Lesson 4: Three Phase Circuits

ET 332b

EEE 302: Electrical Circuit II

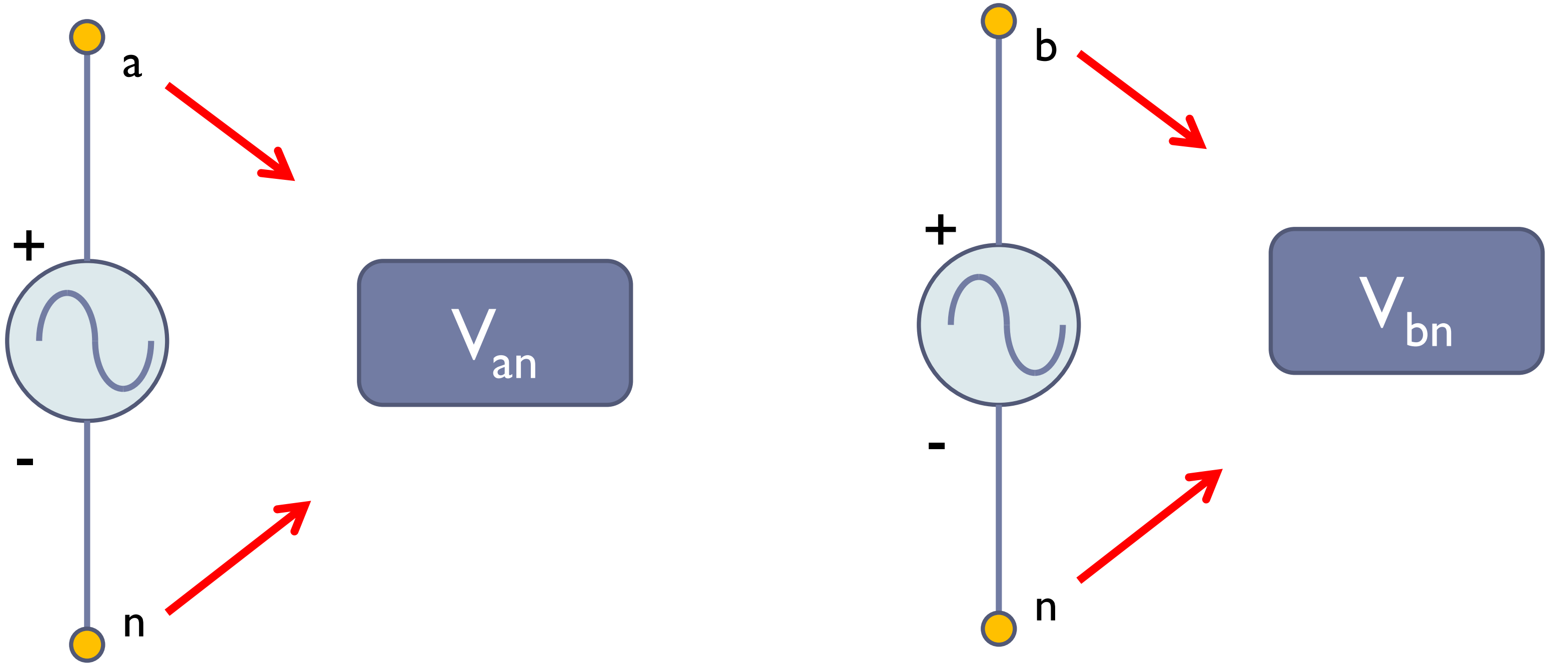
Learning Objectives

After this presentation you will be able to:

- Identify circuit impedances, voltages and currents using the double subscript notation.
- Familiar with different three-phase configurations.
- Know the difference between balanced and unbalanced circuits.
- Perform calculations on wye and delta connected three-phase sources and loads.
- Learn about power in a balanced three-phase system.
- Know how to analyze unbalanced three-phase systems.

Double Subscript Notation

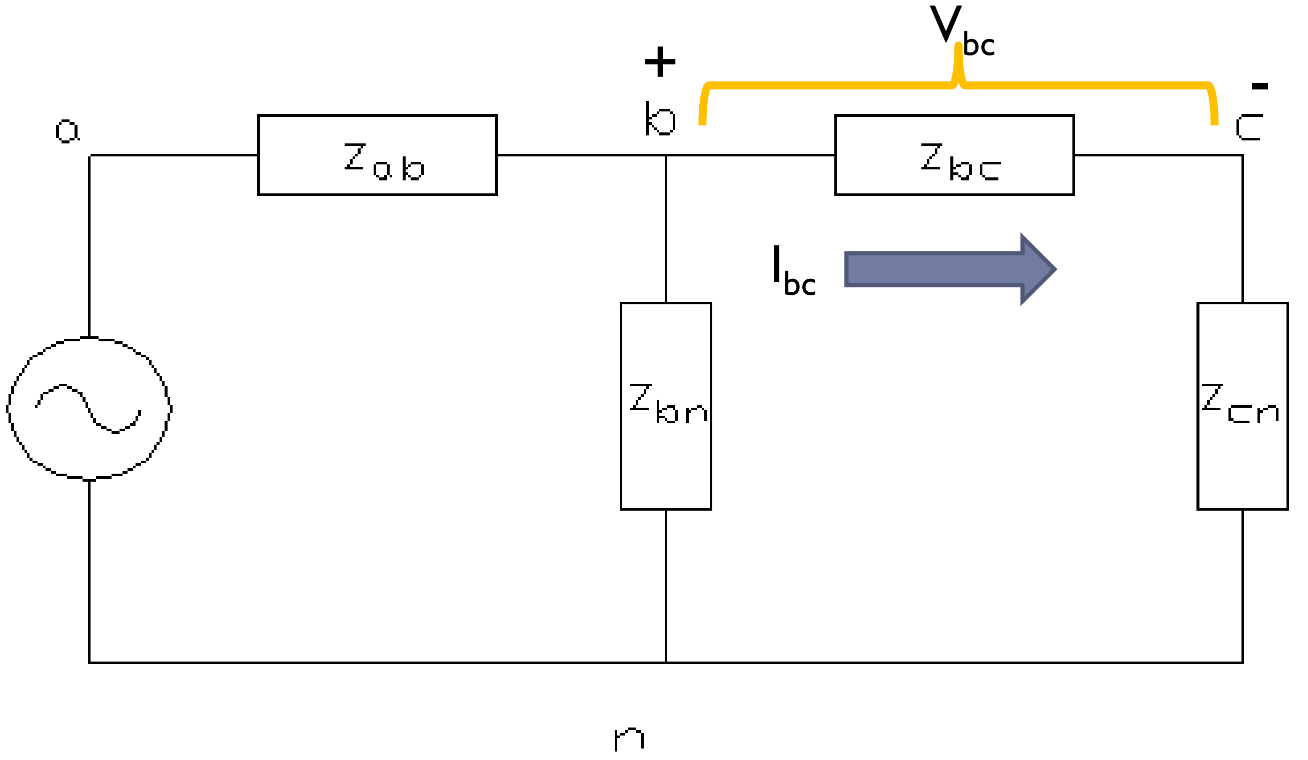
Sources and voltage drops are defined by the terminal letter. Voltage drop and polarity defined by order of subscripts



Voltages considered positive if first node subscript is higher potential than second node subscript

Double Subscript Notation

Identify current flow from point b to c



$V_{cb}=-V_{bc}$

$$I_{bc}=\frac{V_{bc}}{Z_{bc}}$$

V_{bc} = difference in potential between points b and c. If voltage at point b is taken as the reference point, then the polarity is reversed.

Three Phase Power Systems

Fig. 4.1: Single phase systems two-wire type

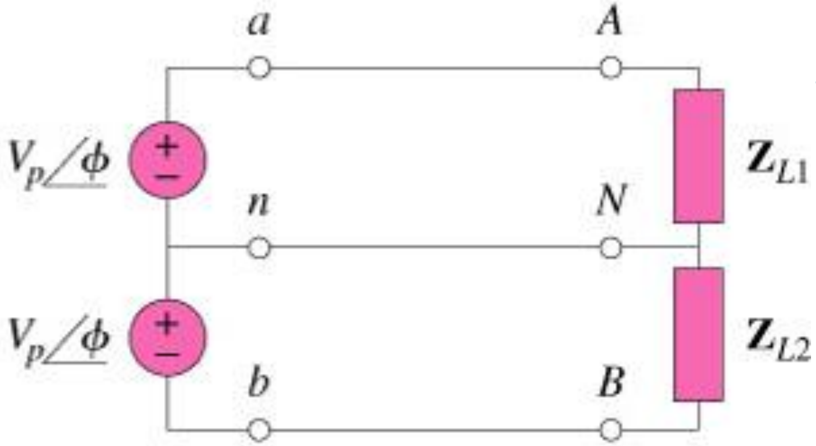


Fig. 42: Single phase systems three-wire type.

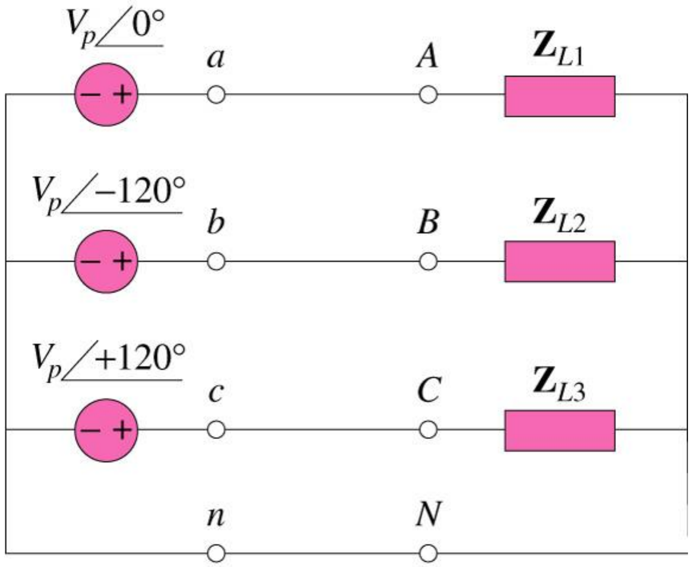


Fig. 4.4: Three-phase four-wire system.

