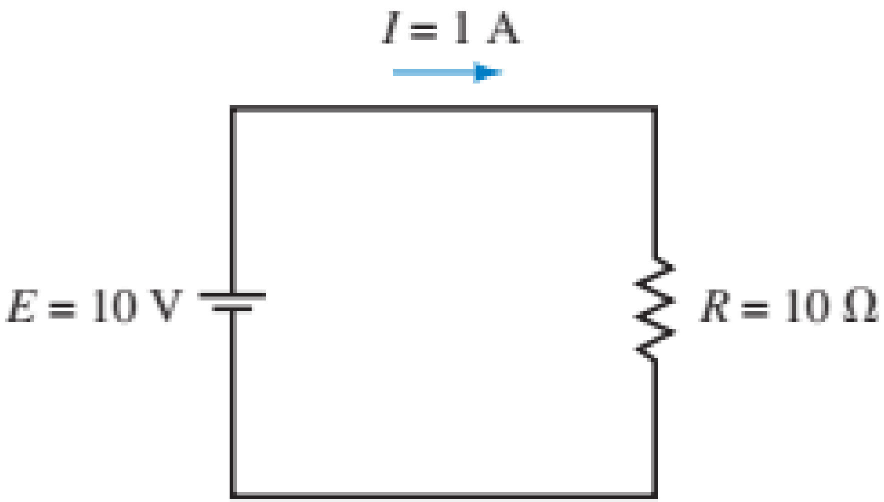
Lesson 2: Laws of Circuit Analysis

COURSE CODE: EEE 201

COURSE TITLE: ELECTRICAL ENGINEERING

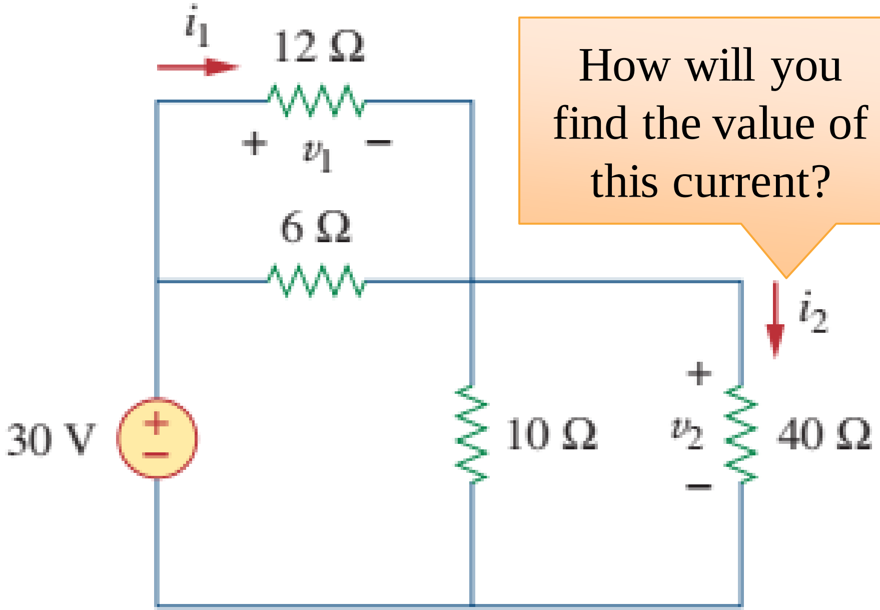
Simple circuit, Current calculated using ohm's law.



(a) $I = \frac{10\ \text{V}}{10\ \Omega} = 1\ \text{A}$

How will you find the value of this current?

How will you find the value of this current?



