CSE221: Algorithms
Fall 2024

A useful tool for making graphs:
https://csacademy.com/app/graph_editor/

Task 01:

You are given a weighted, directed graph with N nodes and M edges. Each edge is represented as a triple (u, v, w), where u and v are the nodes connected by the edge and w is the weight of the edge.

Your task is to find the shortest path from a source node S to all other nodes in the graph using Dijkstra's algorithm. You should output the shortest distance from the source node to each of the other nodes in the graph. If a node is unreachable from the source node, its distance should be represented as -1.

Input

The first line of the input contains two integers, N and M (1 <= N <= 1000, 1 <= M <= 100000) denoting the number of nodes and edges in the graph, respectively.

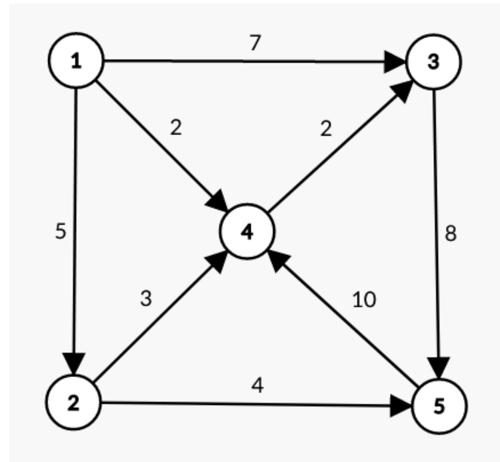
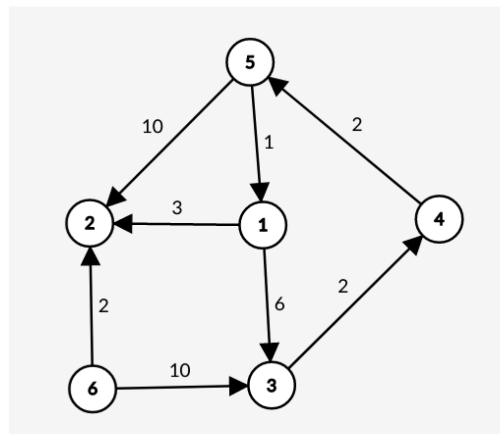
The next M lines each contain three integers, u, v (1 <= u, v <= N), and w (1 <= w <= 100) denoting an edge from node u to node v with weight w.

The last line of the input contains an integer S (1 <= S <= N) denoting the source node.

Output

Output N space-separated integers, where the i-th integer represents the shortest distance from the source node to node i. If a node is not reachable from the source node, output -1 instead.

Sample Input 1	Sample Output 1	Sample Graph 1

5 8 1 2 5 1 3 7 1 4 2 2 4 3 2 5 4 3 5 8 4 3 2 5 4 10 1	0 5 4 2 9	
Sample Input 2	Sample Output 2	Sample Graph 2
6 8 1 2 3 1 3 6 3 4 2 4 5 2 5 2 10 5 1 1 6 2 2 6 3 10 3	5 8 0 2 4 -1	

Explanation of Sample Input 1

Using Dijkstra's algorithm, the shortest path from node 1 to all other nodes in the given graph is:

- Distance from node 1 to node 1 is 0.
- Distance from node 1 to node 2 is 5.
- Distance from node 1 to node 3 is 4.
- Distance from node 1 to node 4 is 2.
- Distance from node 1 to node 5 is 9.

Task 02:

Alice and Bob are in a hurry to meet each other and have to traverse through a directed graph with weighted edges. Alice starts from node S and Bob starts from node T. They want to find a common node in the graph, where they can meet each other in the minimum amount of time. Alice or Bob can wait at any node if they want to.

Input

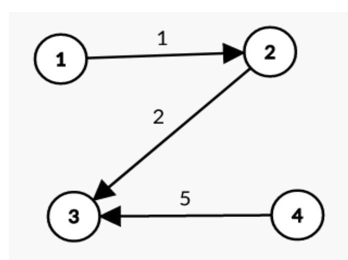
The first line contains two integers N (1 ≤ N ≤ 10000) and M (1 ≤ M ≤ 100000) separated by a space, denoting the number of nodes and edges in the graph, respectively.

