Operational Amplifier (Op –Amp)

(Applications, Frequency and Phase Response)

Summing Amplifier:

The summing amplifier, shown in Fig. 5.21, is a variation of the inverting amplifier. It takes advantage of the fact that the inverting configuration can handle many inputs at the same time. We keep in mind that the current entering each op amp input is zero. Applying KCL at node a gives

$$i = i_1 + i_2 + i_3 \tag{5.13}$$

But

$$i_1 = \frac{v_1 - v_a}{R_1}, \quad i_2 = \frac{v_2 - v_a}{R_2}$$
$$i_3 = \frac{v_3 - v_a}{R_3}, \quad i = \frac{v_a - v_o}{R_f} \tag{5.14}$$

We note that $v_a = 0$ and substitute Eq. (5.14) into Eq. (5.13). We get

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right) \tag{5.15}$$

indicating that the output voltage is a weighted sum of the inputs. For this reason, the circuit in Fig. 5.21 is called a *summer*. Needless to say, the summer can have more than three inputs.

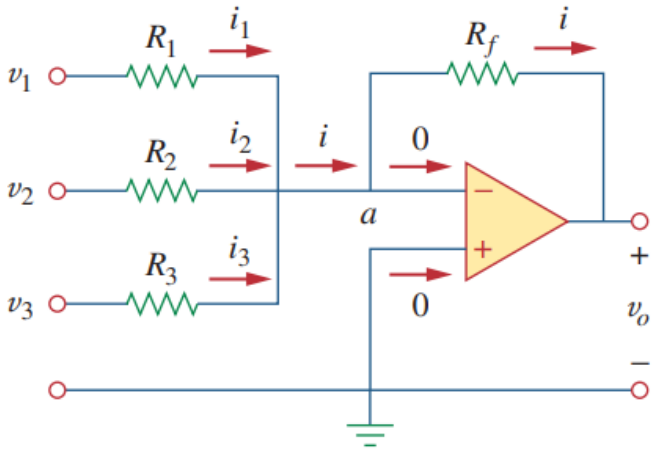


Figure 5.21
The summing amplifier.

A summing amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.

Example 5.6

Calculate v_o and i_o in the op amp circuit in Fig. 5.22.

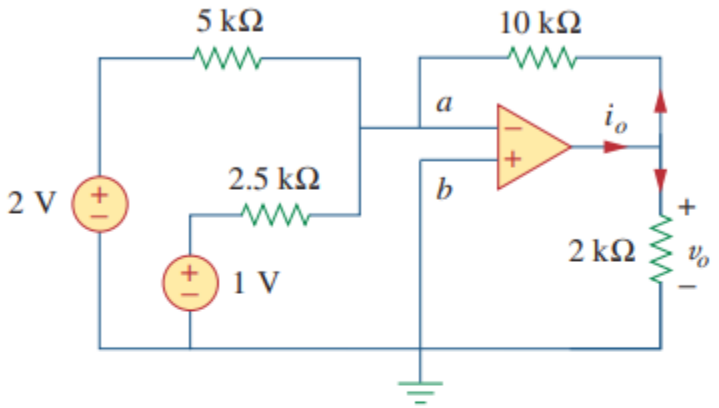


Figure 5.22
For Example 5.6.

Solution:

This is a summer with two inputs. Using Eq. (5.15) gives

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4 + 4) = -8\ \text{V}$$

The current i_o is the sum of the currents through the 10-kΩ and 2-kΩ resistors. Both of these resistors have voltage $v_o = -8$ V across them, since $v_a = v_b = 0$. Hence,

$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2}\text{mA} = -0.8 - 4 = -4.8\ \text{mA}$$

Practice Problem 5.6

Find v_o and i_o in the op amp circuit shown in Fig. 5.23.

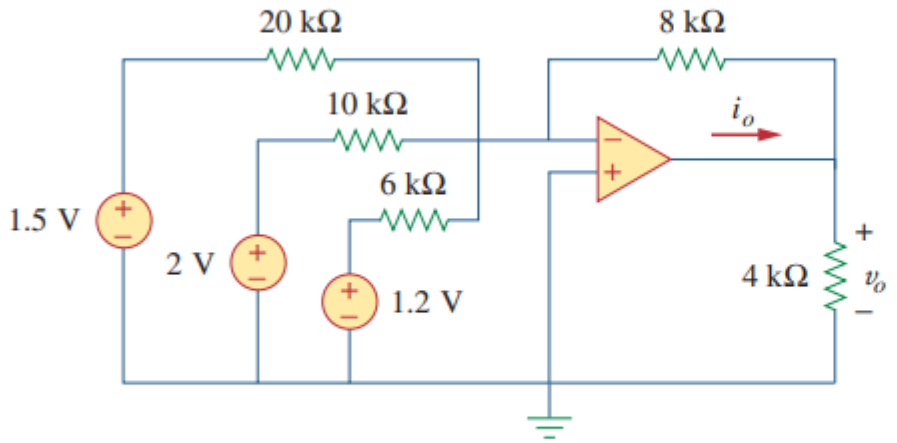


Figure 5.23
For Practice Prob. 5.6.

Answer: −3.8 V, −1.425 mA.

