Power System

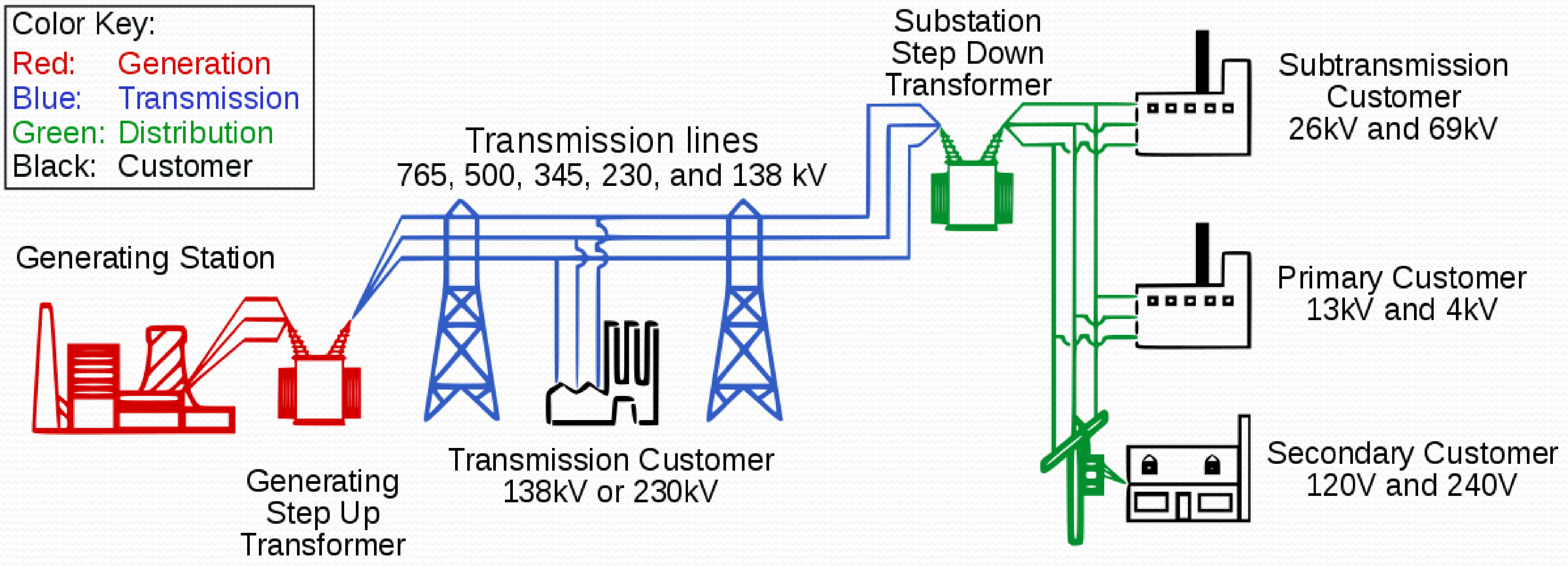


Fig 04: Power System Network

EEE203: Electrical Circuit II

AC Circuit

- Driven by alternating voltage or current source.
- Excitation by sinusoid.

A sinusoid is a signal that has the form of the sine or cosine function.

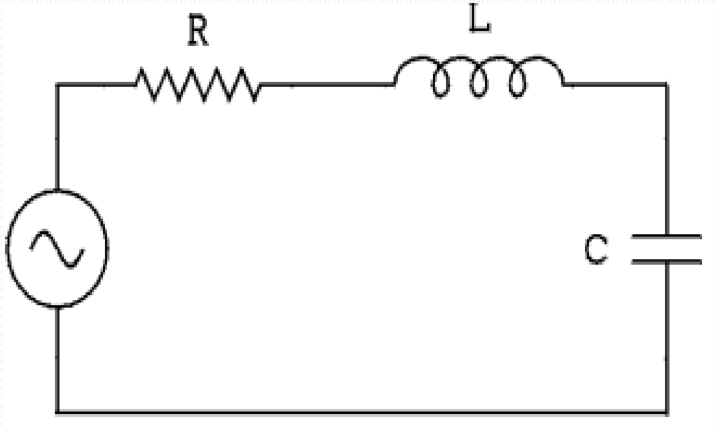


Fig 01: Simple ac circuit.

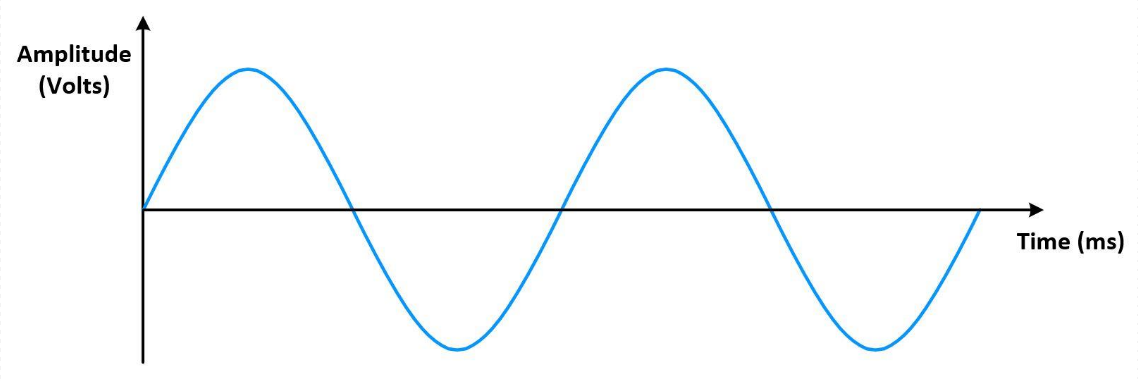


Fig 02: Sinusoidal signal.

Lesson 6: Sinusoids and Phasors

Course Code: EEE 203
Course Title: Electrical Circuit II

Learning Objectives

After this session you will be able to

- Explain the generation of alternating voltage and current.
- Identify different form of emf equations of alternating voltage.
- Define terminology of alternating wave.
- Sketch alternating voltage waveform.

AC Source

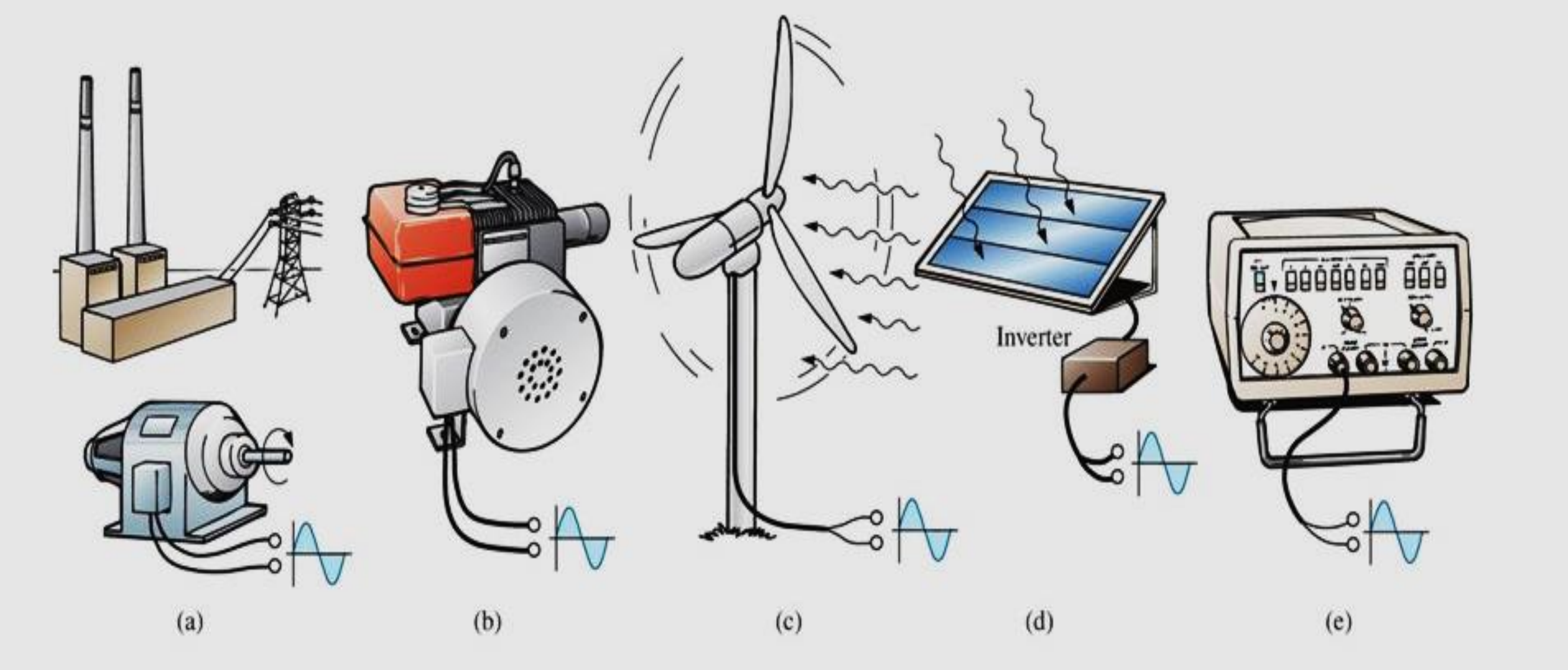


Fig 03: Various sources of ac power (a) generating plant (b) portable ac generator (c) wind-power station (d) solar panel and (e) function generator

AC Generation

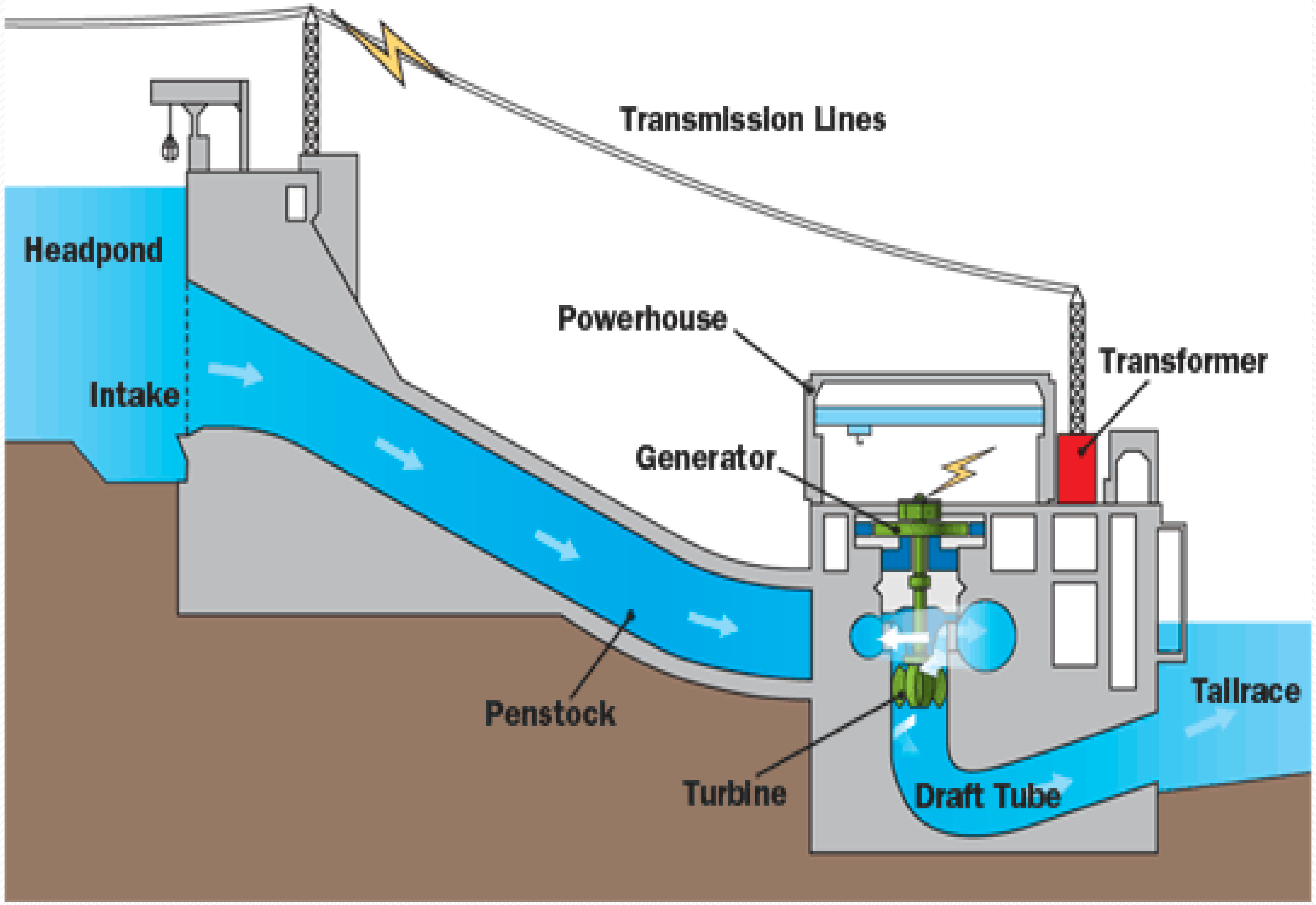


Fig 05: Hydroelectric Power Plant.

AC Generation

AC Generator (Alternator)

- Converts mechanical energy to electrical energy.
- Principle: Faraday's laws of electromagnetic induction.

Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil.

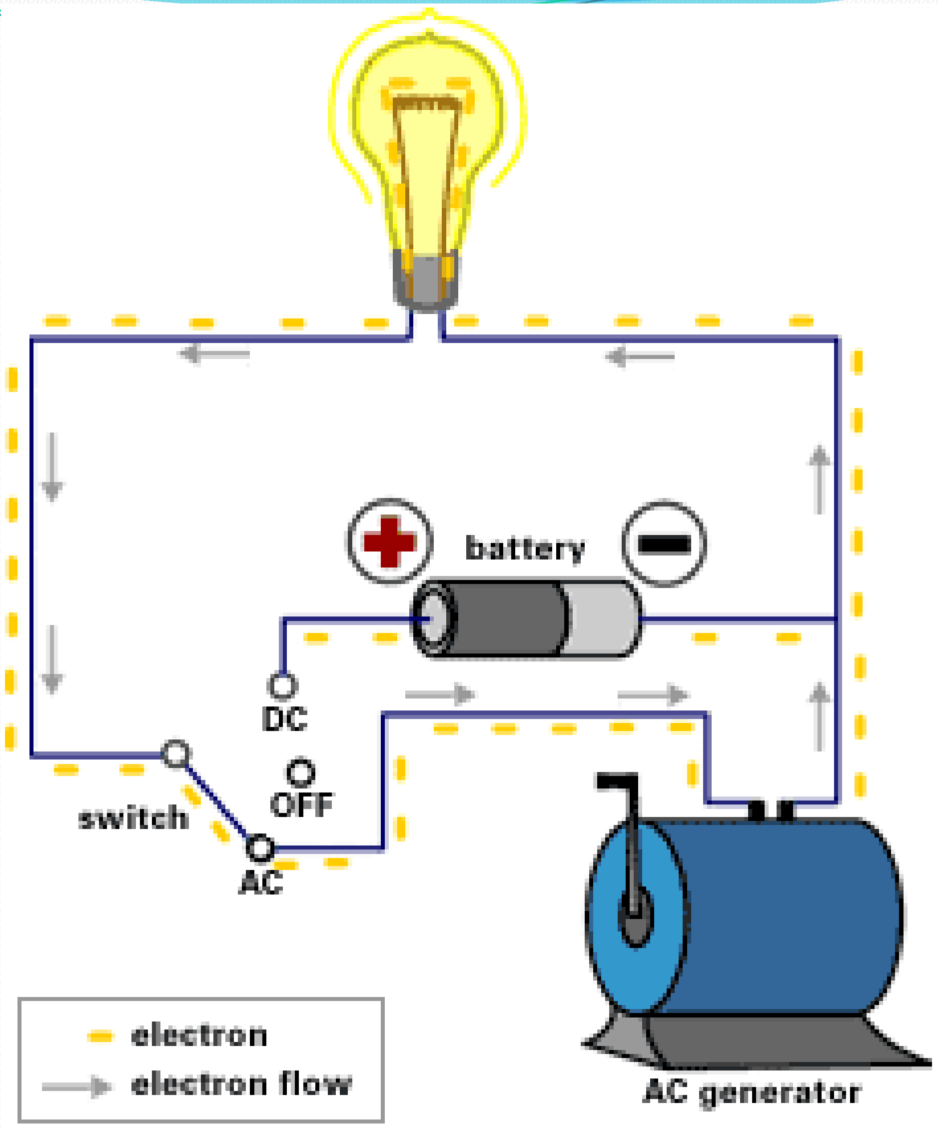
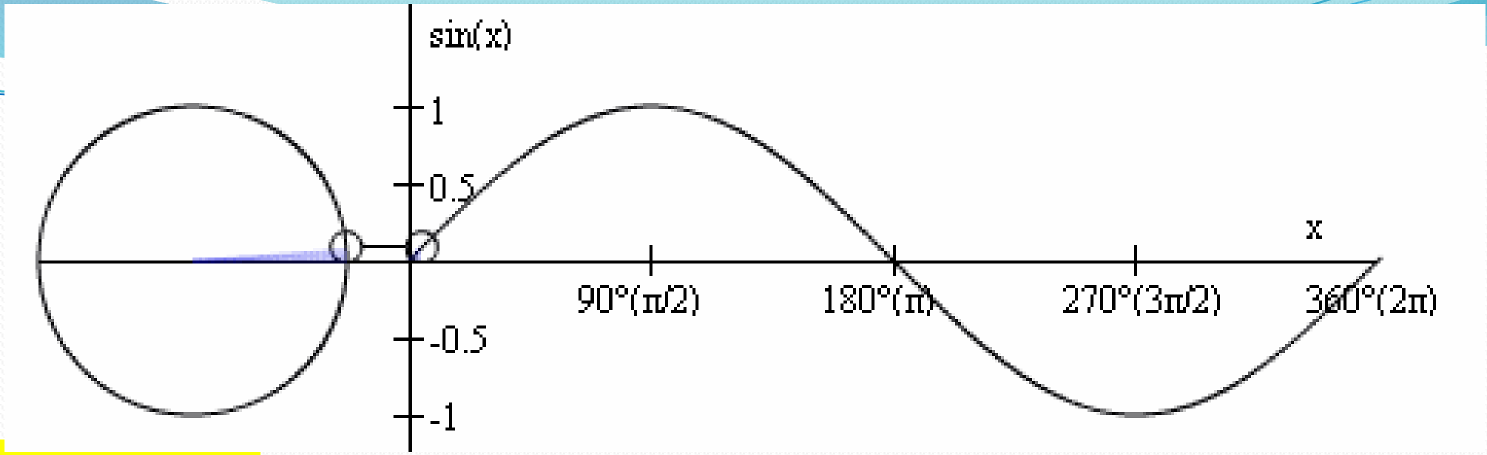


Fig 06: Generating alternating current from ac generator.

