Lesson 4: Network Theorems

COURSE CODE: EEE 201

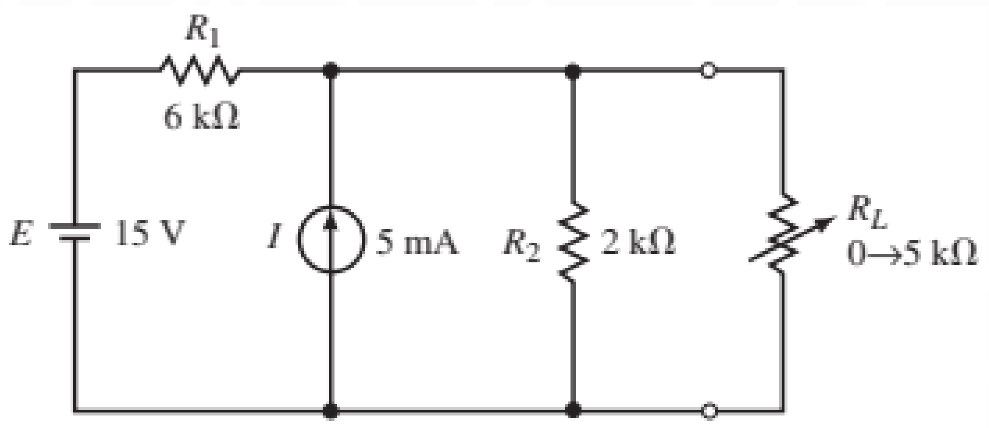
COURSE TITLE: ELECTRICAL ENGINEERING

Introduction

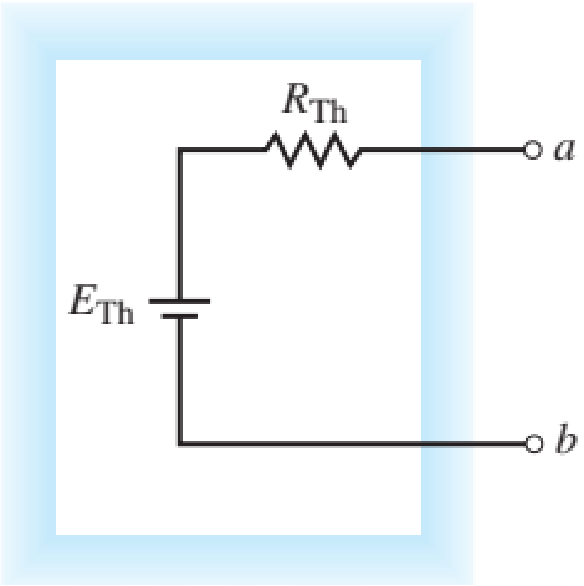
- Kirchhoff's laws:
  Advantages: circuit can analyze without tampering its original configuration.
  Disadvantage: for a large, complex circuit, tedious computation is involved.
- To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis. Such theorems include Thevenin's and Norton's theorems.
- These theorems are applicable to *linear* circuits.

A **linear circuit** is one whose output is linearly related (or directly proportional) to its input.

Thevenin's Theorem



If we wanted to find the current through the variable load resistor when $R_L = 0\Omega$ $R_L = 2\ k\Omega$ *and* $R_L = 5\ k\Omega$ using existing methods, we would need to analyze the entire circuit three separate times.



However, if we can reduce the entire circuit external to the load resistor to a single voltage source in series with a resistor, the solution becomes very easy.

Thevenin's Theorem

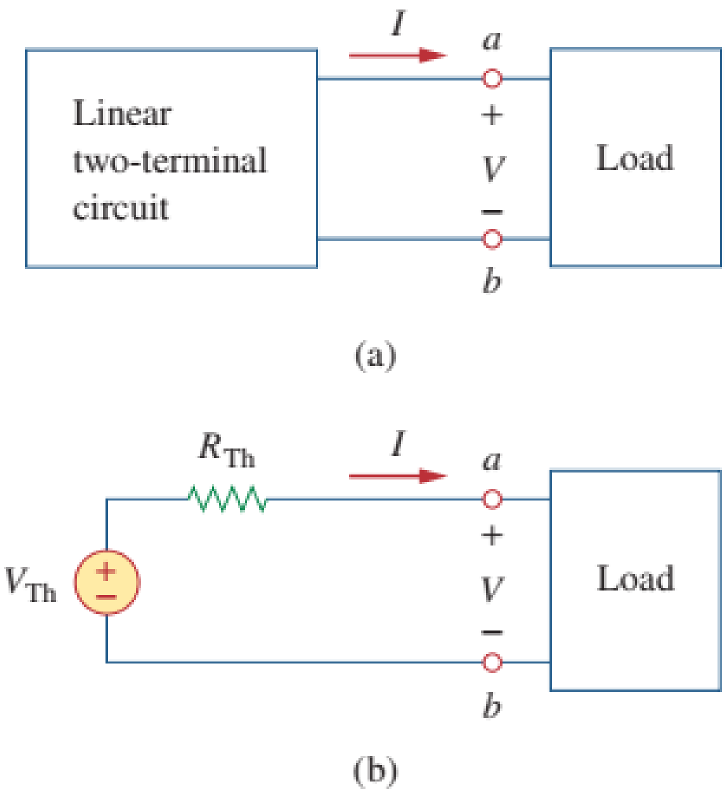
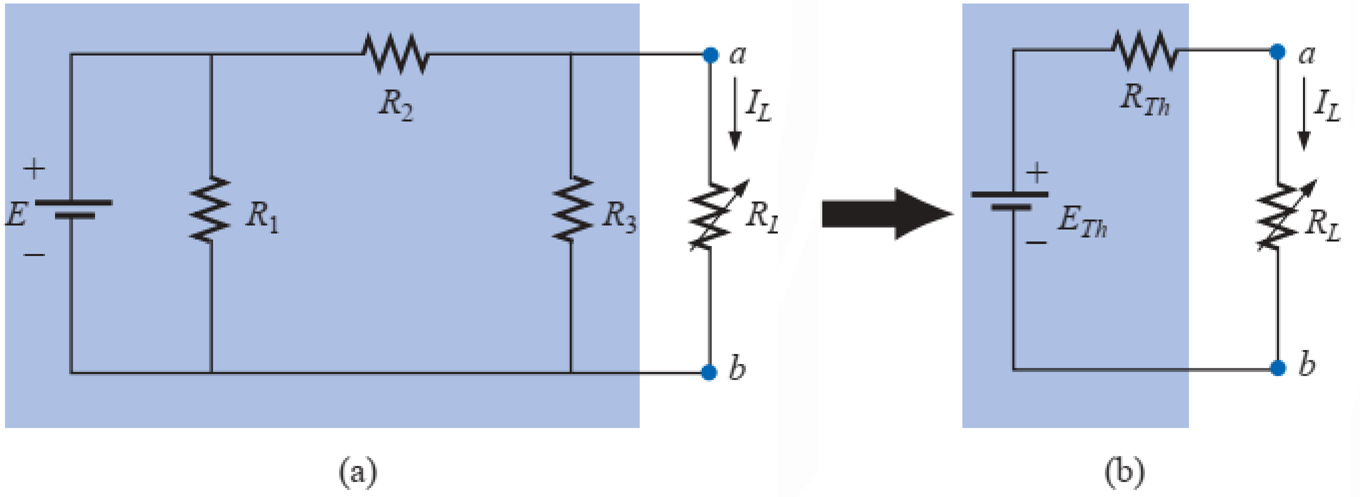


Figure 4.23
Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.



Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

Fig. 4.23(b) is known as the *Thevenin equivalent circuit*; it was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

Thevenin's Theorem

Example 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16,$ and $36\ \Omega$.

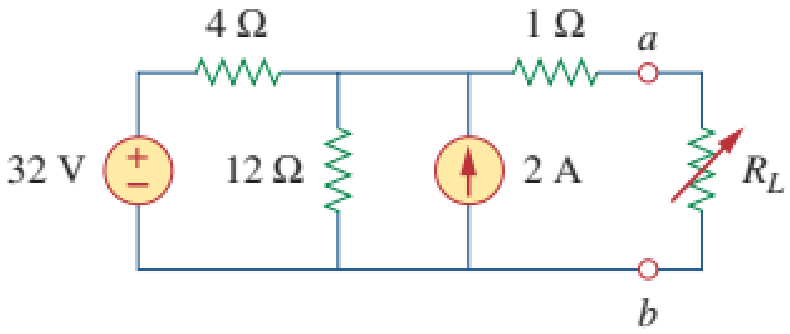
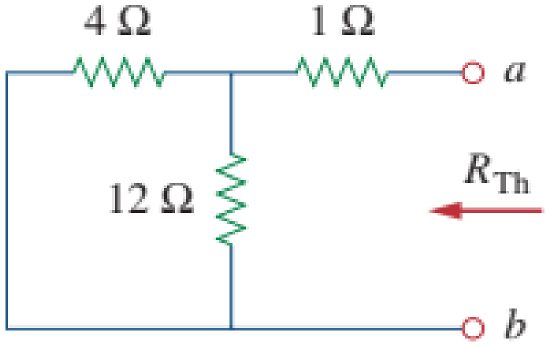


Figure 4.27
For Example 4.8.

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,



(a)

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

Thevenin's Theorem

Example 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16,$ and $36\ \Omega$.

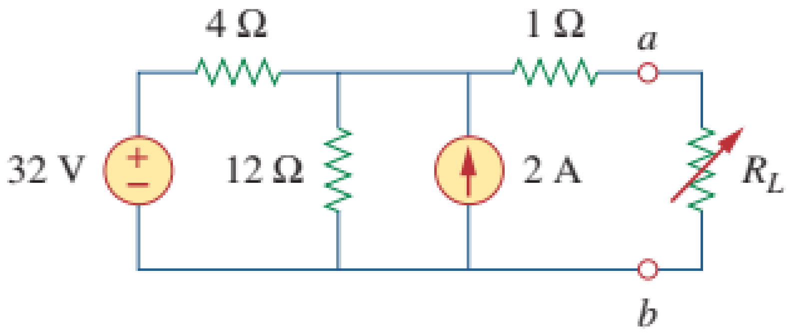
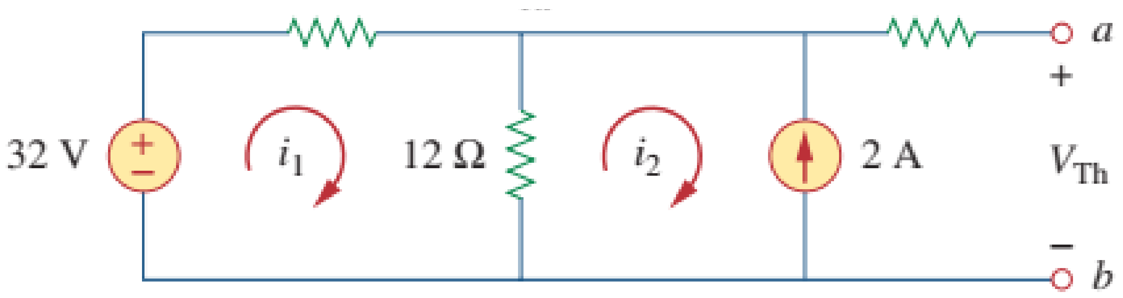


Figure 4.27
For Example 4.8.



(b)

To find V_{Th}, consider the circuit in Fig. 4.28(b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\text{ V}$$

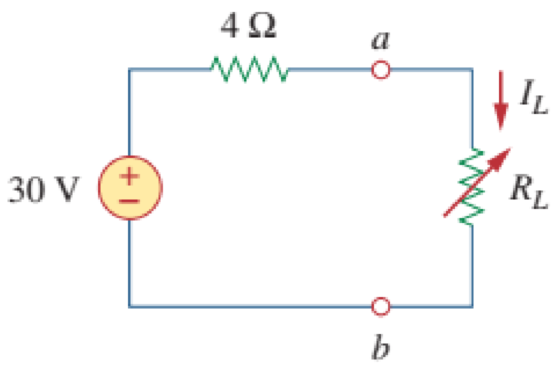


Figure 4.29
The Thevenin equivalent circuit for Example 4.8.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3\text{ A}$$

Thevenin's Theorem

Example 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16,$ and $36\ \Omega$.

