Lesson 5: Capacitors and Inductors

COURSE CODE: EEE 201

COURSE TITLE: ELECTRICAL ENGINEERING

Introduction

- In this chapter, we shall introduce two new and important passive linear circuit elements: the capacitor and the inductor.
- Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy,
  which can be retrieved at a later time. For this reason, capacitors and inductors are called *storage* elements.
- The application of resistive circuits is quite limited. With the introduction of capacitors and inductors in this chapter, we will be able to analyze more important and practical circuits.
- We begin by introducing capacitors and describing how to combine them in series or in parallel. Later, we do the same for inductors.

Capacitor

- A capacitor is a passive element designed to store energy in its electric field.
- Used extensively in electronics, communications, computers, and power systems.

A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

- In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.
- The capacitor is said to store the electric charge. The amount of charge stored, represented by *q*, is directly proportional to the applied voltage so that

$$\boxed{q = Cv} \qquad C = \frac{\epsilon A}{d}$$

where *C*, the constant of proportionality, is known as the *capacitance* of the capacitor. The unit of capacitance is the farad (F), in honor of the English physicist Michael Faraday (1791–1867).

Capacitance is the amount of charge stored per plate for a unit voltage difference in a capacitor

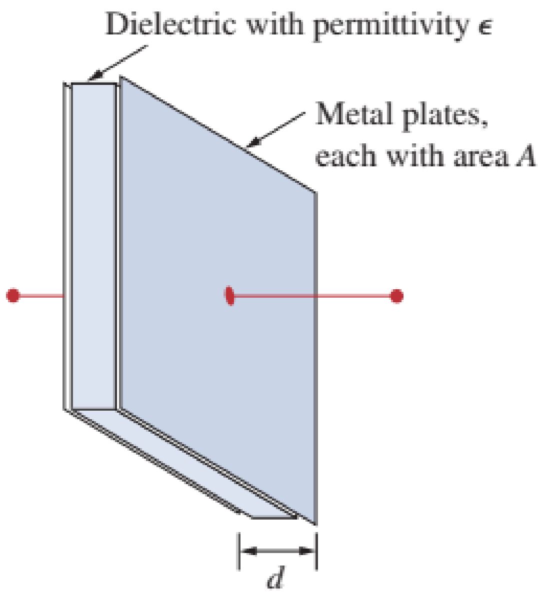


Figure 6.1
A typical capacitor.

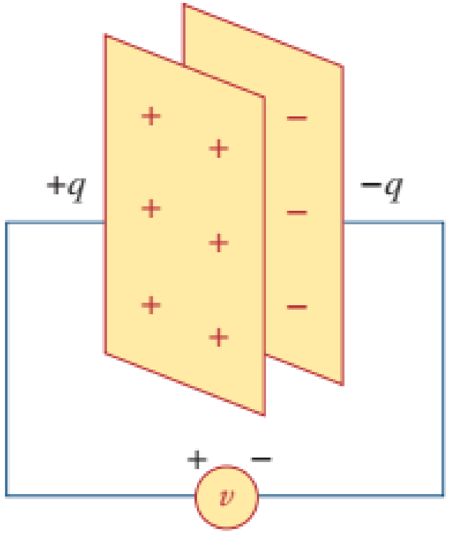
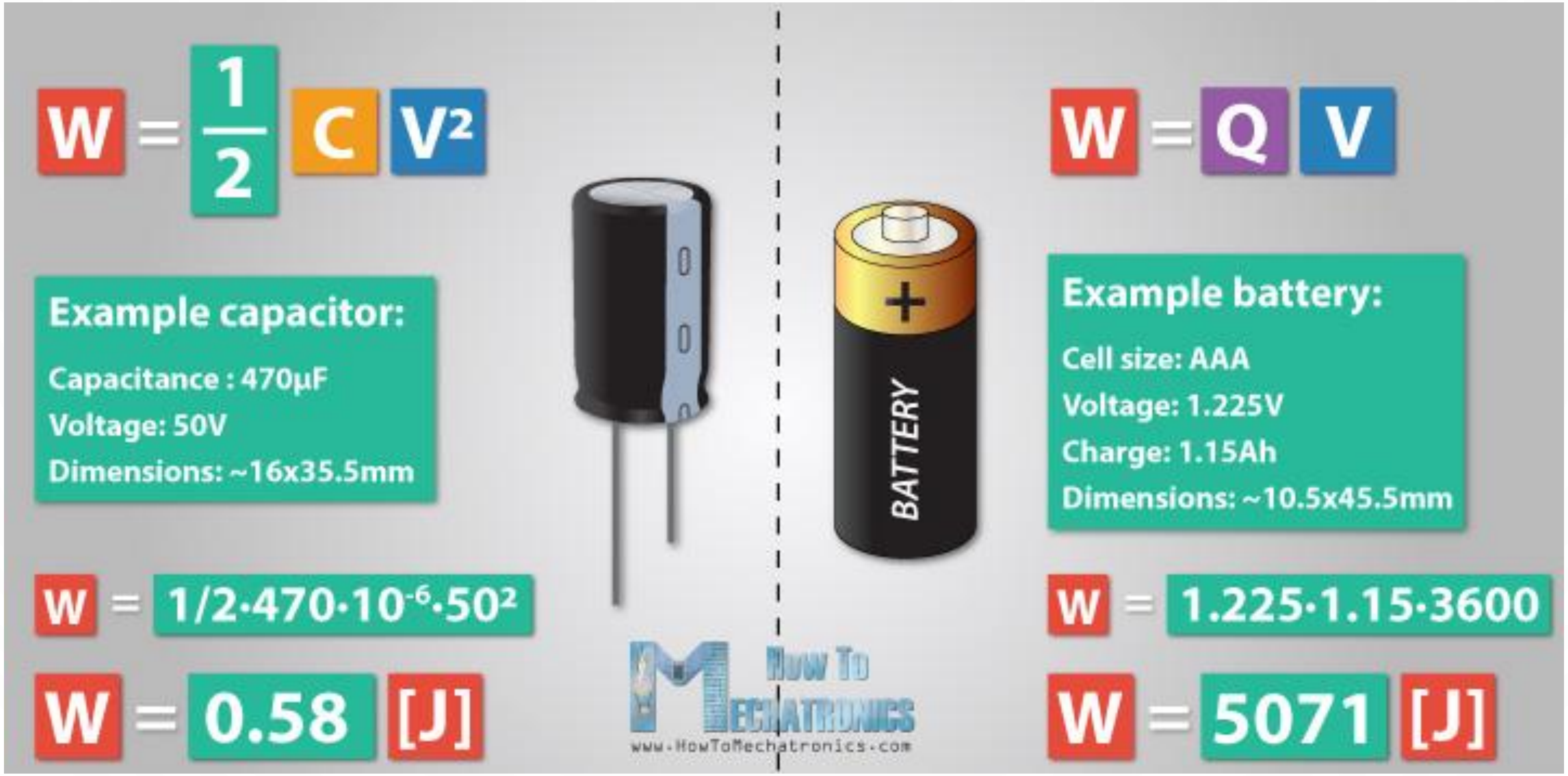


Figure 6.2
A capacitor with applied voltage v.