AC Generation



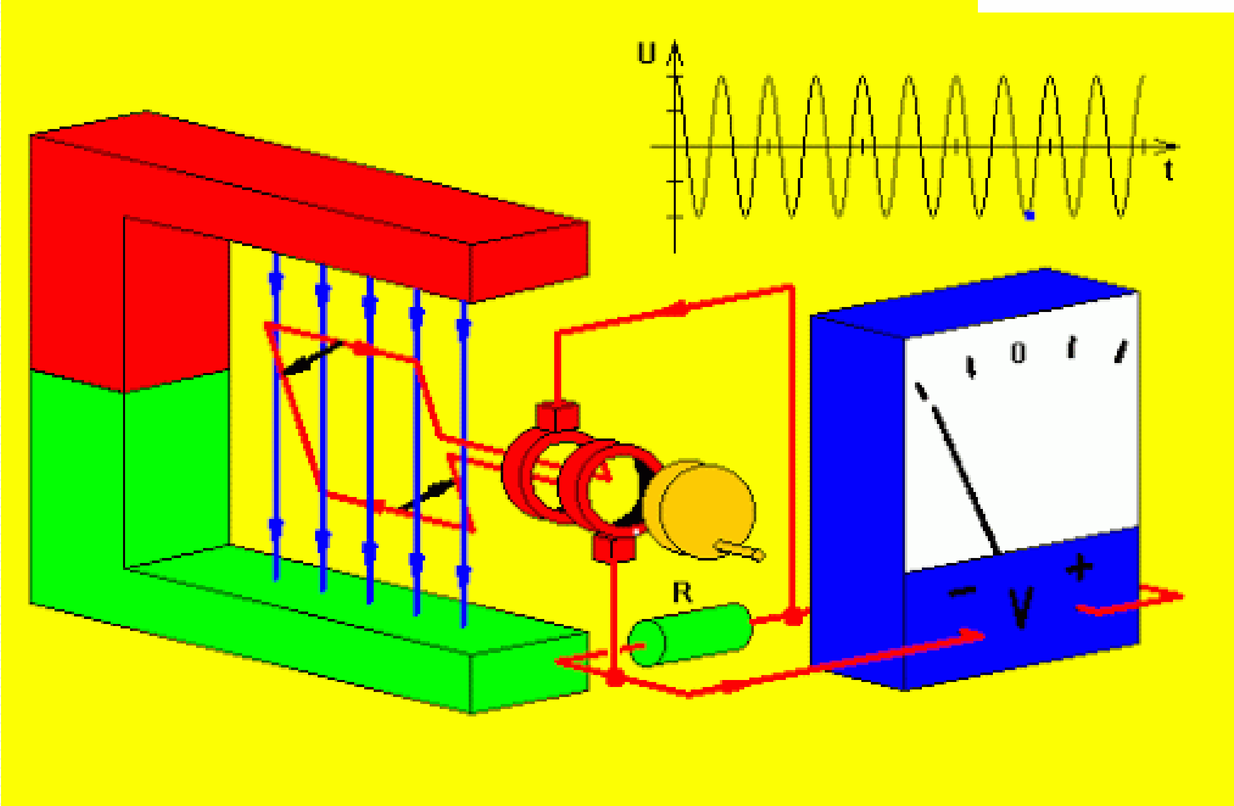


Fig 07: Simple ac generator.

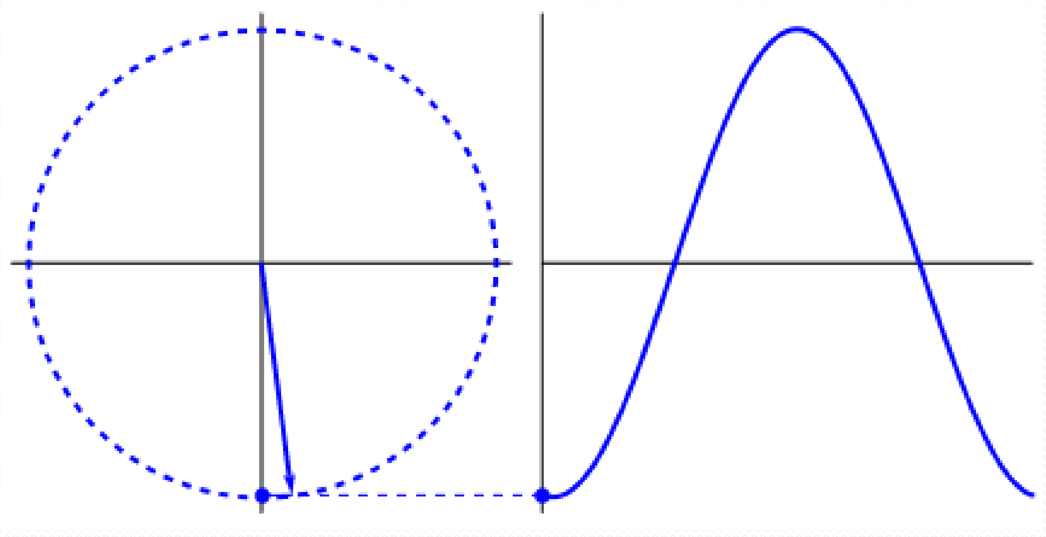


Fig 08: Internal generated voltage wave shape of ac generator.

Sinusoidal Alternating Current and Voltage

According to Faraday's and Lenz' s law.

Alternating Voltage,

- $e = -N_C \frac{d\varphi(t)}{dt} = -N_C \frac{d}{dt}(\varphi_m \cos \omega t)$
- $e = N_C \omega \varphi_m sin\ \omega t$
- $\boldsymbol{e = E_m \sin \omega t}$ **Volt**

$$\begin{aligned} \boldsymbol{e} &= \boldsymbol{E_m \sin \theta} \\ &= \boldsymbol{E_m \sin 2\pi f t} \\ &= \boldsymbol{E_m \sin \frac{2\pi}{T} t} \end{aligned}$$

Similarly for Alternating Current,

- $\boldsymbol{i = I_m \sin \omega t\ \ Amp}$

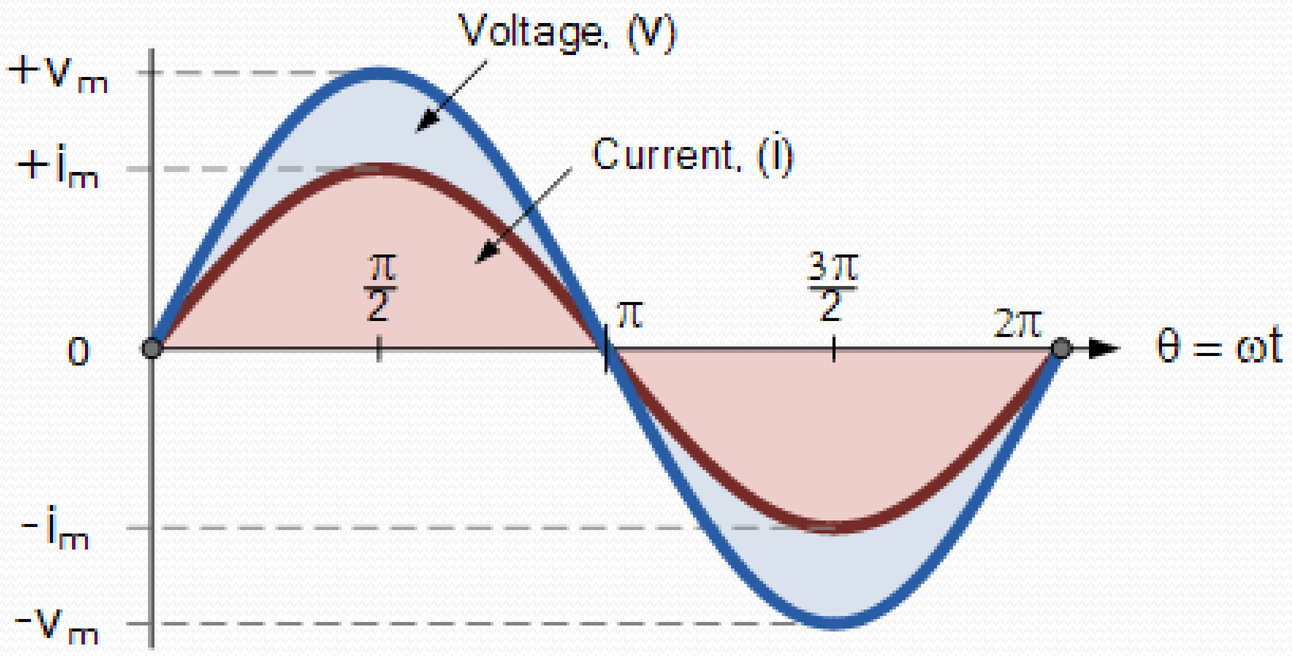


Fig 09: Sinusoidal alternating current and voltage.

Important AC Terminology

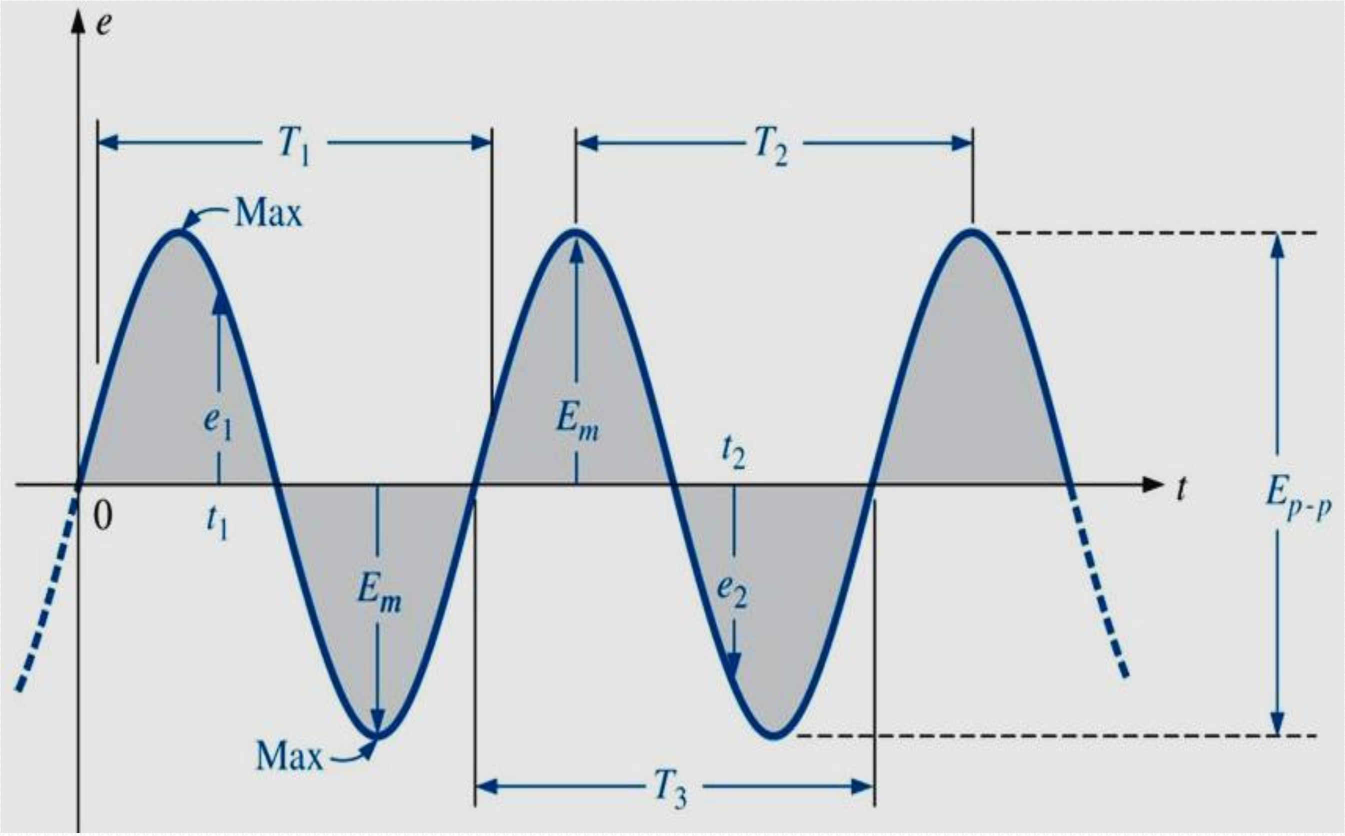


Fig 10: Important parameters for a sinusoidal voltage.

- Instantaneous Value
- Peak amplitude
- Peak value
- Peak to peak value
- Period & Frequency

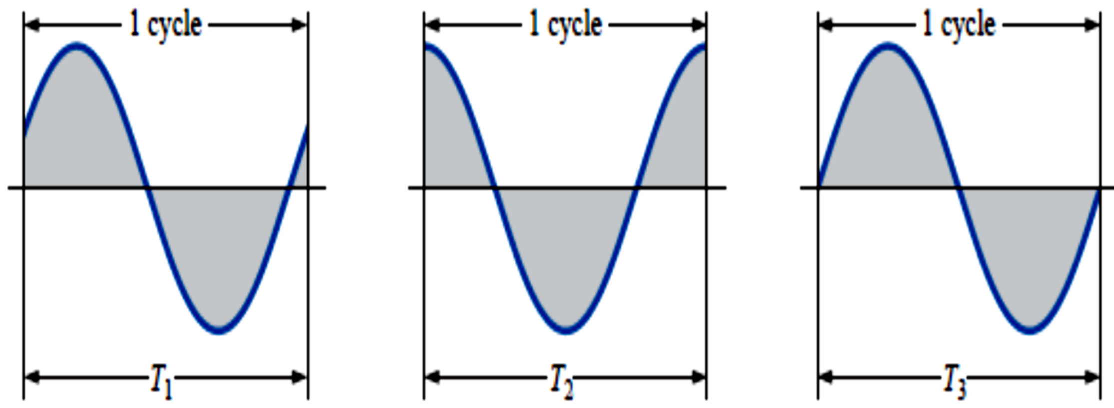


Fig 1.10: Defining the cycle and period of a sinusoidal waveform.

Important Relations Between Sinusoidal Parameters

- Time period and frequency, $T = \frac{1}{f}$
- $angular\ velocity, \omega = \frac{angle\ turned}{time\ taken} = \frac{2\pi}{T}$
- Frequency and speed, $f = \frac{NP}{120}$
- Radian and degree

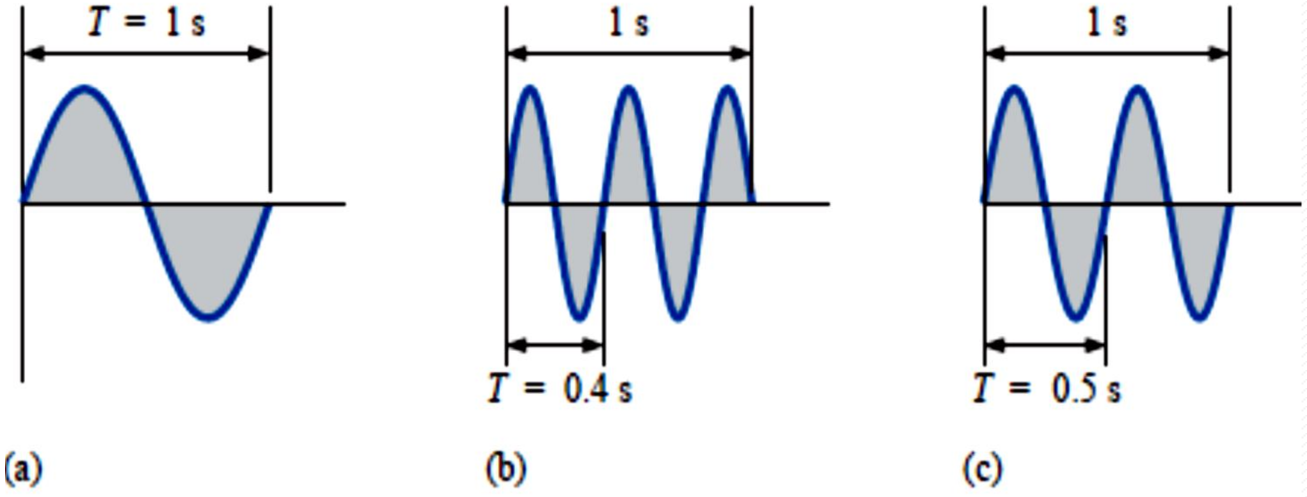


Fig 12: Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

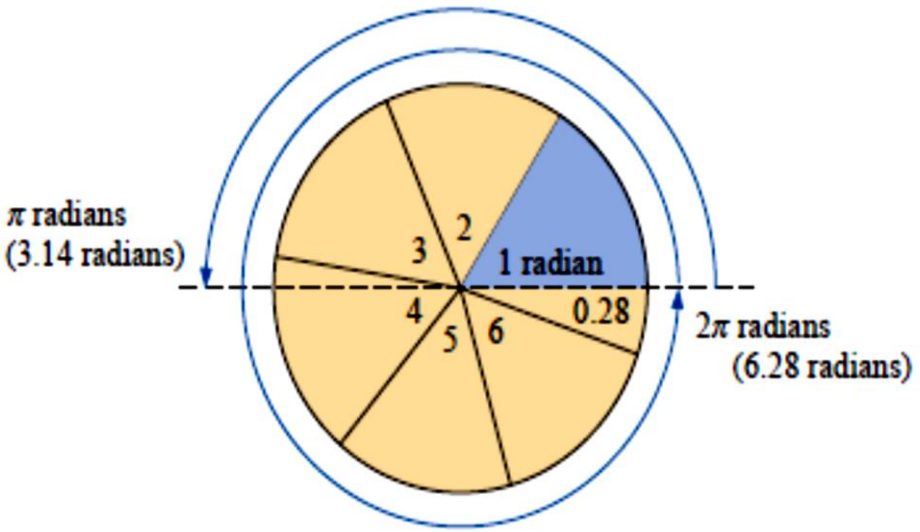


Fig 1.13: There are 2π radians in one full circle of 360°.