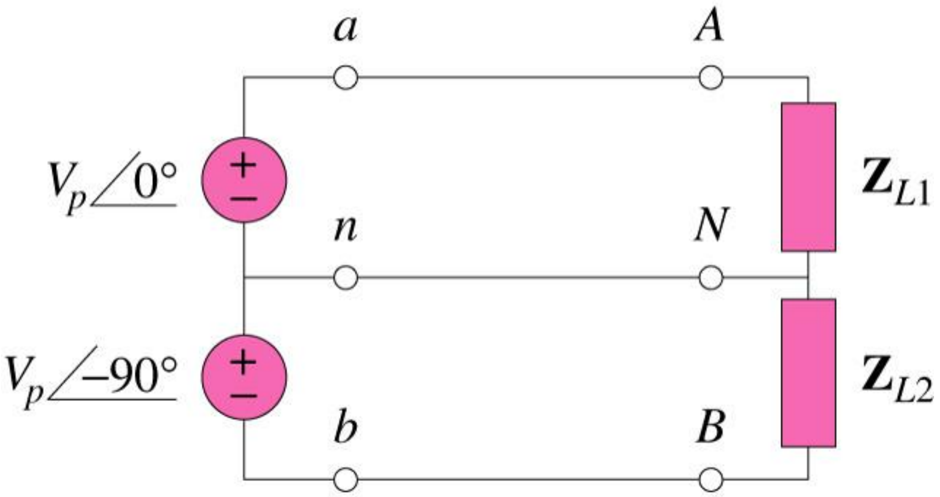
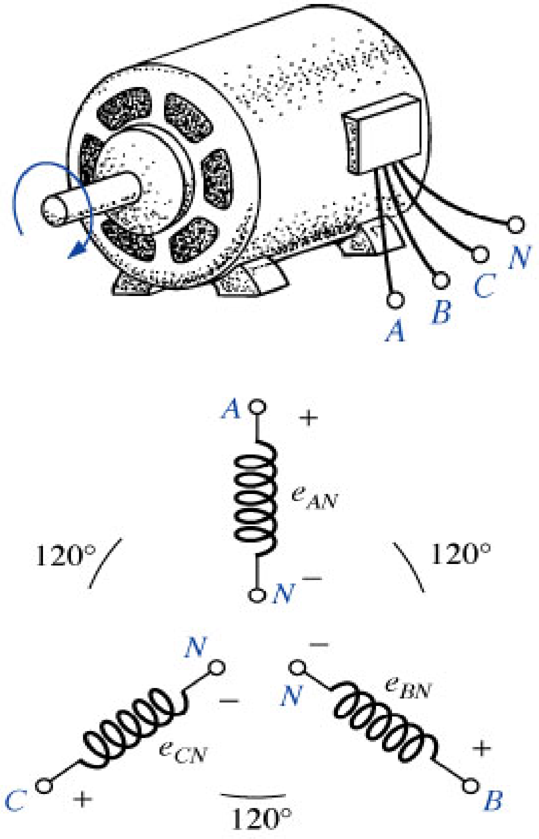


Fig. 4.3: Two-phase three-wire system.

Three Phase Power Systems

- An AC generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a **single-phase AC generator.** If the number of coils on the rotor is increased in a specified manner, the result is a **Polyphase AC generator**, which develops more than one AC phase voltage per rotation of the rotor.



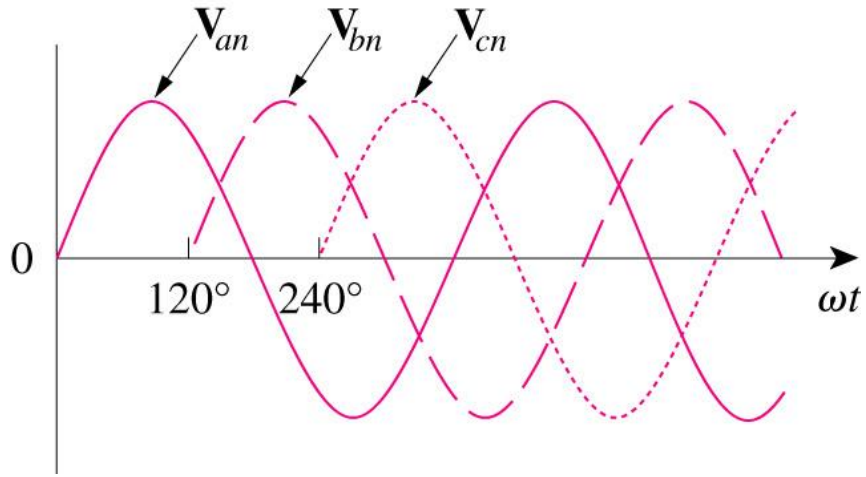


Fig. 4.6: Voltages having 120° phase difference.

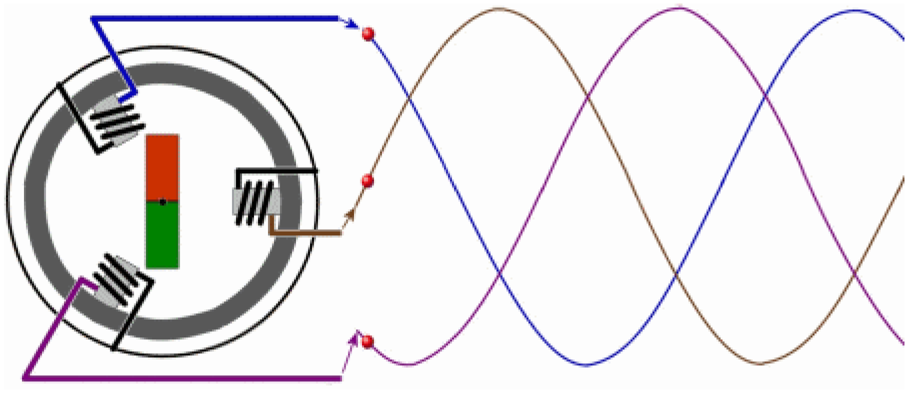


Fig. 4.5: A Three-phase Generator.

Q01: Why three-phase systems are preferred over single-phase systems for the transmission of power ?

Answer:

1. When one phase or two-phase inputs are required, they are taken from the three-phase system rather than generated independently.
2. The instantaneous power in a three-phase system can be constant (not pulsating). This result in uniform power transmission and less vibration of three-phase machines.
3. Thinner conductors can be used to transmit the same kVA at the same voltage, which reduces the amount of copper required (typically about 25% less).
4. The lighter lines are easier to install, and the supporting structures can be less massive and farther apart.
5. In general, most larger motors are three phase because they are essentially self-starting and do not require a special design or additional starting circuitry.

Balanced three-phase voltage sources

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

Characteristics:	Three phasor voltages Equal voltage magnitudes Phase shift equally spaced 120 degrees apart

Time equations for balanced three-phase voltage sources

$$v_{an}(t) = V_p \cdot \sin(2\pi \cdot f \cdot t + 0^\circ)$$

$$v_{bn}(t) = V_p \cdot \sin(2\pi \cdot f \cdot t - 120^\circ)$$

$$v_{cn}(t) = V_p \cdot \sin(2\pi \cdot f \cdot t - 240^\circ)$$

Balanced three-phase voltage sources

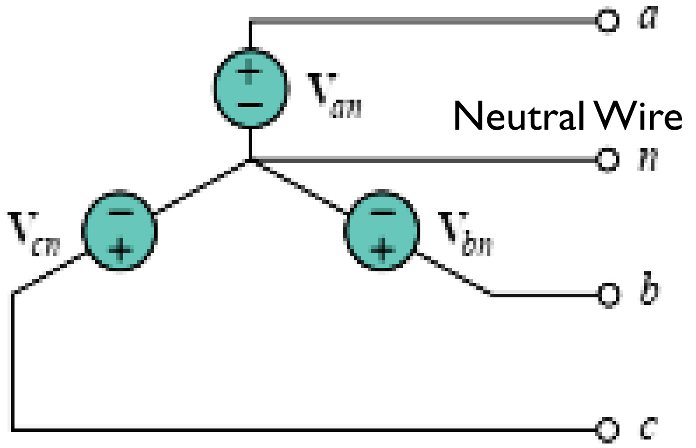


Fig. 4.7: Wye Connected Source.

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

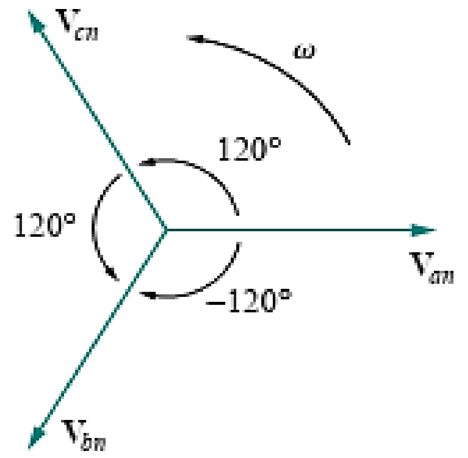


Fig. 4.8: Positive or abc phase sequence.

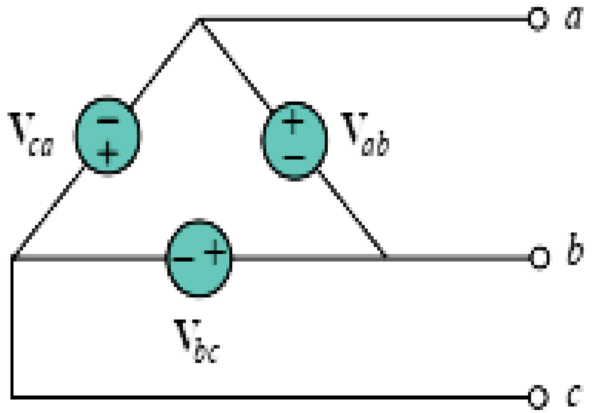


Fig. 4.8: Delta Connected Source.

