Lesson 2: Laws of Circuit Analysis

COURSE CODE: EEE 201

COURSE TITLE: ELECTRICAL ENGINEERING

Introduction

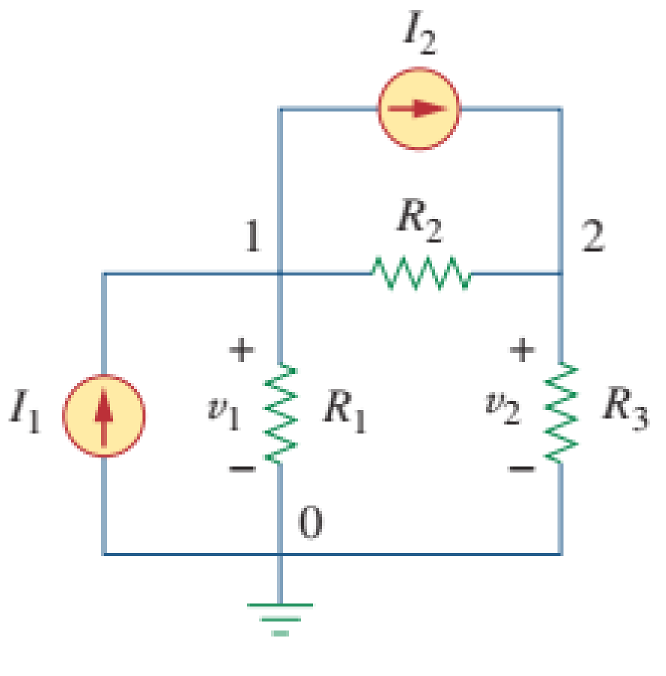
- Two powerful techniques for circuit analysis: nodal analysis, based on KCL & mesh analysis, based on KVL.
- These the two techniques used in linear circuit to obtain a set of simultaneous equations of current or voltage.
- Equations are solved by Cramer's rule or using *MATLAB*
- All the methods can be applied to *linear bilateral* networks.
- The term *linear* indicates that the characteristics of the network elements (such as the resistors) are independent of the voltage across or current through them.
- The second term, *bilateral,* refers to the fact that there is no change in the behavior or characteristics of an element if the current through or voltage across the element is reversed.

Nodal Analysis (without voltage sources)

Nodal analysis is also known as the *node-voltage method*.

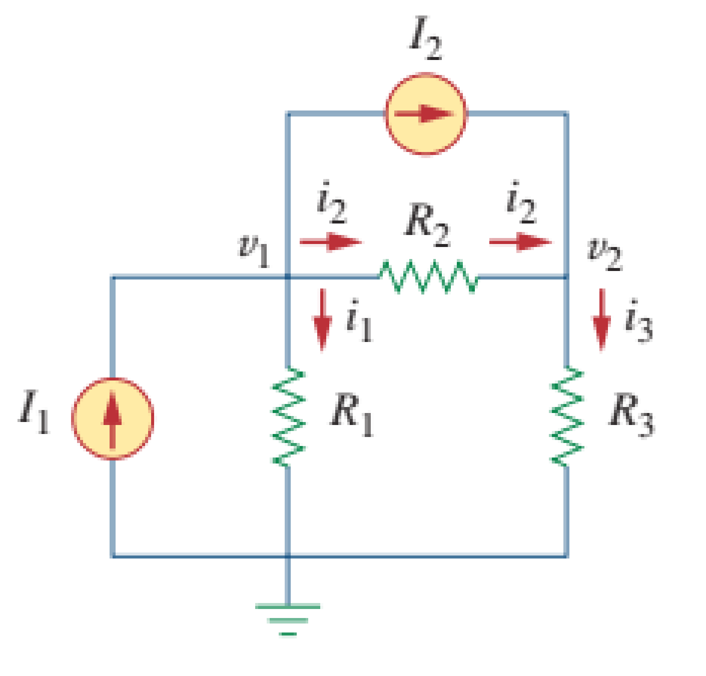
Steps:

1. Select a node as the reference node (Fig: a)
2. Apply KCL to each of the nonreference nodes. (Fig: b)
3. Solve the resulting simultaneous equations.



