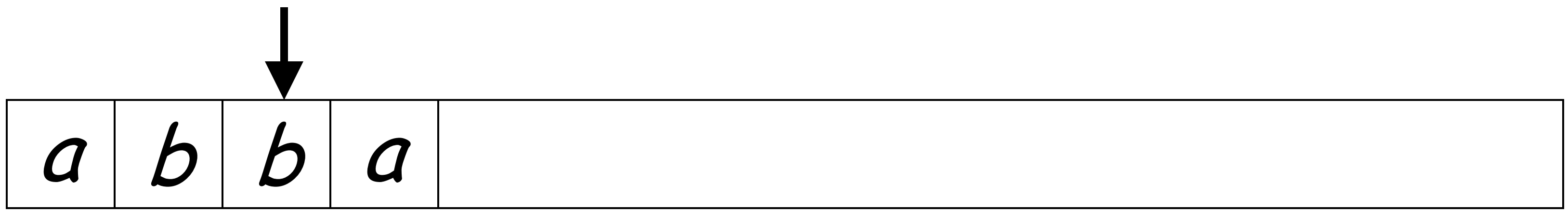
Scanning the Input

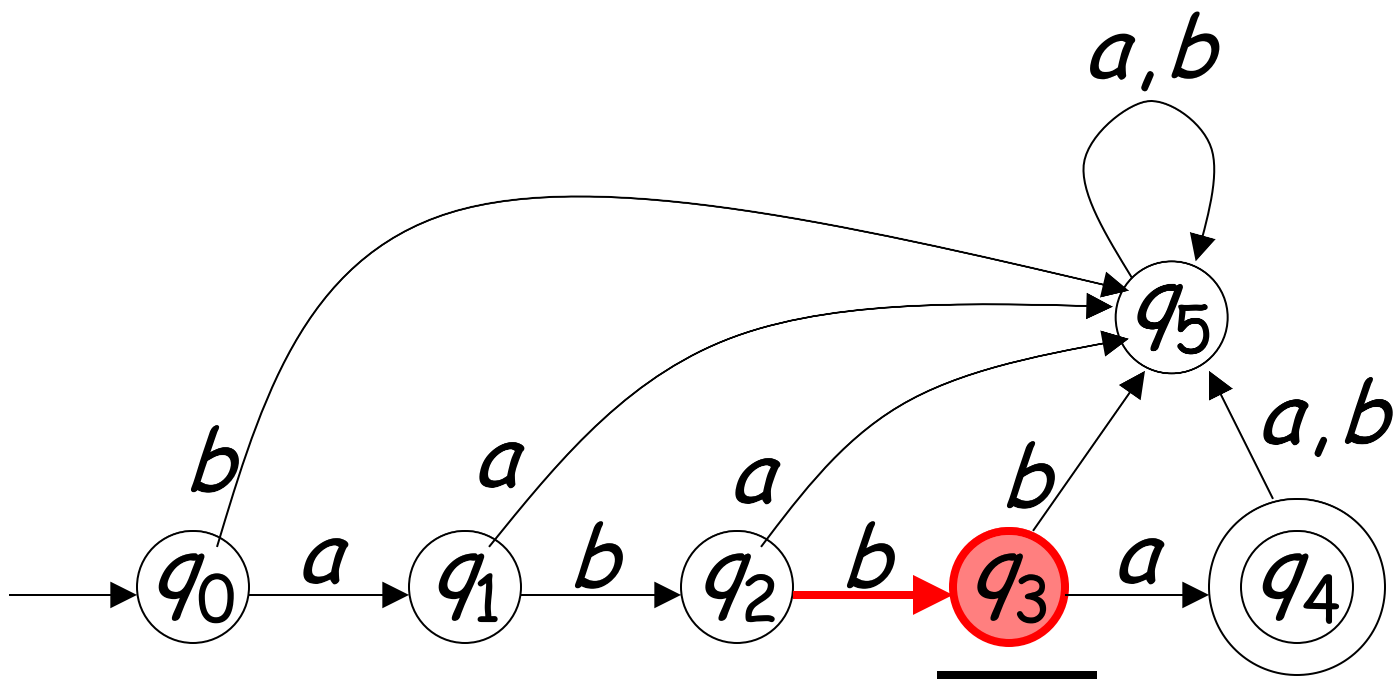




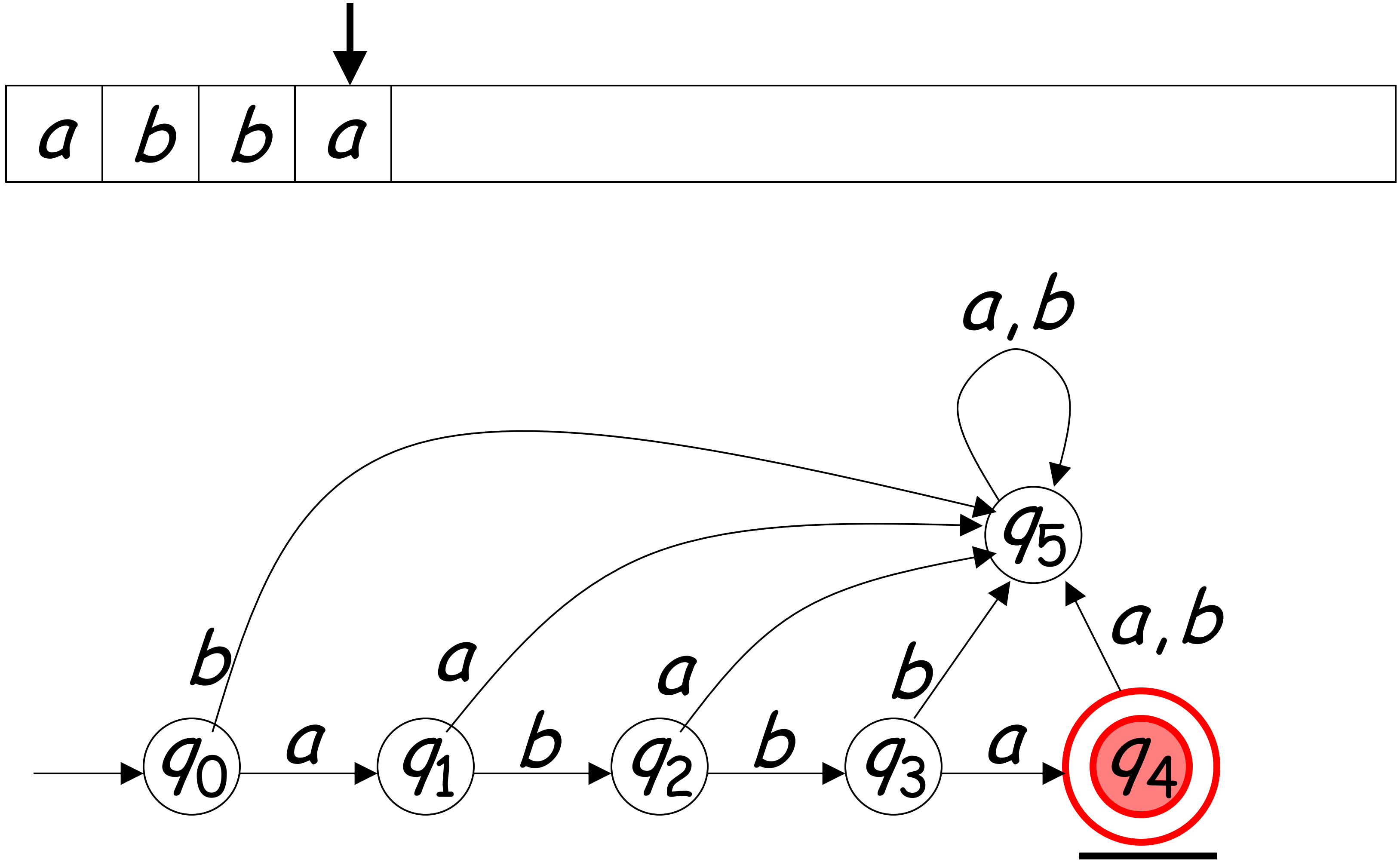

a
b
b
a


a, b
b
a
a
b
a, b
a
b
b
a
q0
q1
q2
q3
q4
q5


a
b
b
a

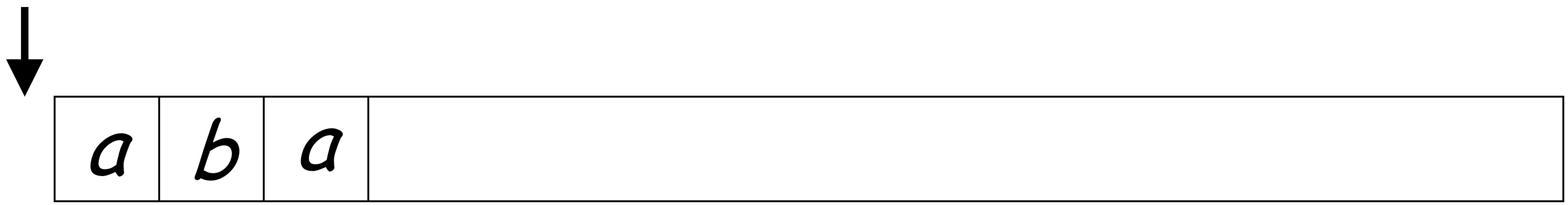

a,b
q_5
b
a
a
b
a,b
q_0
a
q_1
b
q_2
b
q_3
a
q_4

Input finished

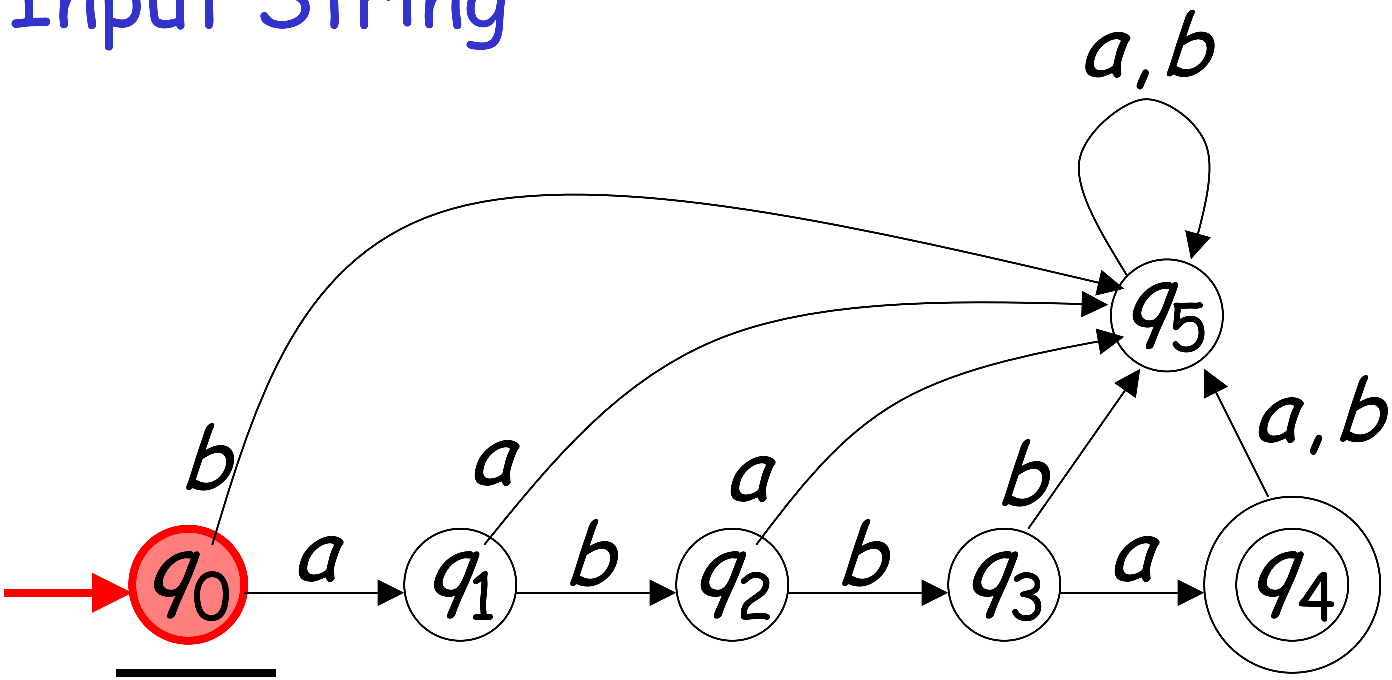


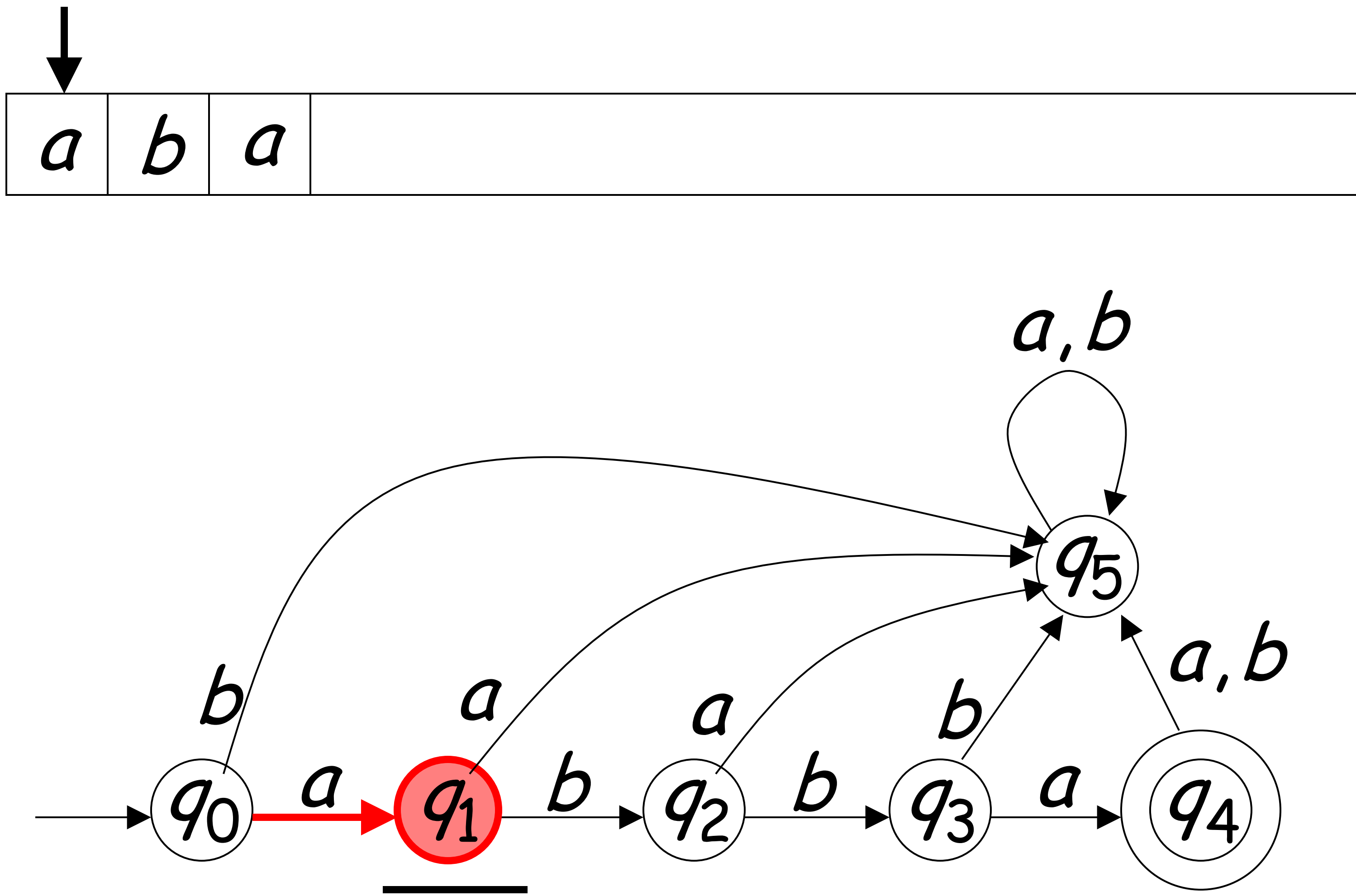
accept

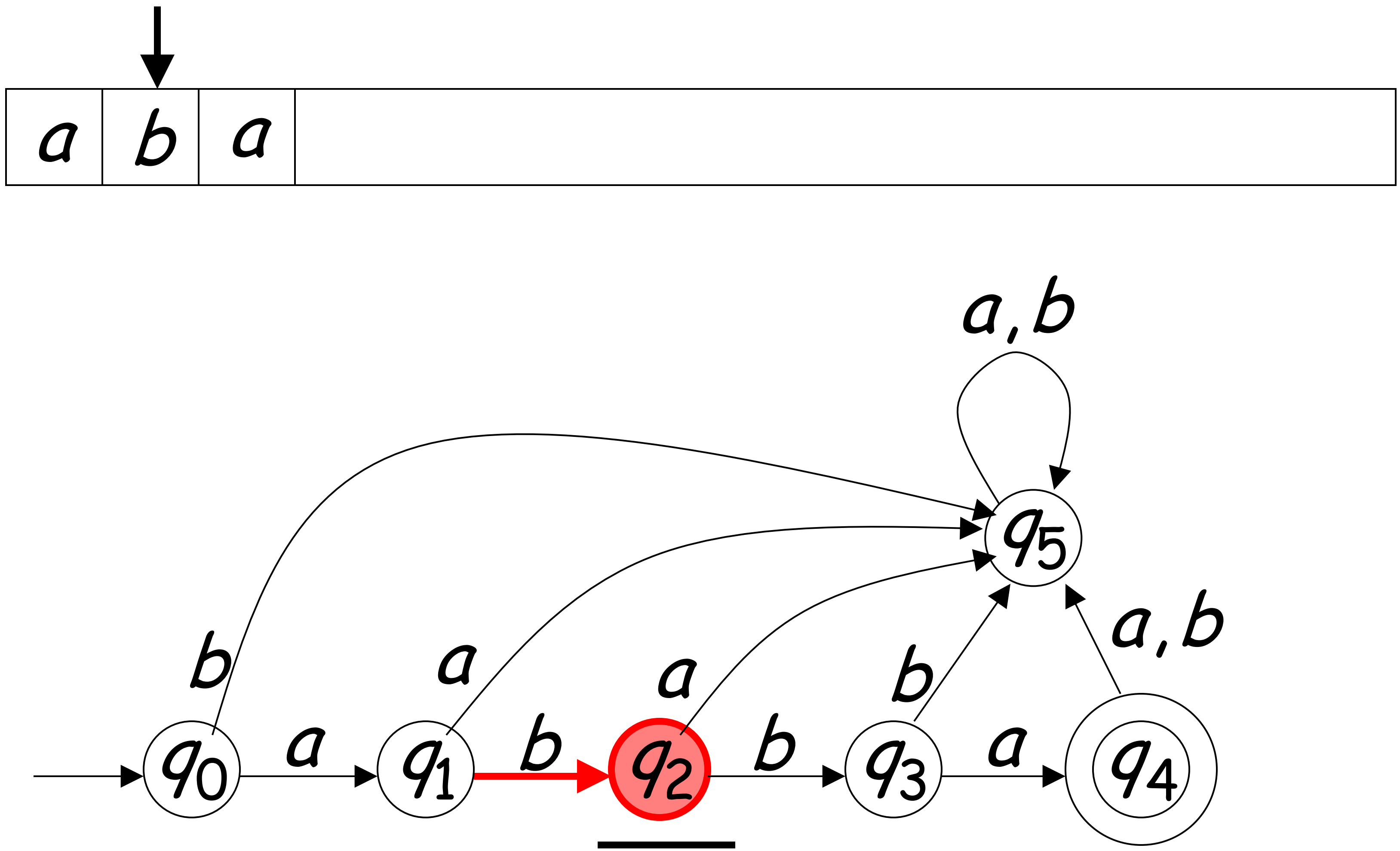