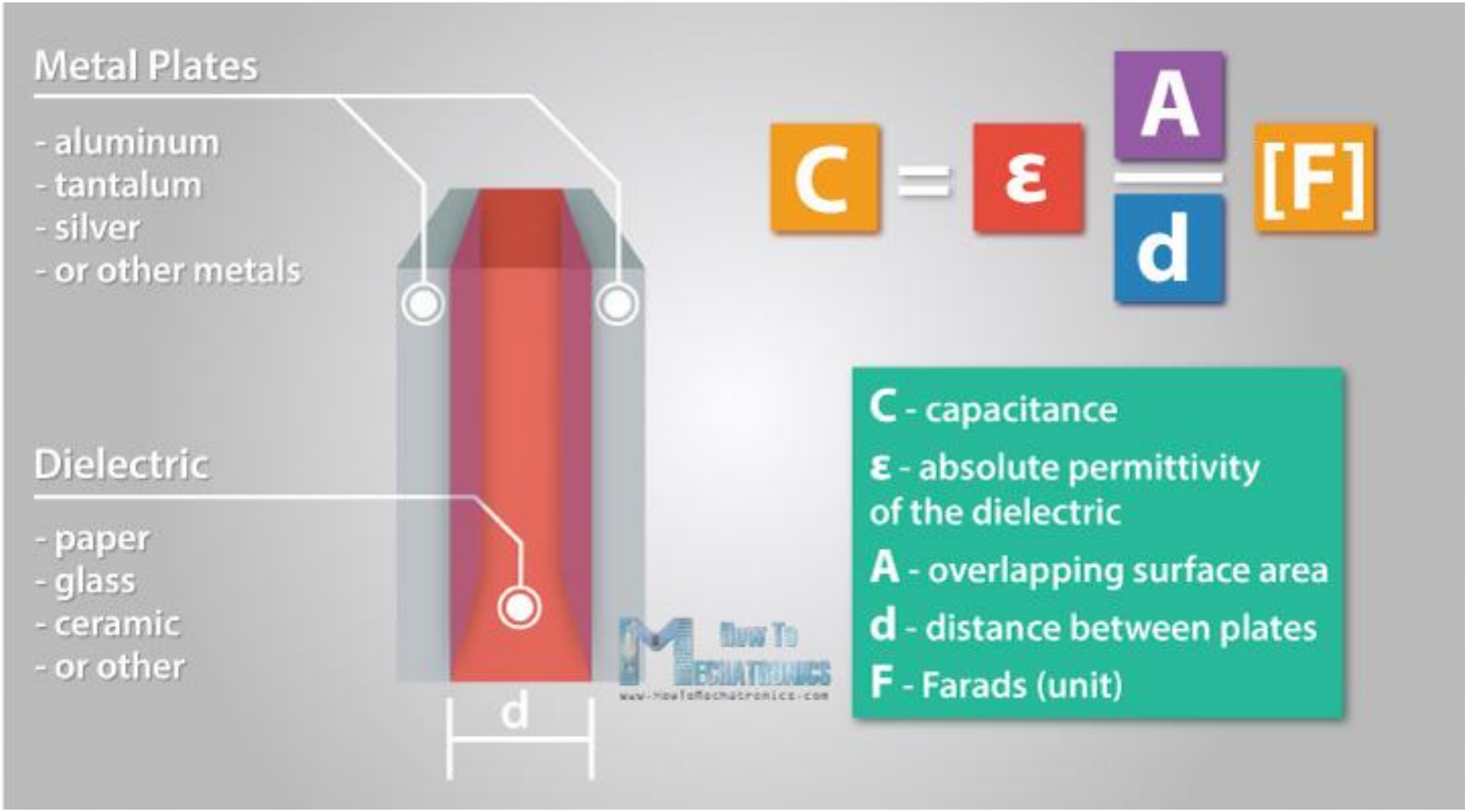
Capacitor

Compared to a same size battery, a capacitor can store much smaller amount of energy, around 10 000 times smaller, but useful enough for so many circuit designs.



Capacitor Construction

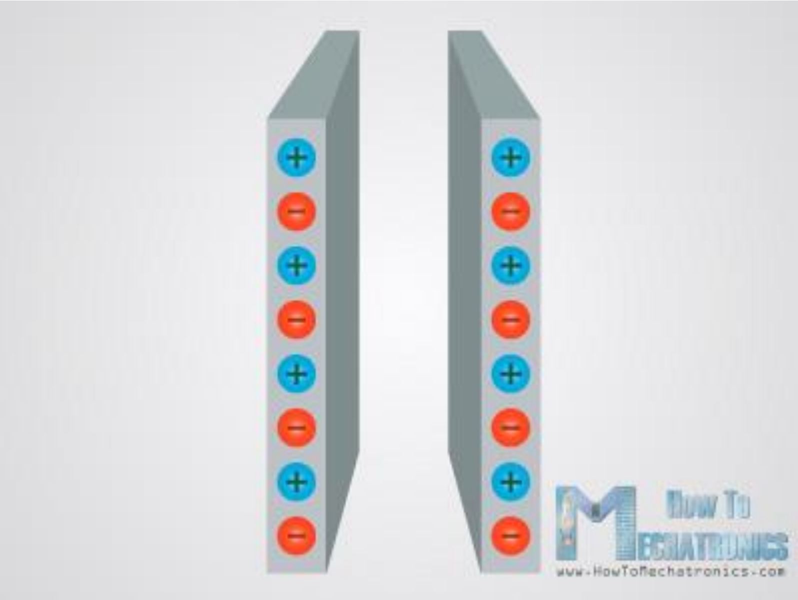
A capacitor is constructed out of two metal plates, separated by an insulating material called dielectric.