(a)

$$i = \frac{v_{higher} - v_{lower}}{R}$$



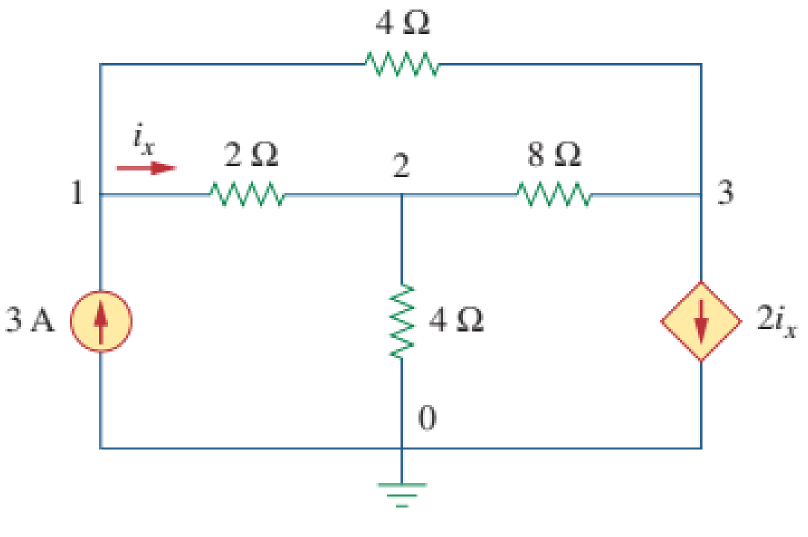
(b)

Current flows from **a higher** potential to **a lower** potential in a resistor.

$$i_1 = \frac{v_1 - 0}{R_1} \qquad i_2 = \frac{v_1 - v_2}{R_2} \qquad i_3 = \frac{v_2 - 0}{R_3}$$

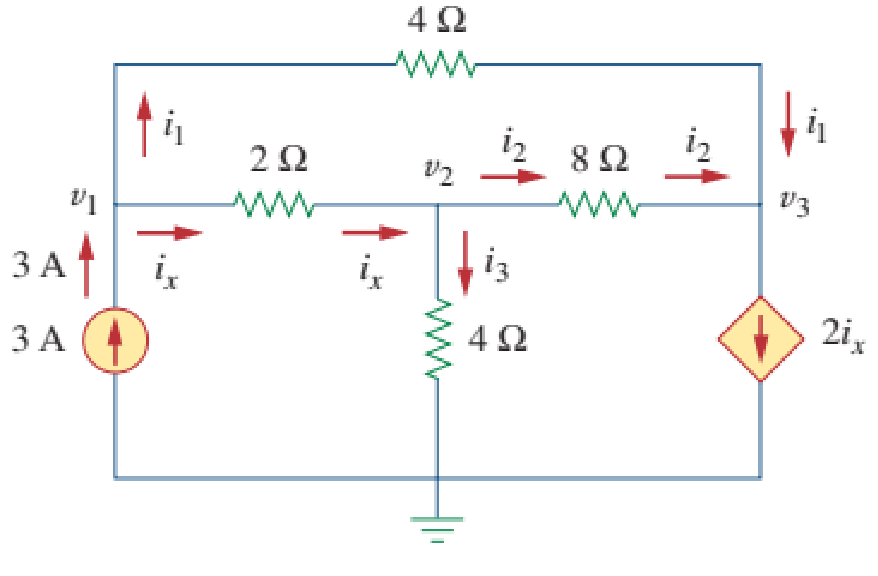
Nodal Analysis (without voltage sources)

Example 3.2 Determine the voltages at the nodes in Fig. 3.5(a).



(a)