Integrator:

If the feedback resistor R_f in the familiar inverting amplifier of Fig. 6.35(a) is replaced by a capacitor, we obtain an ideal integrator, as shown in Fig. 6.35(b). It is interesting that we can obtain a mathematical representation of integration this way. At node a in Fig. 6.35(b),

$$i_R = i_C \tag{6.32}$$

But

$$i_R = \frac{v_i}{R}, \qquad i_C = -C\frac{dv_o}{dt}$$

Substituting these in Eq. (6.32), we obtain

$$\frac{v_i}{R} = -C\frac{dv_o}{dt} \tag{6.33a}$$

$$dv_o = -\frac{1}{RC}v_i\,dt \tag{6.33b}$$

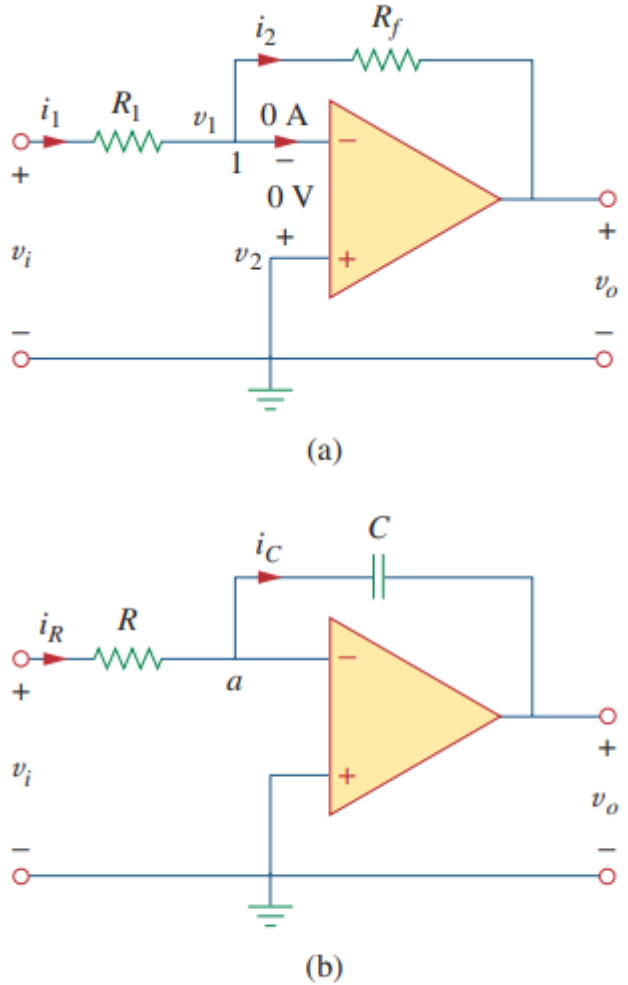


Figure 6.35
Replacing the feedback resistor in the inverting amplifier in (a) produces an integrator in (b).

An integrator is an op amp circuit whose output is proportional to the integral of the input signal.

Integrating both sides gives

$$v_o(t) - v_o(0) = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau \tag{6.34}$$

To ensure that $v_o(0) = 0$, it is always necessary to discharge the integrator's capacitor prior to the application of a signal. Assuming $v_o(0) = 0$,

$$v_o = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau \tag{6.35}$$

which shows that the circuit in Fig. 6.35(b) provides an output voltage proportional to the integral of the input. In practice, the op amp integrator requires a feedback resistor to reduce dc gain and prevent saturation. Care must be taken that the op amp operates within the linear range so that it does not saturate.

Example 6.13

If $v_1 = 10\cos 2t$ mV and $v_2 = 0.5t$ mV, find v_o in the op amp circuit in Fig. 6.36. Assume that the voltage across the capacitor is initially zero.

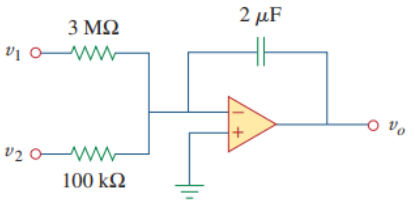


Figure 6.36
For Example 6.13.

Solution:
This is a summing integrator, and

$$v_o = -\frac{1}{R_1C}\int v_1\,dt - \frac{1}{R_2C}\int v_2\,dt$$

$$= -\frac{1}{3\times 10^6 \times 2\times 10^{-6}}\int_0^t 10\cos(2\tau)\,d\tau$$

$$-\frac{1}{100\times 10^3 \times 2\times 10^{-6}}\int_0^t 0.5\tau\,d\tau$$

$$= -\frac{1}{6}\frac{10}{2}\sin 2t - \frac{1}{0.2}\frac{0.5t^2}{2} = -0.833\sin 2t - 1.25t^2\ \text{mV}$$

Practice Problem 6.13

The integrator in Fig. 6.35(b) has $R = 100\ \text{k}\Omega$, $C = 20\ \mu\text{F}$. Determine the output voltage when a dc voltage of 2.5 mV is applied at $t = 0$. Assume that the op amp is initially nulled.

Answer: $-1.25t$ mV.