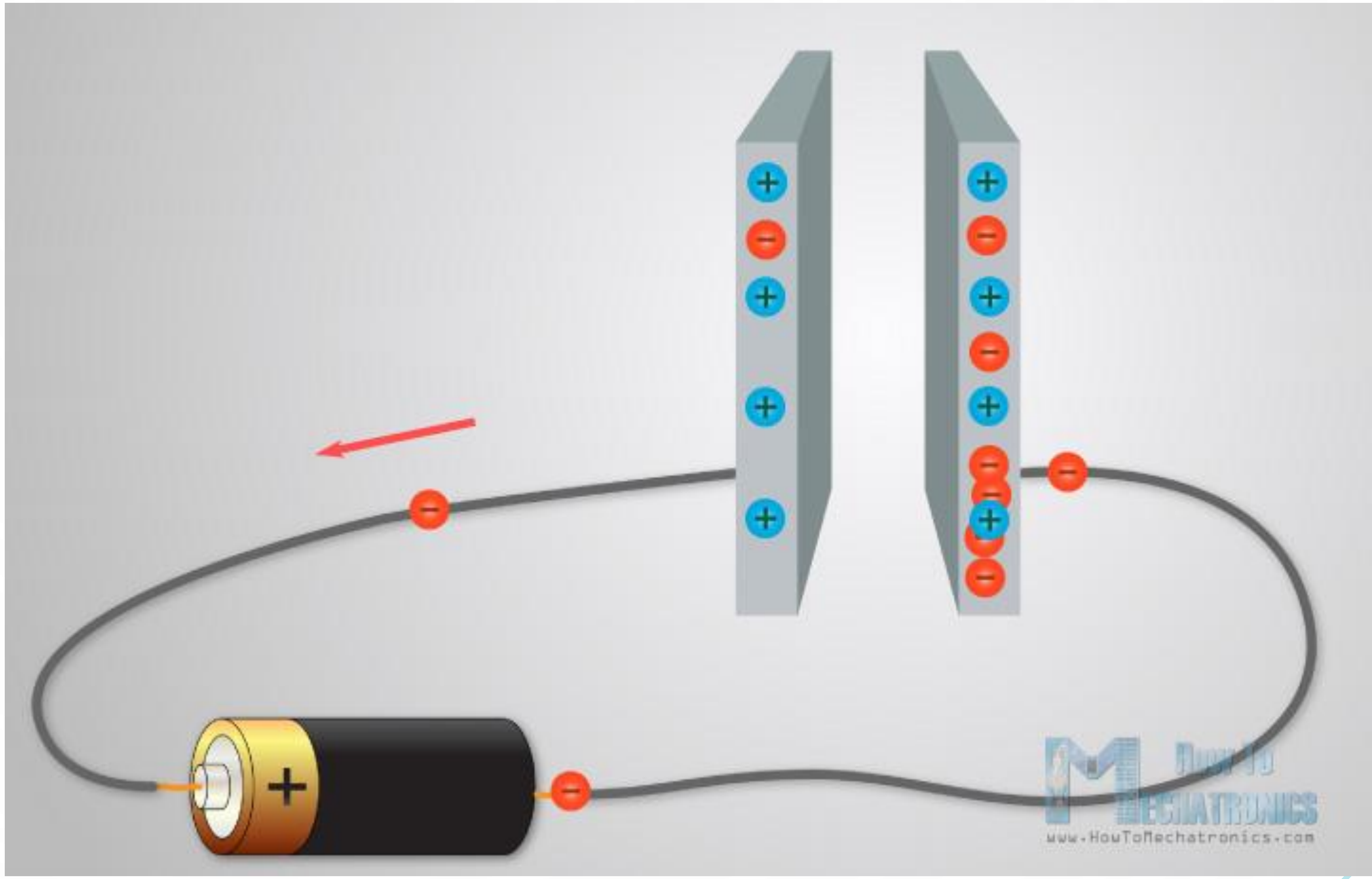


The capacitance of a capacitor is directly proportional to the surface area of the two plates, as well as the permittivity ε of the dielectric, while the smaller distance between the plates the greater capacitance.

How Capacitor Works

A metal typically has an equal amount of positively and negatively charged particles, which means it's electrically neutral.



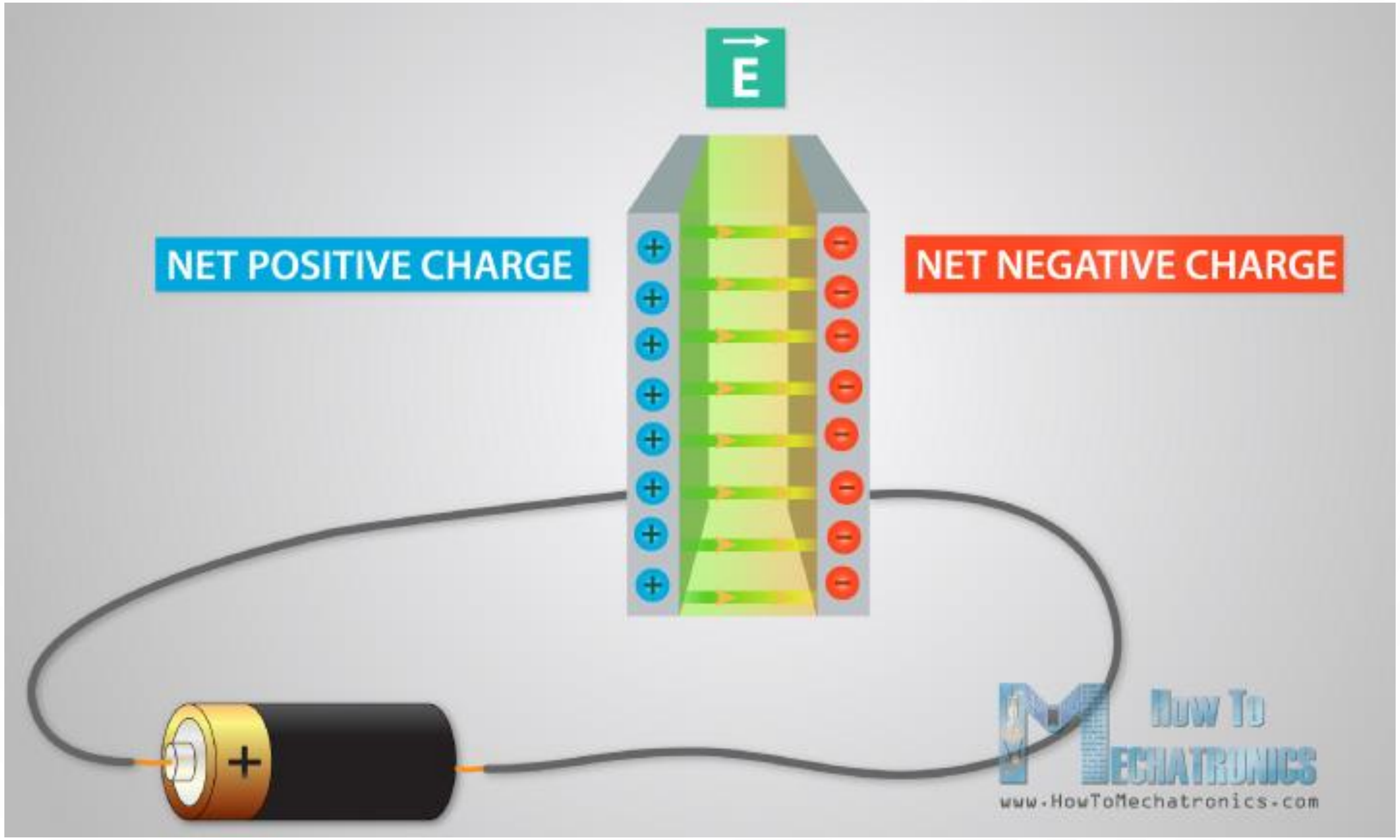


If we connect a power source or a battery to the metal plates, the electrons from the plate connected to the positive lead of the battery will start moving to the plate connected to the negative lead of the battery. However, because of the dielectric between the plates, the electrons won't be able to pass through the capacitor, so they will start accumulating on the plate.