$\therefore 1\ rad \cong 57.3°$

Simple and Complex Wave

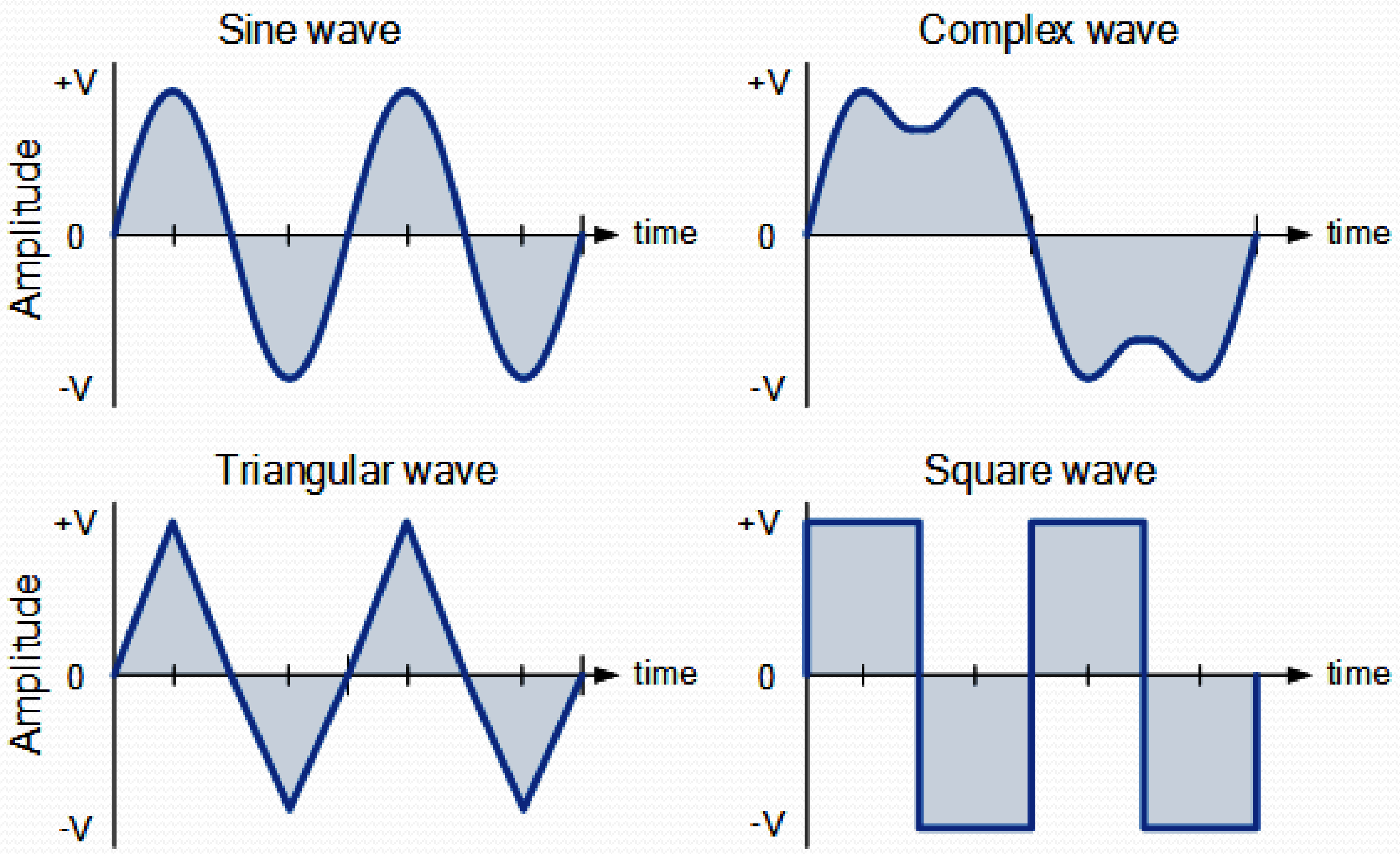


Fig 1.14: Simple wave

Simple and Complex Wave

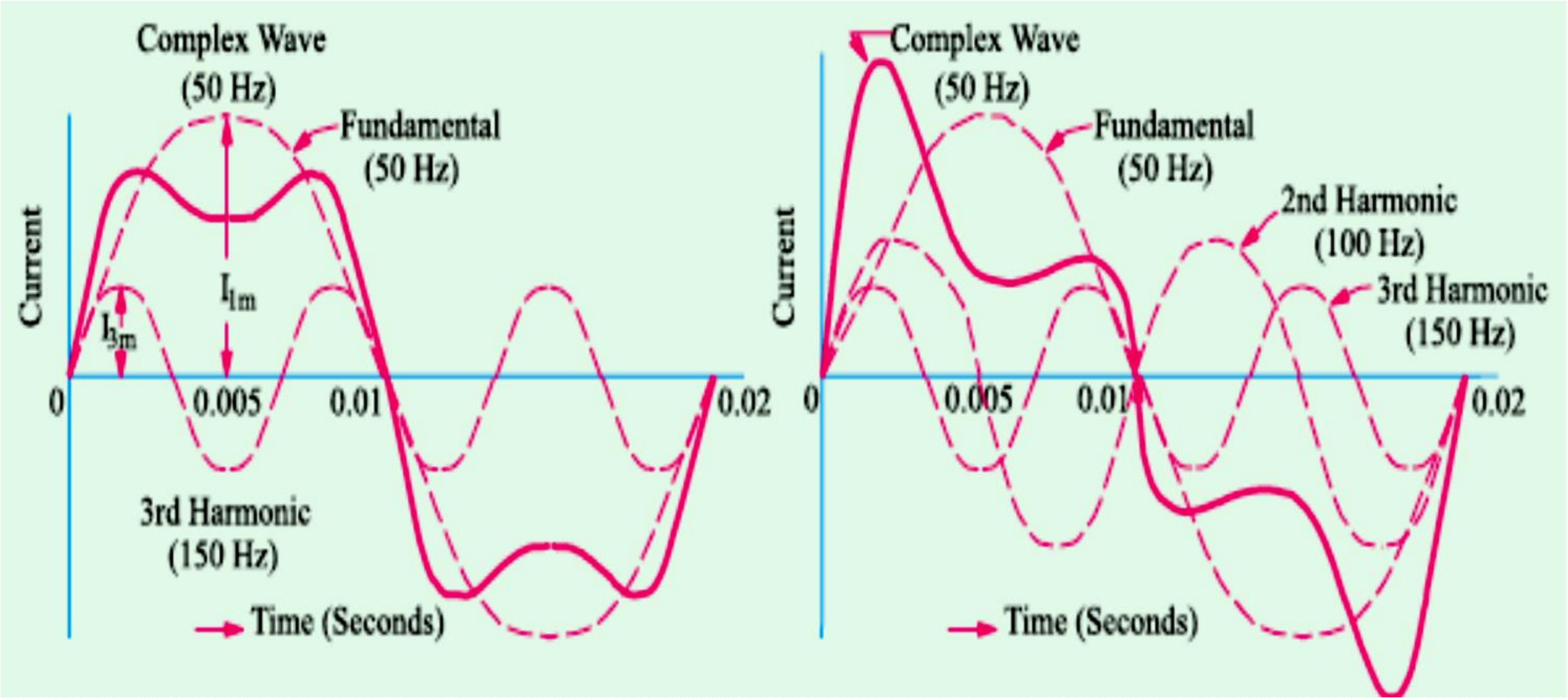


Fig 1.15: Complex wave

Problem Solving

Example 01: [Boylestad: **Example 13.10]**

Sketch e = 10 sin 314t with the abscissa (a) angle α in degrees, (b) angle α in radians and (c) time t in seconds.

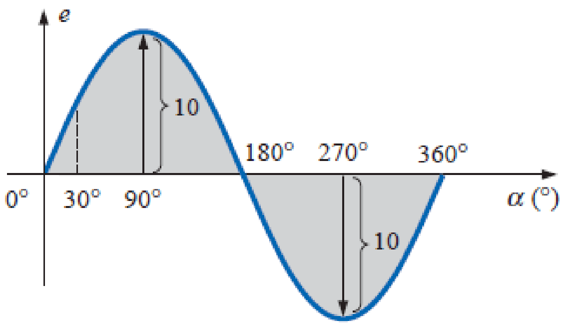


FIG. 13.20
Example 13.10, horizontal axis in degrees.

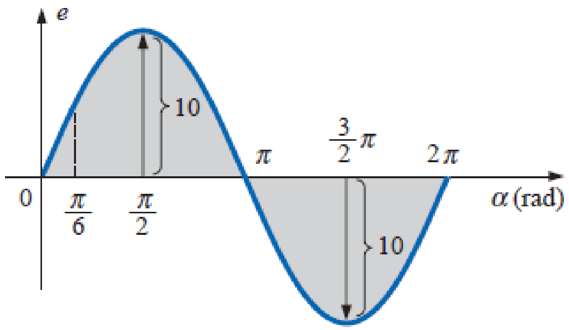


FIG. 13.21
Example 13.10, horizontal axis in radians.

c. $360°$: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{314} = 20\text{ ms}$

$180°$: $\dfrac{T}{2} = \dfrac{20\text{ ms}}{2} = 10\text{ ms}$

$90°$: $\dfrac{T}{4} = \dfrac{20\text{ ms}}{4} = 5\text{ ms}$

$30°$: $\dfrac{T}{12} = \dfrac{20\text{ ms}}{12} = 1.67\text{ ms}$

See Fig. 13.22.

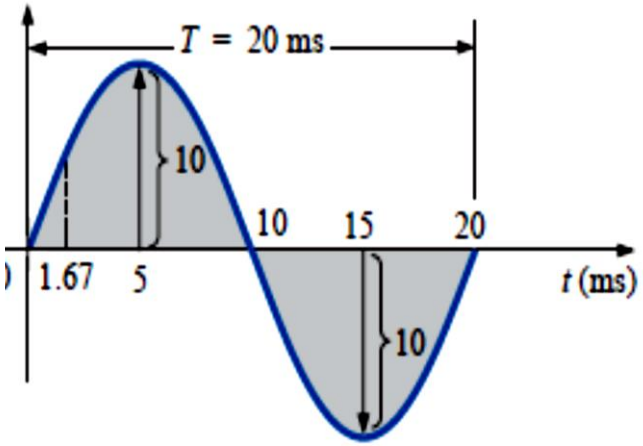


FIG. 13.22
Example 13.10, horizontal axis in milliseconds.

Problem Solving

Example 02: [B.L.Theraja: **Example 11.3]**

An alternating current of frequency 60 Hz has a maximum value of 120 A. Write down the equation for its instantaneous value. Reckoning time from the instant the current is zero and is becoming positive, find ***(a)*** *the instantaneous value after 1/360 second and* ***(b)*** *the time taken to reach 96 A for the first time.*

Solution. The instantaneous current equation is

$$i = 120 \sin 2\pi f t = 120 \sin 120\pi t$$

Now when $t = 1/360$ second, then

(*a*)
$$i = 120 \sin(120 \times \pi \times 1/360) \quad \text{...angle in radians}$$
$$= 120 \sin(120 \times 180 \times 1/360) \quad \text{...angle in degree}$$
$$= 120 \sin 60^\circ = 103.9 \text{ A}$$

(*b*)
$$96 = 120 \times \sin 2 \times 180 \times 60 \times t \quad \text{...angle in degree}$$

or
$$\sin(360 \times 60 \times t) = 96/120 = 0.8 \quad \therefore\ 360 \times 60 \times t = \sin^{-1} 0.8 = 53^\circ \text{ (approx)}$$

$\therefore$
$$t = \theta/2\pi f = 53/360 \times 60 = \mathbf{0.00245\ second.}$$

Phase Relation

Phase: Specific location within a cycle with respect to a fixed reference.

What will be the phase of maximum positive value for fig 1.16?

General expression of sinusoid,

$$\boxed{A(t) = A_m \sin(\omega t \pm \phi)}$$

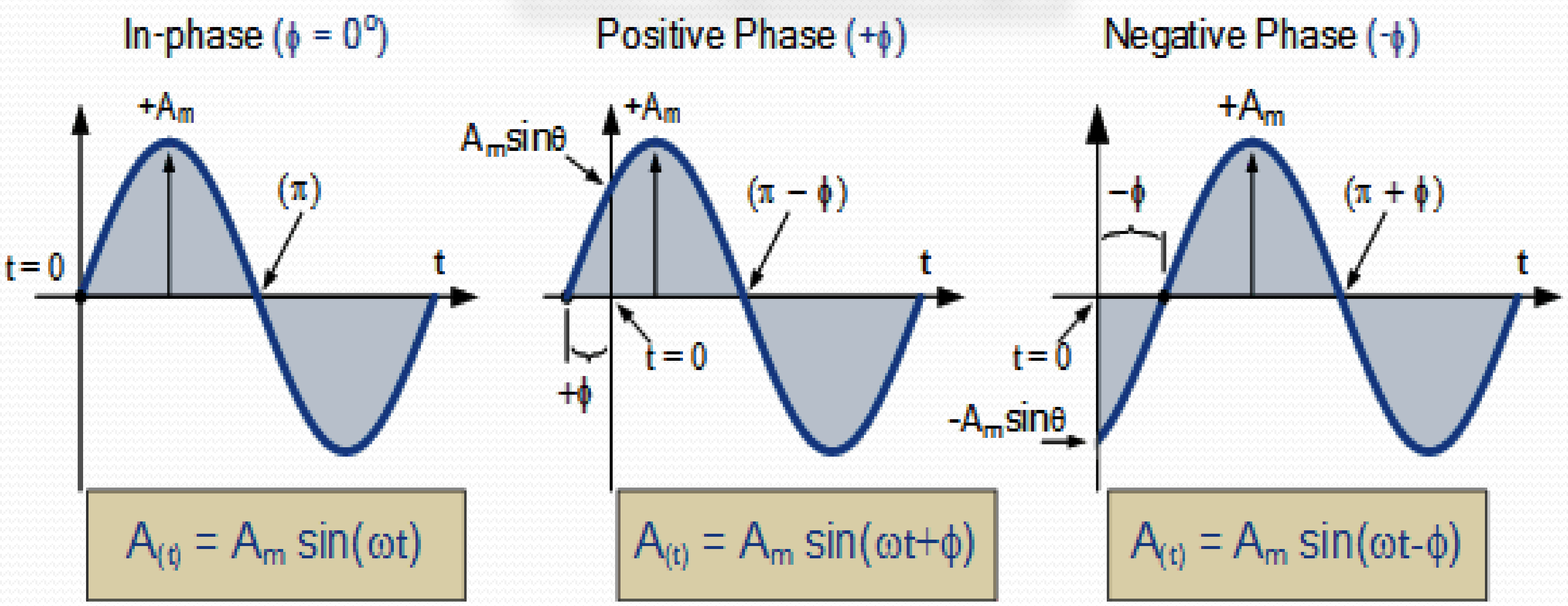


Fig 1.17: Phase shift of sinusoidal function..

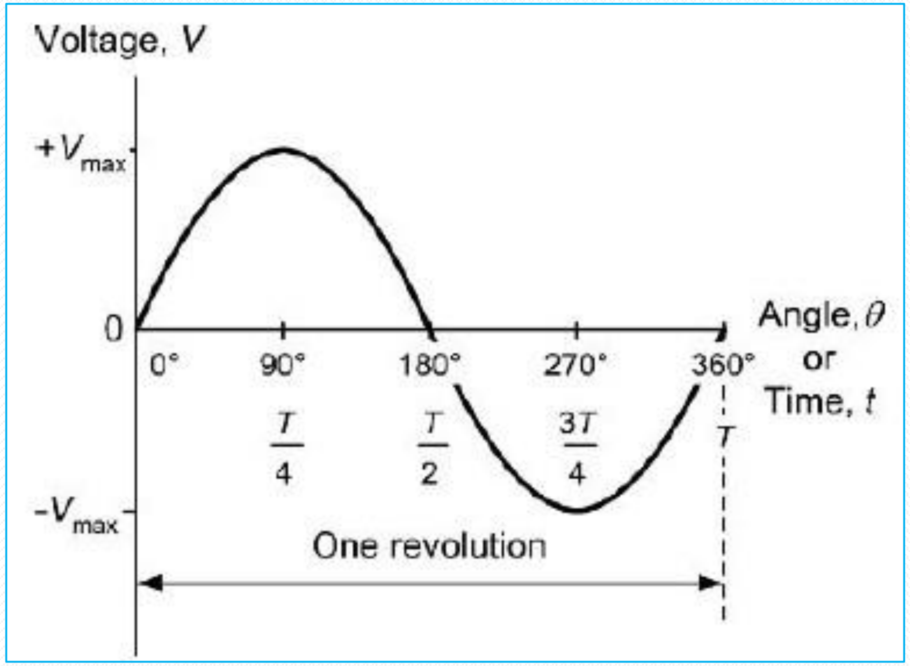


Fig 1.16: Phase of a sinusoid.

Phase Relation

Phase difference: Two alternating quantities of the same frequency have different zero points.