Differentiator:

In Fig. 6.35(a), if the input resistor is replaced by a capacitor, the resulting circuit is a differentiator, shown in Fig. 6.37. Applying KCL at node a,

$$i_R = i_C \tag{6.36}$$

But

$$i_R = -\frac{v_o}{R}, \qquad i_C = C\frac{dv_i}{dt}$$

Substituting these in Eq. (6.36) yields

$$\boxed{v_o = -RC\frac{dv_i}{dt}} \tag{6.37}$$

showing that the output is the derivative of the input. Differentiator circuits are electronically unstable because any electrical noise within the circuit is exaggerated by the differentiator. For this reason, the differentiator circuit in Fig. 6.37 is not as useful and popular as the integrator. It is seldom used in practice.

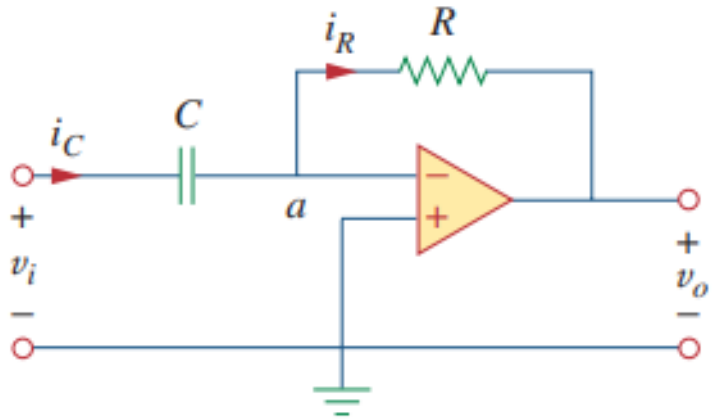
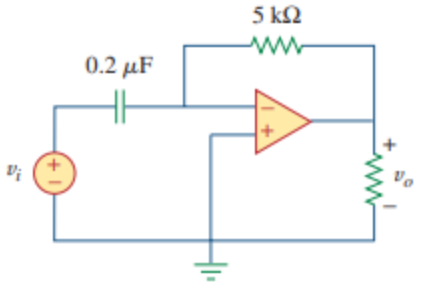


Figure 6.37
An op amp differentiator.

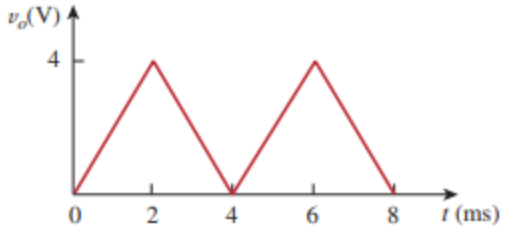
A **differentiator** is an op amp circuit whose output is proportional to the rate of change of the input signal.

Example 6.14

Sketch the output voltage for the circuit in Fig. 6.38(a), given the input voltage in Fig. 6.38(b). Take $v_o = 0$ at $t = 0$.



(a)



(b)

Figure 6.38
For Example 6.14.

Solution:
This is a differentiator with

$$RC = 5 \times 10^3 \times 0.2 \times 10^{-6} = 10^{-3}\ \text{s}$$

For $0 < t < 4$ ms, we can express the input voltage in Fig. 6.38(b) as

$$v_i = \begin{cases} 2000t & 0 < t < 2\ \text{ms} \\ 8 - 2000t & 2 < t < 4\ \text{ms} \end{cases}$$

This is repeated for $4 < t < 8$ ms. Using Eq. (6.37), the output is obtained as

$$v_o = -RC\frac{dv_i}{dt} = \begin{cases} -2\ \text{V} & 0 < t < 2\ \text{ms} \\ 2\ \text{V} & 2 < t < 4\ \text{ms} \end{cases}$$

Thus, the output is as sketched in Fig. 6.39.

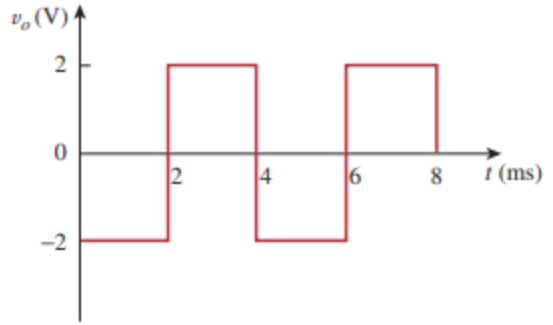


Figure 6.39
Output of the circuit in Fig. 6.38(a).

Practice Problem 6.14

The differentiator in Fig. 6.37 has $R = 100$ kΩ and $C = 0.1$ μF. Given that $v_i = 1.25t$ V, determine the output v_o.

Answer: −12.5 mV.

Difference Amplifier:

Difference (or differential) amplifiers are used in various applications where there is a need to amplify the difference between two input signals. They are first cousins of the *instrumentation amplifier*, the most useful and popular amplifier, which we will discuss in Section 5.10.

A difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.

