AC POWER ANALYSIS

Text: Fundamentals of Electric Circuits by Alexander and Sadiku

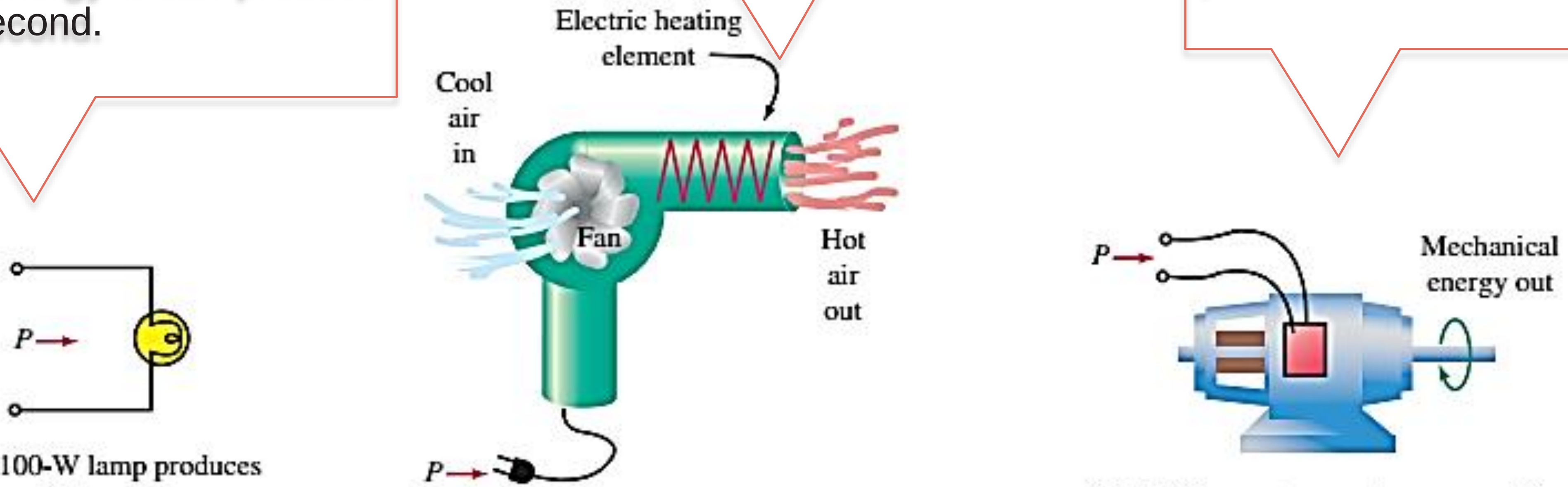
Chapter 11

POWER

The greater the power rating of the light, the more light energy it can produce per second.

The greater the power rating of the heater, the more heat energy it can produce per second.

The larger the power rating of the motor, the more mechanical work it can do per second.



(a) A 100-W lamp produces more light energy per second than a 40-W lamp

(b) Hair dryer

(c) A 10-hp motor can do more work in a given time than a ½-hp motor

Fig 01: Energy conversion, Power P is a measure of the rate of energy conversion.

POWER

- **Power** is an indication of how much work can be done in a specified amount of time.

Or a *rate* of doing work.

Or the rate of transfer of energy.

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t}$$

$$I = \frac{Q}{t}$$

$$\boxed{P = VI} \qquad \text{(watts)}$$

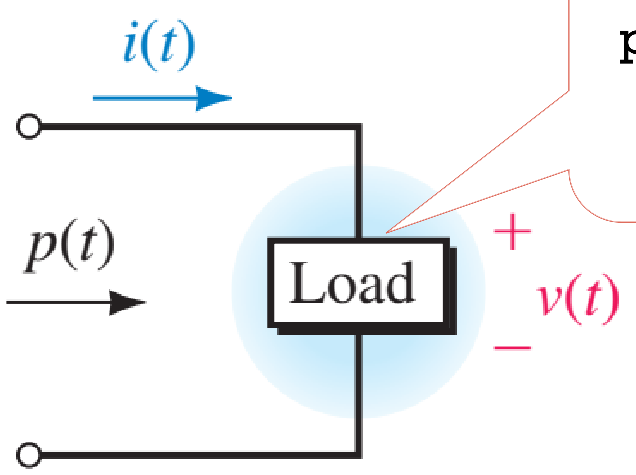
Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where

$$1 \text{ Wh} = 3{,}600 \text{ J}$$

- The SI unit of power is the watt (W) or joules per second (J/s).
- In dc circuits, the only power relationship is $p = VI = I^2R = V^2/R$
- This power is referred to as real power or active power and is the power that does useful work.

INSTANTANEOUS POWER



The instantaneous power is the power at any instant of time.
$p(t) = i(t)v(t)$

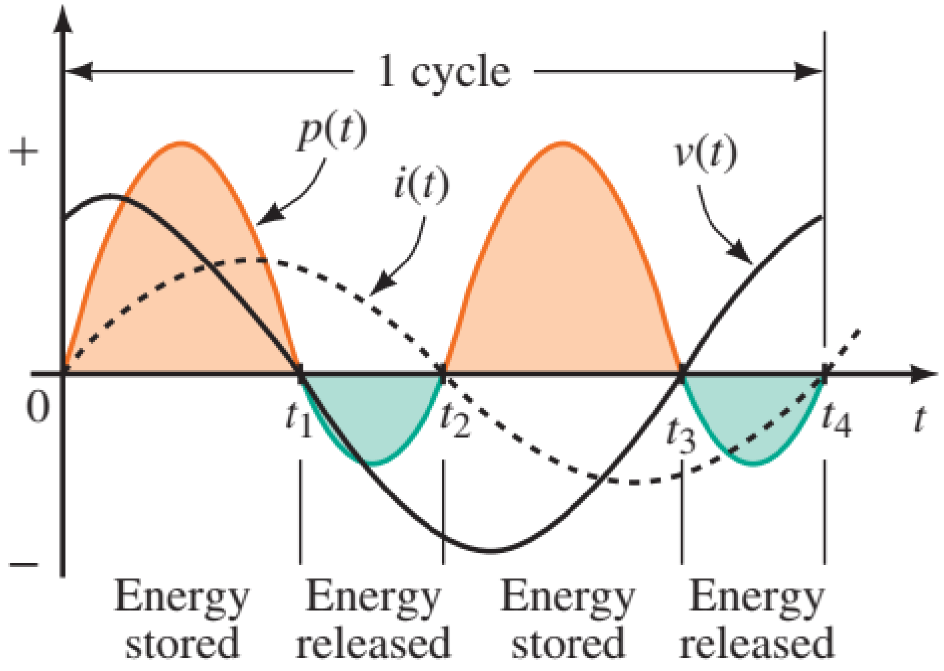


Fig 02: Instantaneous power in an ac circuit. Positive p represents power to the load; negative p represents power returned from the load.

- The waveform p(t) of Fig 02 is the actual power waveform.
- The key aspects of power flow can be described in terms of
  1. Active power,
  2. Reactive power and
  3. Apparent power.

ACTIVE POWER (P)

- Actually consumed or utilized in an AC Circuit and measured in watts (W) or kW of MW.
- Since p(t) in Fig 02 represents the power flowing to the load, its average will be the average power (P) to the load.
- If P is positive, then, on average, more power flows to the load than is returned from it.
- If P is zero, all power sent to the load is returned.
- Thus, if P has a positive value, it represents the power that is really dissipated by the load. For this reason, P is called real power or active power.
- Thus, active power is the average value of the instantaneous power, and the terms real power, true power, active power, and average power mean the same thing.

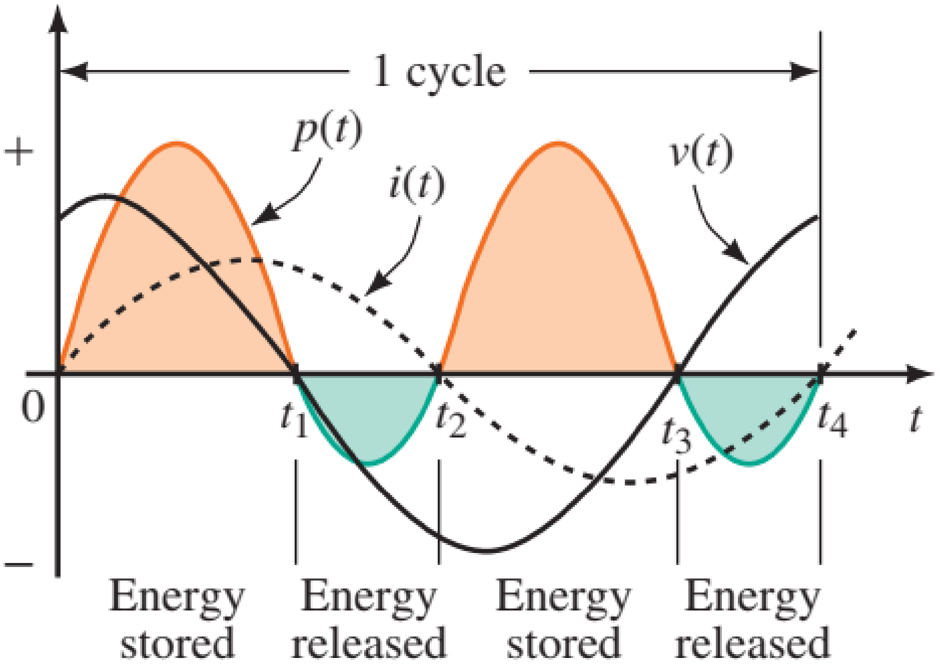


Fig 02: Instantaneous power in an ac circuit. Positive p represents power to the load; negative p represents power returned from the load.

REACTIVE POWER (Q)

- Flows back and forth in the circuit and measured in volt ampere reactive (VAR) or kVAR or MVAR.
- If p(t) is negative, then power is being returned from the load.
- The portion of power that flows into the load then back out is called reactive power (Q).
- Since it first flows one way then the other, its average value is zero.
- Although reactive power does no useful work, it cannot be ignored. Extra current is required to create reactive power, and this current must be supplied by the source; this also means that conductors, circuit breakers, switches, transformers, and other equipment must be made physically larger to handle the extra current. This increases the cost of a system. (This is one of the reasons that reactive power is a major concern of power system engineers.)
- Reactive Power represent that the energy is first stored and then released in the form of magnetic field or electrostatic field in case of inductor and capacitor respectively.

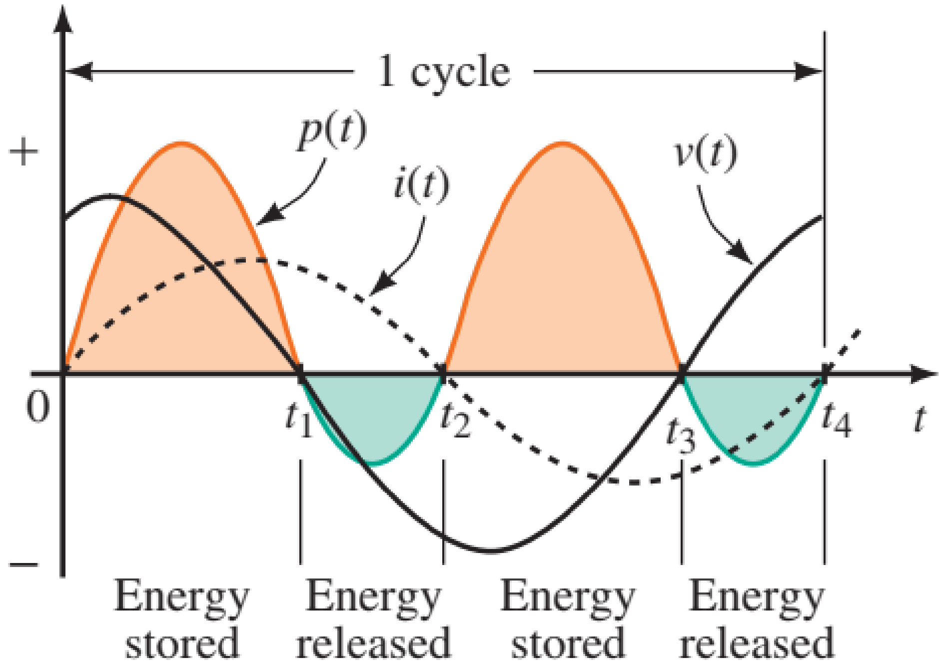


Fig 02: Instantaneous power in an ac circuit. Positive p represents power to the load; negative p represents power returned from the load.

APPARENT POWER (S)

- The combination of reactive power and active power is called apparent power (S) and computed by multiplying the root-mean-square (rms) current by the root-mean-square voltage.
- Apparent power is measured in the unit of Volt-Amps (VA) or kVA.