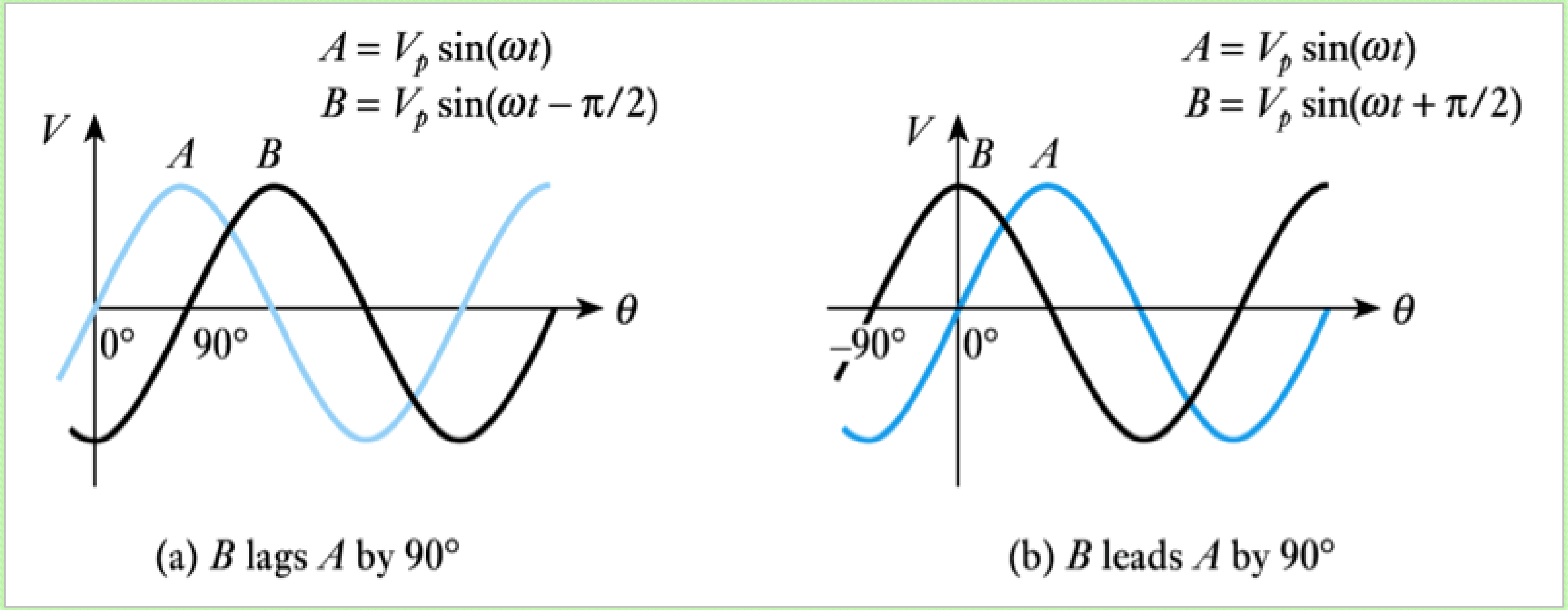


Fig 1.17: Phase shift of sinusoidal function..

Phase Relation

Example 03: [Boylestad, Example 13.12, Ed: 10th]

What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10\sin(\omega t + 30^\circ)$
$i = 5\sin(\omega t + 70^\circ)$

b. $i = 15\sin(\omega t + 60^\circ)$
$v = 10\sin(\omega t - 20^\circ)$

c. $i = 2\cos(\omega t + 10^\circ)$
$v = 3\sin(\omega t - 10^\circ)$

d. $i = -\sin(\omega t + 30^\circ)$
$v = 2\sin(\omega t + 10^\circ)$

e. $i = -2\cos(\omega t - 60^\circ)$
$v = 3\sin(\omega t - 150^\circ)$

Solutions:

a. See Fig. 13.27.
***i* leads *v* by 40°, or *v* lags *i* by 40°.**

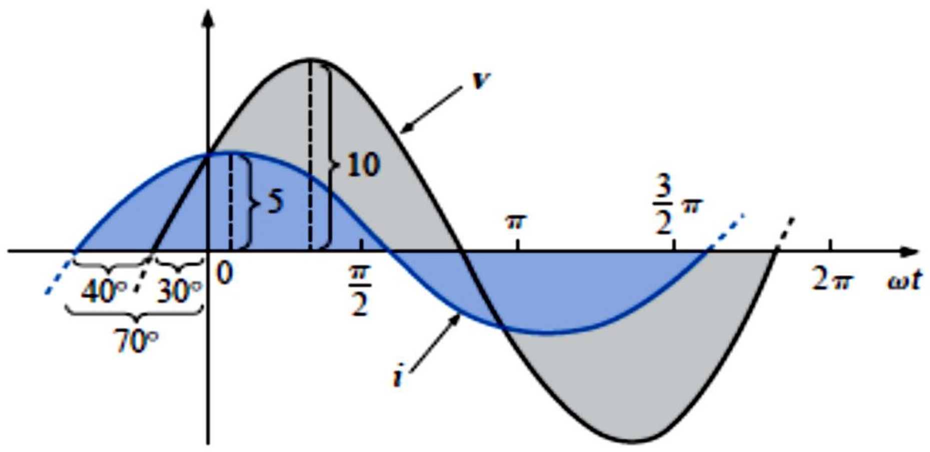


FIG. 13.27
Example 13.12; i leads v by 40°.

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$

$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$

$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$

$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

Phase Relation

Example 03: [Boylestad, Example 13.12, Ed: 10th]

What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10\sin(\omega t + 30°)$
$\quad i = 5\sin(\omega t + 70°)$

b. $i = 15\sin(\omega t + 60°)$
$\quad v = 10\sin(\omega t - 20°)$

c. $i = 2\cos(\omega t + 10°)$
$\quad v = 3\sin(\omega t - 10°)$

d. $i = -\sin(\omega t + 30°)$
$\quad v = 2\sin(\omega t + 10°)$

e. $i = -2\cos(\omega t - 60°)$
$\quad v = 3\sin(\omega t - 150°)$

c. See Fig. 13.29.

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$$
$$= 2\sin(\omega t + 100°)$$

***i* leads *v* by 110°, or *v* lags *i* by 110°.**

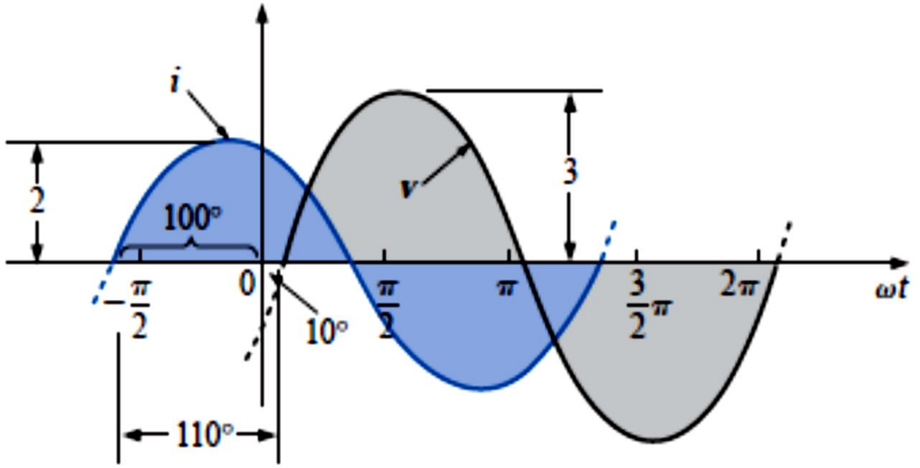


FIG. 13.29

Example 13.12; i leads v by 110°.

$$\sin(\omega t \pm 180°) = -\sin\omega t$$
$$\cos(\omega t \pm 180°) = -\cos\omega t$$
$$\sin(\omega t \pm 90°) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90°) = \mp\sin\omega t$$

Phase Relation

Example 03: [Boylestad, Example 13.12, Ed: 10th]
What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
$\quad i = 5 \sin(\omega t + 70°)$
b. $i = 15 \sin(\omega t + 60°)$
$\quad v = 10 \sin(\omega t - 20°)$
c. $i = 2 \cos(\omega t + 10°)$
$\quad v = 3 \sin(\omega t - 10°)$
d. $i = -\sin(\omega t + 30°)$
$\quad v = 2 \sin(\omega t + 10°)$
e. $i = -2 \cos(\omega t - 60°)$
$\quad v = 3 \sin(\omega t - 150°)$

d. See Fig. 13.30.

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° - \overset{\text{Note}}{180°})$$
$$= \sin(\omega t - 150°)$$

***v* leads *i* by 160°, or *i* lags *v* by 160°.**

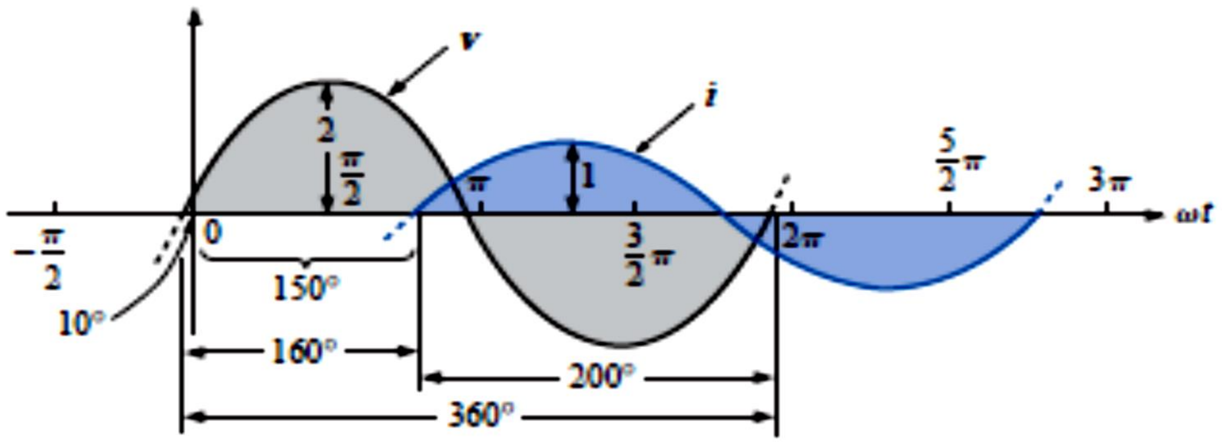


FIG. 13.30
Example 13.12; v leads i by 160°.

$$\sin(\omega t \pm 180°) = -\sin \omega t$$
$$\cos(\omega t \pm 180°) = -\cos \omega t$$
$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90°) = \mp \sin \omega t$$

Phase Relation

Example 03: [Boylestad, Example 13.12, Ed: 10th]
What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
   $i = 5 \sin(\omega t + 70°)$
b. $i = 15 \sin(\omega t + 60°)$
   $v = 10 \sin(\omega t - 20°)$
c. $i = 2 \cos(\omega t + 10°)$
   $v = 3 \sin(\omega t - 10°)$
d. $i = -\sin(\omega t + 30°)$
   $v = 2 \sin(\omega t + 10°)$
e. $i = -2 \cos(\omega t - 60°)$
   $v = 3 \sin(\omega t - 150°)$

e. See Fig. 13.31.

$$i = -2\cos(\omega t - 60°) = 2\cos(\omega t - 60° - \underset{\text{By choice}}{180°}) = 2\cos(\omega t - 240°)$$

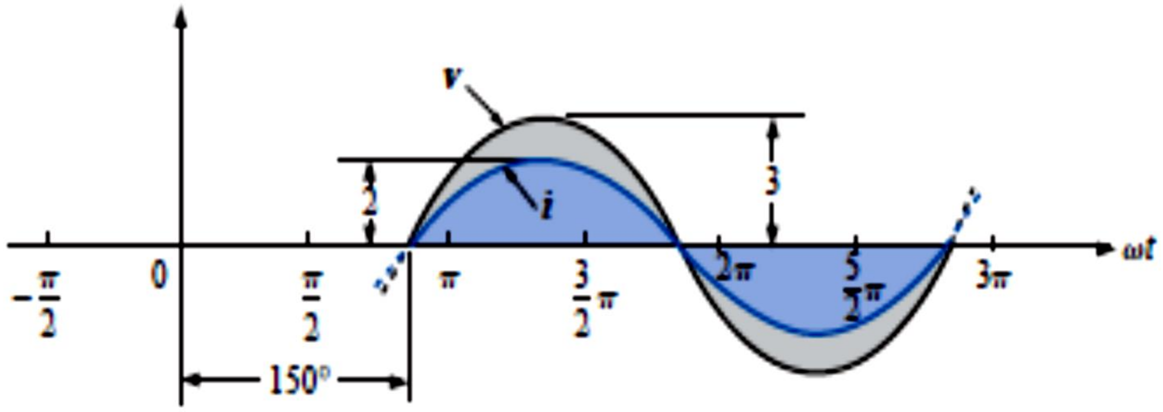


FIG. 13.31
Example 13.12; v and i are in phase.

$$\sin(\omega t \pm 180°) = -\sin \omega t$$
$$\cos(\omega t \pm 180°) = -\cos \omega t$$
$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90°) = \mp \sin \omega t$$

Average value of AC

$$V_{avg} = \frac{area\ of\ half\ cycle}{base\ length\ of\ half\ cycle}$$

$$= \frac{\int_0^\pi v\, d\theta}{\pi}$$

$$= \frac{\int_0^\pi V_m\, sin\theta\, d\theta}{\pi}$$

$$= \frac{V_m}{\pi} |-\cos\theta|_0^\pi$$

$$= \frac{2V_m}{\pi}$$

$$V_{avg} = 0.637V_m$$

Similarly, $I_{avg} = 0.637I_m$

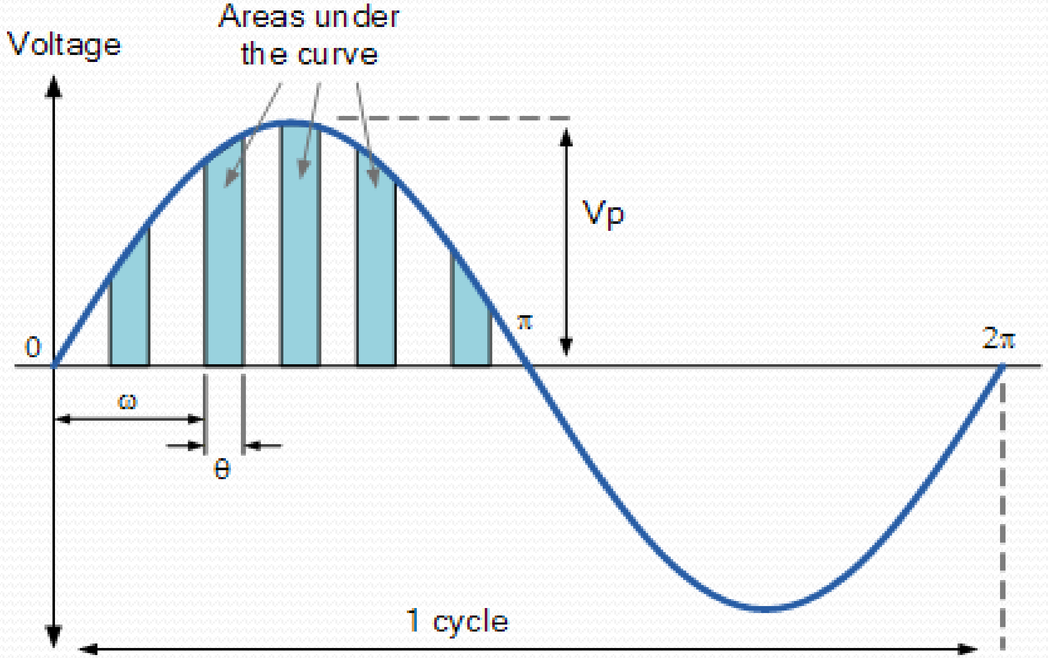


Fig 1.18: Sinusoidal waveform

RMS value of AC

From equations (i) and (ii)

$$I_{eff} = \sqrt{(i^2)_{avg}}$$

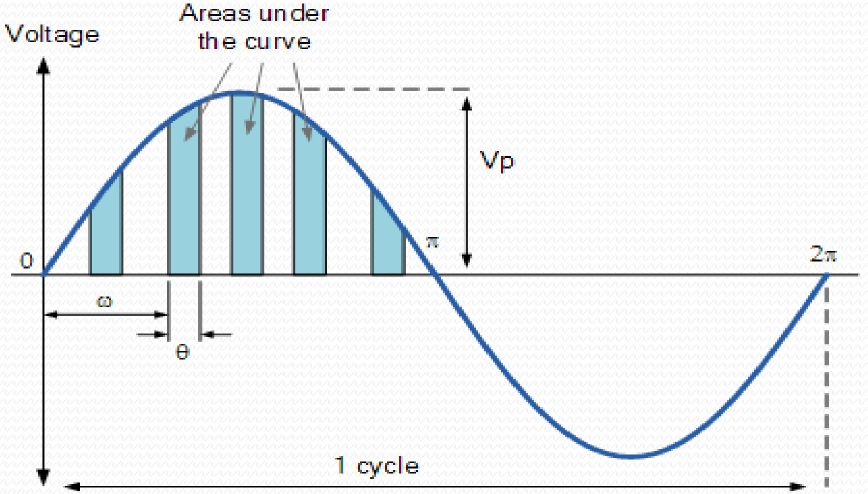
$$I_{eff} = I_{rms} = \sqrt{(I_m^2 sin^2\theta)_{avg}}$$

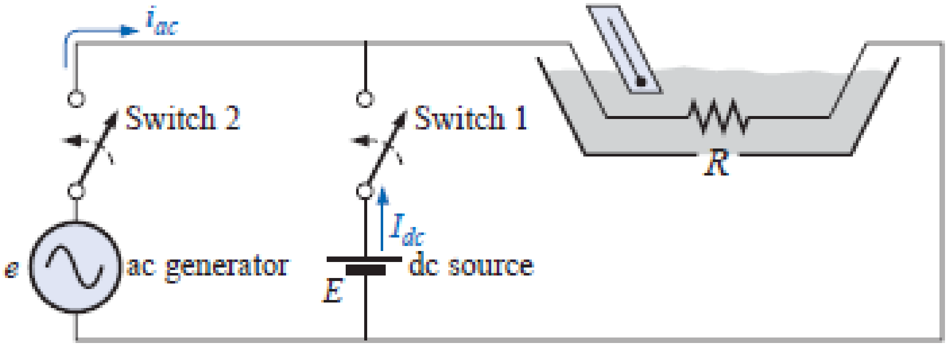
$$= \sqrt{\frac{\int_0^{2\pi} I_m^2 sin^2\theta \, d\theta}{2\pi}} = \sqrt{\left(\frac{I_m^2}{4\pi}\right)\int_0^{2\pi}(1-\cos 2\theta)\, d\theta}$$

$$= \sqrt{\left(\frac{I_m^2}{4\pi}\right)\left|\theta - \frac{sin2\theta}{2}\right|_0^{2\pi}} = \sqrt{\left(\frac{I_m^2}{4\pi}\right)(2\pi)}$$

$$\boldsymbol{I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m}$$

Similarly, $\boldsymbol{V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 V_m}$





