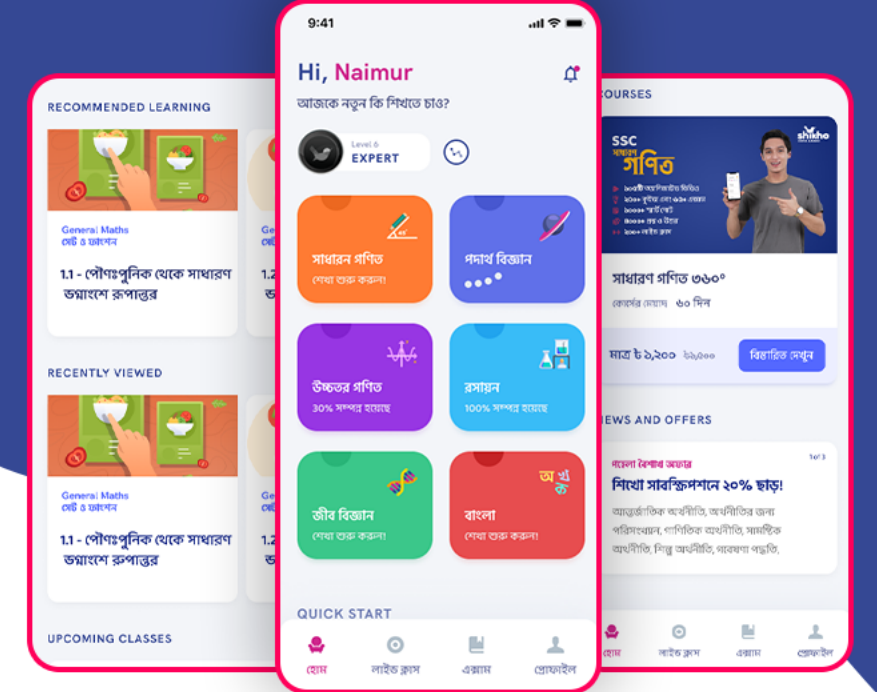


# HSC পদার্থবিজ্ঞান ১ম পত্র

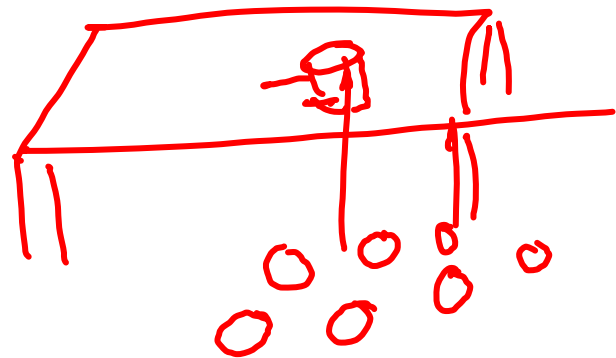
অধ্যায় ২:  
ভেক্টর  
পর্ব: ৩



# আজকে আমরা যা শিখবো

- নদী পারাপার সংক্রান্ত সমস্যা
- সামান্তরিক সূত্র সংক্রান্ত সমস্যা
- সৃজনশীল সমস্যার সমাধান

ଫିକ୍ସ ଡାଇନାମିକ୍:



\* ଫିକ୍ସ ଡାଇନାମିକ୍  
ଓବରଲ୍ୟୁ ଡାଇନାମିକ୍  
ଓଗ କରା ଥାଏ।

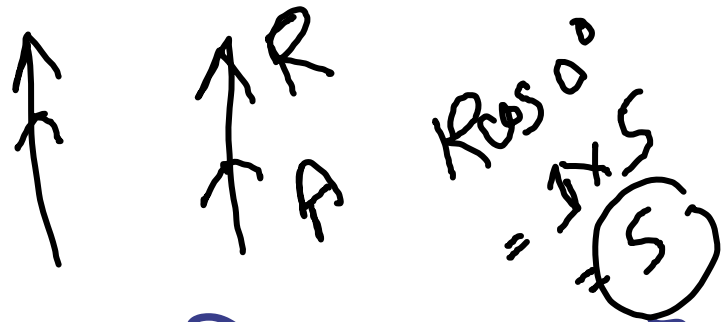
• LIVE

Math 2nd - Chap - 8.

ସମସ୍ତ ଜଣିଆ ଜଣା:



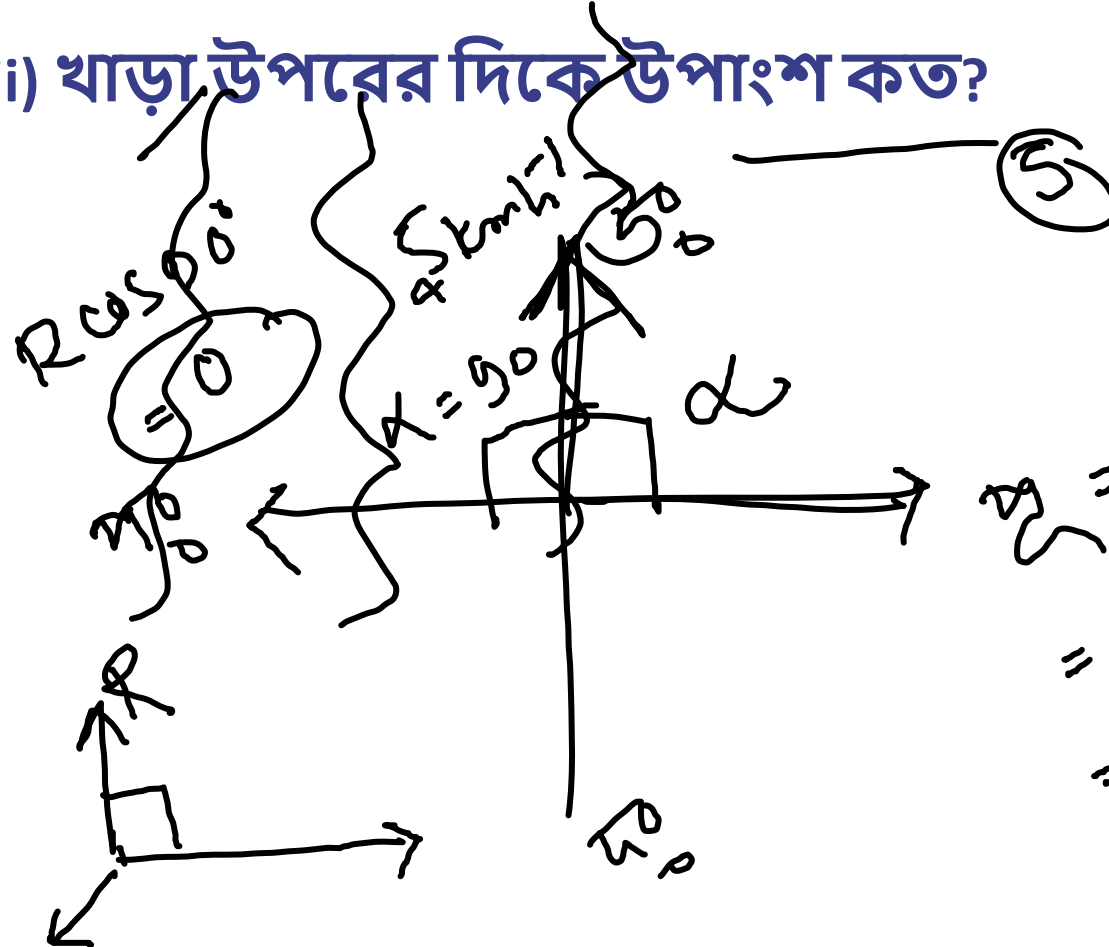
$$\frac{OA}{\sin \gamma} = \frac{AB}{\sin \alpha} = \frac{OB}{\sin \beta}$$



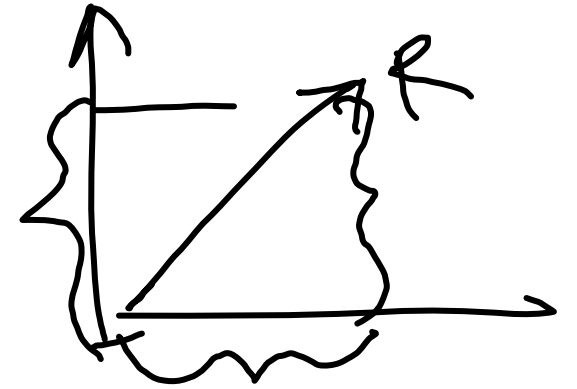
## Poll Question - 1

বায়ু ভূমির সমান্তরালে উত্তর দিকে 5 km/hr বেগে প্রবাহিত হলে (i) পূর্বদিকে, (ii) পশ্চিম দিকে, (iii) খাড়া উপরের দিকে উপাংশ কত?

- ✓ a) 0,0,5
- ~~b) 5,0,0~~
- ~~c) 5,5,0~~
- ✓ d) 0,0,0



$$\begin{aligned} R \cos \alpha &= R \cos 90^\circ \\ &= 0 \end{aligned}$$



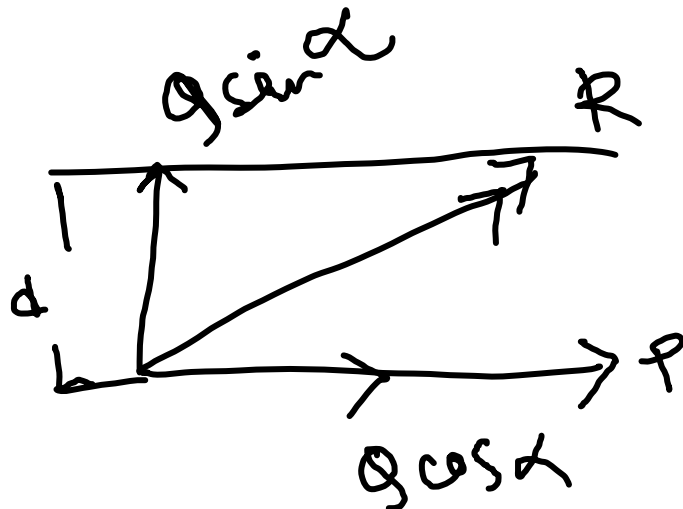
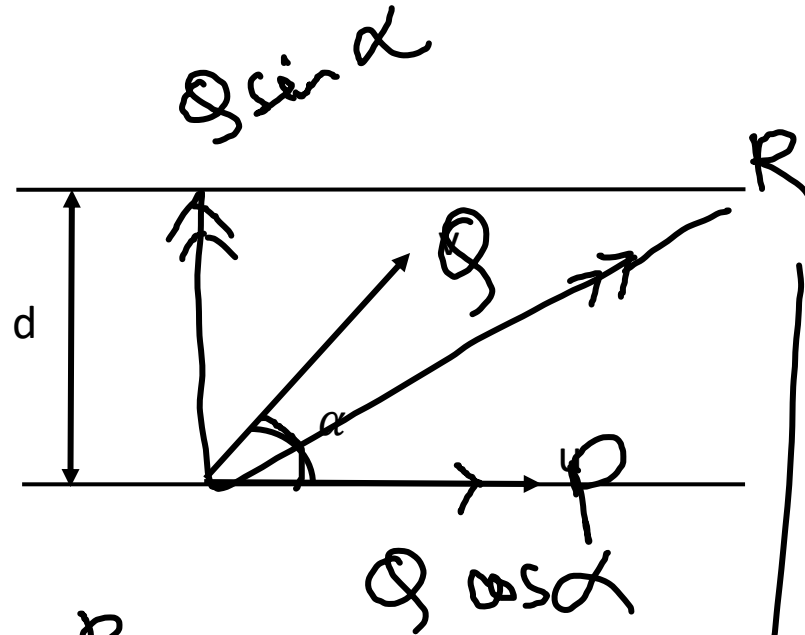
# সর্বনিম্ন সময়ে নদী পারাপার

স্রোতের বেগ =  $u$

নৌকার বেগ =  $v$

নদীর প্রস্থ =  $d$

স্রোত ↓↓  
river



Case: 1  
কোন দিকে/কত কোণে  
 $\Rightarrow \alpha = \arcsin \frac{u}{v}$  - চলা

Case: 2  
কত কোণে চলা হবে?

$$R = \sqrt{p^2 + q^2 + 2pq \cos \alpha}$$

Ques: 3

# সর্বনিম্ন সময়ে নদী পারাপার

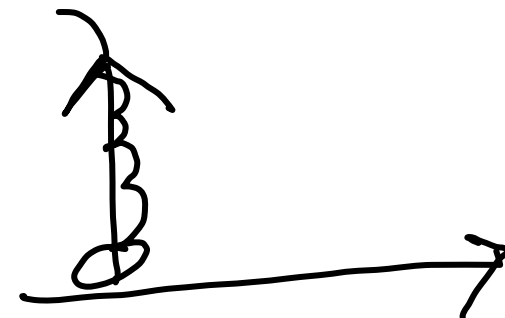
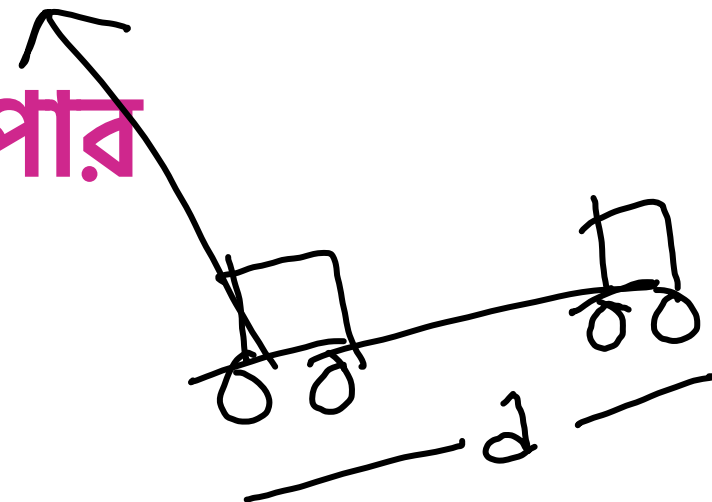
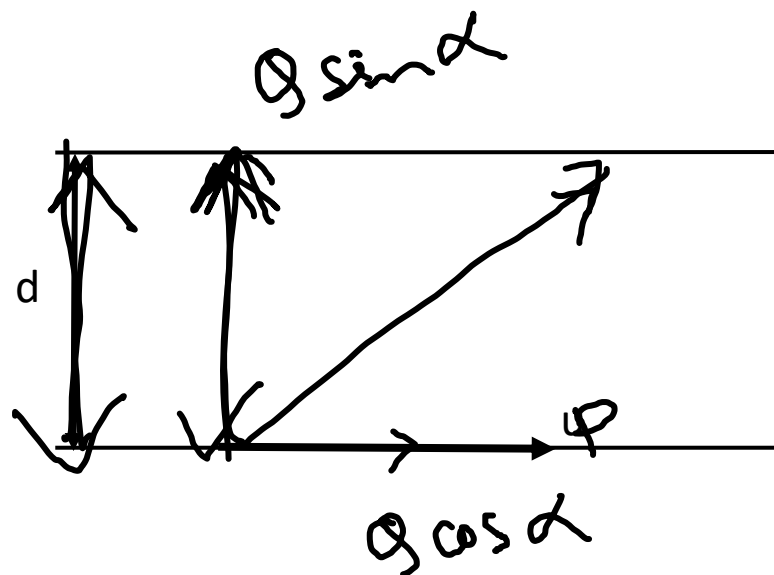
সমাধি:

$$S = vt$$

$$\Rightarrow d = v \sin \alpha t$$

$$\Rightarrow t = \frac{d}{v \sin \alpha}$$

তবে সর্বনিম্ন সময়ে  
নদী পারাপার  
করা  
 $\sin \alpha = 1$   
 $\alpha = 90^\circ$

