Lesson 6: Magnetic Circuits- Quantities and Definitions

EEE 1201: Electrical Engineering

Texts:

- T1: ‘Introductory Circuit Analysis’- Robert L. Boylestad, 10^{th} Edition.

Learning Objectives

After this presentation you will be able to:

- Explain how magnetic flux lines emanate from permanent magnets and produce force
- Define flux density, magnetomotive force, magnetic field intensity, permeability, and reluctance using mathematical equations.
- Perform calculations using magnetic quantities

Introduction

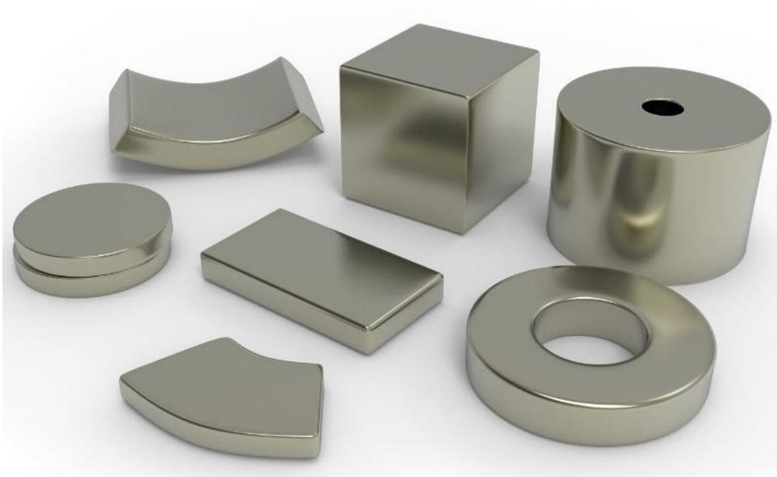


Fig. 1: The Different Types of Permanent Magnets

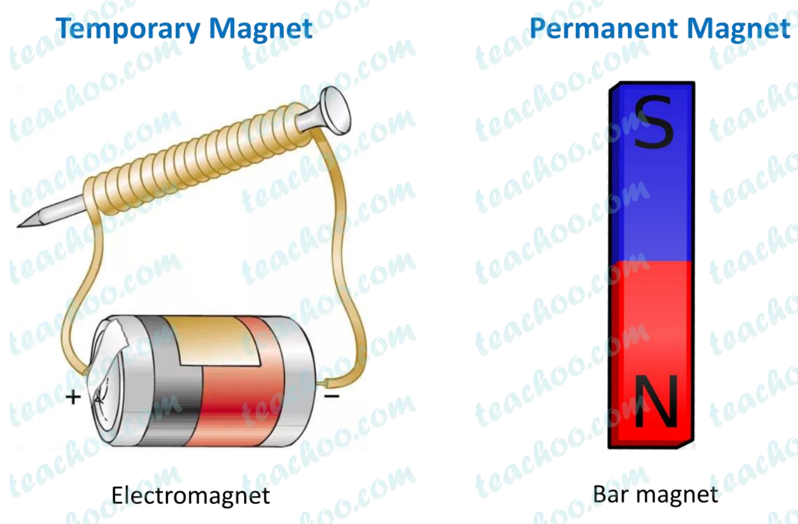


Fig. 2: Electromagnet and Bar Magnet.

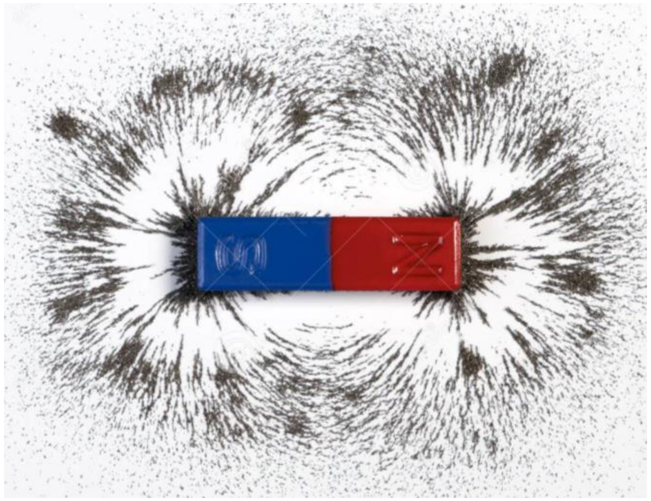


Fig. 3: Magnetic Field Around Magnets With Iron Filings

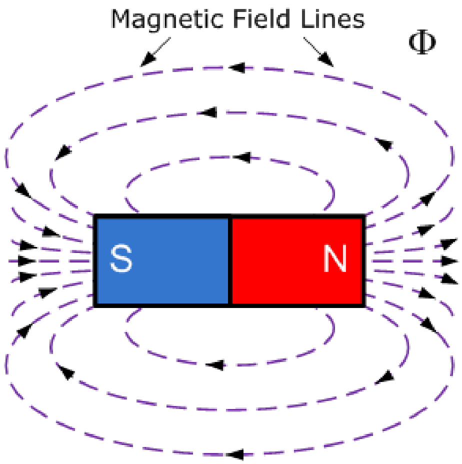


Fig. 4: Magnetic Field Lines.

Magnetic Field

Magnetic field is defined as The region around a magnet in which the force of attraction or repulsion produced by the magnet can be detected is magnetic field.

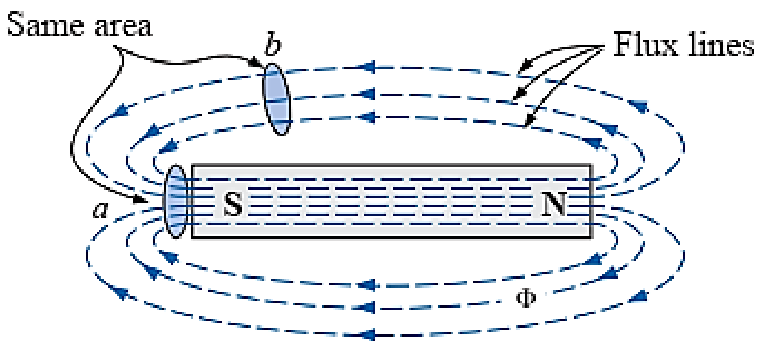


FIG. 11.1
Flux distribution for a permanent magnet.

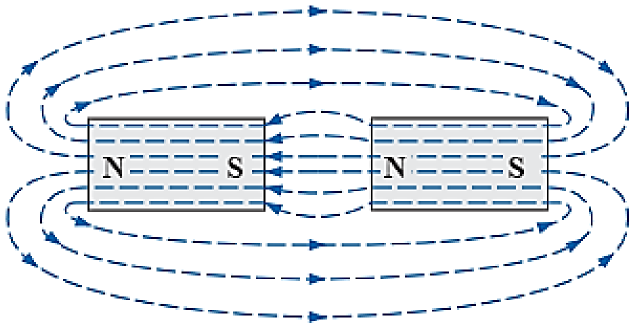


FIG. 11.2
Flux distribution for two adjacent, opposite poles.