A Rejection Case



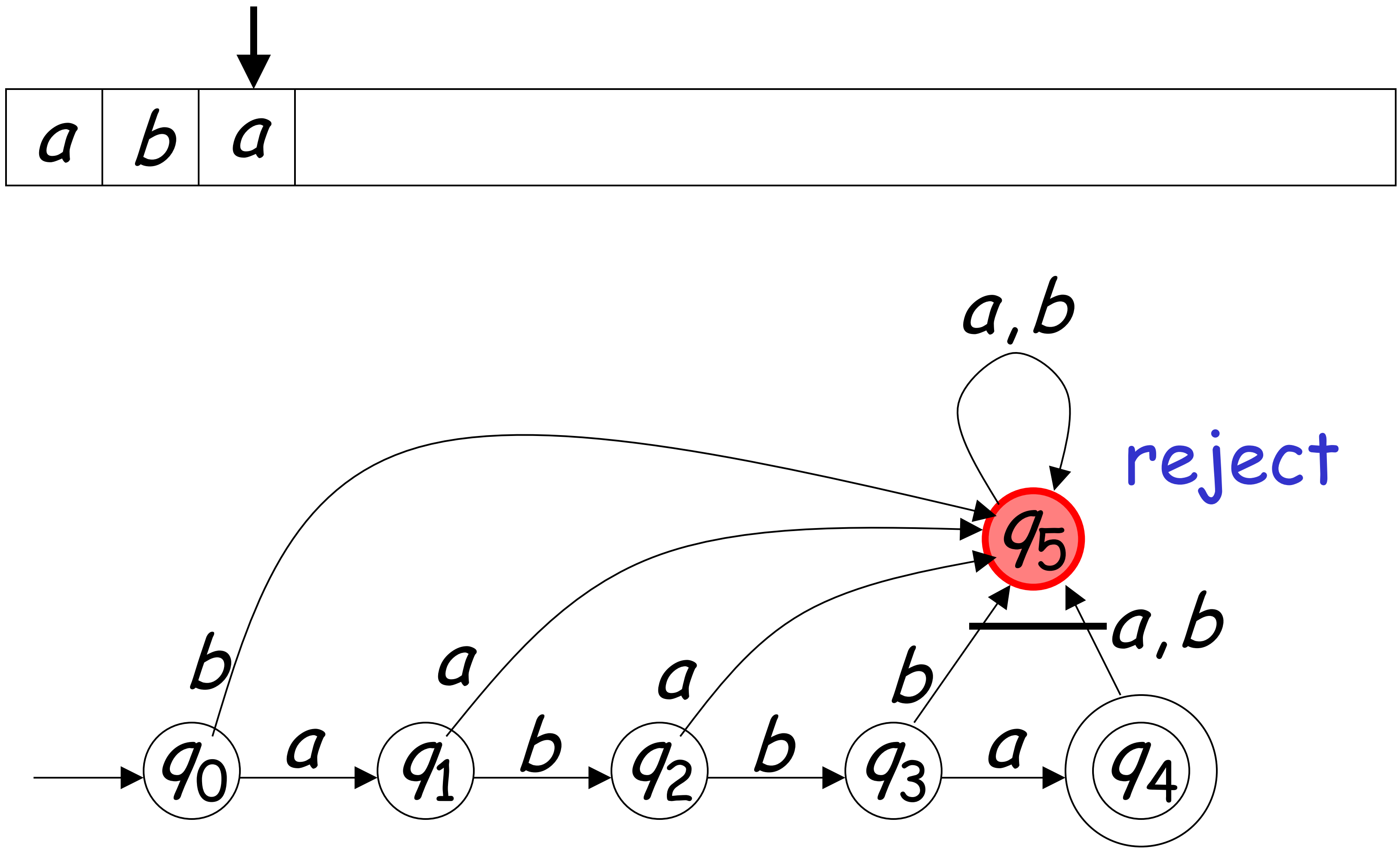
Input String




a
b
a
a,b
b
a
a
b
a,b
a
b
b
a
q_0
q_1
q_2
q_3
q_4
q_5


a
b
a
a,b
b
a
a
b
b
b
a
a,b
q_0
q_1
q_2
q_3
q_4
q_5

Input finished

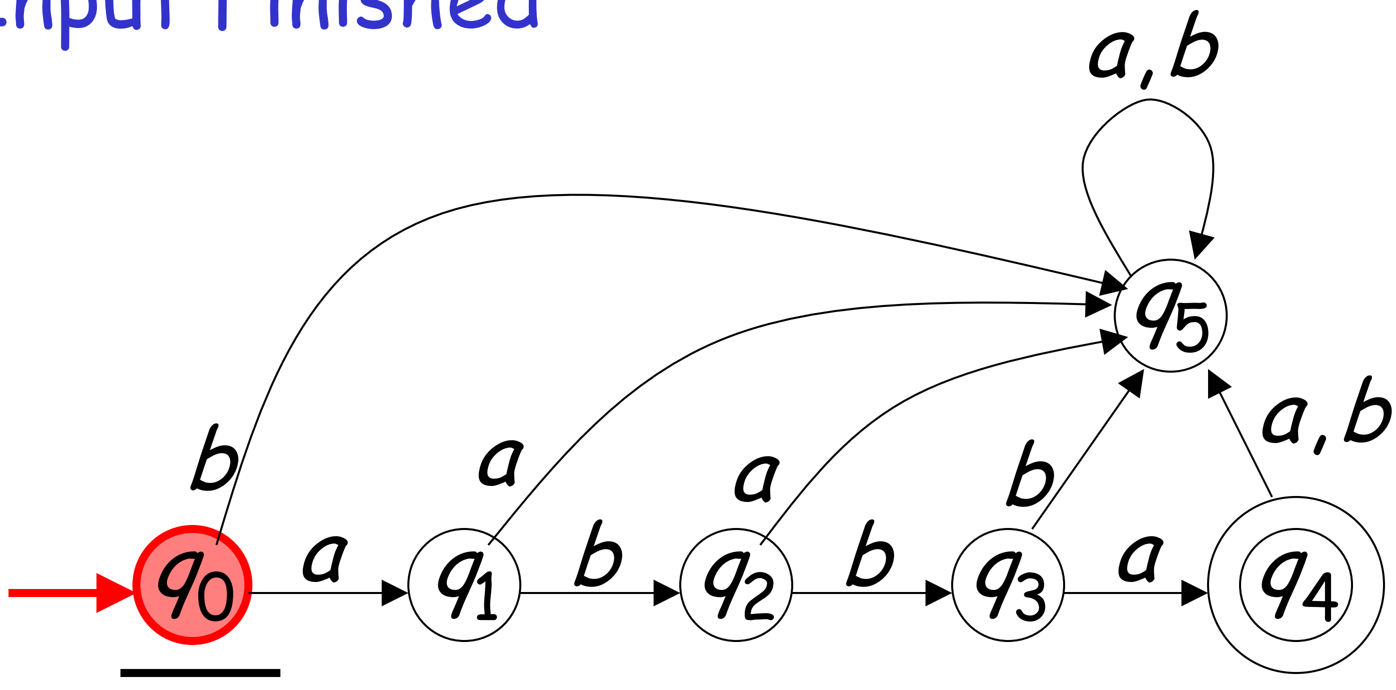


Another Rejection Case

Tape is empty

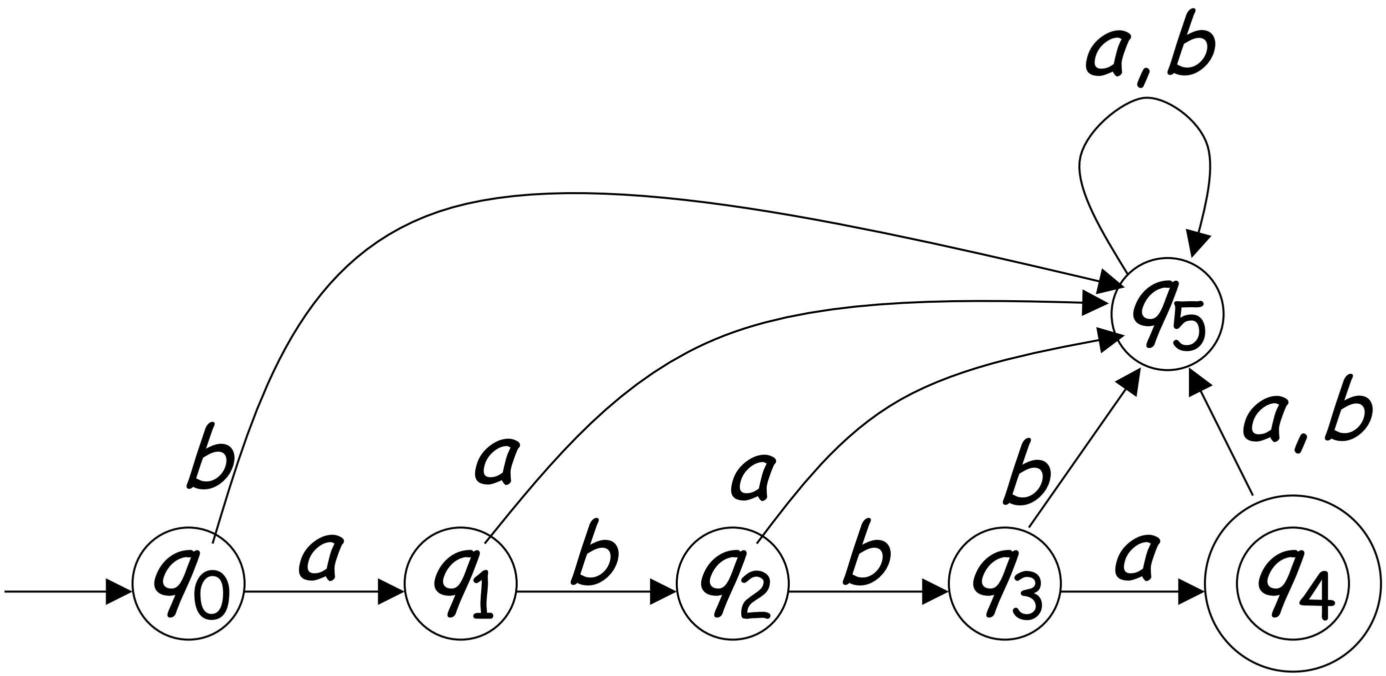
(ε)

Input Finished



reject

Language Accepted: $L = \{abba\}$



To accept a string:

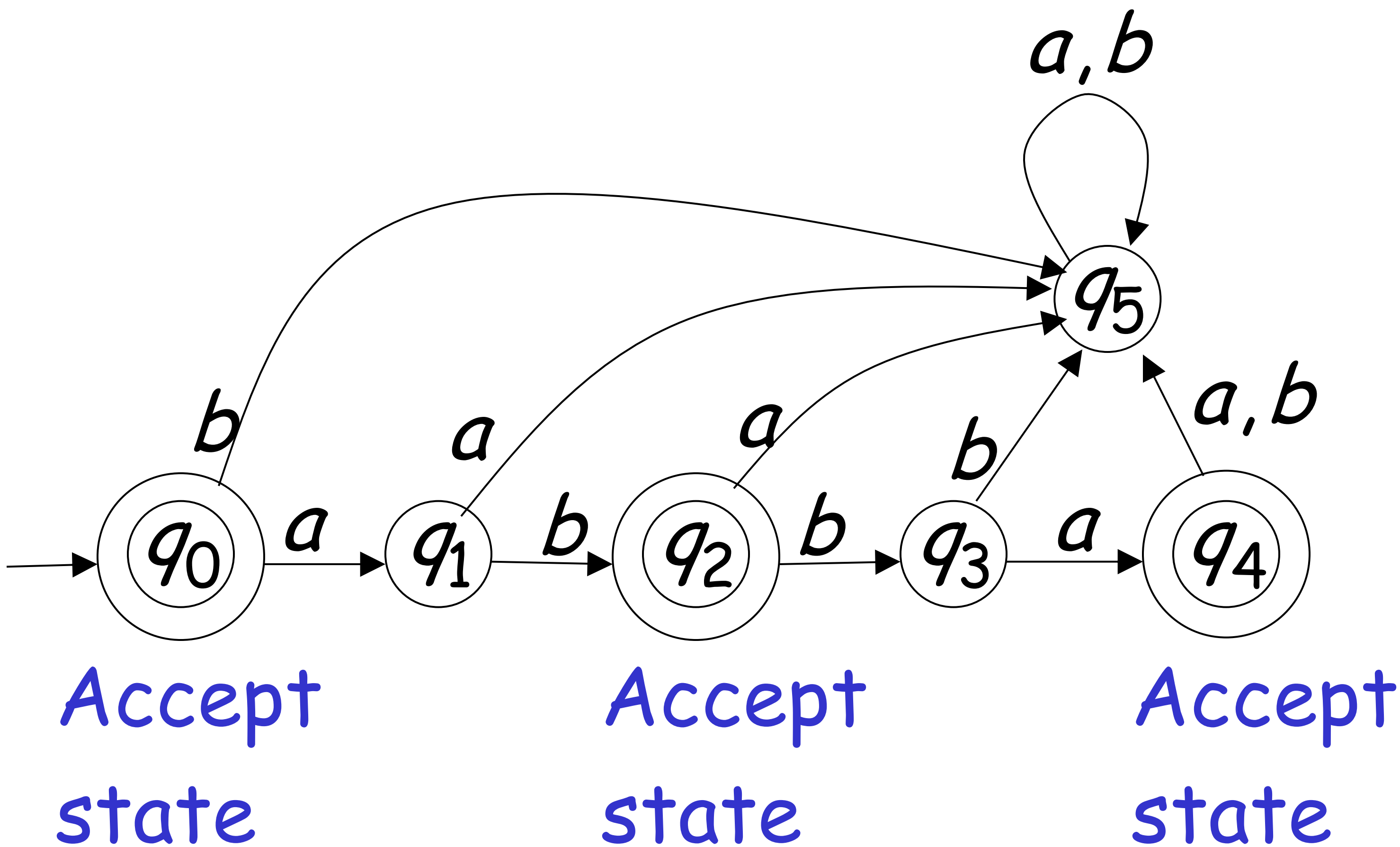
all the input string is scanned
and the last state is an accepting
state

To reject a string:

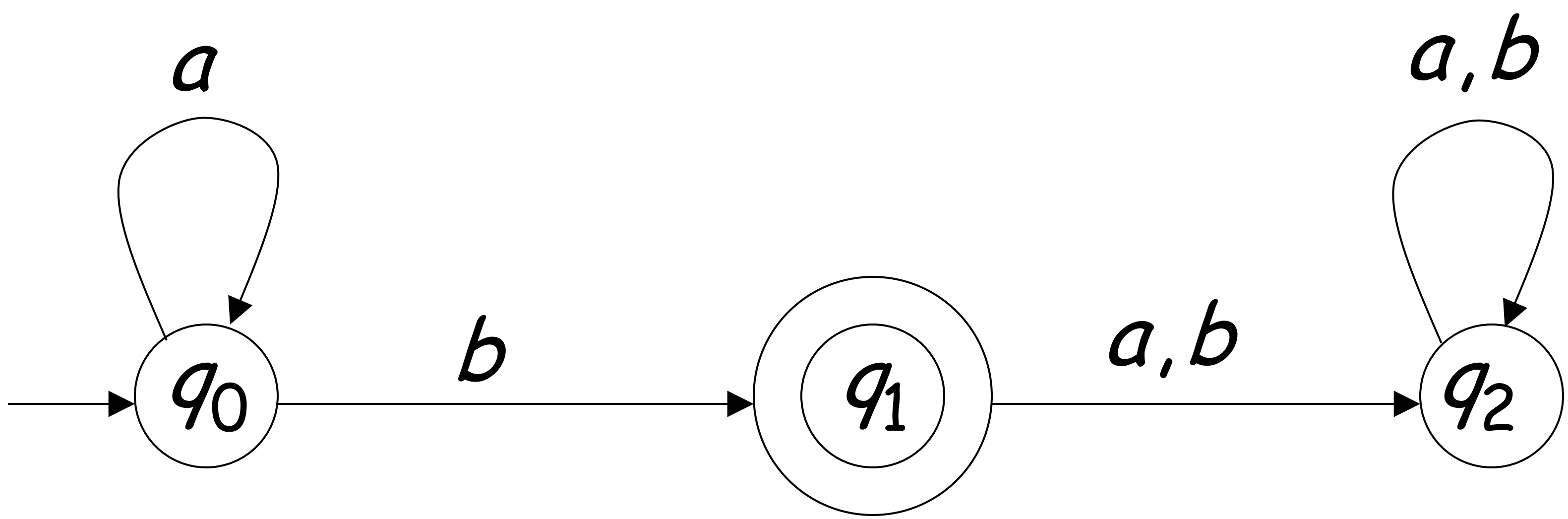
all the input string is scanned
and the last state is not an accepting
state