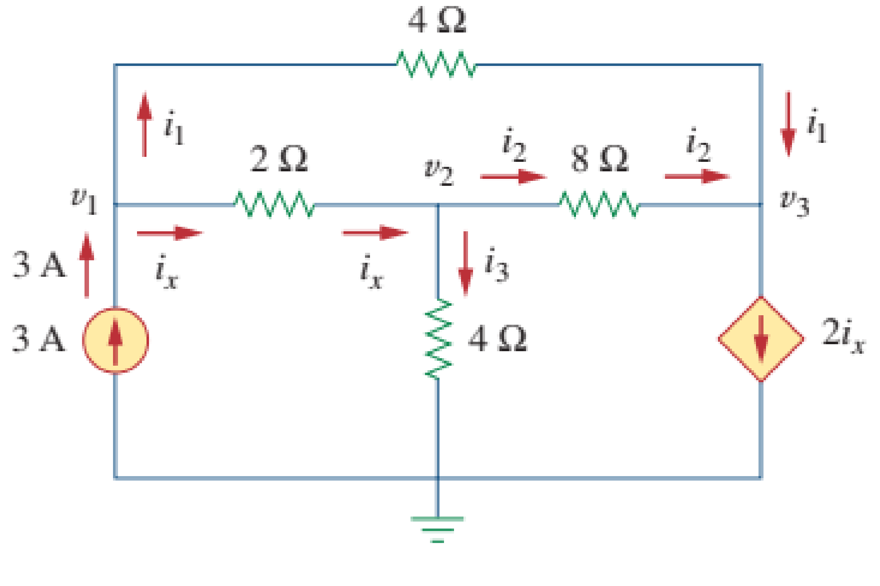
Figure 3.5



(b)

Nodal Analysis (without voltage sources)

Example 3.2 Determine the voltages at the nodes in Fig. 3.5(a).



(b)

At node 1,

$$3 = i_1 + i_x \qquad \Rightarrow \qquad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

$$3v_1 - 2v_2 - v_3 = 12$$

At node 2,

$$i_x = i_2 + i_3 \qquad \Rightarrow \qquad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

$$-4v_1 + 7v_2 - v_3 = 0$$

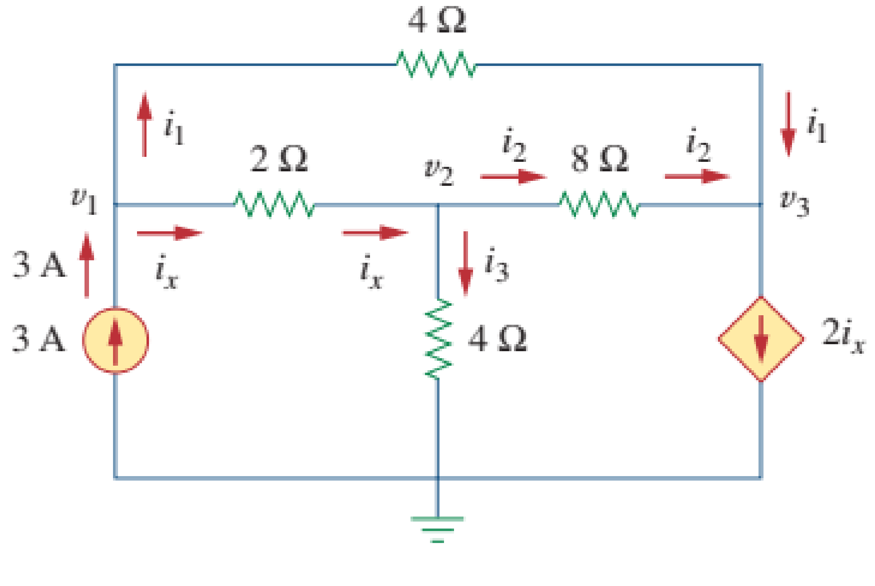
At node 3,

$$i_1 + i_2 = 2i_x \qquad \Rightarrow \qquad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

$$2v_1 - 3v_2 + v_3 = 0$$

Nodal Analysis (without voltage sources)

Example 3.2 Determine the voltages at the nodes in Fig. 3.5(a).



(b)

$$3v_1 - 2v_2 - v_3 = 12$$

$$-4v_1 + 7v_2 - v_3 = 0$$

$$2v_1 - 3v_2 + v_3 = 0$$

Nodal Analysis (without voltage sources)

Example 3.2 Determine the voltages at the nodes in Fig. 3.5(a).

$$3v_1 - 2v_2 - v_3 = 12$$

$$-4v_1 + 7v_2 - v_3 = 0$$

$$2v_1 - 3v_2 + v_3 = 0$$

METHOD 2 To use Cramer's rule, we put Eqs. (3.2.1) to (3.2.3) in matrix form.