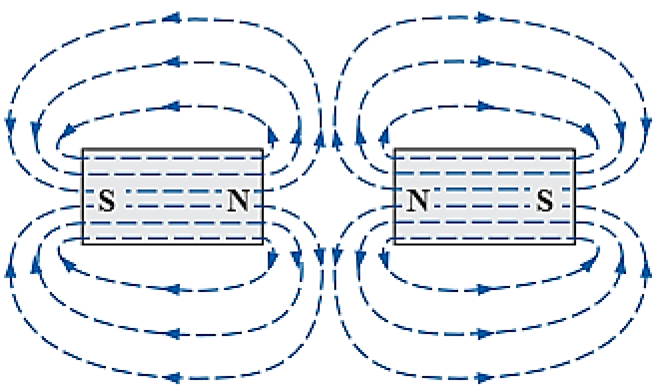


FIG. 11.3
Flux distribution for two adjacent, like poles.

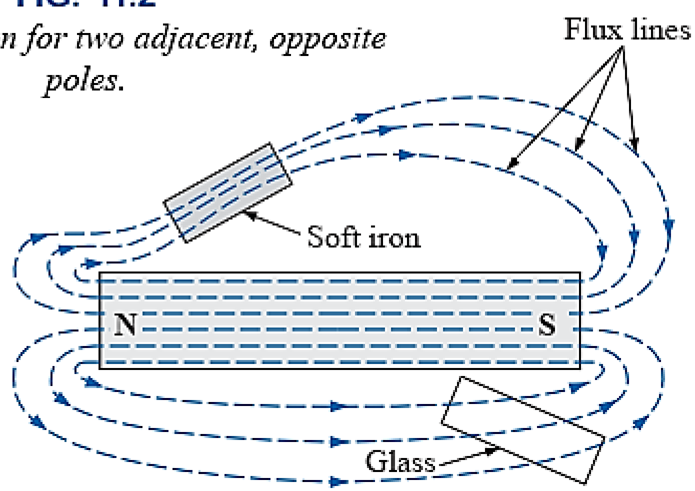


FIG. 11.4
Effect of a ferromagnetic sample on the flux distribution of a permanent magnet.

Magnetic Field

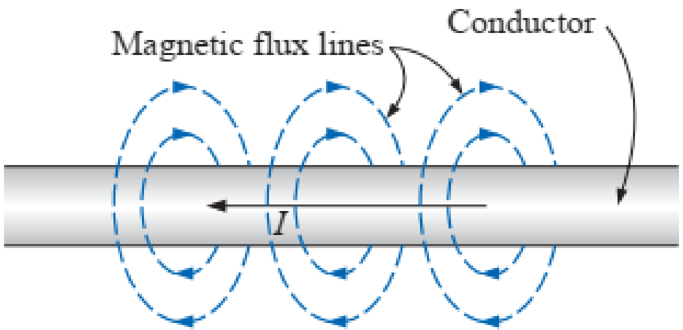


FIG. 11.6
Magnetic flux lines around a current-carrying conductor.

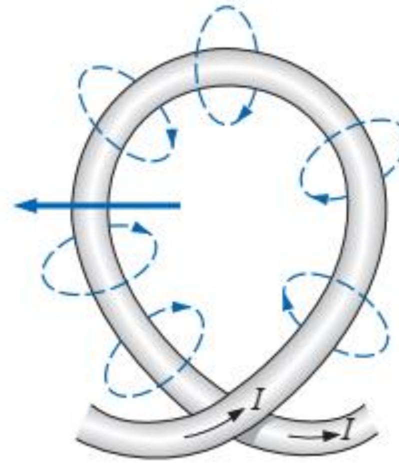


FIG. 11.7
Flux distribution of a single-turn coil.

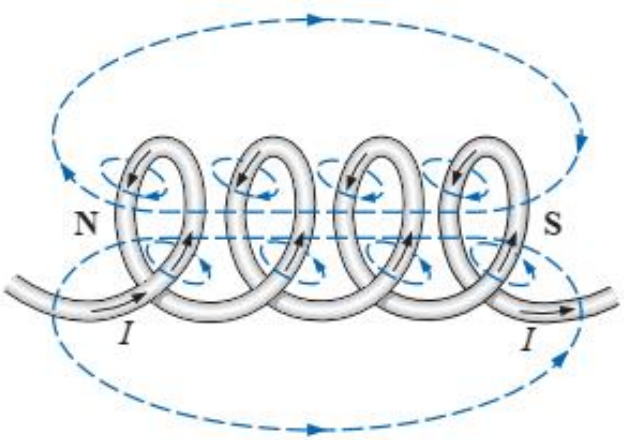


FIG. 11.8
Flux distribution of a current-carrying coil.

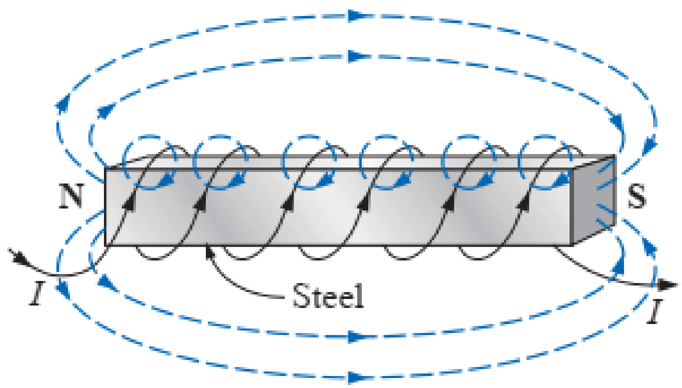
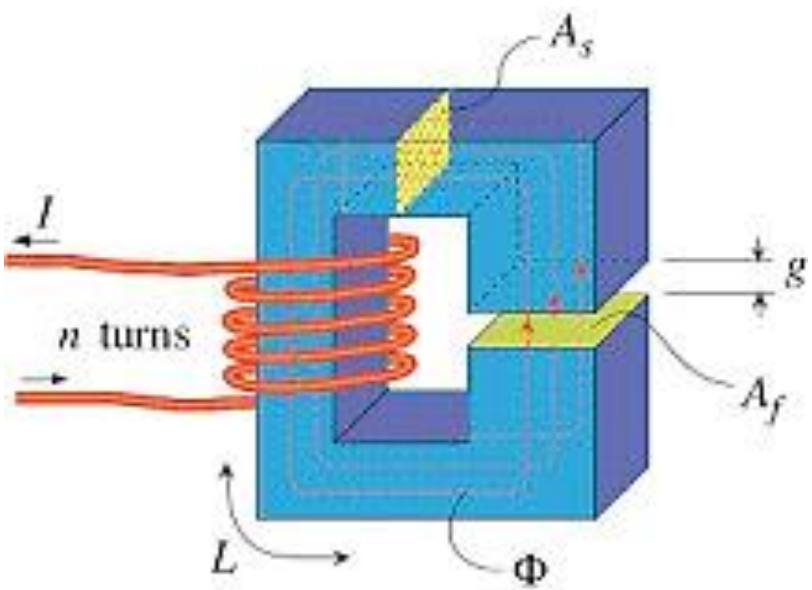


FIG. 11.9
Electromagnet.