Instantaneous power, $p = i^2R$

Average power will be, $P_{ac} = (i^2R)_{avg}$

$$\therefore P_{ac} = R(i^2)_{avg} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

And , power delivered by dc sources

$$P_{dc} = (I_{eff})^2 R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Form and Peak factor of AC

$$Form\ factor = \frac{rms\ value}{average\ value} = 1.11$$

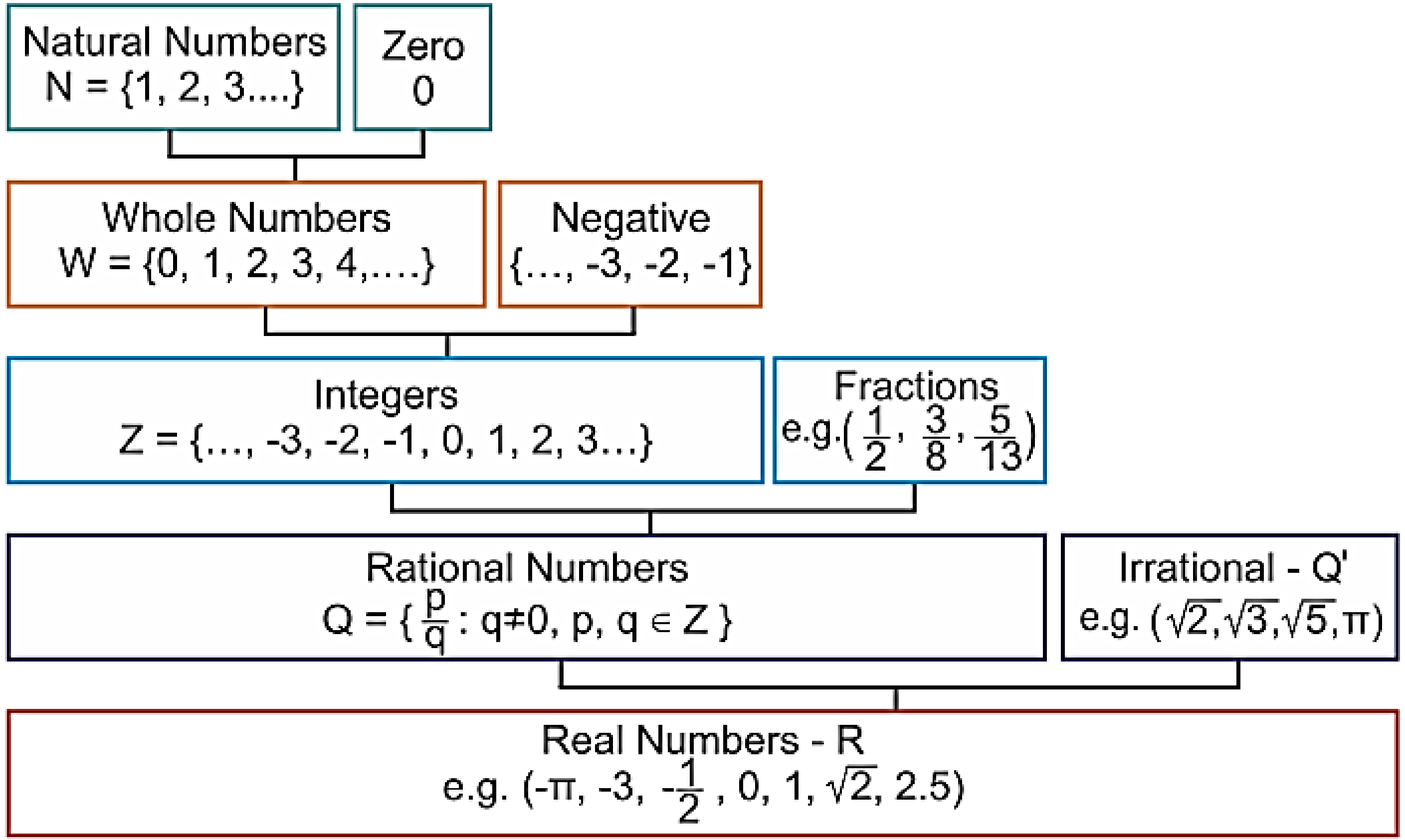
$$Peak\ factor = \frac{\max value}{rms\ value} = 1.41$$

Representation of AC

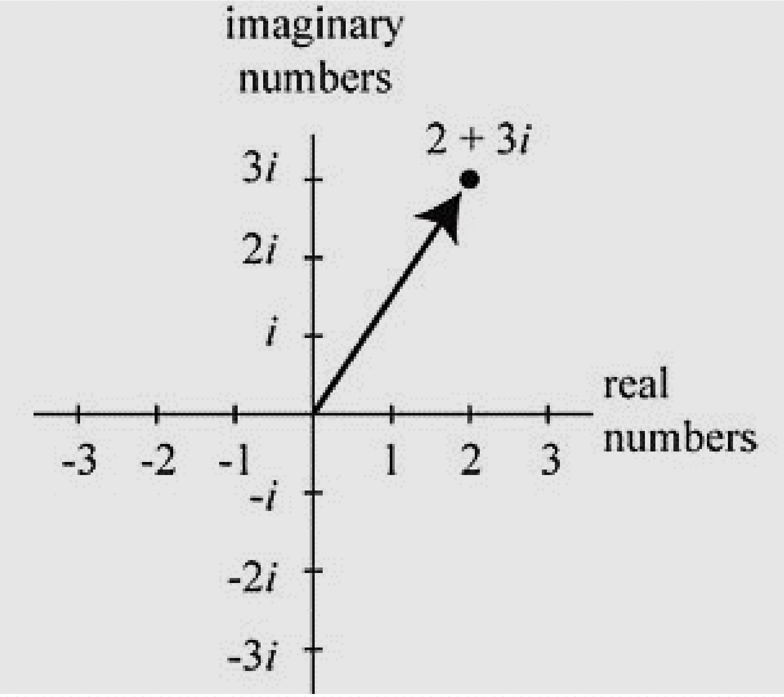
An AC can be represented in the form of
(i) waves and
(ii) equations.

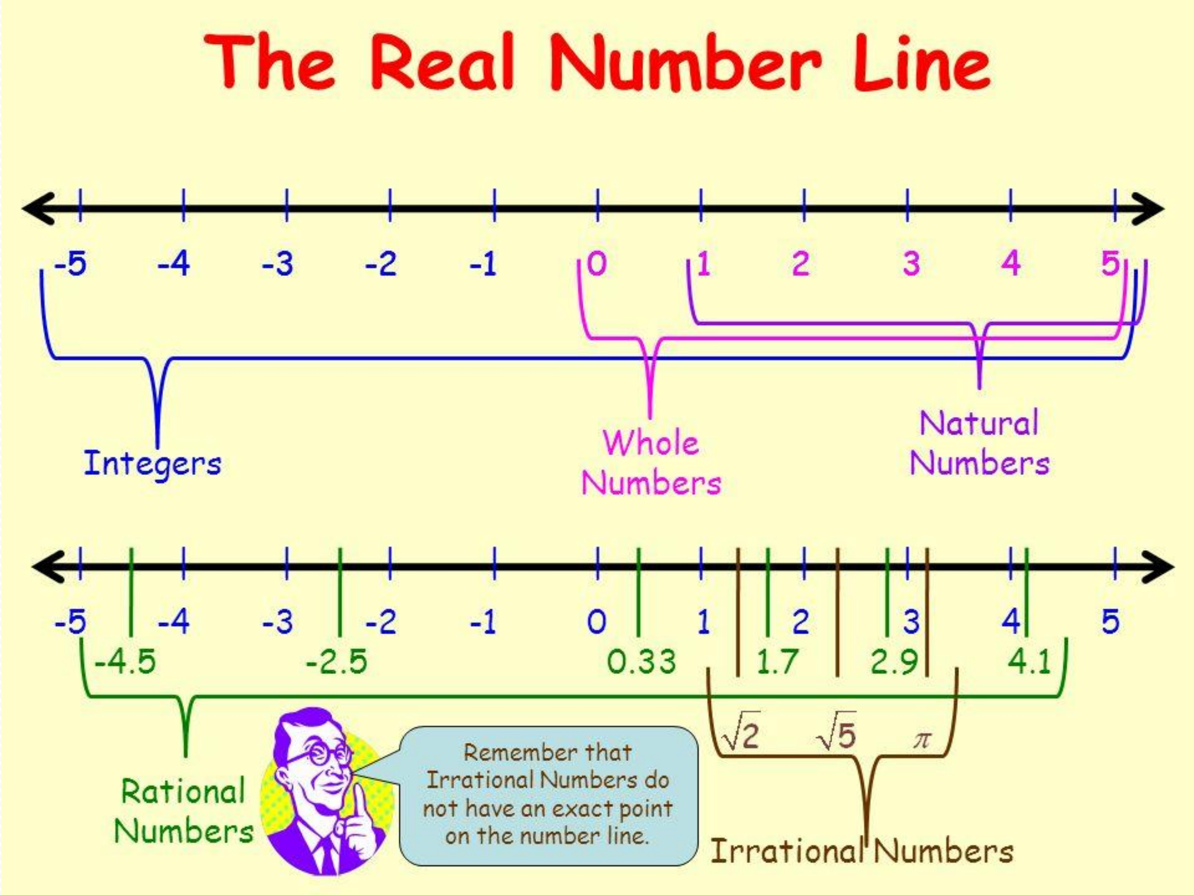
Convenient Method
- Phasor Representation.

Number System



Representation of AC





A **complex number** is a number that can be expressed in the form $a + bi$, where a and b are real numbers i is a solution of the equation $x^2 = -1$. Because no real number satisfies this equation, i is called an imaginary number.

Representation of AC

A complex number z can be represented in three ways:

(i) $z = x + jy \ldots \ldots \ldots Rectangular\ form$
(ii) $z = r\angle\phi \ldots \ldots \ldots \ldots Polar\ form.$
(iii) $z = re^{j\phi} \ldots \ldots \ldots .. Exponential\ form$

Where,

x and y is the real and imaginary part of z respectively.

r is the magnitude of z, and ϕ is the phase of z.

Euler's identity

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$

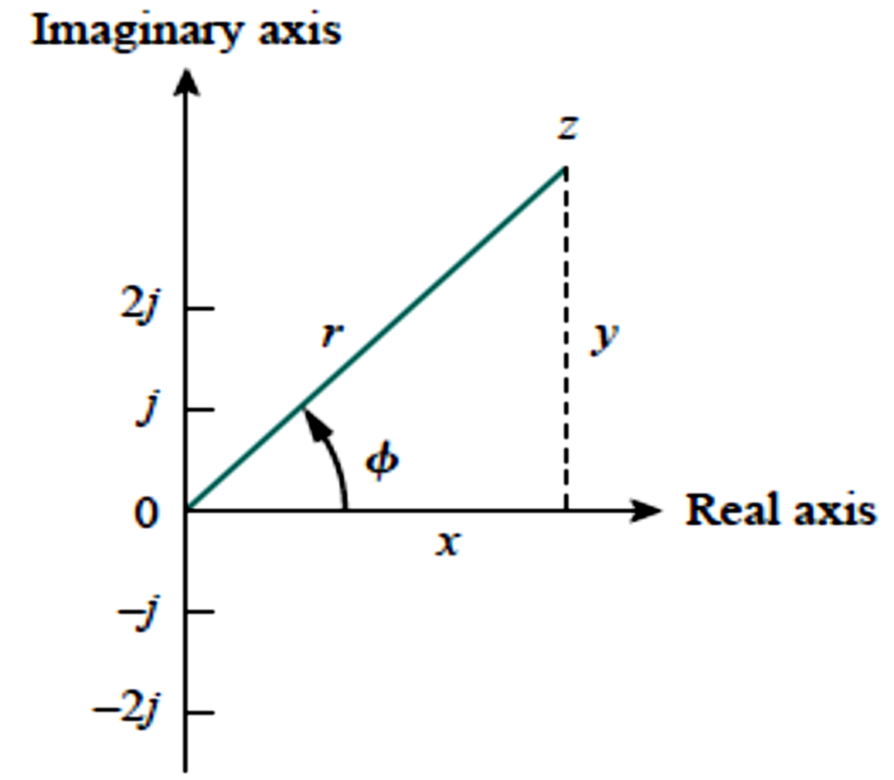


Fig 1.19: Representation of a complex number, z = $\boldsymbol{x + jy = r\angle\theta}$.

Representation of AC

An AC can be represented in the form of
(i) waves and (ii) equations.

Convenient Method
- Phasor Representation.

Phasor: A line of definite length rotating in anticlockwise direction at a constant angular velocity (ω).

$$\boldsymbol{A} = A_m \angle \phi$$

Here, $\boldsymbol{A}$ is the phasor representation of the sinusoid $A(t)$.

A phasor is a complex number that represents the amplitude and phase of a sinusoid.

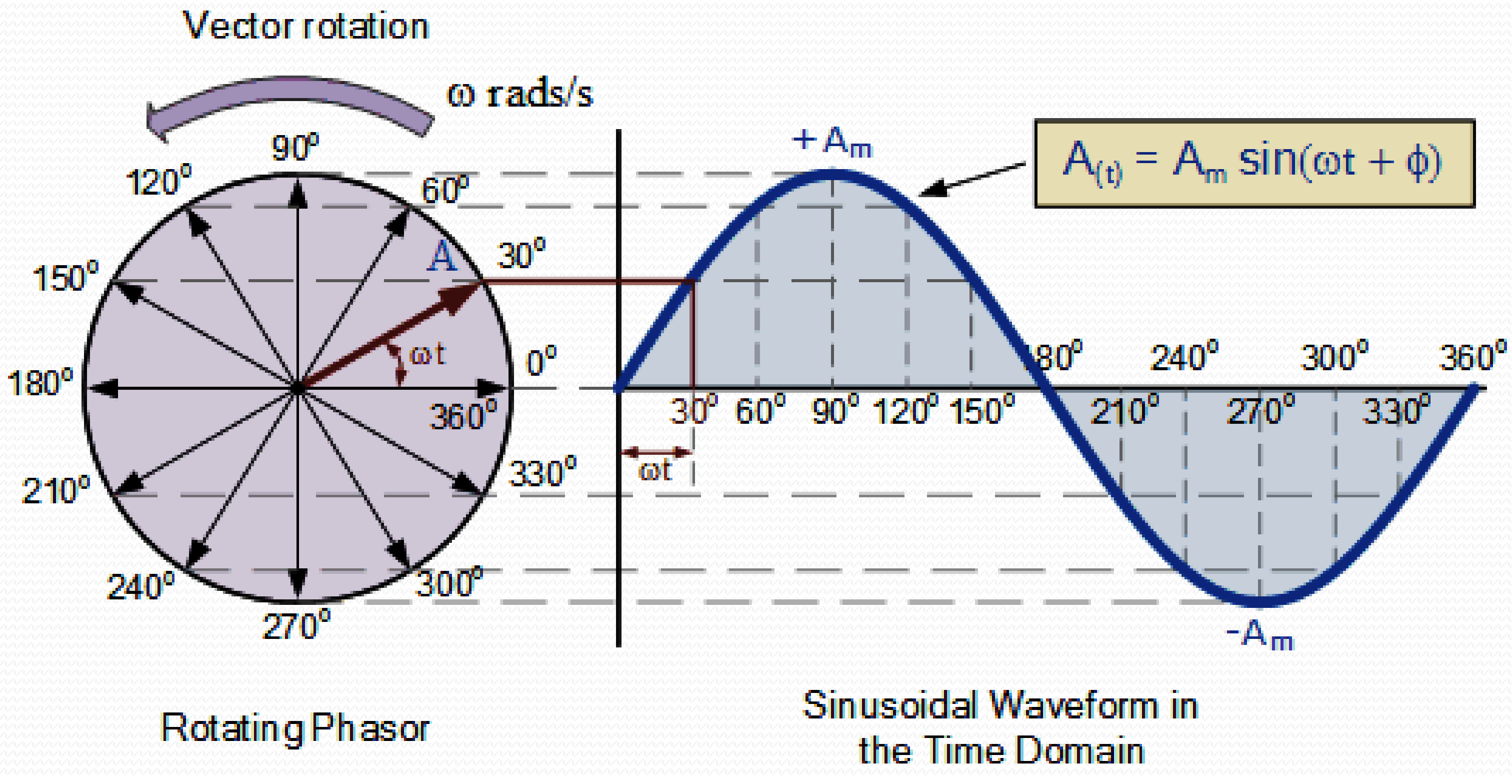


Fig 1.19: Rotating phasor from sinusoidal.

Representation of AC

An AC can be represented in the form of
(i) waves and (ii) equations.

Convenient Method

- Phasor Representation.

Phasor: A line of definite length rotating in anticlockwise direction at a constant angular velocity (ω).

$$\boldsymbol{A} = A_m \angle \phi$$

Here, $\boldsymbol{A}$ is the phasor representation of the sinusoid $A(t)$.

A phasor is a complex number that represents the amplitude and phase of a sinusoid.

Phasor diagram: It is a graphical way of representing two or more phasors.