How Capacitor Works

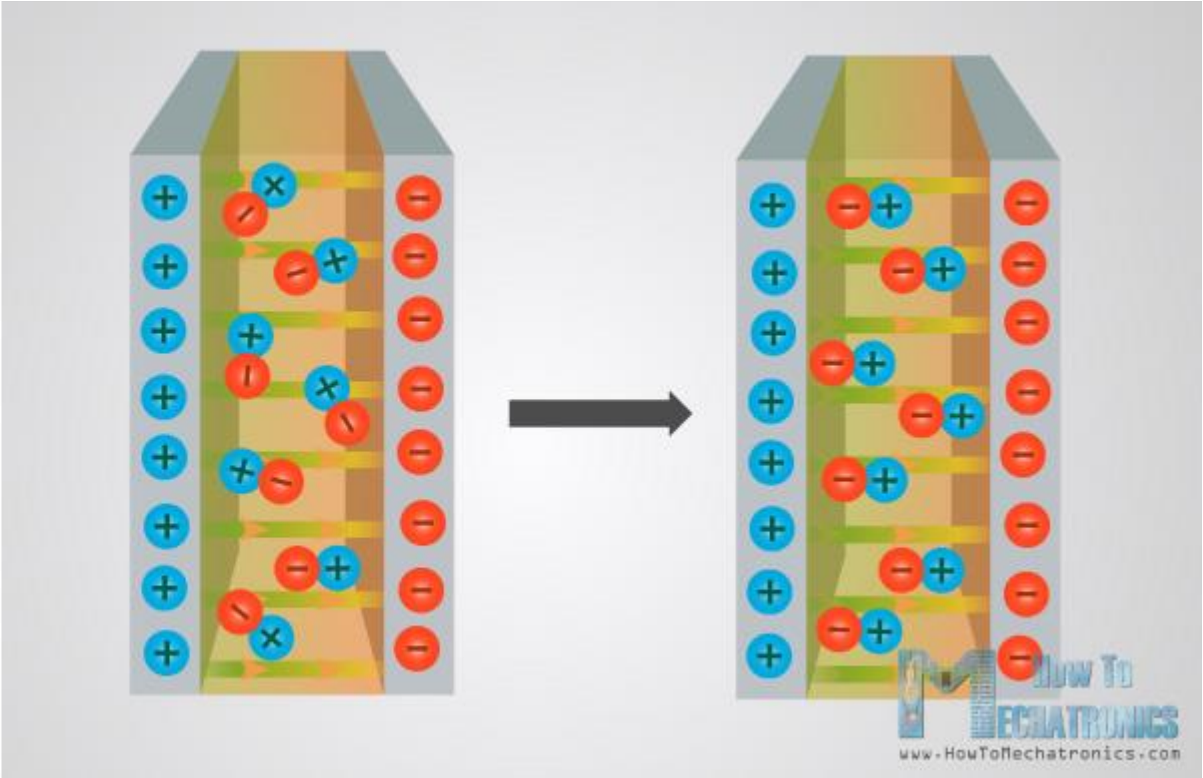
After a certain number of electronics accumulated on the plate, the battery will have insufficient energy to push any new electronics to enter the plate because of the repulsion of those electronics which are already there. At this point, the capacitor is actually fully charged.

The first plate has developed a net negative charge, and the second plate has developed an equal net positive charge, creating an electric field with an attractive force between them which holds the charge of the capacitor.



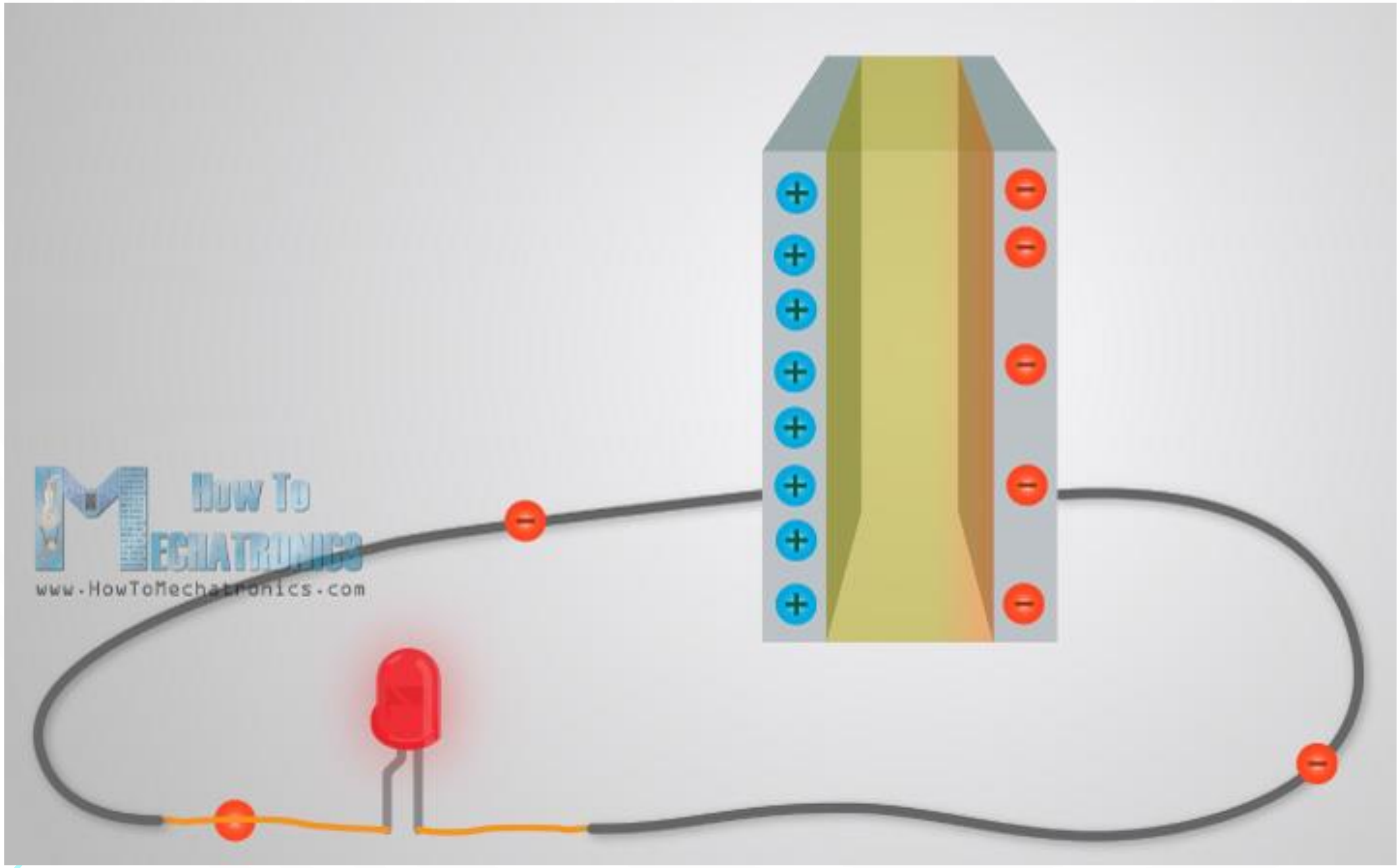
Capacitor Dielectric Working Principle

- The dielectric can increase the capacitance of the capacitor.
- A dielectric contains molecules that are polar which means that they can change their orientation based on the charges on the two plates.
- So the molecules align themselves with the electric field in such a way enabling more electrons to be attracted to the negative plate, while repelling more electrons out of the positive plate.
- So, once the it is fully charged, if we remove the battery, it will hold the electric charge for a long time, acting as energy storage.



Capacitor Dielectric Working Principle

- Now, if we shorten the two ends of the capacitor through a load, a current will start flowing through the load. The accumulated electrons from the first plate will start moving to the second plate, until both plates become back again electrically neutral.