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

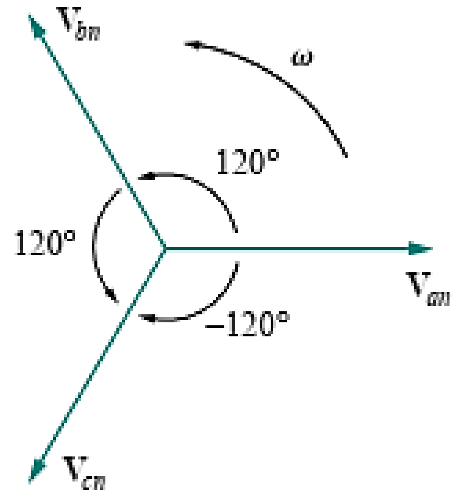


Fig. 4.9: Negative or acb phase sequence.

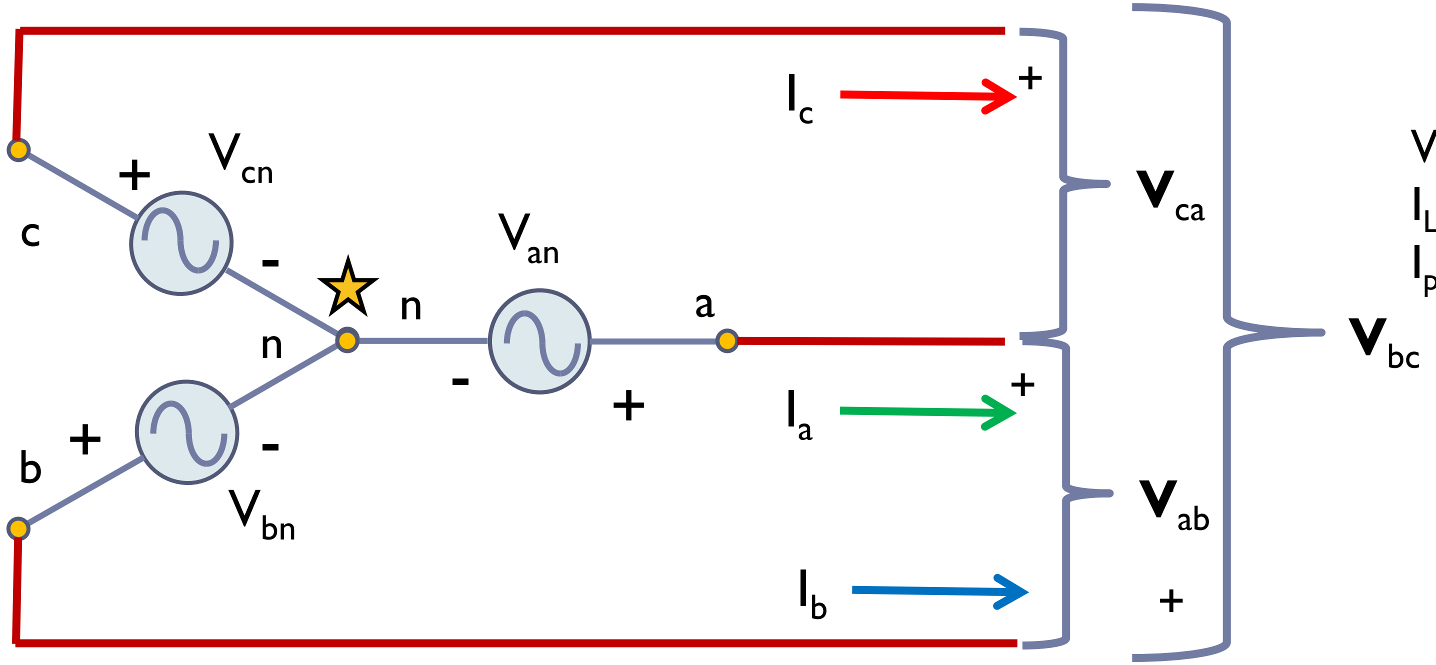
Three-Phase Source Connections

- Phase voltages are:

$$\bar{V}_{an} = V_p \angle 0°$$
$$\bar{V}_{bn} = V_p \angle -120°$$
$$\bar{V}_{cn} = V_p \angle -240°$$

- The three conductors connected from a, b and c are called LINES.
- The voltage from one line to another is called a **LINE voltage**
- Line voltages are: V_{ab}, V_{bc} and V_{ca}

Wye – Connected, three-phase 3ϕ sources



In wye connection:

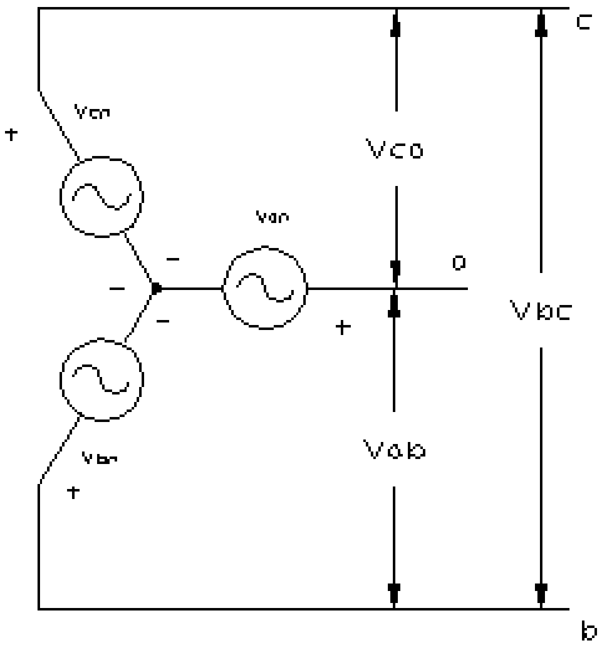
$I_L = I_p$

Where:
I_L = line current
I_p = phase current

Determine the relationship between the magnitude and phase shift of each source voltage and the current and voltage at the terminals of the connection

Wye Connected Sources

Voltage relationships



Perform phasor subtraction to find the values

Line-to-line voltage phasors

$$\bar{V}_{ab} = \bar{V}_{an} + \bar{V}_{nb} = \bar{V}_{an} - \bar{V}_{bn}$$
$$= V_P \angle 0° - V_P \angle -120°$$
$$= V_P \left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$
$$= \sqrt{3} V_P \angle 30°$$

Similarly

$$\bar{V}_{bc} = \bar{V}_{bn} - \bar{V}_{cn}$$
$$= \sqrt{3} V_p \angle -90°$$
$$\bar{V}_{ca} = \bar{V}_{cn} - \bar{V}_{an}$$
$$= \sqrt{3} V_p \angle -210°$$

In balanced systems $\left|\bar{V}_{an}\right| = \left|\bar{V}_{bn}\right| = \left|\bar{V}_{cn}\right|$

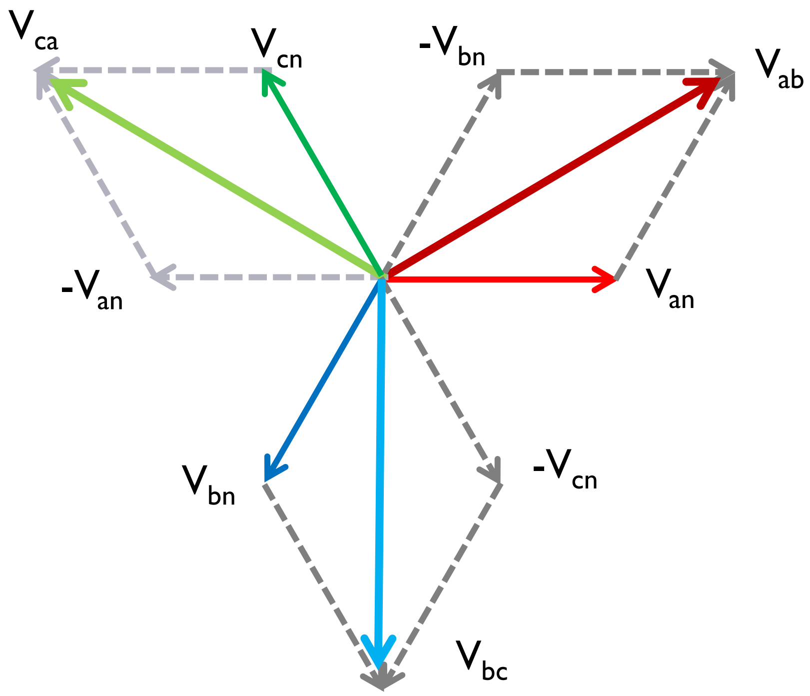
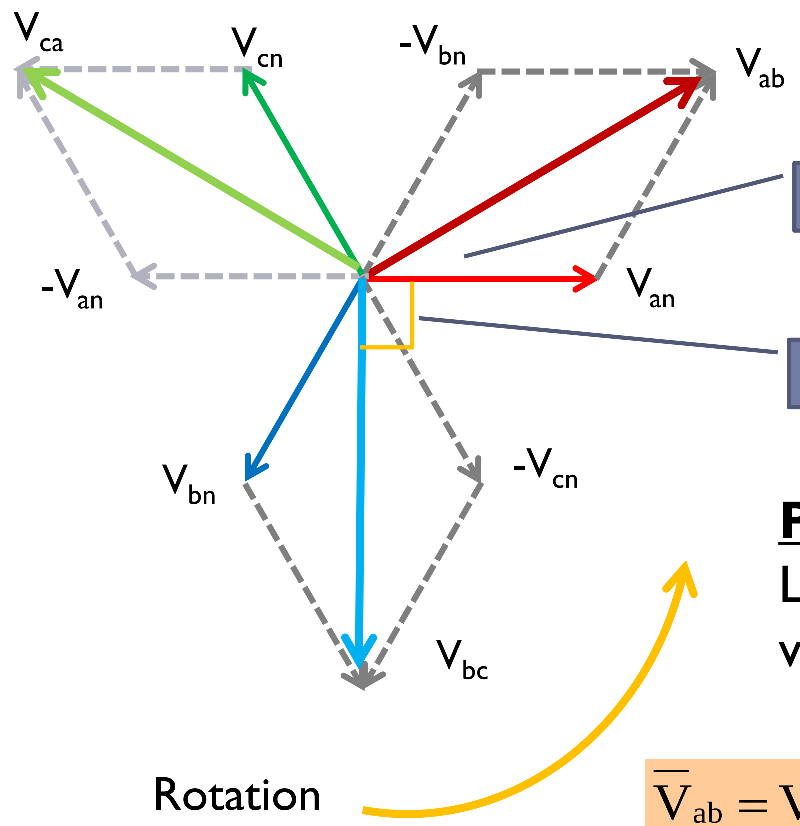


Fig. 4.10: Phasor diagram of Y-connected source in abc sequence.