Another Example

$$L = \{\varepsilon, ab, abba\}$$



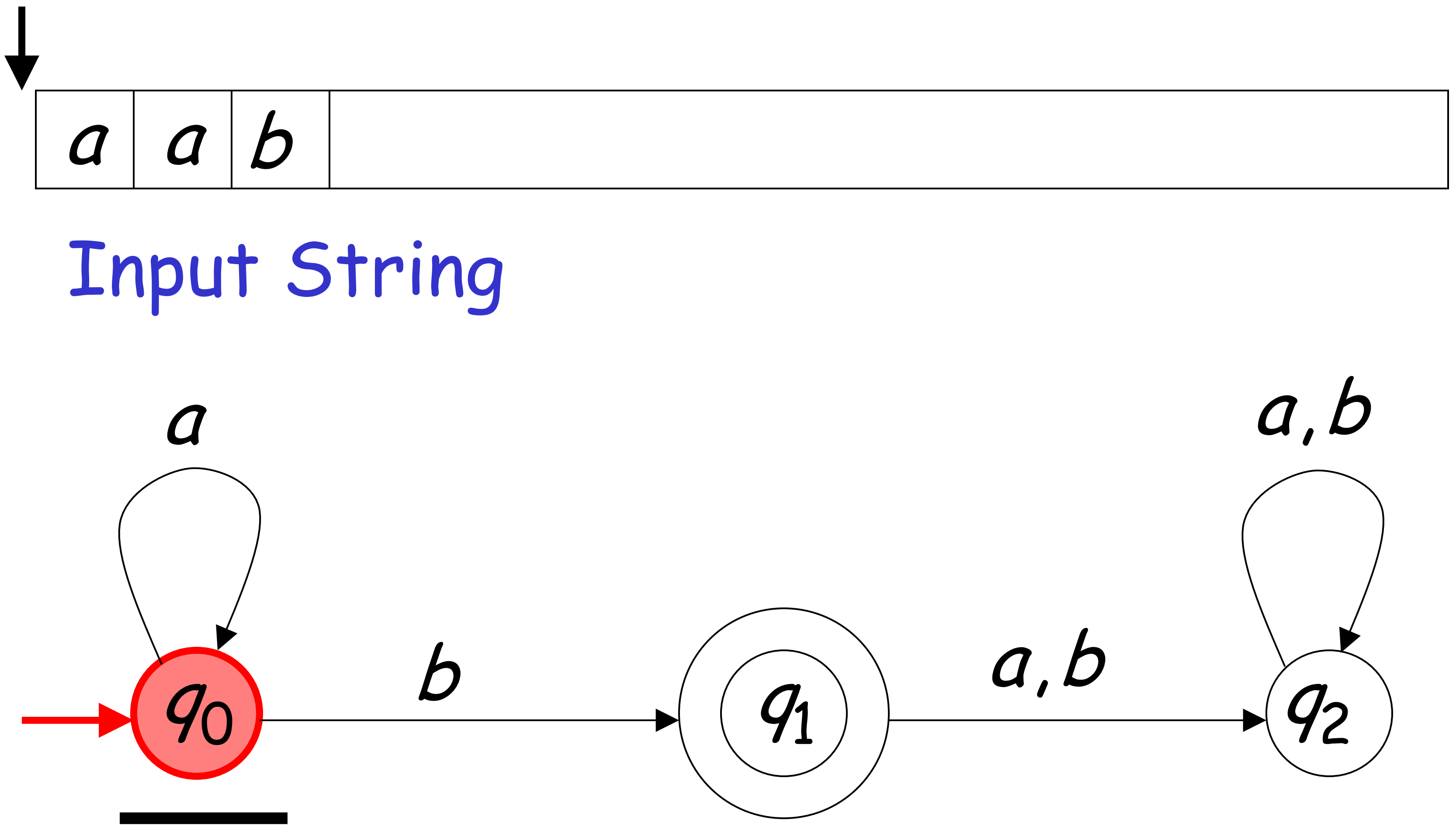
Another Example

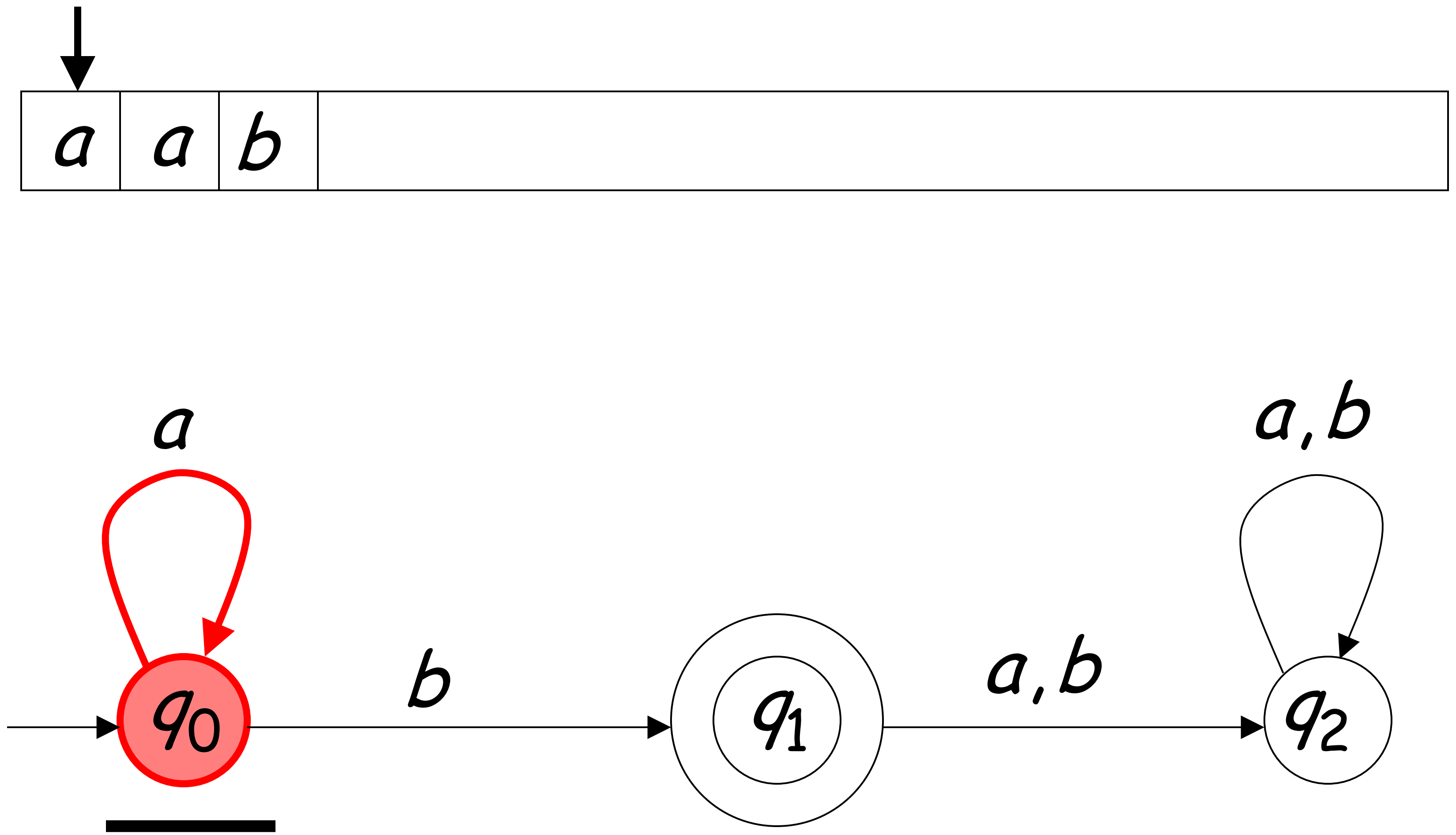


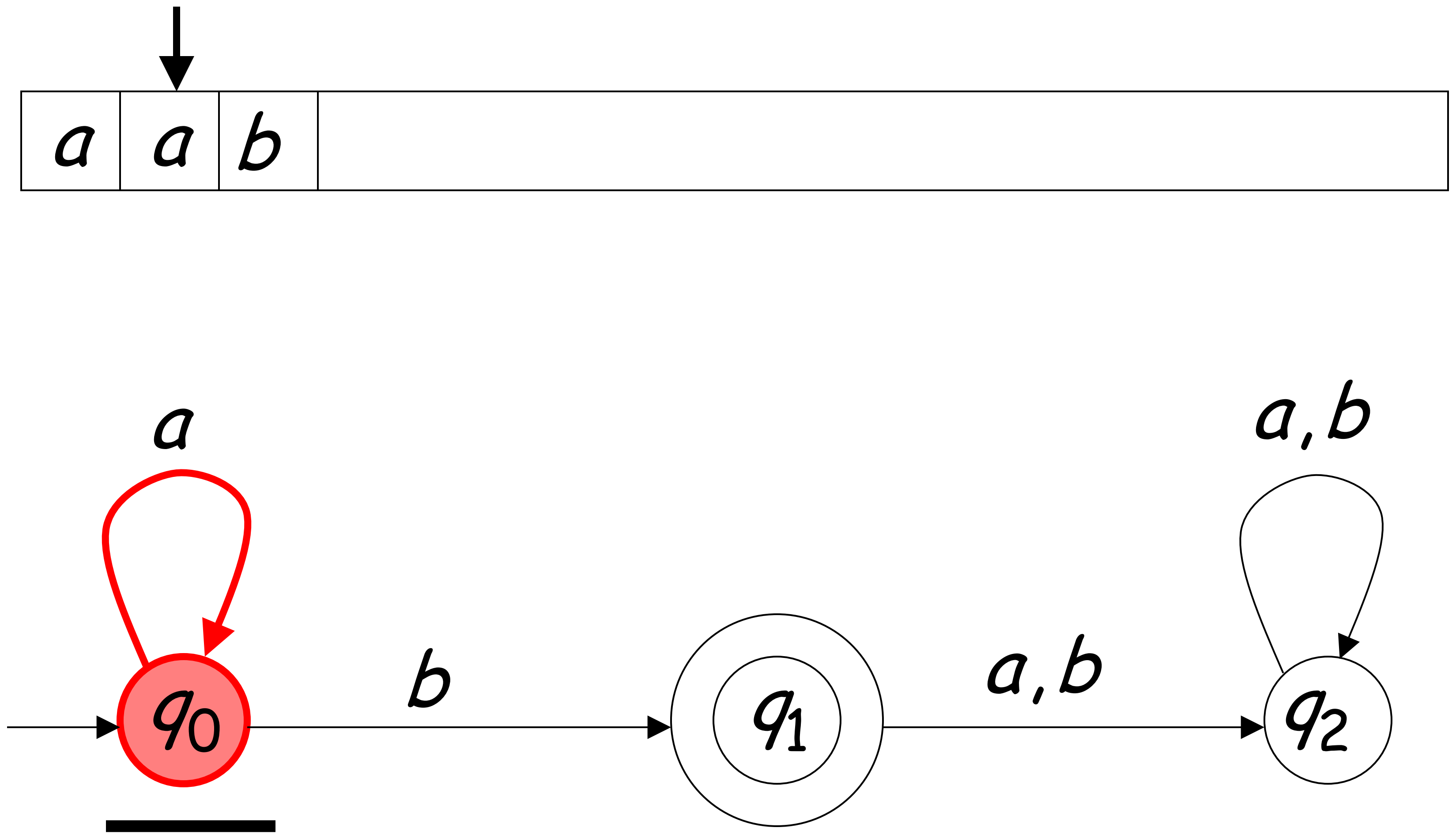
Accept state

trap state

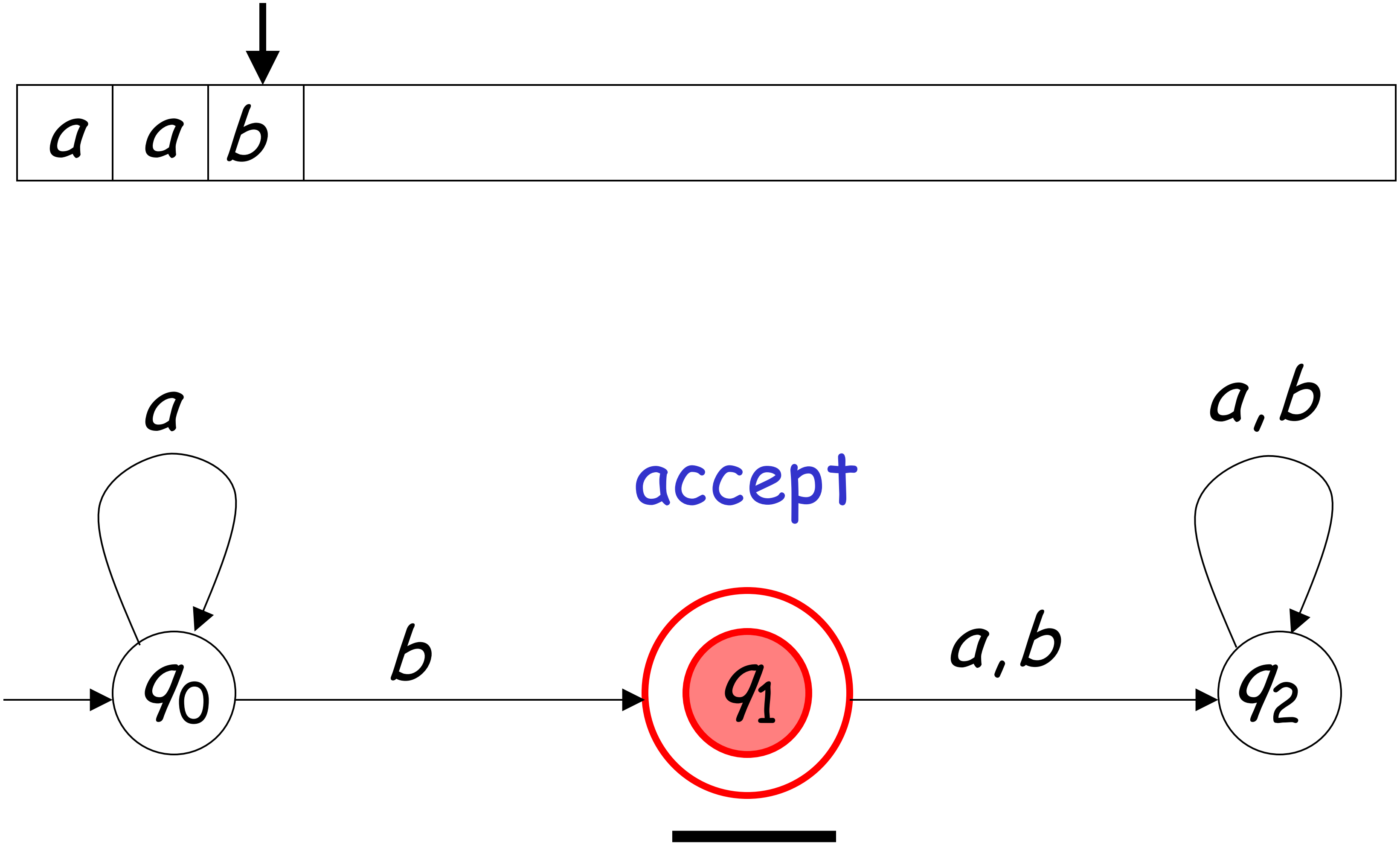
If a transition goes to a state from which it can never escape.