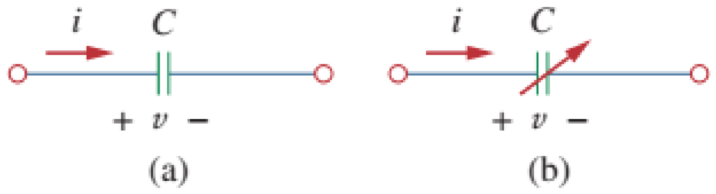
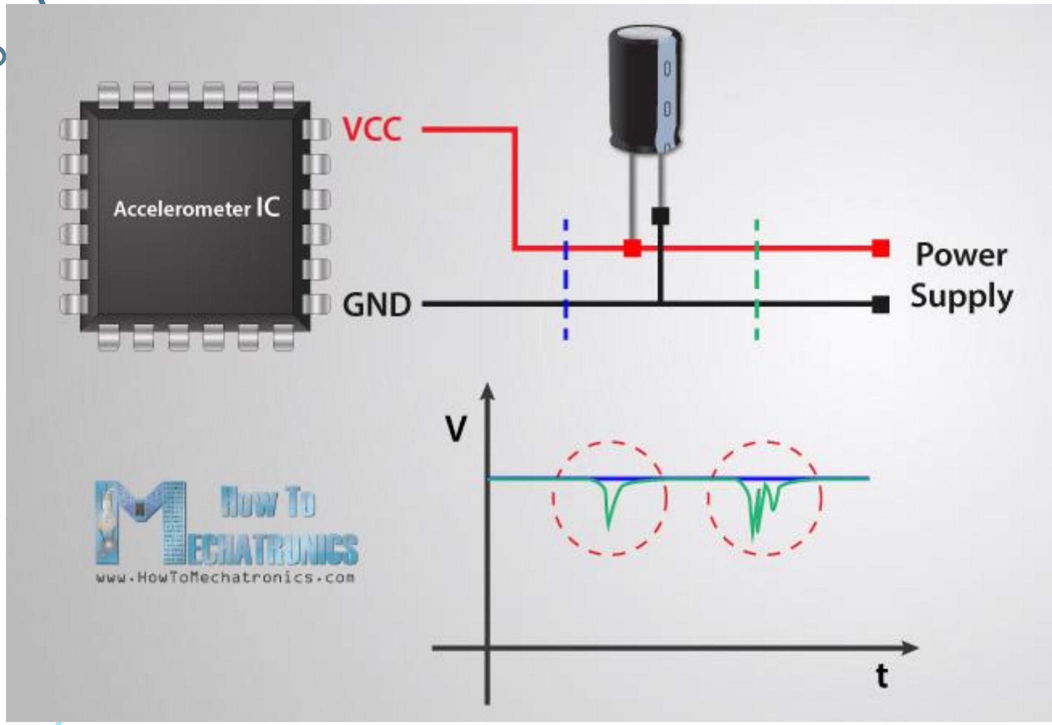


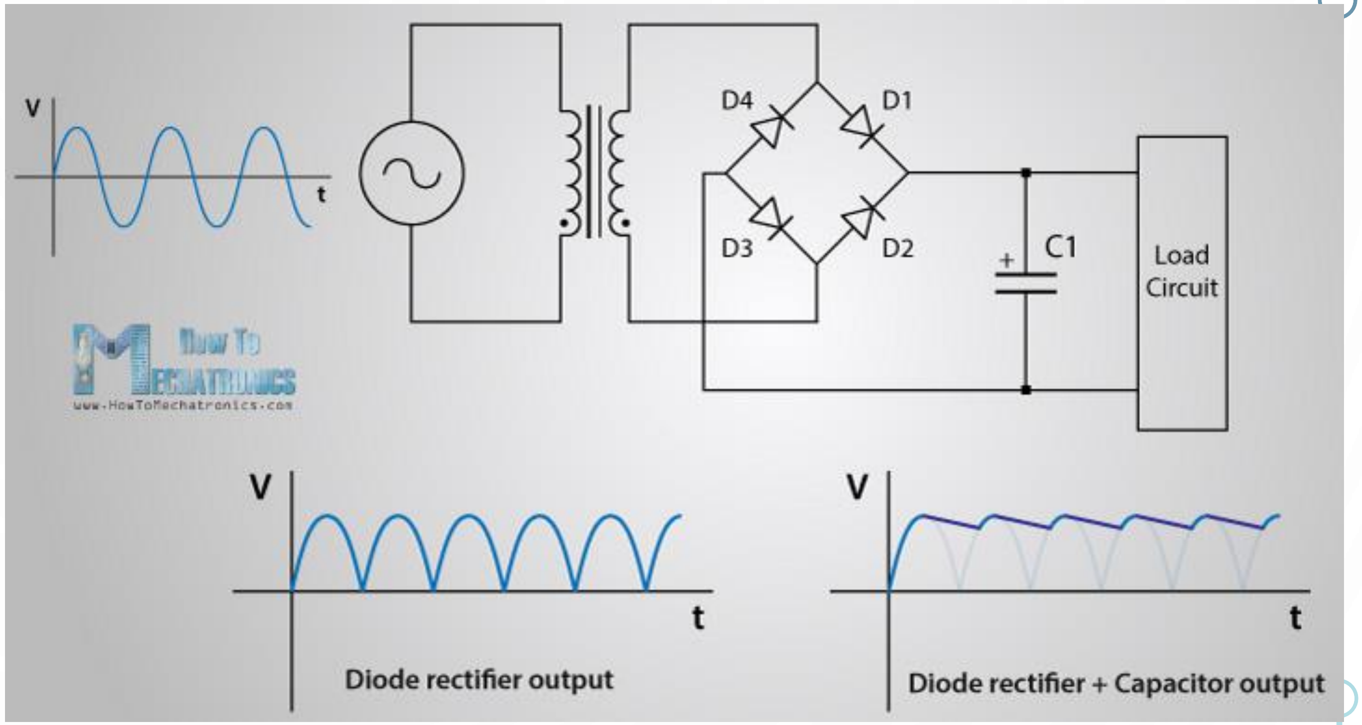
Figure 6.3
Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

Applications

Decoupling (Bypass) Capacitors



AC to DC Converter



Capacitors as Energy Storage

Another rather obvious use of the capacitors is for energy storage and supply. Although they can store considerably lower energy compared to a same size battery, their lifespan is much better and they are capable of delivering energy much faster which makes them more suitable for applications where high burst of power is needed.

	Conventional Capacitor	Supercapacitor	Lithium-ion Battery
Energy (Wh/kg)	< 0.1	1 to 10	100 to 350
Cycle Life	> 500,000	> 500,000	300 to 10,000
Charge Time	10^{-3} to 10^{-6}	0.3 to 30 s	1 to 5 h
Discharge Time	10^{-3} to 10^{-6}	0.3 to 30 s	0.3 to 3 h

Properties of Capacitor

We should note the following important properties of a capacitor:
1. Capacitor is an open circuit to dc.
2. The voltage on a capacitor cannot change abruptly.
3. The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.
4. A real, nonideal capacitor has a parallel-model leakage resistance, as shown in Fig. 6.8. The leakage resistance may be as high a 100Ω and can be neglected for most practical applications.

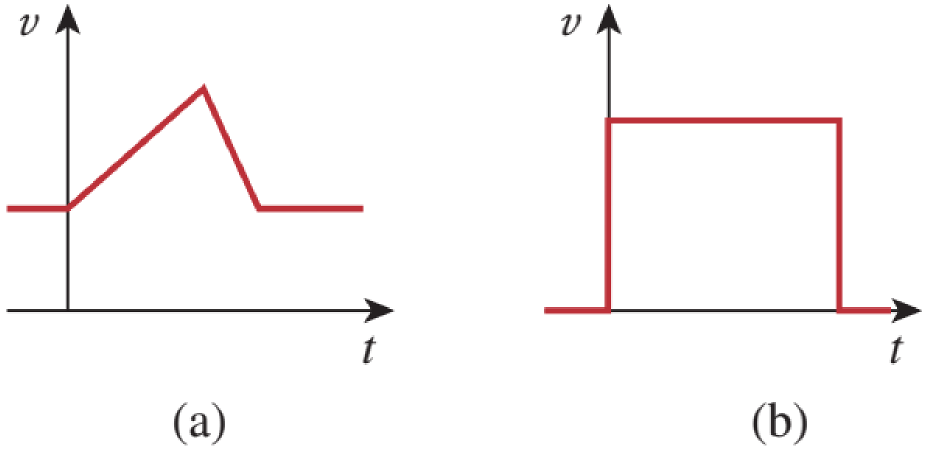


Figure 6.7
Voltage across a capacitor: (a) allowed, (b) not allowable; an abrupt change is not possible.