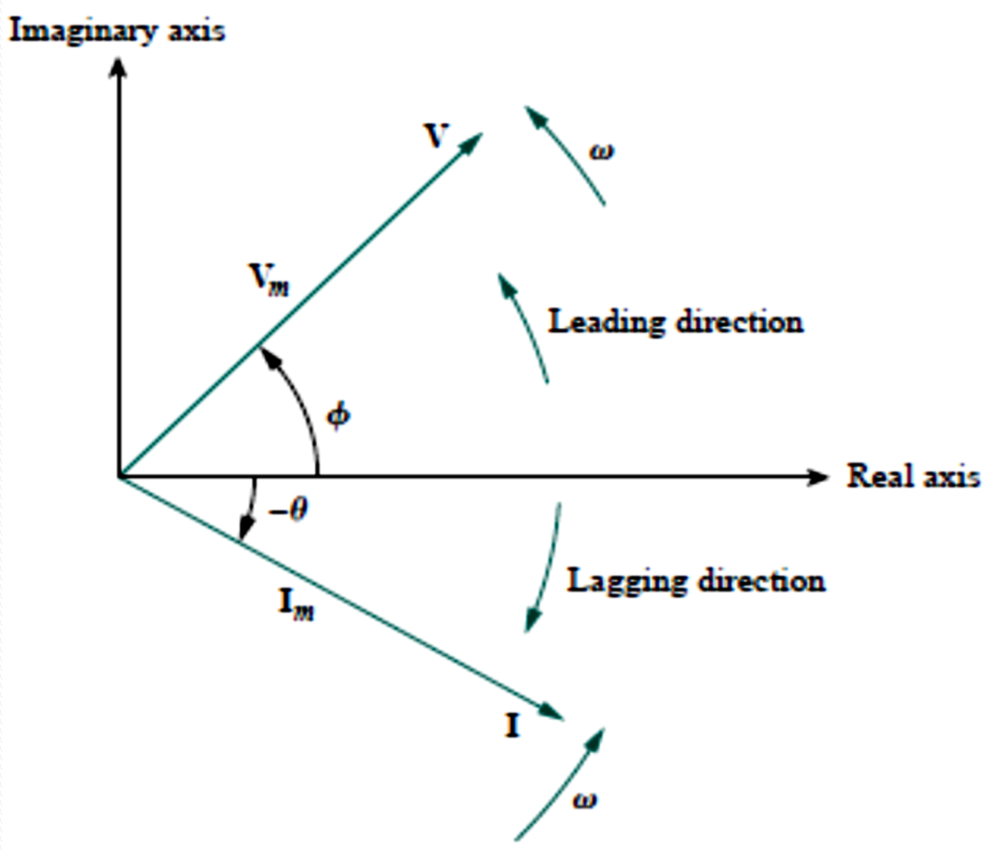


Fig 1.20: A phasor diagram shown, $\boldsymbol{V} = V_m \angle \phi$ and $\boldsymbol{I} = I_m \angle -\phi$

$v(t)$ vs $\boldsymbol{V}$

$v(t)$	$\boldsymbol{V}$
Instantaneous or time domain representation	Frequency or phasor domain representation
Time dependent	Not time dependent
Always real with no complex term	Generally complex

TABLE 9.1 Sinusoid-phasor transformation.

Time-domain representation	Phasor-domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

$\sin(\omega t \pm 180^\circ) = -\sin\omega t$
$\cos(\omega t \pm 180^\circ) = -\cos\omega t$

$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$
$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$

Problems

Example 04: [Charles K. Alexander, Example 9.4, Ed: 5th]	Example 05: [Charles K. Alexander, Example 9.5, Ed: 5th]
Transform these sinusoid to phasors:	Find the sinusoid represent by these phasors:
(a) $v = -4\sin(30t + 50°)$ (b) $i = 6\cos(50t - 40°)$	(a) $\boldsymbol{V} = j8e^{-j20}$ (b) $\boldsymbol{I} = -3 + j4$

Solution:

(a) Since $-\sin A = \cos(A + 90^\circ)$,

$$v = -4\sin(30t + 50^\circ) = 4\cos(30t + 50^\circ + 90^\circ)$$
$$= 4\cos(30t + 140^\circ)$$

The phasor form of v is

$$\mathbf{V} = 4\angle 140^\circ$$

(b) $i = 6\cos(50t - 40^\circ)$ has the phasor

$$\mathbf{I} = 6\angle -40^\circ$$

Solution:

(a) Since $j = 1\angle 90^\circ$,

$$\mathbf{V} = j8\angle -20^\circ = (1\angle 90^\circ)(8\angle -20^\circ)$$
$$= 8\angle 90^\circ - 20^\circ = 8\angle 70^\circ \text{ V}$$

Converting this to the time domain gives

$$v(t) = 8\cos(\omega t + 70^\circ) \text{ V}$$

(b) $\mathbf{I} = -3 + j4 = 5\angle 126.87^\circ$. Transforming this to the time domain gives

$$i(t) = 5\cos(\omega t + 126.87^\circ) \text{ A}$$

Problems

$$\sin(\omega t \pm 180°) = -\sin \omega t$$
$$\cos(\omega t \pm 180°) = -\cos \omega t$$

$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90°) = \mp \sin \omega t$$

Example 06: [Charles K. Alexander, Example 9.6, Ed: 5th]

Given $i_1 = 4\ \cos(\omega t + 30\,°)$ and $i_2 = 5\ \sin(\omega t - 20°)$, find their sum.

Solution:

Here is an important use of phasors—for summing sinusoids of the same frequency. Current $i_1(t)$ is in the standard form. Its phasor is

$$\mathbf{I}_1 = 4\angle 30°$$

We need to express $i_2(t)$ in cosine form. The rule for converting sine to cosine is to subtract 90°. Hence,

$$i_2 = 5\cos(\omega t - 20° - 90°) = 5\cos(\omega t - 110°)$$

and its phasor is

$$\mathbf{I}_2 = 5\angle -110°$$

If we let $i = i_1 + i_2$, then

$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 &= 4\angle 30° + 5\angle -110° \\ &= 3.464 + j2 - 1.71 - j4.698 = 1.754 - j2.698 \\ &= 3.218\angle -56.97°\ \text{A}\end{aligned}$$

Transforming this to the time domain, we get

$$i(t) = 3.218\cos(\omega t - 56.97°)\ \text{A}$$

Of course, we can find $i_1 + i_2$ using Eqs. (9.9), but that is the hard way.

Home Work

Practice Problem 9.4

Express these sinusoids as phasors:

(a) $v = 7\cos(2t + 40^\circ)$ V
(b) $i = -4\sin(10t + 10^\circ)$ A

Answer: (a) $\mathbf{V} = 7\angle 40^\circ$ V, (b) $\mathbf{I} = 4\angle 100^\circ$ A.

Practice Problem 9.5

Find the sinusoids corresponding to these phasors:

(a) $\mathbf{V} = -25\angle 40^\circ$ V
(b) $\mathbf{I} = j(12 - j5)$ A

Answer: (a) $v(t) = 25\cos(\omega t - 140^\circ)$ V or $25\cos(\omega t + 220^\circ)$ V,
(b) $i(t) = 13\cos(\omega t + 67.38^\circ)$ A.

Practice Problem 9.6

If $v_1 = -10\sin(\omega t - 30^\circ)$ V and $v_2 = 20\cos(\omega t + 45^\circ)$ V, find $v = v_1 + v_2$.

Answer: $v(t) = 29.77\cos(\omega t + 49.98^\circ)$ V.

$Phasor\ Relatinship\ (R, L\ and\ C)$

Resistive element:

$$i = I_m \cos(\omega t + \phi)$$

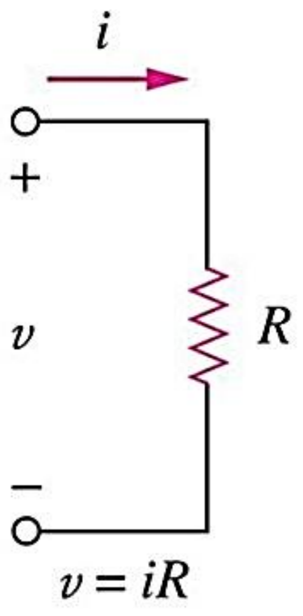
$$\boldsymbol{I} = I_m \angle \phi$$

The voltage across the resistor i given by Ohm's law as

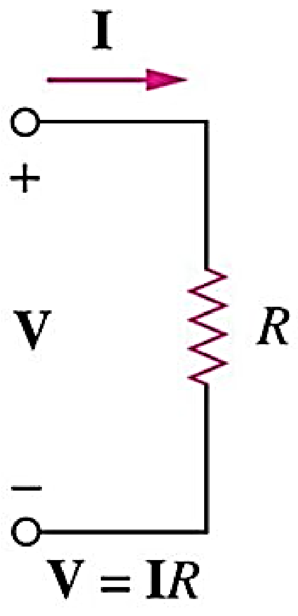
$$v = iR = RI_m \cos(\omega t + \phi)$$

$$\boldsymbol{V} = RI_m \angle \phi$$

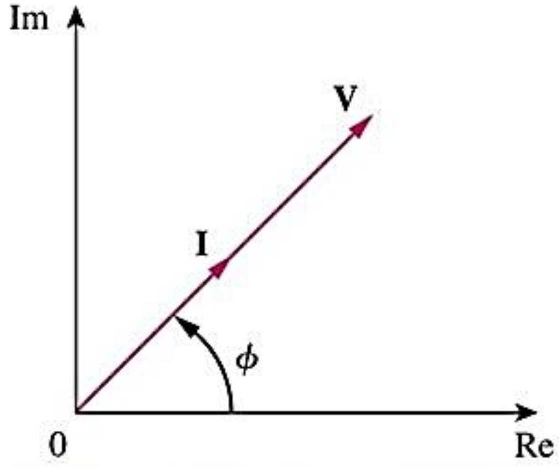
$$\boldsymbol{V} = R\mathbf{I}$$



Time Domain



Frequency Domain



Phasor voltage and current of a resistor are in phase

$Phasor\ Relatinship\ (R, L\ and\ C)$

<u>Inductive Element:</u>

$$i = I_m \cos(\omega t + \phi)$$

$$\boldsymbol{I} = I_m \angle \phi = I_m e^{j\phi}$$

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$$

$$= \omega L I_m \cos(\omega t + \phi + 90)$$

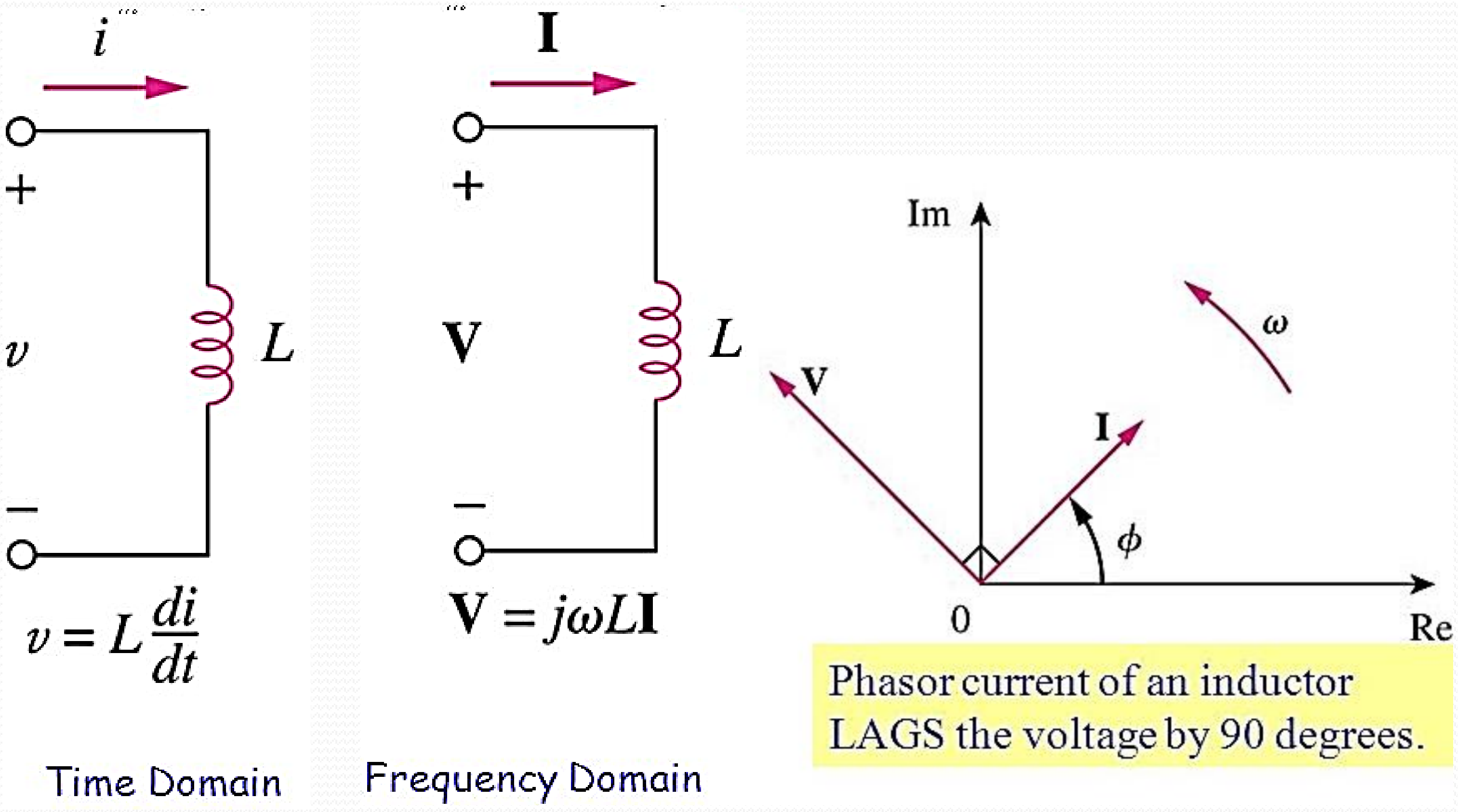
$$\boldsymbol{V} = \omega L I_m \angle(\phi + 90°) = \omega L I_m e^{j(\phi + 90°)}$$

$$= \omega L I_m e^{j\phi} e^{j90°}$$

$$= \omega L I_m e^{j\phi}(\cos 90° + j\sin 90°)$$

$$= j\omega L I_m e^{j\phi} = j\omega L I_m \angle \phi$$

$$\boldsymbol{V} = j\omega L \boldsymbol{I}$$



$Phasor\ Relatinship\ (R, L\ and\ C)$

<u>Capacitive Element:</u>

$$v = V_m \cos(\omega t + \phi)$$

$$\boldsymbol{V} = V_m \angle \phi$$

$$i = C\frac{dv}{dt} = -\omega C V_m \sin(\omega t + \phi)$$

$$= \omega C V_m \cos(\omega t + \phi + 90)$$

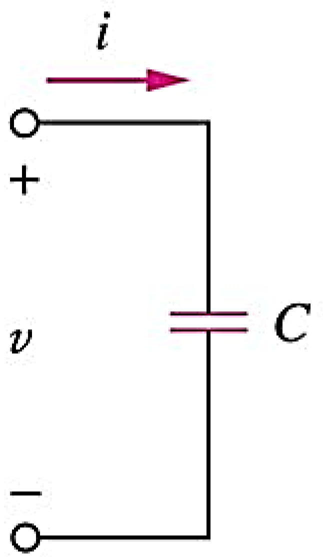
$$\boldsymbol{I} = \omega C V_m \angle(\phi + 90°)$$

$$= \omega C V_m e^{j(\phi + 90°)} = \omega C V_m e^{j\phi} e^{j90°}$$
$$= \omega C V_m e^{j\phi}(\cos 90° + j\sin 90°)$$

$$= j\omega C V_m e^{j\phi} = j\omega C V_m \angle \phi$$

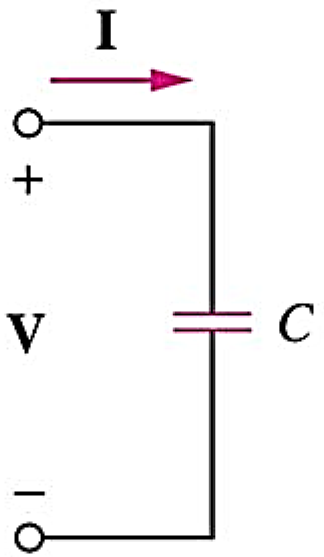
$$\boldsymbol{I} = j\omega C \boldsymbol{V}$$

$$\boldsymbol{V} = \frac{\boldsymbol{I}}{j\omega C}$$



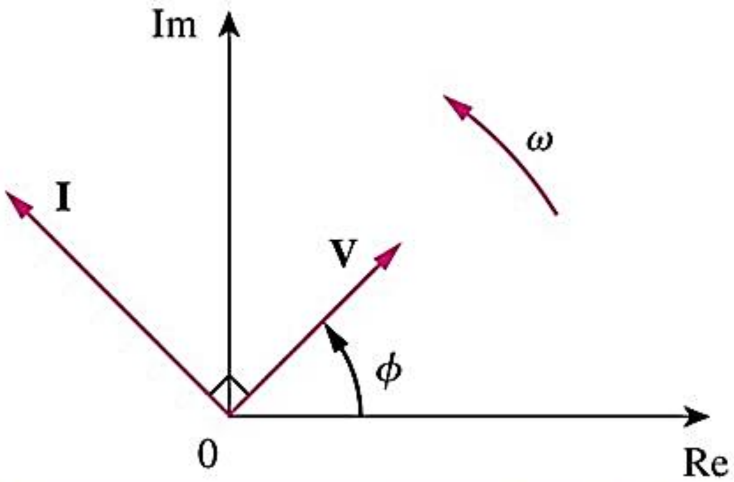
$$i = C\frac{dv}{dt}$$

Time Domain



$$\mathbf{I} = j\omega C\mathbf{V}$$

Frequency Domain



Phasor current of a capacitor LEADS the voltage by 90 degrees.

Impedance & Admittance

From phasor relationship of circuit elements:

$\boldsymbol{V} = R\boldsymbol{I}$	$\boldsymbol{V} = j\omega L\boldsymbol{I}$	$\boldsymbol{V} = \frac{\boldsymbol{I}}{j\omega C}$
$\frac{\boldsymbol{V}}{\boldsymbol{I}} = R$	$\frac{\boldsymbol{V}}{\boldsymbol{I}} = j\omega L = \mathbf{Z}$	$\frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{1}{j\omega C} = \boldsymbol{Z}$

Ohm's law in phasor form for any type of element as

$$\frac{\boldsymbol{V}}{\boldsymbol{I}} = \boldsymbol{Z} \ or\ \boldsymbol{V} = \boldsymbol{Z}\boldsymbol{I}$$

Where, Z is a frequency dependent quantity known as **impedance**, measured in ohms.

Impedance: Opposition to the flow of sinusoidal current.

Admittance: The reciprocal of impedance

$$\therefore Admitance, \boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}}$$

As a complex quantity, the impedance may be expressed in rectangular form as

$$\boldsymbol{Z} = R + jX$$

Where R = Re **Z** is the resistance and X = Im **Z** is the reactance.

Inductive Impedance: **Z** = $R + jX$

Capacitive Impedance: **Z** = $R - jX$

As a complex quantity, we may write **Y** as

$$\boldsymbol{Y} = G + jB$$

Where G =Re **Y** is called the *conductance* and B =Im **Y** is called the *susceptance*.