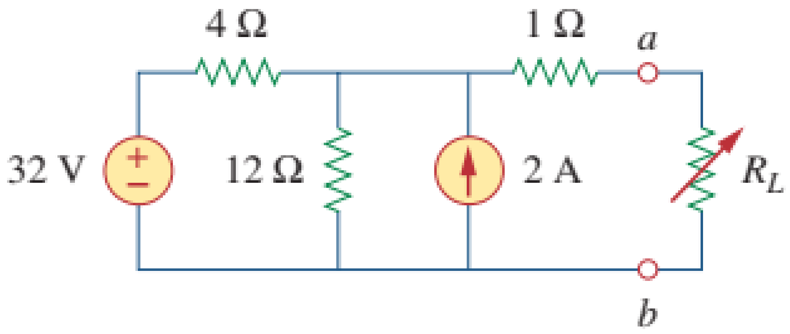
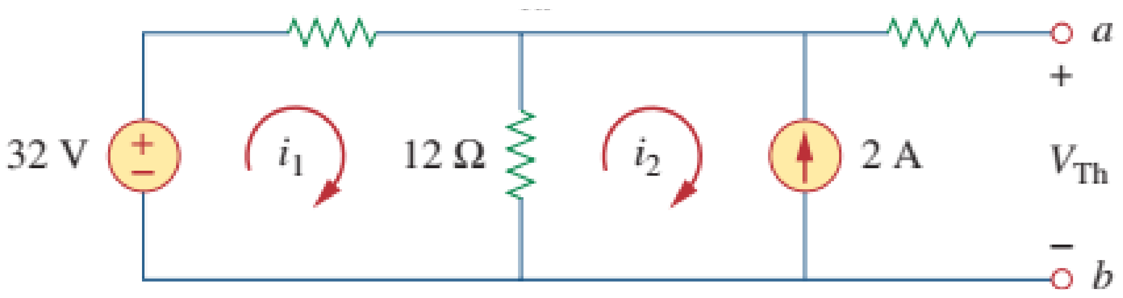


Figure 4.27
For Example 4.8.



Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

or

$$96 - 3V_{Th} + 24 = V_{Th} \quad \Rightarrow \quad V_{Th} = 30\ \text{V}$$

as obtained before. We could also use source transformation to find V_{Th}.

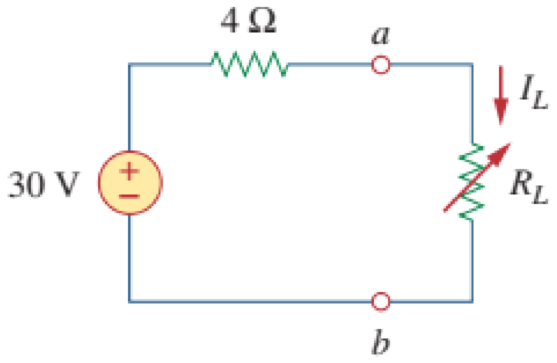


Figure 4.29
The Thevenin equivalent circuit for Example 4.8.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$, $\quad I_L = \frac{30}{10} = 3\ \text{A}$

Thevenin's Theorem

EXAMPLE 9.9 Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network of Fig. 9.44.

From Text: B-2

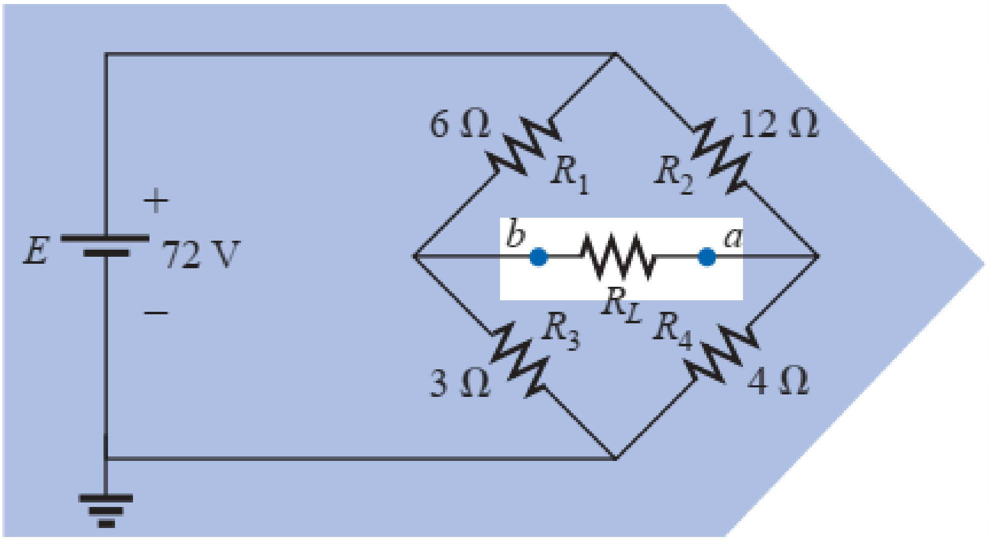


FIG. 9.44
Example 9.9.

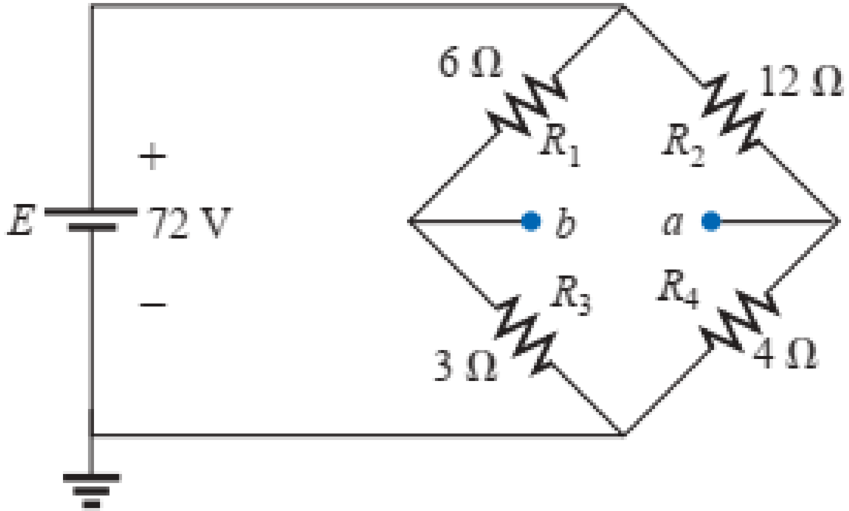
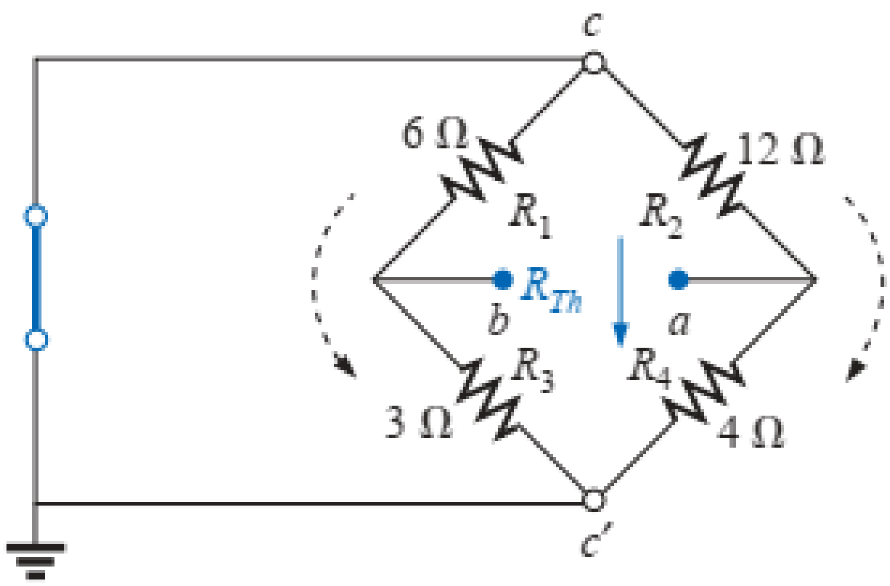


FIG. 9.45
Identifying the terminals of particular interest for the network of Fig. 9.44.

