Chapter 05

Computer Arithmetic

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Learning Objectives

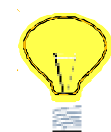
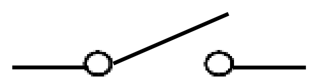
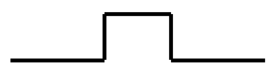
In this chapter you will learn about:

- Reasons for using binary instead of decimal numbers
- Basic arithmetic operations using binary numbers
  - Addition (+)
  - Subtraction (-)
  - Multiplication (*)
  - Division (/)

Binary over Decimal

- Information is handled in a computer by electronic/ electrical components
- Electronic components operate in binary mode (can only indicate two states – on (1) or off (0)
- Binary number system has only two digits (0 and 1), and is suitable for expressing two possible states
- In binary system, computer circuits only have to handle two binary digits rather than ten decimal digits causing:
  - Simpler internal circuit design
  - Less expensive
  - More reliable circuits
- Arithmetic rules/processes possible with binary numbers

Examples of a Few Devices that work in Binary Mode

Binary State	On (1)	Off (0)
Bulb		
Switch		
Circuit Pulse		

Binary Arithmetic

- Binary arithmetic is simple to learn as binary number system has only two digits – 0 and 1

- Following slides show rules and example for the four basic arithmetic operations using binary numbers

Binary Addition

Rule for binary addition is as follows:

0 + 0 = 0
0 + 1 = 1
1 + 0 = 1
1 + 1 = 0 plus a carry of 1 to next higher column

Binary Addition (Example 1)

Example

Add binary numbers 10011 and 1001 in both decimal and binary form

Solution

Binary	Decimal
carry 11	carry 1
10011	19
+1001	+9
11100	28

In this example, carry are generated for first and second columns

Binary Addition (Example 2)

Example

Add binary numbers 100111 and 11011 in both decimal and binary form

Solution

	Binary	Decimal
carry	11111	carry 1
	100111	39
	+11011	+27
	1000010	66

The addition of three 1s can be broken up into two steps. First, we add only two 1s giving 10 (1 + 1 = 10). The third 1 is now added to this result to obtain 11 (a 1 sum with a 1 carry). Hence, 1 + 1 + 1 = 1, plus a carry of 1 to next higher column.

Binary Subtraction

Rule for binary subtraction is as follows:

0 - 0 = 0
0- 1 = 1 with a borrow from the next column
1- 0 = 1
1 - 1 = 0

Binary Subtraction (Example)

Example

Subtract 01110_2 from 10101_2

Solution

```
  12
 0202
 10101
-01110
 _____
 00111
 _____
```

Note: Go through explanation given in the book

Complement of a Number

Number of digits in the number

$$C = B^n - 1 - N$$

Complement of the number

Base of the number

The number

Complement of a Number (Example 1)

Example

Find the complement of 37_{10}

Solution

Since the number has 2 digits and the value of base is 10,

$(\text{Base})^n - 1 = 10^2 - 1 = 99$

Now $99 - 37 = 62$

Hence, complement of $37_{10} = 62_{10}$

Complement of a Number (Example 2)

Example

Find the complement of 6_8

Solution

Since the number has 1 digit and the value of base is 8,

$(\text{Base})^n - 1 = 8^1 - 1 = 7_{10} = 7_8$

Now $7_8 - 6_8 = 1_8$

Hence, complement of $6_8 = 1_8$

Complement of a Binary Number

Complement of a binary number can be obtained by transforming all its 0's to 1's and all its 1's to 0's

Example

Complement of 1 0 1 1 0 1 0 is

↓ ↓ ↓ ↓ ↓ ↓ ↓

0 1 0 0 1 0 1

Note: Verify by conventional complement

Complementary Method of Subtraction

Involves following 3 steps:

Step 1: Find the complement of the number you are subtracting (subtrahend)

Step 2: Add this to the number from which you are taking away (minuend)

Step 3: If there is a carry of 1, add it to obtain the result; if there is no carry, recomplement the sum and attach a negative sign

Complementary subtraction is an additive approach of subtraction

Complementary Subtraction (Example 1)

Example:

Subtract 56_{10} from 92_{10} using complementary method.

Solution

Step 1: Complement of 56_{10}
$= 10^2 - 1 - 56 = 99 – 56 = 43_{10}$

Step 2: 92 + 43 (complement of 56)
= 135 (note 1 as carry)

Step 3: 35 + 1 (add 1 carry to sum)

Result = 36

The result may be verified using the method of normal subtraction:

92 - 56 = 36

Complementary Subtraction (Example 2)

Example

Subtract 35_{10} from 18_{10} using complementary method.