Consider the op amp circuit shown in Fig. 5.24. Keep in mind that zero currents enter the op amp terminals. Applying KCL to node a,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

or

$$v_o = \left(\frac{R_2}{R_1} + 1\right)v_a - \frac{R_2}{R_1}v_1 \qquad \textbf{(5.16)}$$

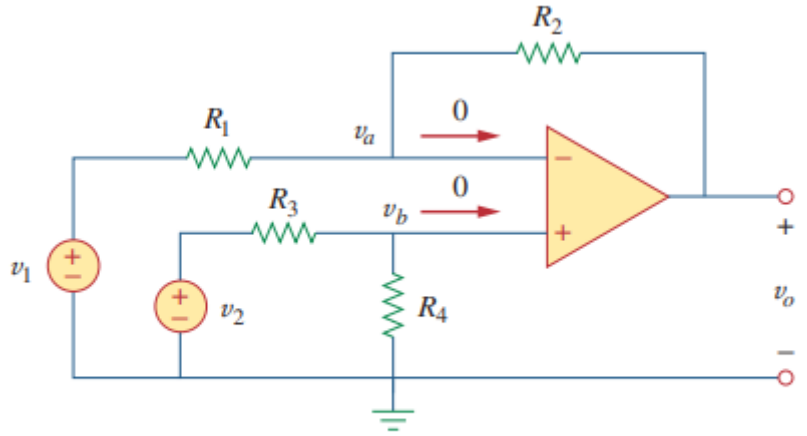


Figure 5.24
Difference amplifier.

Applying KCL to node b,

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

or

$$v_b = \frac{R_4}{R_3 + R_4} v_2 \tag{5.17}$$

But $v_a = v_b$. Substituting Eq. (5.17) into Eq. (5.16) yields

$$v_o = \left(\frac{R_2}{R_1} + 1\right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$

or

$$\boxed{v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1} \tag{5.18}$$

Since a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $v_o = 0$ when $v_1 = v_2$. This property exists when

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \tag{5.19}$$

Thus, when the op amp circuit is a difference amplifier, Eq. (5.18) becomes

$$v_o = \frac{R_2}{R_1}(v_2 - v_1) \tag{5.20}$$

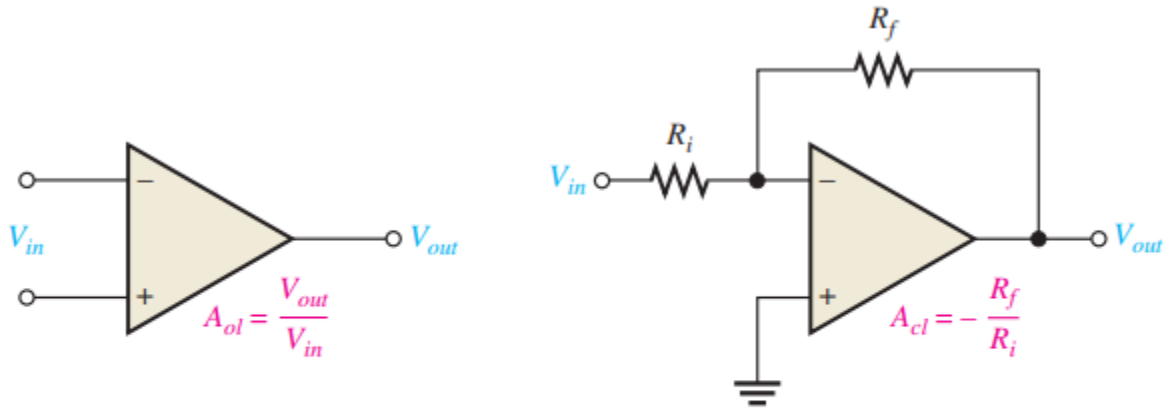
If $R_2 = R_1$ and $R_3 = R_4$, the difference amplifier becomes a *subtractor*, with the output

$$v_o = v_2 - v_1 \tag{5.21}$$

Solve the problems from exercise:
6.74, 6.75, 6.77

Gain and frequency response of 741 Op-Amp

Open Loop Frequency & Phase Responses:



(a) Open-loop

(b) Closed-loop (inverting configuration)

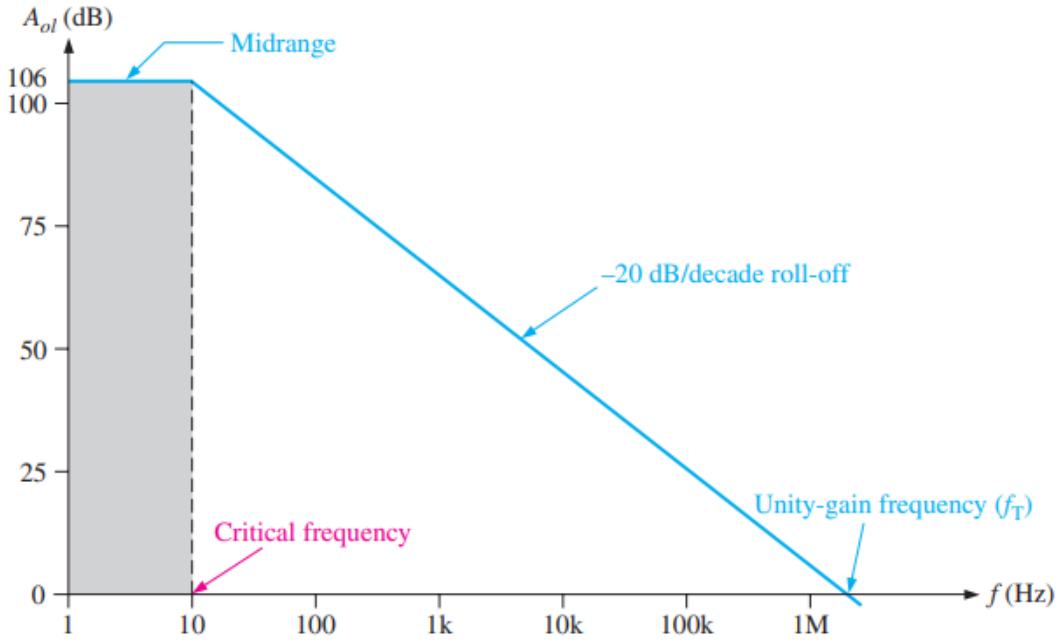
▲ FIGURE 12–35

Open-loop and closed-loop op-amp configurations.