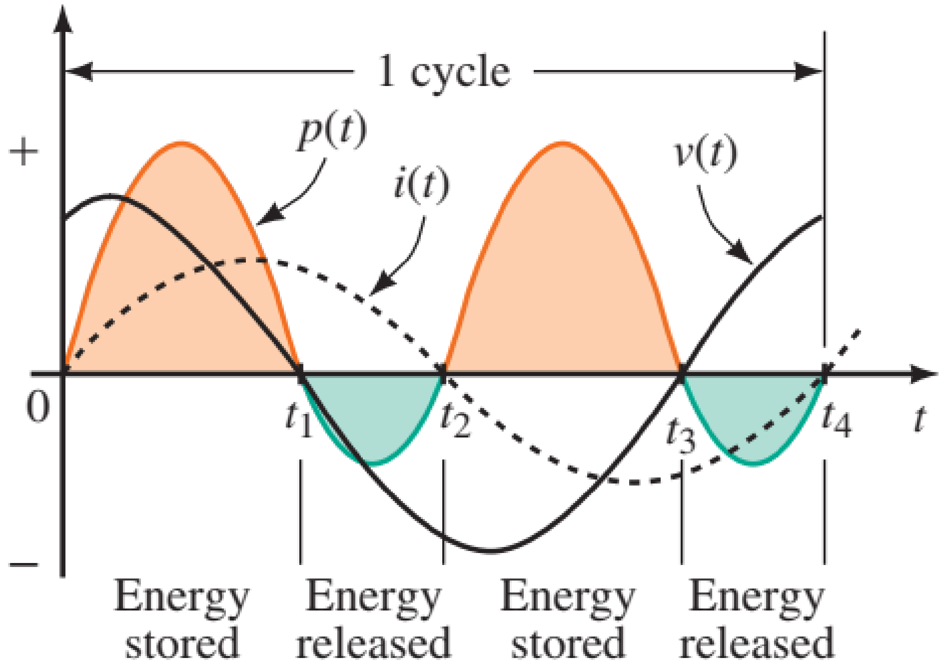


Fig 02: Instantaneous power in an ac circuit. Positive p represents power to the load; negative p represents power returned from the load.

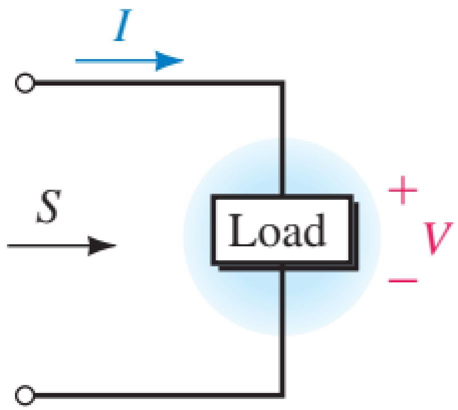


Fig 03: Apparent power, S = VI

Q1: What is real power? What is reactive power? Which power, real or reactive, has an average value of zero?

AVERAGE POWER OF RESISTIVE AND REACTIVE CIRCUIT

Suppose a network's (Fig 03) voltage and current are

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

Then its instantaneous power is $p(t) = v(t)i(t)$

$$p(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$p(t) = \underbrace{\frac{1}{2} V_m I_m \cos\,(\theta_v - \theta_i)}_{\text{Constant term}} + \underbrace{\frac{1}{2} V_m I_m \cos\,(2\omega t + \theta_v + \theta_i)}_{\text{Sinusoid whose amplitude } = \frac{1}{2} V_m I_m.}$$

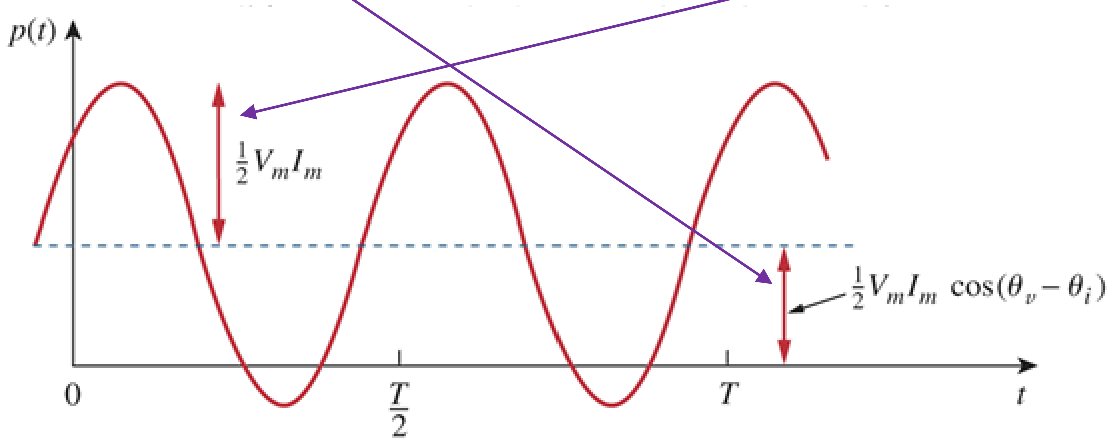


Fig 04: The Instantaneous power p(t) entering a circuit.

NOTES....

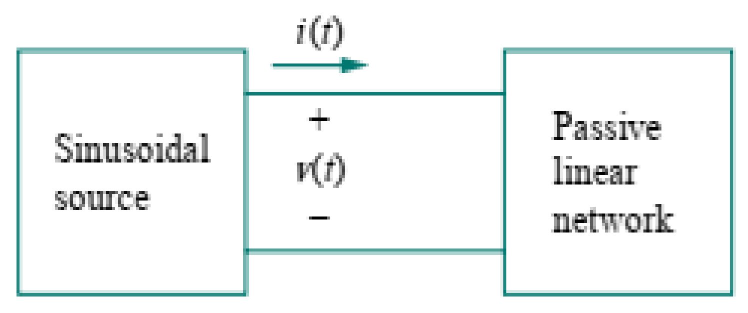


Fig 03: Sinusoidal source and passive linear network.

Trigonometric Identity

- $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

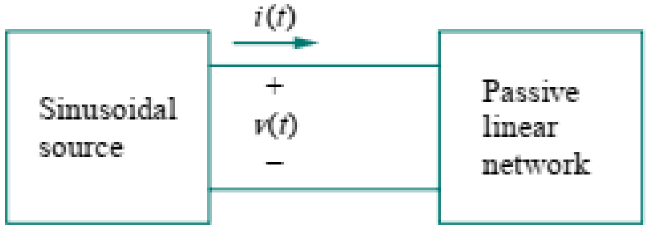
The instantaneous power p(t) has two parts.

1. Constant Part: Does not depend on time (t). We call it the average power (P).
2. Sinusoid Part: Frequency is twice the frequency of v(t) and i(t).

AVERAGE POWER OF RESISTIVE AND REACTIVE CIRCUIT

Let, $v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = I_m \cos(\omega t + \theta_i)$



Average power is given by

$$P = \frac{\int_0^T p(t)dt}{T} = \frac{\int_0^T [v(t)i(t)]dt}{T} = \frac{1}{T}\int_0^T [V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)]dt$$

$$P = \frac{1}{T}\int_0^T \left[\frac{1}{2}V_m I_m \cos\ (\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos\ (2\omega t + \theta_v + \theta_i)\right] dt$$

$$= \frac{1}{2}V_m I_m \cos\ (\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos\ (2\omega t + \theta_v + \theta_i)\, dt$$

Average of a sinusoid over its period is zero.

$$P = \frac{1}{2}V_m I_m \cos\ (\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + 0 == \frac{1}{2}V_m I_m\, cos\ (\theta_v - \theta_i)\frac{1}{T}[T - 0]$$

$$\boxed{P = \frac{1}{2}V_m I_m\, cos\ (\theta_v - \theta_i)}$$

NOTES....

- The instantaneous power changes with time and is therefore difficult to measure.
- The *average* power is more convenient to measure.
- In fact, the wattmeter, the instrument for measuring power, responds to average power.
- The average power is the average of the instantaneous power over one period
- The first integrand is constant, and the average of a constant is the same constant. The second integrand is a sinusoid.

AVERAGE POWER OF RESISTIVE AND REACTIVE CIRCUIT

For purely resistive circuit, Phase differences, $\theta = (\theta_v - \theta_i) = 0$,
So average power becomes,

$$P = \frac{1}{2} V_m I_m \cos\ (\theta_v - \theta_i) = \frac{1}{2} V_m I_m \cos\ 0° = \frac{1}{2} V_m I_m$$

$$P = \frac{1}{2} V_m I_m \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

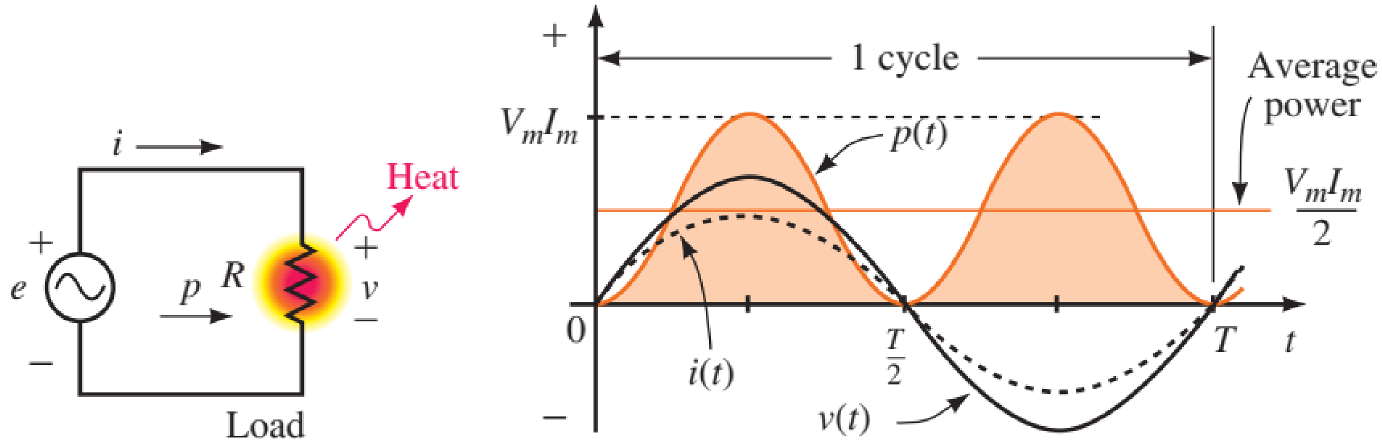


Fig 05: Power to a purely resistive load. The peak value of p(t) is $V_m I_m$

So, equation (1) shows that a purely resistive circuit absorbs power at all times.

NOTES....

$$P = \frac{1}{2} V_m I_m \cos\ (\theta_v - \theta_i)$$
$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\ \theta$$
$$= V_{rms} I_{rms} \cos\theta$$
$$\boxed{P = VI\cos\theta}$$

For resistive circuit, current and voltage are in phase.

$$P = VI = I^2 R = \frac{V^2}{R}$$

So, Active power $P_R = VI$

Reactive power $Q_R = 0$

AVERAGE POWER OF RESISTIVE AND REACTIVE CIRCUIT

For purely reactive circuit (L or C), Phase differences, $\theta = (\theta_v - \theta_i) = \pm 90°$
So average power becomes,

$$P = \frac{1}{2} V_m I_m \cos\ (\theta_v - \theta_i) = \frac{1}{2} V_m I_m \cos\ \pm 90° = 0 \ldots \ldots . . (2)$$

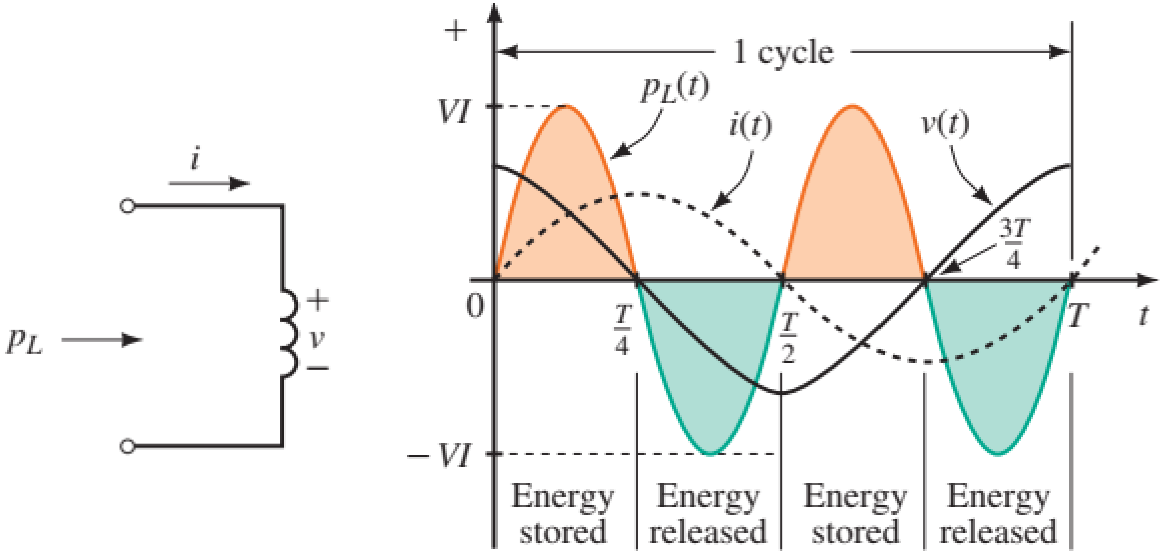


Fig 06: Power to a purely inductive load.