Wye Connected Sources



Wye connected line voltage magnitudes

$$\left|\overline{V}_{ab}\right| = \sqrt{3}.V_p = \sqrt{3}\cdot\left|\overline{V}_{an}\right|$$

$$\left|\overline{V}_{bc}\right| = \sqrt{3}\cdot\left|\overline{V}_{bn}\right|$$

$$\left|\overline{V}_{ca}\right| = \sqrt{3}\cdot\left|\overline{V}_{cn}\right|$$

$$\overline{V}_{ab} = \sqrt{3}V_P\angle 30°$$

$$\overline{V}_{bc} = \sqrt{3}V_p\angle -90°$$

$$\overline{V}_{ca} = \sqrt{3}V_p\angle -210°$$

Phase shifts

Line-to-line (line) voltages lead phase voltages by 30 degrees for CCW rotation

$$\overline{\mathrm{V}}_{\mathrm{ab}} = \mathrm{V}_{\mathrm{ab}}\angle 30^\circ \qquad \overline{\mathrm{V}}_{\mathrm{bc}} = \mathrm{V}_{\mathrm{bc}}\angle -90^\circ \qquad \overline{\mathrm{V}}_{\mathrm{ca}} = \mathrm{V}_{\mathrm{ca}}\angle -210^\circ$$

Delta Connected Sources

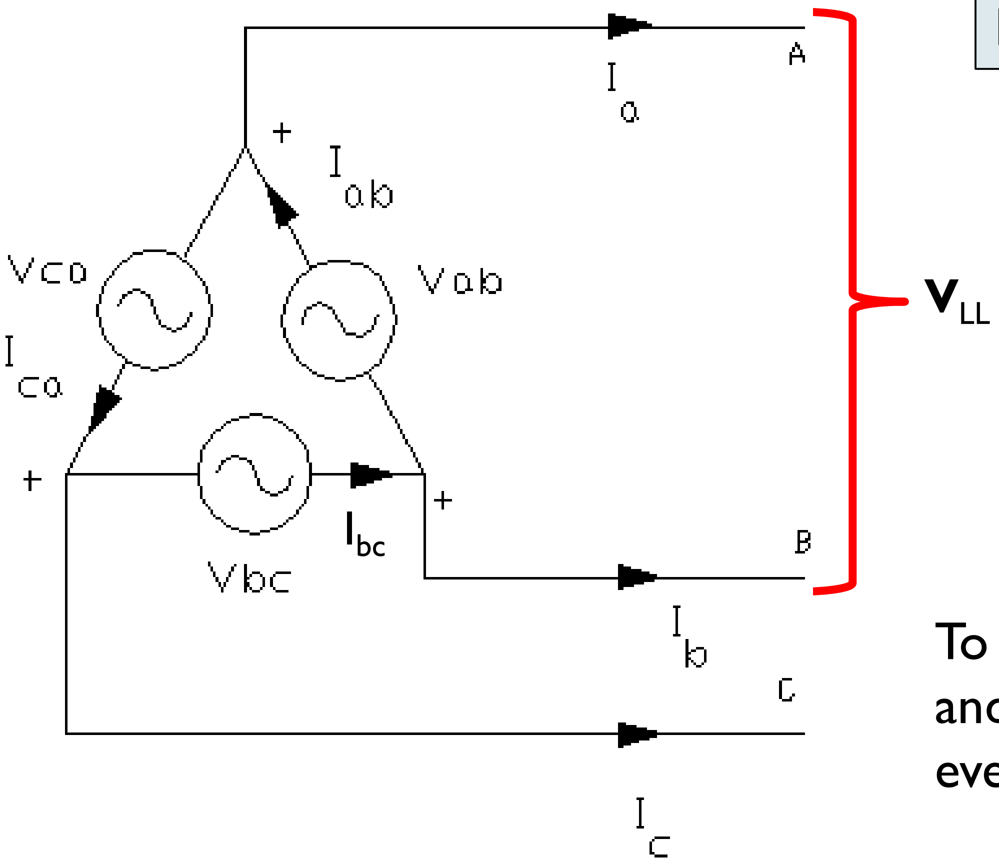
- Phase voltages are:

$$\overline{V}_{ab} = V_p \angle 0°$$

$$\overline{V}_{bc} = V_p \angle -120°$$

$$\overline{V}_{ca} = V_p \angle -240°$$

- The three conductors connected from a to A, b to B and c to C are called LINES.
- The voltage from one line to another is called a **LINE voltage**
- Line voltages are: V_{ab}, V_{bc} and V_{ca}



Phase voltages are equal to line-to-line voltages in delta connections

$$\left|\overline{V}_{LL}\right| = \left|\overline{V}_{ab}\right| = \left|\overline{V}_{P}\right|$$

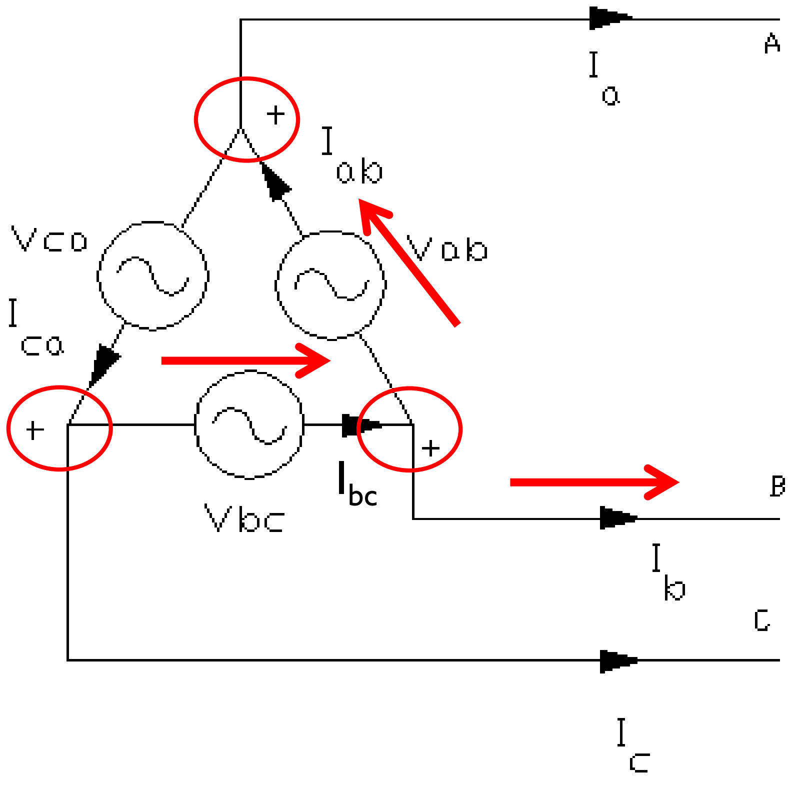
Where: V_{LL} = line-to-line voltage
V_p = phase voltage

So $\left|\overline{V}_{ab}\right| = \left|\overline{V}_{bc}\right| = \left|\overline{V}_{ca}\right|$

To find relationship between phase and line currents, perform KCL at every corner node