Bandwidth Limitations

In the previous sections, all of the voltage gain expressions were based on the midrange gain and were considered independent of the frequency. The midrange open-loop gain of an op-amp extends from zero frequency (dc) up to a critical frequency at which the gain is 3 dB less than the midrange value. This concept should be familiar from your study of Chapter 10. Op-amps are dc amplifiers (no capacitive coupling between stages), and therefore, there is no lower critical frequency. This means that the midrange gain extends down to zero frequency (dc), and dc voltages are amplified the same as midrange signal frequencies.

An open-loop response curve (Bode plot) for a certain op-amp is shown in Figure 12–36. Most op-amp datasheets show this type of curve or specify the midrange open-loop gain. Notice that the curve rolls off (decreases) at −20 dB per decade (−6 dB per octave). The midrange gain is 200,000, which is 106 dB, and the critical (cutoff) frequency is approximately 10 Hz.



▲ FIGURE 12–36

Ideal plot of open-loop voltage gain versus frequency for a typical op-amp. The frequency scale is

3 dB Open-Loop Bandwidth Recall from Chapter 10 that the bandwidth of an ac amplifier is the frequency range between the points where the gain is 3 dB less than the midrange gain. In general, the bandwidth equals the upper critical frequency (f_{cu}) minus the lower critical frequency (f_{cl}).

$$BW = f_{cu} - f_{cl}$$

Since f_{cl} for an op-amp is zero, the bandwidth is simply equal to the upper critical frequency.

$$\boldsymbol{BW = f_{cu}}$$

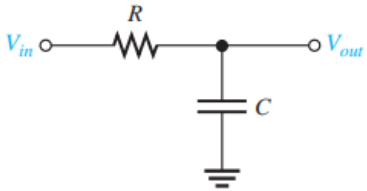
From now on, we will refer to f_{cu} as simply f_c; and we will use open-loop (ol) or closed-loop (cl) subscript designators, for example, $f_{c(ol)}$.

Unity-Gain Bandwidth Notice in Figure 12–36 that the gain steadily decreases to a point where it is equal to unity (1 or 0 dB). The value of the frequency at which this unity gain occurs is the *unity-gain frequency* designated f_T. f_T is also called the *unity-gain bandwidth.*

Gain-Versus-Frequency Analysis

The *RC* lag (low-pass) circuits within an op-amp are responsible for the roll-off in gain as the frequency increases, just as was discussed for the discrete amplifiers in Chapter 10. From basic ac circuit theory, the attenuation of an *RC* lag circuit, such as in Figure 12–37, is expressed as

$$\frac{V_{out}}{V_{in}} = \frac{X_C}{\sqrt{R^2 + X_C^2}}$$



▲ FIGURE 12–37

RC lag circuit.

Dividing both the numerator and denominator to the right of the equals sign by X_C,

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + R^2/X_C^2}}$$

The critical frequency of an *RC* circuit is

$$f_c = \frac{1}{2\pi RC}$$

Dividing both sides by f gives

$$\frac{f_c}{f} = \frac{1}{2\pi RCf} = \frac{1}{(2\pi fC)R}$$

Since $X_C = 1/(2\pi fC)$, the previous expression can be written as

$$\frac{f_c}{f} = \frac{X_C}{R}$$

Substituting this result in the previous equation for V_{out}/V_{in} produces the following expression for the attenuation of an *RC* lag circuit in terms of frequency:

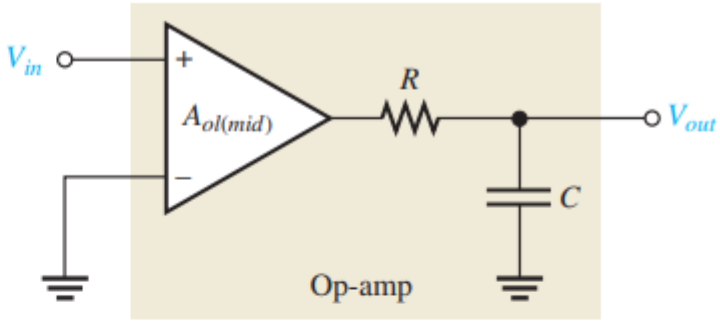
$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + f^2/f_c^2}}$$

Equation 12–18

If an op-amp is represented by a voltage gain element with a gain of $A_{ol(mid)}$ plus a single *RC* lag circuit, as shown in Figure 12–38, it is known as a compensated op-amp. The total open-loop gain of the op-amp is the product of the midrange open-loop gain, $A_{ol(mid)}$, and the attenuation of the *RC* circuit.

▶ **FIGURE 12–38**

Op-amp represented by a gain element and an internal *RC* circuit.