Branches, Nodes and Loops

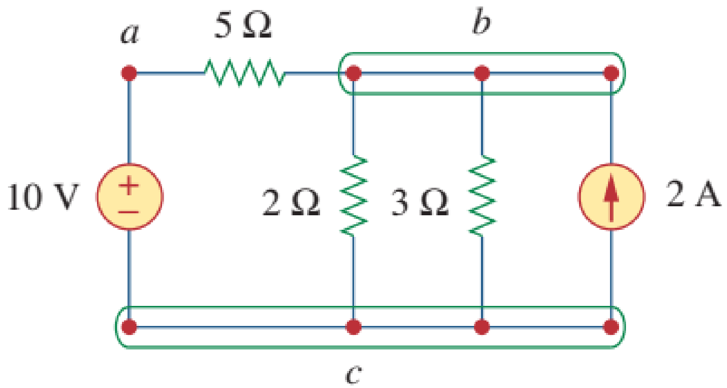


Figure 2.10
Nodes, branches, and loops.

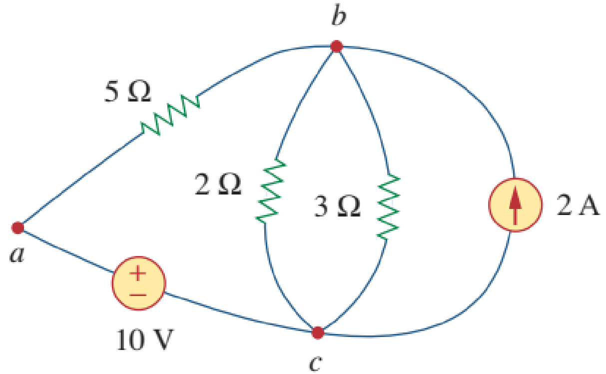


Figure 2.11
The three-node circuit of Fig. 2.10 is redrawn.

A **branch** represents a single element such as a voltage source or a resistor.

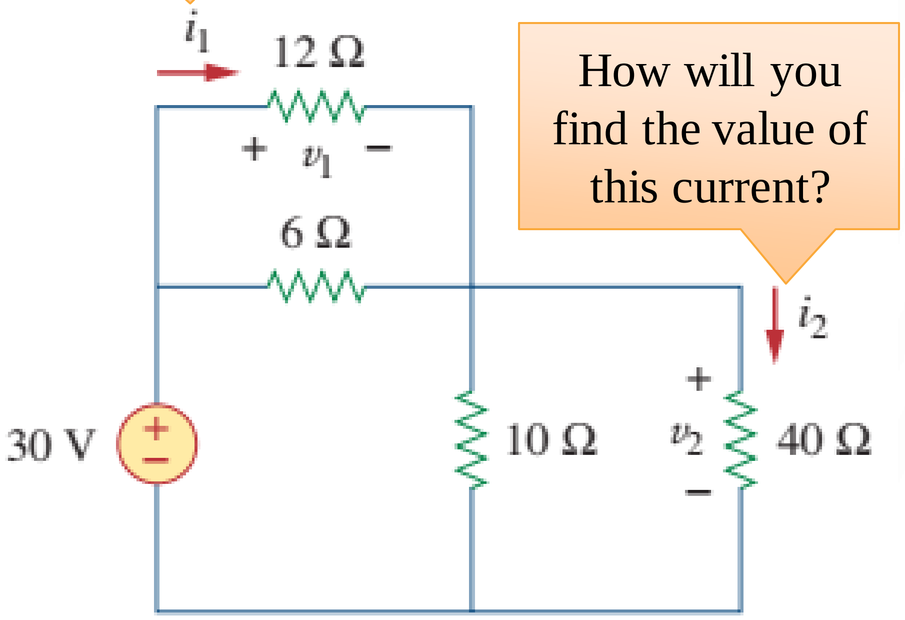
In other words, a branch represents any two-terminal element. The circuit in Fig. 2.10 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

A **node** is the point of connection between two or more branches.

A **loop** is any closed path in a circuit.

How will you find the value of this current?

How will you find the value of this current?



- Find out the number of branches, nodes and loops in the above circuit.