Delta Connected Sources

Current relationships between line and phase



For node B

$$\bar{I}_{bc} - \bar{I}_{ab} - \bar{I}_{b} = 0$$

$$\bar{I}_{bc} - \bar{I}_{ab} = \bar{I}_{b}$$

For node A

$$\bar{I}_{ab} - \bar{I}_{ca} - \bar{I}_{a} = 0$$

$$\bar{I}_{ab} - \bar{I}_{ca} = \bar{I}_{a}$$

For node C

$$\bar{I}_{ca} - \bar{I}_{bc} - \bar{I}_{c} = 0$$

$$\bar{I}_{ca} - \bar{I}_{bc} = \bar{I}_{c}$$

$$Since, I_{ca} = I_{ab}\angle -240°$$

$$\bar{\bar{I}}_a = \bar{I}_{ab} - \bar{I}_{ca}$$

$$= \bar{I}_{ab}(1 - 1\angle -240°)$$

$$= \bar{I}_{ab}(1 + 0.5 - j0.866)$$

$$= \bar{I}_{ab}\sqrt{3}\angle -30°$$

Phasor subtraction gives the current magnitude

$$\boxed{I_L = \sqrt{3}\, I_P}$$

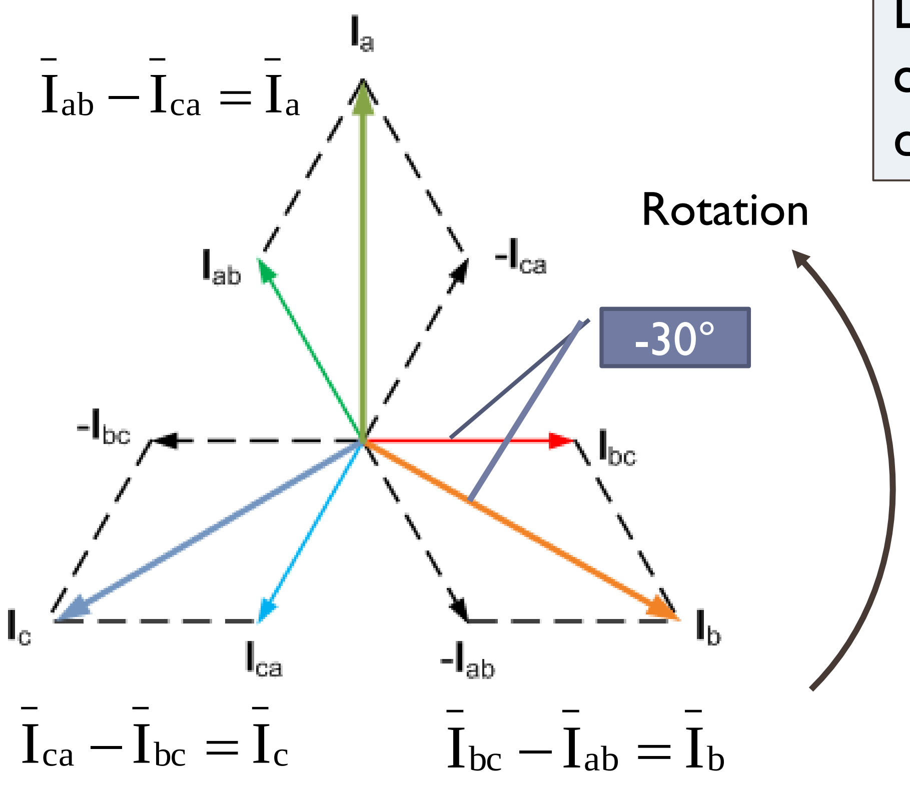
Where:

$$line\ current, I_L = |\bar{I}_a| = |\bar{I}_b| = |\bar{I}_c|$$

$$phase\ current, I_P = |\bar{I}_{ab}| = |\bar{I}_{bc}| = |\bar{I}_{ca}|$$

Delta Connected Sources

Phasor diagram of delta currents



Line current phasors lag phase currents by 30 degrees in balanced delta connection

$$\bar{I}_L = \sqrt{3} \cdot \left|\bar{I}_P\right| \angle -30^\circ$$

Above hold for all phases with I_p as reference phasor

Fig. 4.11: Phasor diagram of delt connected source in abc sequence.

Q02: Determine the relationship between phase voltage and line voltage of a balanced wye connected source. Also draw the phasor diagram. Assume that the phase sequence is positive.

Answer: Follow the slide no. 11

Q02: For a positive phase sequence, determine the relationship between phase current and line current of a balanced delta connected source. Also draw the phasor diagram.

Answer: Follow the slide no. 14 & 15

Balanced three-phase Loads

- A Balanced load has equal impedances on all the phases.

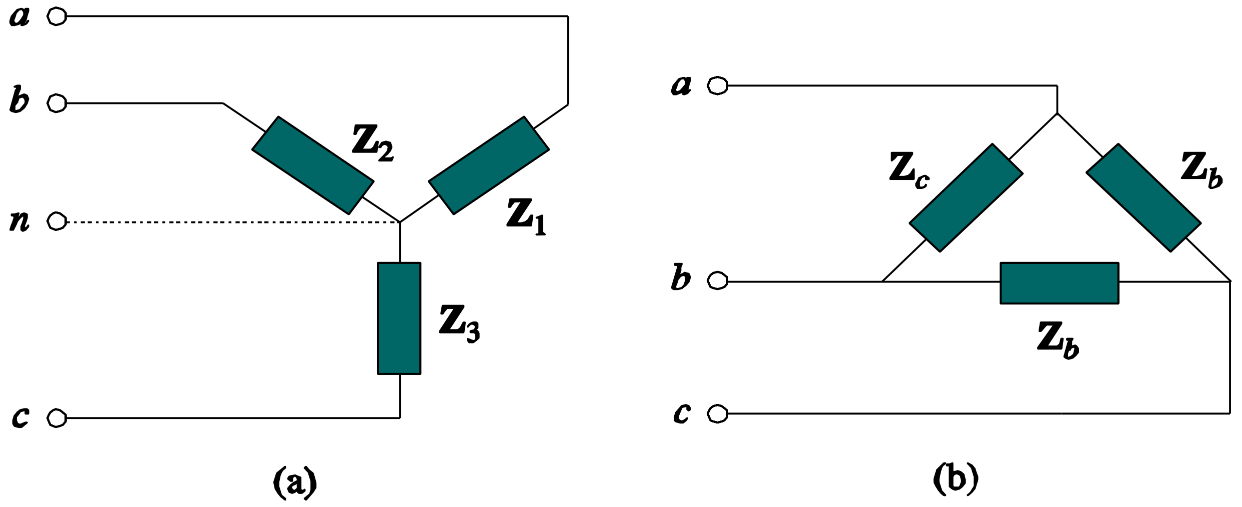
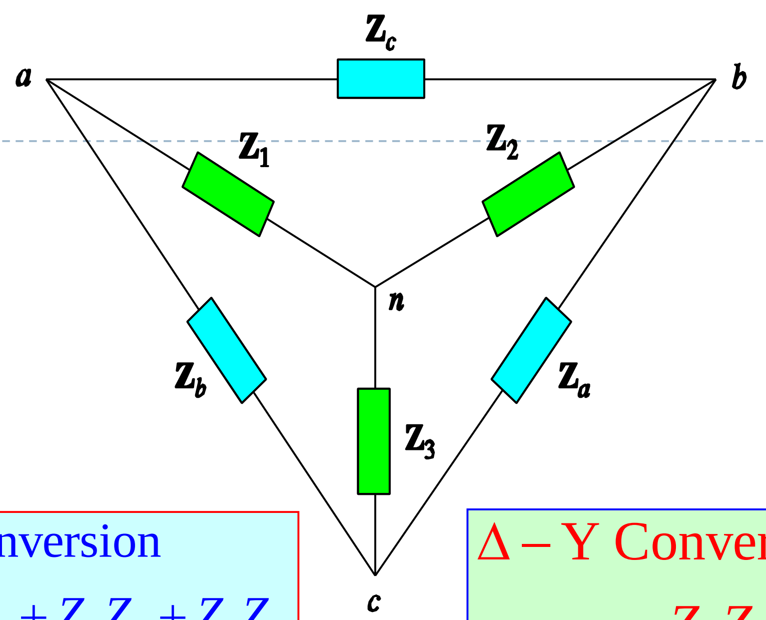


Fig. 4.12: (a) Wye-connected load and (b) Delta-connected load



- Balanced Impedance Conversion:

$$Z_Y = Z_1 = Z_2 = Z_3$$

$$Z_\Delta = Z_a = Z_b = Z_c$$

$$Z_\Delta = 3Z_Y \qquad Z_Y = \frac{1}{3} Z_\Delta$$

$Y - \Delta$ Conversion

$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

$\Delta - Y$ Conversion

$$Z_1 = \frac{Z_bZ_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_aZ_c}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_aZ_b}{Z_a + Z_b + Z_c}$$

Three-phase Connections

- We have 4 possible connection types.
  - Y-Y connection (i.e. , Y-connected source with a Y-connected load).
  - Y-Δ connection.
  - Δ-Δ connection.
  - Δ-Y connection.
- Balanced Δ connected load is more common.
- Y connected sources are more common.

Y-Y connection

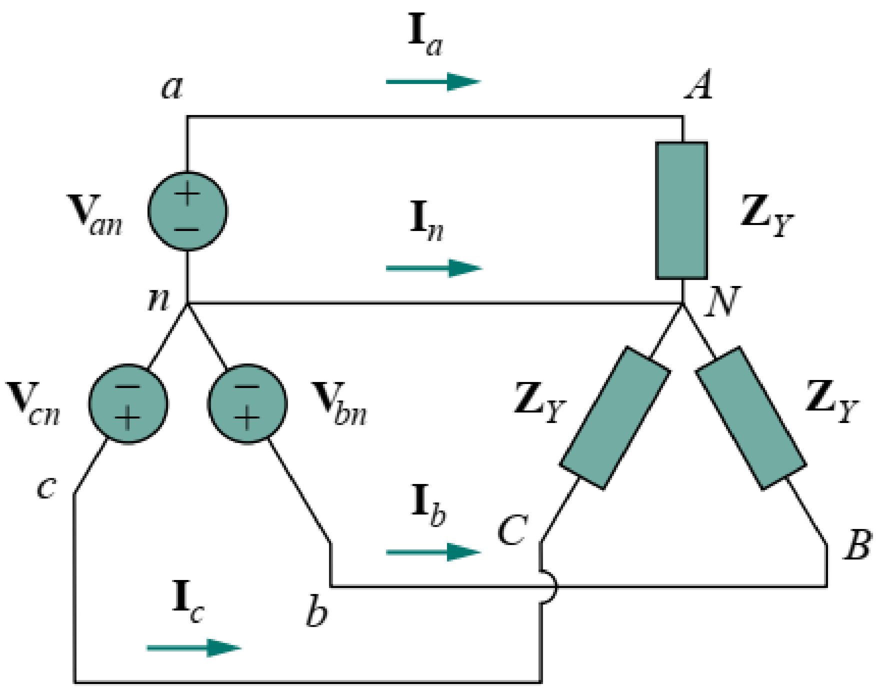


Fig. 4.13: Y-connected source with Y-connected load.