Equation 12–19

$$A_{ol} = \frac{A_{ol(mid)}}{\sqrt{1 + f^2/f_c^2}}$$

As you can see from Equation 12–19, the open-loop gain equals the midrange gain when the signal frequency f is much less than the critical frequency f_c and drops off as the frequency increases. Since f_c is part of the open-loop response of an op-amp, we will refer to it as $f_{c(ol)}$.

The following example demonstrates how the open-loop gain decreases as the frequency increases above $f_{c(ol)}$.

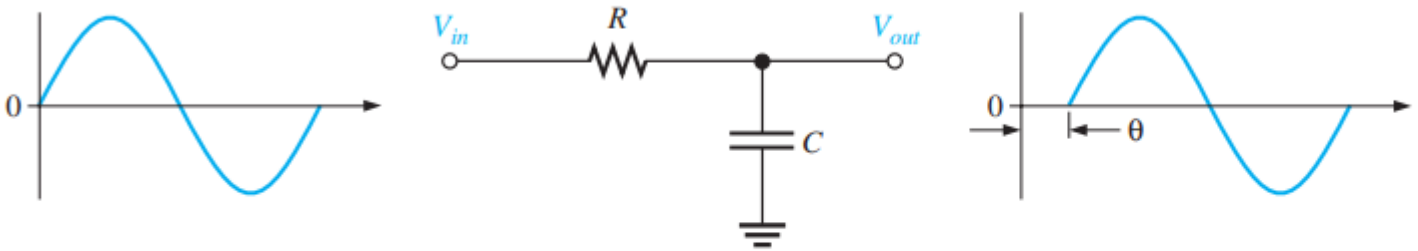
Phase Shift

As you know from Chapter 10, an *RC* circuit causes a propagation delay from input to output, thus creating a **phase shift** between the input signal and the output signal. An *RC* lag circuit such as found in an op-amp stage causes the output signal voltage to lag the input, as shown in Figure 12–39. From basic ac circuit theory, the phase shift, θ, is

$$\theta = -\tan^{-1}\left(\frac{R}{X_C}\right)$$

Since $R/X_C = f/f_c$,

$$\boldsymbol{\theta = -\tan^{-1}\left(\frac{f}{f_c}\right)}$$

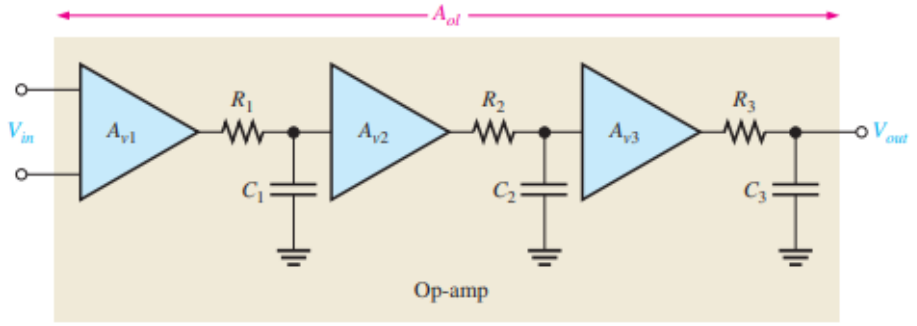


◀ FIGURE 12–39
Output voltage lags input voltage.

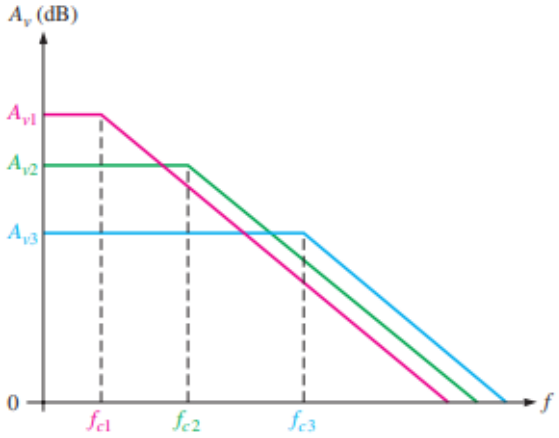
The negative sign indicates that the output lags the input. This equation shows that the phase shift increases with frequency and approaches $-90°$ as f becomes much greater than f_c.

Overall Frequency Response

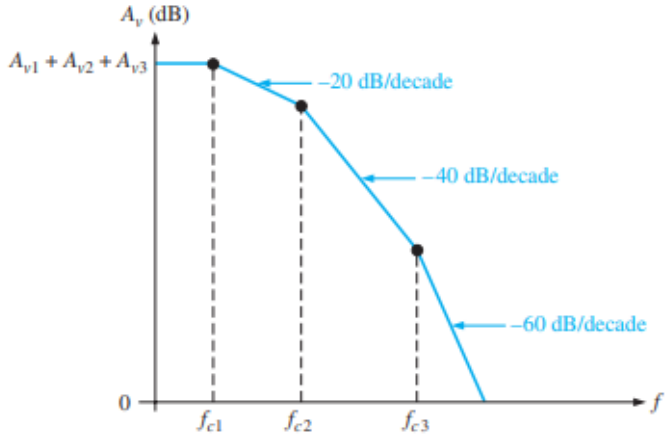
Previously, an op-amp was defined to have a constant roll-off of −20 dB/decade above its critical frequency. For most op-amps this is the case; for some, however, the situation is more complex. The more complex IC operational amplifier may consist of two or more cascaded amplifier stages. The gain of each stage is frequency dependent and rolls off at −20 dB/decade above its critical frequency. Therefore, the total response of an op-amp is a composite of the individual responses of the internal stages. As an example, a three-stage op-amp is represented in Figure 12–41(a), and the frequency response of each stage is shown in Figure 12–41(b). As you know, dB gains are added so that the total op-amp frequency response is as shown in Figure 12–41(c). Since the roll-off rates are additive, the total roll-off rate increases by −20 dB/decade (−6 dB/decade) as each critical frequency is reached.



