



University of Chittagong

Department of Mathematics

First year B.Sc. (Honours) Examination - 2022

Course Title: Calculus-1

Course Code: Math-102

Time: 4 Hours

Full Marks: 75

[**Instruction:** Answer any **05 (Five)** questions. The questions are of equal marks and figures in the margin indicate full marks. **Answer the several parts of a question sequentially.**]

Q1. a) Discuss the relationship between relation and function. Define One to One function, Identity function and Even function with examples.

b) If $f : R - \left\{ \frac{5}{4} \right\} \rightarrow R - \left\{ \frac{1}{2} \right\}$ is defined by the formula $f(x) = \frac{2x+3}{4x-5}$ then find $y = f^{-1}(x)$.

c) A function is given-

$$f(x) = \begin{cases} x^2 & \text{when } x < 0 \\ x & \text{when } 0 \leq x \leq 1 \\ \frac{1}{x} & \text{when } x \geq 1 \end{cases}$$

i) Draw the graph of the given function $f(x)$.

ii) Find the Domain and range of $f(x)$.

iii) Describe the properties of the graph of $f(x)$.

Q2. a) Define limit of function using $(\epsilon-\delta)$. Write the difference between $\lim_{x \rightarrow a} f(x)$ and $f(a)$.

b) Show that the function $f(x) = \begin{cases} x + \frac{1}{3} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$ continuous but $f'(x)$ does not exist at $x = 0$.

c) Using the fundamental theorem of differentiability find differential coefficient of $\tan ax$.

Q3. a) Find $\frac{dy}{dx}$, if (i) $y = (\tan x)^{(\cot x)}$

(ii) $x^2y + xy^2 + \sqrt{xy} = 1$

b) State and prove Leibnitz theorem for the n th derivative of the product of two functions.

c) If $x = \sin \left(\frac{1}{m} \log y \right)$, show that $(a - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$

Q4. a) State and prove the Mean value theorem and also justify this theorem for the function $f(x) = 3 + 2x - x^2$ in the interval $(0, 1)$

b) Show that (Any Two)-

- i) $\lim_{x \rightarrow 0} \frac{x^2 - \sin x^2}{x^6} = \frac{1}{6}$
- ii) $\lim_{\theta \rightarrow \pi/4} \frac{\sqrt{2} - \cos \theta - \sin \theta}{(4\theta - \pi)^2} = \frac{1}{16\sqrt{2}}$
- iii) $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right) = \frac{1}{2}$
- c) Find differential coefficient of $\tan^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right)$ with respect to $\sec^{-1} \left(\frac{1}{2x^2 - 11} \right)$

Q5. a) If u be a homogenous function of degree n in x and y then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial y \partial x} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

b) If u be a homogenous function of degree n in x, y and z then verify the relation

$$x \frac{\partial u}{\partial y} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = nu$$

c) If $z = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$, then prove that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \sin 2z$.

Q6. a) Integrate of the following (Any two)

i) $\int \frac{dx}{\sqrt{1-x^2} \sqrt{\sin^{-1} x}}$ i) $\int \frac{\cos x}{a^2 + b^2 \sin^2 x} dx$

iii) $\int x^2 \cos x dx$ iv) $\int \frac{dx}{5 + 4 \cos x}$

b) Find the reduction formula of $\int \cos^n x dx$. Hence show that

$$\int_0^{\frac{\pi}{2}} \cos^7 x dx = \frac{16}{35}$$

Q7. a) Discuss the geometrical interpretation of definite integral.

b) Integrate of the following (Any two)

i) $\int_0^1 \frac{\ln x}{\sqrt{1-x^2}} dx$ ii) $\int_2^3 \frac{x^2+1}{(2x+1)(x^2-1)} dx$

iii) $\int_0^{\frac{\pi}{4}} \ln \sin 2\theta d\theta$

c) Find the length of an arc of the curve $y = \log \left(\frac{e^x - 1}{e^x + 1} \right)$ from $x = 1$ to $x = 2$.

Q8. a) Define Gamma and Beta functions. Show that, $\Gamma(n+1) = n\Gamma n$.

b) Show that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$

c) Evaluate: $\int_0^{\frac{\pi}{6}} \cos^4 3\theta \sin^2 6\theta d\theta$