61

Problem: Determine the equation of the curve $2x^2+3y^2-8x+6y-7=0$ ~~then~~ when the origin is transferred to the point $(2,-1)$

Solution: Given equation is $2x^2+3y^2-8x+6y-7=0$

and the point is $(2,-1)$.

We know, when the origin is trasferred to the point $(2,-1)$ we put $x=x'+2$, $y=y'-1$ in the above equation.

Then we have

$$2(x'+2)^2+3(y'-1)^2-8(x'+2)+6(y'-1)-7=0$$

$$\Rightarrow 2x'^2+8x'+8+3y'^2-6y'+3-8x'-16+6y'-6-7=0$$

$$\Rightarrow 2x'^2+3y'^2-18=0$$

$$\therefore 2x'^2+3y'^2=18$$

Now removing suffixes the equation referred to new axes is

$$2x^2+3y^2=18$$

which is the required equation of the curve.

Ans.

6.

Problem: Determine the equation of the parabola $x^2-2xy+y^2+2x-4y+3=0$ after rotating of the axes through $45°$.

Solution: Given equation is

$$x^2-2xy+y^2+2x-4y+3=0 \quad (1)$$ and angle $45°$.

We know, when the axes have been rotated through an angle $45°$, then

$$x = x'\cos 45° - y'\sin 45° = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}} = (x'-y')/\sqrt{2}$$

and $y = x'\sin 45° + y'\cos 45° = (x'+y')/\sqrt{2}$

putting these in (1), we get

$$\left(\frac{x'-y'}{\sqrt{2}}\right)^2 - 2.\frac{x'-y'}{\sqrt{2}}.\frac{x'+y'}{\sqrt{2}} + \left(\frac{x'+y'}{\sqrt{2}}\right)^2 + 2\frac{x'-y'}{\sqrt{2}} - 4\frac{x'+y'}{\sqrt{2}} + 3 = 0$$

$$\Rightarrow \left\{\frac{x'-y'}{\sqrt{2}} - \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right\}^2 + \sqrt{2}(x'-y') - 2\sqrt{2}(x'+y') + 3 = 0$$

$$\Rightarrow \left(-\frac{2y'}{\sqrt{2}}\right)^2 + \sqrt{2}x' - \sqrt{2}y' - 2\sqrt{2}x' - 2\sqrt{2}y' + 3 = 0$$

$$\Rightarrow 2y'^2 - \sqrt{2}x' - 3\sqrt{2}y' + 3 = 0$$

Now removing the suffixes, the equation is

$$2y^2 - \sqrt{2}x - 3\sqrt{2}y + 3 = 0$$

which is required equation of the parabola.

(Ans.)

Assignment

If the direction of axes is turned through an angle 30° then find the trastormation equation of $x^2+2\sqrt{3}xy-y^2-2a^2=0$. Ans: $x^2-y^2=a^2$

Transform to parallel axes through the new origin of the equations.

(i) Origin $(1,-2)$, equation $2x^2+y^2-4x+4y=0$

(ii) Origin $(3,1)$, equation $x^2-6x+2y^2+7=0$

Transform to axes inclined at 45° to the original axes the equations.

Ans: $\Rightarrow 2xy=a^2$

(i) $x^2-y^2=a^2$

Assign(ii) $x^2-y^2-2\sqrt{2}x-10\sqrt{2}y+2=0$ Ans: $xy+6x+4y=1$

~~# Transform the axes inclined at 30° to the original axes the equation $x^2+2\sqrt{3}xy+y^2=2a^2$~~