a
a
b
Input String
a
a,b
b
a,b
q_0
q_1
q_2

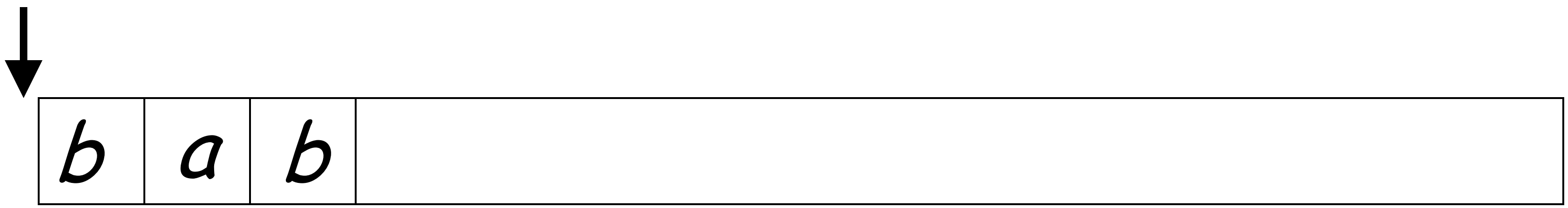

a
a
b
a
a,b
b
a,b
q_0
q_1
q_2


a
a
b
a
a,b
b
a,b
q_0
q_1
q_2

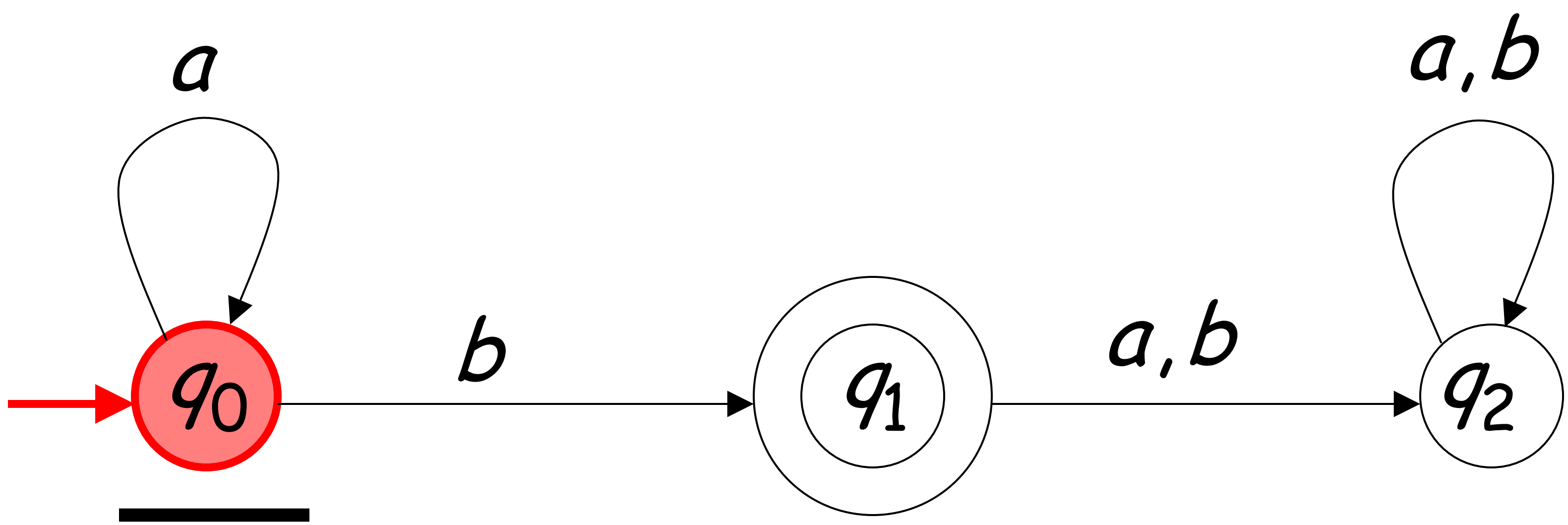
Input finished

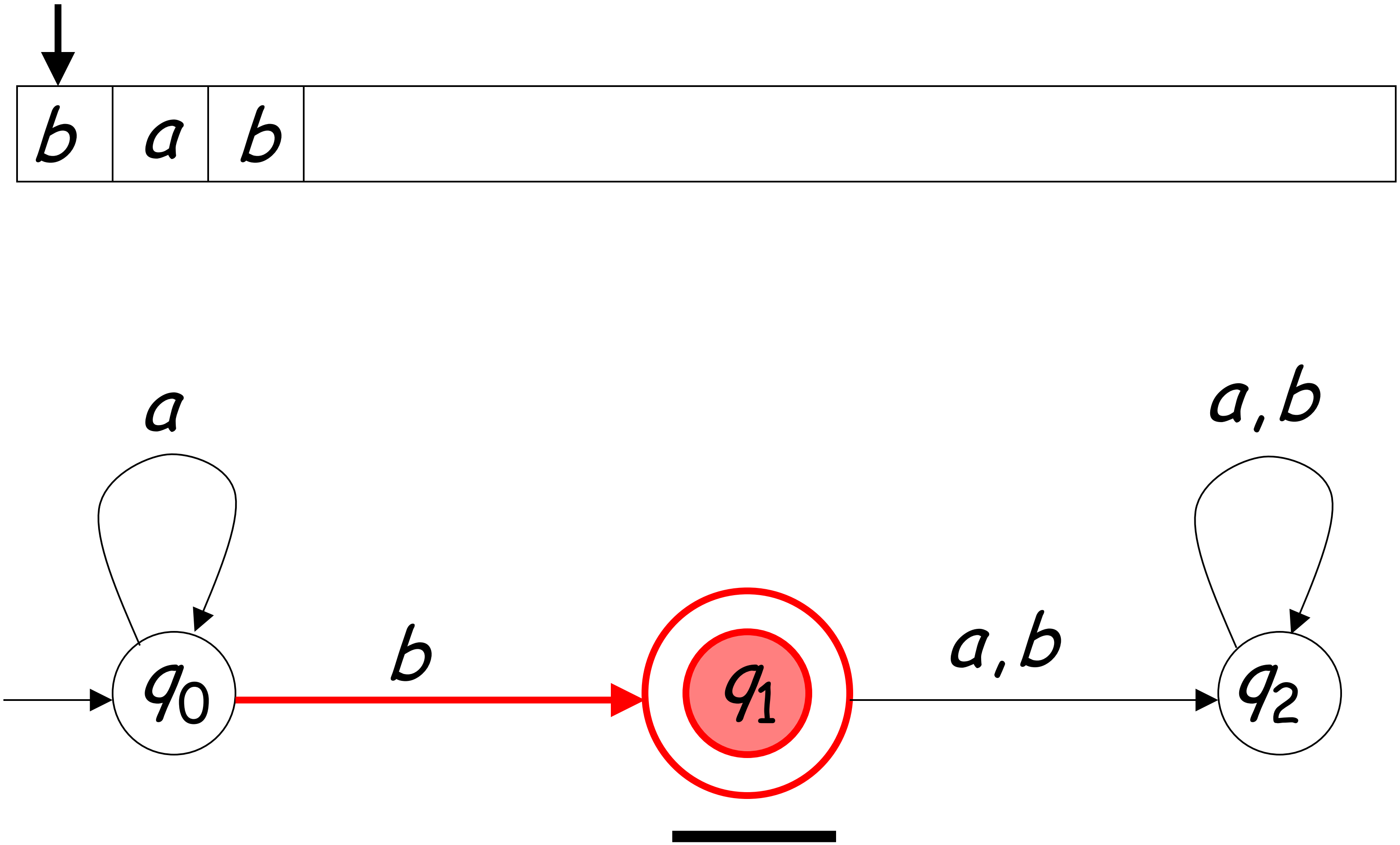


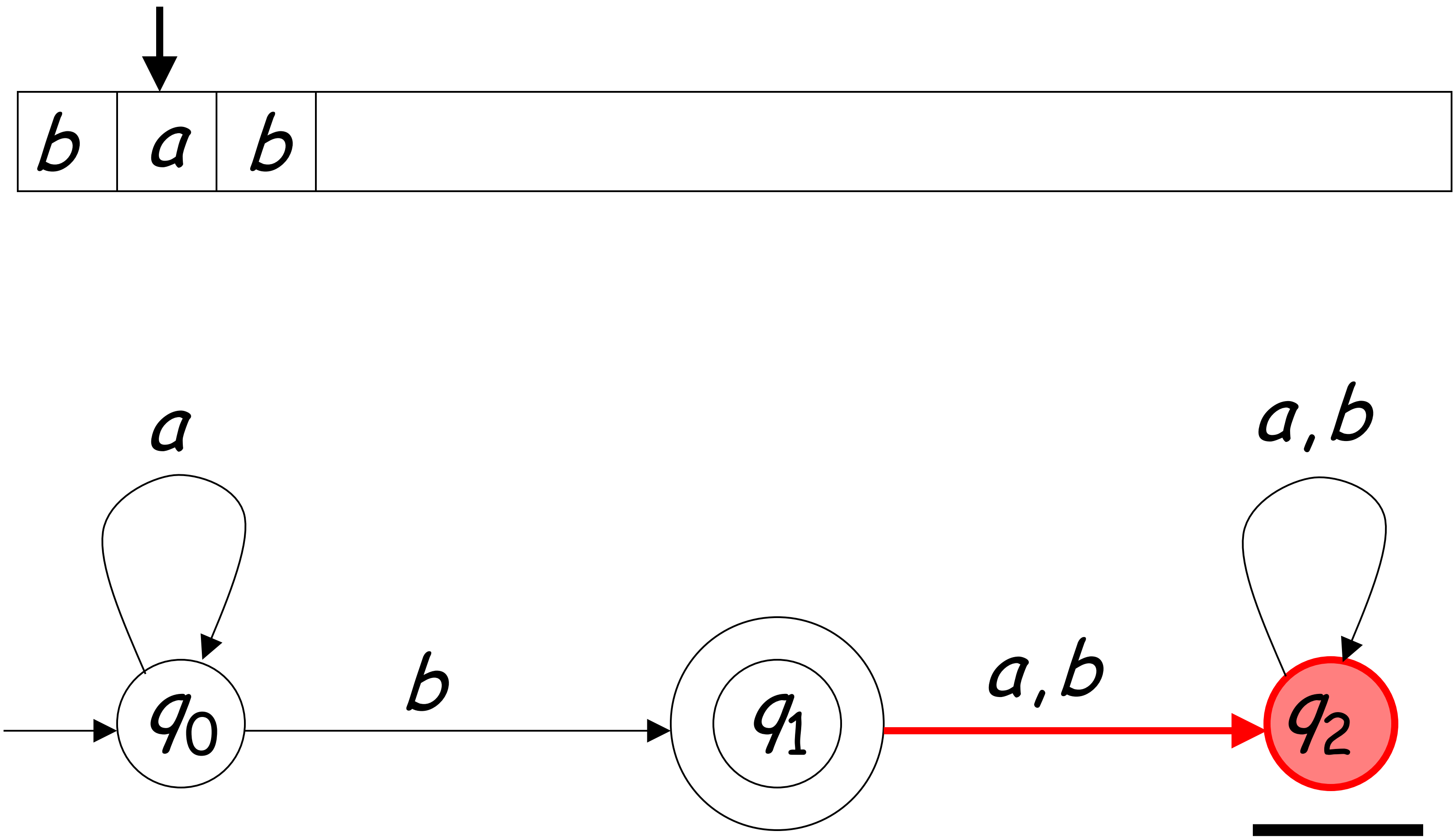
A rejection case



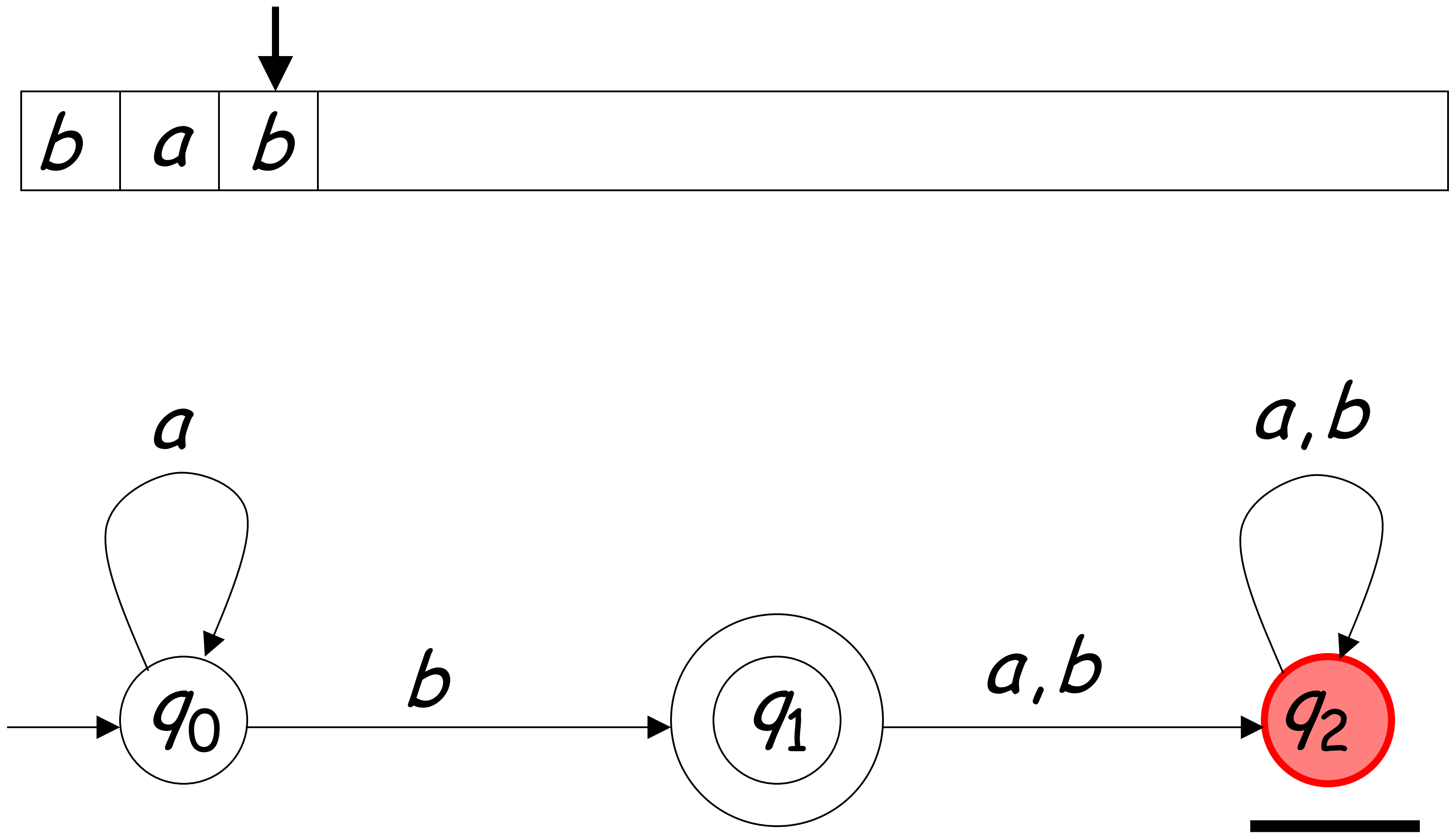
Input String




b
a
b
a
b
a,b
a,b
q_0
q_1
q_2

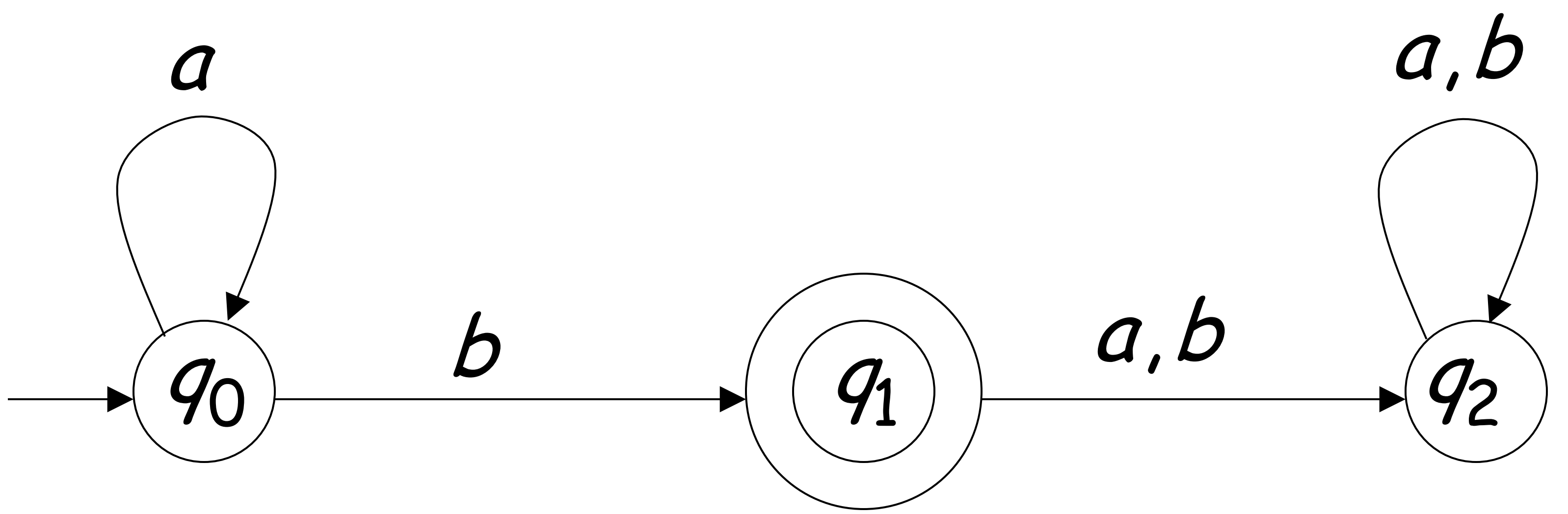

b
a
b
a
a,b
b
a,b
q_0
q_1
q_2

Input finished



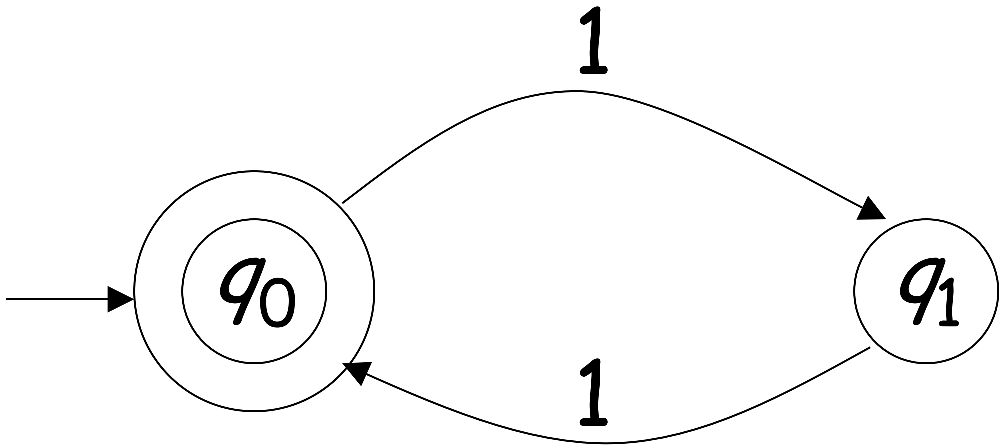
reject

Language Accepted: $L = \{a^n b : n \geq 0\}$



Another Example

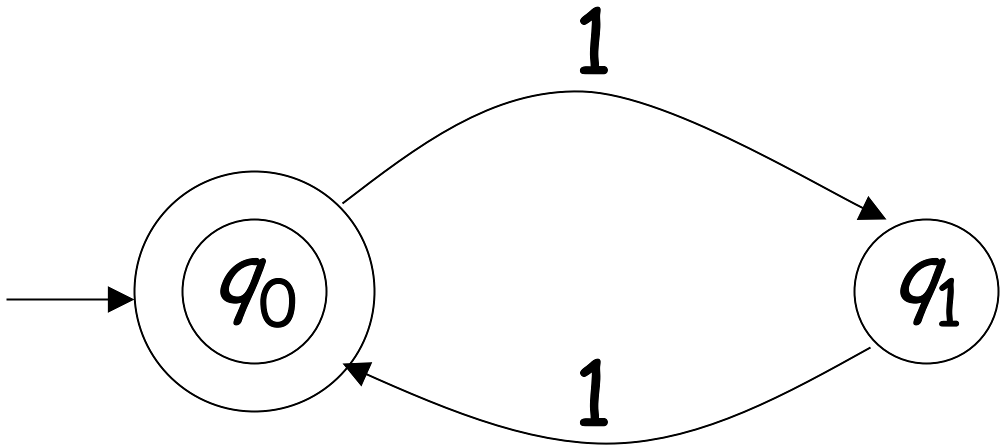
Alphabet: $\Sigma = \{1\}$



Language Accepted: ????????

Another Example

Alphabet: $\Sigma = \{1\}$



Language Accepted:

$$EVEN = \{x : x \in \Sigma^* \text{ and } |x| \text{ is even}\}$$

$$= \{\varepsilon, 11, 1111, 111111, \ldots\}$$

Formal Definition

Deterministic Finite Automaton (DFA)