Kirchhoff's Laws

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

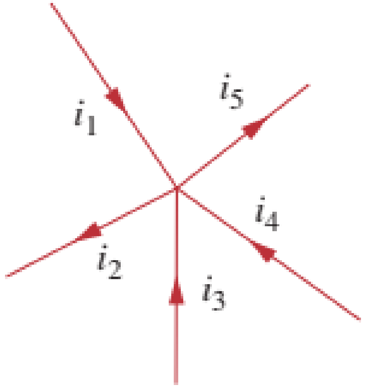


Figure 2.16
Currents at a node illustrating KCL.

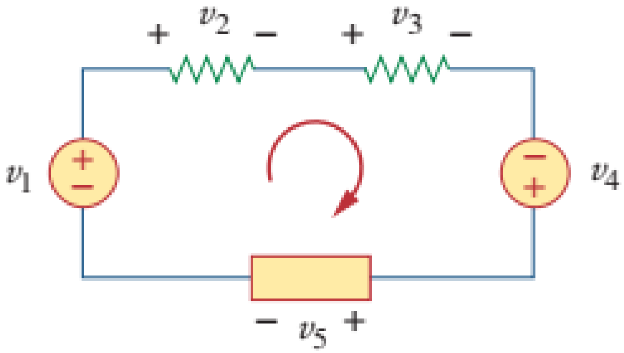
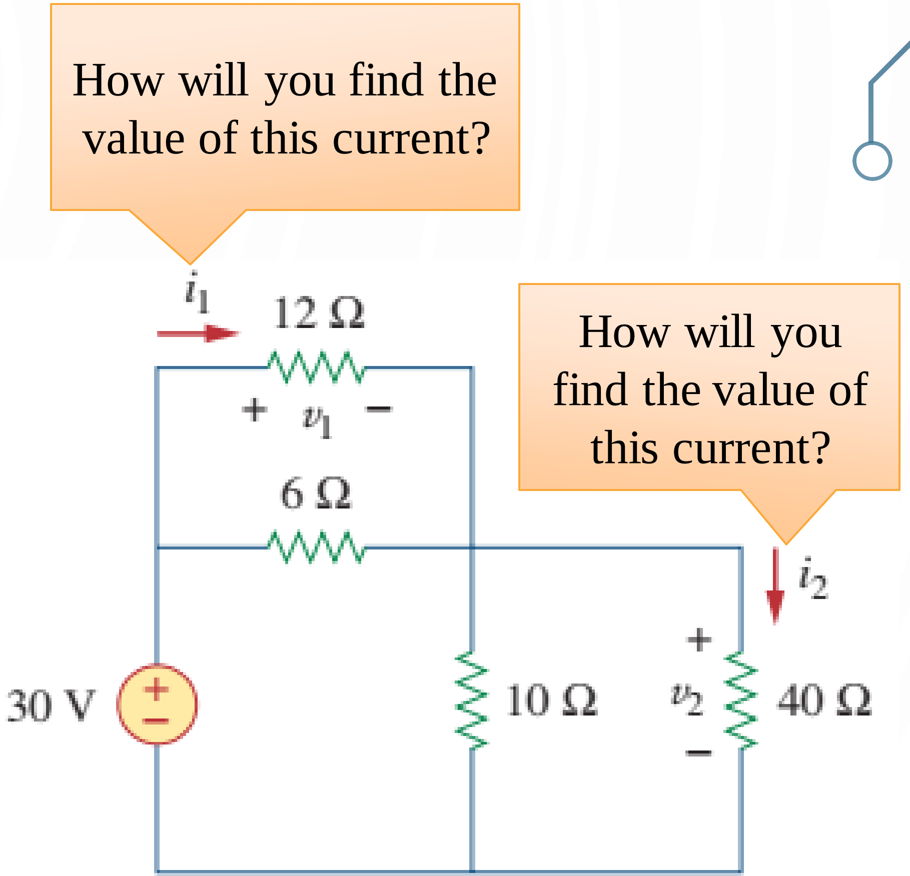


Figure 2.19
A single-loop circuit illustrating KVL.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0 \quad \text{.......KCL}$$

