Difference of Submodular Functions

f, g as submodular,

Problem: $\min_{X \subseteq V} h(X) = f(X) - g(X)$

Example applications: (1) given SF, f, $I_f(A;B) = f(A) + f(B) - f(A \cup B)$

Given a subset B, $I_f(X;B)$ is a Diff of SFs

(2) Max-margin learning (3) $I(X_A; Y)$ for feature sel.

How to solve?

Hardness Result:

$$\textcircled{1} \quad \min_{X \subseteq V} [f(X) - g(X)] \quad \Big\} \text{ even if } h(X) \geq 0$$

$$\textcircled{2} \quad \max_{X \subseteq V} [f(X) - g(X)]$$

Problems ① & ② are NP hard to approximate!!

Any set fn $\equiv$ DS functions

Given any set $v(X)$, $\exists$ submod fns f & g s.t $v(X) = f(X) - g(X)$

Under certain conditions, it is possible to get bounded factors, depending of κ_f & κ_g

Majorization-Minimization & Minorization-Maximization Algorithms

Given functions f & g: $\min_{X \subseteq V} [f(X) - g(X)]$

1. Start with $X_t = X_0$
2. Compute $\underbrace{m^f_{X_t}}_{\text{upper bound}}$ & $\underbrace{m_g^{X_t}}_{\text{lower bound}}$

Still some open Q^s on obtaining approx factors.

3.
   a. $\min_{X \subseteq V} [f(X) - m_g^{X_t}(X)]$ Sub-Sup
   b. $\min_{X \subseteq V} [m^f_{X_t}(X) - g(X)]$ Sup-Sub
   c. $\min_{X \subseteq V} [m^f_{X_t}(X) - m_g^{X_t}(X)]$ Mod-Mod

UAI → 2005 (NB)
→ 2012 (IB)

$$\min \; f(x)$$
$$\text{s.t.} \; g(x) \geq C$$

SCSC

$$\max \; g(x)$$
$$\text{s.t.} \; f(x) \leq B$$

SCSK.

NP-hard.

1. SCSC $\equiv$ SCSK.
   Alg(SCSC), achieve Alg'(SCSK)
   SCSC & SCSK. have similar approx factors

2. Framework of Algs, using mod upper/lower bounds

Algorithms for SCSC / SCSK.

$$\min f(x) \quad \text{s.t.} \quad g(x) \geq c. \quad \Rightarrow \quad \min m^f_{x^t}(x) \quad \text{s.t.} \quad g(x) \geq c.$$

← mod fn

[submodular set cover]

~ log n approx factor!

Submod Min: $f(x) \leq m^f_{x^t}(x) \leq \alpha f(x)$

Achieve approx algorithm.

IB 2013 (NIPS)