City University
Dept. of Computer Science and Engineering
SE 401 Computer Simulation and Modelling
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Class Lecture Notes (SE401)

Auto-Correlation Test (Algorithm)

Step-1: Define the hypothesis for uniformity.

H_0: $\rho_i = 0$ → No's independent

H_1: $\rho_i \neq 0$ → No's aren't independent

Step-2: Find **i** and **lag m**.

Here, **i=** initial no.

lag m= constant (harmony projection)

Geometry projection

Step-3: Using **i**, **m**, **N**

Find **M→ largest integer**

by **i+(M+1)m ≤ N**

N→ total no of values in the sequence

Step-4: $\hat{\rho}_{im} = \frac{1}{M+1}, \left[\sum_{k=0}^{M} R_{i+k_m}, R_{i+[k+1]m}\right] - 0.25$

Step-5: Find the S.D pf the estimator,

$$\sigma_{\hat{p}_m} = \frac{\sqrt{13M+7}}{12(M+1)}$$

Step-6: $Z_0 = \frac{\hat{\rho}im}{\sigma_{\hat{p}_m}} = ?$

Step-7: Determine $+Z_{\alpha/2}$, $-Z_{\alpha/2}$

Step-8: If $-Z_{\alpha/2} \leq Z_0 \leq +Z_{\alpha/2}$ → H_0 isn't rejected.

Auto-Correlation Test (Example)

0.12, 0.01, 0.23, 0.28, 0.89, 0.31, 0.64, 0.28, 0.83, 0.93, 0.99, 0.15, 0.33, 0.35, 0.91, 0.41, 0.60, 0.27, 0.75, 0.88, 0.68, 0.49, 0.05, 0.43, 0.95, 0.58, 0.19, 0.36, 0.69, 0.87. (**α=0.025, Test No=3 at position 3rd, 8th, 13th are auto correlated**)

N=30

S1: Define Hypothesis,

S2: i=3, lag m=5

S3: Find **M**, **i+(M+1)m ≤ N**

3+(M+1)5 ≤ 30

→ (M+1)5 <= 27

→ (M+1) <= 5.4

→ M <= 4.4 [So, M= max (4, 3, 2…0)]

M= 4

S4: $\hat{\rho}_{im} = \frac{1}{M+1} \cdot \left[\sum_{k=0}^{M} R_{i+k_m} . R_{i+[k+1]m}\right] - 0.25$

$\hat{\rho}_{35} = \frac{1}{4+1} \left[\sum_{k=0}^{4} R_{3+5k} . R_{3+5[k+1]}\right] - 0.25$

$= \frac{1}{5}$ [R_3. R_8 + R_8. R_{13} + R_{13}. R_{18} + R_{18}. R_{23} + R_{23}. R_{28}]-0.25

$= \frac{1}{5}$ [0.23*0.28 + 0.28*0.33 + 0.33*0.27 + 0.27*0.05 + 0.05*0.36]

$= \frac{1}{5}$ (0.2774) - 0.25

= 0.05548 – 0.25

= -0.19452, **This is the estimator**

S5: $\sigma_{\hat{p}_m} = \frac{\sqrt{13(4)}+7}{12(4+1)} = \frac{\sqrt{52}+7}{60}$ = 0.128

S6: $Z_0 = \frac{\hat{\rho}im}{\sigma_{\hat{p}_m}} = \frac{-0.19452}{0.128}$ = **-1.51**

S7: $Z_{0.025}$ = 1.96

S8: $-Z_{\alpha/2} \leq Z_0 \leq +Z_{\alpha/2}$

$-1.96 \leq -1.51 \leq 1.96$, H_0 is accepted.

Auto-Correlation Test (Example)

0.19, 0.16, 0.82, 0.63, 0.04, 0.16, 0.30, 0.22, 0.88, 0.48, 0.29, 0.56, 0.44, 0.05, 0.81, 0.38, 0.59, 0.37, 0.71, 0.43, 0.92, 0.45, 0.57, 0.99, 0.20, 0.14, 0.64, 0.50, 0.73, 0.15, 0.02, 0.49, 0.86, 0.24, 0.90, 0.74, 0.41, 0.09, 0.80, 0.42. (**α=0.025, $Z_{0.025}$ =** 1.96. **Test No=3 at position 2nd, 7th, 12th are auto correlated**)

N=40

S1: Define Hypothesis,

S2: i=2, lag m=5

S3: Find **M**, **i+(M+1)m ≤ N**

$2+(M+1)5 \leq 40$

→ $(M+1)5 <= 38$

→ $(M+1) <= 7.6$

→ $M <= 6.6$ [So, M= max (6,5, 4…0)]

M= 6

S4: $\hat{\rho}_{im} = \frac{1}{M+1}, \left[\sum_{k=0}^{M} R_{i+k_m} . R_{i+[k+1]m}\right] - 0.25$

$\hat{\rho}_{25} = \frac{1}{6+1} \left[\sum_{k=0}^{6} R_{2+5k} . R_{2+5[k+1]}\right] - 0.25$

$= \frac{1}{7} [R_2 . R_7 + R_7 . R_{12} + R_{12} . R_{17} + R_{17} . R_{22} + R_{22} . R_{27} + R_{27} . R_{32} + R_{32} . R_{37}] - 0.25$

$= \frac{1}{7} [0.16*0.30 + 0.30*0.56 + 0.56*0.59 + 0.59*0.45 + 0.45*0.64 + 0.64*0.49 + 0.49*0.41]$

$= \frac{1}{7} (1.6144) - 0.25$

$= 0.23063 - 0.25$

= -0.0193, **This is the estimator**

S5: $\sigma_{\hat{p}_m} = \frac{\sqrt{13(6)}+7}{12(6+1)} = \frac{\sqrt{78}+7}{84}$ = 0.10975

S6: $Z_0 = \frac{\hat{\rho}im}{\sigma_{\hat{p}_m}} = \frac{-0.0193}{0.10975} = -0.17$

S7: $Z_{0.025}$ = 1.96

S8: $-Z_{\alpha/2} \leq Z_0 \leq +Z_{\alpha/2}$

-1.96 ≤ **-0.17** ≤ 1.96**,** H_0 is accepted**.**