NOTES....

- For a purely inductive, current lags voltage by 90.
- From the instantaneous power, $p(t) = \frac{1}{2} V_m I_m \cos\ (\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos\ (2\omega t + \theta_v + \theta_i)$

$$p(t) = 0 + \frac{1}{2} V_m I_m \cos\ (2\omega t - 90°)$$

$$p(t) = 0 + V_{rms} I_{rms} \sin 2\omega t$$

$$p(t) = VI\ \sin 2\omega t$$

- So, Active power $P_L = 0$
- Reactive power

$$Q_L = VI$$

$$= I^2 X_L = \frac{V^2}{X_L} \ldots \ldots \ldots VAR\ (ind.)$$

AVERAGE POWER OF RESISTIVE AND REACTIVE CIRCUIT

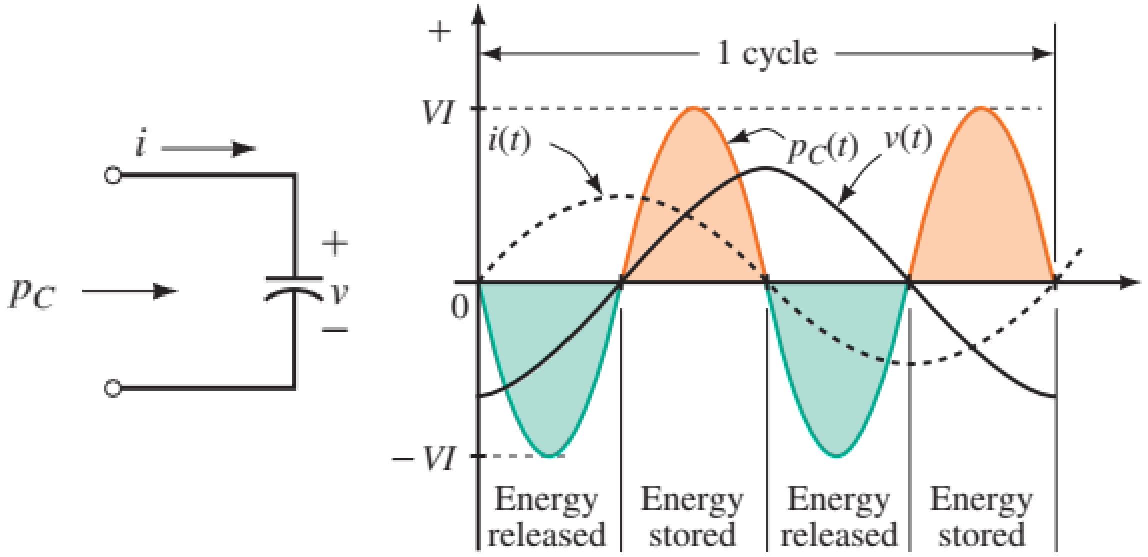


Fig 07: Power to a purely Capacitive load.

So, equation (3) shows that a purely reactive circuit absorbs zero average power.

Q02: Show that a purely resistive circuit (R) absorbs power at all times, while a purely reactive circuit (L or C) absorbs zero average power.

NOTES….

- For a purely capacitive load, current leads voltage by 90.
- From the instantaneous power, $p(t) = \frac{1}{2} V_m I_m \cos\ (\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos\ (2\omega t + \theta_v + \theta_i)$

$$p(t) = 0 + \frac{1}{2} V_m I_m \cos\ (2\omega t + 90°)$$

$$p(t) = 0 - V_{rms} I_{rms} \sin 2\omega t$$

$$p(t) = -VI\ \sin 2\omega t$$

- So, Active power $P_C = 0$
- Reactive power

$$Q_C = -VI$$

$$= -(I^2 X_C) = -\left(\frac{V^2}{X_C}\right) \dots \dots ..VAR$$

Or, $Q_C = VI \dots \dots VAR\ (cap.)$

APPARENT POWER & POWER FACTOR

As we know that, average power

$$P = \frac{1}{2} V_m I_m \cos\ (\theta_v - \theta_i)$$
$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

The product $V_{rms} I_{rms}$ is known as apparent power, S=$V_{rms} I_{rms}$=VI

The factor $\cos(\theta_v - \theta_i)$ is called the power factor (pf) and $(\theta_v - \theta_i)$ is called power factor angle.

$$P = S \cos(\theta_v - \theta_i)$$

$$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

NOTES....

- If the voltage and current of a circuit are:
  $v(t) = V_m \cos(\omega t + \theta_v)$
  $i(t) = I_m \cos(\omega t + \theta_i)$
- Power factor angle $(\theta_v - \theta_i)$ is equal to the angle of load impedance.

$$\vec{Z} = \frac{\vec{V}}{\vec{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i}$$

$$\vec{Z} = \frac{\sqrt{2} V_{rms} \angle \theta_v}{\sqrt{2} I_{rms} \angle \theta_i} = \frac{V_{rms}}{I_{rms}} \angle(\theta_v - \theta_i)$$

APPARENT POWER & POWER FACTOR

Power Factor: The cosine of angle between voltage and current in an a.c. circuit. Denoted by pf or F_p

$$\boxed{pf = cos(\theta_v - \theta_i) = cos\,\theta}$$

- For a purely resistive load, $\theta_v - \theta_i = 0, pf = 1\ and\ P = S$. This case (pf = 1) is referred to as **unity power factor**.
- For a purely reactive load, $\theta_v - \theta_i = 90°, pf = 0\ and\ P = 0$.
- For a load containing only resistance and inductance, the load current lags voltage. The power factor in this case is described as ***lagging power factor.***
- On the other hand, for a load containing only resistance and capacitance, current leads voltage and the power factor is described as ***leading power factor.***

NOTES….

- Resistive load,
  $P = S\,cos(\theta_v - \theta_i) = Scos0°$
- *An inductive circuit has a lagging power factor, while a capacitive circuit has a leading power factor.*
- A load's power factor (pf) shows how much of its apparent power is actually consumed.

$$pf = \frac{P}{S}$$

COMPLEX POWER, ($\bar{S}$)

➢Complex power is the product of the V_{rms} phasor and the complex conjugate of the I_{rms} phasor.

$$\bar{S} = \bar{V}_{rms}\bar{I}^{*}_{rms} = \bar{I}_{rms}\bar{Z} \times \bar{I}^{*}_{rms} = (I_{rms}\angle\theta_i \times I_{rms}\angle{-\theta_i})(R + jX) = I^2_{rms}(R + jX)$$

$$\bar{S} = \underbrace{I^2_{rms}R}_{} + j\underbrace{I^2_{rms}X}_{} = P + jQ \;\ldots\ldots(3)$$

Imaginary Part: Reactive power, represents a lossless interchange between the load and the source

Real Part: Active power, only useful power dissipated by the load.

Again, $\bar{S} = \bar{V}_{rms}\bar{I}^{*}_{rms} = V_{rms}\angle\theta_v \times I_{rms}\angle-\theta_i = V_{rms}I_{rms}\angle(\theta_v - \theta_i)$

$$\bar{S} = \underbrace{V_{rms}I_{rms}\cos(\theta_v - \theta_i)}_{P} + j\underbrace{V_{rms}I_{rms}\sin(\theta_v - \theta_i)}_{Q} \;\ldots\ldots(4)$$

Comparing equation (3) & (4)

$$P = V_{Rms}I_{Rms}\cos(\theta_v - \theta_i) = \mathrm{Re}\{\mathbf{S}\} = I^2_{Rms}R$$

$$Q = V_{Rms}I_{Rms}\sin(\theta_v - \theta_i) = \mathrm{Im}\{\mathbf{S}\} = I^2_{Rms}X$$

NOTES....

- *Consider, the current and voltage as follows:*

  $v(t) = V_m\cos(\omega t + \theta_v)$

  $i(t) = I_m\cos(\omega t + \theta_i)$

- And the phasor forms are

  $\bar{V} = V_m\angle\theta_v$

  $\bar{I} = I_m\angle\theta_I$

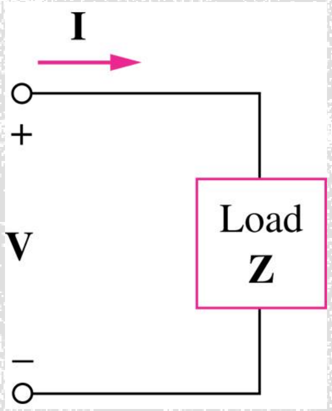


Fig 08: The voltage and current phasors associated with a load.

COMPLEX POWER, ($\bar{S}$)

NOTES....

- The COMPLEX Power contains all the information pertaining to the power absorbed by a given load.

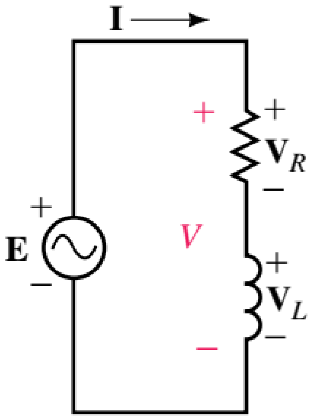
$$\text{Complex Power} = \mathbf{S} = P + jQ = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$
$$= V_{\text{rms}} I_{\text{rms}} \underline{/\theta_v - \theta_i}$$
$$\text{Apparent Power} = S = |\mathbf{S}| = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2}$$
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$
$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

Quantity	Symbol	SI Unit	Symbol for the Unit
Instantaneous power	$p(t)$	watt	W
Average power (also called real power)	P	watt	W
Apparent power	S	volt-ampere	VA
Complex power	$\mathbf{S}$	volt-ampere	VA
Reactive power	Q	volt-ampere reactive	VAR

RELATIONSHIP BETWEEN P, Q AND S

Development of Power triangle:



KVL, $\overline{V} = \overline{V}_R + \overline{V}_L$

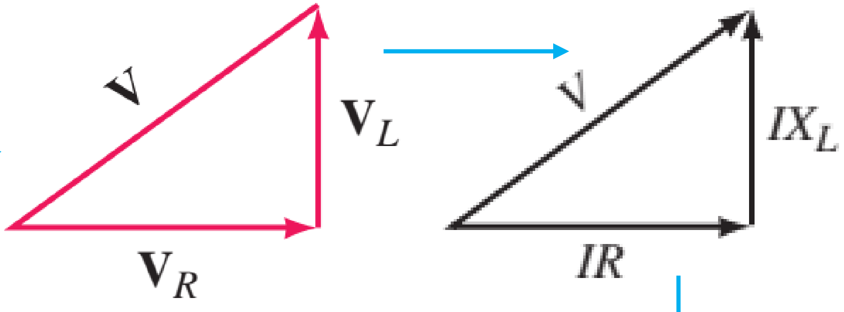
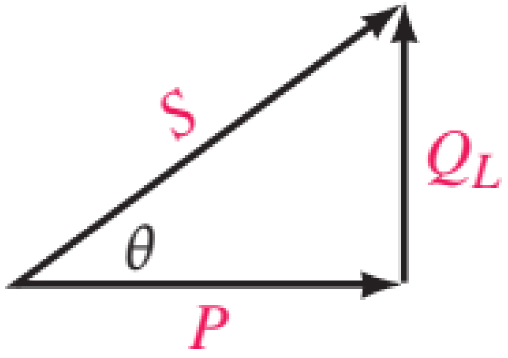


Fig 10: Consider a series circuit.

Fig 11: Phasor Diagram.



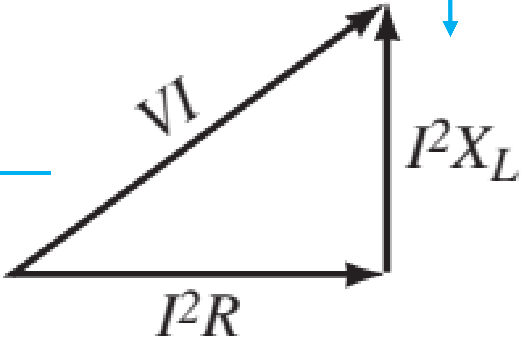


Fig 13: Resultant power triangle.

Fig 12: Phasor Diagram, Multiplied by I.

NOTES….

Standard practice to represent S, P and Q in the form of a triangle, known as the power triangle, shown in Fig. 09.