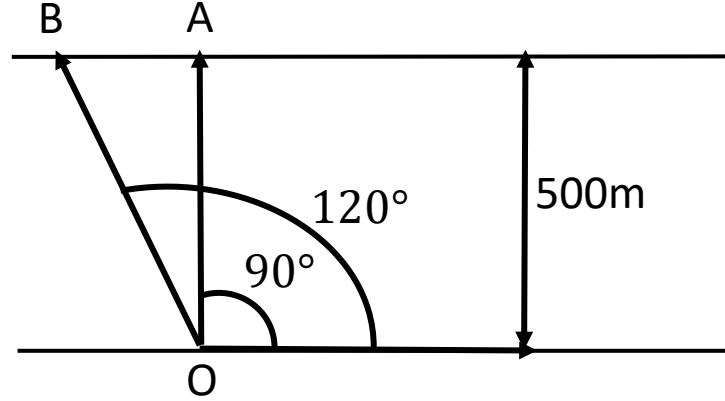


$$t_{\min} = \frac{d}{v}$$

## CQ Solving

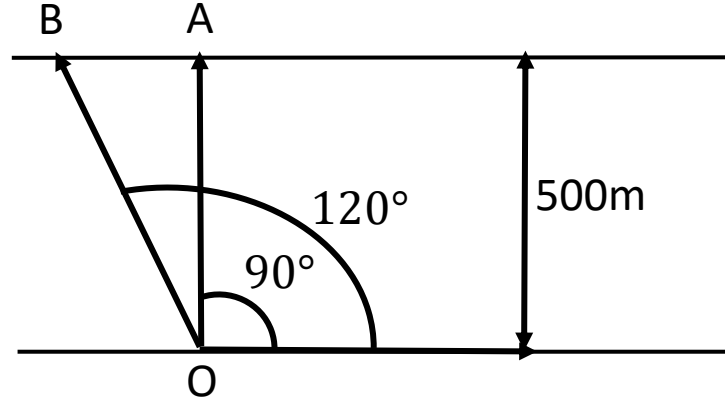
স্রোতের বেগ = 1 m/s  
OB বরাবর করিমের বেগ = 2  
m/s  
OA বরাবর রহিমের বেগ = 7.5  
m/s

গ) করিম কত বেগে অপর পাড়ে পৌঁছাল? নির্ণয় কর।



## CQ Solving

স্রোতের বেগ = 1 m/s  
OB বরাবর করিমের বেগ = 2  
m/s  
OA বরাবর রহিমের বেগ = 7.5  
m/s



ঘ) রহিম ও করিমের মধ্যে কে আগে অপর পাড়ে পৌছাবে? গাণিতিক বিশ্লেষণসহ মতামত দাও।

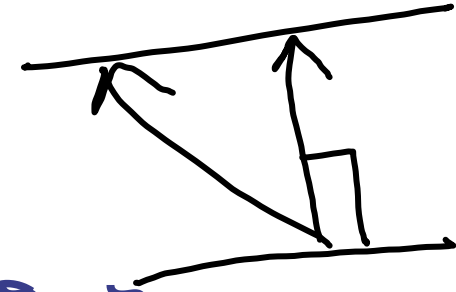


## Poll Question - 2

দু'টি ভেক্টরের বৃহত্তম ও ক্ষুদ্রতম লঙ্ঘিরমান যথাক্রমে 10 ও 2 একক।  
ভেক্টরদ্বয় কোন বিন্দুতে  $60^\circ$  কোণে ক্রিয়াকরলে লঙ্ঘিরমান কত?

- a) 6.732
- b) 8.718
- c) 9.101
- d) 8.987

## Poll Question - 3



4 km/hr বেগে প্রবাহিত নদী মোজাসুজি পাড়ি দিতে একটি নৌকার কত সময় লাগবে? [যখন নৌকার বেগ 5km/hr নদীর প্রস্থ 1000m]  $\Rightarrow 1\text{km}$

a)  $1/(\sqrt{41})$  hr

b)  $\sqrt{40}$  hr

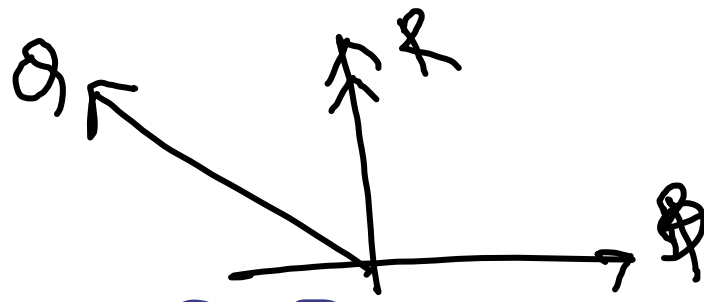
c) 0.9 hr

~~d)  $1/3$  hr~~

$$\frac{d}{\sqrt{q^2 - p^2}} = t$$

$$\Rightarrow t = \frac{1}{\sqrt{25 - 16}} \text{ hr}$$

$$t = \frac{1}{\sqrt{9}}$$
$$t = \frac{1}{3} \text{ hr}$$



## Poll Question - 4

$$\alpha = 120^\circ$$

একটি নদীতে স্রোতের বেগ  $x$  km/hr, যদি একটি নৌকাকে  $2x$  km/hr বেগে চালনা করা হয়, তবে নৌকাটিকে স্রোতের সাথে কত কোণে চালালে তা সোজাসুজি অপর পাড়ে পৌঁছাবে?

$$\begin{aligned} \alpha &= \cos^{-1} \left( \frac{-x}{2x} \right) \\ &= \cos^{-1} \left( \frac{-1}{2} \right) \\ &= \cos^{-1} \left( -\frac{1}{2} \right) \end{aligned}$$

$\Rightarrow \alpha = 120^\circ$

$$\alpha = 120^\circ$$

## Poll Question - 5

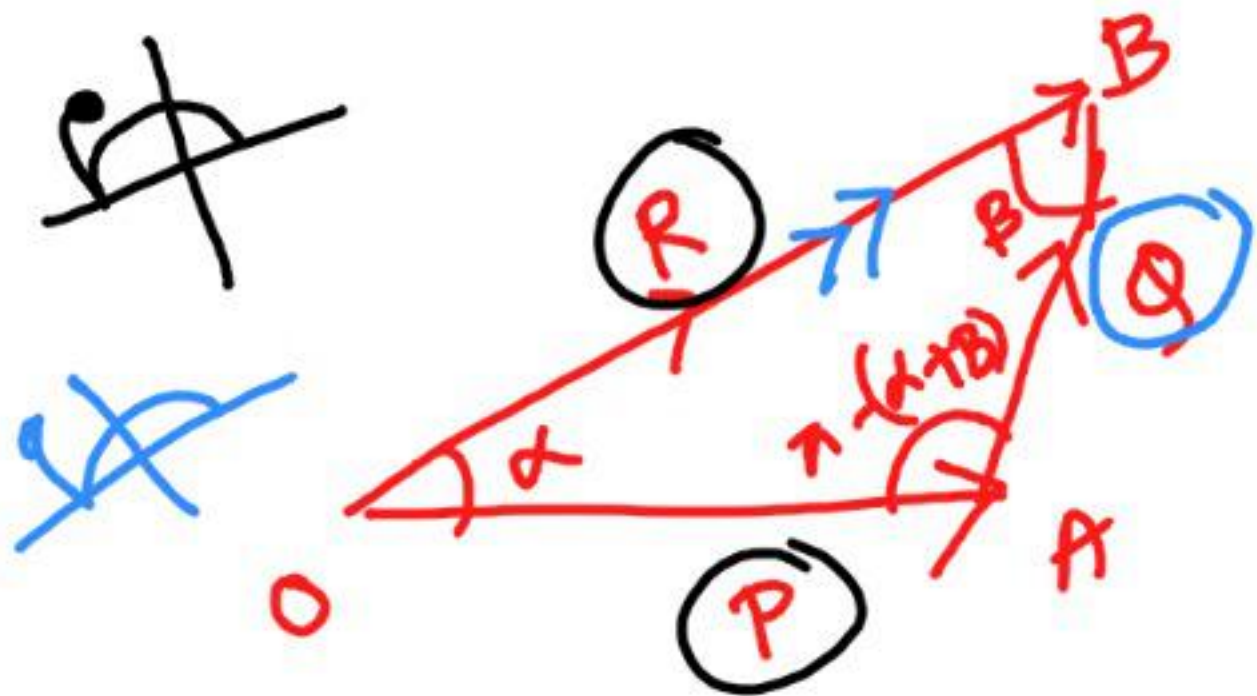
দুটি ভেক্টর  $\vec{P}$  ও  $\vec{Q}$  এর লব্ধি  $\vec{R}$  এর মান -

- ~~a) সর্বদা  $R > P + Q$~~
- ~~b) সর্বদা  $R < P + Q$~~
- ~~c) সর্বদা  $R = P + Q$~~
- d)  $P + Q \leq R \leq P + Q$

$$\begin{array}{l} \vec{P} + \vec{Q} \leq \vec{R} \leq \vec{P} + \vec{Q} \\ \vec{R}_{\min} \leq \vec{R} \leq \vec{R}_{\max} \end{array}$$

**ANY  
QUESTION**





$$\frac{OA}{\sin \beta} = \frac{OB}{\sin(\pi - (\alpha + \beta))} = \frac{AB}{\sin \alpha}$$

$$\frac{P}{\sin \beta} = \frac{R}{\sin(\alpha + \beta)} = \frac{Q}{\sin \alpha}$$

$$\frac{P}{\sin \beta} = \frac{R}{\sin(\alpha + \beta)}$$

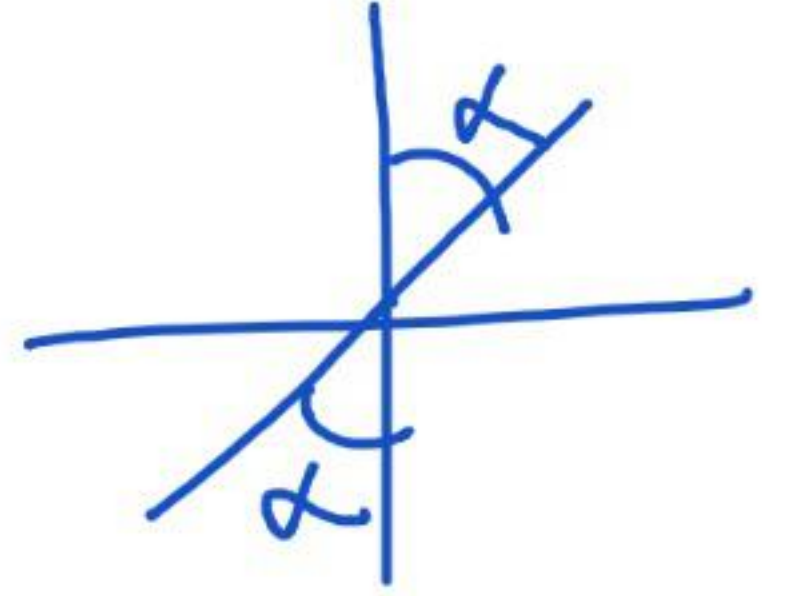
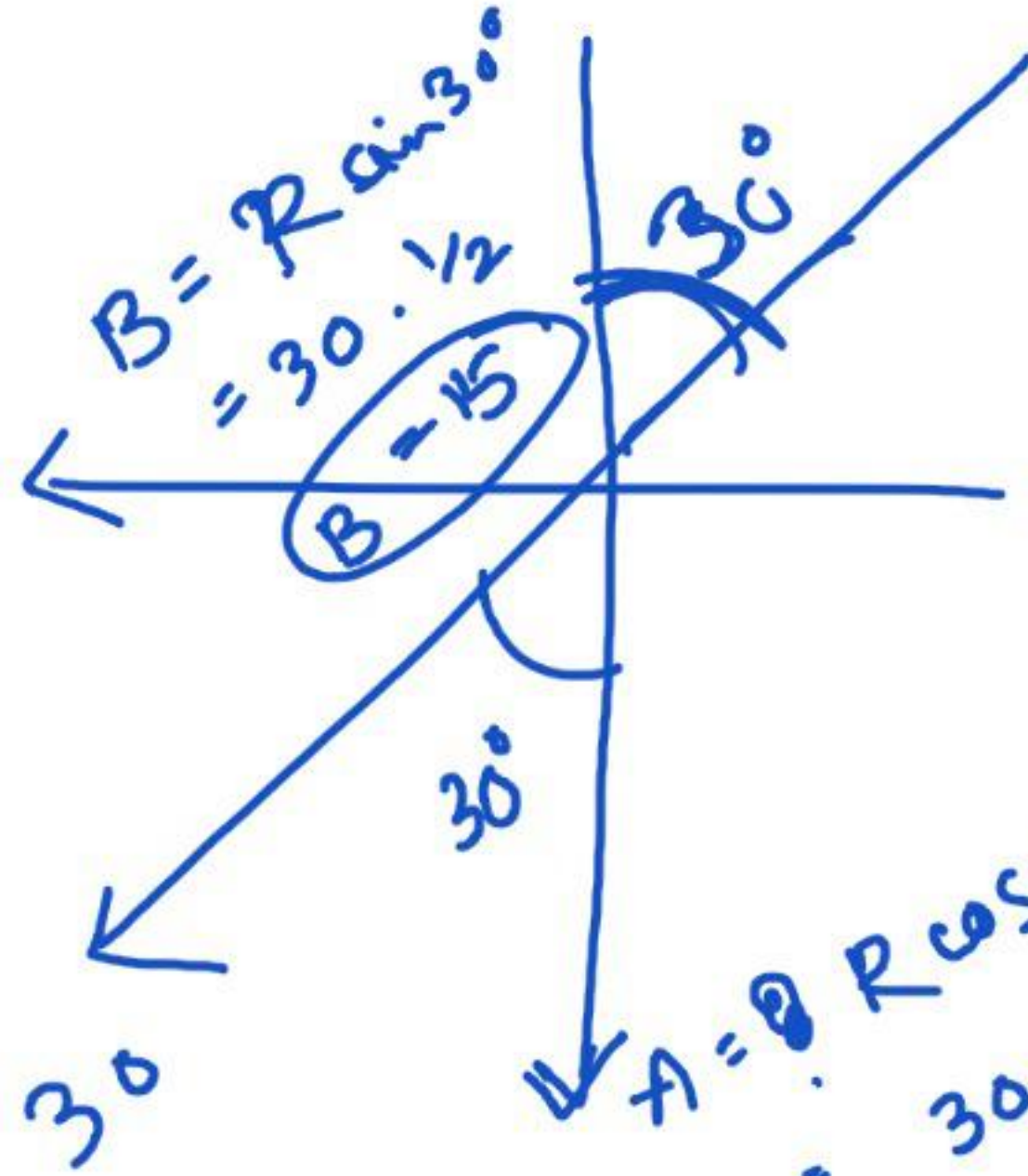
$$P = \frac{R \sin \beta}{\sin(\alpha + \beta)}$$