Magnetic Circuit

A magnetic circuit is made up of one or more closed loop paths containing a magnetic flux. The flux is usually generated by permanent magnets or electromagnets and confined to the path by magnetic cores consisting of ferromagnetic materials like iron, although there may be air gaps or other materials in the path. Magnetic circuits are employed to efficiently channel magnetic fields in many devices such as electric motors, generators, transformers, relays, lifting electromagnets, SQUIDs, galvanometers, and magnetic recording heads.



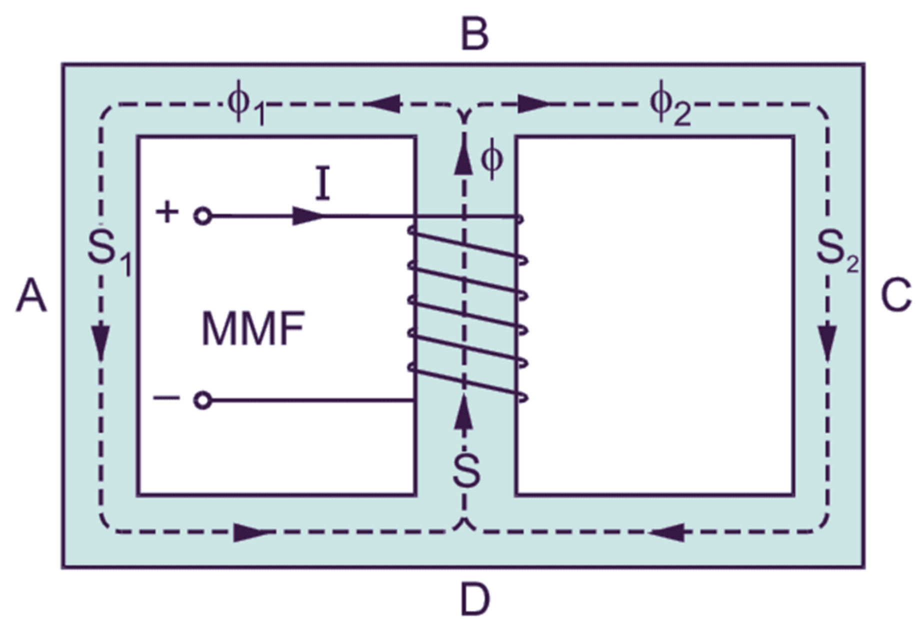


Fig. 5: Example of Magnetic Circuits.

Flux Density

In the SI system of units, magnetic flux is measured in *webers* (note Fig. 11.12) and has the symbol Φ. The number of flux lines per unit area is called the **flux density,** is denoted by the capital letter B, and is measured in *teslas* (note Fig. 11.15). Its magnitude is determined by the following equation:

$$\boxed{B = \frac{\Phi}{A}}$$

B = teslas (T)
Φ = webers (Wb)
A = square meters (m^2)

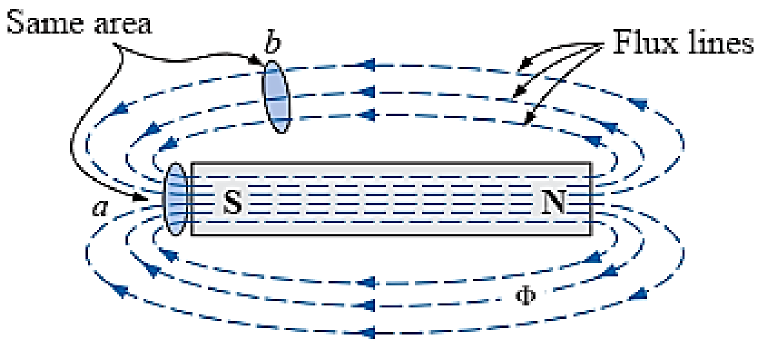


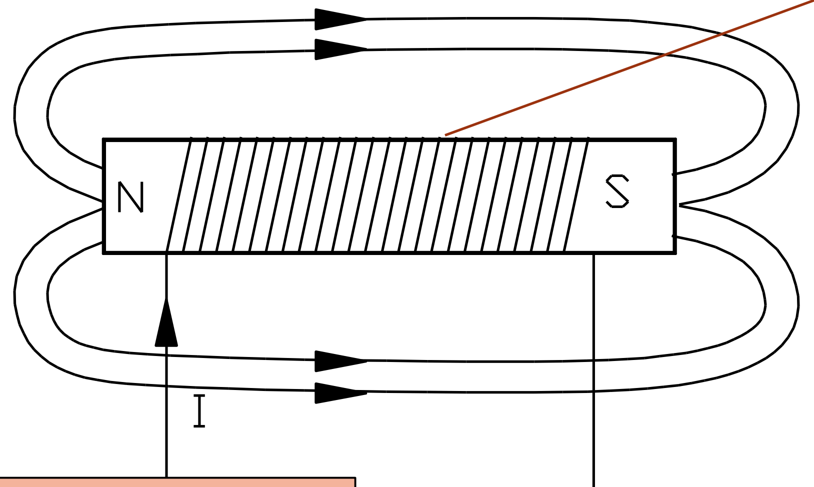
FIG. 11.1
Flux distribution for a permanent magnet.

where Φ is the number of flux lines passing through the area A (Fig. 11.13). The flux density at position a in Fig. 11.1 is twice that at b because twice as many flux lines are passing through the same area.
By definition,

Magnetomotive Force (MMF)

Current through a coil creates magnetic flux. The greater the current or the greater the number of turns, the greater will be the flux. This flux-producing ability of a coil is called its magnetomotive force (mmf) and is measured in ampere-turns.