Thevenin's Theorem

EXAMPLE 9.9 Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network of Fig. 9.44.

From Text: B-2



(a)

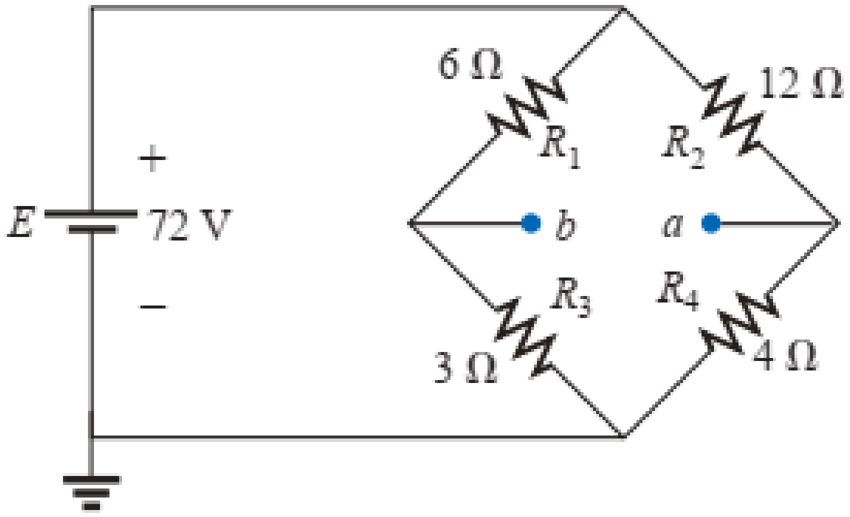


FIG. 9.45

Identifying the terminals of particular interest for the network of Fig. 9.44.

Thevenin's Theorem

EXAMPLE 9.9 Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network of Fig. 9.44.

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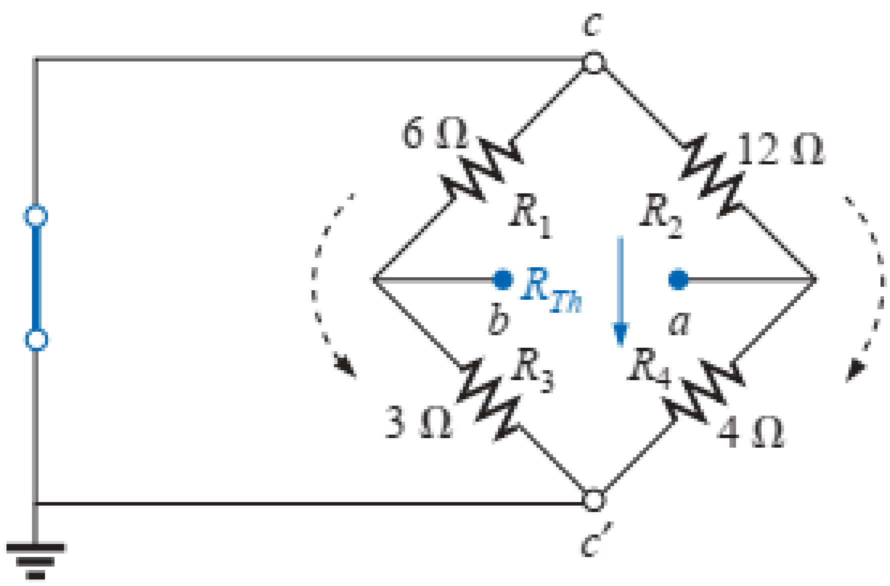
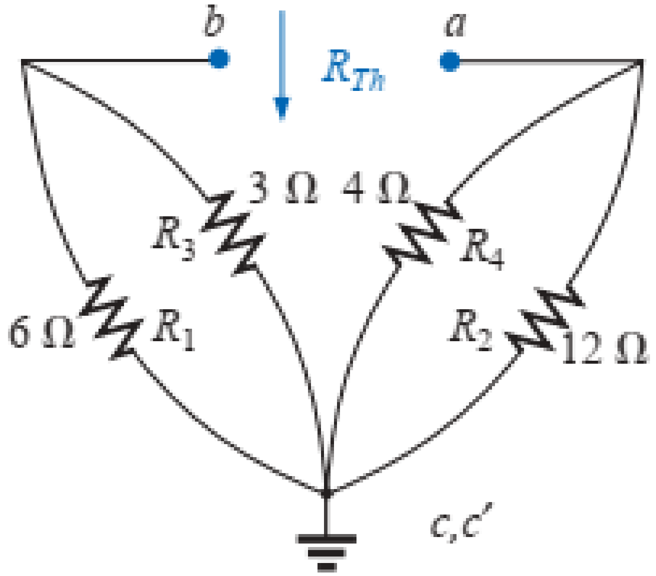


(a)