$$\frac{Q}{\sin \alpha} = \frac{R}{\sin(\alpha + \beta)}$$

$$\Rightarrow Q = \frac{R \sin \alpha}{\sin(\alpha + \beta)}$$

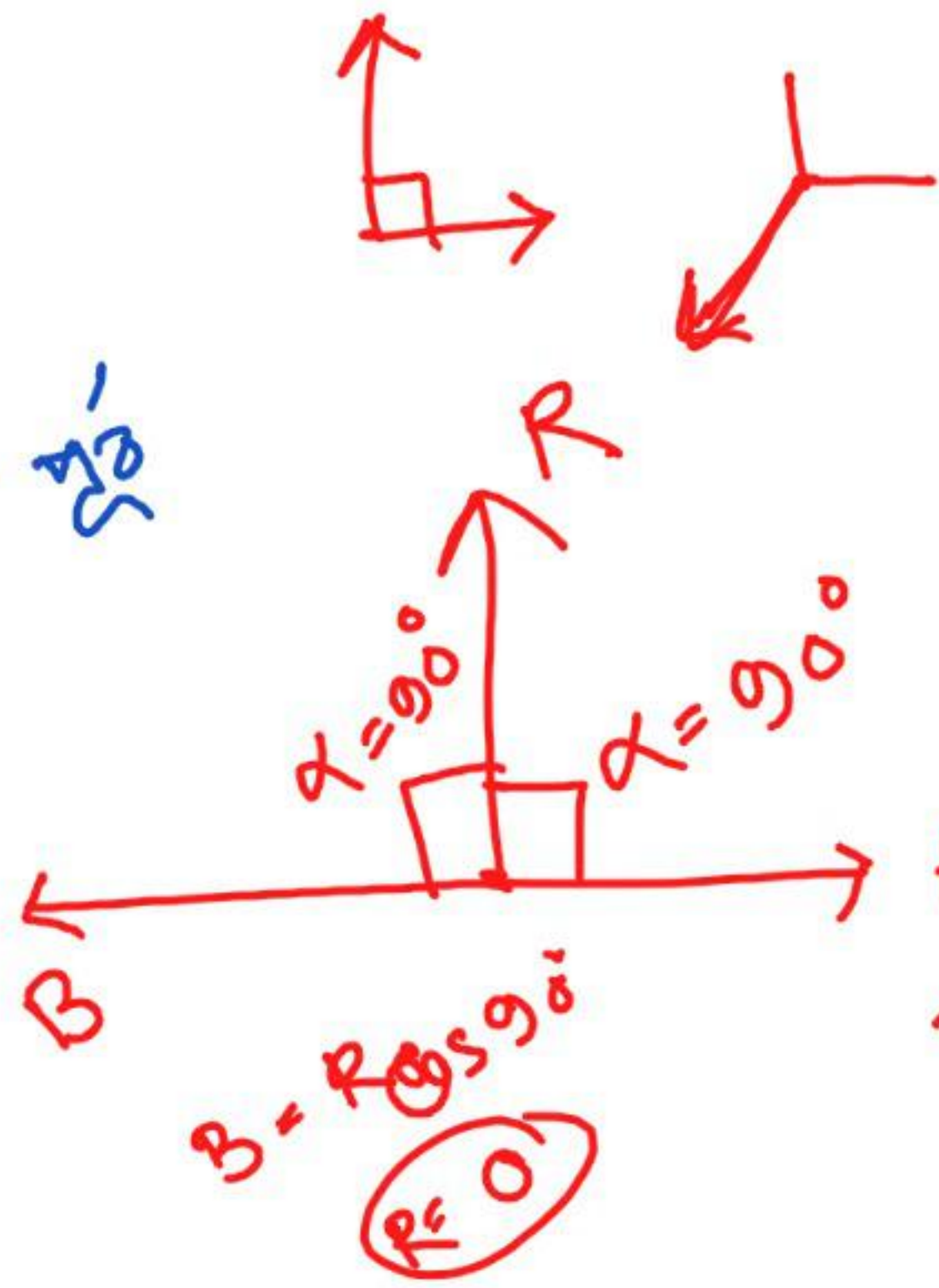
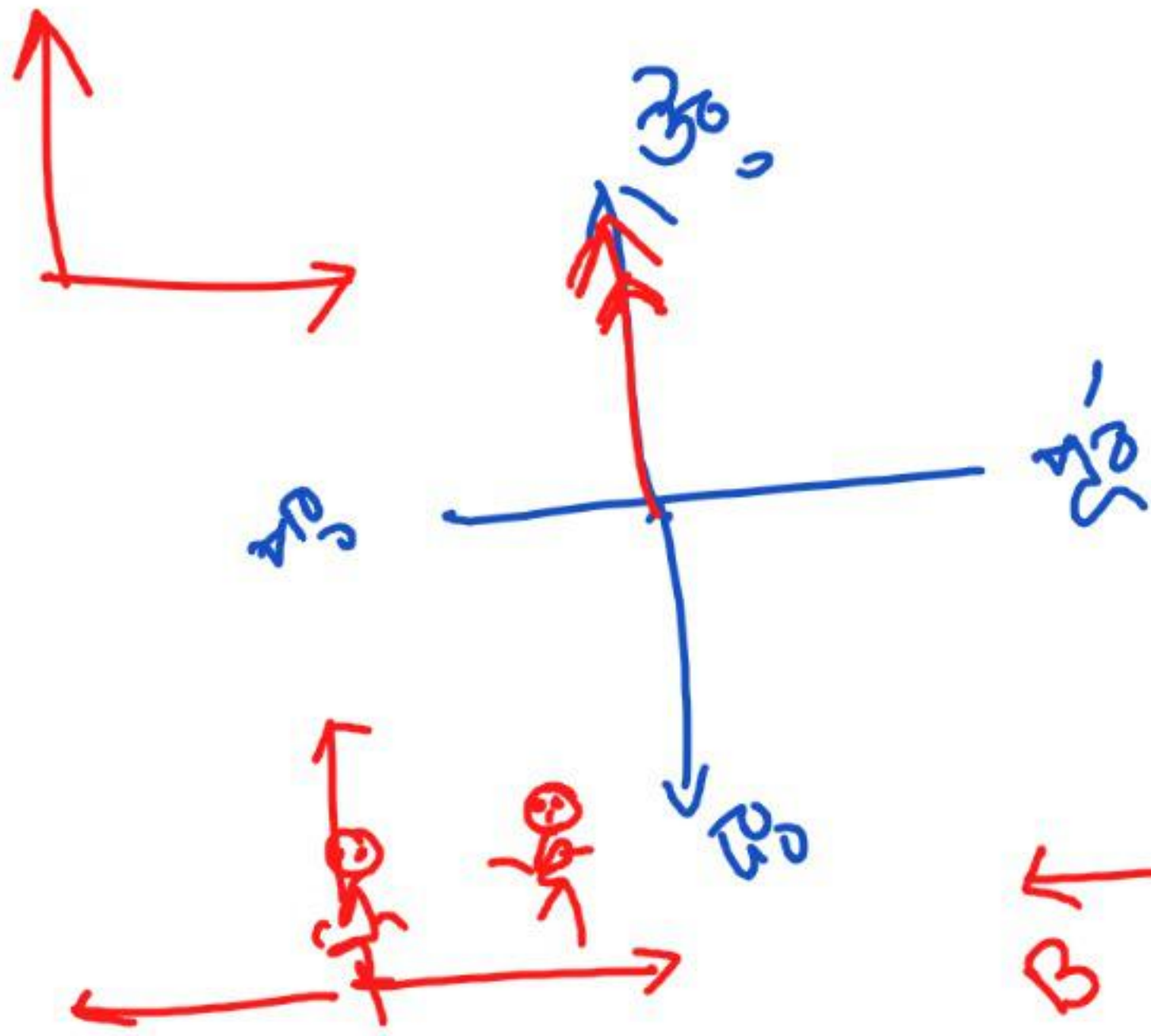
ADM





A

$$\begin{aligned}
 A &= R \cos 30^\circ \\
 &= 30 \cos 30^\circ \\
 &= 30 \cdot \frac{\sqrt{3}}{2} \\
 15\sqrt{3} &= A
 \end{aligned}$$