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \quad \text{.......KVL}$$



- ➢ Write the KCL and KVL equations for the above circuit

Series Resistors and Voltage Division

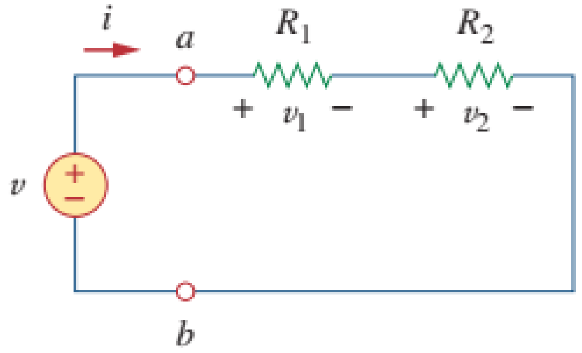


Figure 2.29
A single-loop circuit with two resistors in series.

$$R_{eq} = R_1 + R_2$$

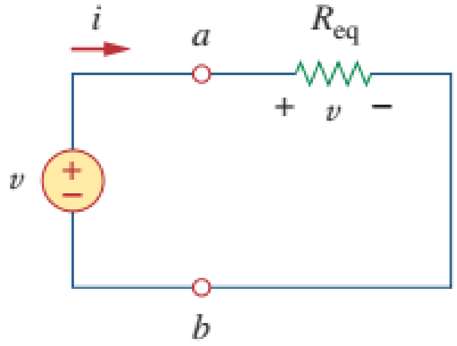


Figure 2.30
Equivalent circuit of the Fig. 2.29 circuit.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

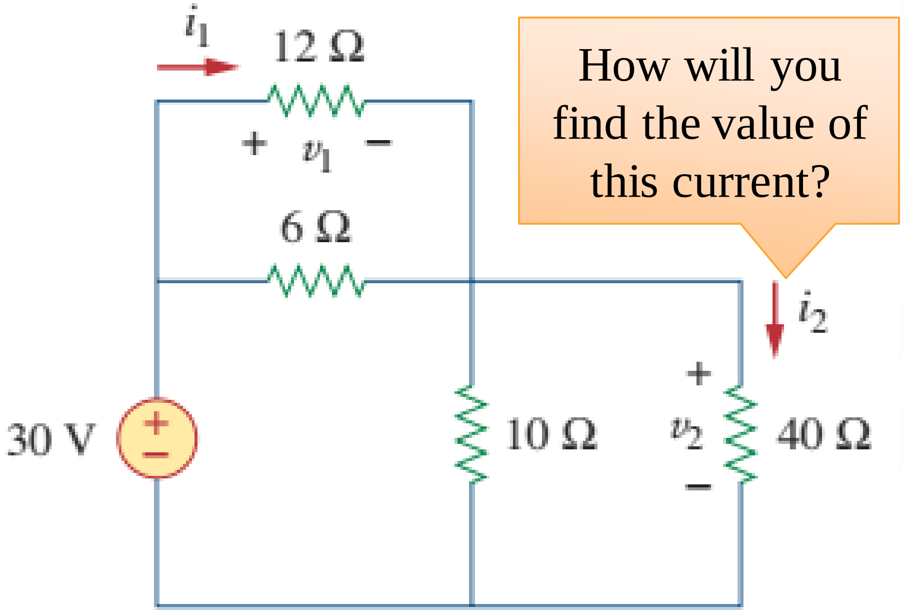
Voltage Divider Rule:

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

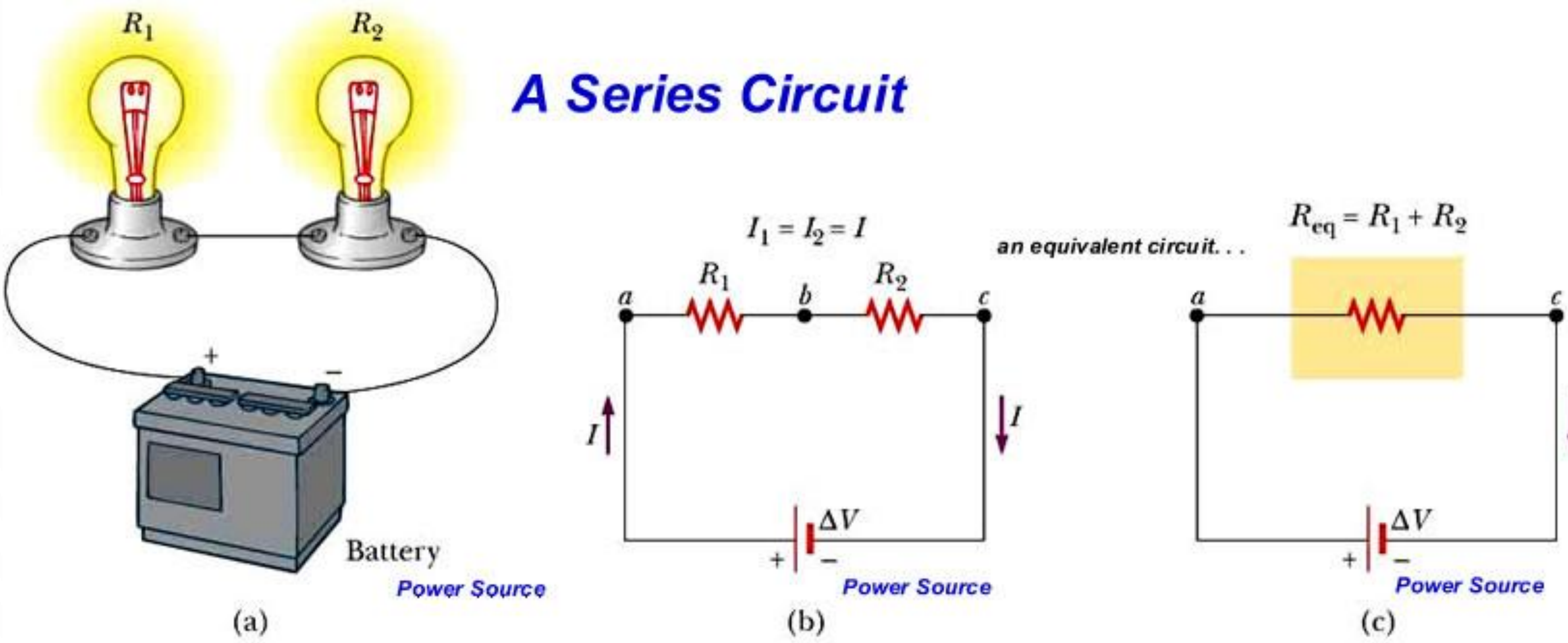
$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

How will you find the value of this current?

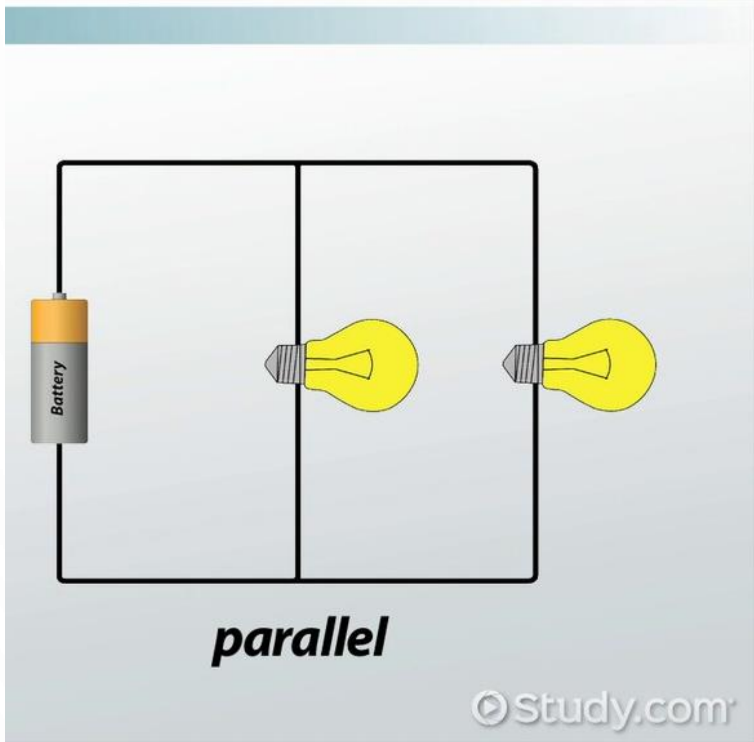
How will you find the value of this current?