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \tag{3.2.6}$$

From this, we obtain

$$v_1 = \frac{\Delta_1}{\Delta}, \qquad v_2 = \frac{\Delta_2}{\Delta}, \qquad v_3 = \frac{\Delta_3}{\Delta}$$

where Δ, Δ_1, Δ_2, and Δ_3 are the determinants to be calculated as follows. As explained in Appendix A, to calculate the determinant of a 3 by 3 matrix, we repeat the first two rows and cross multiply.

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \begin{array}{c} \\ \\ \\ - \\ - \\ - \end{array} \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \\ 3 & -2 & -1 \\ -4 & 7 & -1 \end{vmatrix} \begin{array}{c} \\ \\ \\ + \\ + \\ + \end{array}$$

$$= 21 - 12 + 4 + 14 - 9 - 8 = 10$$

$$\Delta_1 = \begin{array}{c} \\ \\ \\ - \\ - \\ - \end{array} \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \\ 12 & -2 & -1 \\ 0 & 7 & -1 \end{vmatrix} \begin{array}{c} \\ \\ \\ + \\ + \\ + \end{array} = 84 + 0 + 0 - 0 - 36 - 0 = 48$$

Nodal Analysis (without voltage sources)

Example 3.2 Determine the voltages at the nodes in Fig. 3.5(a).

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{v_1 = \frac{\Delta_1}{\Delta}, \qquad v_2 = \frac{\Delta_2}{\Delta}, \qquad v_3 = \frac{\Delta_3}{\Delta}}$$

$$\Delta_2 = \begin{array}{c|ccc|c} & 3 & 12 & -1 & \\ & -4 & 0 & -1 & \\ & 2 & 0 & 1 & \\ - & 3 & 12 & -1 & + \\ - & -4 & 0 & -1 & + \\ - & & & & + \end{array} = 0 + 0 - 24 - 0 - 0 + 48 = 24$$

$$\Delta_3 = \begin{array}{c|ccc|c} & 3 & -2 & 12 & \\ & -4 & 7 & 0 & \\ & 2 & -3 & 0 & \\ - & 3 & -2 & 12 & + \\ - & -4 & 7 & 0 & + \\ - & & & & + \end{array} = 0 + 144 + 0 - 168 - 0 - 0 = -24$$

Thus, we find

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8 \text{ V}, \qquad v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4 \text{ V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4 \text{ V}$$

Nodal Analysis (without voltage sources)

Practice Problem 3.2

Find the voltages at the three nonreference nodes in the circuit of Fig. 3.6.

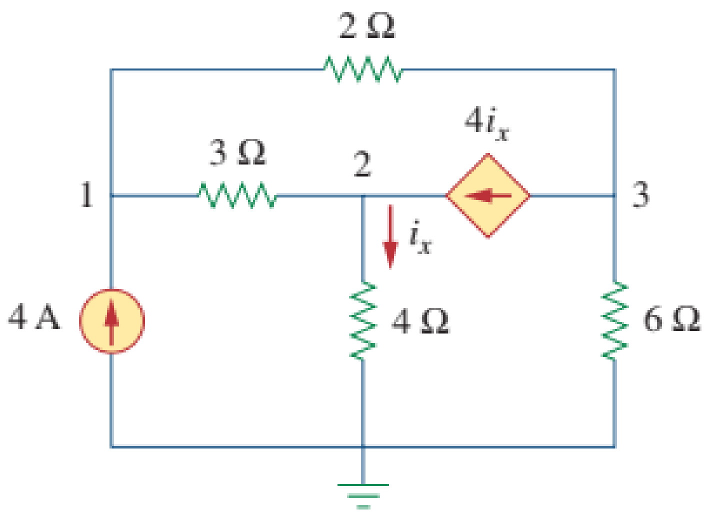


Figure 3.6
For Practice Prob. 3.2.

Mesh Analysis

- A loop is a closed path with no node passed more than once.
- A mesh is a loop that does not contain any other loop within it.
- Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is *planar*.