Line current I_n add up to zero.
Neutral current is zero:

$I_n = -(I_a + I_b + I_c) = 0$

- Magnitude of line voltages is √3 times the magnitude of phase voltages. $V_L = \sqrt{3}\, V_P$

Y-Δ connection

- Three phase sources are usually Wye connected and three phase loads are Delta connected.
- There is no neutral connection for the Y-Δ system.

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

$$I_a = I_{AB} - I_{CA} = I_{AB}\sqrt{3}\angle -30^\circ$$

$$I_b = I_{BC} - I_{AB} = I_{BC}\sqrt{3}\angle -30^\circ$$

$$I_c = I_{CA} - I_{BC} = I_{CA}\sqrt{3}\angle -30^\circ$$

$$I_L = |I_a| = |I_b| = |I_c|$$

$$I_p = |I_{AB}| = |I_{BC}| = |I_{CA}|$$

$$I_L = \sqrt{3} I_p$$

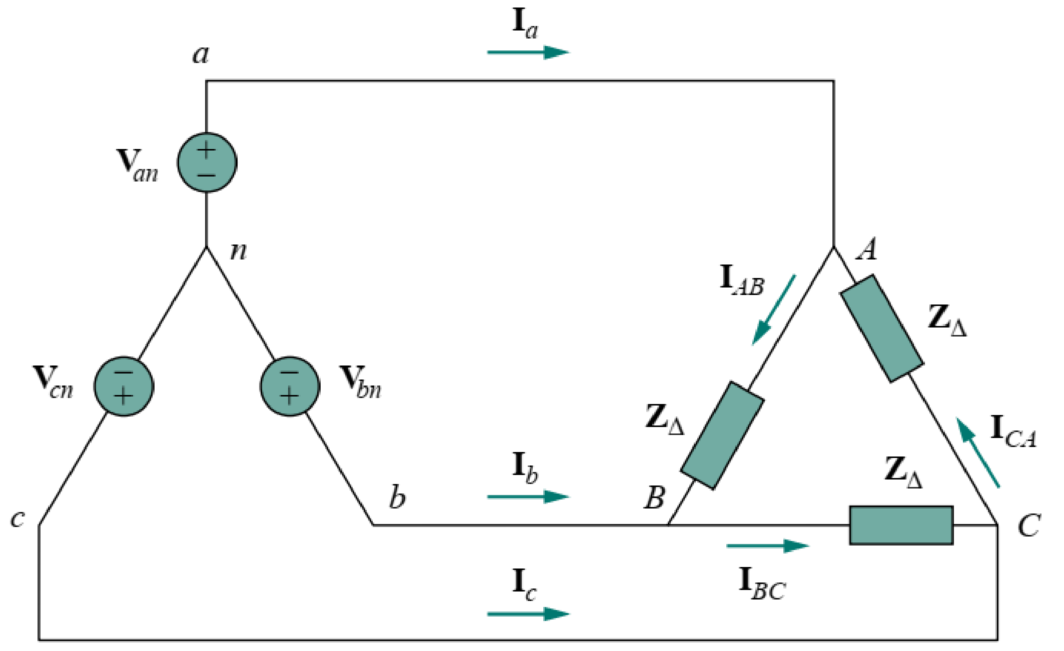
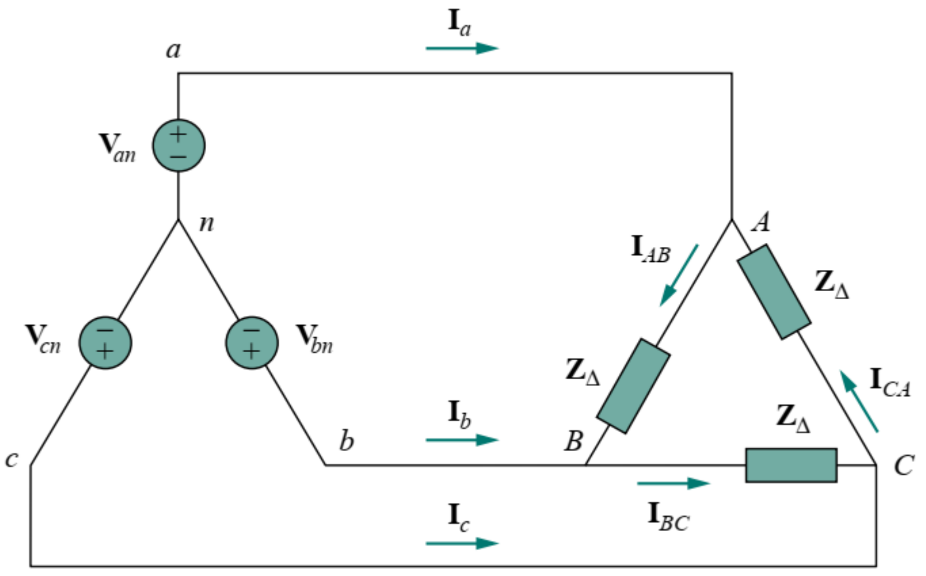


Fig. 4.14: Y-connected source feeding a delta connected load.

Y-Δ connection

EXAMPLE 12.3

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase. Calculate the phase and line currents.



Solution: The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\angle 26.57^\circ\ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\angle 10^\circ$, then the line voltage is

$$\bar{V}_{ab} = \bar{V}_{an} + \bar{V}_{nb} = \bar{V}_{an} + \bar{V}_{bn}$$
$$= 100\angle 10^\circ - 100\angle 10^\circ - 120^\circ = 173.2\angle 40^\circ V$$
$$\bar{V}_{AB} = 173.2\angle 40^\circ V$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\angle 40^\circ}{8.944\angle 26.57^\circ} = 19.36\angle 13.43^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 19.36\angle -106.57^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 19.36\angle 133.43^\circ \text{ A}$$

$$\bar{I}_a = \bar{I}_{AB} - \bar{I}_{CA}$$
$$= 19.36\angle 13.43^\circ - 19.36\angle 133.43^\circ$$
$$= 35.53\angle -16.57^\circ V$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 33.53\angle -136.57^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 33.53\angle 103.43^\circ \text{ A}$$

Δ-Δ connection

➢ Both the source and load are Delta connected and balanced.

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}, \quad I_{BC} = \frac{V_{BC}}{Z_\Delta}, \quad I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

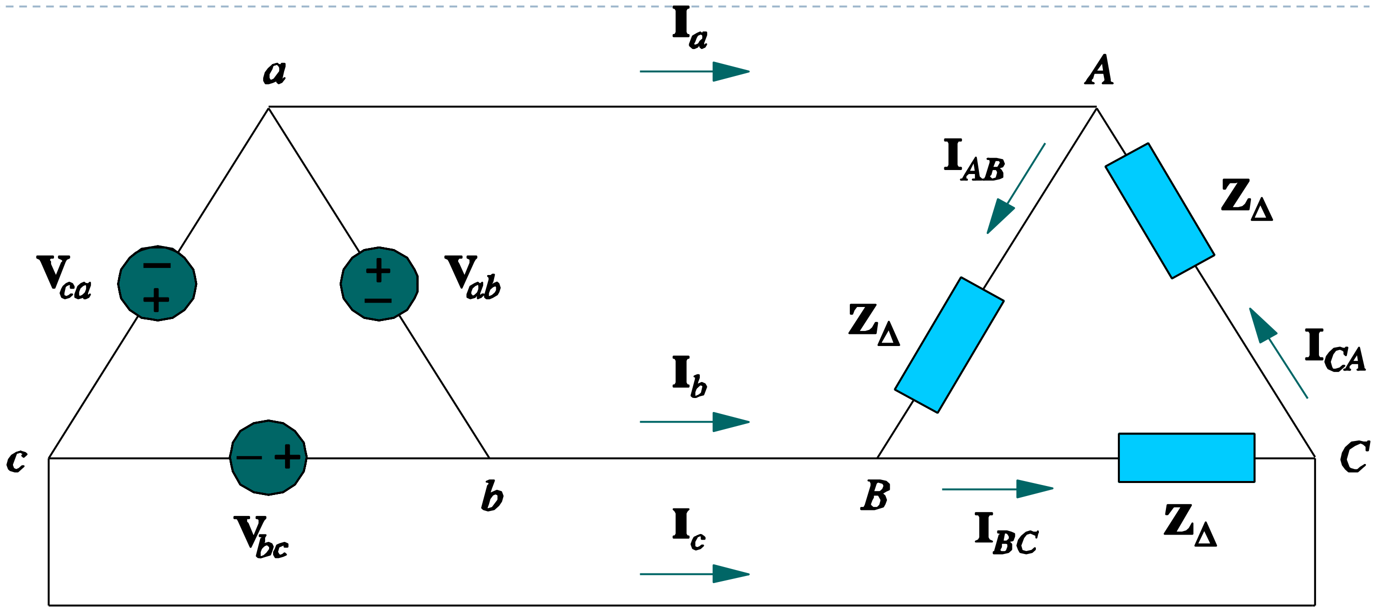


Fig. 4.15: Delta-connected source with a delta connected load.

$$I_a = I_{AB} - I_{CA}, \quad I_b = I_{BC} - I_{AB}, \quad I_c = I_{CA} - I_{BC}$$