Solution

Step 1: Complement of 35_{10}

$= 10^2 - 1 - 35$

$= 99 - 35$

$= 64_{10}$

Step 2:

$$\begin{array}{r l} 18 & \\ +\ 64 & \text{(complement of 35)} \\ \hline 82 & \\ \hline \end{array}$$

Step 3: Since there is no carry, re-complement the sum and attach a negative sign to obtain the result.

Result $= -(99 - 82)$

$= -17$

The result may be verified using normal subtraction:

$18 - 35 = -17$

Binary Subtraction Using Complementary Method (Example 1)

Example

Subtract 0111000_2 (56_{10}) from 1011100_2 (92_{10}) using complementary method.

Solution

$$
\begin{array}{rl}
1011100 & \\
+1000111 & \text{(complement of 0111000)} \\
\hline
10100011 & \\
\hookrightarrow 1 & \text{(add the carry of 1)} \\
\hline
0100100 & \\
\hline
\end{array}
$$

Result = $0100100_2 = 36_{10}$

Binary Subtraction Using Complementary Method (Example 2)

Example

Subtract 100011_2 (35_{10}) from 010010_2 (18_{10}) using complementary method.

Solution

$$\begin{array}{r} 010010 \\ +011100 \\ \hline 101110 \\ \hline \end{array} \quad \text{(complement of 100011)}$$

Since there is no carry, we have to complement the sum and attach a negative sign to it. Hence,

Result = -010001_2 (complement of 101110_2)
= -17_{10}

Binary Multiplication

Table for binary multiplication is as follows:

0 x 0 = 0

0 x 1 = 0

1 x 0 = 0

1 x 1 = 1

Binary Multiplication (Example 1)

Example

Multiply the binary numbers 1010 and 1001

Solution

```
      1010    Multiplicand
     x1001    Multiplier
   ________
      1010    Partial Product
     0000     Partial Product
    0000      Partial Product
   1010       Partial Product
   ________
   1011010    Final Product
   ________
```

(Continued on next slide)

Binary Multiplication (Example 2)

(Continued from previous slide..)

Whenever a 0 appears in the multiplier, a separate partial product consisting of a string of zeros need not be generated (only a shift will do). Hence,

```
      1010
     x1001
  ________
      1010
   1010SS   (S = left shift)
  ________
   1011010
  ________
```

Binary Division

Table for binary division is as follows:

$1 \div 0 =$ Divide by zero error
$0 \div 1 = 0$
$1 \div 0 =$ Divide by zero error
$1 \div 1 = 1$

As in the decimal number system (or in any other number system), division by zero is meaningless

The computer deals with this problem by raising an error condition called 'Divide by zero' error

Rules for Binary Division

1. Start from the left of the dividend
2. Perform a series of subtractions in which the divisor is subtracted from the dividend
3. If subtraction is possible, put a 1 in the quotient and subtract the divisor from the corresponding digits of dividend
4. If subtraction is not possible (divisor greater than remainder), record a 0 in the quotient
5. Bring down the next digit to add to the remainder digits. Proceed as before in a manner similar to long division

Binary Division (Example 1)

Example

Divide 100001_2 by 110_2

Solution

0101 (Quotient)		
110) 100001 (Dividend)		
110	1 ←	Divisor greater than 100, so put 0 in quotient
1000	2 ←	Add digit from dividend to group used above
110	3 ←	Subtraction possible, so put 1 in quotient
100	4 ←	Remainder from subtraction plus digit from dividend
110	5 ←	Divisor greater, so put 0 in quotient
1001	6 ←	Add digit from dividend to group
110	7 ←	Subtraction possible, so put 1 in quotient
11		Remainder

Additive Method of Multiplication and Division

Most computers use the additive method for performing multiplication and division operations because it simplifies the internal circuit design of computer systems

Example

$4 \times 8 = 8 + 8 + 8 + 8 = 32$

Rules for Additive Method of Division

- Subtract the divisor repeatedly from the dividend until the result of subtraction becomes less than or equal to zero
- If result of subtraction is zero, then:
  - quotient = total number of times subtraction was performed
  - remainder = 0
- If result of subtraction is less than zero, then:
  - quotient = total number of times subtraction was performed minus 1
  - remainder = result of the subtraction previous to the last subtraction

Additive Method of Division (Example)

Example

Divide 33_{10} by 6_{10} using the method of addition

Solution:

33 - 6 = 27	
27 - 6 = 21	Since the result of the last
21 - 6 = 15	subtraction is less than zero,
15 - 6 = 9	
9 - 6 = 3	Quotient = 6 - 1 (ignore last
3 - 6 = -3	subtraction) = 5
Total subtractions = 6	Remainder = 3 (result of previous subtraction)

Key Words/Phrases

- Additive method of division
- Additive method of multiplication
- Additive method of subtraction
- Binary addition
- Binary arithmetic
- Binary division
- Binary multiplication
- Binary subtraction
- Complement
- Complementary subtraction
- Computer arithmetic