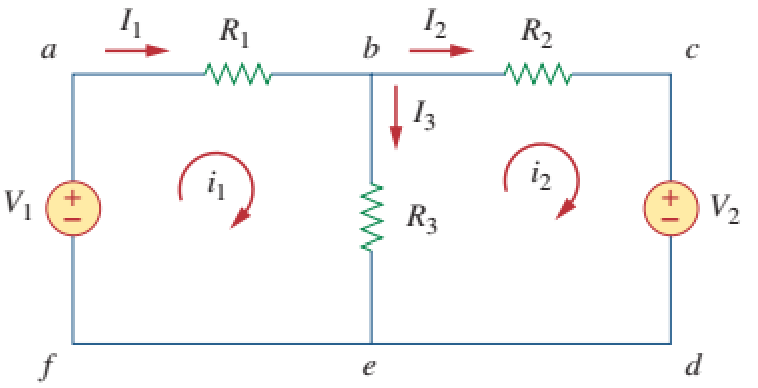
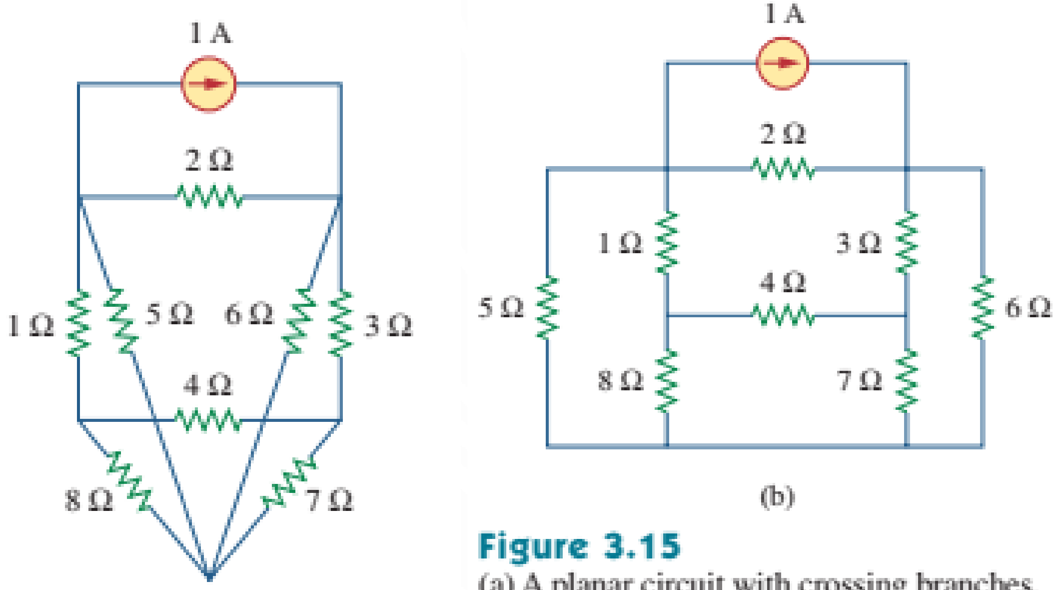


Figure 3.17
A circuit with two meshes.



(a)

(b)

Figure 3.15
(a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.

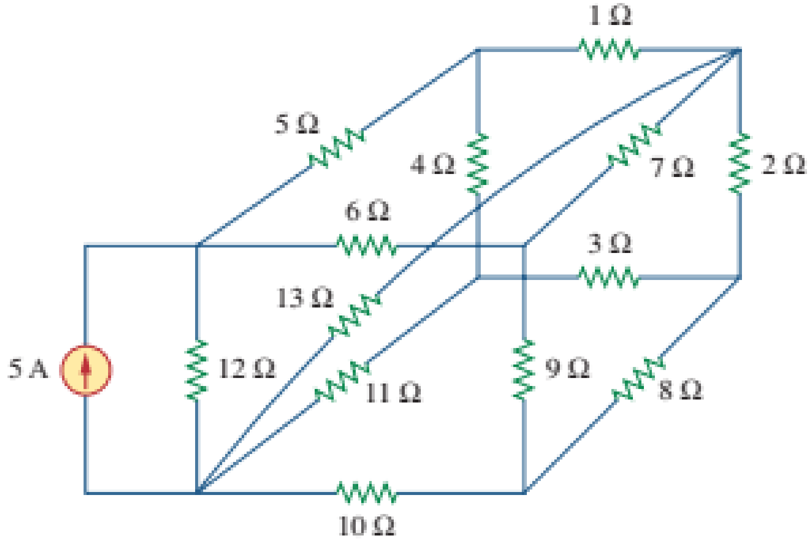


Figure 3.16
A nonplanar circuit.