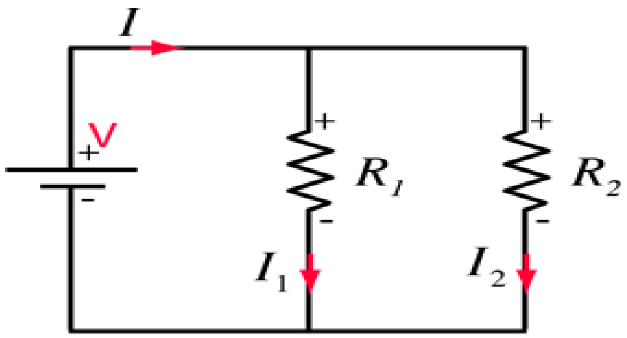


DC SERIES CIRCUIT: PROPERTIES



DC PARALLEL CIRCUIT: PROPERTIES





Parallel resistors

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Parallel Resistors and Current Division

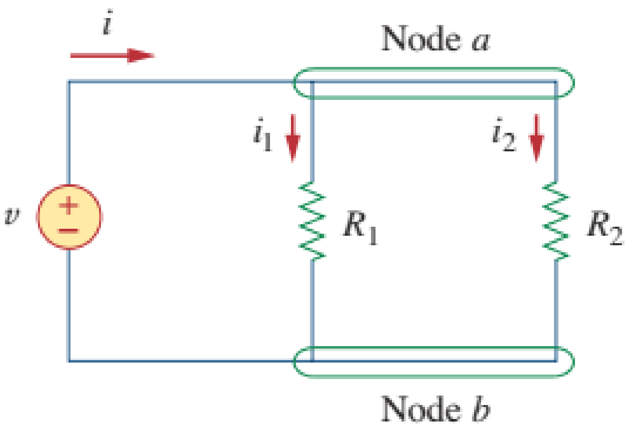


Figure 2.31
Two resistors in parallel.

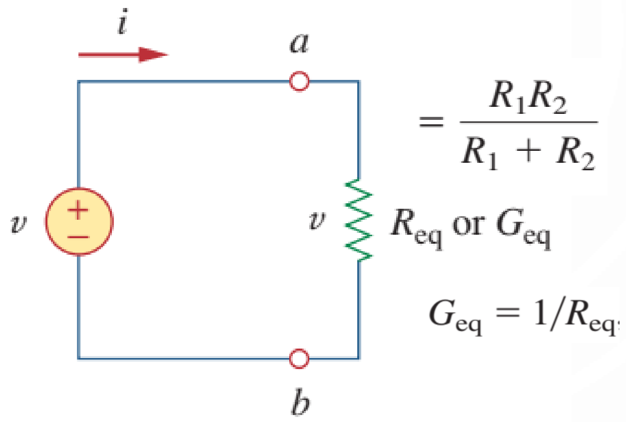


Figure 2.32
Equivalent circuit to Fig. 2.31.