(a) Representation of an op-amp with three internal stages



(b) Individual responses



(c) Composite response

▲ FIGURE 12–41

Op-amp open-loop frequency response

Overall Phase Response

In a multistage amplifier, each stage contributes to the total phase lag. As you have seen, each *RC* lag circuit can produce up to a −90° phase shift. Since each stage in an op-amp includes an *RC* lag circuit, a three-stage op-amp, for example, can have a maximum phase lag of −270°. Also, the phase lag of each stage is less than −45° when the frequency is below the critical frequency, equal to −45° at the critical frequency, and greater than −45° when the frequency is above the critical frequency. The phase lags of the stages of an op-amp are added to produce a total phase lag, according to the following formula for three stages:

$$\theta_{tot} = -\tan^{-1}\left(\frac{f}{f_{c1}}\right) - \tan^{-1}\left(\frac{f}{f_{c2}}\right) - \tan^{-1}\left(\frac{f}{f_{c3}}\right)$$

EXAMPLE 12–8

Determine A_{ol} for the following values of f. Assume $f_{c(ol)} = 100$ Hz and $A_{ol(mid)} = 100{,}000$.

(a) $f = 0$ Hz **(b)** $f = 10$ Hz **(c)** $f = 100$ Hz **(d)** $f = 1000$ Hz

Solution

(a) $A_{ol} = \dfrac{A_{ol(mid)}}{\sqrt{1 + f^2/f_{c(ol)}^2}} = \dfrac{100{,}000}{\sqrt{1 + 0}} = \mathbf{100{,}000}$

(b) $A_{ol} = \dfrac{100{,}000}{\sqrt{1 + (0.1)^2}} = \mathbf{99{,}503}$

(c) $A_{ol} = \dfrac{100{,}000}{\sqrt{1 + (1)^2}} = \dfrac{100{,}000}{\sqrt{2}} = \mathbf{70{,}710}$

(d) $A_{ol} = \dfrac{100{,}000}{\sqrt{1 + (10)^2}} = \mathbf{9950}$

Related Problem Find A_{ol} for the following frequencies. Assume $f_{c(ol)} = 200$ Hz and $A_{ol(mid)} = 80{,}000$.

(a) $f = 2$ Hz **(b)** $f = 10$ Hz **(c)** $f = 2500$ Hz

EXAMPLE 12–10

A certain op-amp has three internal amplifier stages with the following gains and critical frequencies:

Stage 1: $A_{v1} = 40$ dB, $f_{c1} = 2$ kHz

Stage 2: $A_{v2} = 32$ dB, $f_{c2} = 40$ kHz

Stage 3: $A_{v3} = 20$ dB, $f_{c3} = 150$ kHz

Determine the open-loop midrange gain in decibels and the total phase lag when $f = f_{c1}$.

Solution

$$A_{ol(mid)} = A_{v1} + A_{v2} + A_{v3} = 40\,\text{dB} + 32\,\text{dB} + 20\,\text{dB} = 92\,\text{dB}$$

$$\theta_{tot} = -\tan^{-1}\left(\frac{f}{f_{c1}}\right) - \tan^{-1}\left(\frac{f}{f_{c2}}\right) - \tan^{-1}\left(\frac{f}{f_{c3}}\right)$$

$$= -\tan^{-1}(1) - \tan^{-1}\left(\frac{2}{40}\right) - \tan^{-1}\left(\frac{2}{150}\right) = -45° - 2.86° - 0.76° = \mathbf{-48.6°}$$

Related Problem The internal stages of a two-stage amplifier have the following characteristics: $A_{v1} = 50$ dB, $A_{v2} = 25$ dB, $f_{c1} = 1500$ Hz, and $f_{c2} = 3000$ Hz. Determine the open-loop midrange gain in decibels and the total phase lag when $f = f_{c1}$.

EXAMPLE 12–9

Calculate the phase shift for an *RC* lag circuit for each of the following frequencies, and then plot the curve of phase shift versus frequency. Assume $f_c = 100$ Hz.