Mesh Analysis

Steps to Determine Mesh Currents:

1. Assign mesh currents to the *n* meshes.
2. Apply KVL to each of the *n* meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting *n* simultaneous equations to get the mesh currents.

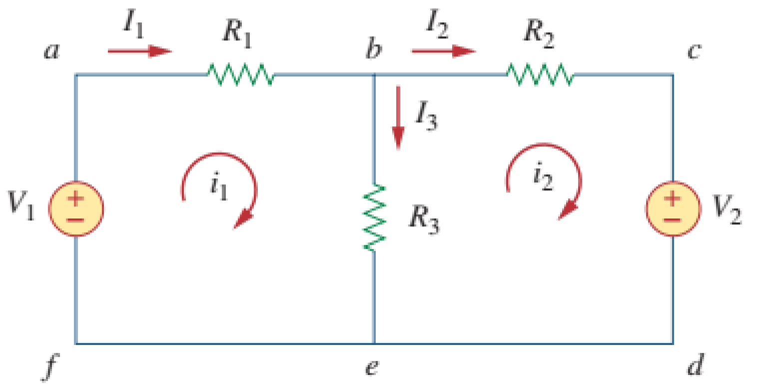


Figure 3.17
A circuit with two meshes.

To illustrate the steps, consider the circuit in Fig. 3.17. The first step requires that mesh currents and are assigned to meshes 1 and 2. Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.

As the second step, we apply KVL to each mesh. Applying KVL to mesh 1, we obtain

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

For mesh 2, applying KVL gives

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

$$\boxed{\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}}$$

Mesh Analysis

Example 3.6 Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.

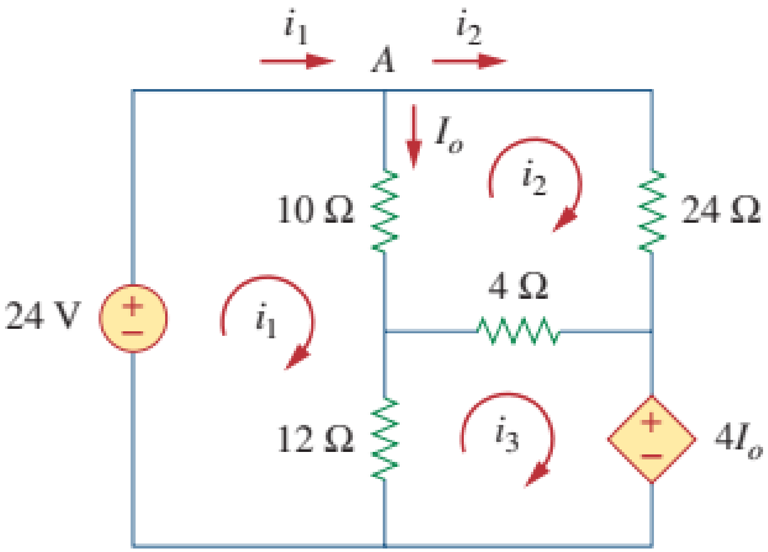


Figure 3.20
For Example 3.6.

Solution:
We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad (3.6.1)$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad (3.6.2)$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \qquad (3.6.3)$$

Mesh Analysis

Example 3.6 Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.

Solution:

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \tag{3.6.1}$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0 \tag{3.6.2}$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \tag{3.6.3}$$

In matrix form, Eqs. (3.6.1) to (3.6.3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A},$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $I_o = i_1 - i_2 = 1.5$ A.

Mesh Analysis

Practice Problem 3.6 Using mesh analysis, find I_o in the circuit of Fig. 3.21.