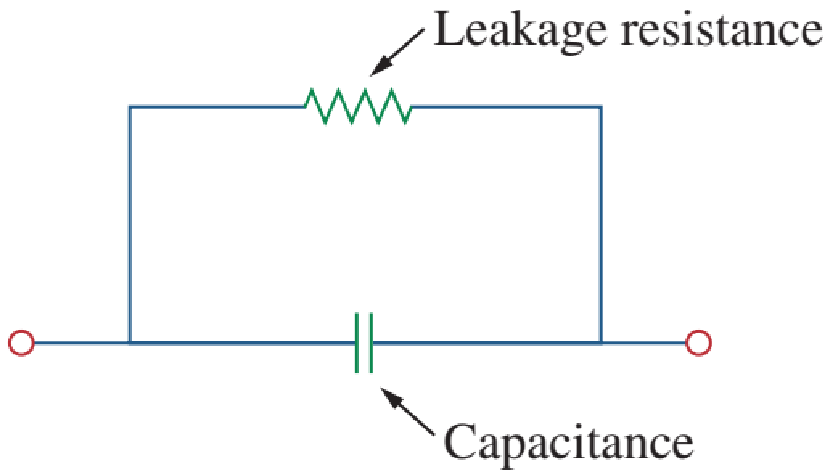
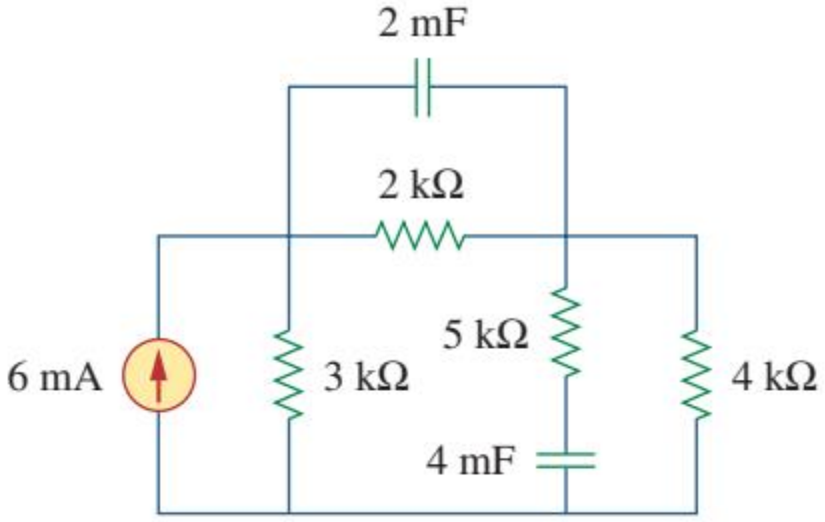


Figure 6.8
Circuit model of a nonideal capacitor.

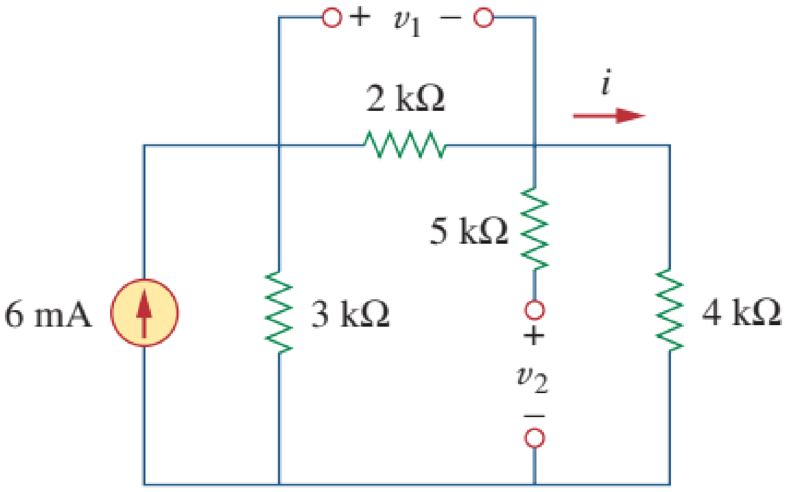
Example (B-1)

Example 6.5

Obtain the energy stored in each capacitor in Fig. 6.12(a) under dc conditions.



(a)



(b)

Figure 6.12
For Example 6.5.

Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3 + 2 + 4}(6 \text{ mA}) = 2 \text{ mA}$$

$$v_1 = 2000i = 4 \text{ V}$$
$$v_2 = 4000i = 8 \text{ V}$$

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

Series and Parallel Capacitor

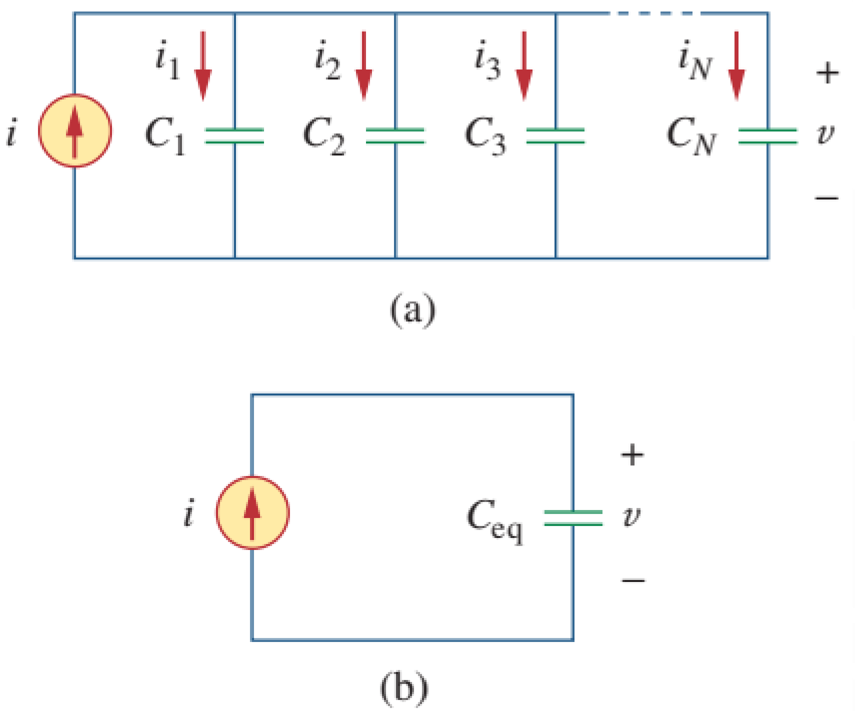


Figure 6.14
(a) Parallel-connected N capacitors, (b) equivalent circuit for the parallel capacitors.