Unit [**Y**, G and B]: Siemens (or mhos).

Impedance & Admittance

Impedances and admittances of passive elements.

Element	Impedance	Admittance
Resistor, R	$\mathbf{Z = R}$	$\mathbf{Y} = \dfrac{\mathbf{1}}{\boldsymbol{R}}$
Inductor, L	$\mathbf{Z} = j\omega L = \mathrm{j}2\pi f L$	$\mathbf{Y} = \dfrac{\mathbf{1}}{\mathrm{j}2\pi f L}$
Capacitor, C	$\boldsymbol{Z} = \dfrac{1}{j\omega C} = \dfrac{1}{j2\pi f C}$	$\mathbf{Y} = \dfrac{\mathbf{1}}{\mathrm{j}2\pi f \mathrm{C}}$

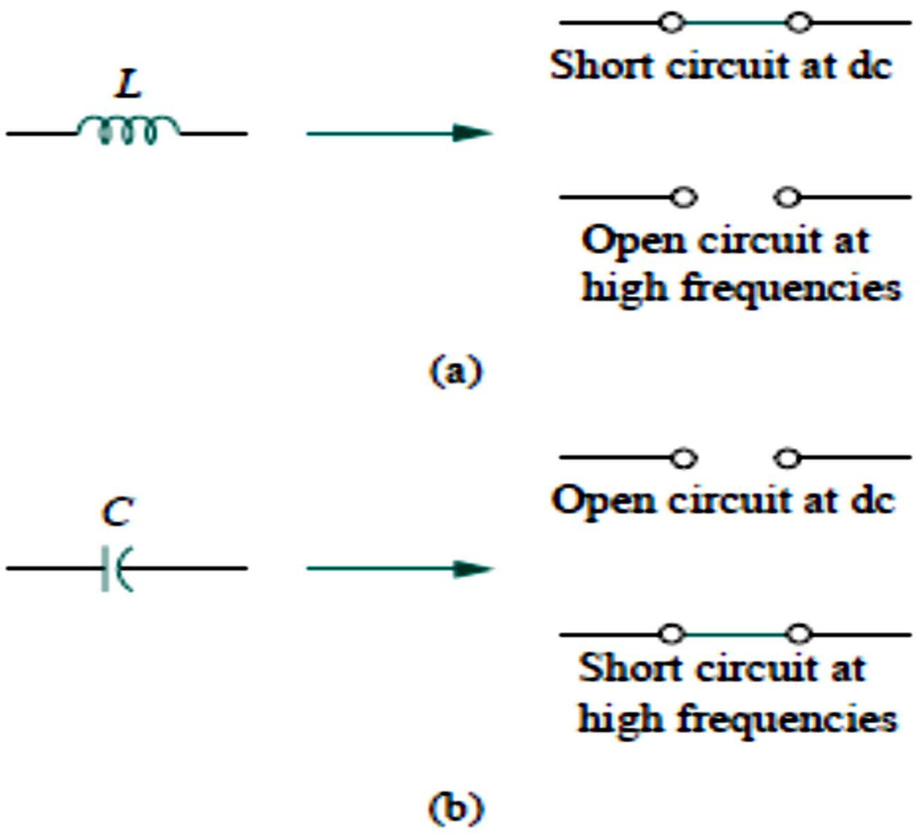


Figure 9.15 Equivalent circuits at dc and high frequencies: (a) inductor, (b) capacitor.

Problems

Example 07: [Charles K. Alexander, Example 9.11]
Determine $v_0(t)$ in the circuit of following Fig 1.21.

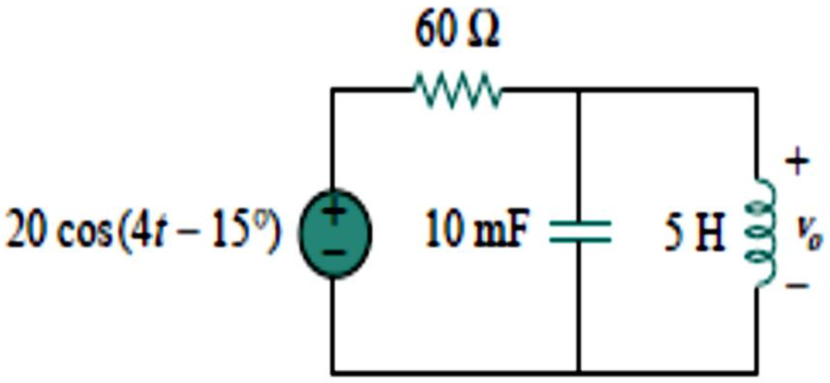


Fig 1.21: Example 07

Solution:

$$v_s = 20\cos(4t - 15^\circ) \quad \Longrightarrow \quad \mathbf{V}_s = 20\angle -15^\circ \text{ V}, \qquad \omega = 4$$

$$10 \text{ mF} \quad \Longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\ \Omega$$

$$5 \text{ H} \quad \Longrightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

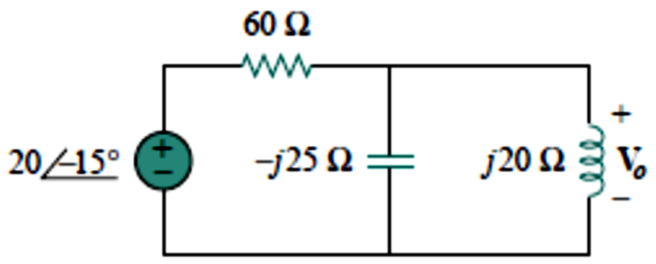


Figure 9.26 The frequency-domain equivalent of the circuit in Fig. 9.25.

Let

$\mathbf{Z}_1$ = Impedance of the 60-Ω resistor

$\mathbf{Z}_2$ = Impedance of the parallel combination of the 10-mF capacitor and the 5-H inductor

Then $\mathbf{Z}_1 = 60\ \Omega$ and

$$\mathbf{Z}_2 = -j25 \parallel j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$

By the voltage-division principle,

$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100}(20\angle -15^\circ)$$

$$= (0.8575\angle 30.96^\circ)(20\angle -15^\circ) = 17.15\angle 15.96^\circ \text{ V}.$$

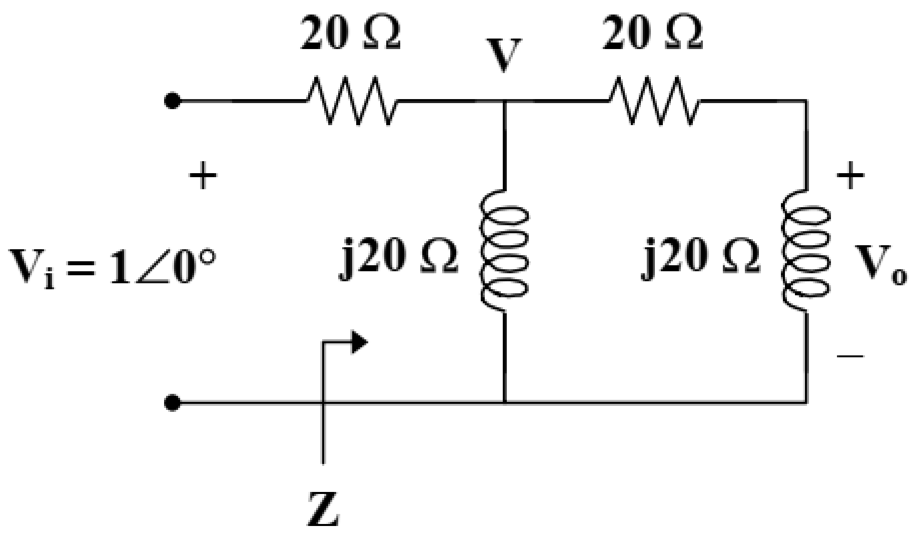
We convert this to the time domain and obtain

$$v_o(t) = 17.15\cos(4t + 15.96^\circ)\text{V}$$

Example 8

Design an *RL* circuit to provide a 90^{o} leading phase shift.

One such RL circuit is shown below.



We now want to show that this circuit will produce a 90° phase shift.

$$\mathbf{Z} = j20 \,||\, (20 + j20) = \frac{(j20)(20 + j20)}{20 + j40} = \frac{-20 + j20}{1 + j2} = 4(1 + j3)$$

$$\mathbf{V} = \frac{\mathbf{Z}}{\mathbf{Z} + 20}\mathbf{V}_i = \frac{4 + j12}{24 + j12}(1\angle 0°) = \frac{1 + j3}{6 + j3} = \frac{1}{3}(1 + j)$$

$$\mathbf{V}_o = \frac{j20}{20 + j20}\mathbf{V} = \left(\frac{j}{1 + j}\right)\left(\frac{1}{3}(1 + j)\right) = \frac{j}{3} = 0.3333\angle 90°$$

This shows that the output leads the input by 90°.

Assignment # 3

Q1: Alternating voltage available in our homes has a frequency of 50 Hz. What does it mean?
Q2: What are the differences between a direct current and an alternating current?
Q3: Why are alternating voltages and currents expressed in rms values and not average values?
Q4: Our homes are supplied with sinusoidal voltage whose equations should be $v = V_m \sin\theta$. But we always say that ac voltage at home is 230V. How do you explain this difference?
Q5: What quantities should be located to know an alternating quantity completely?
Q6: Is alternating voltage or current a vector quantity?
Q7: List the differences between a dc circuit and ac circuit.
Q8: Write down the advantages and disadvantages of ac over dc.

Problems [Text: Introductory Circuit Analysis by Boylestad, Edition 10th]

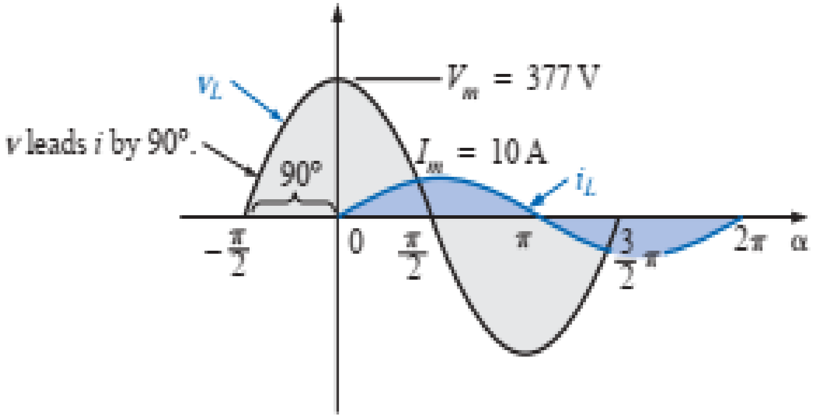


FIG. 14.15
Example 14.3(a).

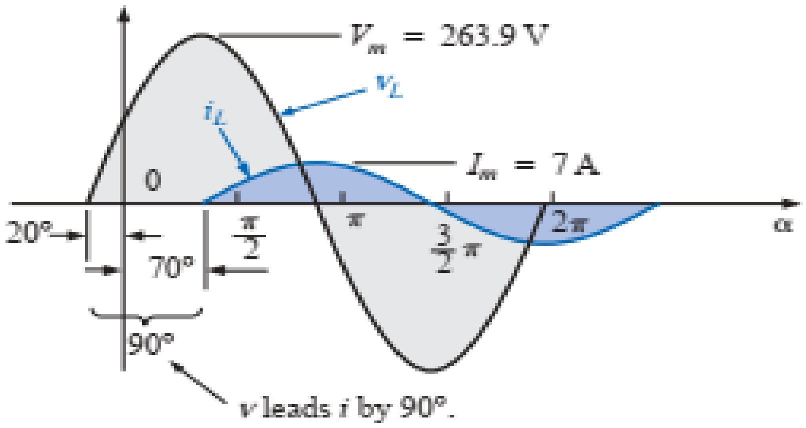


FIG. 14.16
Example 14.3(b).

- **EXAMPLE 14.1** The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 **Ω**. Sketch the curves for *v* and *i*.
  - *a.* $v = 100 \sin 377t$
  - b. b. $v = 25 \sin(377t + 60°)$
- **EXAMPLE 14.2** The current through a 5-**Ω** resistor is given. Find the sinusoidal expression for the voltage across the resistor for $I = 40 \sin(377t + 30°)$.
- **EXAMPLE 14.3** The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the *v* and *I* curves.
  - *a.* $i = 10 \sin 377t$
  - b. $i = 7 \sin(377t \; 70°)$
- **EXAMPLE 14.5** The voltage across a 1-mF capacitor is $v = 30 \sin 400t$. What is the sinusoidal expression for the current? Sketch the *v* and *i* curves.

[Text: Introductory Circuit Analysis by Boylestad, Edition 10th]

- **EXAMPLE 14.3** The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the *v* and *I* curves.
    a. $i = 10 \sin 377t$
    b. $i = 7 \sin(377t \;\; 70°)$

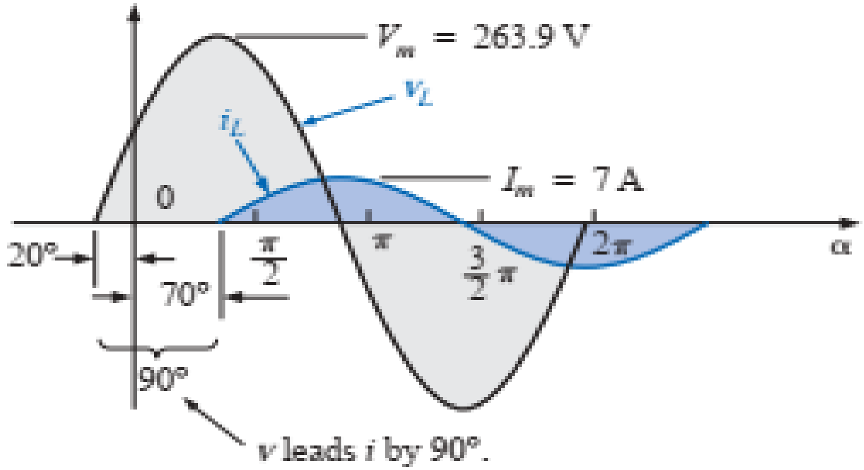


FIG. 14.16
Example 14.3(b).

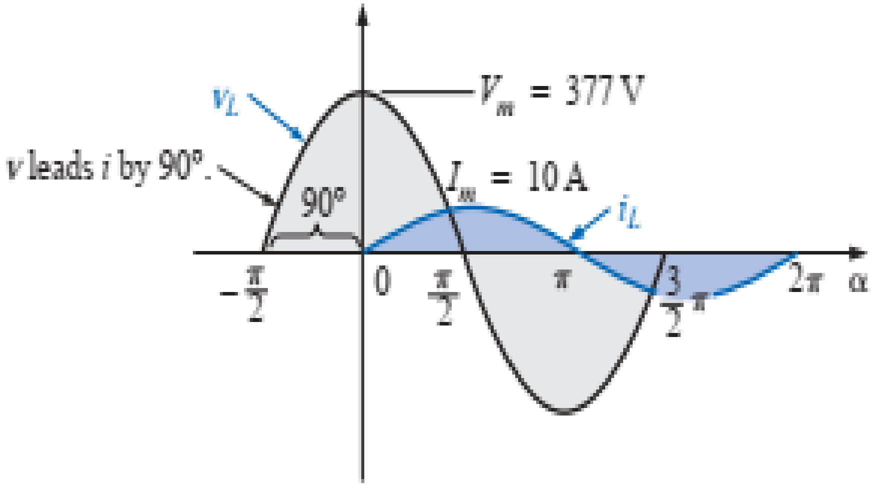


FIG. 14.15
Example 14.3(a).

Problems [Text: Introductory Circuit Analysis by Boylestad, Edition 10th]

EXAMPLE 14.7 For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor, and determine the value of *C*, *L*, or *R* if sufficient data are provided (Fig. 14.18):

a. $v = 100 \sin(\omega t + 40^\circ)$
   $i = 20 \sin(\omega t + 40^\circ)$

b. $v = 1000 \sin(377t + 10^\circ)$
   $i = 5 \sin(377t - 80^\circ)$

c. $v = 500 \sin(157t + 30^\circ)$
   $i = 1 \sin(157t + 120^\circ)$

d. $v = 50 \cos(\omega t + 20^\circ)$
   $i = 5 \sin(\omega t + 110^\circ)$

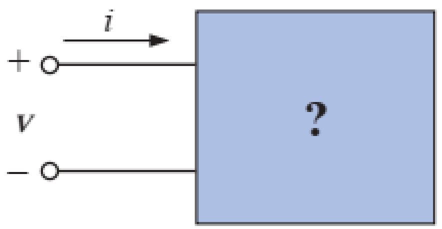


FIG. 14.18
Example 14.7.

Problems [Text: Introductory Circuit Analysis by Boylestad, Edition 10th]

EXAMPLE 16.5 The network of Fig. 16.10 is frequently encountered in the analysis of transistor networks. The transistor equivalent circuit includes a current source **I** and an output impedance *Ro*. The resistor R_c is a biasing resistor to establish specific dc conditions, and the resistor R_i represents the loading of the next stage. The coupling capacitor is designed to be an open circuit for dc and to have as low an impedance as possible for the frequencies of interest to ensure that V_L is a maximum value. The frequency range of the example includes the entire audio (hearing) spectrum from 100 Hz to 20 kHz. The purpose of the example is to demonstrate that, for the full audio range, the effect of the capacitor can be ignored. It performs its function as a dc blocking agent but permits the ac to pass through with little disturbance.

a. Determine $\mathbf{V}_L$ for the network of Fig. 16.10 at a frequency of 100 Hz.
b. Repeat part (a) at a frequency of 20 kHz.
c. Compare the results of parts (a) and (b).

