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课程名称: Advanced Mathematics A(2) 试卷满分 100 分

考试时间: June 4, 2020 (14th Week Thursday)

考试形式: Open-book exam (开闭卷)

题	号	_	 三	四	<u>Б</u> і.	六	七	八	九	+	总分
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Please answer in English or Bilingualism!

Directions: There are 10 questions in this part. Evaluate the following questions and write steps: $(10 \times 10 = 100 \text{ points})$

- (1) Find symmetric equation for the line that passes through (3, -1, 6) and is parallel to both of the planes x 2y + z = 2 and 2x + y 3z = 5.
 - (2) Evaluate the triple integral $\iiint_{\Omega} z dx dy dz$ where the closed set Ω is

bounded by $z = x^2 + y^2$ and z = 4.

- (3) Use Green's Theorem to evaluate the line integral $\oint_C \frac{xdy-ydx}{x^2+y^2}$, where C is the boundary of the square with vertices A(1,0), B(0,1), C(-1,0), and D(0,-1) with the anti-clockwise direction.
- (4) Evaluate $\iint_G xyzdS$, where G is the portion of the cone $z^2 = x^2 + y^2$ between the planes z = 1 and z = 4.
 - (5) Does $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ converge or diverge? Give your reason.
- (6) Find the first order and second order partial derivatives of the implicit function z determined by the equation $x^2 2y^2 + z^2 4x + 2z 5 = 0$.

- (7) Use Gauss's Divergence Theorem to calculate $\iint_{\partial S} \vec{F} \cdot \vec{n} \, dS$, where $\vec{F}(x, y, z) = z\vec{i} + \vec{j}$ $x\vec{j} + y\vec{k}$; S is the hemisphere $0 \le z \le \sqrt{9 - x^2 - y^2}$ with outward orientation \vec{n} .
 - (8) What are the dimensions of the rectangular box, open at the top, t hat has maximum volume when the surface area is 48?
- (9) Compute the double integral

$$I = \iint_D |y - x^2| d\sigma$$

- a) D= $\{(x,y)|0 \le x \le 1, 0 \le y \le 1\}$ (6 points)
- b) D= $\{(x,y)| -1 \le x \le 1, 0 \le y \le 1\}$ (4 points)
- (10) Discuss the convergence or divergence of the following series. If a series converges, is it absolutely convergent or conditionally convergen t?

a)
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+1}$$
 b) $\sum_{n=1}^{n} \frac{n}{2^n} \cos \frac{n\pi}{3}$

$$b)\sum_{n=1}^{n} \frac{n}{2^n} \cos \frac{n\pi}{3}$$