Given a coil



N = number of turns in coil

Magnetomotive Force (MMF) $\mathscr{F}$

$\mathscr{F} = NI$

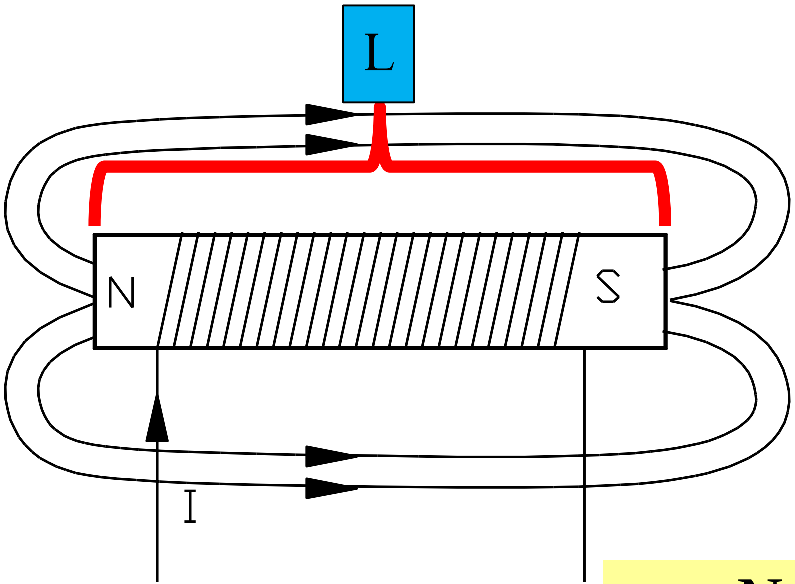
Where $\mathscr{F}$ = MMF (A-t)

N = number of turns in coil (t)

I = current in coil (A)

The magnetic pressure, which sets up the magnetic flux in a magnetic circuit is called Magnetomotive Force.

Magnetic Field Intensity



Magnetic field intensity, H:

A measure of the "effort" that a current is putting into the establishment of a magnetic field.

Mathematically

$$H = \frac{N \cdot I}{L} = \frac{\mathcal{F}}{L}$$

Where: H = Magnetic field intensity (Oersteds, A-t/m)

L = path length (meters)

Ampere's Circuital Law

One of the key relationships in magnetic circuit theory is **Ampere's circuital law,** Equation 12–10. This law was determined experimentally and is a generalization of the relationship $\mathscr{F} = NI = H\ell$ that we developed earlier. Ampere showed that the algebraic sum of mmfs around a closed loop in a magnetic circuit is zero, regardless of the number of sections or coils. That is,

$$\sum_{\circlearrowright} \mathscr{F} = 0 \qquad \textbf{(12–10)}$$

Applying Ampère's circuital law, we have

$$\Sigma_{\circlearrowright}\ \mathscr{F} = 0$$

$$\underbrace{+NI}_{\text{Rise}} - \underbrace{H_{ab}l_{ab}}_{\text{Drop}} - \underbrace{H_{bc}l_{bc}}_{\text{Drop}} - \underbrace{H_{ca}l_{ca}}_{\text{Drop}} = 0$$

$$\underbrace{NI}_{\substack{\text{Impressed}\\ \text{mmf}}} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{\text{mmf drops}}$$

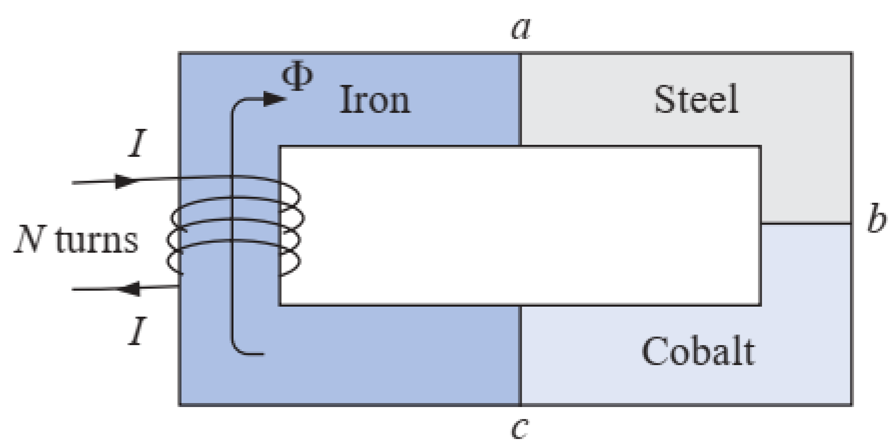


FIG. 11.27

Series magnetic circuit of three different materials.

Magnetic Permeability

Permeability - Amount of magnetic field intensity required to produce a given flux density for a given material.

Mathematically $$\mu = \frac{B}{H}$$

Where: B = flux density (Wb/m^2)
H = magnetic field intensity (A-t/m)
μ = permeability (Wb/A-t-m)

Materials in which flux lines can readily be set up are said to be magnetic and to have high permeability. The permeability (m) of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material.

Characteristics:

- similar to resistivity in conductors
- not a constant for a given material
- larger μ, less H required to produce given B

Magnetic Permeability

Relative permeability - Ratio of material's permeability to that of free space

$$\mu_r = \frac{\mu}{\mu_0}$$

Where:

μ_0 = permeability of free space ($4\pi \times 10^{-7}$ Wb/A-t-m)

μ = permeability of material (Wb/A-t-m)

μ_r = relative permeability (dimensionless)

Substances	Relative magnetic permeability
Air	About 1
Copper	About 1
Aluminum	About 1
Nickel	About 600
MnZn ferrite	About 2,000
Iron	About 5,000
Pure iron	About 200,000

Magnetization Curve (B-H curve)

The B-H curve is a graphical representation that describes the magnetic properties of a material. It shows the relationship between the magnetic field strength (H) and the magnetic flux density (B) of a material.