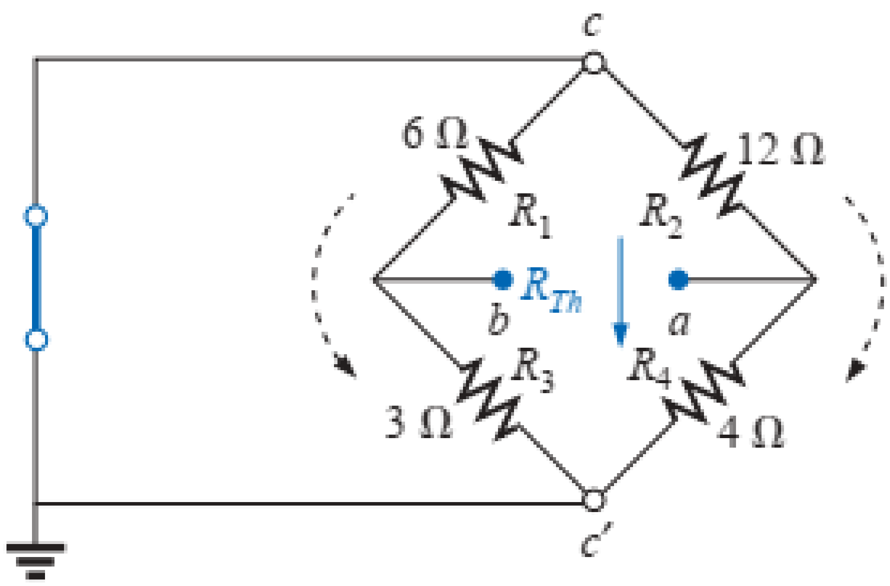
(b)

$$
\begin{aligned}
R_{Th} = R_{a\text{-}b} &= R_1 \parallel R_3 + R_2 \parallel R_4 \\
&= 6\ \Omega \parallel 3\ \Omega + 4\ \Omega \parallel 12\ \Omega \\
&= 2\ \Omega + 3\ \Omega = \mathbf{5\ \Omega}
\end{aligned}
$$

Thevenin's Theorem

EXAMPLE 9.9 Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network of Fig. 9.44.

From Text: B-2



(a)

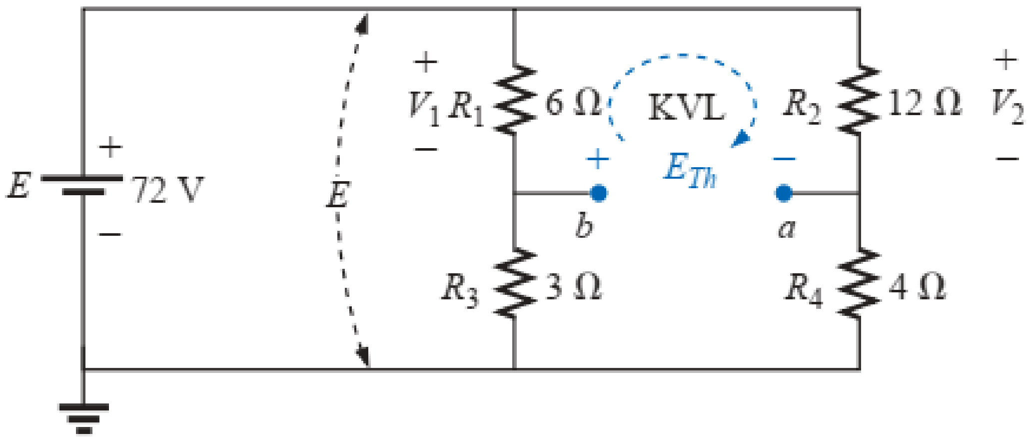


FIG. 9.47
Determining E_{Th} for the network of Fig. 9.45.

Thevenin's Theorem

EXAMPLE 9.9 Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network of Fig. 9.44.

From Text: B-2

$$V_1 = \frac{R_1E}{R_1 + R_3} = \frac{(6\ \Omega)(72\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{432\ \text{V}}{9} = 48\ \text{V}$$

$$V_2 = \frac{R_2E}{R_2 + R_4} = \frac{(12\ \Omega)(72\ \text{V})}{12\ \Omega + 4\ \Omega} = \frac{864\ \text{V}}{16} = 54\ \text{V}$$

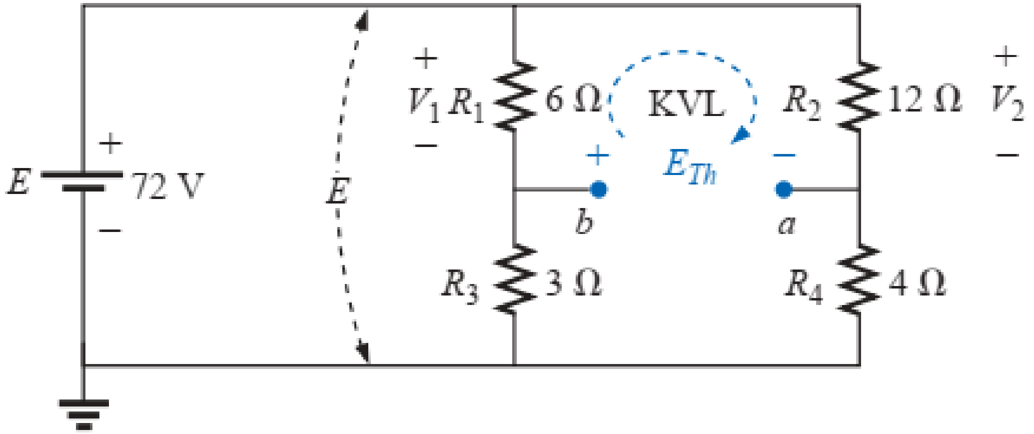


FIG. 9.47
Determining E_{Th} for the network of Fig. 9.45.

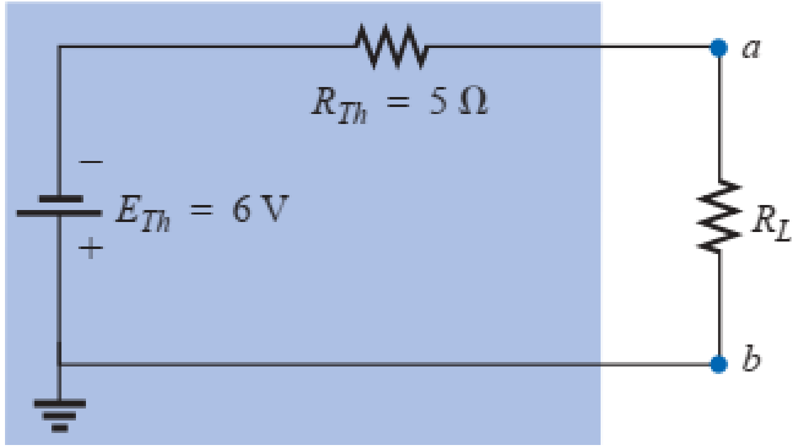


FIG. 9.48
Substituting the Thévenin equivalent circuit for the network external to the resistor R_L of Fig. 9.44.