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

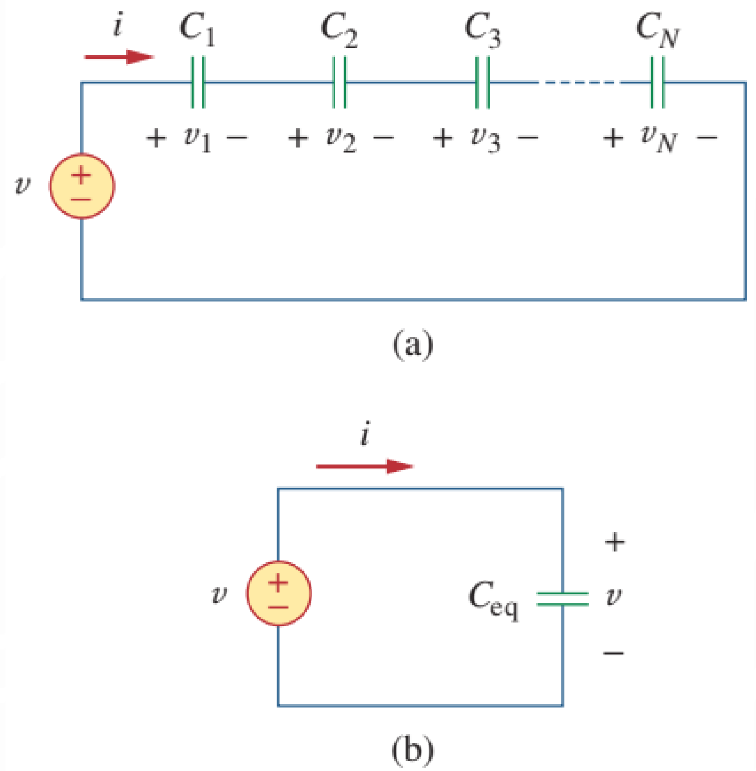


Figure 6.15
(a) Series-connected N capacitors, (b) equivalent circuit for the series capacitor.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

Series and Parallel Capacitor

For the circuit in Fig. 6.18, find the voltage across each capacitor.

Example 6.7

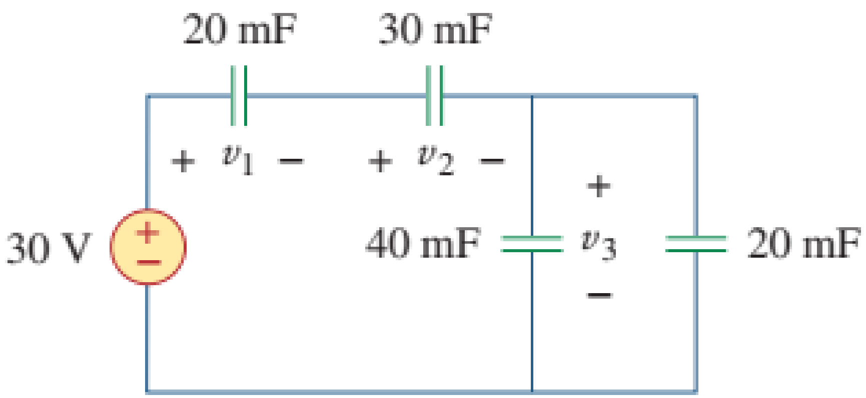


Figure 6.18
For Example 6.7.

Solution:

We first find the equivalent capacitance C_{eq}, shown in Fig. 6.19. The two parallel capacitors in Fig. 6.18 can be combined to get 40 + 20 = 60 mF. This 60-mF capacitor is in series with the 20-mF and 30-mF capacitors. Thus,

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

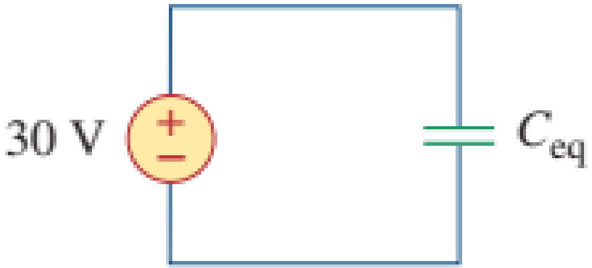


Figure 6.19
Equivalent circuit for Fig. 6.18.

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

Inductor

- An inductor is a passive element designed to store energy in its magnetic field.
- Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown in Fig. 6.21.

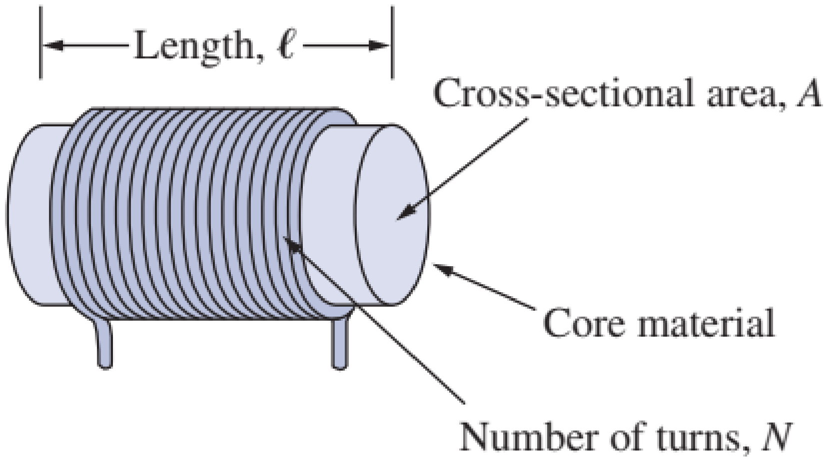


Figure 6.21
Typical form of an inductor.

An **inductor** consists of a coil of conducting wire.

$$\boxed{v = L\frac{di}{dt}}$$

where L is the constant of proportionality called the *inductance* of the inductor.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

$$L = \frac{N^2 \mu A}{\ell}$$

The energy stored is

$$\boxed{w = \frac{1}{2}Li^2}$$

Properties of Inductor

We should note the following important properties of an inductor.

1. An inductor acts like a short circuit to dc.
2. The current through an inductor cannot change instantaneously.
3. Like the ideal capacitor, the ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.
4. A practical, nonideal inductor has a significant resistive component, as shown in Fig. 6.26. This is due to the fact that the inductor is made of a conducting material such as copper, which has some resistance. This resistance is called the *winding resistance* , and it appears in series with the inductance of the inductor. The nonideal inductor also has a *winding capacitance* due to the capacitive coupling between the conducting coils.

Since an inductor is often made of a highly conducting wire, it has a very small resistance.

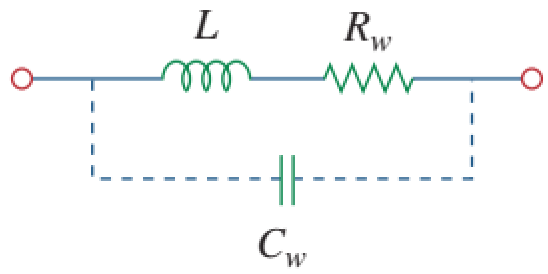


Figure 6.26
Circuit model for a practical inductor.

Example (B-1)

Example 6.10

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.