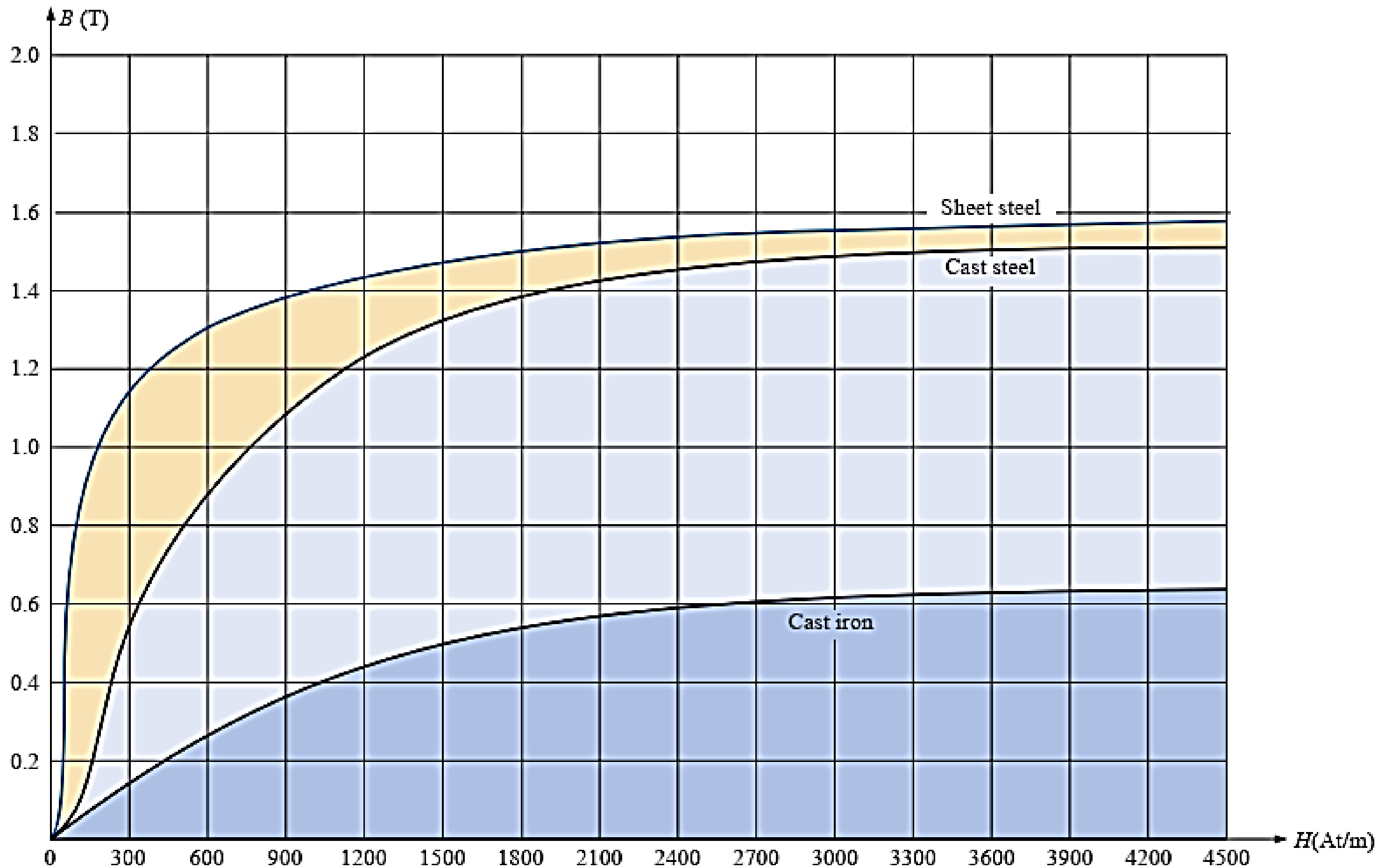
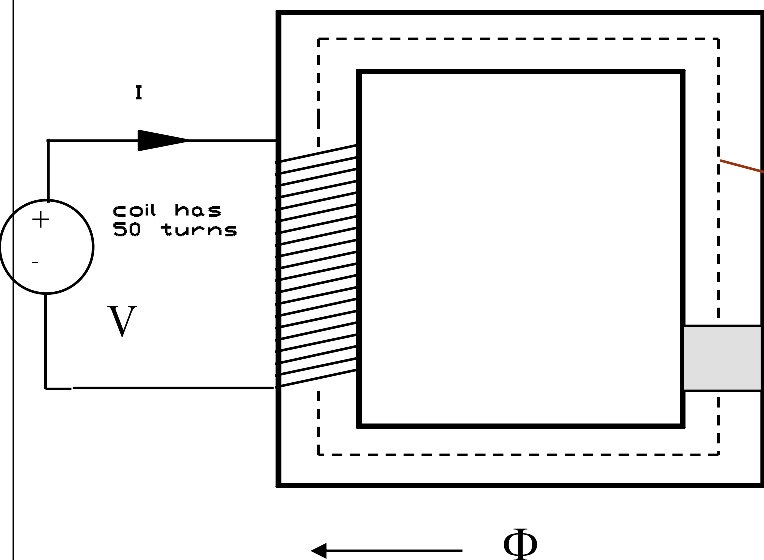


FIG. 11.23

Normal magnetization curve for three ferromagnetic materials.

Reluctance of Magnetic Circuits

Reluctance –opposition to flux. Similar to R in dc electric circuit



Ferromagnetic core (iron)

L = mean path length (m)

Core has cross sectional area A (m^2) = LxW

L

W

Magnetic Circuit Relationship

$$\Phi = \frac{\mathfrak{F}}{\Re} = \frac{N \cdot I}{\Re}$$

So

$$\Re = \frac{\mathfrak{F}}{\Phi}$$

Where: $\mathfrak{F}$ = MMF (A-t)
$\Re$ = Reluctance (A-t/Wb)
Φ = flux (Wb)
N = coil turns (t)
I = coil current (A)

Reluctance of Magnetic Circuits

Coil reluctance related to core geometry and material

From previous math relationships

$$\mathscr{F} = H \cdot L$$

$$\Phi = B \cdot A$$

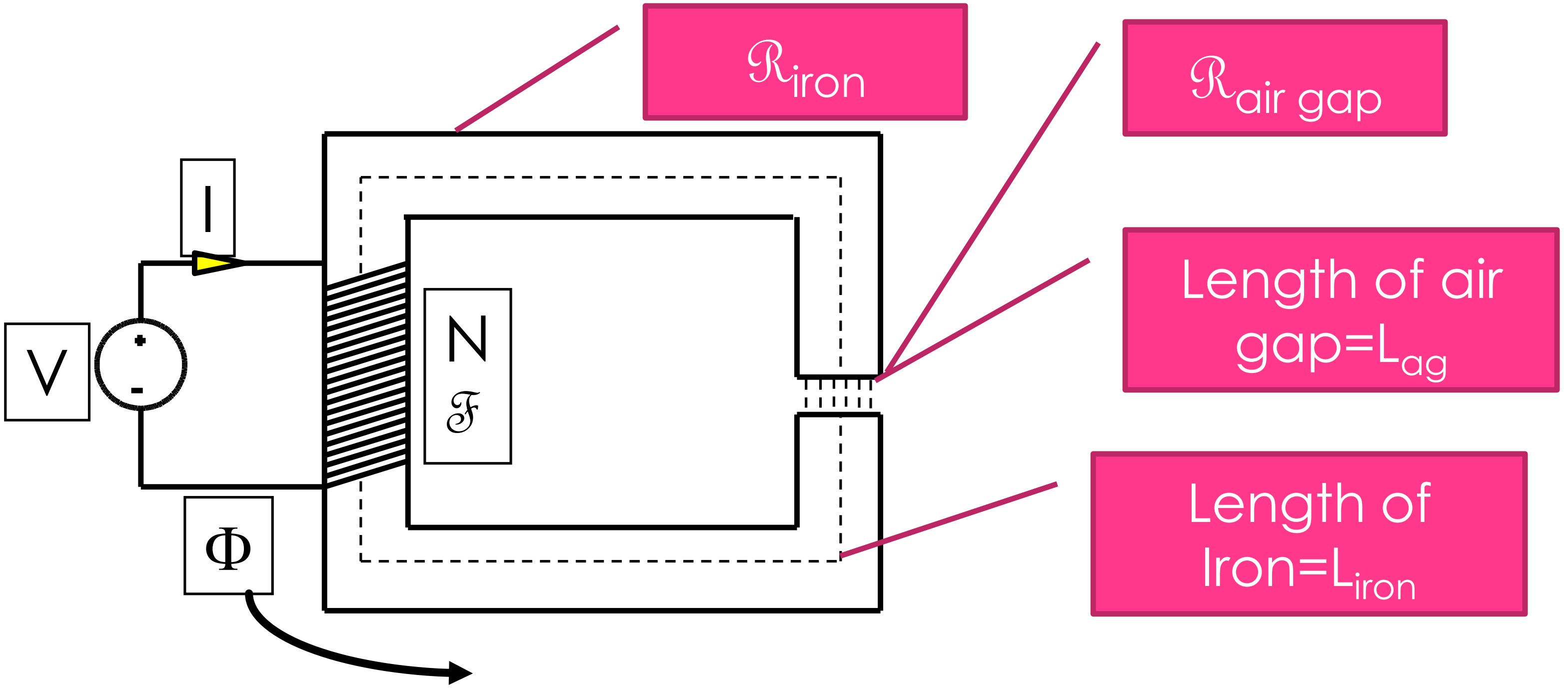
Substitute into equation from last slide and simplify