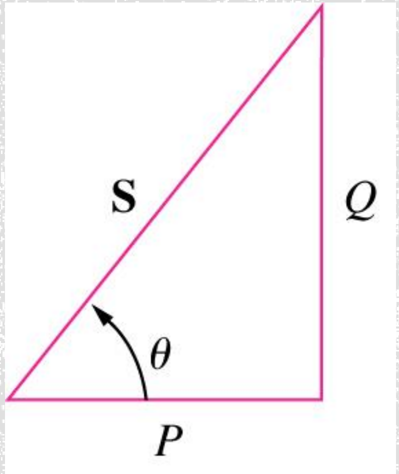


Fig 09: Power triangle which has four items P, Q, S and θ.

RELATIONSHIP BETWEEN P, Q AND S

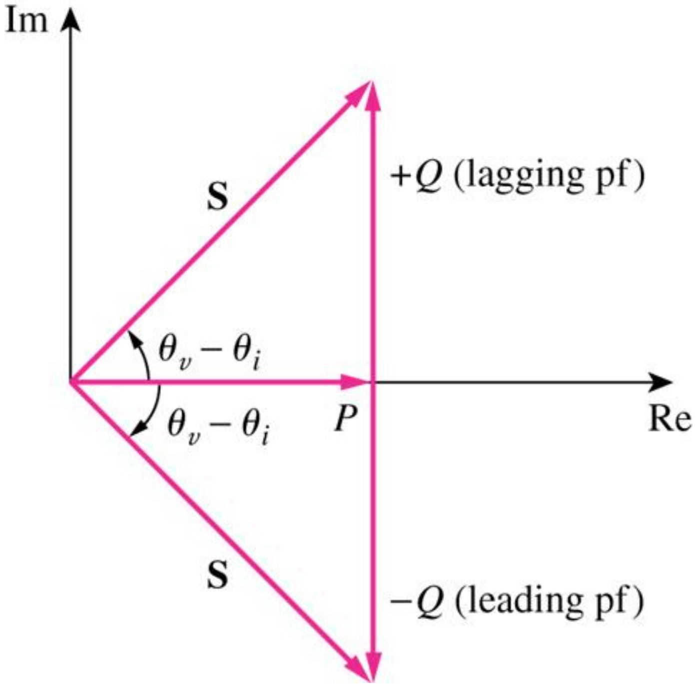


Fig 14: Power triangle of Inductive and capacitive load.

$$\bar{S} = P + jQ \; for \; inductive$$
$$\bar{S} = P - jQ \; for \; capacitive$$
$$\bar{S} = S\angle\theta$$

From the geometry of power triangle,

$$S = \sqrt{P^2 + Q^2}$$
$$\tan\theta = \frac{Q}{P}$$
$$\cos\theta = \frac{P}{S} = pf$$
$$\sin\theta = \frac{Q}{S} = pf$$

NOTES....

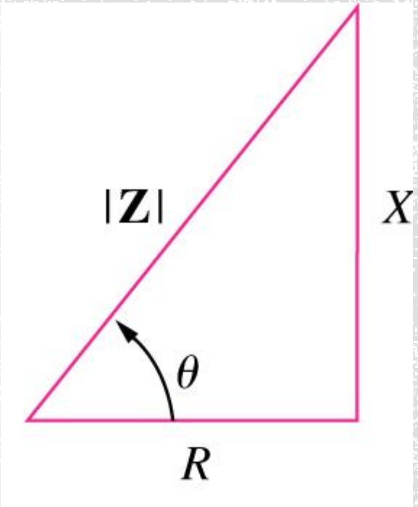


Fig 15: Impedance triangle.

CAUSES OF LOW POWER FACTOR

The following are the causes of low power factor:

(i) Most of the a.c. motors are of induction type (1φ and 3φ induction motors) which have low lagging power factor. These motors work at a power factor which is extremely small on light load (0·2 to 0·3) and rises to 0·8 or 0·9 at full load.

(ii) Arc lamps, electric discharge lamps and industrial heating furnaces operate at low lagging power factor.

(iii) The load on the power system is varying ; being high during morning and evening and low at other times. During low load period, supply voltage is increased which increases the magnetization current. This results in the decreased power factor.

DISADVANTAGES OF LOW POWER FACTOR

1. Large kVA rating of equipment: Power equipment is always rated in volt-amperes (VA).

$$S = \frac{P}{\cos\theta} \; or \; VA = \frac{kW}{\cos\theta}$$

At low power factor, the kVA rating of the equipment has to be made more, making the equipment larger and expensive.

2. Greater conductor size: To transmit or distribute a fixed amount of power at constant voltage, the conductor will have to carry more current ($I = \frac{P}{V\cos\theta}$) at low power factor. This necessitates large conductor size.

3. Large copper losses: The large current at low power factor causes more I^2R losses in all the elements of the supply system. This results in poor efficiency.

4. Poor voltage regulation: The large current at low lagging power factor causes greater voltage drops in alternators, transformers, transmission lines and distributors. This results in the decreased voltage available at the supply end, thus impairing the performance of utilization devices.

5. Reduced handling capacity of system: The lagging power factor reduces the handling capacity of all the elements of the system. It is because the reactive component of current prevents the full utilization of installed capacity.

POWER FACTOR CORRECTION

Increasing the power factor without altering the voltage or current to the load is called Power Factor Improvement (PFI) or Correction.

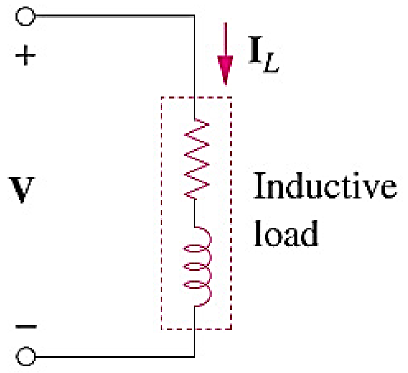


Fig 15: Original Inductive Load

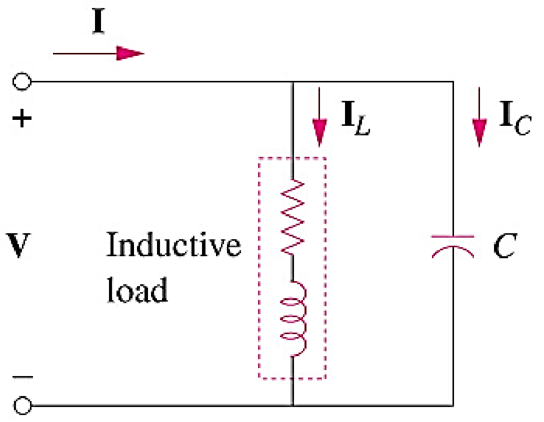


Fig 16: Inductive Load with improved power factor correction

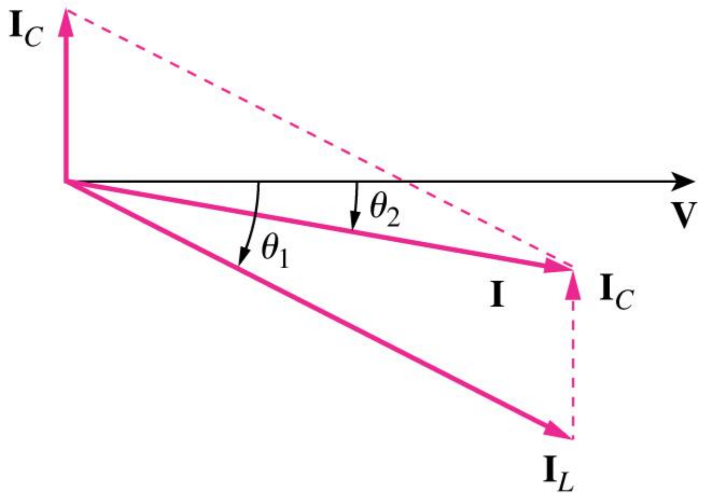


Fig 17: Effect of capacitor on total current.

NOTES….

- Transmission system is very sensitive to the magnitude of the current in the lines.
- Increased currents result in increased power losses.
- Heavier currents also require larger conductors.
- Most domestic and industrial loads are inductive.
- The inductive nature of the load cannot be changed, we can increase its power factor.
- PFI Process: Devices which taking leading power should be connected in parallel with the load.

POWER FACTOR CORRECTION

Increasing the power factor without altering the voltage or current to the load is called Power Factor Improvement (PFI) or Correction.

$$Q_C = Q_1 - Q_2$$

$$\frac{V^2}{X_C} = P\tan\theta_1 - P\tan\theta_2$$

$$\omega C V^2 = P(\tan\theta_1 - \tan\theta_2)$$

$$\therefore C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V^2}$$

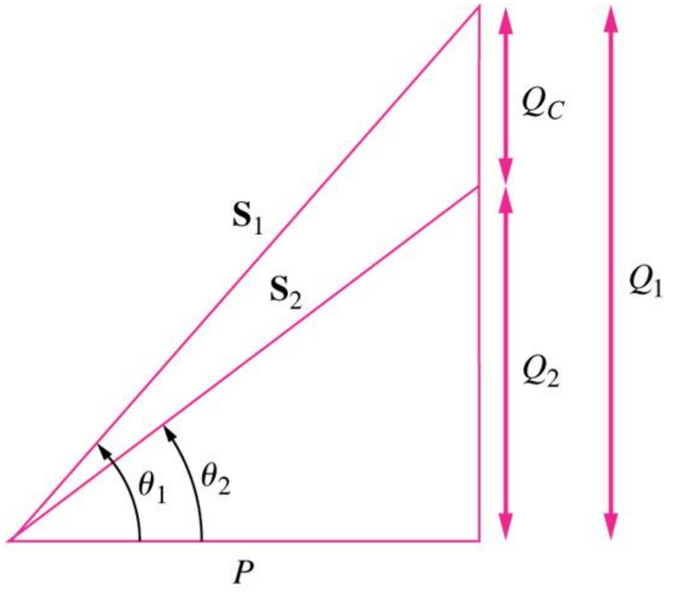


Fig 18: Power triangle of power factor correction.

Similarly the inductance value needed to change the pf angle from θ_1 to θ_2 for a capacitive load.

$$Q_L = Q_1 - Q_2 \qquad Q_L = \frac{V^2}{X_L} = \frac{V^2}{\omega L}$$

$$L = \frac{V^2}{\omega Q_L}$$

Q03: WHY CAPACITOR IS CONNECTED IN PARALLEL NOT IN SERIES FOR POWER FACTOR IMPROVEMENT?

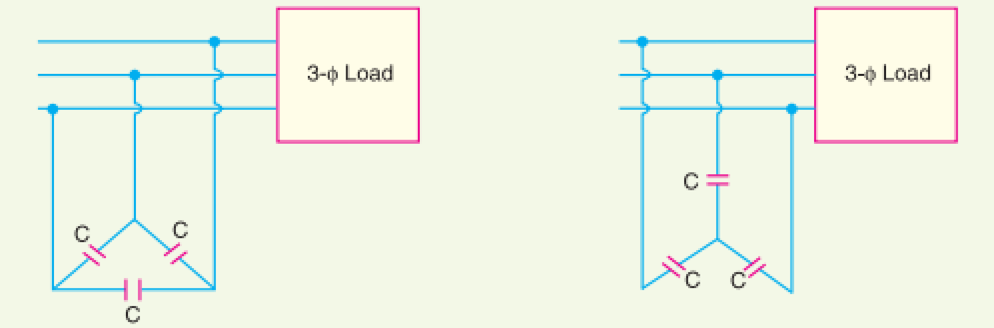
Answer:

1. In **series connection c**urrent is constant and voltage is varying but in **parallel connection**, voltage is constant and current is varying. To keep constant the voltage across the load, parallel connection is required.
2. Connecting a series capacitor will actually boost up the voltage at the consumer end by compensating for the transmission line inductance (Series Compensation). Using too small a capacitance value in series will result in overcompensation and voltages larger than the source voltage may appear across the load (Ferranti Effect). So, one can connect capacitors both in series and shunt. Series compensation is used to improve voltage regulation at the load and shunt compensation is used for PF improvement.
3. If we connect the capacitor in series with the load then if short circuit fault occurs in the load then the total voltage will be applied to the capacitor, which may blow them.
4. In case of series connection, if we want to connect additional capacitor then we need to open the whole circuit. However, in case of parallel connection, we can easily connect an additional capacitor in parallel with the existing capacitor.

POWER FACTOR IMPROVEMENT EQUIPMENT

1. **Static capacitors**.
2. Synchronous condenser.
3. Phase advancers.

1. Static capacitor. The power factor can be improved by connecting capacitors in parallel with the equipment operating at lagging power factor. For three-phase loads, the capacitors can be connected in delta or star.



Advantages

(*i*) Low losses.
(*ii*) Require little maintenance as there are no rotating parts.
(*iii*) Can be easily installed.