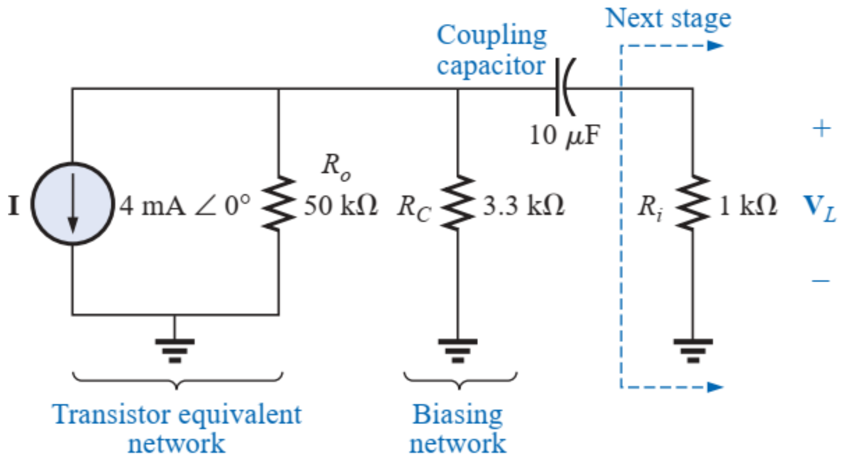


FIG. 16.10
Basic transistor amplifier.

Problems [Text: Introductory Circuit Analysis by Boylestad, Edition 10th]

Solutions (a):

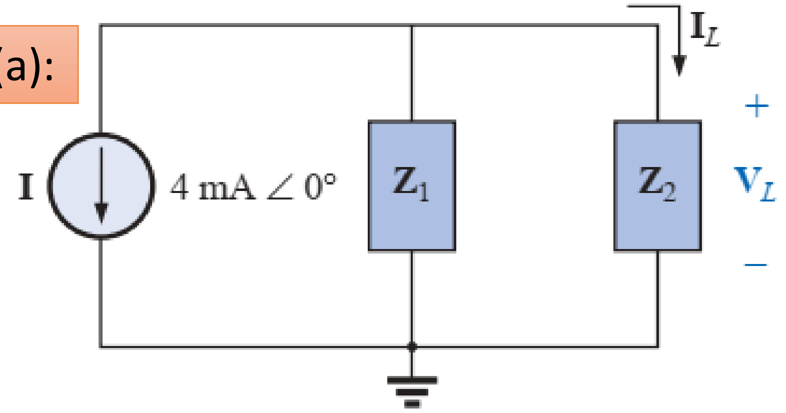


FIG. 16.11

Network of Fig. 16.10 following the assignment of the block impedances.

EXAMPLE 16.5 The network of Fig. 16.10 is frequently encountered in the analysis of transistor networks. The transistor equivalent circuit includes a current source **I** and an output impedance *Ro*. The resistor R_C is a biasing resistor to establish specific dc conditions, and the resistor R_i represents the loading of the next stage. The coupling capacitor is designed to be an open circuit for dc and to have as low an impedance as possible for the frequencies of interest to ensure that V_L is a maximum value. The frequency range of the example includes the entire audio (hearing) spectrum from 100 Hz to 20 kHz. The purpose of the example is to demonstrate that, for the full audio range, the effect of the capacitor can be ignored. It performs its function as a dc blocking agent but permits the ac to pass through with little disturbance.

a. Determine $\mathbf{V}_L$ for the network of Fig. 16.10 at a frequency of 100 Hz.
b. Repeat part (a) at a frequency of 20 kHz.
c. Compare the results of parts (a) and (b).

$$\mathbf{Z}_1 = 50\text{ k}\Omega \angle 0° \parallel 3.3\text{ k}\Omega \angle 0° = 3.096\text{ k}\Omega \angle 0°$$

$$\mathbf{Z}_2 = R_i - j X_C$$

$$\text{At } f = 100\text{ Hz: } X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(100\text{ Hz})(10\ \mu\text{F})} = 159.16\ \Omega$$

and

$$\mathbf{Z}_2 = 1\text{ k}\Omega - j\,159.16\ \Omega$$

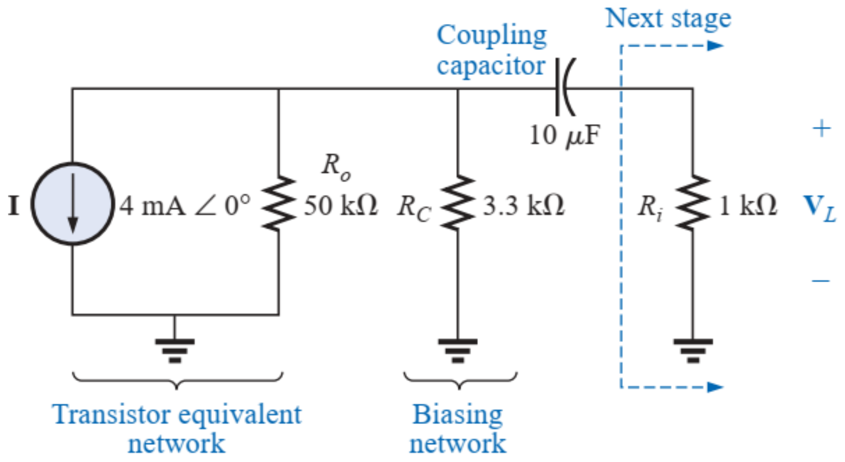


FIG. 16.10

Basic transistor amplifier.

Problems [Text: Introductory Circuit Analysis by Boylestad, Edition 10th]

Solutions (a):

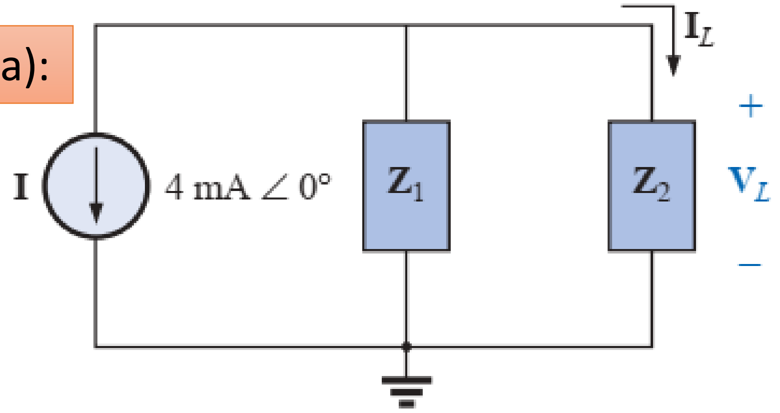


FIG. 16.11

Network of Fig. 16.10 following the assignment of the block impedances.

$$\mathbf{Z}_1 = 50\text{ k}\Omega \angle 0° \parallel 3.3\text{ k}\Omega \angle 0° = 3.096\text{ k}\Omega \angle 0°$$

$$\mathbf{Z}_2 = R_i - j\,X_C$$

At $f = 100$ Hz: $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(100\text{ Hz})(10\ \mu\text{F})} = 159.16\ \Omega$

and $\mathbf{Z}_2 = 1\text{ k}\Omega - j\,159.16\ \Omega$

Current divider rule:

$$\mathbf{I}_L = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(3.096\text{ k}\Omega \angle 0°)(4\text{ mA} \angle 0°)}{3.096\text{ k}\Omega + 1\text{ k}\Omega - j\,159.16\ \Omega}$$

$$= \frac{12.384\text{ A} \angle 0°}{4096 - j\,159.16} = \frac{12.384\text{ A} \angle 0°}{4099 \angle -2.225°}$$

$$= 3.021\text{ mA} \angle 2.225°$$

and

$$\mathbf{V}_L = \mathbf{I}_L\mathbf{Z}_R$$

$$= (3.021\text{ mA} \angle 2.225°)(1\text{ k}\Omega \angle 0°)$$

$$= \mathbf{3.021\ V \angle 2.225°}$$

Problems [Text: Introductory Circuit Analysis by Boylestad, Edition 10th]

Solutions (b):

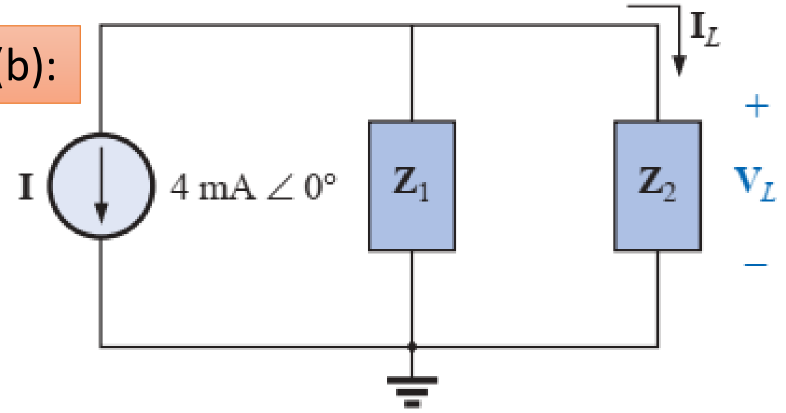


FIG. 16.11

Network of Fig. 16.10 following the assignment of the block impedances.

$$\mathbf{Z}_1 = 50\ \text{k}\Omega \angle 0^\circ \parallel 3.3\ \text{k}\Omega \angle 0^\circ = 3.096\ \text{k}\Omega \angle 0^\circ$$

$$\mathbf{Z}_2 = R_i - j\,X_C$$

$$\text{At } f = 20\ \text{kHz: } X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(20\ \text{kHz})(10\ \mu\text{F})} = 0.796\ \Omega$$

$$\mathbf{Z}_2 = 1\ \text{k}\Omega - j\,0.796\ \Omega$$

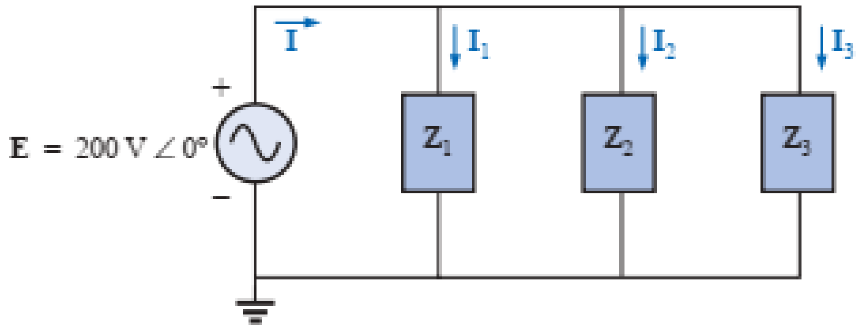
Current divider rule:

$$\begin{aligned}\mathbf{I}_L &= \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(3.096\ \text{k}\Omega \angle 0^\circ)(4\ \text{mA} \angle 0^\circ)}{3.096\ \text{k}\Omega + 1\ \text{k}\Omega - j\,0.796\ \Omega} \\ &= \frac{12.384\ \text{A} \angle 0^\circ}{4096 - j\,0.796\ \Omega} = \frac{12.384\ \text{A} \angle 0^\circ}{4096 \angle -0.011^\circ} \\ &= 3.023\ \text{mA} \angle 0.011^\circ\end{aligned}$$

$$\begin{aligned}\mathbf{V}_L &= \mathbf{I}_L\mathbf{Z}_R \\ &= (3.023\ \text{mA} \angle 0.011^\circ)(1\ \text{k}\Omega \angle 0^\circ) \\ &= \mathbf{3.023\ V \angle 0.011^\circ}\end{aligned}$$

c. The results clearly indicate that the capacitor had little effect on the frequencies of interest. In addition, note that most of the supply current reached the load for the typical parameters employed.

Problems [Text: Introductory Circuit Analysis by Boylestad, Edition 10th]



a. Redrawing the circuit as in Fig. 16.15 reveals a strictly parallel network where

$$\mathbf{Z}_1 = R_1 = 10\ \Omega \angle 0°$$
$$\mathbf{Z}_2 = R_2 + j X_{L_1} = 3\ \Omega + j\,4\ \Omega$$
$$\mathbf{Z}_3 = R_3 + j X_{L_2} - j X_C = 8\ \Omega + j\,3\ \Omega - j\,9\ \Omega = 8\ \Omega - j\,6\ \Omega$$

The total admittance is

$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 \\
&= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{10\ \Omega} + \frac{1}{3\ \Omega + j\,4\ \Omega} + \frac{1}{8\ \Omega - j\,6\ \Omega} \\
&= 0.1\ \text{S} + \frac{1}{5\ \Omega \angle 53.13°} + \frac{1}{10\ \Omega \angle -36.87°} \\
&= 0.1\ \text{S} + 0.2\ \text{S} \angle -53.13° + 0.1\ \text{S} \angle 36.87° \\
&= 0.1\ \text{S} + 0.12\ \text{S} - j\,0.16\ \text{S} + 0.08\ \text{S} + j\,0.06\ \text{S} \\
&= 0.3\ \text{S} - j\,0.1\ \text{S} = 0.316\ \text{S} \angle -18.435°
\end{aligned}$$

EXAMPLE 16.7 For the network of Fig. 16.14:

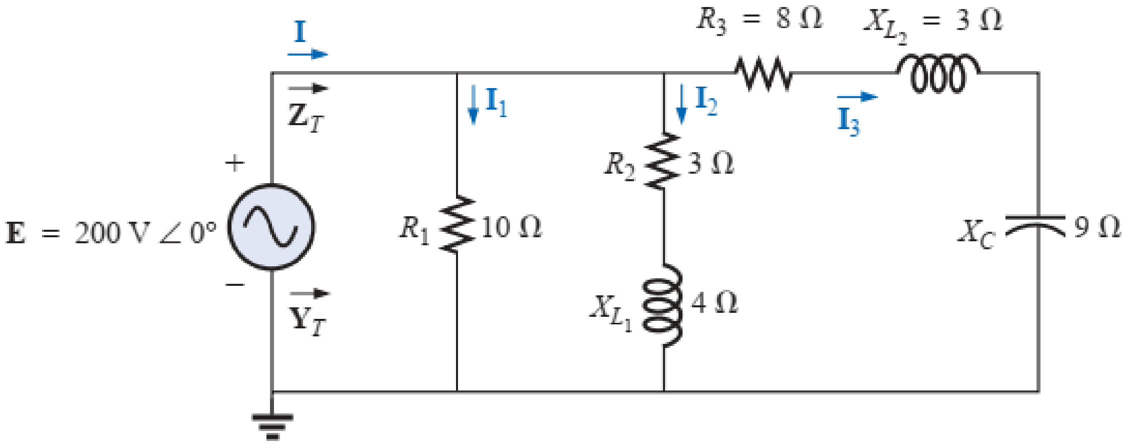


FIG. 16.14
Example 16.7.

a. Compute **I**.
b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
c. Verify Kirchhoff's current law by showing that

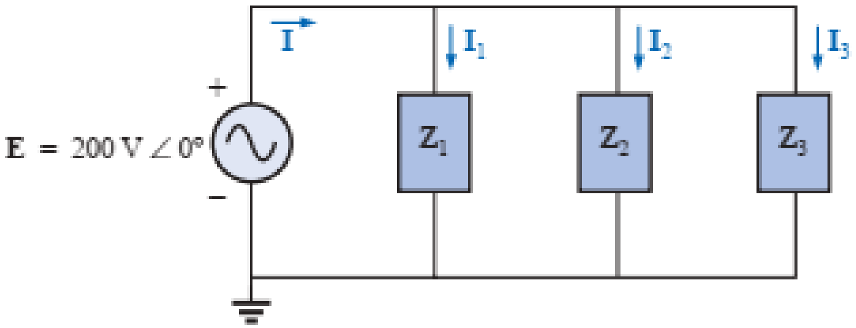
$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.

The current **I**:

$$\mathbf{I} = \mathbf{E}\mathbf{Y}_T = (200\ \text{V} \angle 0°)(0.316\ \text{S} \angle -18.435°)$$
$$= \mathbf{63.2\ A \angle -18.435°}$$

Problems [Text: Introductory Circuit Analysis by Boylestad, Edition 10th]



b. Since the voltage is the same across parallel branches,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{200\text{ V}\angle 0°}{10\ \Omega\angle 0°} = \mathbf{20\ A\angle 0°}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{200\text{ V}\angle 0°}{5\ \Omega\angle 53.13°} = \mathbf{40\ A\angle -53.13°}$$

$$\mathbf{I}_3 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{200\text{ V}\angle 0°}{10\ \Omega\angle -36.87°} = \mathbf{20\ A\angle +36.87°}$$

c. $$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

$$60 - j\,20 = 20\angle 0° + 40\angle -53.13° + 20\angle +36.87°$$
$$= (20 + j\,0) + (24 - j\,32) + (16 + j\,12)$$
$$60 - j\,20 = 60 - j\,20 \quad \text{(checks)}$$

d. $$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.316\text{ S}\angle -18.435°}$$
$$= \mathbf{3.165\ \Omega\angle 18.435°}$$

EXAMPLE 16.7 For the network of Fig. 16.14:

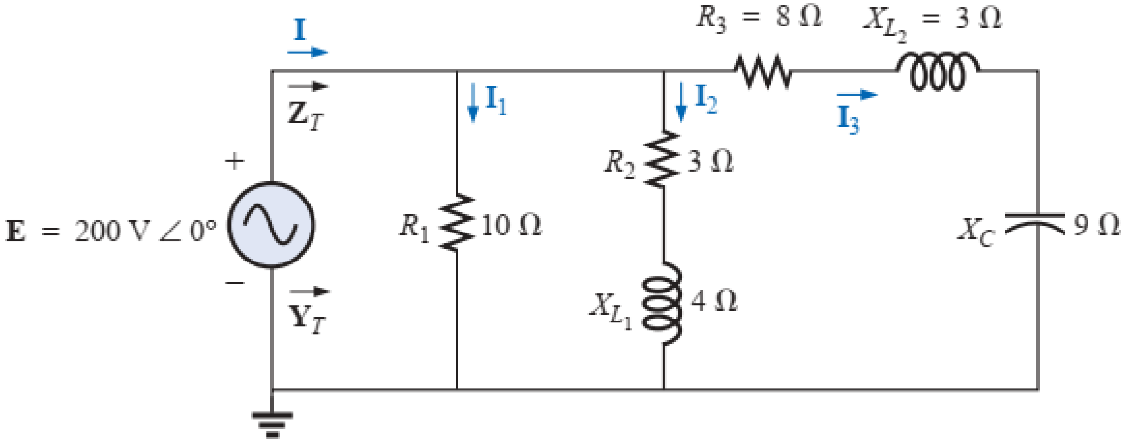


FIG. 16.14
Example 16.7.

a. Compute **I**.
b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
c. Verify Kirchhoff's current law by showing that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.

Next Chapter…………

- Sinusoidal Steady State Analysis
- Fundamentals of Electric Circuit by Charles K. Alexander, Chapter: 09, Ed: 5th