$$\mathfrak{R} = \frac{L}{\left(\frac{B}{H}\right) \cdot A}$$

$$\mathfrak{R} = \frac{L}{\mu \cdot A}$$

$$\mathfrak{R} = \frac{L}{\mu_0 \cdot \mu_r}$$

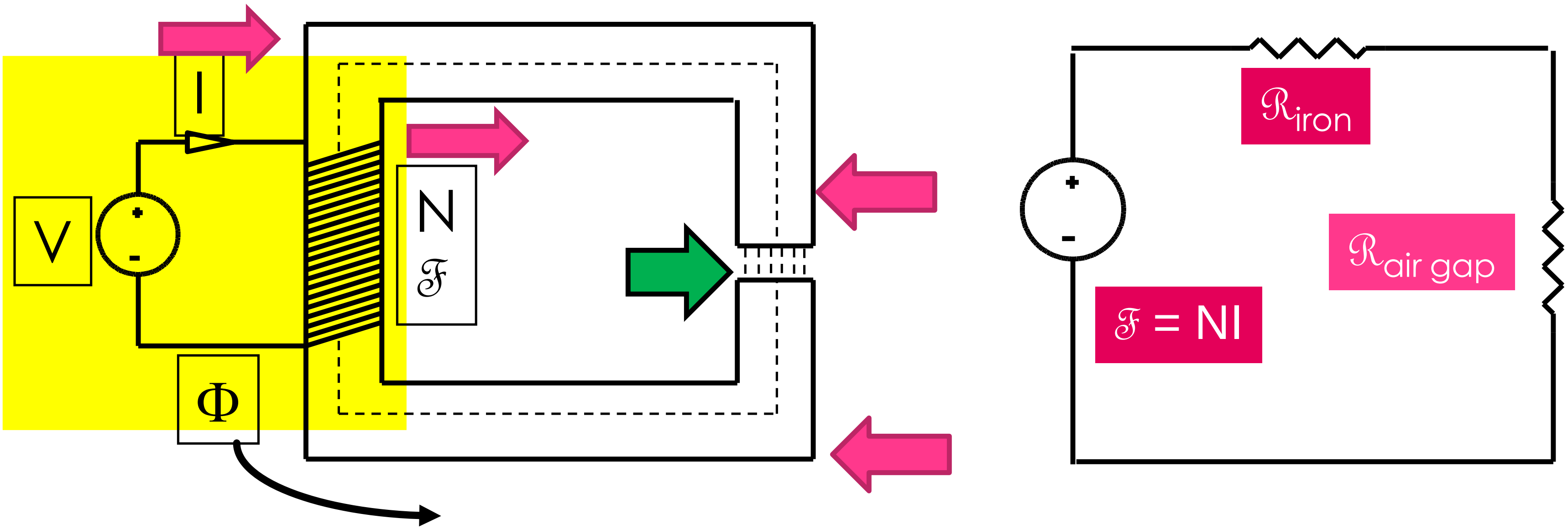
Solving Magnetic Circuits



$$\mathcal{R}_{ag} = \frac{L_{ag}}{\mu_0 \cdot A}$$

$$\mathcal{R}_{iron} = \frac{L_{iron}}{\mu_{iron} \cdot A}$$

Solving Magnetic Circuits



Voltage source and coil form MMF source

Iron in magnetic structure has reluctance

Air gap has reluctance

Series Magnetic Circuits

EXAMPLE 11.3 For the series magnetic circuit of Fig. 11.29:

a. Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$ Wb.

b. Determine μ and μ_r for the material under these conditions.

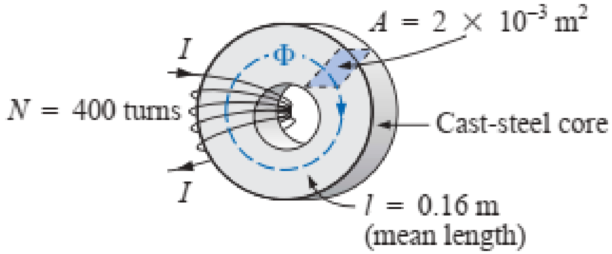


FIG. 11.29
Example 11.3.

Solutions: The magnetic circuit can be represented by the system shown in Fig. 11.30(a). The electric circuit analogy is shown in Fig. 11.30(b). Analogies of this type can be very helpful in the solution of magnetic circuits. Table 11.2 is for part (a) of this problem. The table is fairly trivial for this example, but it does define the quantities to be found.

a. The flux density B is

$$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4}\ \text{Wb}}{2 \times 10^{-3}\ \text{m}^2} = 2 \times 10^{-1}\ \text{T} = 0.2\ \text{T}$$

Using the B-H curves of Fig. 11.24, we can determine the magnetizing force H:

$$H\ (\text{cast steel}) = 170\ \text{At/m}$$

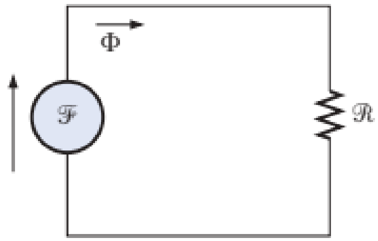
Will be provided in the question.