(a) $f = 1$ Hz **(b)** $f = 10$ Hz **(c)** $f = 100$ Hz
(d) $f = 1000$ Hz **(e)** $f = 10{,}000$ Hz

Solution

(a) $\theta = -\tan^{-1}\left(\frac{f}{f_c}\right) = -\tan^{-1}\left(\frac{1\text{ Hz}}{100\text{ Hz}}\right) = \mathbf{-0.573°}$

(b) $\theta = -\tan^{-1}\left(\frac{10\text{ Hz}}{100\text{ Hz}}\right) = \mathbf{-5.71°}$

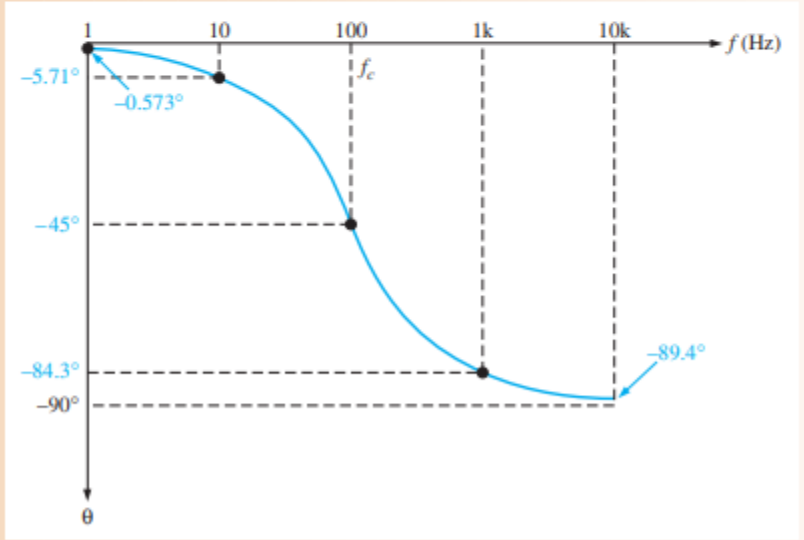
(c) $\theta = -\tan^{-1}\left(\frac{100\text{ Hz}}{100\text{ Hz}}\right) = \mathbf{-45°}$

(d) $\theta = -\tan^{-1}\left(\frac{1000\text{ Hz}}{100\text{ Hz}}\right) = \mathbf{-84.3°}$

(e) $\theta = -\tan^{-1}\left(\frac{10{,}000\text{ Hz}}{100\text{ Hz}}\right) = \mathbf{-89.4°}$

The phase shift-versus-frequency curve is plotted in Figure 12–40. Note that the frequency axis is logarithmic.

▶ **FIGURE 12–40**

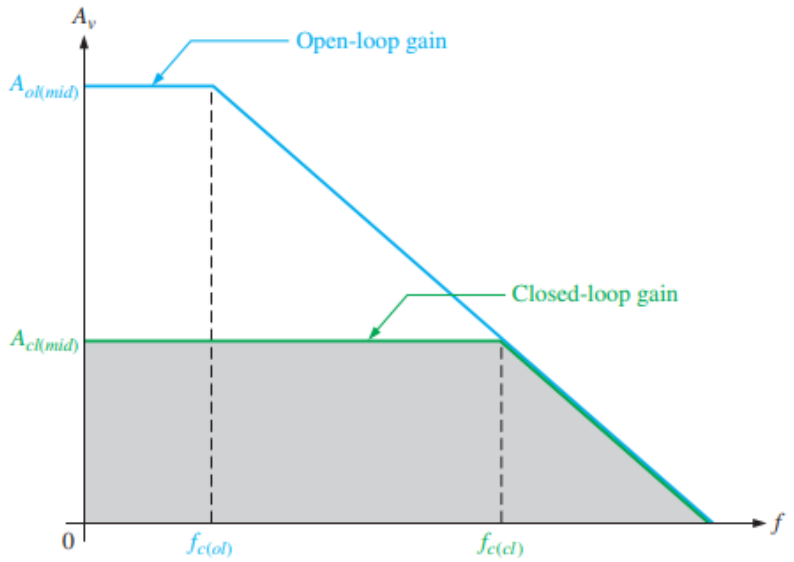


Related Problem At what frequency, in this example, is the phase shift 60°?

Closed Loop Frequency & Phase Responses:

FIGURE 12–42

Closed-loop gain compared to open-loop gain.



Effect of Negative Feedback on Bandwidth

You know how negative feedback affects the gain; now you will learn how it affects the amplifier's bandwidth. The closed-loop critical frequency of an op-amp is

$$f_{c(cl)} = f_{c(ol)}(1 + BA_{ol(mid)})$$

This expression shows that the closed-loop critical frequency, $f_{c(cl)}$, is higher than the open-loop critical frequency $f_{c(ol)}$ by the factor $1 + BA_{ol(mid)}$. You will find a derivation of Equation 12–21 in "Derivations of Selected Equations" at www.pearsonhighered.com/floyd.

Since $f_{c(cl)}$ equals the bandwidth for the closed-loop amplifier, the closed-loop bandwidth (BW_{cl}) is also increased by the same factor.

$$BW_{cl} = BW_{ol}(1 + BA_{ol(mid)})$$

EXAMPLE 12–11

A certain amplifier has an open-loop midrange gain of 150,000 and an open-loop 3 dB bandwidth of 200 Hz. The attenuation (B) of the feedback loop is 0.002. What is the closed-loop bandwidth?

Solution

$$BW_{cl} = BW_{ol}(1 + BA_{ol(mid)}) = 200\,\text{Hz}[1 + (0.002)(150{,}000)] = \mathbf{60.2\,kHz}$$

Related Problem

If $A_{ol(mid)} = 200{,}000$ and $B = 0.05$, what is the closed-loop bandwidth?


Thank
you