Assuming the polarity shown for E_{Th} and applying Kirchhoff's voltage law to the top loop in the clockwise direction will result in

$$\Sigma_{C}\ V = +E_{Th} + V_1 - V_2 = 0$$

and

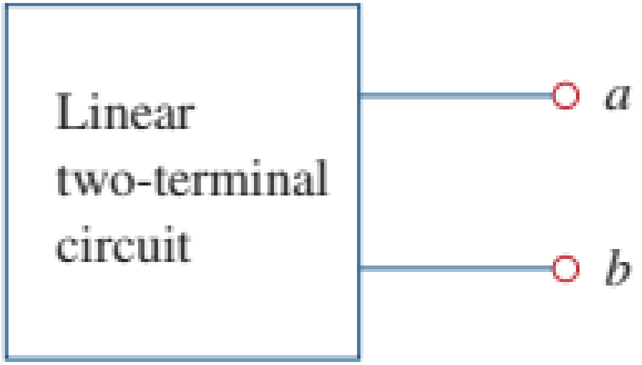
$$E_{Th} = V_2 - V_1 = 54\ \text{V} - 48\ \text{V} = \mathbf{6\ V}$$

Norton’s Theorem

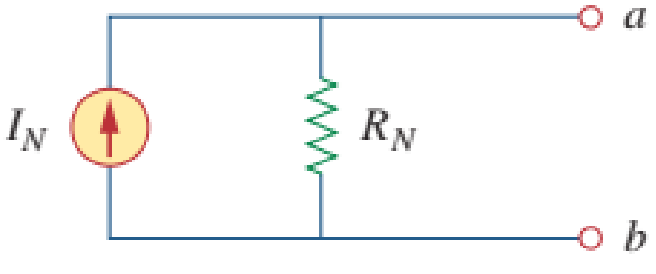
In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Norton’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

$$R_N = R_{Th}$$



(a)



(b)

Figure 4.37
(a) Original circuit, (b) Norton equivalent circuit.

Norton's Theorem

EXAMPLE 9.11 Find the Norton equivalent circuit for the network in the shaded area of Fig. 9.60.

From Text: B-2

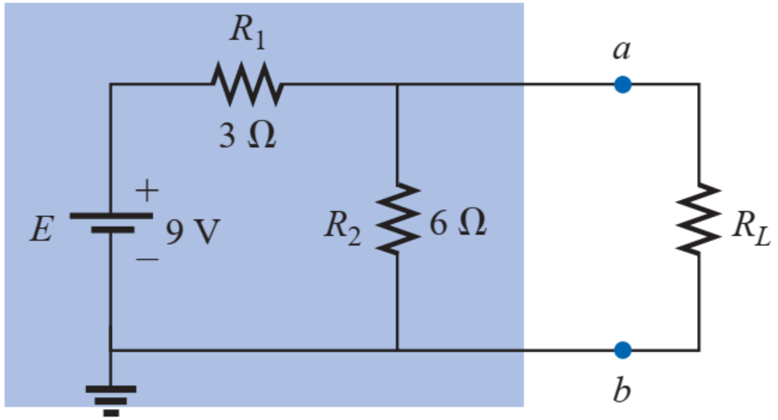


FIG. 9.60
Example 9.11.

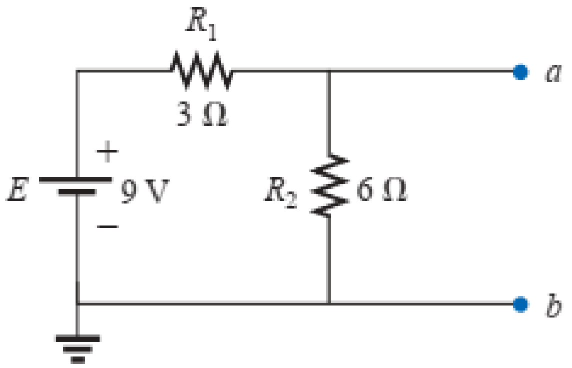


FIG. 9.61
Identifying the terminals of particular interest for the network of Fig. 9.60.

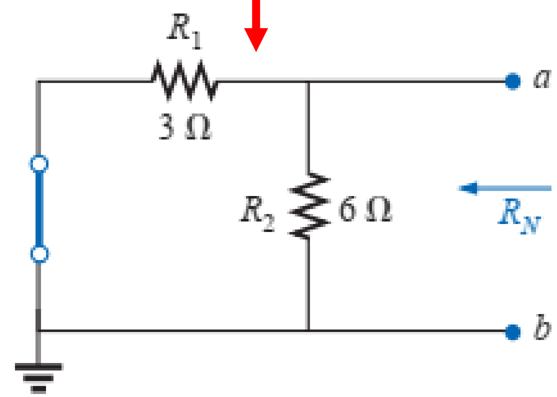


FIG. 9.62
Determining R_N for the network of Fig. 9.61.

$$R_N = R_1 \parallel R_2 = 3\ \Omega \parallel 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = \mathbf{2\ \Omega}$$

Norton's Theorem

EXAMPLE 9.11 Find the Norton equivalent circuit for the network in the shaded area of Fig. 9.60.

From Text: B-2

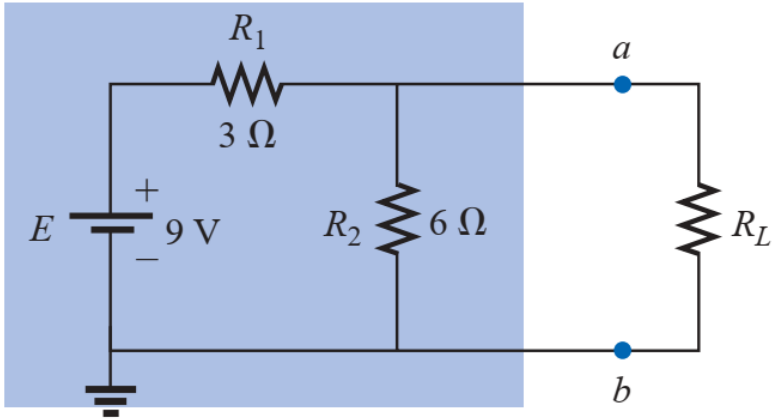


FIG. 9.60
Example 9.11.