Disadvantages

(*i*) Short service life ranging from 8 to 10 years.
(*ii*) Easily damaged if the voltage exceeds the rated value.
(*iii*) Once the capacitors are damaged, their repair is uneconomical.

Fig 19: Static capacitor bank used for PFI.

POWER FACTOR IMPROVEMENT EQUIPMENT

2. Synchronous condenser. A synchronous motor takes a leading current when over-excited and, therefore, behaves as a capacitor and known as *synchronous condenser.*

Advantages

(*i*) Stepless control of power factor.
(*ii*) The motor windings have high thermal stability to short circuit currents.
(*iii*) The faults can be removed easily.

Disadvantages

(*i*) There are considerable losses in the motor.
(*ii*) The maintenance cost is high.
(*iii*) It produces noise.
(*iv*) Except in sizes above 500 kVA, the cost is greater than that of static capacitors of the same rating.
(*v*) As a synchronous motor has no self-starting torque, therefore, an auxiliary equipment has to be provided for this purpose.

1. Static capacitors.
2. **Synchronous condenser.**
3. Phase advancers.

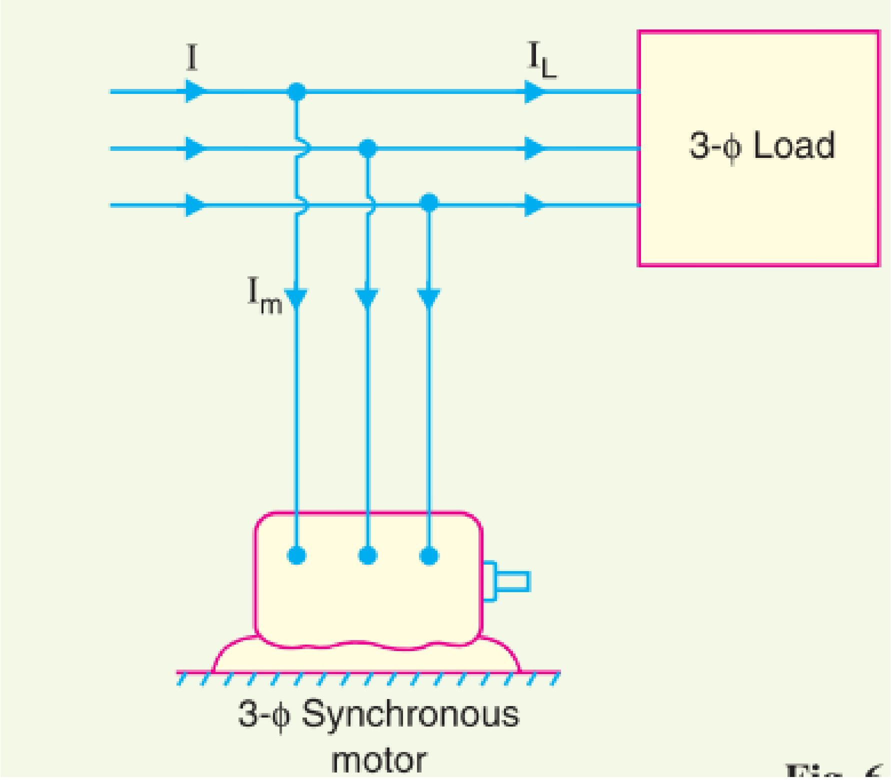


Fig 20: Synchronous condenser used for PFI.

POWER FACTOR IMPROVEMENT EQUIPMENT

3. Phase advancers. Phase advancers which is simply an a.c. exciter are used to improve the power factor of induction motors. It is mounted on the same shaft as the main motor and is connected in the rotor circuit of the motor.

Advantages:

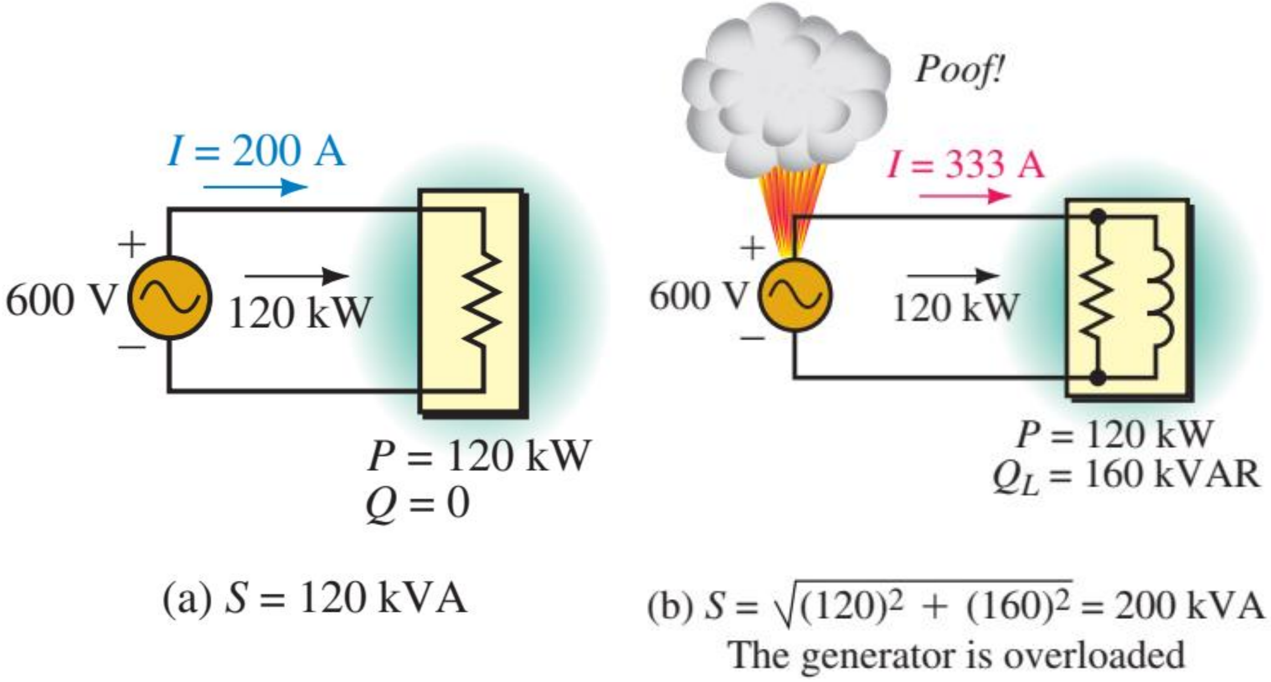
1. As the exciting ampere turns are supplied at slip frequency, therefore, lagging kVAR drawn by the motor are considerably reduced.
2. Phase advancer can be conveniently used where the use of synchronous motors is unadmissible.

Disadvantage: They are not economical for motors below 200 H.P.

1. Static capacitors.
2. Synchronous condenser.
3. **Phase advancers.**

POWER EQUIPMENT RATING

- Power equipment: Generators, interconnecting wires, transformers, etc.
- Rated in volt-amperes, VA or kVA and not in watts.



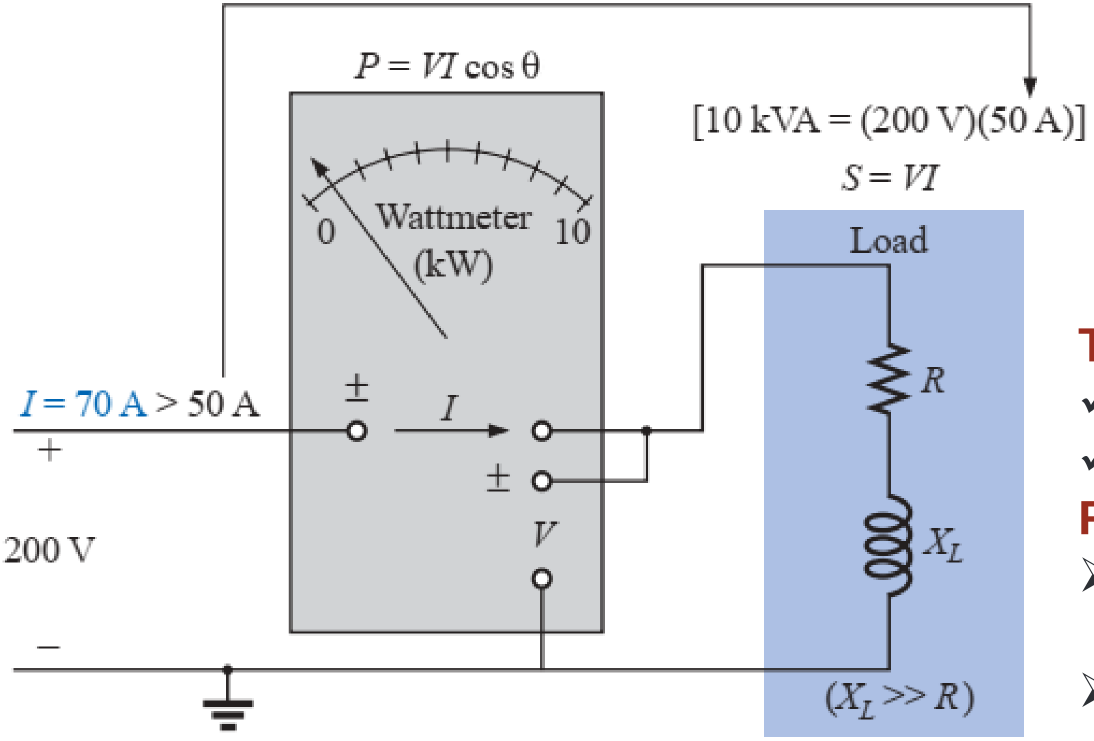


Fig 19: Demonstrating the reason for rating a load in kVA rather than kW.

To determine maximum current rating, it is required to know:

✓ The VA and the rated voltage or

✓ The wattage rating and the power factor

Reasons of VA rating:

➢ The power factor is sometimes not available, or it may vary with the load.

➢ So, the designer doesn't know the actual consumer power factor while manufacturing transformers and generators

PROBLEMS

Example 01: [Fundamentals of Electric Circuits by Alexander and Sadiku: Example 11.3]

For the circuit shown in Fig. 11.3, find the average power supplied by the source and the average power absorbed by the resistor.

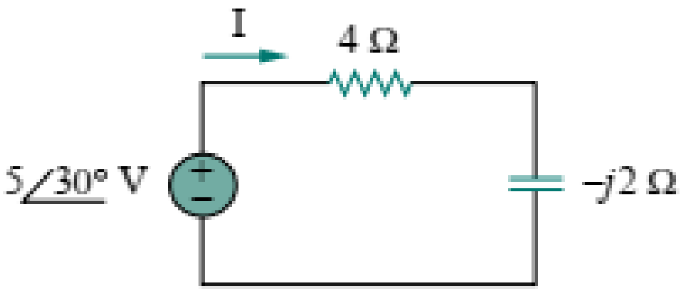


Figure 11.3 For Example 11.3.

PROBLEMS

Example 02: [Fundamentals of Electric Circuits by Alexander and Sadiku: Example 11.4]

Determine the power generated by each source and the average power absorbed by each passive element in the circuit of Fig. 11.5(a).

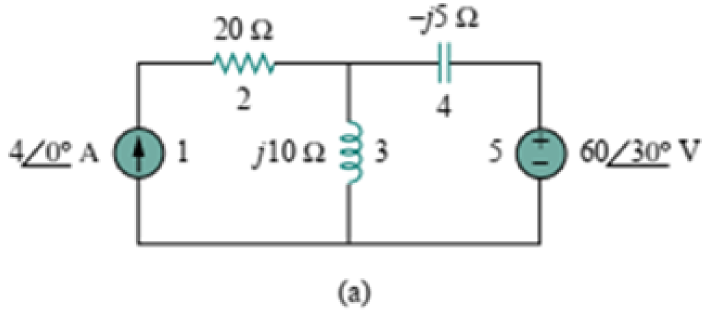


Figure 11.5 For Example 11.4.

Example 03: [Fundamentals of Electric Circuits by Alexander and Sadiku: Example 11.11]

The voltage across a load is $v(t) = 60\cos(\omega t - 10^\circ)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50^\circ)$ A. Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

Solution:

(a) For the rms values of the voltage and current, we write

$$\mathbf{V}_{rms} = \frac{60}{\sqrt{2}}\underline{/-10^\circ}, \qquad \mathbf{I}_{rms} = \frac{1.5}{\sqrt{2}}\underline{/+50^\circ}$$

The complex power is

$$\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = \left(\frac{60}{\sqrt{2}}\underline{/-10^\circ}\right)\left(\frac{1.5}{\sqrt{2}}\underline{/-50^\circ}\right) = 45\underline{/-60^\circ} \text{ VA}$$

The apparent power is

$$S = |\mathbf{S}| = 45 \text{ VA}$$

(b) We can express the complex power in rectangular form as

$$\mathbf{S} = 45\underline{/-60^\circ} = 45[\cos(-60^\circ) + j\sin(-60^\circ)] = 22.5 - j38.97$$

Since $\mathbf{S} = P + jQ$, the real power is

$$P = 22.5 \text{ W}$$

while the reactive power is

$$Q = -38.97 \text{ VAR}$$

(c) The power factor is

$$\text{pf} = \cos(-60^\circ) = 0.5 \text{ (leading)}$$

It is leading, because the reactive power is negative. The load impedance is

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{60\underline{/-10^\circ}}{1.5\underline{/+50^\circ}} = 40\underline{/-60^\circ}\ \Omega$$

which is a capacitive impedance.