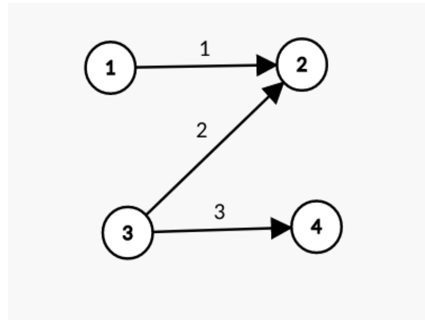
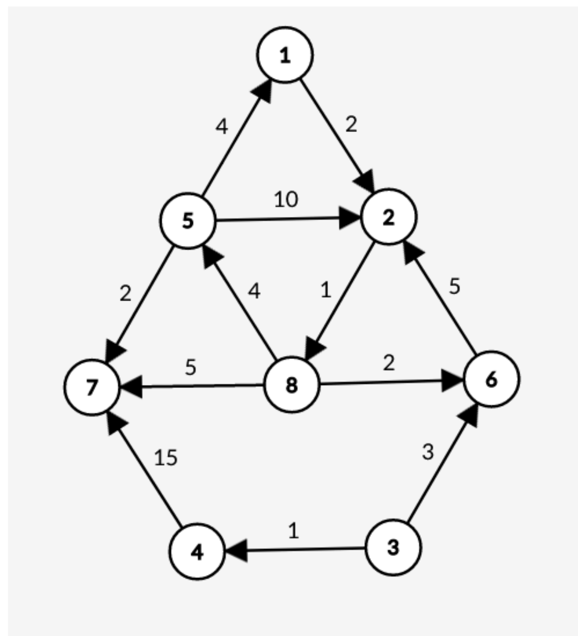
The next M lines each contain three integers, u, v (1 <= u, v <= N), and w (1 <= w <= 100) denoting an edge from node u to node v with weight w.

The last line contains two integers S,T (1 <= S,T <= N) which denotes the starting node of Alice and the starting node of Bob respectively.

Output

Output two integers separated by a space. The first integer is the minimum time it takes for Alice and Bob to meet. The second integer is the vertex where they will meet. If there is no such node, print “Impossible”.

Sample Input 1	Sample Output 1	Sample Graph 1
4 3 1 2 1 2 3 2 4 3 5 1 4	Time 5 Node 3	
Sample Input 2	Sample Output 2	Sample Graph 2

4 3 1 2 1 3 2 2 3 4 3 1 4	Impossible	
Sample Input 3	Sample Output 3	Sample Graph 3
8 12 3 6 3 3 4 1 6 2 5 4 7 15 2 8 1 8 6 2 8 7 5 8 5 4 5 2 10 5 1 4 1 2 2 5 7 2 3 5	Time 8 Node 2	

Task 03:

You are traveling through dangerous wilderness with bridges and rivers. Each of them has a danger level that represents the risk of injury if you attempt to cross it.

Your goal is to find the safest path from your current location to your destination node **N**. You will start your journey from node **1**.

You define the danger level of a path as the maximum danger level of all the edges along the path. To maximize your chances of survival, you decide to take the safest path possible. The safest path is the one with the minimum danger level among all the paths from node 1 to node N.

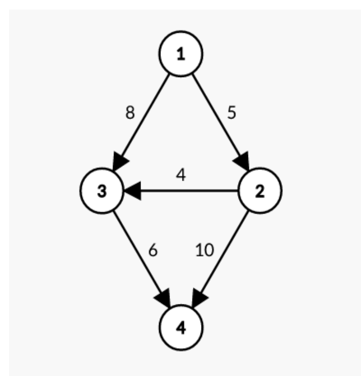
Input

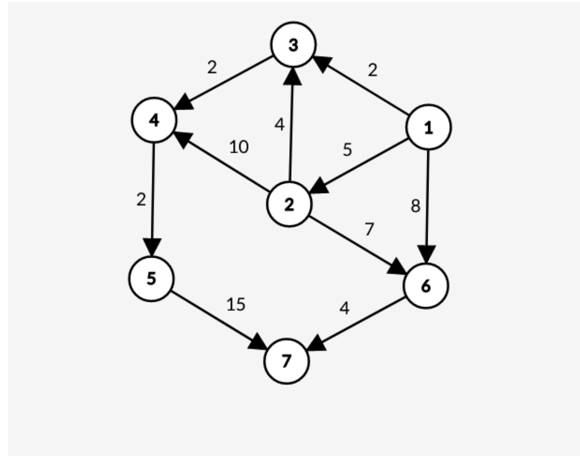
The first line of the input contains two integers, N and M (1 <= N <= 1000, 1 <= M <= 100000) denoting the number of nodes and edges in the graph respectively.

The next M lines each contain three integers, u, v (1 <= u, v <= N), and w (1 <= w <= 100) denoting an edge from node u to node v with weight w.

Output

Find a path with the minimum danger level from source node 1 to destination node N. Print “Impossible” if there is no path from node 1 to N.

Sample Input 1	Sample Output 1	Sample Graph 1
4 5 1 2 5 2 3 4 3 4 6 1 3 8 2 4 10	6	
Sample Input 2	Sample Output 2	Sample Graph 2

7 10 1 2 5 2 3 4 3 4 2 1 3 2 2 4 10 4 5 2 5 7 15 2 6 7 1 6 8 6 7 4	7	

Explanation of Sample Input 1

There are three paths from node 1 to node 4.

a) 1 → 3 → 4
   In path (a), the danger level is maximum(8,6) = 8

b) 1 → 2 → 4
   In path (b), the danger level is maximum(5,10) = 10

c) 1 → 2 → 3 → 4
   In path (c), the danger level is maximum(5,4,6) = 6

Among all the three paths, the minimum danger level is 6 by following the path 1 → 2 → 3 → 4.

Task 04:

There is a group of N people living in a small village. They live in their own house. Although they are all neighbors, they don't all know each other very well.