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : finite set of states

Σ : finite input alphabet $\varepsilon \notin \Sigma$

δ : transition function

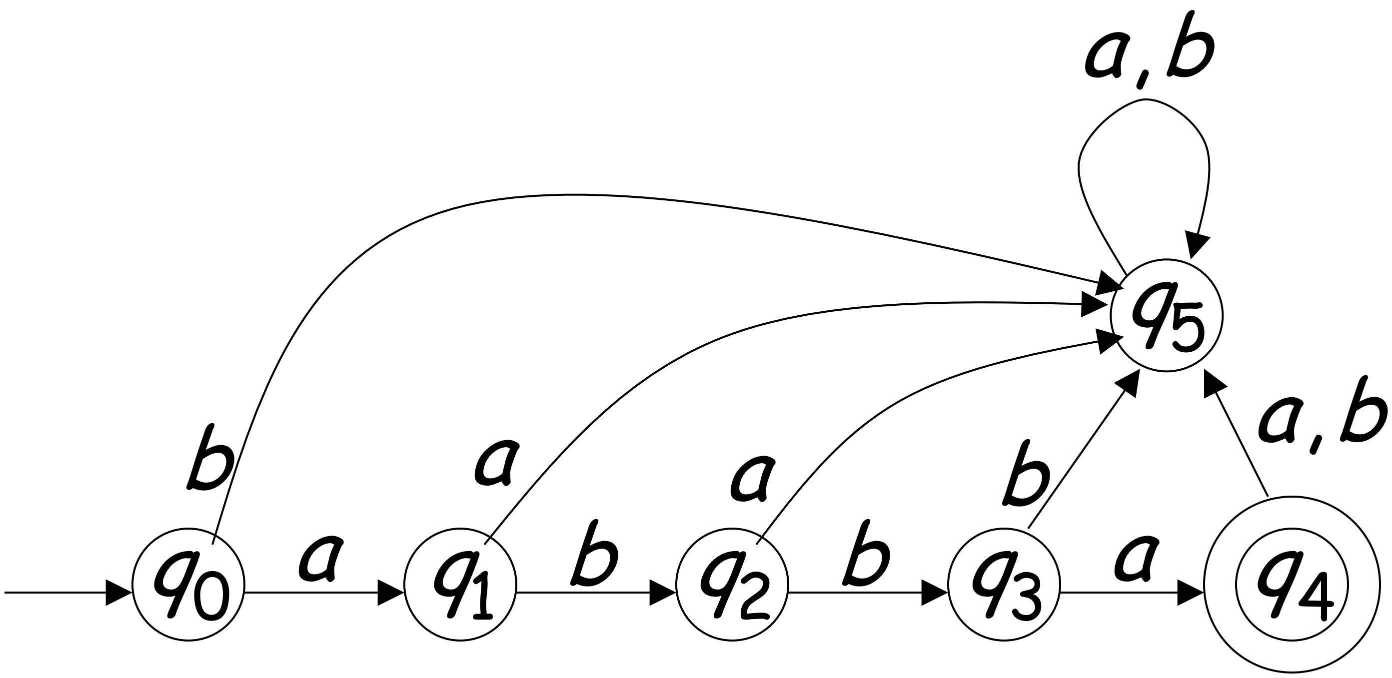
q_0 : initial state

F : set of accepting states

Set of States Q

Example

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

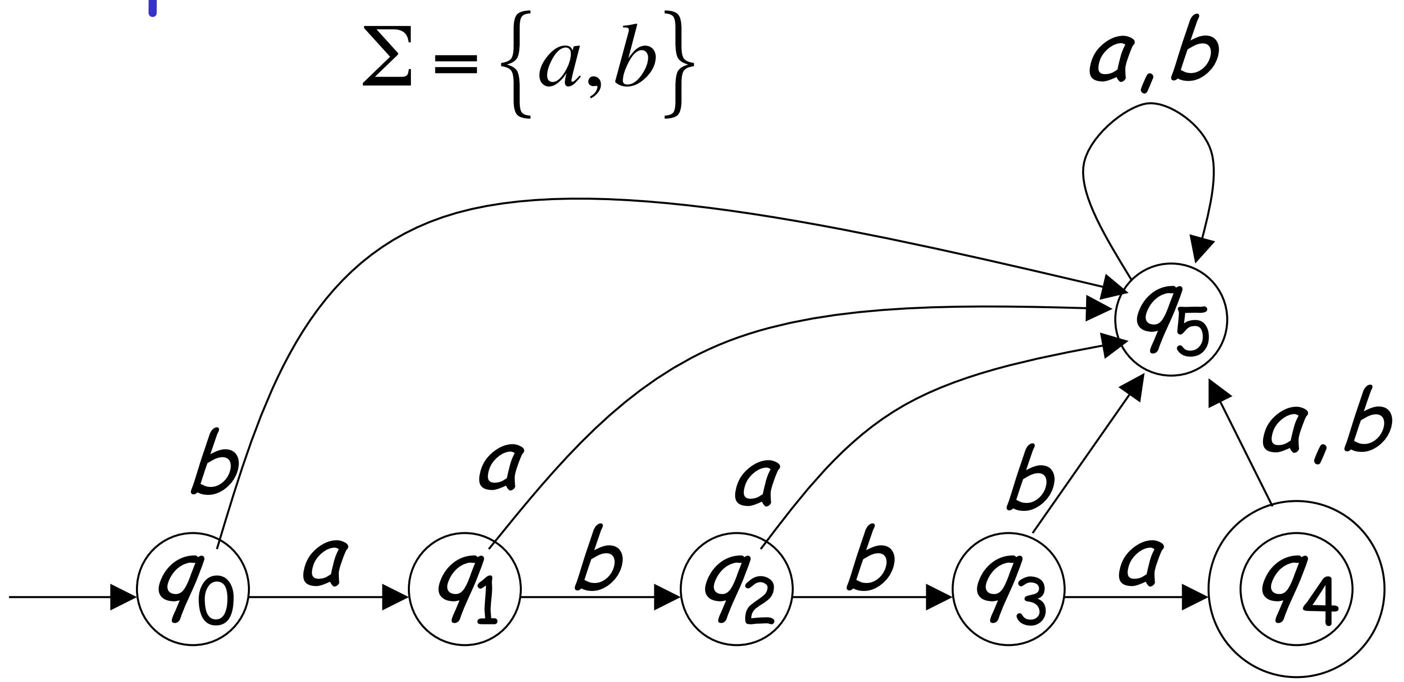


Input Alphabet Σ

$\varepsilon \notin \Sigma$:the input alphabet never contains ε

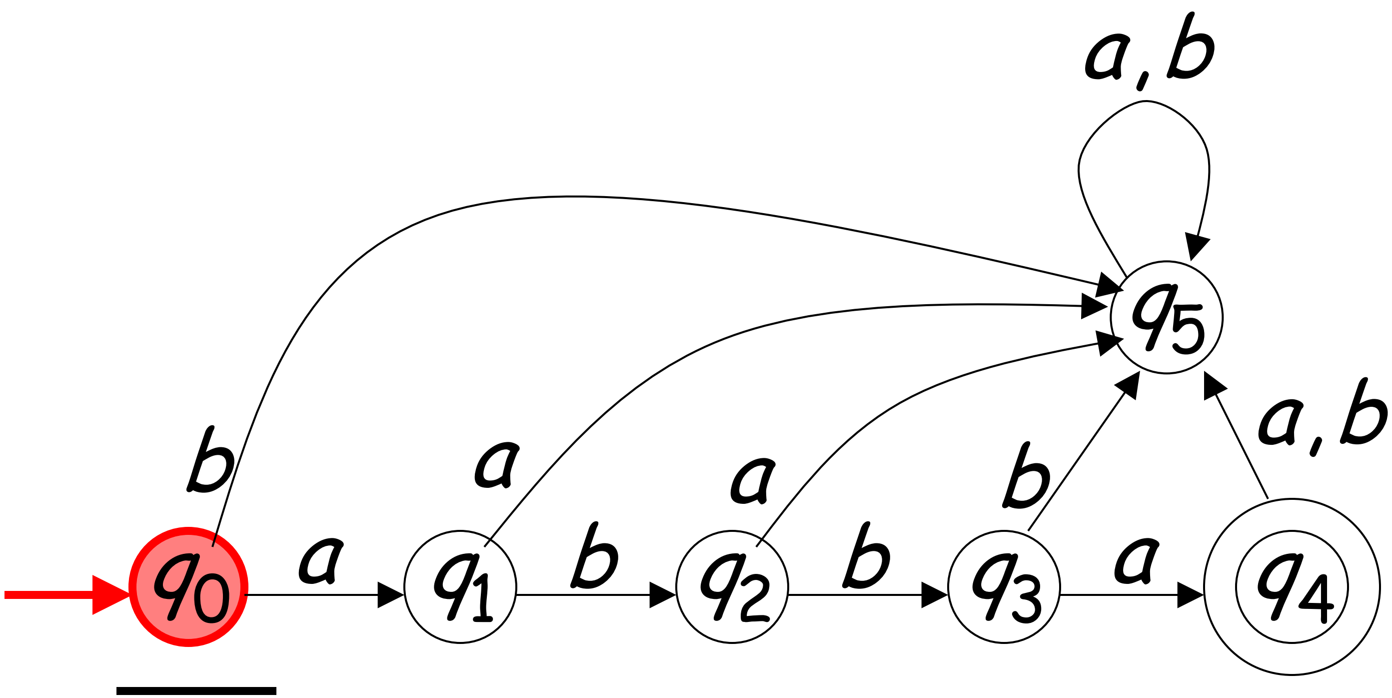
Example

$\Sigma = \{a, b\}$



Initial State q_0

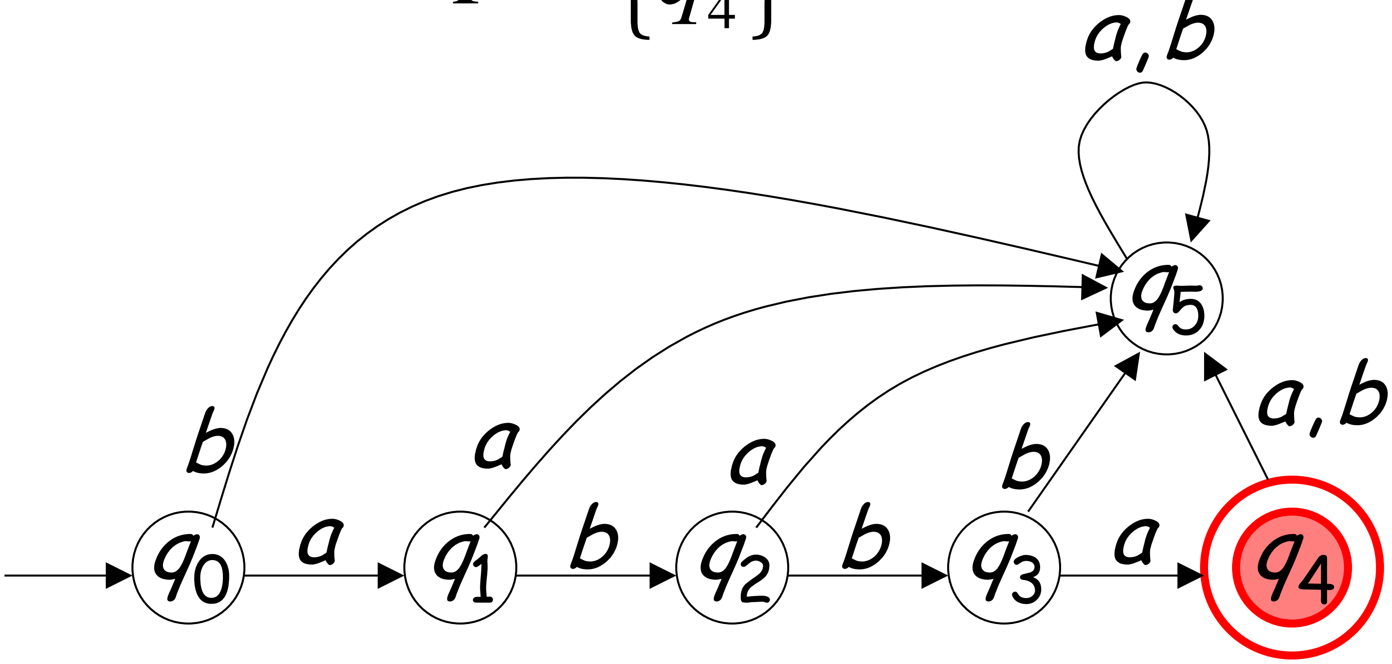
Example



Set of Accepting States $F \subseteq Q$

Example

$$F = \{q_4\}$$



Transition Function $\delta : Q \times \Sigma \rightarrow Q$

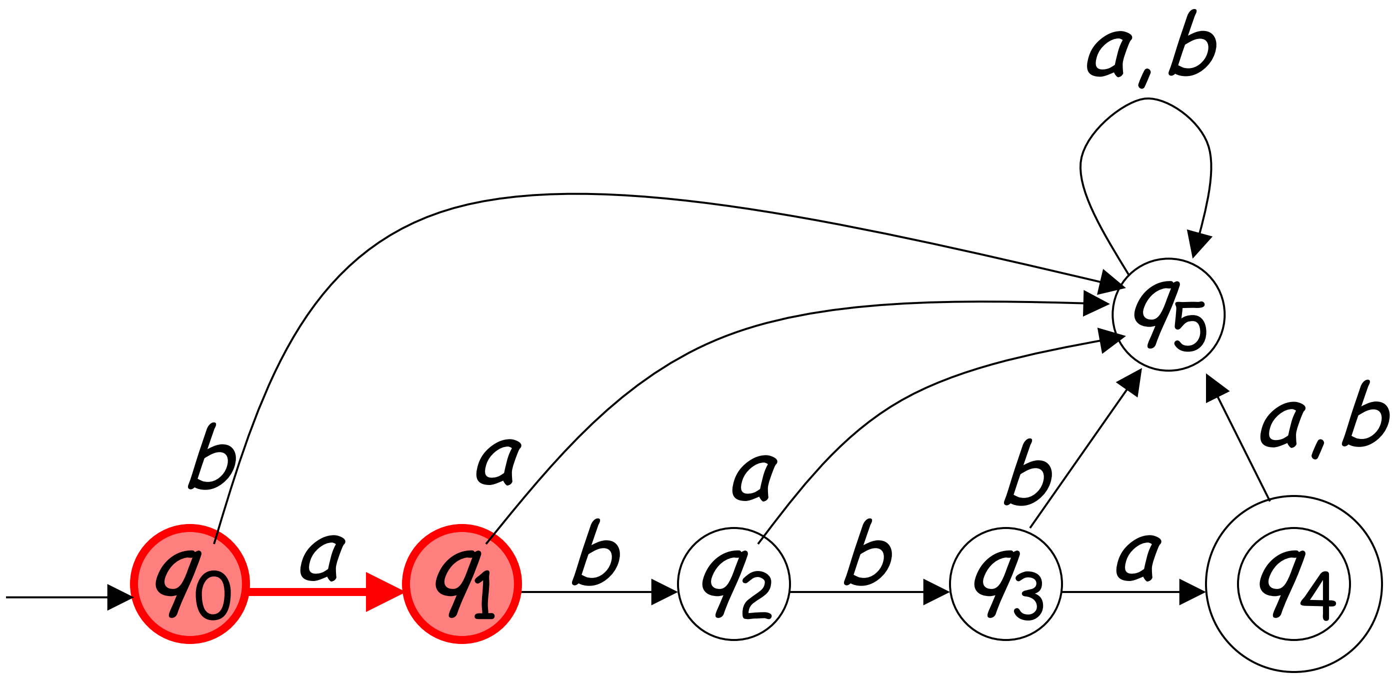
$$\delta(q, x) = q'$$

$$q \xrightarrow{x} q'$$

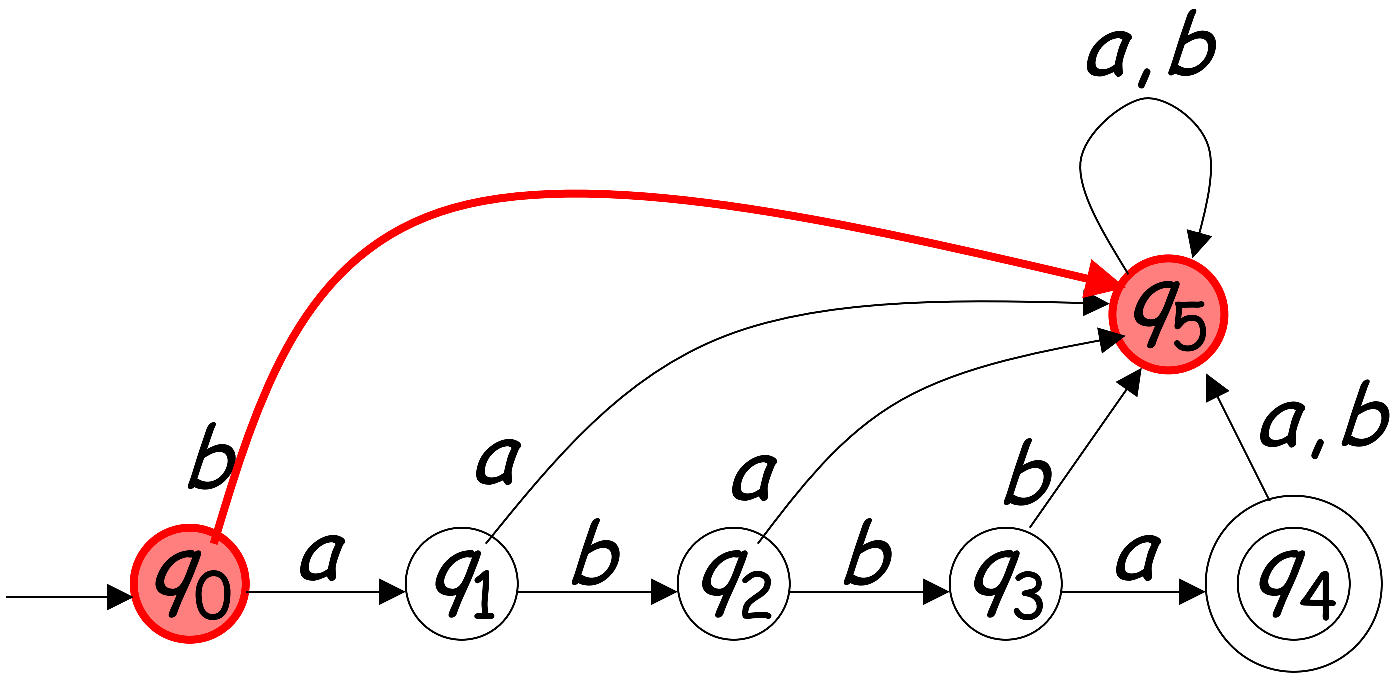
Describes the result of a transition from state q with symbol x

Example:

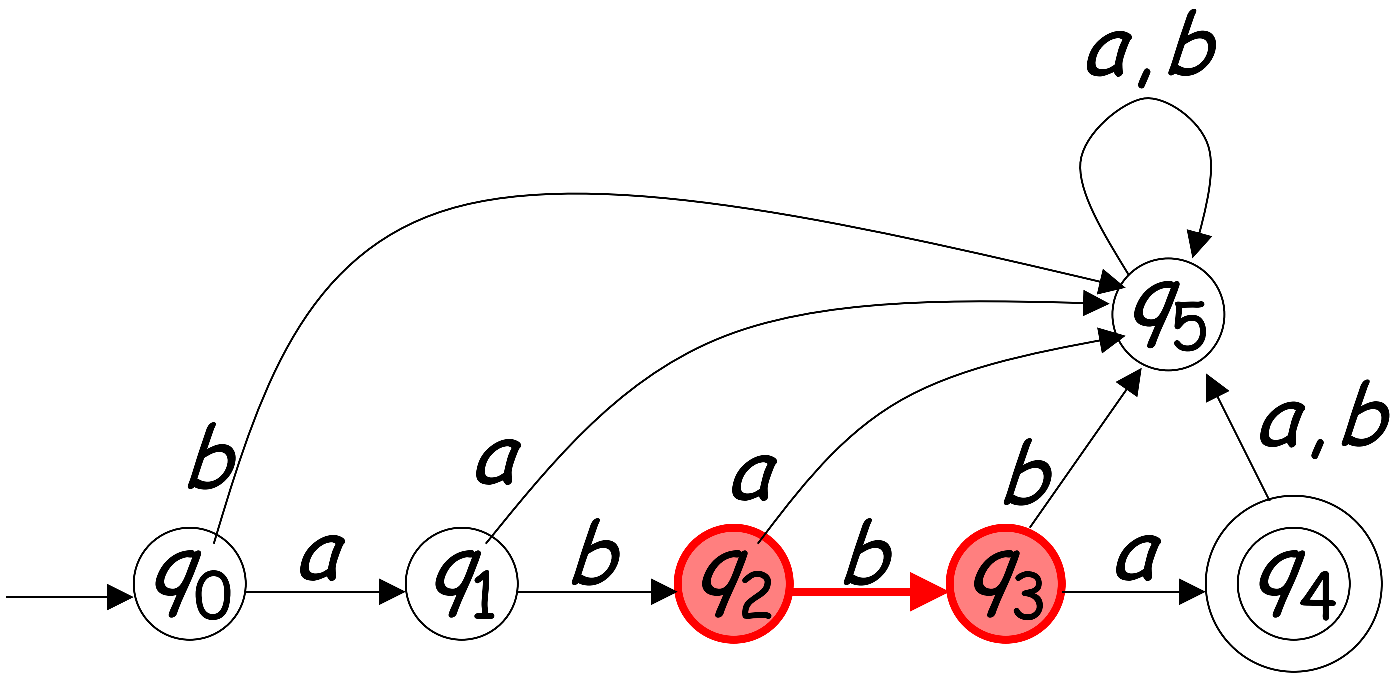
$$\delta(q_0, a) = q_1$$



$$\delta(q_0, b) = q_5$$



$$\delta(q_2, b) = q_3$$

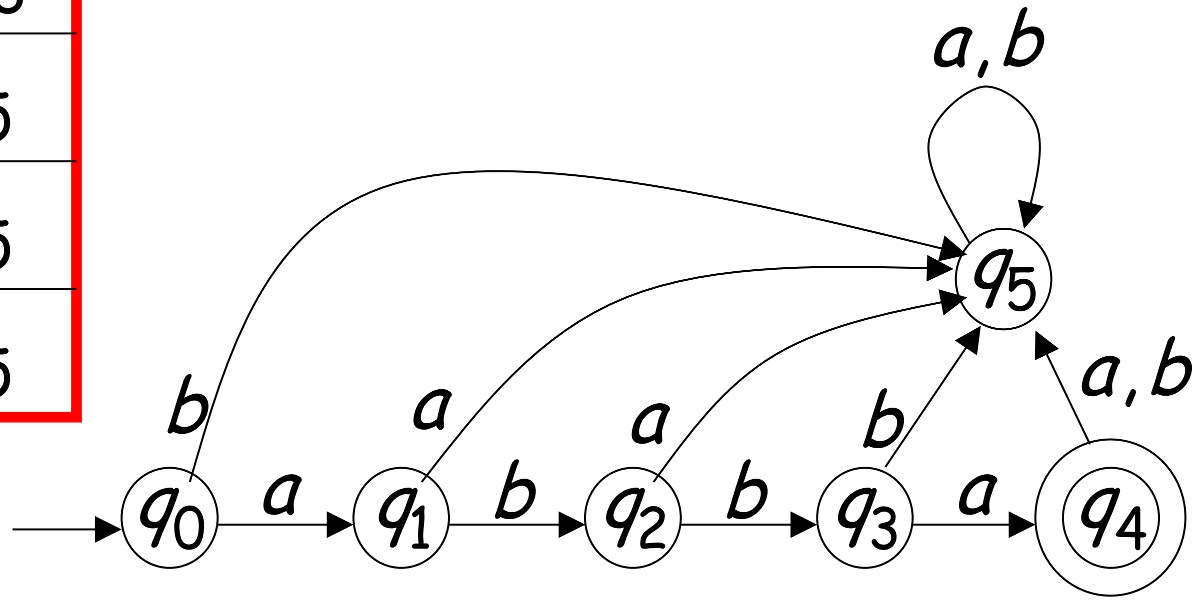


Transition Table for δ

symbols

states

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5	q_5	q_5



Extended Transition Function

$$\hat{\delta} : Q \times \Sigma^* \rightarrow Q$$

$$\hat{\delta}(q, w) = q'$$

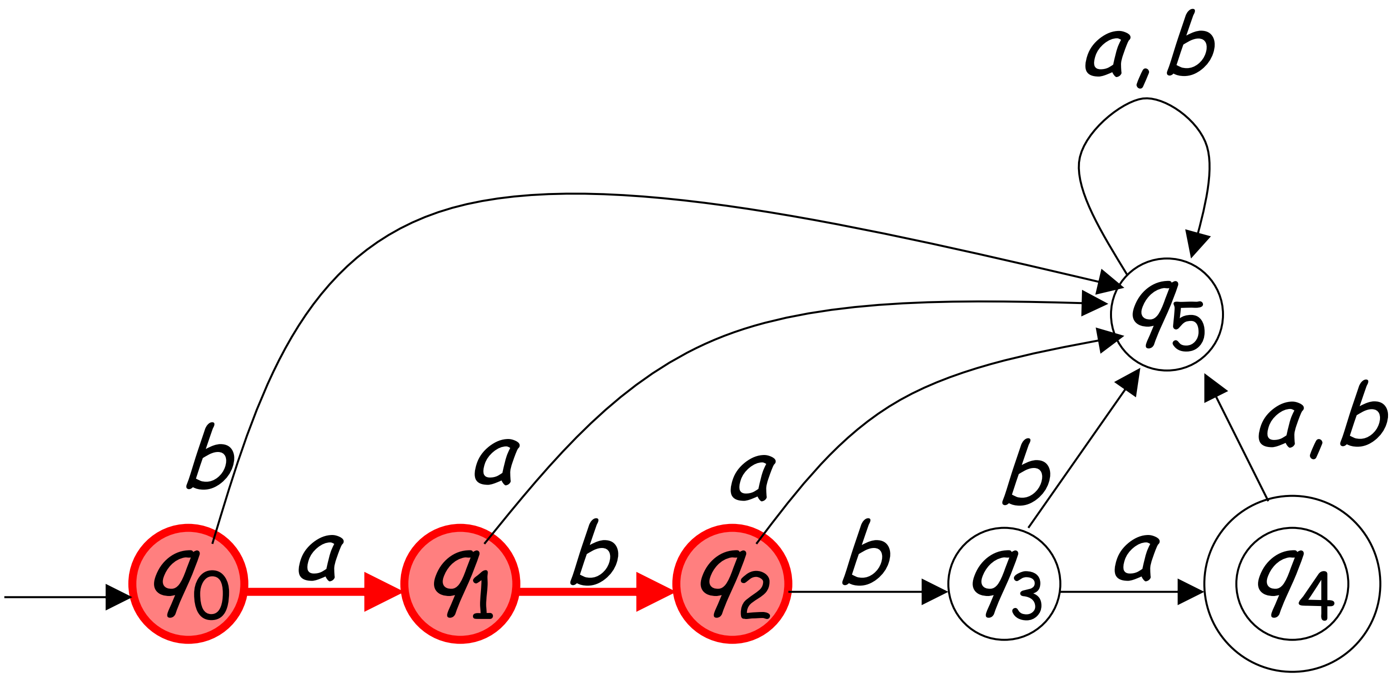
Describes the resulting state
after scanning string w from state q

Definition of $\hat{\delta}$:

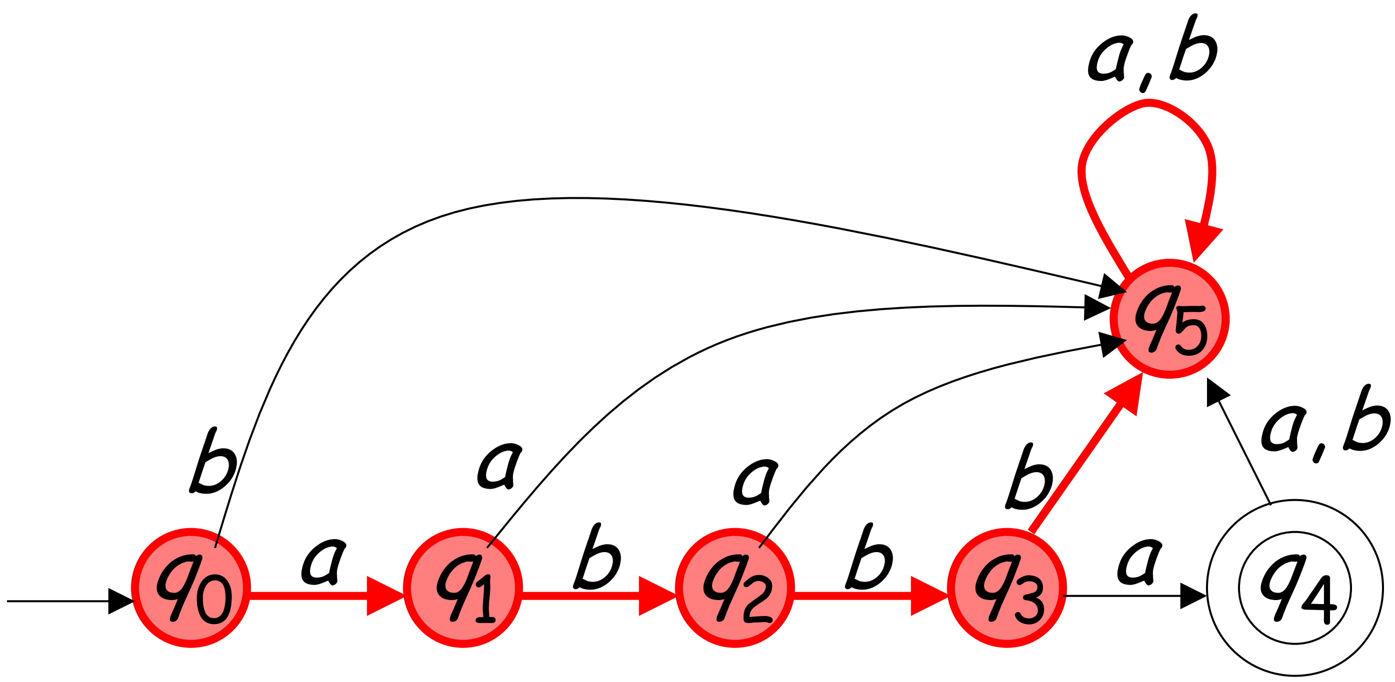
1 $\hat{\delta}(q, wx) = \delta(\hat{\delta}(q, w), x)$

2 $\hat{\delta}\left(q, \varepsilon\right) = q$

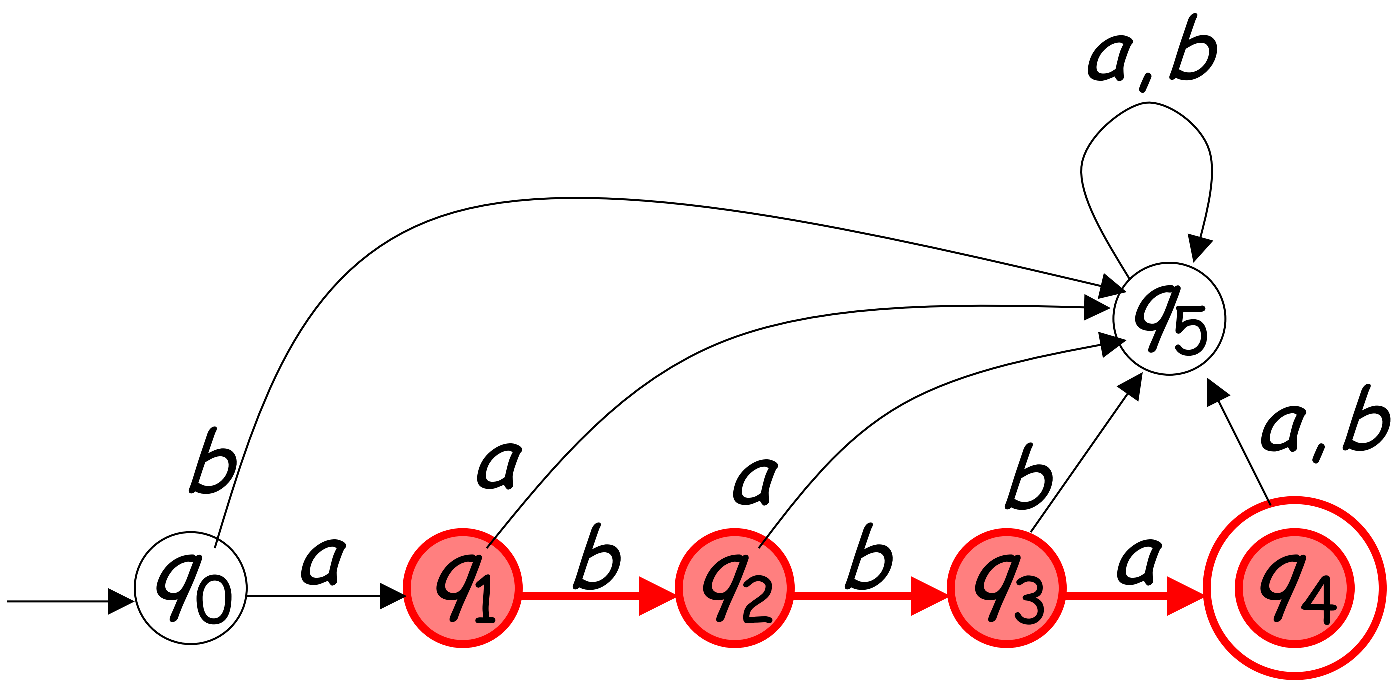
Example: $\hat{\delta}(q_0, ab) = q_2$



$$\hat{\delta}(q_0, abbbaa) = q_5$$



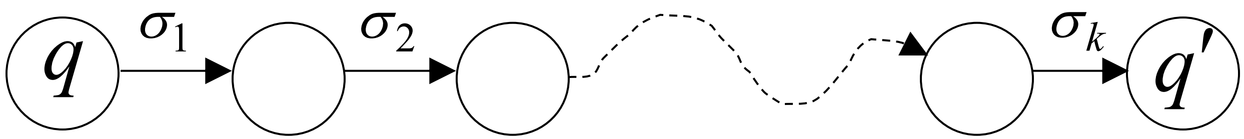
$$\hat{\delta}(q_1, bba) = q_4$$



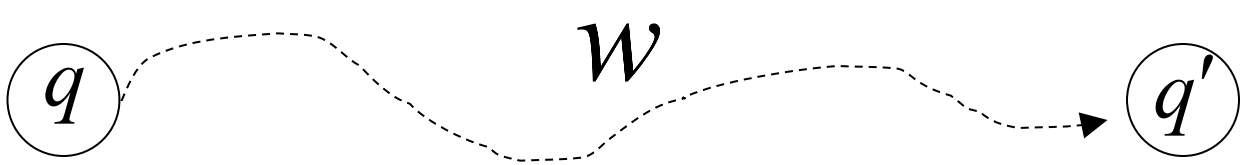
In general: $\hat{\delta}(q, w) = q'$

implies that there is a walk of transitions

$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$



states may be repeated



Language Accepted by DFA

Language of DFA M :