Applying Ampère's circuital law yields

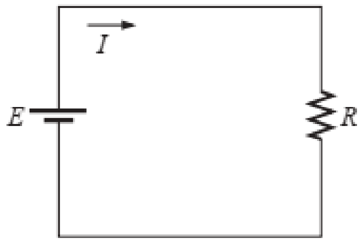
$$NI = Hl$$

and

$$I = \frac{Hl}{N} = \frac{(170\ \text{At/m})(0.16\ \text{m})}{400\ \text{t}} = \mathbf{68\ mA}$$



(a)



(b)

FIG. 11.30
(a) Magnetic circuit equivalent and (b) electric circuit analogy.

Series Magnetic Circuits

EXAMPLE 11.3 For the series magnetic circuit of Fig. 11.29:

a. Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$ Wb.

b. Determine μ and μ_r for the material under these conditions.

Solutions: The magnetic circuit can be represented by the system shown in Fig. 11.30(a). The electric circuit analogy is shown in Fig. 11.30(b). Analogies of this type can be very helpful in the solution of magnetic circuits. Table 11.2 is for part (a) of this problem. The table is fairly trivial for this example, but it does define the quantities to be found.

b. The permeability of the material can be found using Eq. (11.8):

$$\mu = \frac{B}{H} = \frac{0.2\ \text{T}}{170\ \text{At/m}} = \mathbf{1.176 \times 10^{-3}\ Wb/A{\cdot}m}$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{935.83}$$

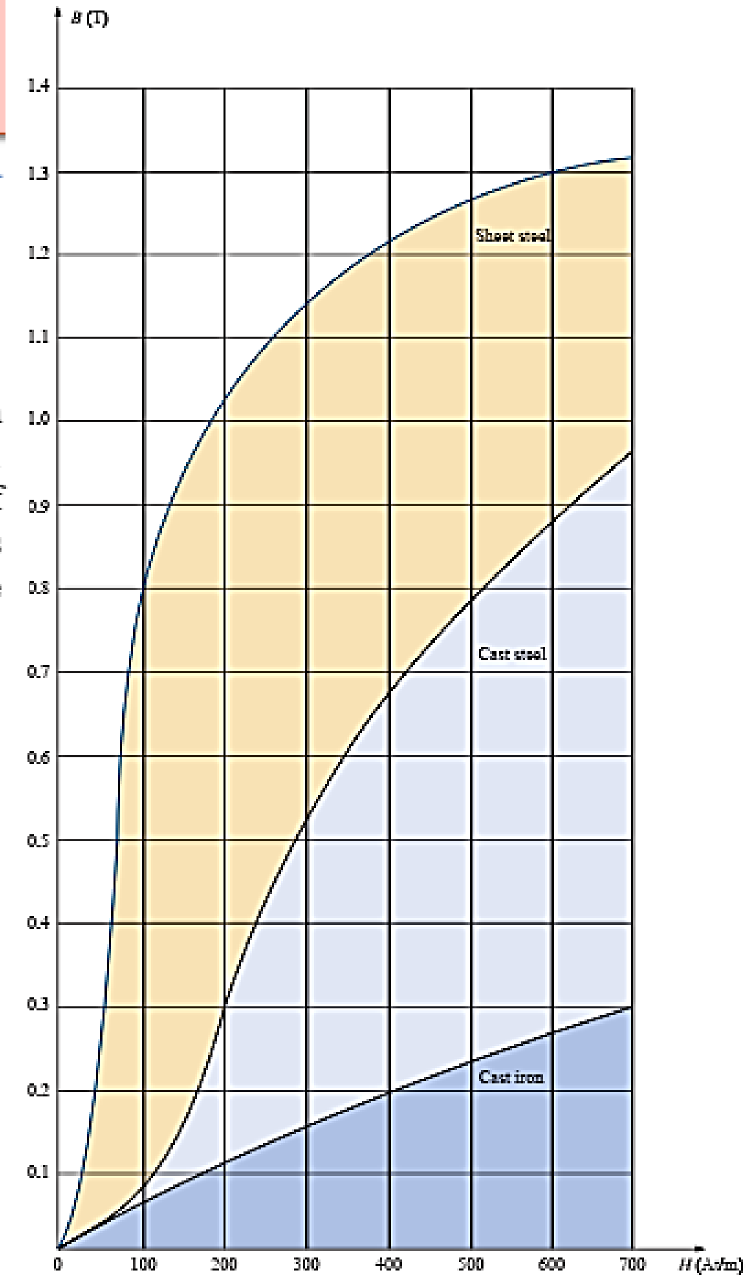


FIG. 11.24
Expanded view of Fig. 11.23 for the low magnetizing force region.

Series Magnetic Circuits

EXAMPLE 11.6 Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4}$ Wb in the series magnetic circuit of Fig. 11.36.

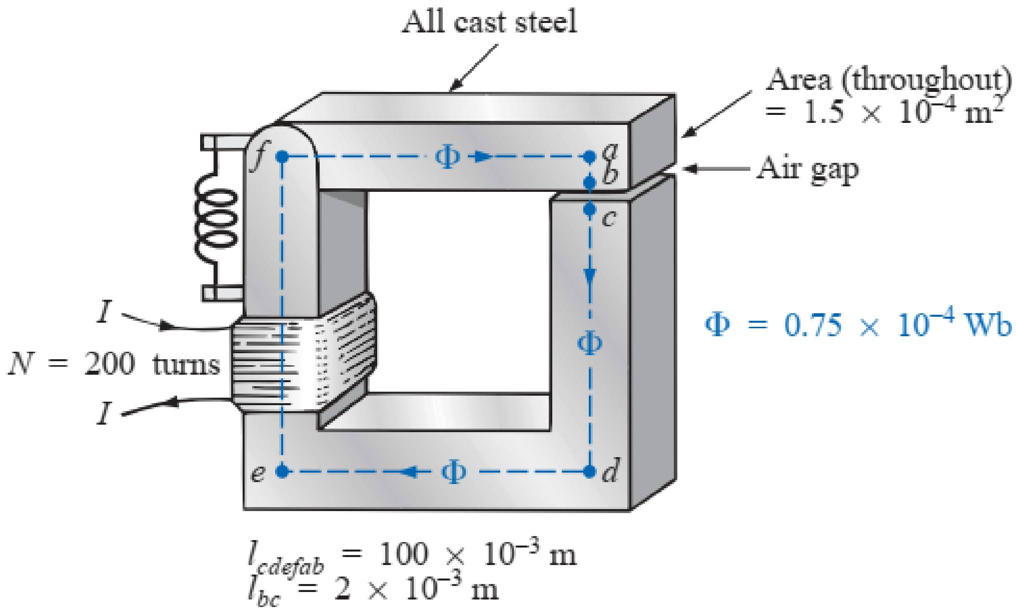


FIG. 11.36
Relay for Example 11.6.