Example 04: [Introductory Circuit Analysis by Boylestad: Example 19.2]

a. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p for the network of Fig. 19.19.
b. Sketch the power triangle.
c. Find the energy dissipated by the resistor over one full cycle of the input voltage if the frequency of the input quantities is 60 Hz.
d. Find the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve for each if the frequency of the input quantities is 60 Hz.

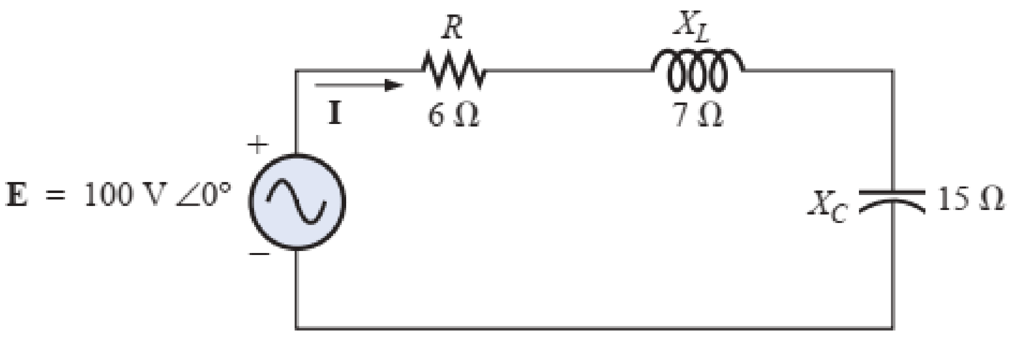


FIG. 19.19
Example 19.2.

Solutions:

a.
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\text{ V}\angle 0°}{6\ \Omega + j\,7\ \Omega - j\,15\ \Omega} = \frac{100\text{ V}\angle 0°}{10\ \Omega\angle -53.13°} = 10\text{ A}\angle 53.13°$$

$$\mathbf{V}_R = (10\text{ A}\angle 53.13°)(6\ \Omega\angle 0°) = 60\text{ V}\angle 53.13°$$
$$\mathbf{V}_L = (10\text{ A}\angle 53.13°)(7\ \Omega\angle 90°) = 70\text{ V}\angle 143.13°$$
$$\mathbf{V}_C = (10\text{ A}\angle 53.13°)(15\ \Omega\angle -90°) = 150\text{ V}\angle -36.87°$$

$$P_T = EI\cos\theta = (100\text{ V})(10\text{ A})\cos 53.13° = \mathbf{600\ W}$$
$$= I^2R = (10\text{ A})^2(6\ \Omega) = \mathbf{600\ W}$$
$$= \frac{V_R^2}{R} = \frac{(60\text{ V})^2}{6} = \mathbf{600\ W}$$

$$S_T = EI = (100\text{ V})(10\text{ A}) = \mathbf{1000\ VA}$$
$$= I^2Z_T = (10\text{ A})^2(10\ \Omega) = \mathbf{1000\ VA}$$
$$= \frac{E^2}{Z_T} = \frac{(100\text{ V})^2}{10\ \Omega} = \mathbf{1000\ VA}$$

$$Q_T = EI\sin\theta = (100\text{ V})(10\text{ A})\sin 53.13° = \mathbf{800\ VAR}$$
$$= Q_C - Q_L$$
$$= I^2(X_C - X_L) = (10\text{ A})^2(15\ \Omega - 7\ \Omega) = \mathbf{800\ VAR}$$

Example 04: [Introductory Circuit Analysis by Boylestad: Example 19.2]

a. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p for the network of Fig. 19.19.
b. Sketch the power triangle.
c. Find the energy dissipated by the resistor over one full cycle of the input voltage if the frequency of the input quantities is 60 Hz.
d. Find the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve for each if the frequency of the input quantities is 60 Hz.

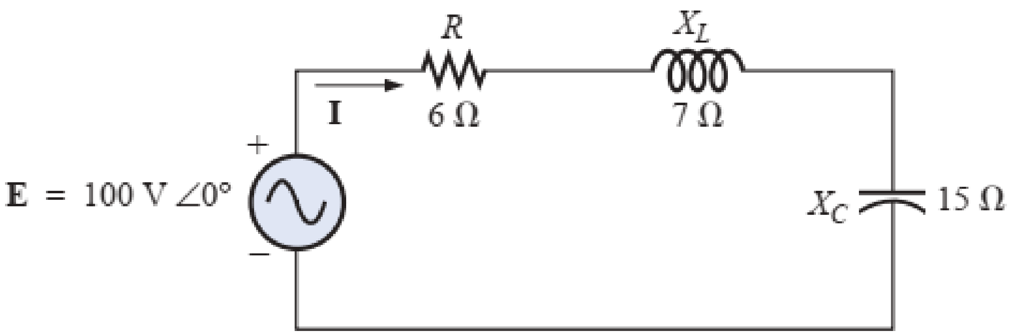


FIG. 19.19
Example 19.2.

Solutions:

$$Q_T = \frac{V_C^2}{X_C} - \frac{V_L^2}{X_L} = \frac{(150\text{ V})^2}{15\ \Omega} - \frac{(70\text{ V})^2}{7\ \Omega}$$

$$= 1500\text{ VAR} - 700\text{ VAR} = \mathbf{800\ VAR}$$

$$F_p = \frac{P_T}{S_T} = \frac{600\text{ W}}{1000\text{ VA}} = \mathbf{0.6\ leading\ (}C\mathbf{)}$$

b. The power triangle is as shown in Fig. 19.20.

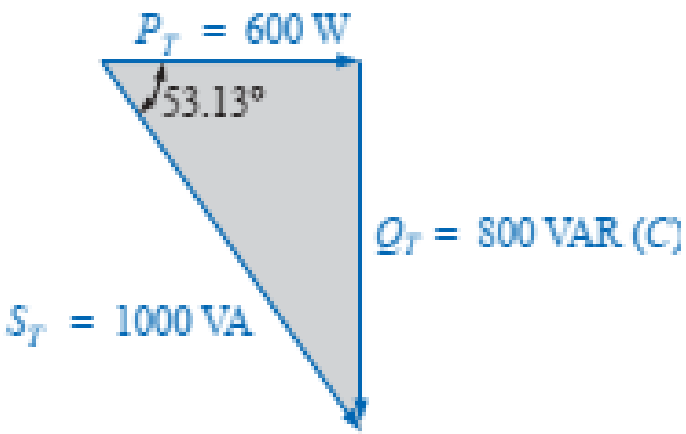


FIG. 19.20
Power triangle for Example 19.2.

c. $$W_R = \frac{V_R I}{f_1} = \frac{(60\text{ V})(10\text{ A})}{60\text{ Hz}} = \mathbf{10\ J}$$

d. $$W_L = \frac{V_L I}{\omega_1} = \frac{(70\text{ V})(10\text{ A})}{(2\pi)(60\text{ Hz})} = \frac{700\text{ J}}{377} = \mathbf{1.86\ J}$$

$$W_C = \frac{V_C I}{\omega_1} = \frac{(150\text{ V})(10\text{ A})}{377\text{ rad/s}} = \frac{1500\text{ J}}{377} = \mathbf{3.98\ J}$$

ENERGY DISSIPATED BY THE RESISTOR

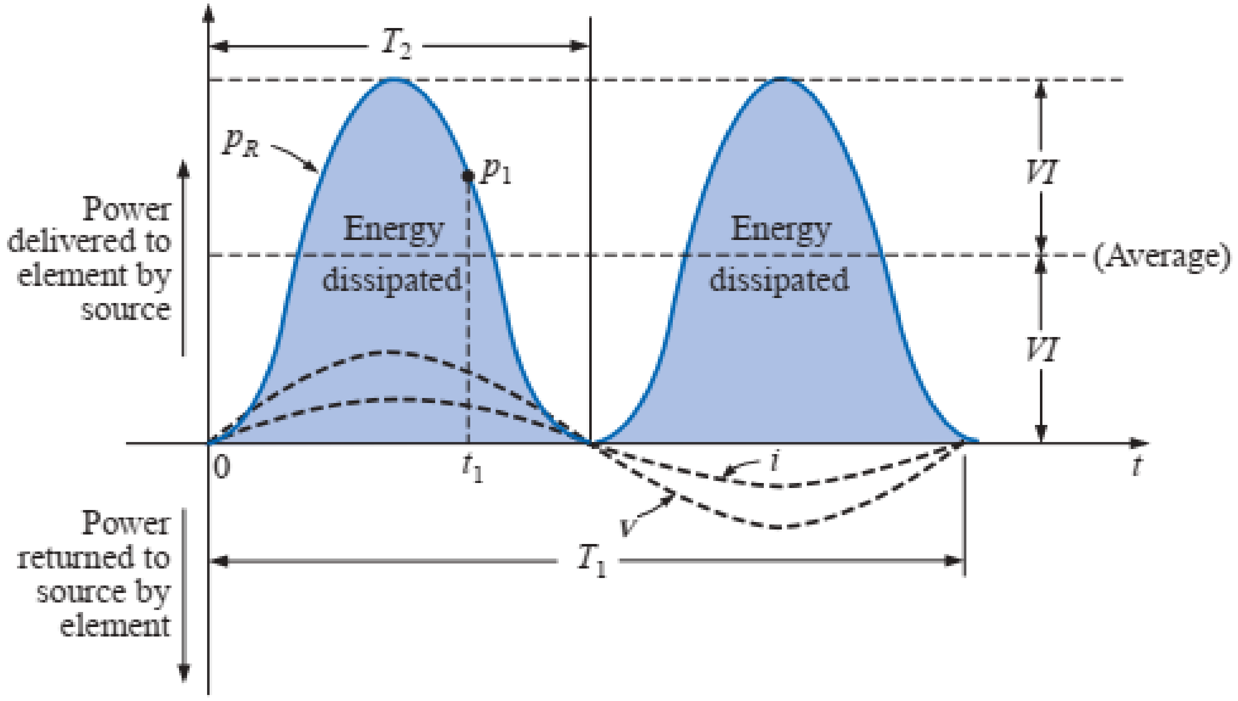


FIG. 19.3
Power versus time for a purely resistive load.

$$\boxed{P = VI = \frac{V_m I_m}{2} = I^2R = \frac{V^2}{R}} \qquad \text{(watts, W)} \qquad \textbf{(19.3)}$$

as derived in Chapter 14.

The energy dissipated by the resistor (W_R) over one full cycle of the applied voltage (Fig. 19.3) can be found using the following equation:

$$W = Pt$$

where P is the average value and t is the period of the applied voltage; that is,

$$\boxed{W_R = VIT_1} \qquad \text{(joules, J)} \qquad \textbf{(19.4)}$$

or, since $T_1 = 1/f_1$,

$$\boxed{W_R = \frac{VI}{f_1}} \qquad \text{(joules, J)} \qquad \textbf{(19.5)}$$

ENERGY STORED BY THE INDUCTOR

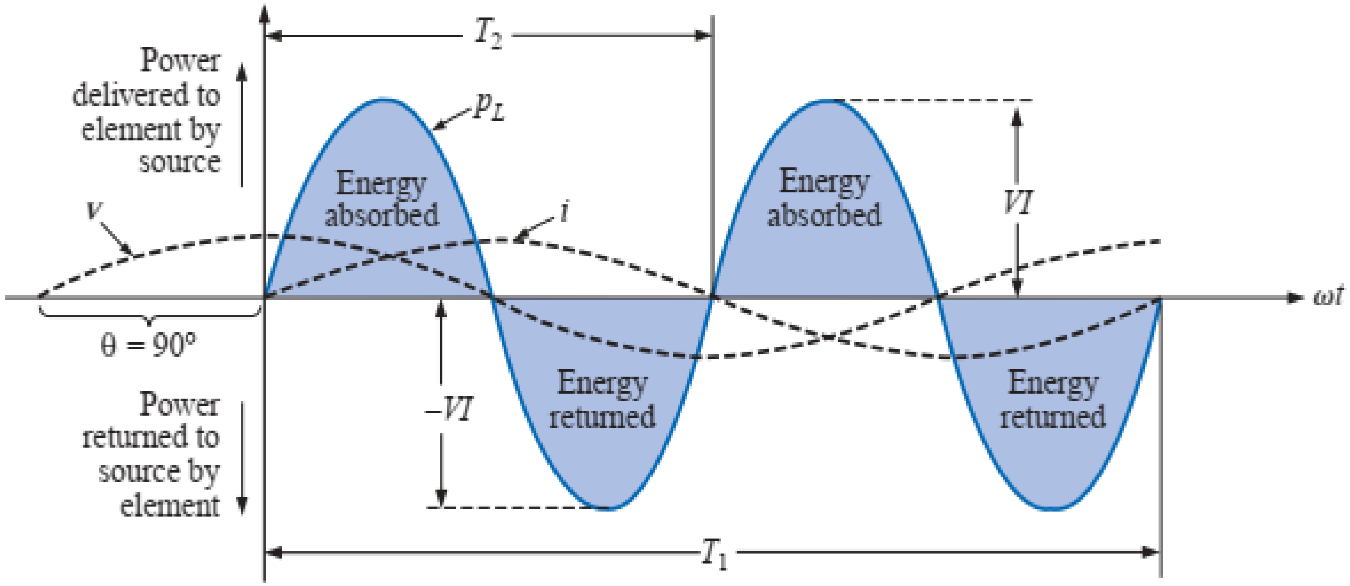


FIG. 19.7

The power curve for a purely inductive load.

$$V_{avg} = \frac{area\ of\ half\ cycle}{base\ length\ of\ half\ cycle}$$

$$= \frac{\int_0^{\pi} v\, d\theta}{\pi} = \frac{\int_0^{\pi} V_m \sin\theta\, d\theta}{\pi}$$

$$= \frac{V_m}{\pi} \left| -\cos\theta \right|_0^{\pi}$$

$$V_{avg} = \frac{2V_m}{\pi} = 0.637V_m$$

The energy stored by the inductor during the positive portion of the cycle (Fig. 19.7) is equal to that returned during the negative portion and can be determined using the following equation:

$$W = Pt$$

where P is the average value for the interval and t is the associated interval of time.