Δ-Y connection

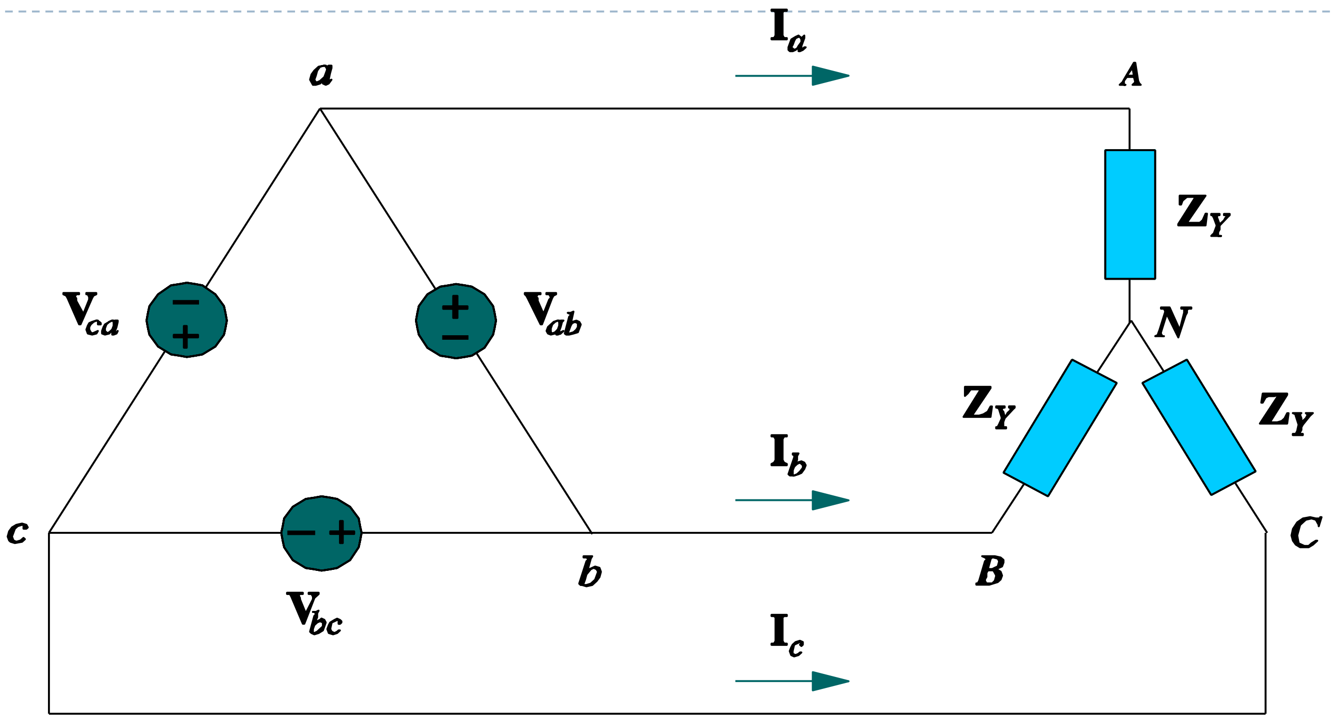


Fig. 4.16: Delta-connected source with a Y-connected load.

Power in a balanced system

Trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

For a Y-connected load, let phase voltages are:

$$v_{AN} = \sqrt{2}V_p \cos \omega t, \qquad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120^\circ)$$
$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120^\circ)$$

If $\overline{Z}_Y = Z\angle\theta$, the phase currents lag behind their corresponding phase voltages by θ. Thus,

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta), \qquad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ)$$
$$i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120^\circ)$$

Here, V_p *and* I_p *are the rms value of phase voltage and current.*

The total instantaneous power in the load,

$$\begin{aligned} p &= p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c \\ &= 2V_pI_p[\cos \omega t \cos(\omega t - \theta) \\ &\quad + \cos(\omega t - 120^\circ)\cos(\omega t - \theta - 120^\circ) \\ &\quad + \cos(\omega t + 120^\circ)\cos(\omega t - \theta + 120^\circ)] \end{aligned}$$

$$\begin{aligned} p &= V_pI_p[3\cos\theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\ &\quad + \cos(2\omega t - \theta + 240^\circ)] \\ &= V_pI_p[3\cos\theta + \cos\alpha + \cos\alpha\cos 240^\circ + \sin\alpha\sin 240^\circ \\ &\quad + \cos\alpha\cos 240^\circ - \sin\alpha\sin 240^\circ] \\ &\quad \text{where } \alpha = 2\omega t - \theta \\ &= V_pI_p\left[3\cos\theta + \cos\alpha + 2\left(-\frac{1}{2}\right)\cos\alpha\right] = 3V_pI_p\cos\theta \end{aligned}$$

$$\boxed{p = 3V_pI_p\cos\theta}$$

The important consequences of the instantenous power equation of a balanced three phase system are:

- The instantaneous power is not function of time.
- The total power behaves similar to DC power.
- This result is true whether the load is Y or Δ connected.
- The average power per phase is obtained as

$$P_p = \frac{P}{3} = V_pI_p\cos\theta$$

Power in a balanced system

$p = 3V_p I_p \cos\theta$ (Total Instantenous Power)

$P_p = \frac{1}{3} p = V_p I_p \cos\theta$ (Average Power per phase)

$Q_p = \frac{1}{3} p = V_p I_p \sin\theta$ (Reactive Power per phase)

$S_p = V_p I_p$ (Apparent Power per phase)

$\mathrm{S_p} = P_p + jQ_p = \mathrm{V_p I_p}^*$ Complex power for each phase

V_p and I_p refer to magnitude values whereas

$\mathrm{V_p}$ and $\mathrm{I_p}$ refer to phasor values (Both magnitude and phase)

$\mathrm{S_p} = P_p + jQ_p = \mathrm{V_p I_p}^*$ Complex power for each phase

$\mathrm{S} = P + jQ = 3\mathrm{S_p} = 3\mathrm{V_p I_p}^*$ Total Complex power for three phase

$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta$

$Q = Q_a + Q_b + Q_c = 3Q_p = 3V_p I_p \sin\theta = \sqrt{3} V_L I_L \sin\theta$

$\mathrm{S} = 3\mathrm{S_p} = 3\mathrm{V_p I_p}^* = 3I_p^{\,2} Z_p = \frac{3V_p^{\,2}}{Z_p^{\,*}}$ Total complex power

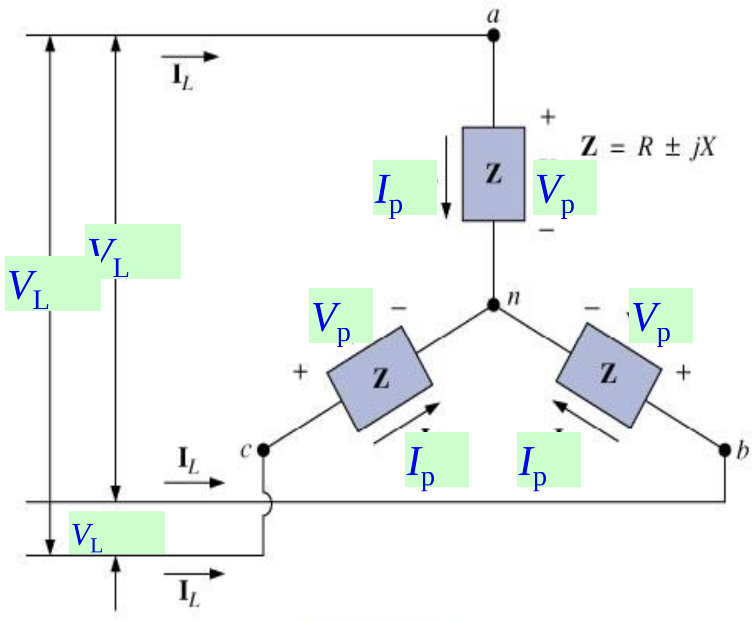
$\mathbf{S} = P + jQ = \sqrt{3} V_L I_L \angle\theta$ Total complex power using line values

V_p, I_p, V_L and I_L are all rms values, θ is the load impedance angle

Relationship between Phase and Line

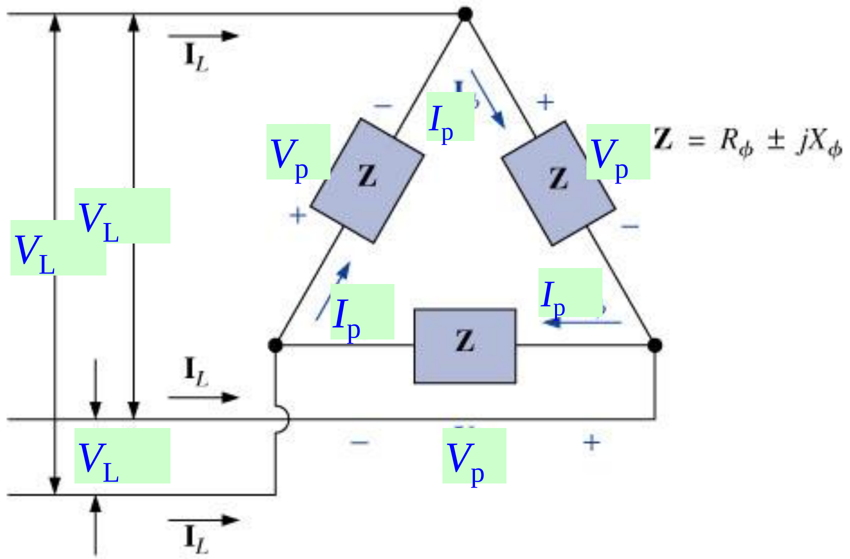
➢ Notice the values of V_p, V_L, I_p, I_L for different load connections.

$$V_L = \sqrt{3}V_p \quad I_L = I_p$$



Y connected load.

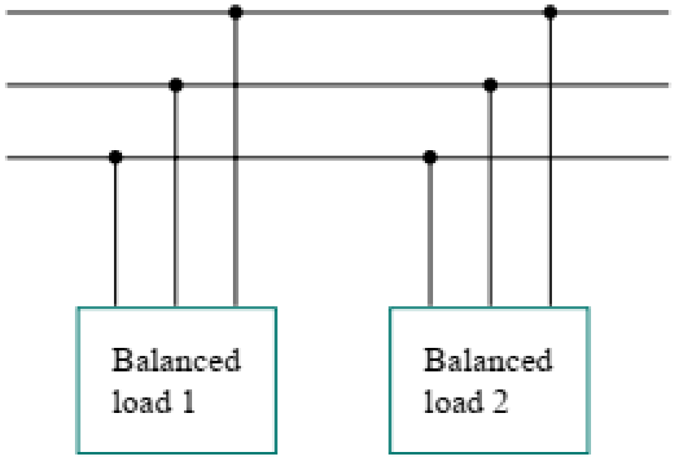
$$V_L = V_p \quad I_L = \sqrt{3}\,I_p$$



Δ connected load.