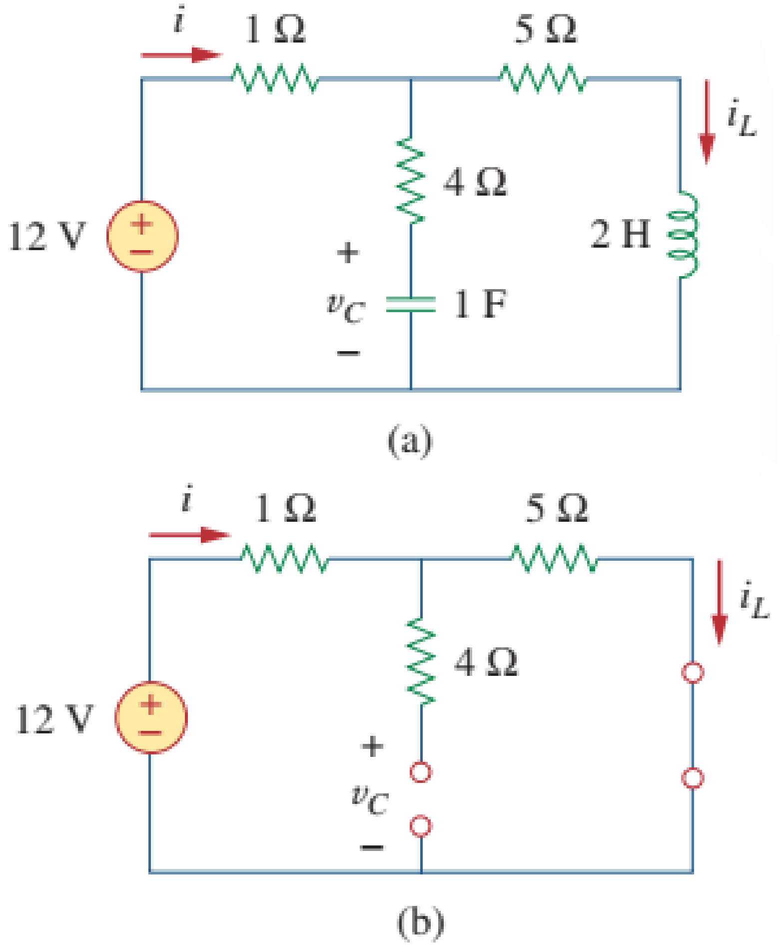


Figure 6.27
For Example 6.10.

Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in Fig. 6.27(b). It is evident from Fig. 6.27(b) that

$$i = i_L = \frac{12}{1 + 5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10 \text{ V}$$

(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

Series & Parallel Inductors

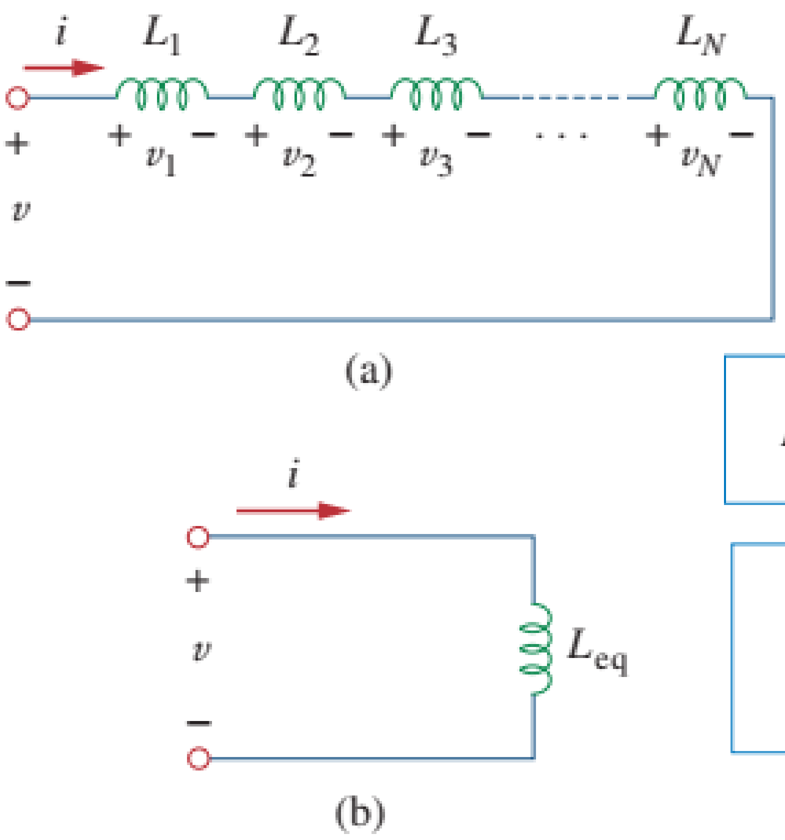


Figure 6.29
(a) A series connection of N inductors, (b) equivalent circuit for the series inductors.

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

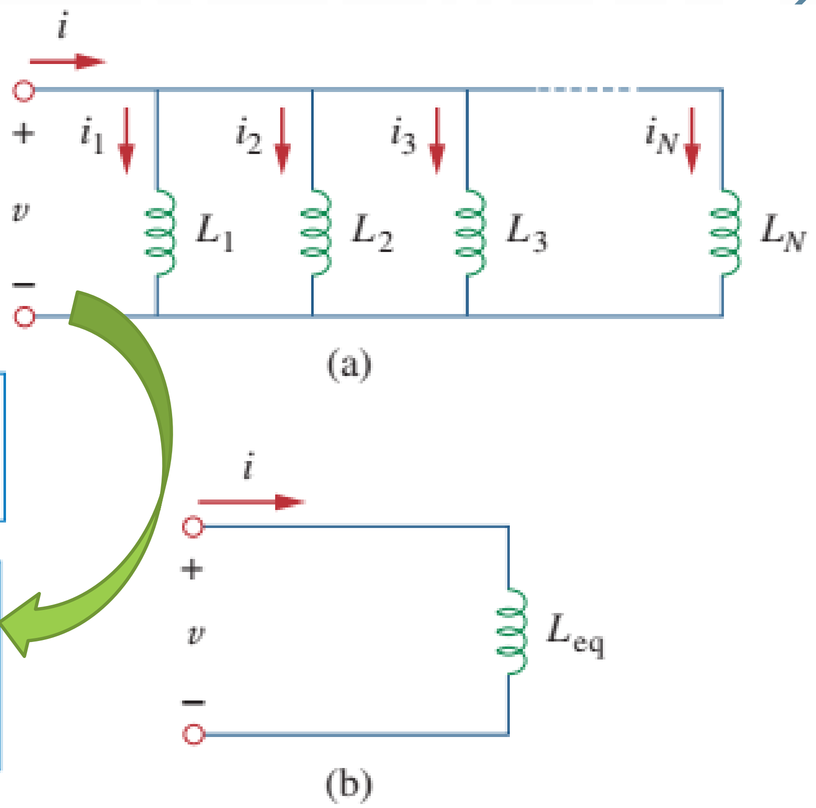


Figure 6.30
(a) A parallel connection of N inductors, (b) equivalent circuit for the parallel inductors.

Series & Parallel Inductors

Example 6.11 Find the equivalent inductance of the circuit shown in Fig. 6.31.

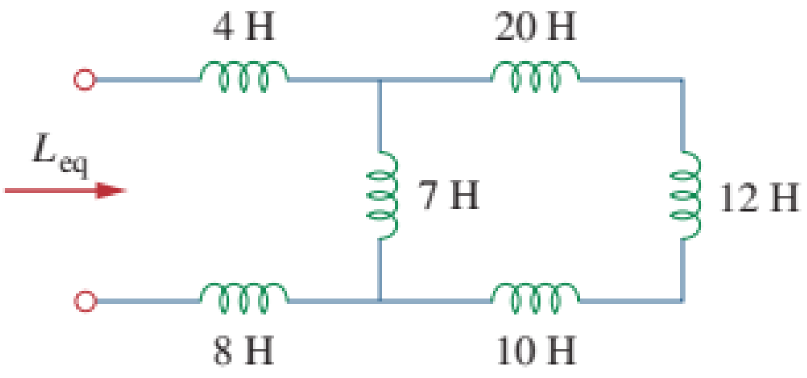


Figure 6.31
For Example 6.11.

Solution:

The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductance. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

$$\frac{7 \times 42}{7 + 42} = 6\text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

$$L_{eq} = 4 + 6 + 8 = 18\text{ H}$$

Important Characteristics

TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

END LESSON 5: CAPACITORS AND INDUCTORS

- Next Lesson…….
- Topics:
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