$$A = R \cos 90^\circ$$

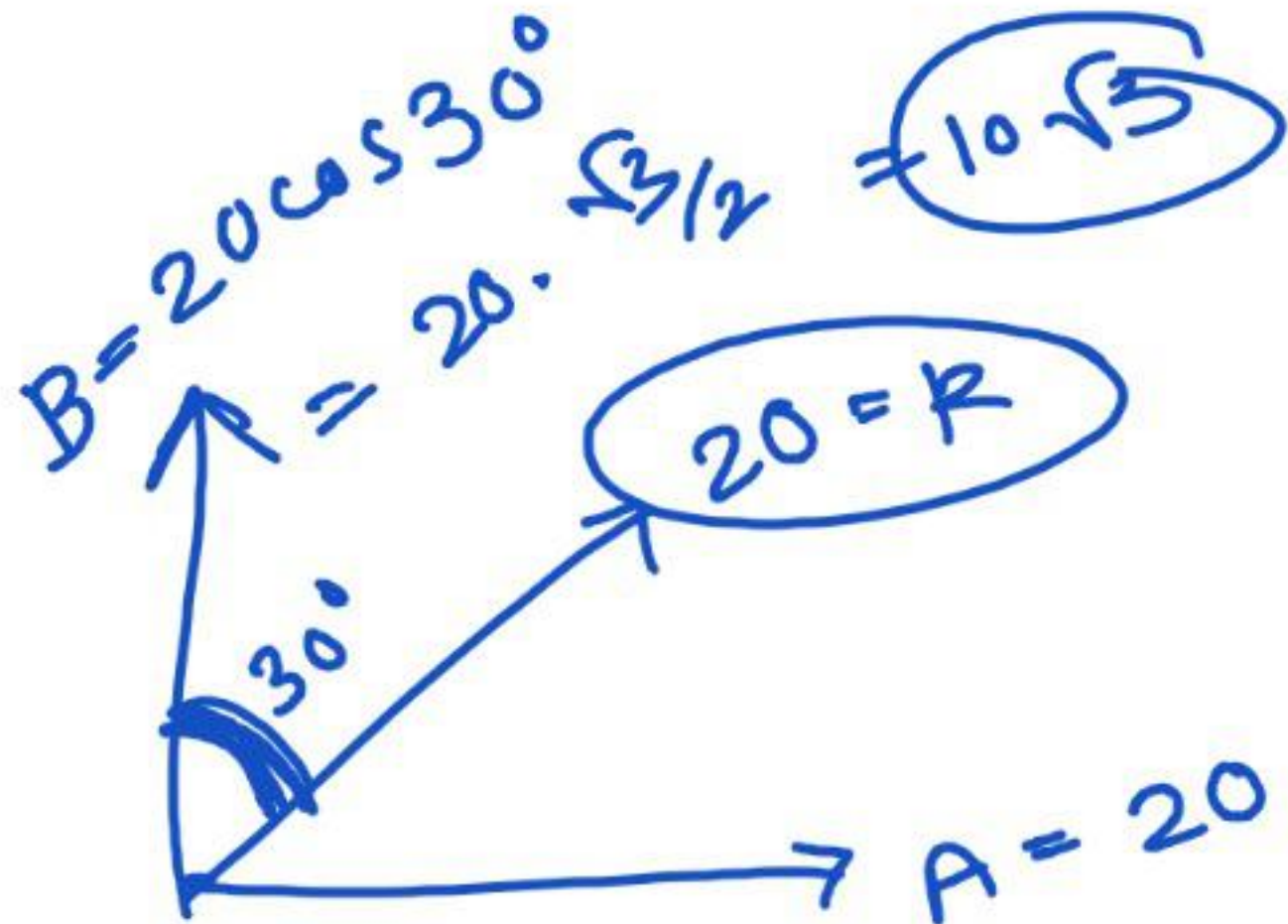
$A = 0$

$$B = R \cos 90^\circ$$

$R = 0$



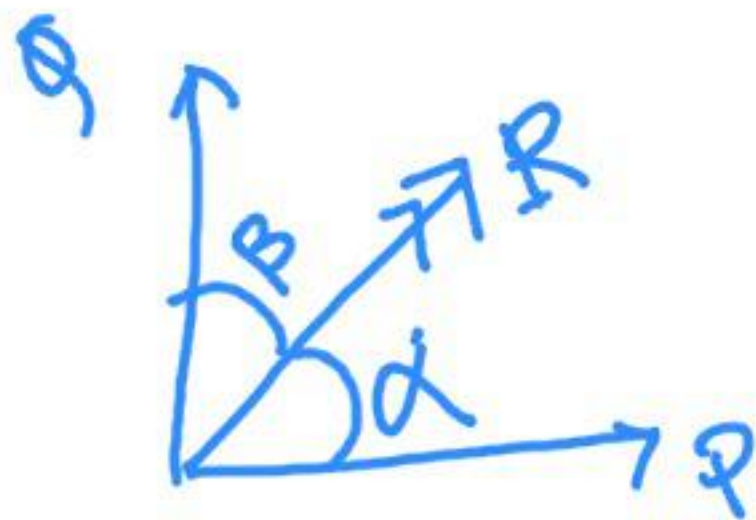
$$A = 10$$
$$B =$$



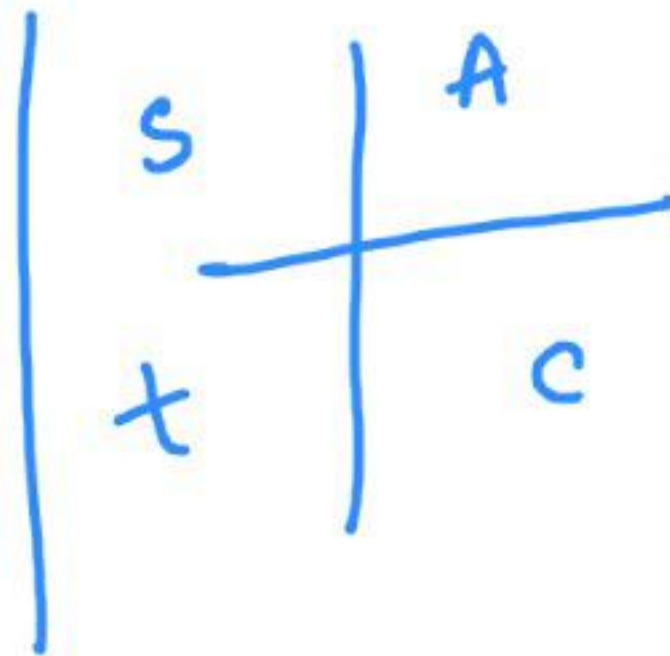
$$= 20 \cdot \frac{1}{2}$$
$$A = 10$$

Special Case:

$$\alpha + \beta = 90^\circ$$



$$\alpha + \beta = 90^\circ$$
$$\beta = (90 - \alpha)$$



$$P = \frac{R \sin \beta}{\sin(\alpha + \beta)}$$

$$\Rightarrow P = \frac{R \sin(90 - \alpha)}{\sin 90^\circ}$$

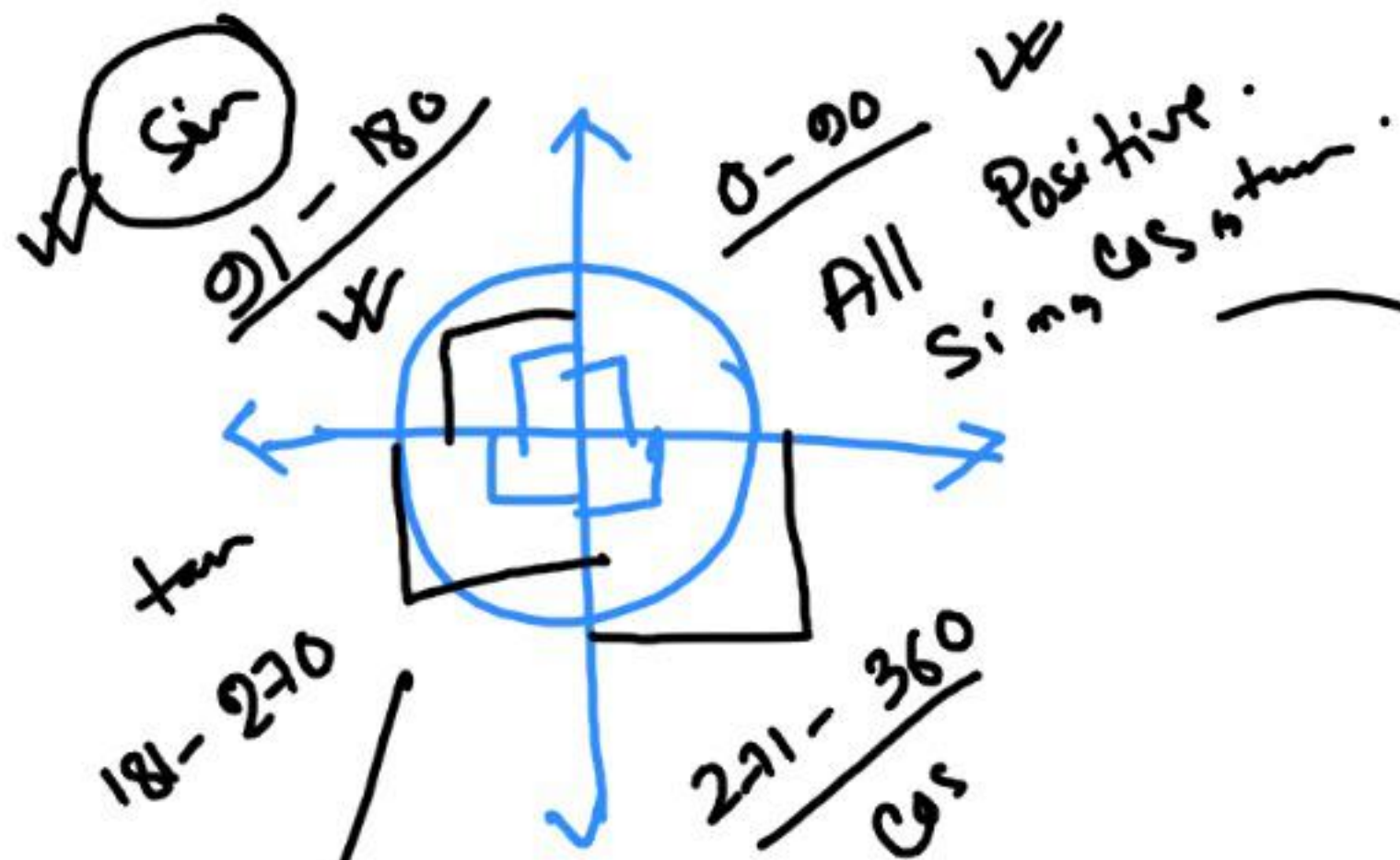
$$P = \frac{R \cos \alpha}{1}$$

$$P = R \cos \alpha$$

$$Q = \frac{R \sin \alpha}{\sin \alpha + \beta}$$

$$Q = R \sin \alpha$$



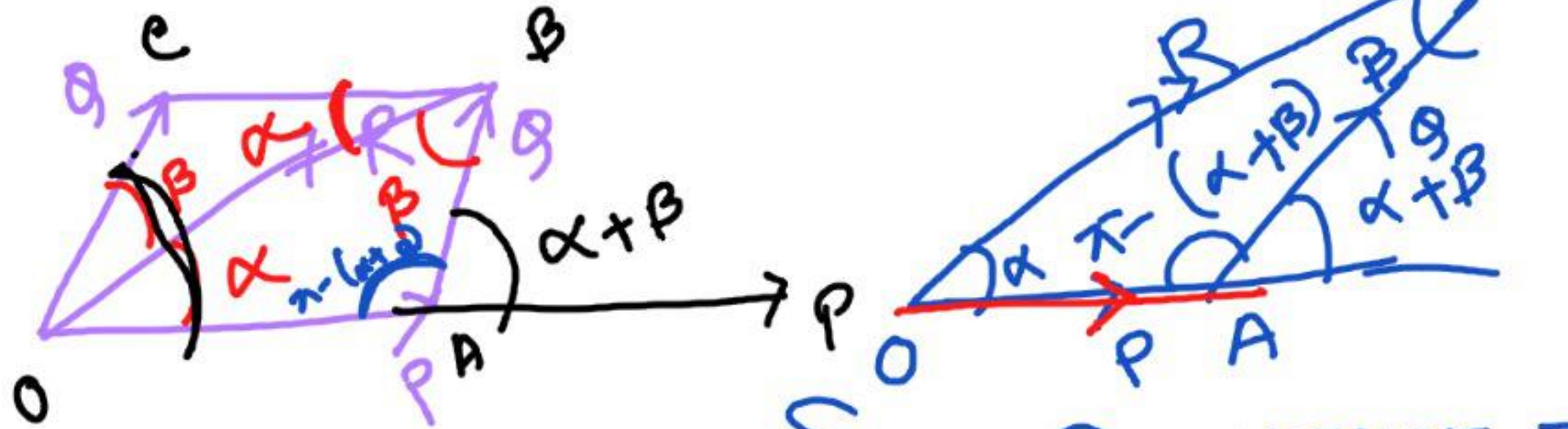


$\sin 155^\circ = (-)$   
 $\cos 155^\circ = (-)$   
 $\tan 155^\circ = (+)$

$\sin 30^\circ = +\frac{1}{2}$   
 $\cos 30^\circ = +\frac{\sqrt{3}}{2}$   
 $\tan 30^\circ = +\frac{1}{\sqrt{3}}$

$\sin 120^\circ = \frac{\sqrt{3}}{2}$   
 $\cos 120^\circ = -\frac{1}{2}$   
 $\tan 120^\circ = -\sqrt{3}$

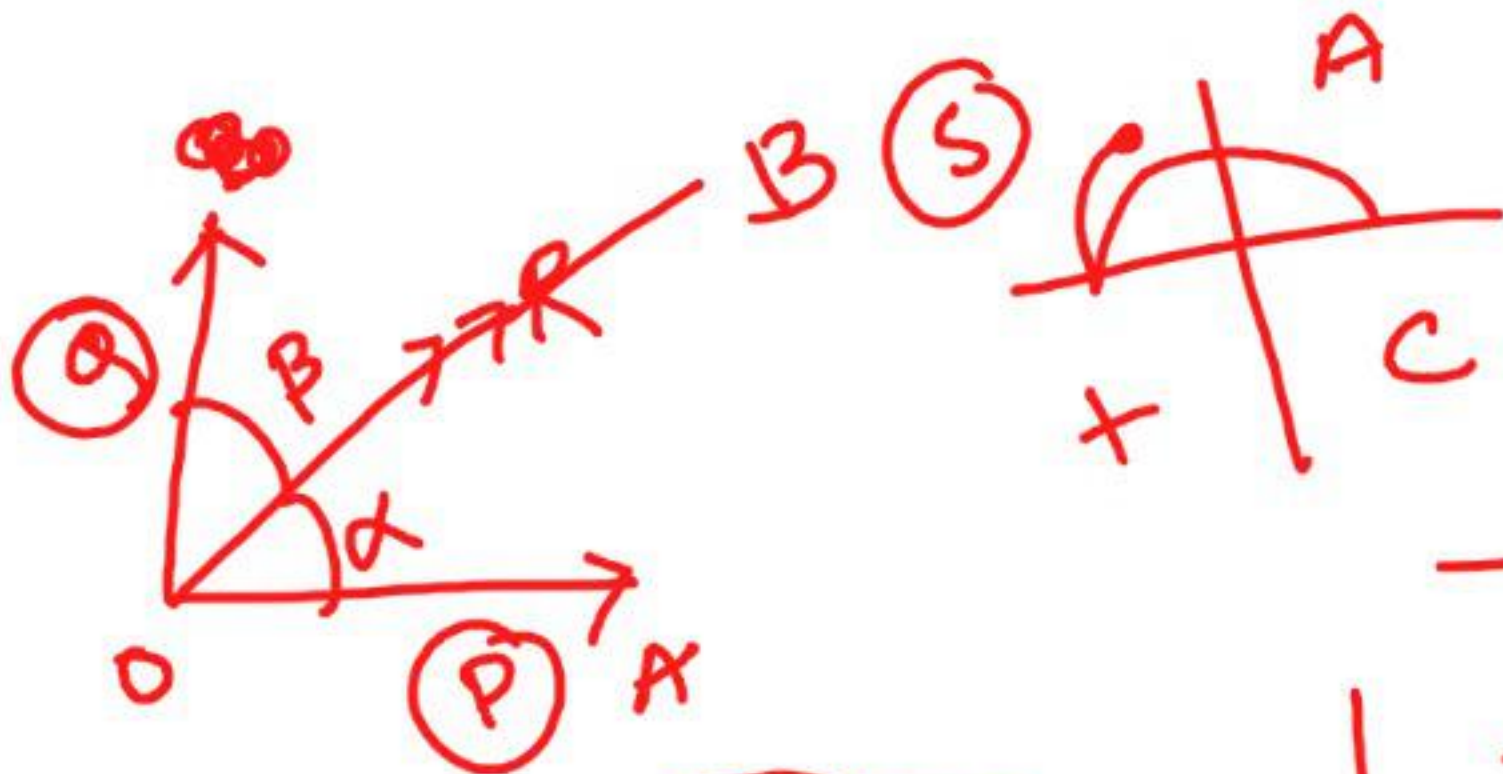
ଦ୍ୱିତୀୟ ଅଙ୍ଗୁଳ:



$\triangle OAB$  .... ସିନୁସ୍ ରୁଲ୍ ଉପରେ ଆଧାରିତ ହେଉଛି ଏହି ସାହାଯ୍ୟ କରୁଛି

$$\frac{OA}{\sin \beta} = \frac{AB}{\sin \alpha} = \frac{OB}{\sin (\pi - (\alpha + \beta))}$$





$$Q = \frac{R \sin \alpha}{\sin(\alpha + \beta)}$$

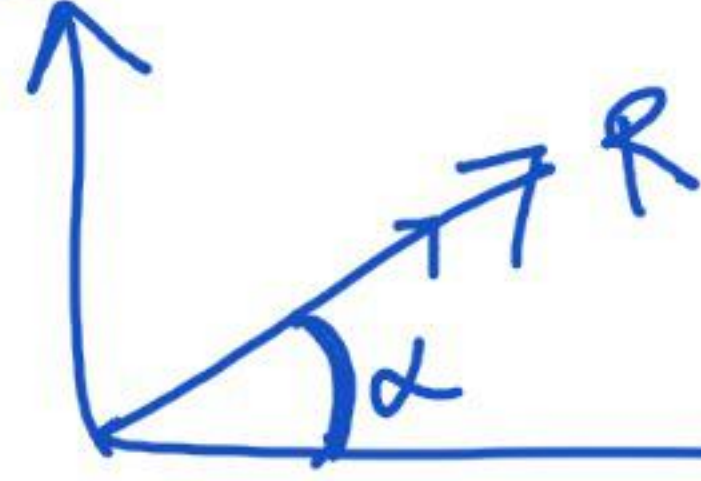
$$\frac{OA}{\sin \beta} = \frac{AB}{\sin \alpha} = \frac{OB}{\sin(\pi - (\alpha + \beta))}$$

$$\Rightarrow \frac{OA}{\sin \beta} = \frac{OB}{\sin(\alpha + \beta)}$$

$$\frac{P}{\sin \beta} = \frac{R}{\sin(\alpha + \beta)}$$

$$P = \frac{R \sin \beta}{\sin(\alpha + \beta)}$$

$$Q = R \sin \alpha$$



$$P = R \cos \alpha$$



\*  $\cos \alpha$   $\alpha$   $\sin \alpha$   $\cos \alpha$   $\sin \alpha$

ଅବସ୍ଥା  $\alpha$   $\cos \alpha$   $\sin \alpha$

ଅବସ୍ଥା  $\alpha$   $\cos \alpha$   $\sin \alpha$

ଅବସ୍ଥା  $\alpha$   $\cos \alpha$   $\sin \alpha$

$$\sin(\alpha)$$

$$\sin(180 - (\alpha + \beta))$$

$$\sin(2 \times 90 - (\alpha + \beta))$$

$$\sin(\alpha + \beta)$$

$$\sin(\alpha + \beta)$$

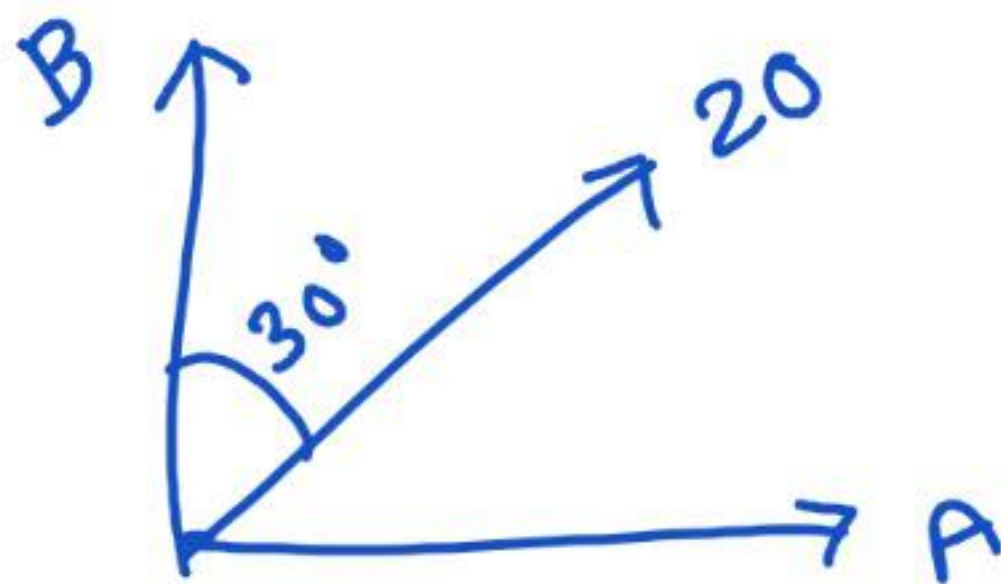
$$\sin(90 - \alpha)$$

$$\sin(1 \times 90 - \alpha)$$

$$\cos \alpha$$



$$A'' = B''$$





120

HSC  
Trig-Formula

$$\sin(A+B) \quad \begin{matrix} 80 \\ 40 \end{matrix}$$

$$\sin(60+60) \quad \begin{matrix} A \\ B \end{matrix}$$

$$= \sin A \cos B + \cos A \sin B$$

$$= \sin 60 \cos 60 + \sin 60 \cos 60$$

$$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \text{○}$$

$$\sin(\pi - (\alpha + \beta))$$

$$\sin 120^\circ$$

$\Rightarrow$

$$\sin(180 - 60^\circ)$$

$\Rightarrow$

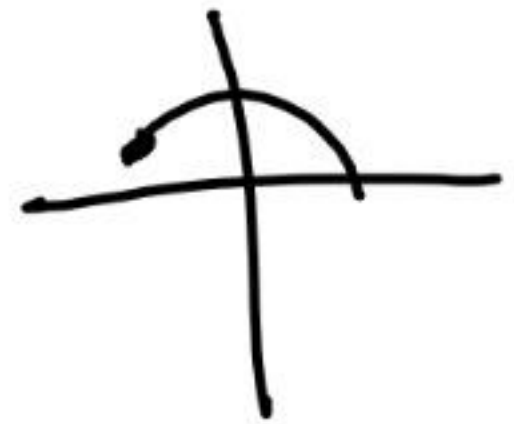
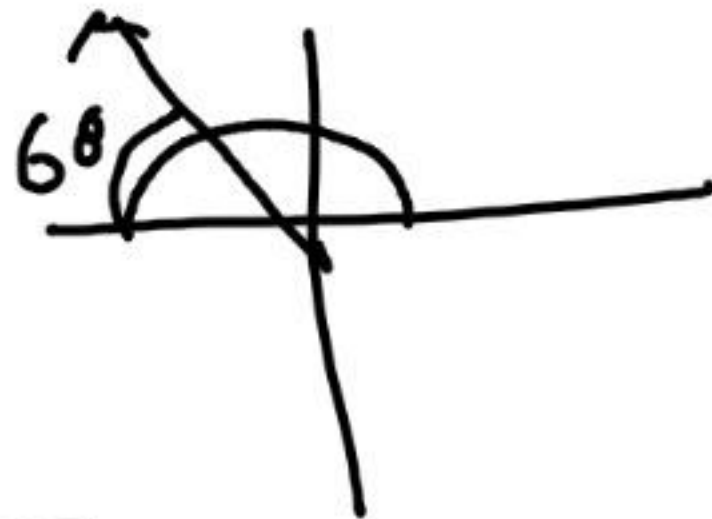
$$\sin(2 \times 90 - 60^\circ)$$

$\Rightarrow$

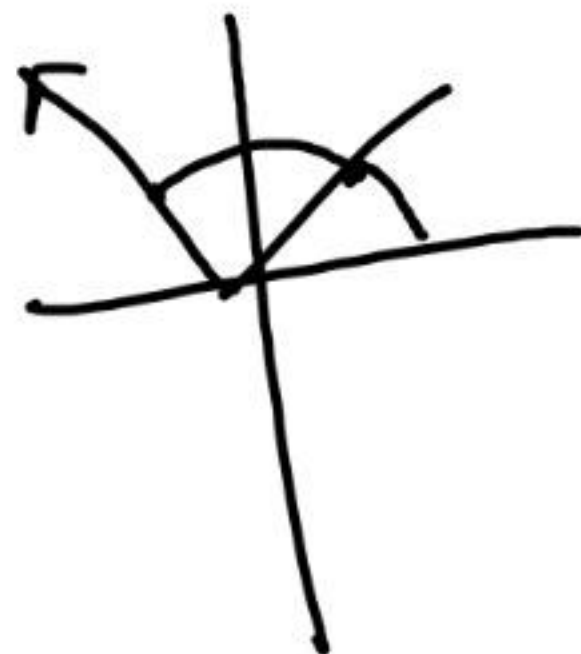
$$\sin 60^\circ$$

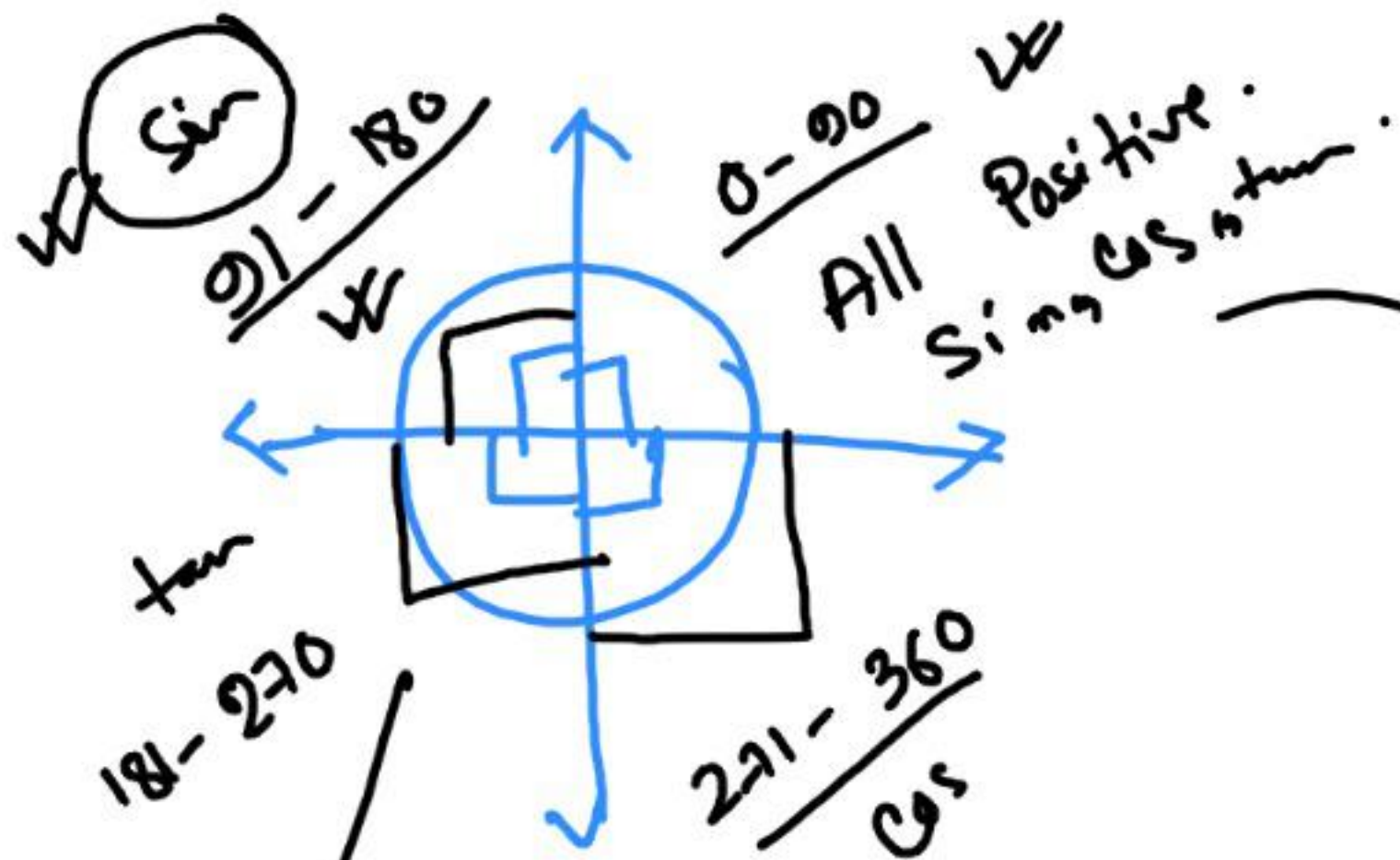
$\Rightarrow$

$$\frac{\sqrt{3}}{2}$$



$$\sin(90 + 30)$$





$\sin 155^\circ = (-)$   
 $\cos 155^\circ = (-)$   
 $\tan 155^\circ = (+)$

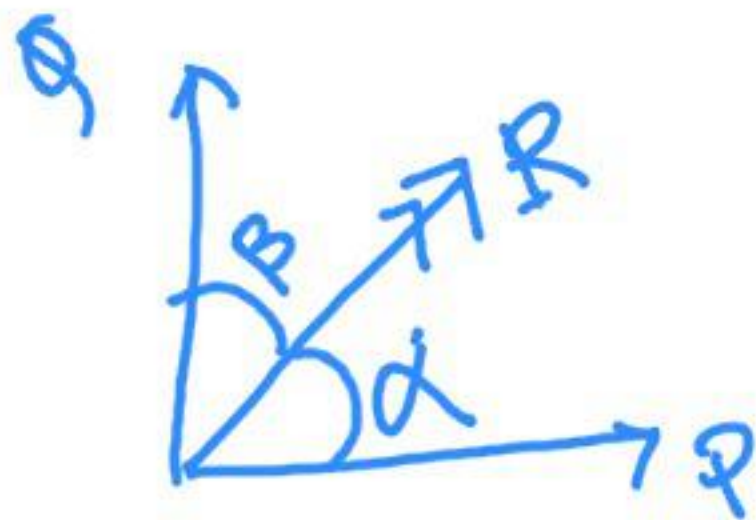
$\sin 30^\circ = +\frac{1}{2}$   
 $\cos 30^\circ = +\frac{\sqrt{3}}{2}$   
 $\tan 30^\circ = +\frac{1}{\sqrt{3}}$

$\sin 120^\circ = \frac{\sqrt{3}}{2}$   
 $\cos 120^\circ = -\frac{1}{2}$   
 $\tan 120^\circ = -\sqrt{3}$