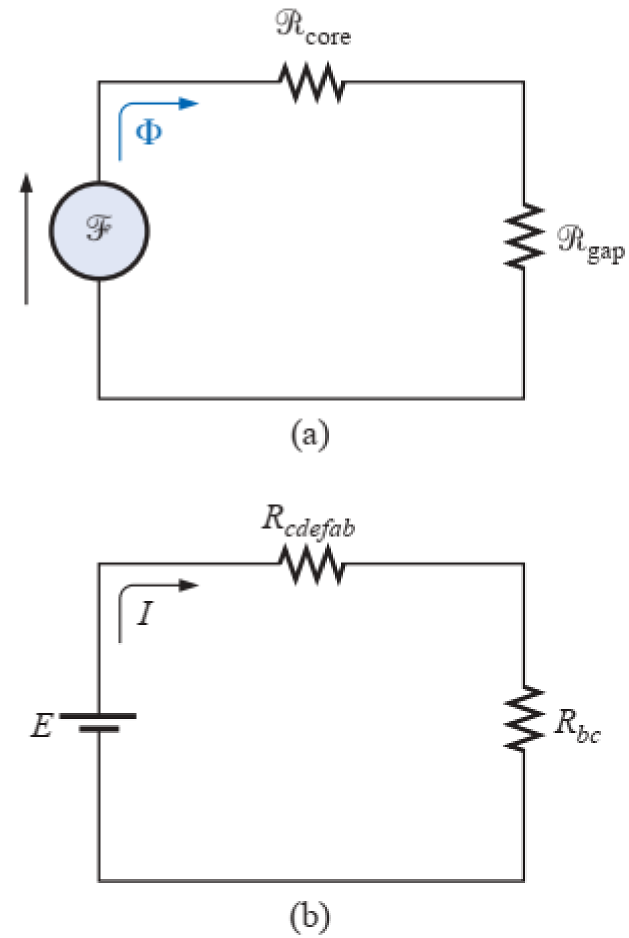


FIG. 11.37
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the relay of Fig. 11.36.

Series Magnetic Circuits

EXAMPLE 11.6 Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4}$ Wb in the series magnetic circuit of Fig. 11.36.

Solution: An equivalent magnetic circuit and its electric circuit analogy are shown in Fig. 11.37.

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4}\,\text{Wb}}{1.5 \times 10^{-4}\,\text{m}^2} = 0.5\,\text{T}$$

From the B-H curves of Fig. 11.24,

$$H\,(\text{cast steel}) \cong 280\,\text{At/m}$$

Applying Eq. (11.15),

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5\,\text{T}) = 3.98 \times 10^5\,\text{At/m}$$

The mmf drops are

$$H_{\text{core}}l_{\text{core}} = (280\,\text{At/m})(100 \times 10^{-3}\,\text{m}) = 28\,\text{At}$$
$$H_g l_g = (3.98 \times 10^5\,\text{At/m})(2 \times 10^{-3}\,\text{m}) = 796\,\text{At}$$

Applying Ampère's circuital law,

$$NI = H_{\text{core}}l_{\text{core}} + H_g l_g$$
$$= 28\,\text{At} + 796\,\text{At}$$
$$(200\,\text{t})I = 824\,\text{At}$$
$$I = \mathbf{4.12\,A}$$

Note from the above that the air gap requires the biggest share (by far) of the impressed NI due to the fact that air is nonmagnetic.

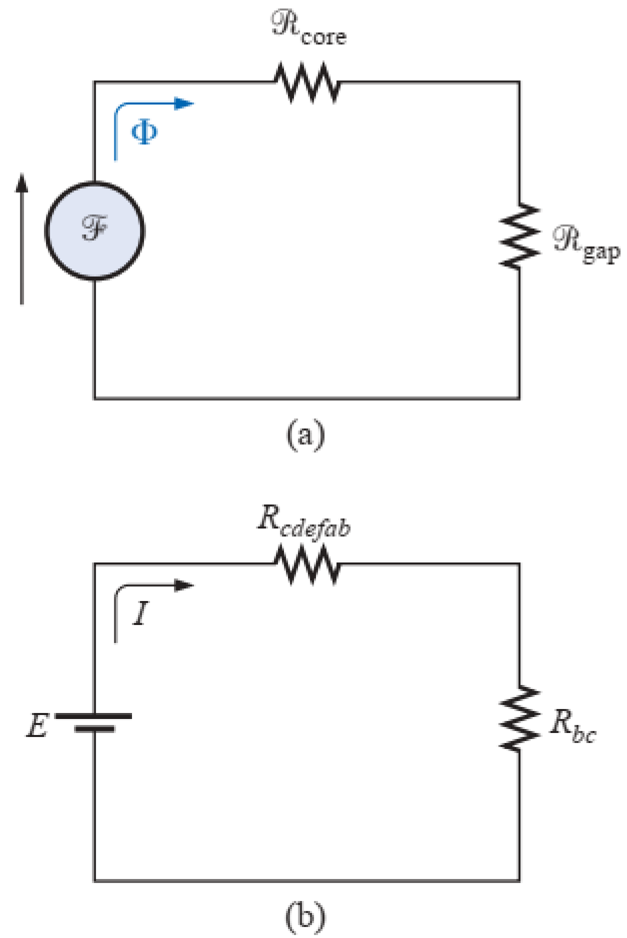


FIG. 11.37
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the relay of Fig. 11.36.

Series Magnetic Circuits

EXAMPLE 11.8 Calculate the magnetic flux Φ for the magnetic circuit of Fig. 11.40.

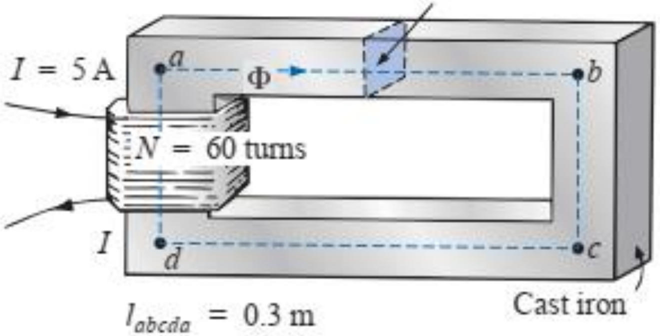


FIG. 11.40
Example 11.8.

Solution: By Ampère's circuital law,

$$NI = H_{abcda}l_{abcda}$$

or

$$H_{abcda} = \frac{NI}{l_{abcda}} = \frac{(60\text{ t})(5\text{ A})}{0.3\text{ m}}$$

$$= \frac{300\text{ At}}{0.3\text{ m}} = 1000\text{ At/m}$$

and

$$B_{abcda}\text{ (from Fig. 11.23)} \cong 0.39\text{ T}$$

Since $B = \Phi/A$, we have

$$\Phi = BA = (0.39\text{ T})(2 \times 10^{-4}\text{ m}^2) = \mathbf{0.78 \times 10^{-4}\ Wb}$$