


Difference of submodular Functions

f, g as submodular,

Problem: $\min_{X \subseteq V} h(X) \triangleq f(X) - g(X)$

Example applications: ① given S, f , $I_f(A; B) = f(A) + f(B) - f(A \cup B)$

Given a subset B , $I_f(X; B)$ is a Diff of SFs

② Max-margin learning ③ $I(X_A; Y)$ for feature sel.

How to solve?

Hardness Result: $\left. \begin{array}{l} \textcircled{1} \min_{x \in V} [f(x) - g(x)] \\ \textcircled{2} \max_{x \in V} [f(x) - g(x)] \end{array} \right\} \begin{array}{l} \text{even if} \\ n(x) \geq 0 \end{array}$

Problems $\textcircled{1}$ & $\textcircled{2}$ are NP hard to approximate!!

Any set fn \equiv DS function

Given any set $v(x)$, \exists subset fns
 f & g s.t $v(x) = f(x) - g(x)$

Under certain conditions, it is possible to get
bounded factors, depending of χ_f & χ_g

Majorization - Minimization & Minorization - Maximization Algorithms

Given functions f & g : $\min_{x \in V} [f(x) - g(x)]$

(1) Start with $x_t = x_0$

(2) Compute $\underbrace{m^f_{x_t}}_{\text{upper bound}}$ & $\underbrace{m^g_{x_t}}_{\text{lower bound}}$

still some open Q's on obtaining approx factors.

(3) (a) $\min_{x \in V} [f(x) - m^g_{x_t}(x)]$ Sub-sup

(b) $\min_{x \in V} [m^f_{x_t}(x) - g(x)]$ Sup-sub

(c) $\min_{x \in V} [m^f_{x_t}(x) - m^g_{x_t}(x)]$ Maj-min

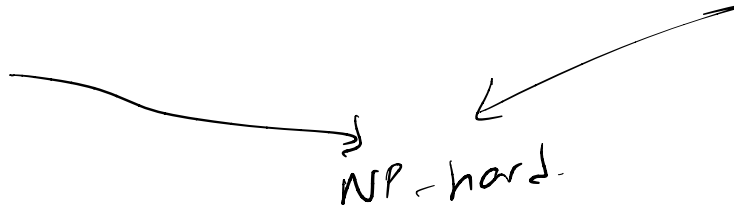
VAI \rightarrow 2005 (NB)
 \rightarrow 2012 (NB)

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & g(x) \geq C \end{aligned}$$

SCSC

$$\begin{aligned} \max \quad & g(x) \\ \text{s.t.} \quad & f(x) \leq B \end{aligned}$$

SCSK.



① $SCSC \equiv SCSK$.
 $Alg(SCSC)$, achieve. $Alg'(SCSK)$
 $SCSC$ & $SCSK$ have similar approx factor

② Framework of Algs, using mod upper/lower bounds

Algorithms for SCS / SCSK.

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & g(x) \geq c. \end{array}$$

\Rightarrow

$$\begin{array}{ll} \min & m_{x^*}^f(x) \leftarrow \text{mod } f^* \\ \text{s.t.} & g(x) \geq c. \end{array}$$

[Submodular Set Cover]
 $\sim \log n$ approx factor!

Submod Min: $f(x) \leq m_{x^*}^f(x) \leq 2 f(x)$

Achieve approx algorithm.

1B 2013 (NIPS)