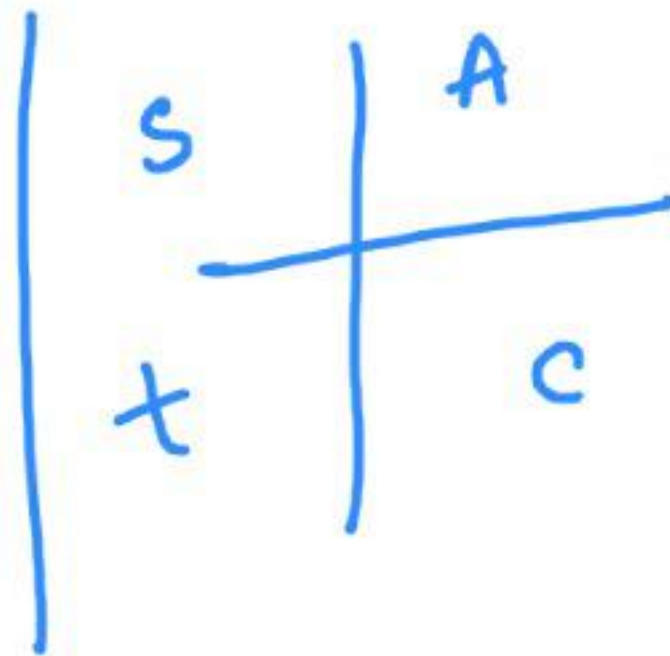


Special Case:

$$\alpha + \beta = 90^\circ$$



$$\alpha + \beta = 90^\circ$$
$$\beta = (90 - \alpha)$$



$$P = \frac{R \sin \beta}{\sin(\alpha + \beta)}$$

$$\Rightarrow P = \frac{R \sin(90 - \alpha)}{\sin 90^\circ}$$

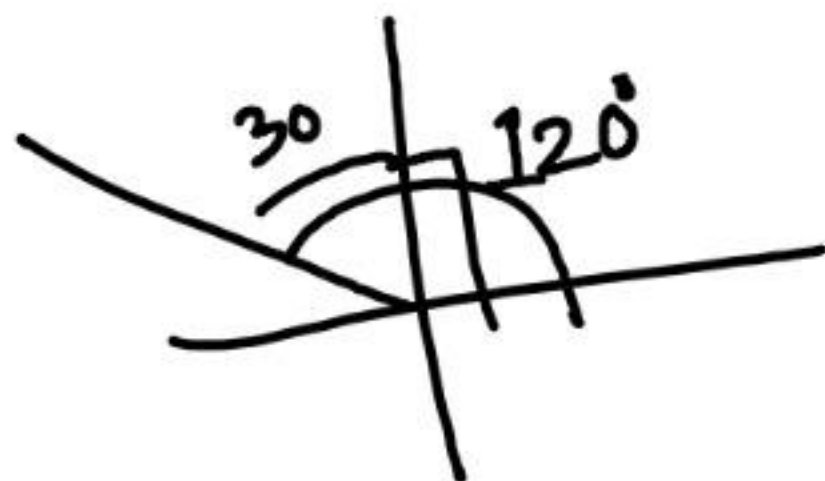
$$P = \frac{R \cos \alpha}{1}$$

$$P = R \cos \alpha$$

$$Q = \frac{R \sin \alpha}{\sin \alpha + \beta}$$

$$Q = R \sin \alpha$$

|                   |              |
|-------------------|--------------|
| Standard<br>sin + | All +        |
| tan +             | One<br>cos + |



|        |                  |
|--------|------------------|
| 1 x 90 | <del>10/10</del> |
| 3 x 90 | <del>10/10</del> |
| 7 x 90 | sin - cos        |
|        | cos - sin        |

Sin 120°

(90 + 30)

+ Sin (1 x 90 + 30)

cos 30°

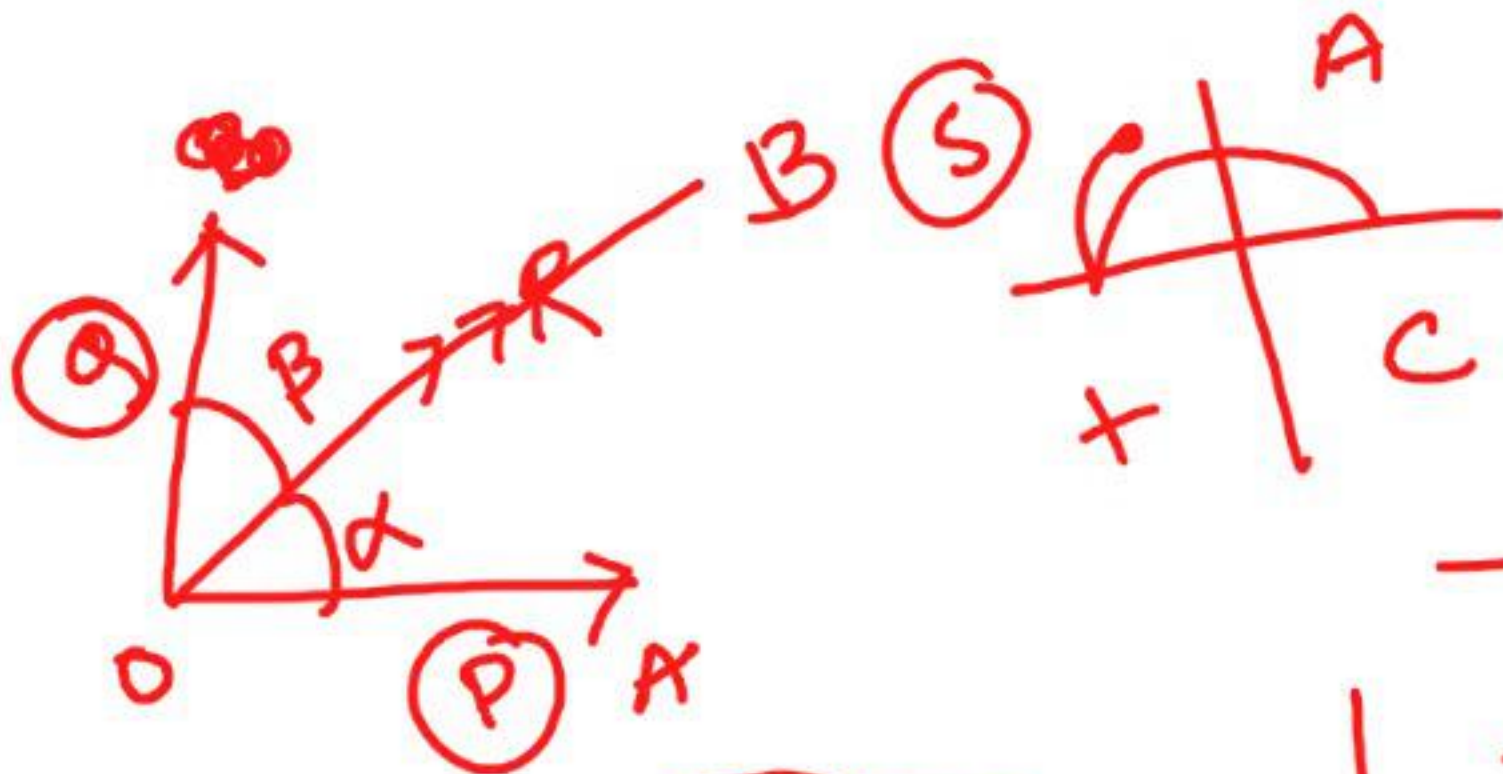
( $\sqrt{3}/2$ )

13/15 (2,4)  
sin - sin  
cos - cos

~~Q~~

|     |   |       |              |              |    |
|-----|---|-------|--------------|--------------|----|
| Sin | 0 | 30    | 45           | 60           | 90 |
|     | 0 | $1/2$ | $1/\sqrt{2}$ | $\sqrt{3}/2$ | 1  |





$$Q = \frac{R \sin \alpha}{\sin(\alpha + \beta)}$$

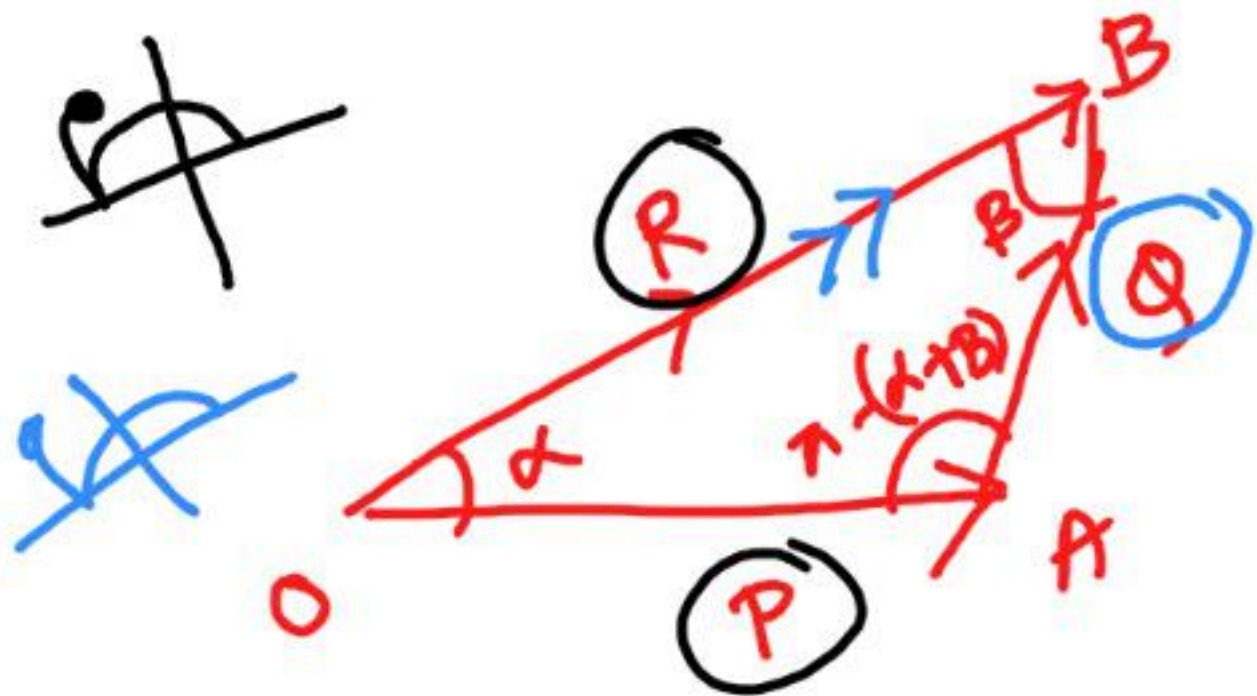
$$\frac{OA}{\sin \beta} = \frac{AB}{\sin \alpha} = \frac{OB}{\sin(\pi - (\alpha + \beta))}$$

$$\Rightarrow \frac{OA}{\sin \beta} = \frac{OB}{\sin(\alpha + \beta)}$$

$$\frac{P}{\sin \beta} = \frac{R}{\sin(\alpha + \beta)}$$

$$P = \frac{R \sin \beta}{\sin(\alpha + \beta)}$$





$$\frac{OA}{\sin \beta} = \frac{OB}{\sin(\pi - (\alpha + \beta))} = \frac{AB}{\sin \alpha}$$

$$\frac{P}{\sin \beta} = \frac{R}{\sin(\alpha + \beta)} = \frac{Q}{\sin \alpha}$$

$$\frac{P}{\sin \beta} = \frac{R}{\sin(\alpha + \beta)}$$

$$P = \frac{R \sin \beta}{\sin(\alpha + \beta)}$$

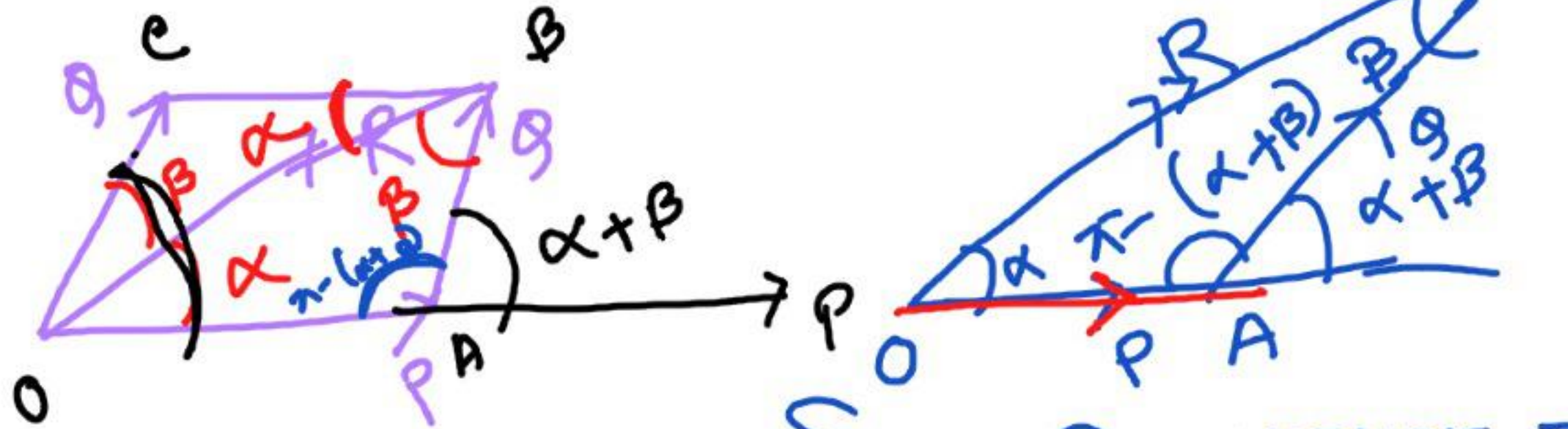
$$\frac{Q}{\sin \alpha} = \frac{R}{\sin(\alpha + \beta)}$$

$$\Rightarrow Q = \frac{R \sin \alpha}{\sin(\alpha + \beta)}$$

ADM



ଦ୍ୱିତୀୟ ଅଙ୍ଗୁଳୀ:

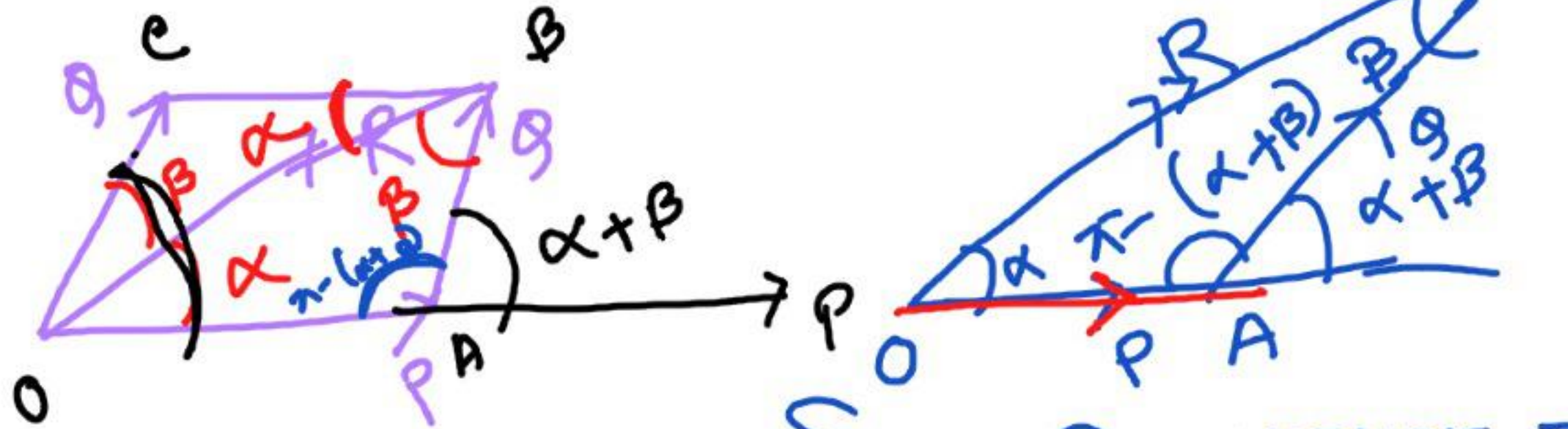


$\triangle OAB$  .... ସିନୁସ୍ ରୁଲ୍ ଉପରେ ଆଧାରିତ ହେଉଛି ଏହି ସମ୍ବନ୍ଧଟି କିପରି ପ୍ରାପ୍ତ ହୁଏ

$$\frac{OA}{\sin \beta} = \frac{AB}{\sin \alpha} = \frac{OB}{\sin (\pi - (\alpha + \beta))}$$