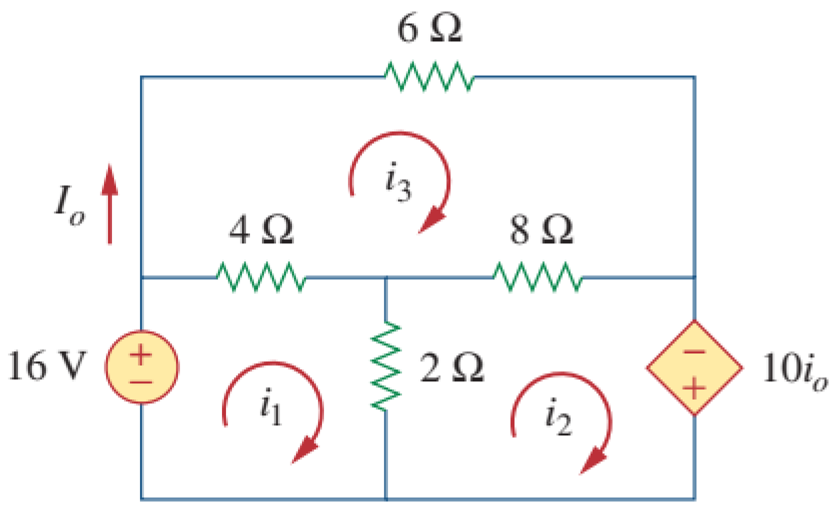


Figure 3.21
For Practice Prob. 3.6.

Home Work (B-1)

3.2 For the circuit in Fig. 3.51, obtain v_1 and v_2.

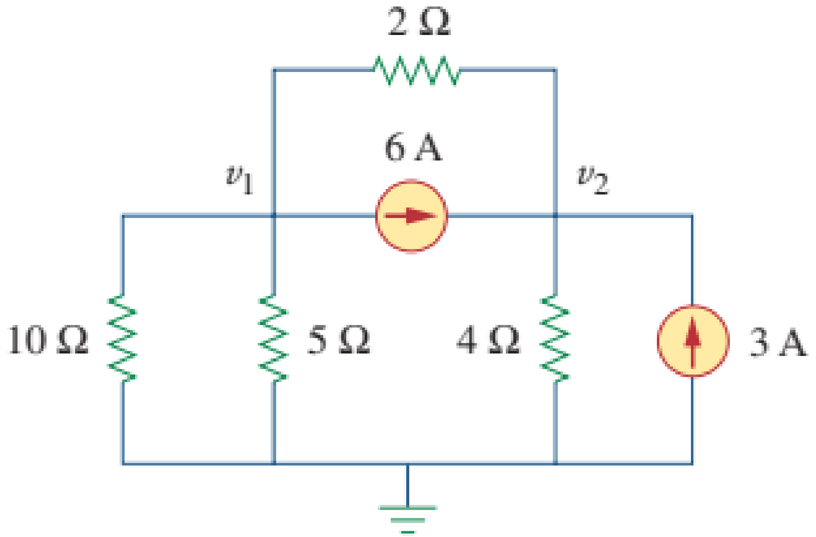


Figure 3.51
For Prob. 3.2.

3.43 Use mesh analysis to find v_{ab} and i_o in the circuit of Fig. 3.89.

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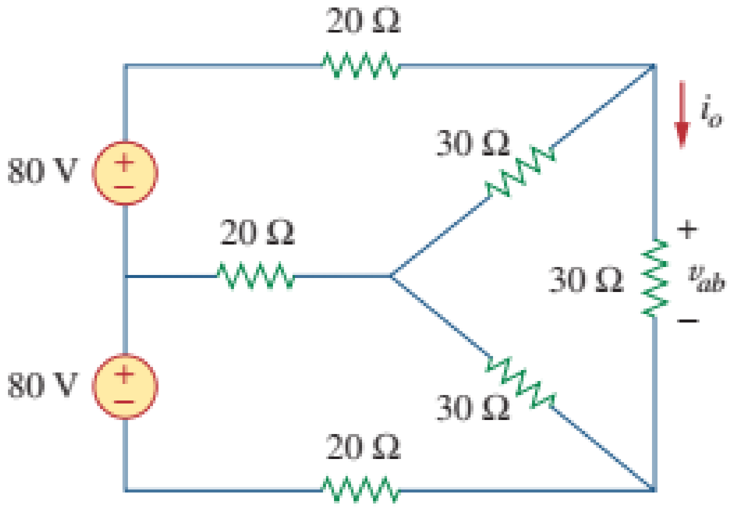

Figure 3.89
For Prob. 3.43.

END LESSON 3: METHODS OF CIRCUIT ANALYSIS

- Next Lesson…….
- Topics: Introduction to Circuit Theorems
- Text: B-1, Chapter 4