University of Chittagong

Department of Mathematics

First year B.Sc. (Honours) Examination - 2022

Course Title: Calculus-1

Course Code: Math-102

Time: 4 Hours **Full Marks: 75**

[**Instruction:** Answer any **05 (Five)** questions. The questions are of equal marks and figures in the margin indicate full marks. **Answer the several parts of a question sequentially.**]

Q1. a) Discuss the relationship between relation and function. Define One to One function, Identity function and Even function with examples.

b) If $f : R-\left\{\frac{5}{4}\right\} \to R-\left\{\frac{1}{2}\right\}$ is defined by the formula $f(x) = \frac{2x+3}{4x-5}$ then find $y = f^{-1}(x)$.

c) A function is given-

$$f(x) = \begin{cases} x^2 & \text{when } x < 0 \\ x & \text{when } 0 \leq x \leq 1 \\ \frac{1}{x} & \text{when } x \geq 1 \end{cases}$$

i) Draw the graph of the given function $f(x)$.

ii) FInd the Domain and range of $f(x)$.

iii) Describe the properties of the graph of $f(x)$.

Q2. a) Define limit of function using $(\epsilon-\delta)$. Write the difference between $\lim_{x\to a} f(x)$ and $f(a)$.

b) Show that the function $f(x) = \begin{cases} x + \frac{1}{3} & \text{when } x \neq 0 \\ 0 & \text{when } x = 1 \end{cases}$ continuous but $f'(x)$ does not exist at $x = 0$.

c) Using the fundamental theorem of differentiability find differential coefficient of $\tan ax$.

Q3. a) Find $\frac{dy}{dx}$, if (i) $y = (\tan x)^{(\cot x)}$

(ii) $x^2y + xy^2 + \sqrt{xy} = 1$

b) State and prove Leibnitz theorem for the nth derivative of the product of two functions.

c) If $x = \sin\left(\frac{1}{m}\log y\right)$, show that
$(a - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$

Q4. a) State and prove the Mean value theorem and also justify this theorem for the function $f(x) = 3 + 2x - x^2$ in the interval $(0, 1)$

b) Show that (Any Two)-

i) $\lim_{x\to 0} \dfrac{x^2 - \sin x^2}{x^6} = \dfrac{1}{6}$

ii) $\lim_{\theta\to \pi/4} \dfrac{\sqrt{2} - \cos\theta - \sin\theta}{(4\theta - \pi)^2} = \dfrac{1}{16\sqrt{2}}$

iii) $\lim_{x\to 1} \left(\dfrac{x}{x-1} - \dfrac{1}{\ln x}\right) = \dfrac{1}{2}$

c) Find differential coefficient of $\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sec^{-1}\left(\dfrac{1}{2x^2-11}\right)$

Q5. a) If u be a homogenous function of degree n in x and y then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial y \partial x} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

b) If u be a homigenous function of degree n in x, y and z then verify the relation

$$x\frac{\partial u}{\partial y} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu$$

c) If $z = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, then prove that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = \sin 2z$.

Q6. a) Integrate of the following (Any two)

i) $\int \dfrac{dx}{\sqrt{1-x^2}\sqrt{\sin^{-1} x}}$ i) $\int \dfrac{\cos x}{a^2 + b^2\sin^2 x}\,dx$

iii) $\int x^2 \cos x\,dx$ iv) $\int \dfrac{dx}{5 + 4\cos x}$

b) Find the reduction formula of $\int \cos^n x\,dx$. Hence show that $\int_0^{\frac{\pi}{2}} \cos^7 x\,dx = \dfrac{16}{35}$

Q7. a)

b)

c)

d)

Q8. a)

b)

c)

d)