Each person in that village has their own unique identity - labeled with an integer value between 1 to N. Initially, the villagers don't have any friends. As time passes by, they begin to make a friendship between themselves.

In this problem, you will be given K friendships. You have to print an integer value that denotes their friend circle's size.

Suppose, five people are living in the village labeled 1,2,3,4 and 5. Initially, the size of each friend circle is one, since friendship has yet to be created.
One day, person 1 and person 2 become friends. So the size of their friend circle becomes two. The next day, person 3 and person 4 become friends and the size of their friendship becomes two as well. After a few days, person 1 and person 4 become friends. Now the size of their friend circle becomes four consisting of persons 1,2,3 and 4.

Input Format:

The input consists of two integers, separated by a space, denoting the number of people in the village, N ($1 \le N \le 10^5$) and the number of queries that will follow, K ($1 \le K \le 10^5$).

The next K lines contain two integers A_i and B_i each ($1 \le A_i, B_i \le N$ and $A_i \ne B_i$), separated by a space, representing two people who have become friends as a result of the query.

Output Format:

For each query, output a single integer on a new line representing the size of the friend circle that the two people belong to after becoming friends.

Sample Input/Output:

Sample Input 1	Sample Output 1
5 3 1 2 3 4 1 4	2 2 4
Sample Input 2	Sample Output 2

8 7 2 4 4 5 3 6 4 7 3 1 2 7 6 2	2 3 2 4 3 4 7

Sample Input Explanation:

In sample input 2,

Query 0: Initially, 8 people in the village do not know each other.

{1} {2} {3} {4} {5} {6} {7} {8}

Query 1: After person 2 and person 4 become friends:

{1} {2,4} {3} {5} {6} {7} {8}

The output is 2, since the size of the friends circle {2,4} is 2.

Query 2: After person 4 and person 5 become friends:

{1} {2,4,5} {3} {6} {7} {8}

The output is 3, since the size of the friends circle {2,4,5} is 3.

Query 3: After person 3 and person 6 becomie friends:

{1} {2,4,5} {3,6} {7} {8}

The output is 2, since the size of the friends circle {3,6} is 2.

Query 4: After person 4 and person 7 become friends:

{1} {2,4,5,7} {3,6} {8}

The output is 4, since the size of the friends circle {2,4,5,7} is 4.

<u>Query 5:</u> After person 3 and person 1 become friends:

{2,4,5,7} {1,3,6} {8}

The output is 3, since the size of the friends circle {1,3,6} is 3.

<u>Query 6:</u> Since person 2 and person 7 are already in the same friend circle, nothing changes:

{2,4,5,7} {1,3,6} {8}

The output is 4, since the size of the friends circle {2,4,5,7} is 4.

<u>Query 7:</u> After person 6 and person 2 become friends:

{2,4,5,7,1,3,6} {8}

The output is 7, since the size of the friends circle {2,4,5,7,1,3,6} is 7.

Task 05:

In the kingdom of Beluga, there are N cities connected by M roads, each with a maintenance cost associated with it. There is at least one path between any two cities. The king is concerned about the increasing maintenance cost and decides to take action.

He calls upon his council, and they suggest that they find a minimum-cost set of roads that connects all cities while minimizing the maintenance cost. Then the king decides to reduce the total maintenance cost by destroying some of the existing roads, instead of building new ones.

Since you are a very good programmer the king calls you. He asks you to find out what the lowest maintenance cost can be achieved after destroying a few roads while ensuring there still exists a path from each city to another.

Input

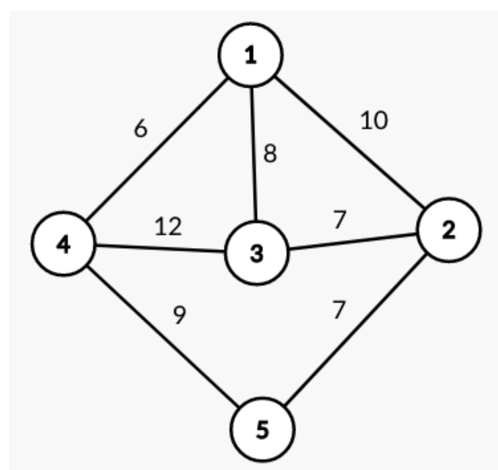
The first line of the input contains two space-separated integers, N and M (1 ≤ N ≤ 10^5, 1 ≤ M ≤ 10^6), representing the number of cities and roads in the kingdom of Beluga, respectively.

The next M lines each contain three space-separated integers, u, v, and w (1 ≤ u, v ≤ N, 1 ≤ w ≤ 10^9), where u and v denote the endpoints of a road and w represents its maintenance cost.

Output

The output should contain a single integer, with the minimum total maintenance cost achievable.

Sample Input/Output:

Sample Input 1	Sample Output 1	Sample Graph 1
5 7 1 2 10 1 3 8 1 4 6 2 3 7 2 5 7 3 4 12 4 5 9	28	
Sample Input 2	Sample Output 2	Sample Graph 2

6 9 1 2 6 2 4 5 2 3 4 1 3 8 4 3 4 2 6 1 2 5 3 5 6 2 5 1 7	17