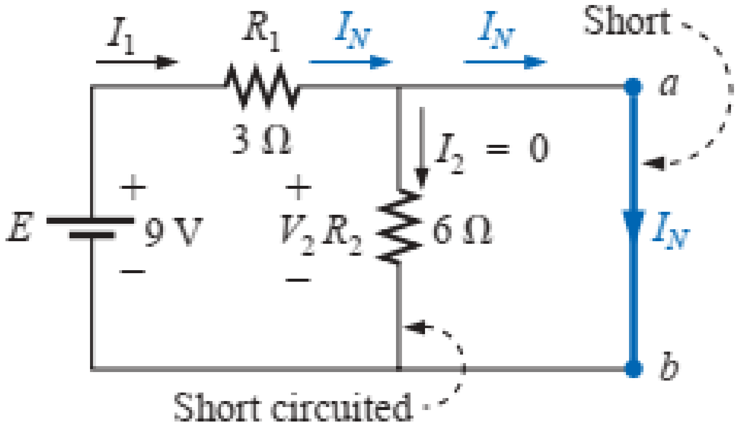


FIG. 9.63
Determining I_N for the network of Fig. 9.61.

$$V_2 = I_2R_2 = (0)6\ \Omega = 0\ \text{V}$$

$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = 3\ \text{A}$$

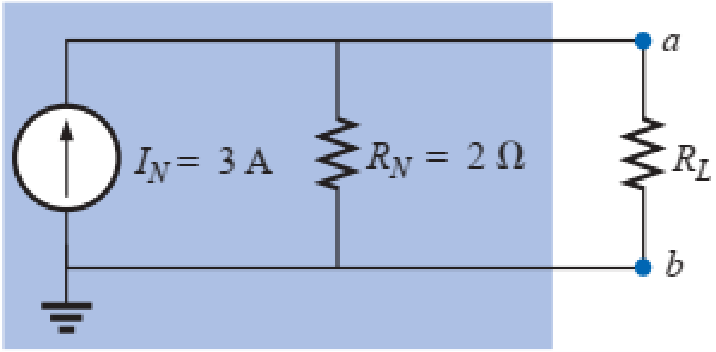


FIG. 9.64
Substituting the Norton equivalent circuit for the network external to the resistor R_L of Fig. 9.60.

Norton's Theorem

From Text: B-2

EXAMPLE 9.12 Find the Norton equivalent circuit for the network external to the 9-Ω resistor in Fig. 9.66.

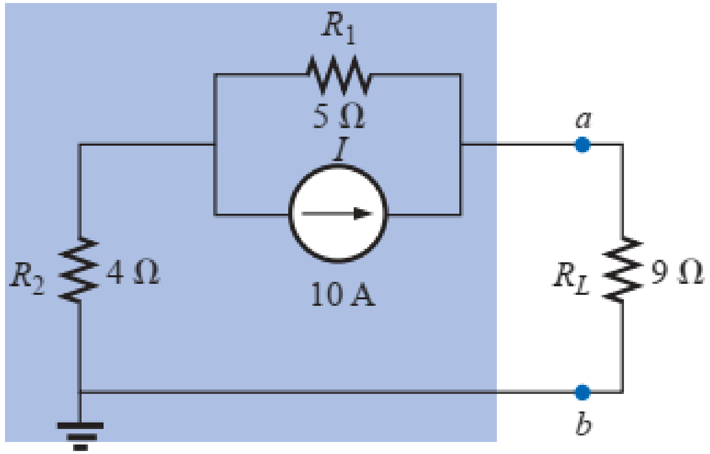


FIG. 9.66
Example 9.12.

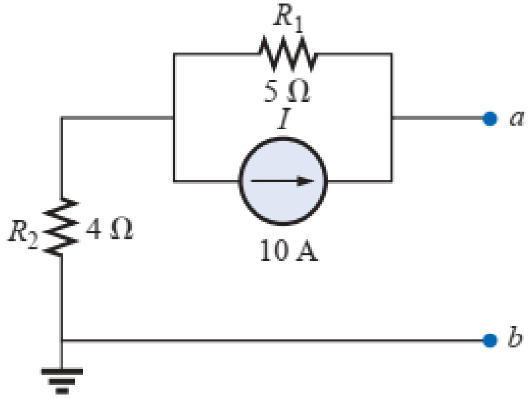


FIG. 9.67
Identifying the terminals of particular interest for the network of Fig. 9.66.

$$R_N = R_1 + R_2 = 5\ \Omega + 4\ \Omega = 9\ \Omega$$

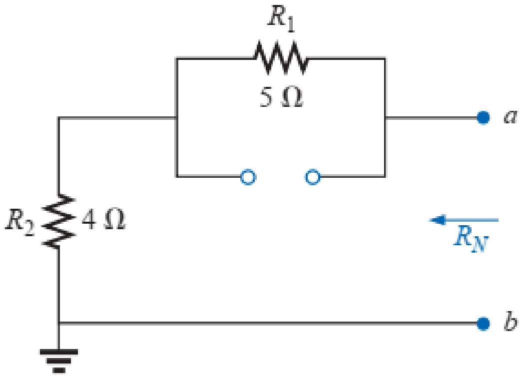


FIG. 9.68
Determining R_N for the network of Fig. 9.67.

Norton's Theorem

From Text: B-2

EXAMPLE 9.12 Find the Norton equivalent circuit for the network external to the 9-Ω resistor in Fig. 9.66.

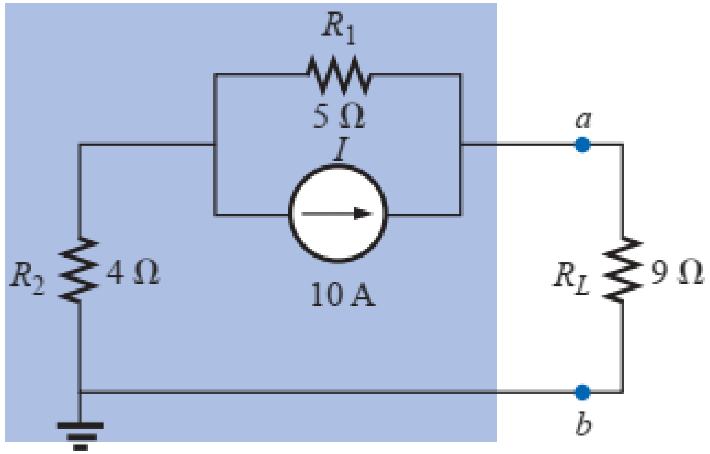


FIG. 9.66
Example 9.12.