Example 1: Text _ Fundamental of Electric Circuits (Sadiku)

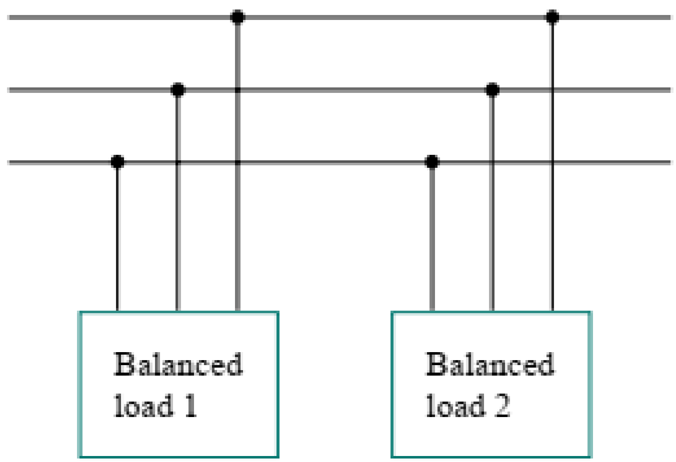
EXAMPLE 12.8



Two balanced loads are connected to a 240-kV rms 60-Hz line, as shown in Fig. 12.22(a). Load 1 draws 30 kW at a power factor of 0.6 lagging, while load 2 draws 45 kVAR at a power factor of 0.8 lagging. Assuming the *abc* sequence, determine: (a) the complex, real, and reactive powers absorbed by the combined load, (b) the line currents, and (c) the kVAR rating of the three capacitors Δ-connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.

Example 1: (Continued)

EXAMPLE 12.8



Solution:

(a) For load 1, given that $P_1 = 30$ kW and $\cos\theta_1 = 0.6$, then $\sin\theta_1 = 0.8$. Hence,

$$S_1 = \frac{P_1}{\cos\theta_1} = \frac{30\text{ kW}}{0.6} = 50\text{ kVA}$$

and $Q_1 = S_1 \sin\theta_1 = 50(0.8) = 40$ kVAR. Thus, the complex power due to load 1 is

$$\mathbf{S}_1 = P_1 + jQ_1 = 30 + j40\text{ kVA} \tag{12.8.1}$$

For load 2, if $Q_2 = 45$ kVAR and $\cos\theta_2 = 0.8$, then $\sin\theta_2 = 0.6$. We find

$$S_2 = \frac{Q_2}{\sin\theta_2} = \frac{45\text{ kVA}}{0.6} = 75\text{ kVA}$$

and $P_2 = S_2 \cos\theta_2 = 75(0.8) = 60$ kW. Therefore the complex power due to load 2 is

$$\mathbf{S}_2 = P_2 + jQ_2 = 60 + j45\text{ kVA} \tag{12.8.2}$$

From Eqs. (12.8.1) and (12.8.2), the total complex power absorbed by the load is

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 90 + j85\text{ kVA} = 123.8\underline{/43.36^\circ}\text{ kVA} \tag{12.8.3}$$

which has a power factor of $\cos 43.36^\circ = 0.727$ lagging. The real power is then 90 kW, while the reactive power is 85 kVAR.

Example 1: (Continued)

(b) Since $S = \sqrt{3} V_L I_L$, the line current is

$$I_L = \frac{S}{\sqrt{3} V_L}$$

We apply this to each load, keeping in mind that for both loads, $V_L = 240$ kV. For load 1,

$$I_{L1} = \frac{50{,}000}{\sqrt{3}\, 240{,}000} = 120.28 \text{ mA}$$

Since the power factor is lagging, the line current lags the line voltage by $\theta_1 = \cos^{-1} 0.6 = 53.13°$. Thus,

$$\mathbf{I}_{a1} = 120.28 \angle -53.13°$$

For load 2,

$$I_{L2} = \frac{75{,}000}{\sqrt{3}\, 240{,}000} = 180.42 \text{ mA}$$

and the line current lags the line voltage by $\theta_2 = \cos^{-1} 0.8 = 36.87°$. Hence,

$$\mathbf{I}_{a2} = 180.42 \angle -36.87°$$

The total line current is

$$\begin{aligned}\mathbf{I}_a = \mathbf{I}_{a1} + \mathbf{I}_{a2} &= 120.28 \angle -53.13° + 180.42 \angle -36.87° \\ &= (72.168 - j96.224) + (144.336 - j108.252) \\ &= 216.5 - j204.472 = 297.8 \angle -43.36° \text{ mA}\end{aligned}$$

Alternatively, we could obtain the current from the total complex power using Eq. (12.8.4) as

$$I_L = \frac{123{,}800}{\sqrt{3}\, 240{,}000} = 297.82 \text{ mA}$$

and

$$\mathbf{I}_a = 297.82 \angle -43.36° \text{ mA}$$

which is the same as before. The other line currents, $\mathbf{I}_{b2}$ and $\mathbf{I}_{ca}$, can be obtained according to the abc sequence (i.e., $\mathbf{I}_b = 297.82 \angle -163.36°$ mA and $\mathbf{I}_c = 297.82 \angle 76.64°$ mA).

Example 1: (Continued)

(c) We can find the reactive power needed to bring the power factor to 0.9 lagging using Eq. (11.59),

$$Q_C = P(\tan\theta_{old} - \tan\theta_{new})$$

where $P = 90$ kW, $\theta_{old} = 43.36°$, and $\theta_{new} = \cos^{-1} 0.9 = 25.84°$. Hence,

$$Q_C = 90{,}000(\tan 43.36° - \tan 25.04°) = 41.4 \text{ kVAR}$$

This reactive power is for the three capacitors. For each capacitor, the rating $Q'_C = 13.8$ kVAR. From Eq. (11.60), the required capacitance is

$$C = \frac{Q'_C}{\omega V_{rms}^2}$$

Since the capacitors are Δ-connected as shown in Fig. 12.22(b), V_{rms} in the above formula is the line-to-line or line voltage, which is 240 kV. Thus,

$$C = \frac{13{,}800}{(2\pi 60)(240{,}000)^2} = 635.5 \text{ pF}$$

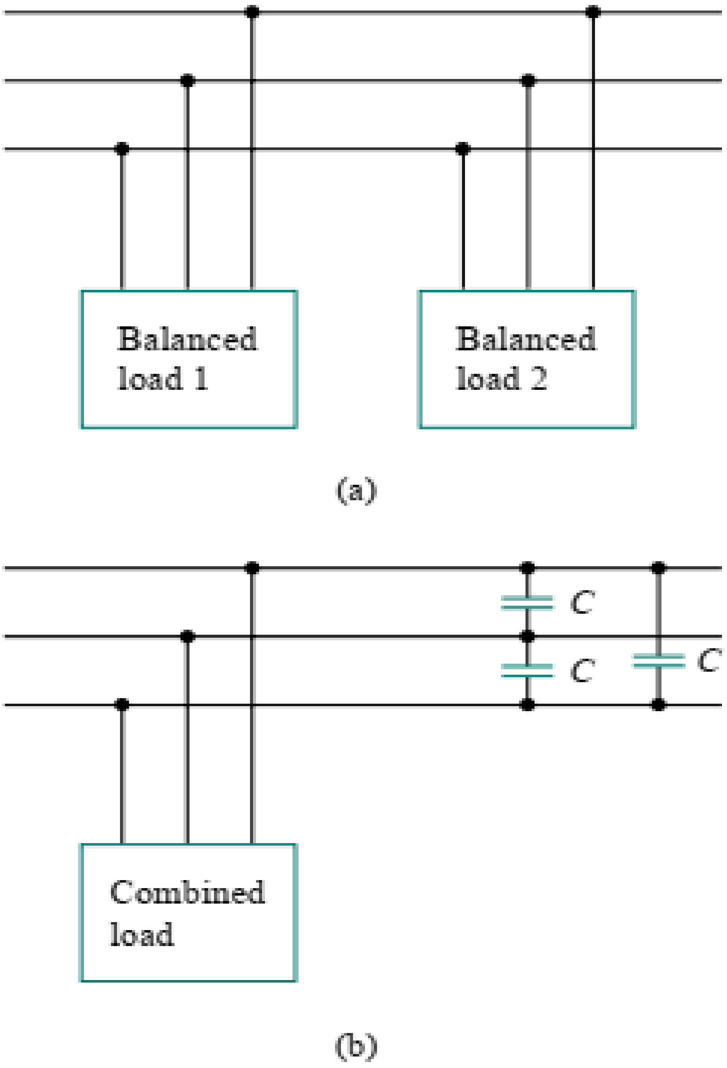


Figure 12.22 For Example 12.8: (a) The original balanced loads, (b) the combined load with improved power factor.

Example 2:

e⇗d Design a three-phase heater with suitable symmetric loads using wye-connected pure resistance. Assume that the heater is supplied by a 240-V line voltage and is to give 27 kW of heat.

Let $\quad Z_Y = R$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{240}{\sqrt{3}} = 138.56 \text{ V}$$

$$P = V_p I_p = \frac{27}{2} = 9 \text{ kW} = \frac{V_p^2}{R}$$

$$R = \frac{V_p^2}{P} = \frac{(138.56)^2}{9000} = 2.133\ \Omega$$

Thus, $\quad Z_Y = \underline{\mathbf{2.133\ \Omega}}$

End Lesson 4: Three Phase Circuits

EEE 203: Electrical Circuit II