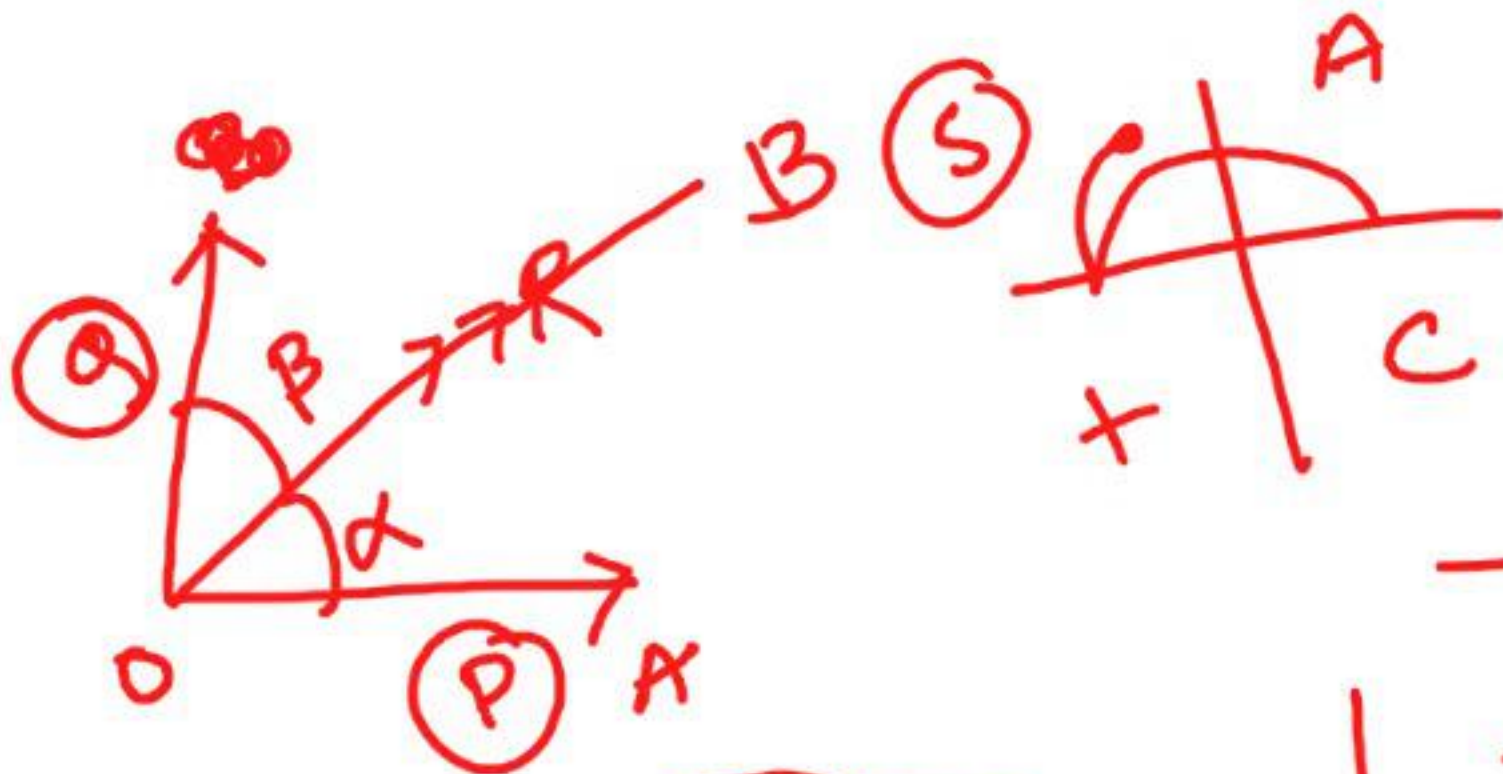


ଦ୍ୱିତୀୟ ଅଙ୍ଗୁଳ:



$\triangle OAB$  .... ସିନୁସ୍ ରୁଲ୍ ଉପରେ ଏହି ସାହାଯ୍ୟ କରୁଛି

$$\frac{OA}{\sin \beta} = \frac{AB}{\sin \alpha} = \frac{OB}{\sin (\pi - (\alpha + \beta))}$$



$$Q = \frac{R \sin \alpha}{\sin(\alpha + \beta)}$$

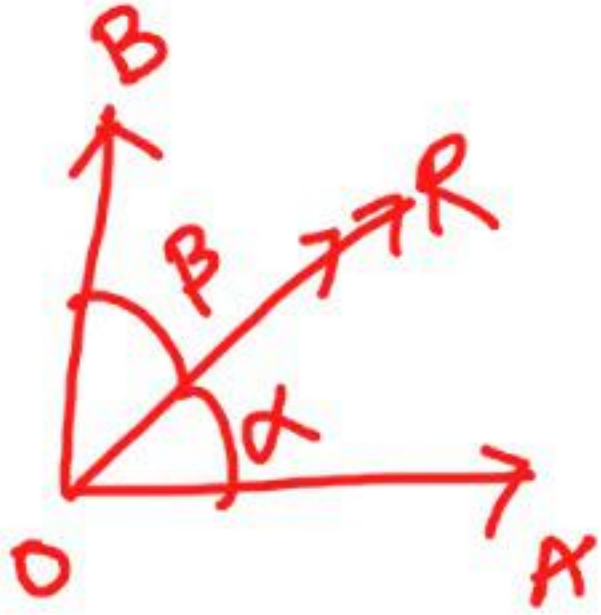
$$\frac{OA}{\sin \beta} = \frac{AB}{\sin \alpha} = \frac{OB}{\sin(\pi - (\alpha + \beta))}$$

$$\Rightarrow \frac{OA}{\sin \beta} = \frac{OB}{\sin(\alpha + \beta)}$$

$$\frac{P}{\sin \beta} = \frac{R}{\sin(\alpha + \beta)}$$

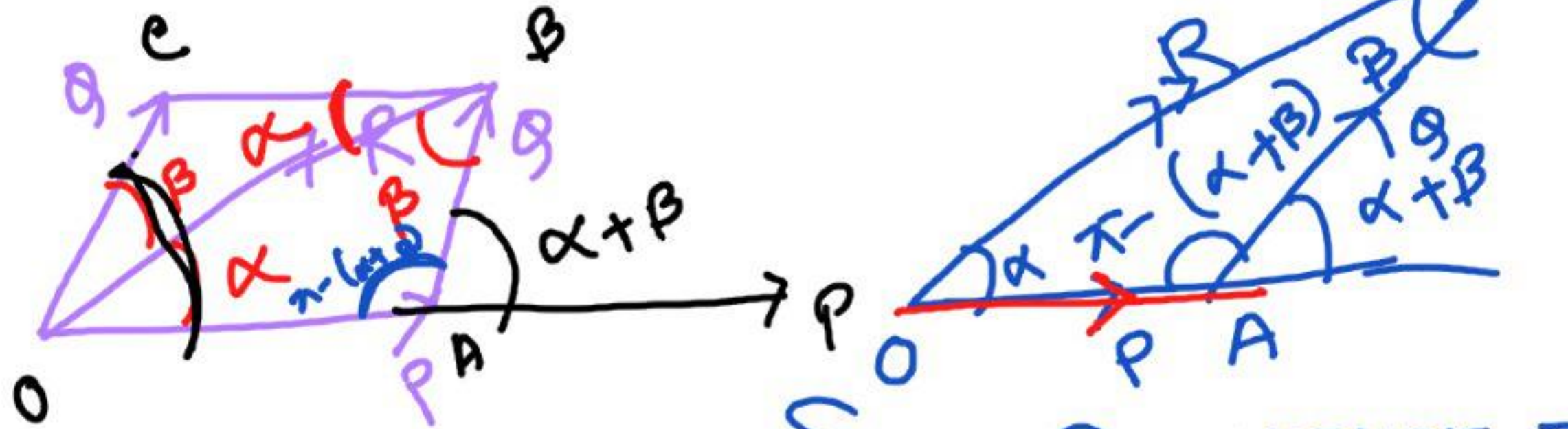
$$P = \frac{R \sin \beta}{\sin(\alpha + \beta)}$$





$$\frac{OA}{\sin \beta} = \frac{AB}{\sin \alpha} = \frac{OB}{\sin(\pi - (\alpha + \beta))}$$

ଦ୍ୱିତୀୟ ଅଙ୍ଗୁଳ:



$\triangle OAB$  .... ସିନୁସ୍ ରୁଲ୍ ଉପରେ ଆଧାରିତ ହେଉଛି ଏହି ସମ୍ବନ୍ଧ ଗଠନ କରି

$$\frac{OA}{\sin \beta} = \frac{AB}{\sin \alpha} = \frac{OB}{\sin (\pi - (\alpha + \beta))}$$