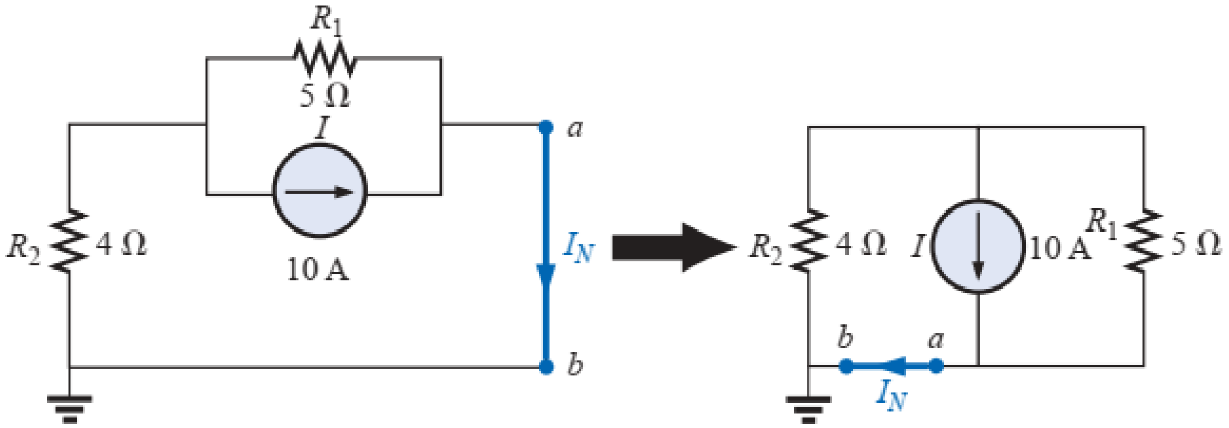


FIG. 9.69
Determining I_N for the network of Fig. 9.67.

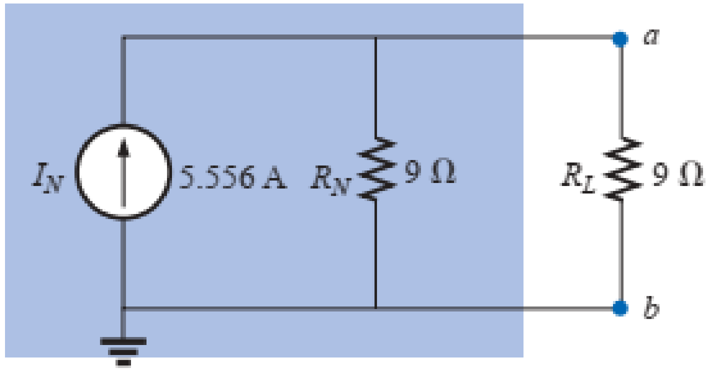


FIG. 9.70
Substituting the Norton equivalent circuit for the network external to the resistor R_L of Fig. 9.66.

$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5\ \Omega)(10\ \text{A})}{5\ \Omega + 4\ \Omega} = \frac{50\ \text{A}}{9} = \mathbf{5.556\ A}$$

Norton's Theorem

Example 4.11

Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals *a*-*b*.

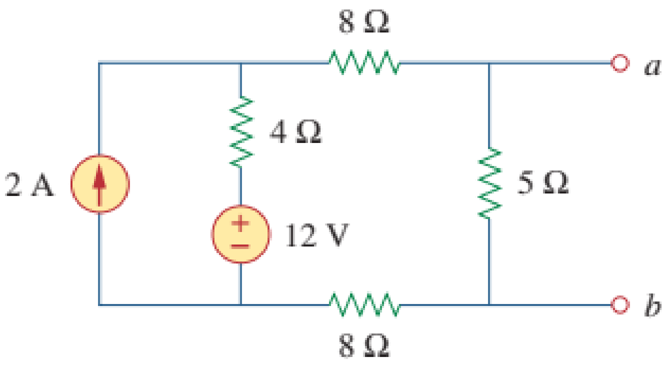
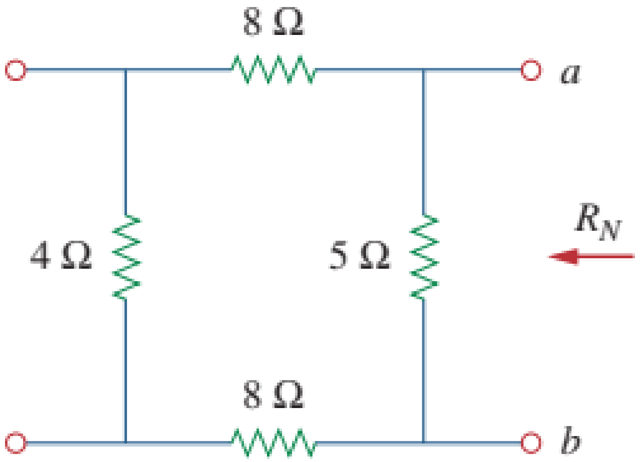
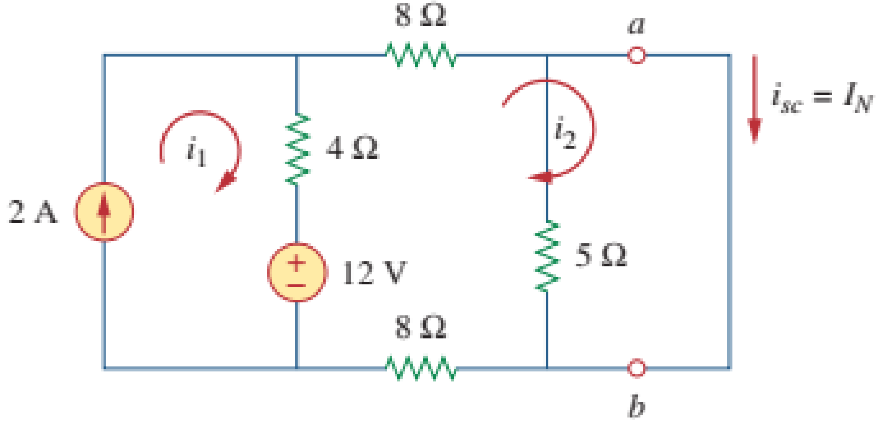


Figure 4.39
For Example 4.11.



(a)

$$R_N = 5 \,\|\, (8 + 4 + 8) = 5 \,\|\, 20 = \frac{20 \times 5}{25} = 4\ \Omega$$



(b)

$$i_1 = 2\ \text{A},$$

$$20i_2 - 4i_1 - 12 = 0$$

$$i_2 = 1\ \text{A} = i_{sc} = I_N$$

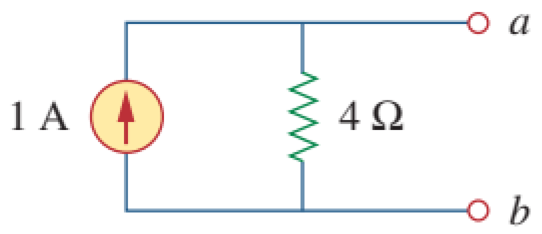


Figure 4.41
Norton equivalent of the circuit in Fig. 4.39.

END LESSON 4: NETWORK THEOREMS

- Next Lesson…….
- Topics: Capacitors and Inductors
- Text: B-1,