Recall from Chapter 14 that the average value of the positive portion of a sinusoid equals 2(peak value/π) and $t = T_2/2$. Therefore,

$$W_L = \left(\frac{2VI}{\pi}\right) \times \left(\frac{T_2}{2}\right)$$

and

$$\boxed{W_L = \frac{VIT_2}{\pi}} \quad \text{(J)} \qquad \textbf{(19.16)}$$

or, since $T_2 = 1/f_2$, where f_2 is the frequency of the p_L curve, we have

$$\boxed{W_L = \frac{VI}{\pi f_2}} \quad \text{(J)} \qquad \textbf{(19.17)}$$

Since the frequency f_2 of the power curve is twice that of the input quantity, if we substitute the frequency f_1 of the input voltage or current, Equation (19.17) becomes

$$W_L = \frac{VI}{\pi(2f_1)} = \frac{VI}{\omega_1}$$

Example 05: [Introductory Circuit Analysis by Boylestad: Example 19.3]

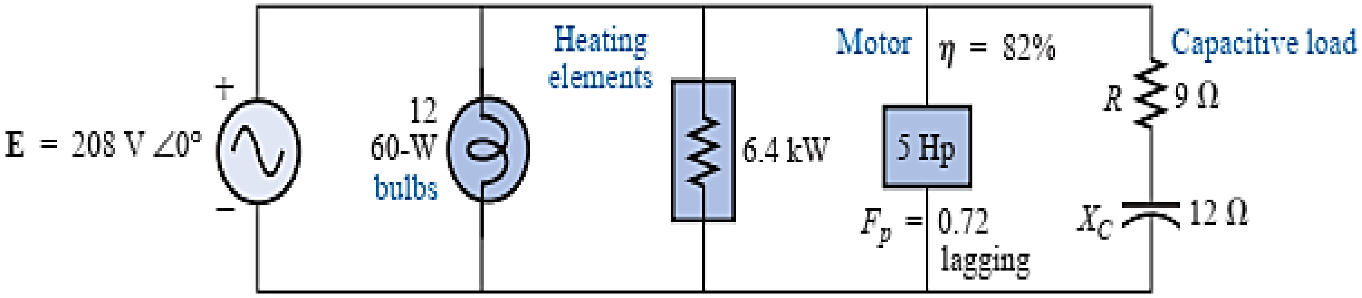


FIG. 19.21

Example 19.3.

a. Find the average power, apparent power, reactive power, and F_p for each branch.
b. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of the system. Sketch the power triangle.
c. Find the source current *I*

Solutions:

a. *Bulbs:*

Total dissipation of applied power

$$P_1 = 12(60\text{ W}) = \mathbf{720\text{ W}}$$
$$Q_1 = \mathbf{0\text{ VAR}}$$
$$S_1 = P_1 = \mathbf{720\text{ VA}}$$
$$F_{p_1} = \mathbf{1}$$

Heating elements:

Total dissipation of applied power

$$P_2 = \mathbf{6.4\text{ kW}}$$
$$Q_2 = \mathbf{0\text{ VAR}}$$
$$S_2 = P_2 = \mathbf{6.4\text{ kVA}}$$
$$F_{p_2} = \mathbf{1}$$

Motor:

$$\eta = \frac{P_o}{P_i} \to P_i = \frac{P_o}{\eta} = \frac{5(746\text{ W})}{0.82} = \mathbf{4548.78\text{ W}} = P_3$$

$$F_p = \mathbf{0.72\text{ lagging}}$$

$$P_3 = S_3 \cos\theta \to S_3 = \frac{P_3}{\cos\theta} = \frac{4548.78\text{ W}}{0.72} = \mathbf{6317.75\text{ VA}}$$

Also, $\theta = \cos^{-1} 0.72 = 43.95°$, so that

$$Q_3 = S_3 \sin\theta = (6317.75\text{ VA})(\sin 43.95°)$$
$$= (6317.75\text{ VA})(0.694) = \mathbf{4384.71\text{ VAR }}(L)$$

Example 05: [Introductory Circuit Analysis by Boylestad: Example 19.3]

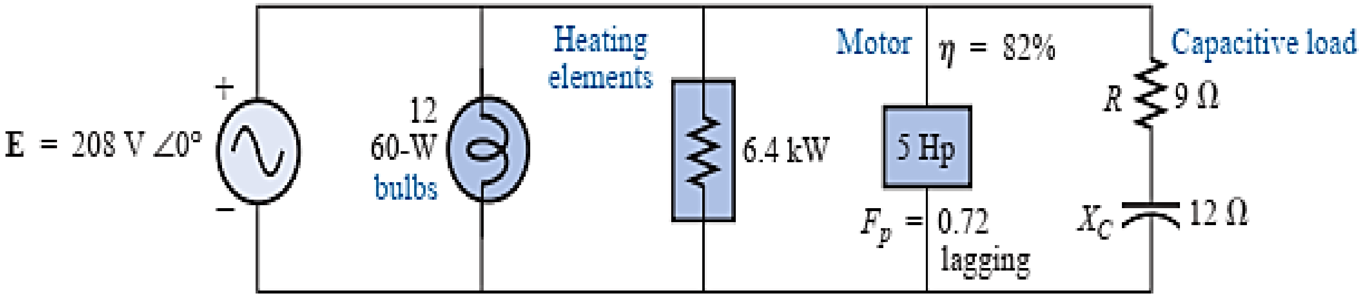


FIG. 19.21
Example 19.3.

a. Find the average power, apparent power, reactive power, and F_p for each branch.
b. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of the system. Sketch the power triangle.
c. Find the source current *I*

Capacitive load:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{208\text{ V}\angle 0^\circ}{9\ \Omega - j\,12\ \Omega} = \frac{208\text{ V}\angle 0^\circ}{15\ \Omega\angle -53.13^\circ} = 13.87\text{ A}\angle 53.13^\circ$$

$$P_4 = I^2R = (13.87\text{ A})^2 \cdot 9\ \Omega = \mathbf{1731.39\ W}$$

$$Q_4 = I^2X_C = (13.87\text{ A})^2 \cdot 12\ \Omega = \mathbf{2308.52\ VAR\ (}\boldsymbol{C}\mathbf{)}$$

$$S_4 = \sqrt{P_4^2 + Q_4^2} = \sqrt{(1731.39\text{ W})^2 + (2308.52\text{ VAR})^2}$$

$$= \mathbf{2885.65\ VA}$$

$$F_p = \frac{P_4}{S_4} = \frac{1731.39\text{ W}}{2885.65\text{ VA}} = \mathbf{0.6\ leading}$$

b. $P_T = P_1 + P_2 + P_3 + P_4$

$$= 720\text{ W} + 6400\text{ W} + 4548.78\text{ W} + 1731.39\text{ W}$$

$$= \mathbf{13{,}400.17\ W}$$

$$Q_T = \pm Q_1 \pm Q_2 \pm Q_3 \pm Q_4$$

$$= 0 + 0 + 4384.71\text{ VAR }(L) - 2308.52\text{ VAR }(C)$$

$$= \mathbf{2076.19\ VAR\ (}\boldsymbol{L}\mathbf{)}$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13{,}400.17\text{ W})^2 + (2076.19\text{ VAR})^2}$$

$$= 13{,}560.06\text{ VA}$$

$$F_p = \frac{P_T}{S_T} = \frac{13.4\text{ kW}}{13{,}560.06\text{ VA}} = \mathbf{0.988\ lagging}$$

$$\theta = \cos^{-1} 0.988 = 8.89^\circ$$

Example 05: [Introductory Circuit Analysis by Boylestad: Example 19.3]

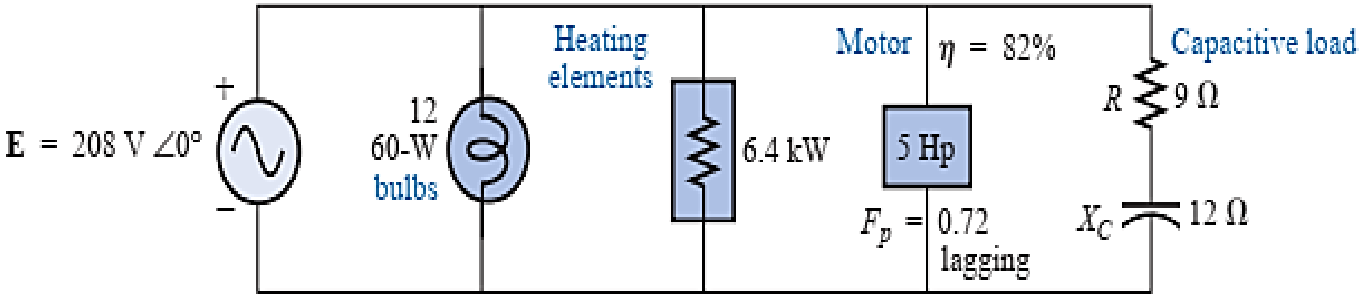


FIG. 19.21
Example 19.3.

a. Find the average power, apparent power, reactive power, and F_p for each branch.
b. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of the system. Sketch the power triangle.
c. Find the source current I

FIG. 19.22
Power triangle for Example 19.3.

c. $S_T = EI \rightarrow I = \dfrac{S_T}{E} = \dfrac{13{,}559.89\ \text{VA}}{208\ \text{V}} = 65.19\ \text{A}$

Lagging power factor: **E** leads **I** by 8.89°, and

$$\mathbf{I} = 65.19\ \text{A} \angle -8.89^\circ$$

Example 06: [Fundamentals of Electric Circuits by Alexander and Sadiku: Example 11.15]

When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

Solution:

If the pf = 0.8, then

$$\cos\theta_1 = 0.8 \qquad \Longrightarrow \qquad \theta_1 = 36.87^\circ$$

where θ_1 is the phase difference between voltage and current. We obtain the apparent power from the real power and the pf as

$$S_1 = \frac{P}{\cos\theta_1} = \frac{4000}{0.8} = 5000 \text{ VA}$$

The reactive power is

$$Q_1 = S_1 \sin\theta = 5000 \sin 36.87 = 3000 \text{ VAR}$$

When the pf is raised to 0.95,

$$\cos\theta_2 = 0.95 \qquad \Longrightarrow \qquad \theta_2 = 18.19^\circ$$

The real power P has not changed. But the apparent power has changed; its new value is

$$S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA}$$

The new reactive power is

$$Q_2 = S_2 \sin\theta_2 = 1314.4 \text{ VAR}$$

The difference between the new and old reactive powers is due to the parallel addition of the capacitor to the load. The reactive power due to the capacitor is

$$Q_C = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6 \text{ VAR}$$

and

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5\ \mu\text{F}$$

Example 07: [Introductory Circuit Analysis by Boylestad: Example 19.6]

EXAMPLE 19.6

a. A small industrial plant has a 10-kW heating load and a 20-kVA inductive load due to a bank of induction motors. The heating elements are considered purely resistive ($F_p = 1$), and the induction motors have a lagging power factor of 0.7. If the supply is 1000 V at 60 Hz, determine the capacitive element required to raise the power factor to 0.95.

b. Compare the levels of current drawn from the supply.