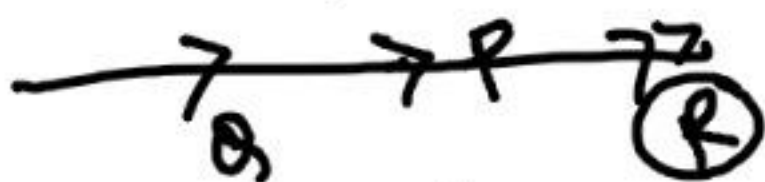
Case: 1

$$d = R t$$

$$\Rightarrow d = (Q + P) t_2 \quad 25$$

$$\Rightarrow Q + P = \frac{d}{t_2} = \frac{100}{4}$$

$$\Rightarrow \boxed{Q + P = 25}$$



Case: 2

$$t_1 = \frac{d}{\sqrt{Q^2 - P^2}}$$

$$\Rightarrow \sqrt{Q^2 - P^2} = \frac{d}{t_1}$$

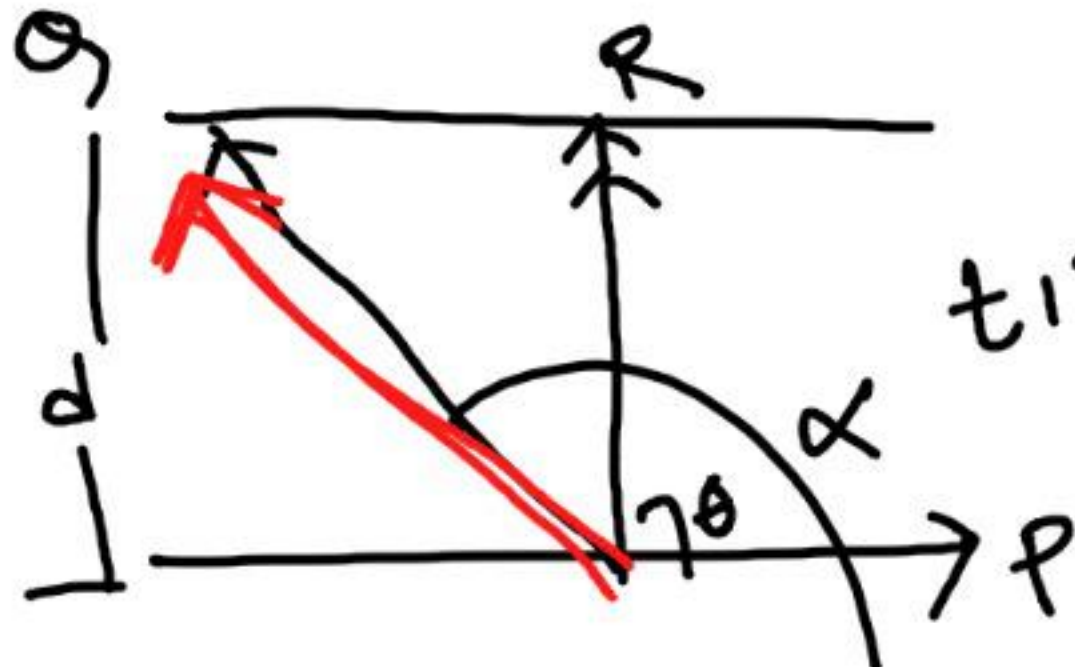
$$\Rightarrow \sqrt{Q^2 - P^2} = 20$$

$$Q^2 - P^2 = 400$$

$$\Rightarrow \sqrt{Q^2 - P^2} = \frac{d}{t_1} = \frac{100}{5} = 20$$

$$\begin{array}{l} Q + P = 25 \\ Q \cdot P = 16 \end{array}$$

$$\frac{Q + P}{P, Q}$$

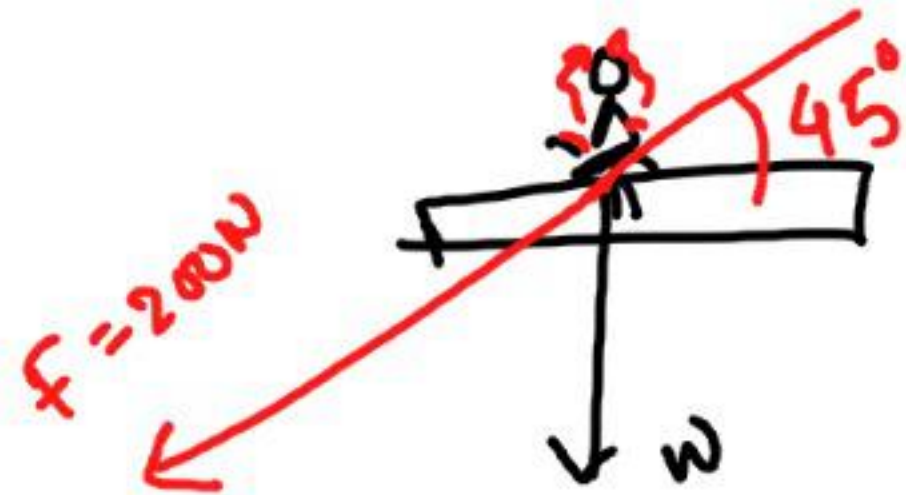


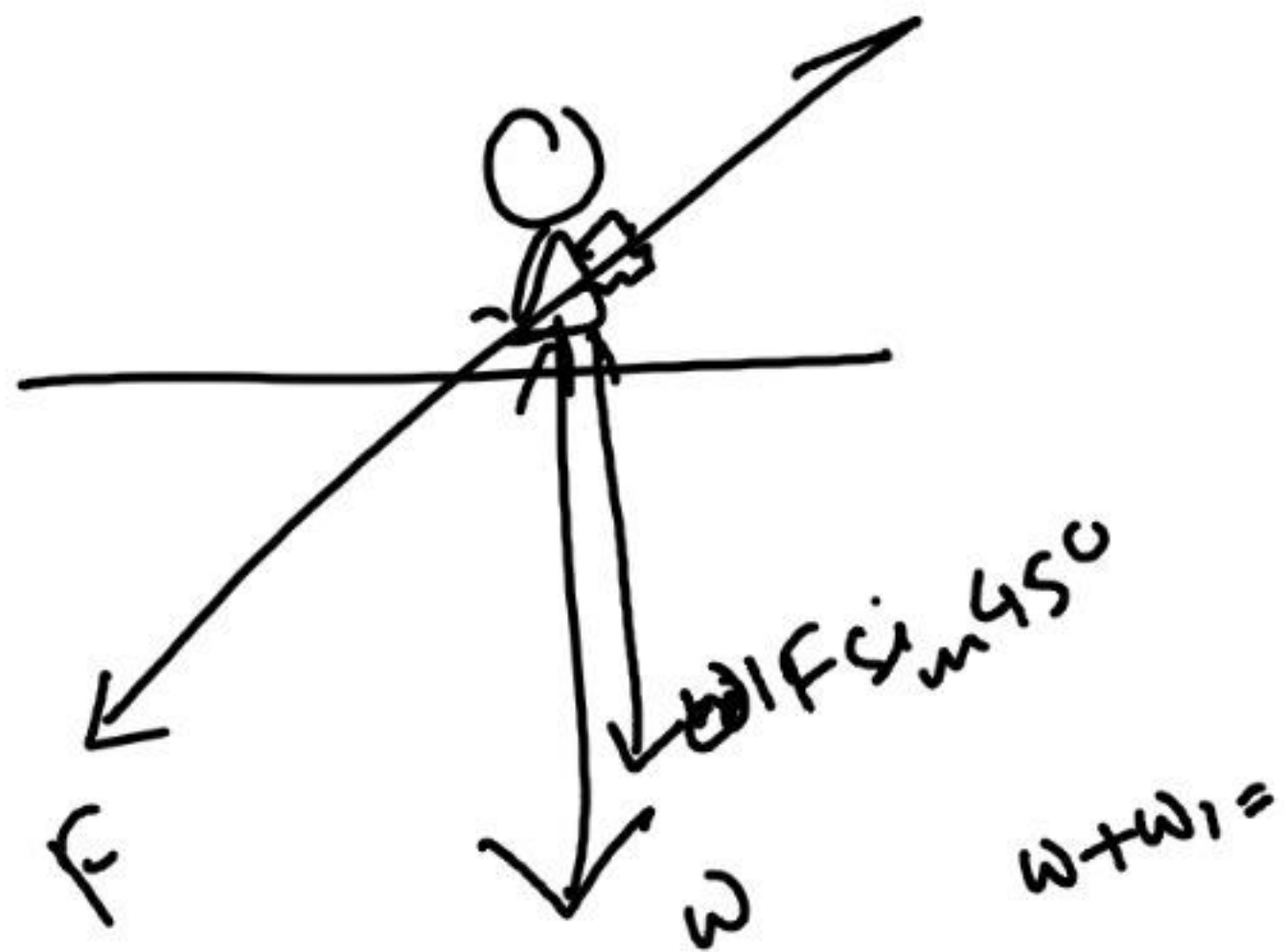
$$t_1 = 5 \text{ min}$$

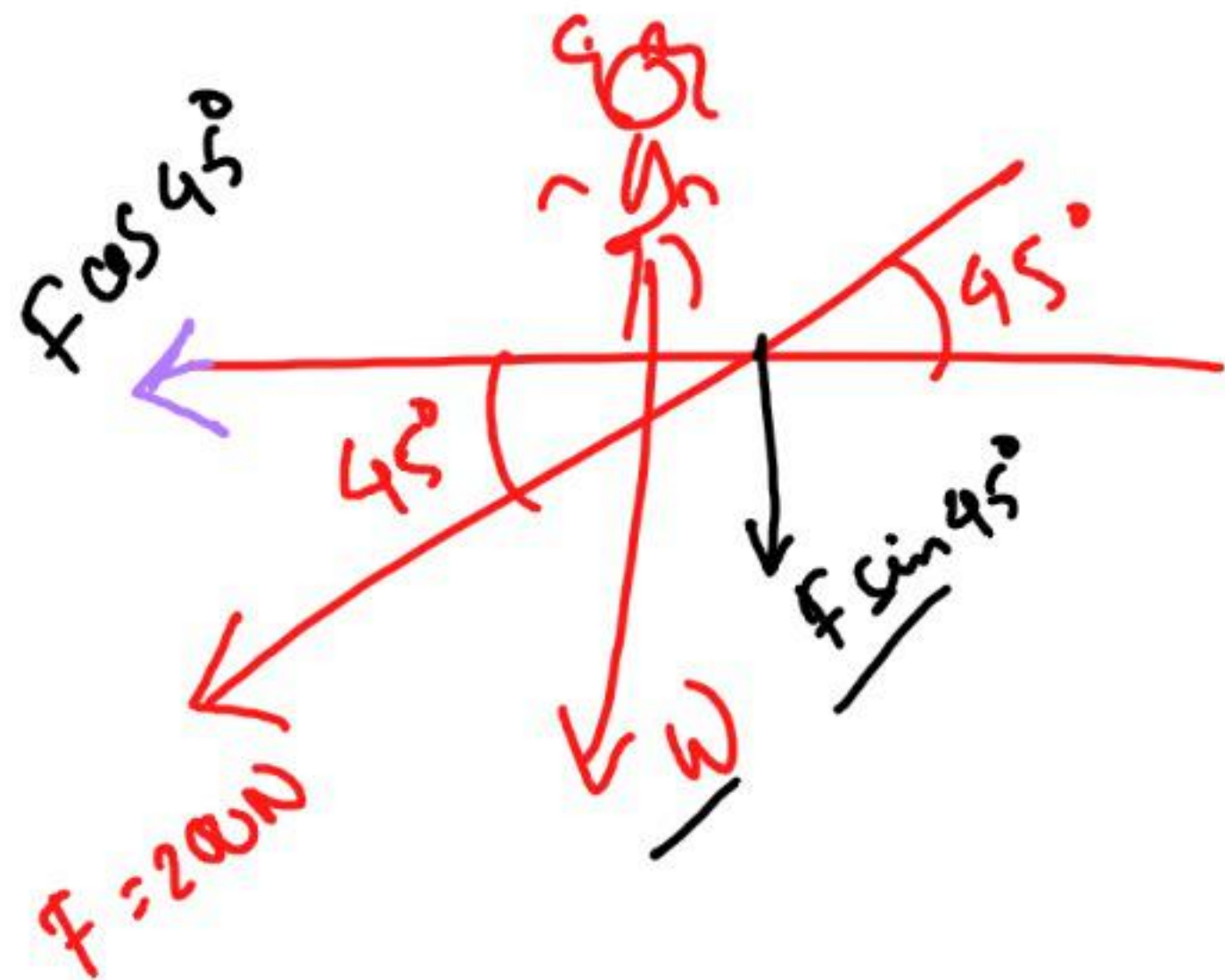
$$\begin{array}{l} Q^2 - P^2 = 400 \\ (Q + P)(Q - P) = 400 \\ 25(Q - P) = 400 \\ Q - P = 16 \end{array}$$



\*
   
 ଦୁଇଜଣା ବ୍ୟକ୍ତି ଓଉନ ମାମାବୁ ଶିକ୍ଷାନ ଦାଞ୍ଜିଏ
   
 ଓଉନ ମାମାବୁ ଉପର, ଖାସିକା ଉତ୍ତମିୟୁ-ମାଥ 45°
   
 ଖାସ 200N ଠଳ ଶ୍ରାୟାସି ଗୁରୁ ବେତ ଉତ୍ତମିୟୁ
   
 ଯି (700N) ଗୁଡି ଦିଲେ ଦୁଇଜଣା ଶ୍ରାୟାସି ଓଉନ ?







Ans:

$$W + F \sin 45^\circ = 700$$

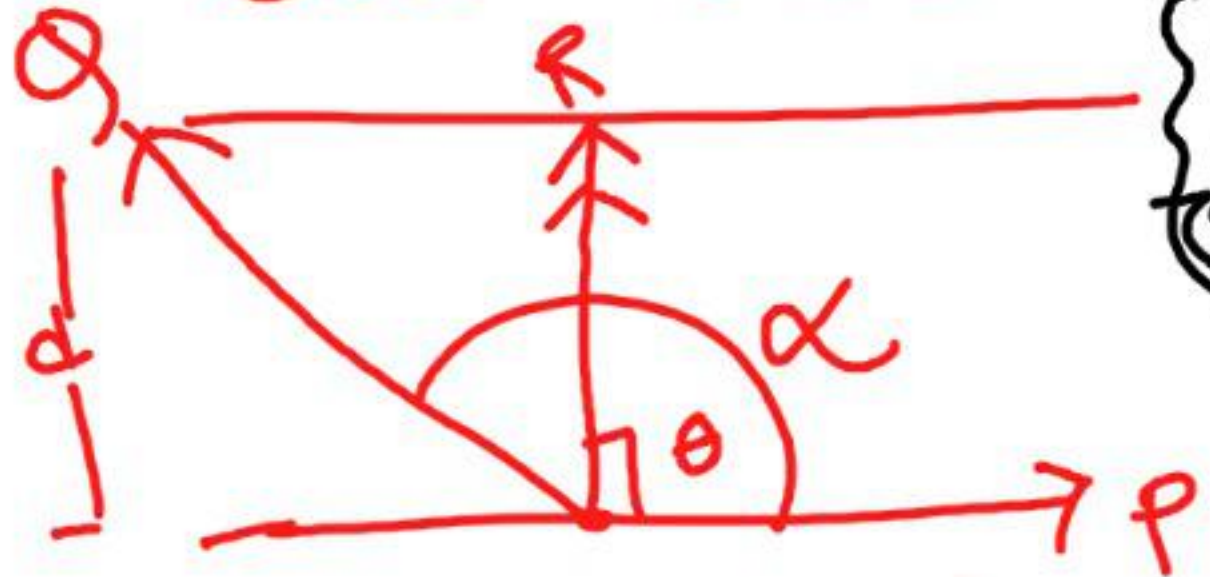
$$W = 700 - 200 \sin 45^\circ$$

$$W = 700 - 200 \cdot \frac{1}{\sqrt{2}}$$

$$\boxed{W = 558.6 \text{ N}}$$



ସମ୍ବନ୍ଧ (ସୂତ୍ର)



{ ୧. ୨. ୩. ୪  
ମୋଡ଼ାଠି  $\theta = 90^\circ$

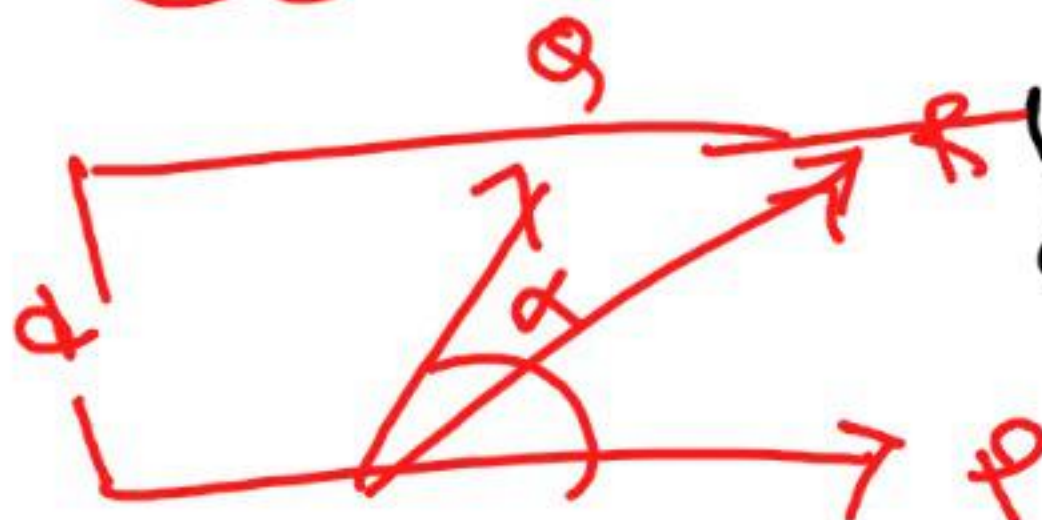
i)  $\alpha = \cos^{-1}(-p/q)$

ii)  $R = \sqrt{q^2 - p^2}$

iii)  $t = \frac{d}{\sqrt{q^2 - p^2}}$

ସମ୍ବନ୍ଧ  
ମୋଡ଼ାଠି  
ଅଥବା

ସମ୍ବନ୍ଧ (ସୂତ୍ର)



୧. ୨. ୩. ୪  
 $\alpha = 90^\circ$   
ମୋଡ଼ାଠି  
 $\theta$

i)  $\alpha = 90$

ii)  $R = \sqrt{p^2 + q^2}$

iii)  $t = \frac{d}{q}$

ସମ୍ବନ୍ଧ  
ଅଥବା