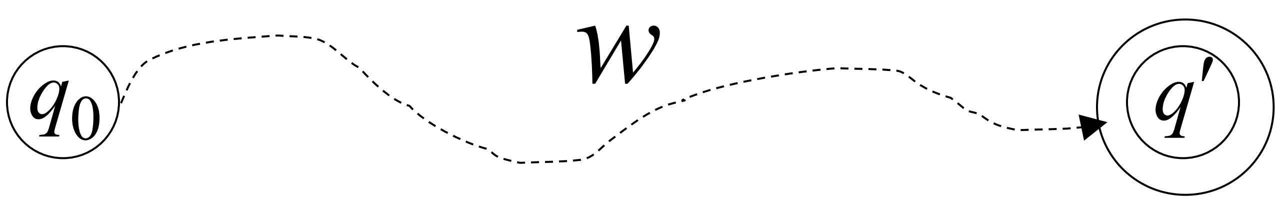
it is denoted as $L(M)$ and contains all the strings accepted by M

We say that a language L' is accepted (or recognized) by DFA M if $L(M) = L'$

For a DFA $M = (Q, \Sigma, \delta, q_0, F)$

Language accepted by M :

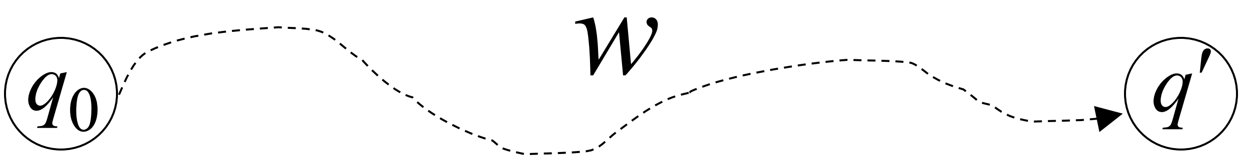
$$L(M) = \left\{ w \in \Sigma^* : \hat{\delta}(q_0, w) \in F \right\}$$



$q' \in F$

Language rejected by M:

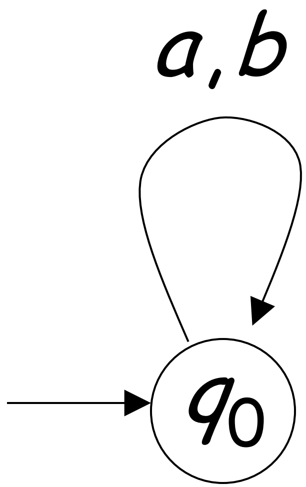
$$\overline{L(M)} = \left\{ w \in \Sigma^* : \hat{\delta}(q_0, w) \notin F \right\}$$

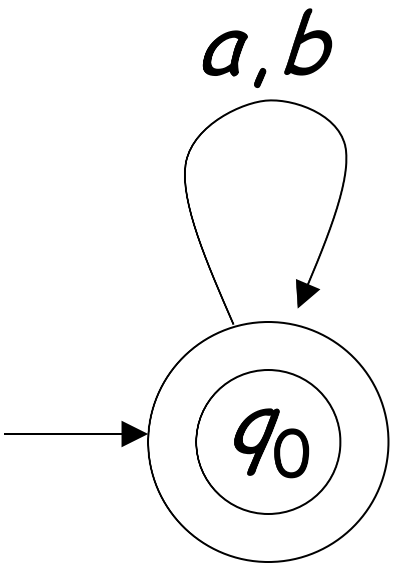


$$q' \notin F$$

More DFA Examples

$$\Sigma = \{a, b\}$$





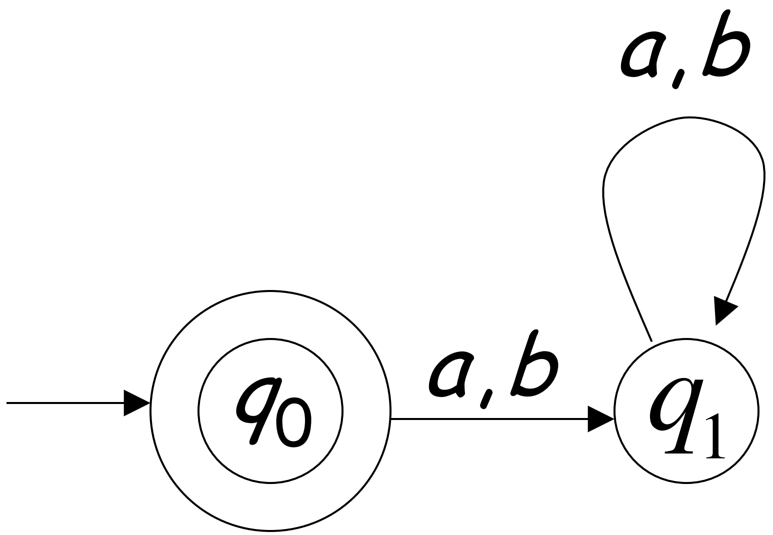
$$L(M) = \{\ \}$$

Empty language

$$L(M) = \Sigma^*$$

All strings

$$\Sigma = \{a, b\}$$

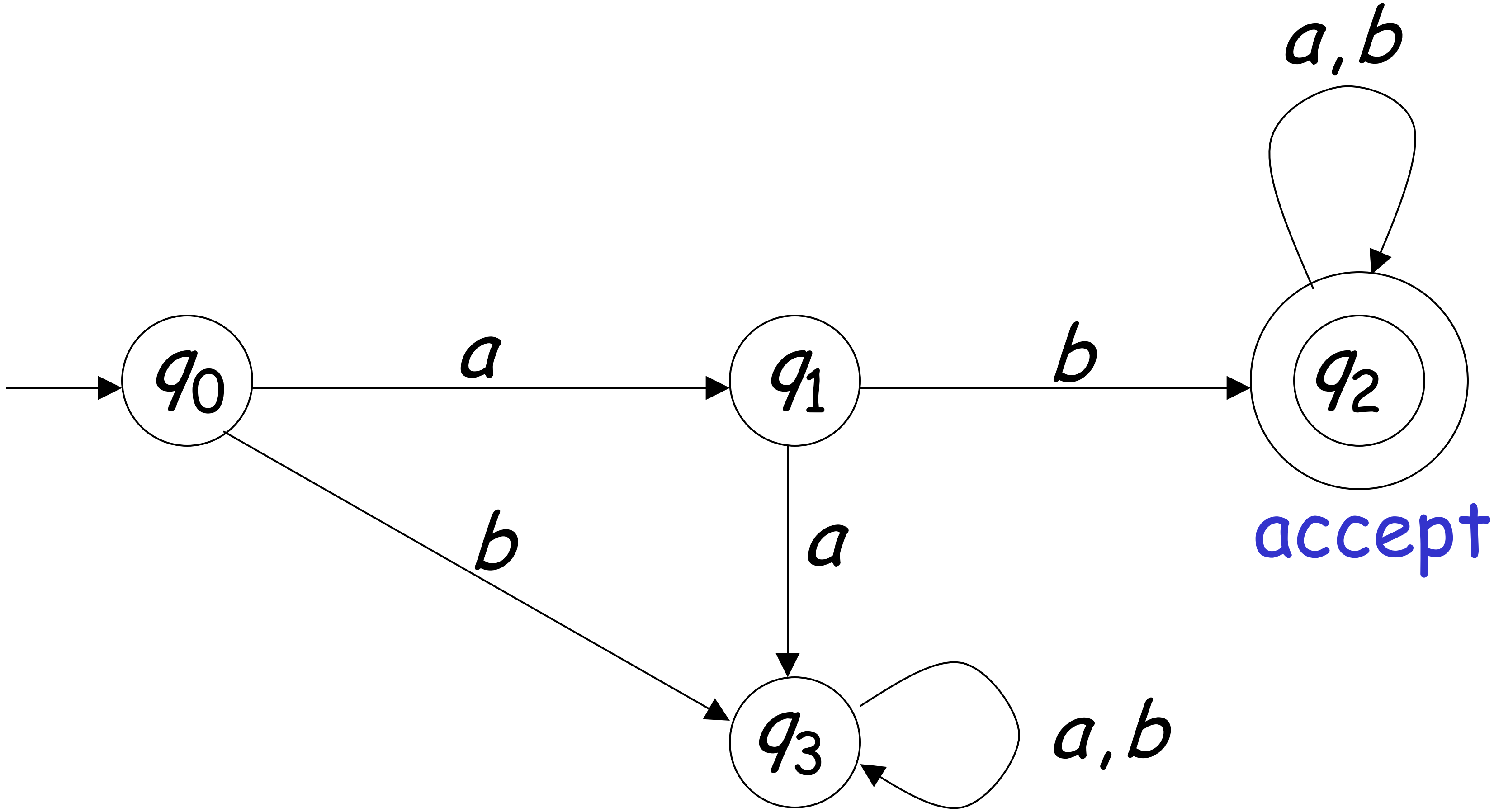


$$L(M) = \{\varepsilon\}$$

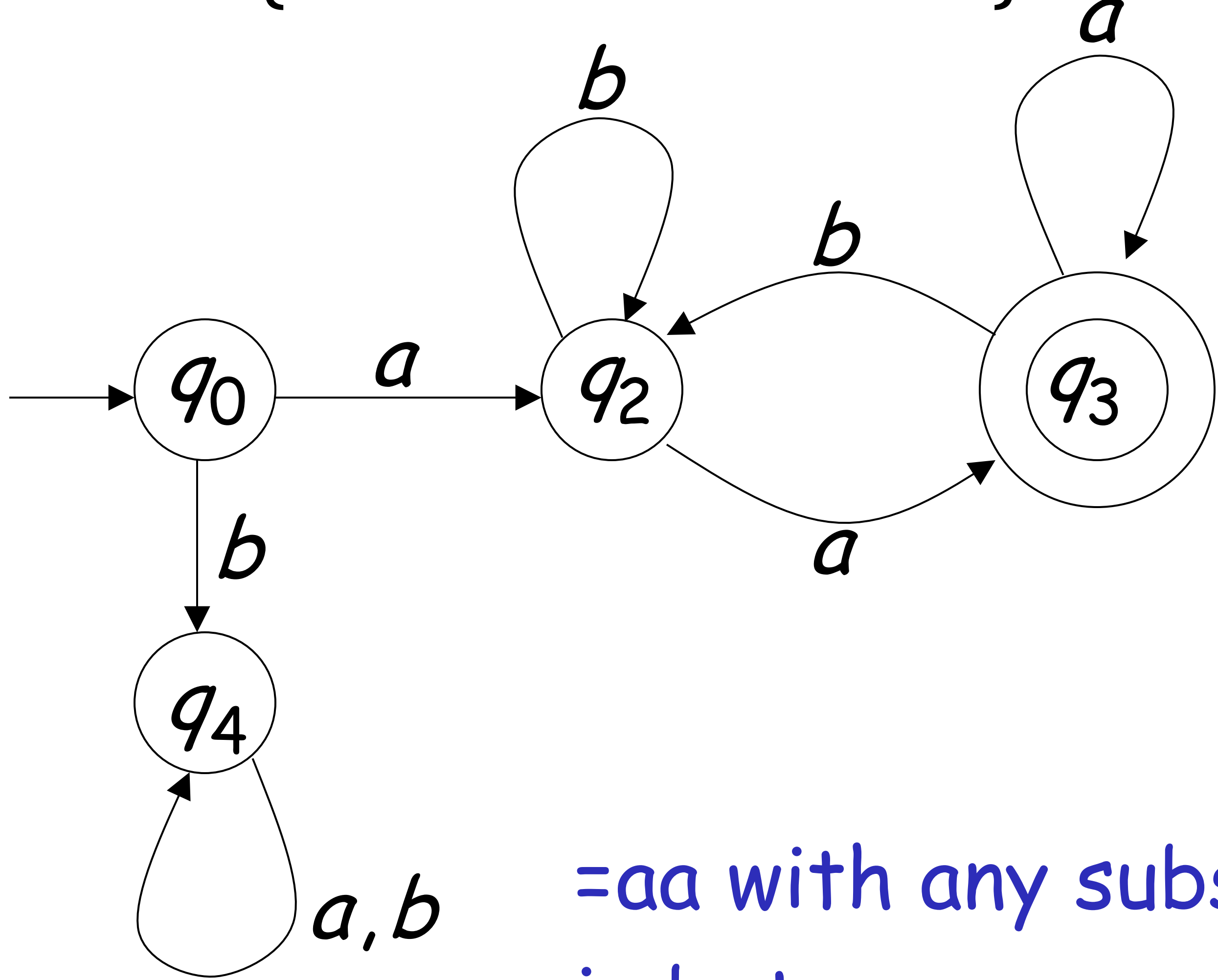
Language of the empty string

$$\Sigma = \{a, b\}$$

$L(M) =$ { all strings with prefix ab }



$$L(M) = \left\{ awa : w \in \{a,b\}^* \right\}$$



=aa with any substring in between

Regular Languages

Definition:

A language L is regular if there is a DFA M that accepts it ($L(M) = L$)

The languages accepted by all DFAs form the family of regular languages

Example regular languages:

$\{abba\}$ $\{\varepsilon, ab, abba\}$

$\{a^n b : n \geq 0\}$ $\left\{awa : w \in \{a,b\}^*\right\}$

{ all strings in {a,b}* with prefix ab }

{ all binary strings without substring 001}

$\{x : x \in \{1\}^*$ and $|x|$ is even$\}$

$\{\ \}$ $\{\varepsilon\}$ $\{a,b\}^*$

There exists a DFA that accept each of these languages

There exist languages which are not Regular:

$$L = \{a^n b^n : n \geq 0\}$$

$$ADDITION = \{|x| + |y| = |z| \;:\; |\mathrm{x}| = 1^{\mathrm{n}}, |\mathrm{y}| = 1^{\mathrm{m}}, |\mathrm{z}| = 1^{k}, \\ \mathrm{n} + m = k\}$$

There is no DFA that accepts these languages

(we will prove this in a later class)

Why define a language?

- We might want to formally define a language for:
  - Natural language (to allow eg automatic analysis of text).
  - A programming language (we need to do this!)
- Finite Automata are not quite expressive enough - we will extend them later.

Why define a language?

Another reason:

- The letters in the alphabet might model events (inputs and outputs).
- A language could then model system behaviour.

- Remember – eg Statecharts and their use for embedded systems.

Why use finite automata?

There are more expressive formalisms but:

- FA are simpler to use/understand.
- FA are simpler to analyse (eg it is feasible to analyse large FA).

- We will see:
  - As formalisms become more expressive, analysis becomes more difficult.