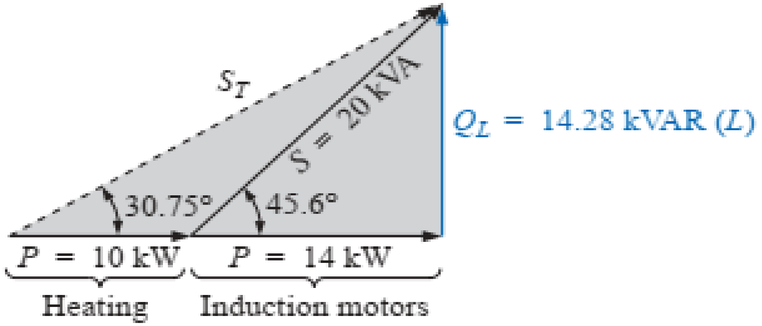


FIG. 19.28

Initial power triangle for the load of Example 19.6.

Solutions:

a. For the induction motors,

$$S = VI = 20 \text{ kVA}$$
$$P = S\cos\theta = (20 \times 10^3 \text{ VA})(0.7) = 14 \times 10^3 \text{ W}$$
$$\theta = \cos^{-1} 0.7 \cong 45.6°$$

and

$$Q_L = VI\sin\theta = (20 \times 10^3 \text{ VA})(0.714) = 14.28 \times 10^3 \text{ VAR } (L)$$

The power triangle for the total system appears in Fig. 19.28. Note the addition of real powers and the resulting S_T:

$$S_T = \sqrt{(24 \text{ kW})^2 + (14.28 \text{ kVAR})^2} = 27.93 \text{ kVA}$$

with $$I_T = \frac{S_T}{E} = \frac{27.93 \text{ kVA}}{1000 \text{ V}} = \mathbf{27.93\ A}$$

The desired power factor of 0.95 results in an angle between S and P of

$$\theta = \cos^{-1} 0.95 = 18.19°$$

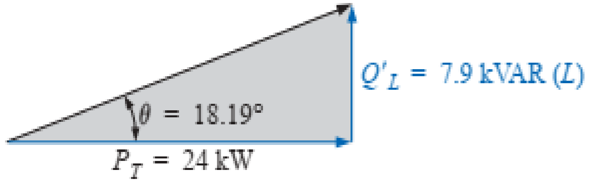


FIG. 19.29

Power triangle for the load of Example 19.6 after raising the power factor to 0.95.

changing the power triangle to that of Fig. 19.29:

$$\text{with } \tan\theta = \frac{Q'_L}{P_T} \rightarrow Q'_L = P_T \tan\theta = (24 \times 10^3 \text{ W})(\tan 18.19°)$$
$$= (24 \times 10^3 \text{ W})(0.329) = 7.9 \text{ kVAR } (L)$$

The inductive reactive power must therefore be reduced by

$$Q_L - Q'_L = 14.28 \text{ kVAR } (L) - 7.9 \text{ kVAR } (L) = 6.38 \text{ kVAR } (L)$$

Therefore, $Q_C = 6.38$ kVAR, and using

$$Q_C = \frac{E^2}{X_C}$$

we obtain

$$X_C = \frac{E^2}{Q_C} = \frac{(10^3 \text{ V})^2}{6.38 \times 10^3 \text{ VAR}} = 156.74\ \Omega$$

and $$C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60 \text{ Hz})(156.74\ \Omega)} = \mathbf{16.93\ \mu F}$$

b. $$S_T = \sqrt{(24 \text{ kW})^2 + [7.9 \text{ kVAR } (L)]^2}$$
$$= 25.27 \text{ kVA}$$

$$I_T = \frac{S_T}{E} = \frac{25.27 \text{ kVA}}{1000 \text{ V}} = \mathbf{25.27\ A}$$

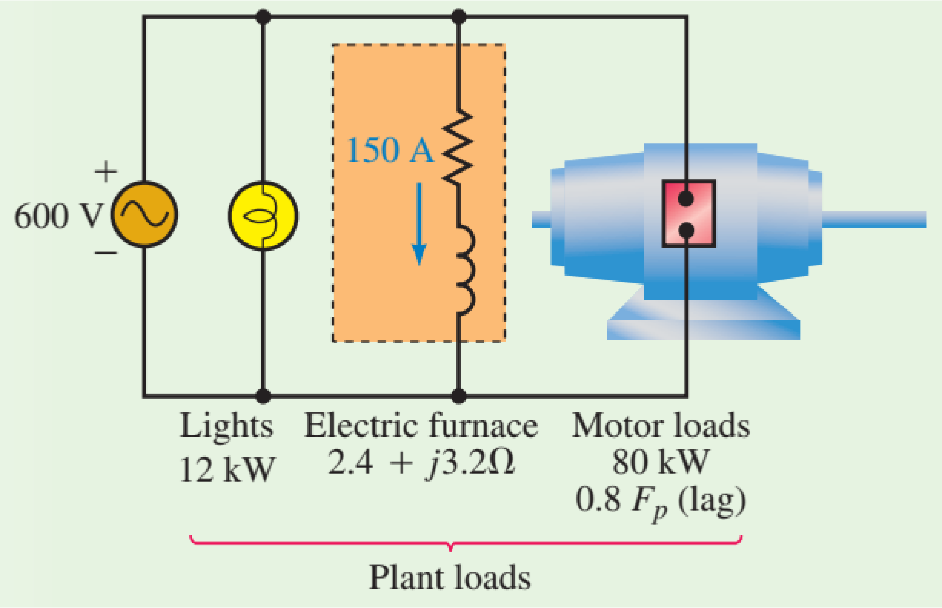
The new I_T is

$$I_T = 25.27 \text{ A} \angle 27.93 \text{ A}$$

Example 08: [Circuit Analysis, Robbins and Miller: Example 17-8]

An industrial client is charged a penalty if the power factor drops below 0.85. The equivalent plant loads are as shown in following figure below. The frequency is 60 Hz.

a) Determine the total active and reactive power.
b) Find the value of capacitance is required to bring the power factor up to 0.85
c) Calculate generator current before and after correction.



Solution: (a)

Connected load	Active Power	Reactive Power	Given that
Lights	$P_1 = 12\text{ kW}$	$Q_1 = 0\text{ kVAR}$	Lights = 12 kW
Furnace	$P_2 = I^2R$ $= (150)^2 \times 2.4$ $= 54\text{ kW}$	$Q_2 = I^2X_L$ $= (150)^2 \times 3.2$ $= 72\text{ kVAR}$	Impedance, $\boldsymbol{Z} = (2.4 + j3.2)\Omega$
Motor	$P_3 = 80\text{ kW}$	$Q_3 = P_3 \tan\theta_3$ $= P_3 \tan(cos^{-1}0.8)$ $= 80 \tan 39.6°$ $= 80\text{ kVAR}$	Motor = 80 kW, 0.8 pf (lag)
Total	$P_T = 146\ kW$	$Q_T = 132\ kVAR$	

Example 08: [Circuit Analysis, Robbins and Miller: Example 17-8] (continued)

Solution: (b)

From power triangle, present apparent power,

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(146)^2+(132)^2} = 196.8\ kVA$$

$$\therefore Present\ pf = \cos\theta = \frac{P}{S} = \frac{146\ kW}{196.8\ kVA} = 0.74\ lagging$$

However, we need to correct power factor to 0.85

$\therefore \cos\theta' = 0.85$

$\theta' = \cos^{-1}(0.85) = 31.78°$

From corrected power triangle,

$Q_T' = P_T \tan\theta' = 146\tan(31.8)° = 90.5\ kVAR$

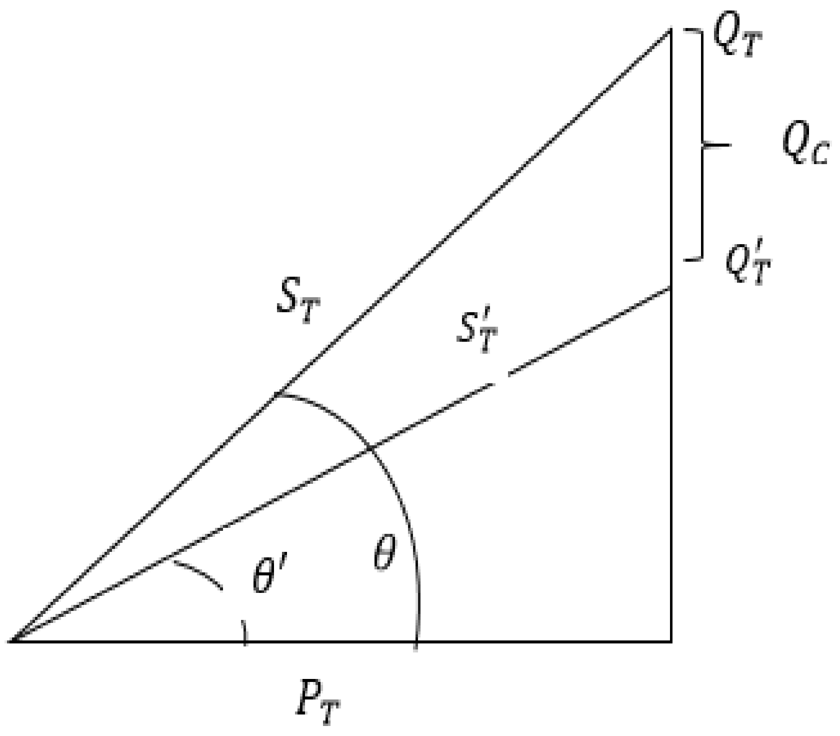


Fig: Power Triangle

Example 08: [Circuit Analysis, Robbins and Miller: Example 17-8] (continued)

Solution: (b) $\therefore Q_C = Q_T - Q'_T = 132 - 90.5 = 41.5\ kVAR$

$$\frac{V_{rms}^2}{X_C} = 41.5\ kVAR$$

$$\frac{1}{\omega C} = \frac{V_{rms}^2}{41.5}$$

$$C = \frac{41.5}{V_{rms}^2 \times \omega} = \frac{41.5}{V_{rms}^2 \times 2\pi f} = \frac{41.5}{(600)^2 \times 2\pi \times 60} = 3.04 \times 10^{-4}$$

$$C = 304\ \mu F$$

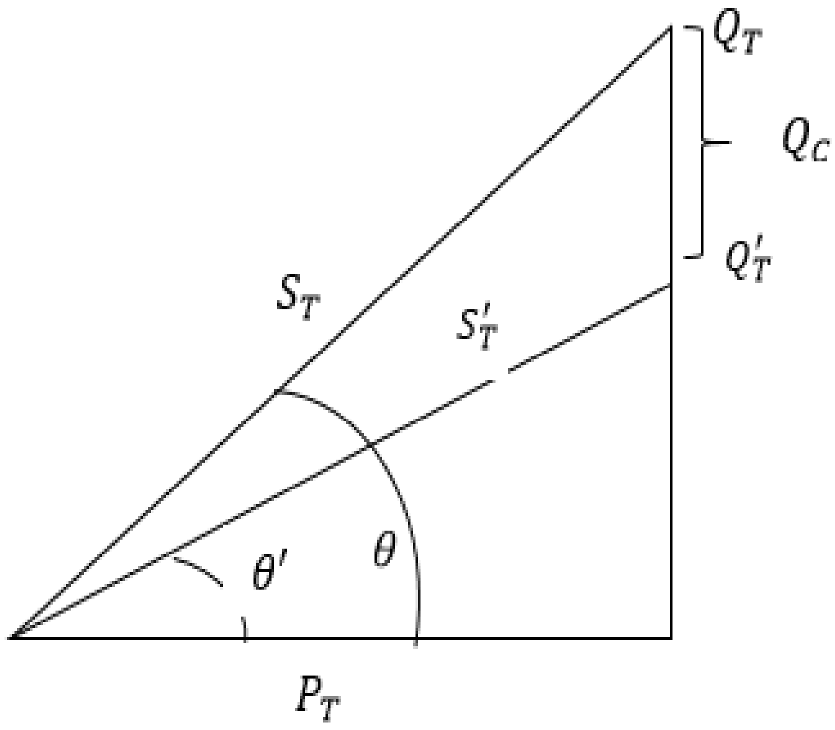


Fig: Power Triangle

Note: capacitors are normally purchased for voltages they expect to see. In this case, the maximum voltage this capacitor will see is about 848.5 V peak. It is suggested to purchase a capacitor with a voltage-rating equal to, say, 900V.

Solution: (c) Before correction,

$$S_T = 196.8\ kVA$$

$$I_{rms} = \frac{196.8\ kVA}{600\ V} = 328\ A$$

After correction, $S'_T = \sqrt{P_T^2 + Q'_T} = \sqrt{(146)^2 + (90.5\ )^2} = 171.8\ kVA$

$$I'_{rms} = \frac{171.8\ kVA}{600\ V} = 286\ A$$