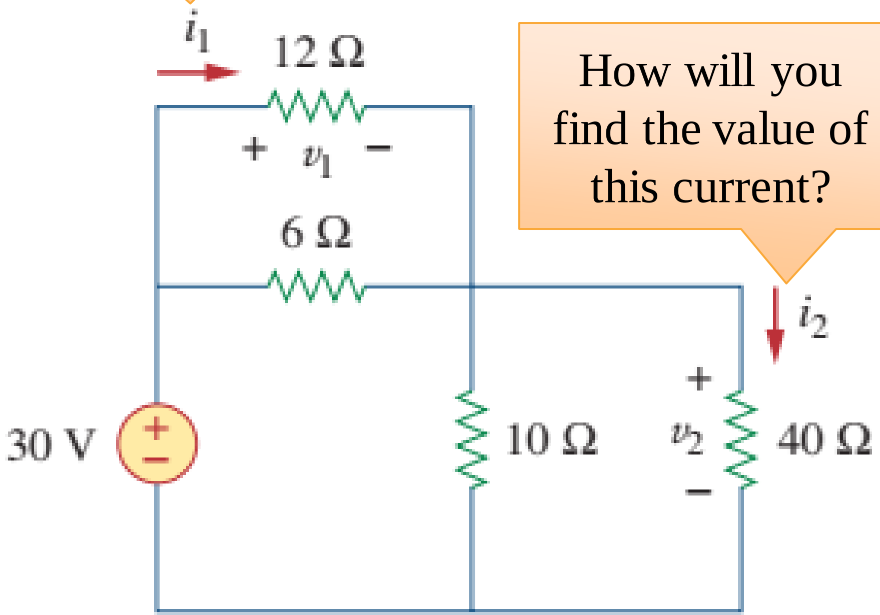
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2} \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Current Divider Rule:

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

How will you find the value of this current?

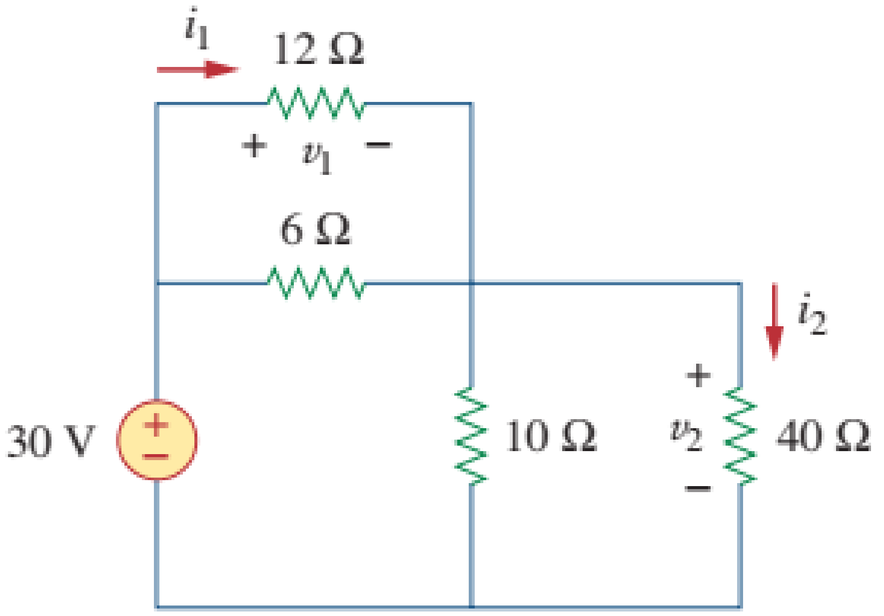
How will you find the value of this current?



- Find the total equivalent resistance of this circuit.

Practice Problem 2.12

Find v_1 and v_2 in the circuit shown in Fig. 2.43. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.



Answer: $v_1 = 10$ V, $i_1 = 833.3$ mA, $p_1 = 8.333$ W, $v_2 = 20$ V, $i_2 =$ 500 mA, $p_2 = 10$ W.

- Now you can solve this problem if you understand KVL, KCL, Series and parallel circuit.

Example 2.6

Determine v_o and i in the circuit shown in Fig. 2.23(a).

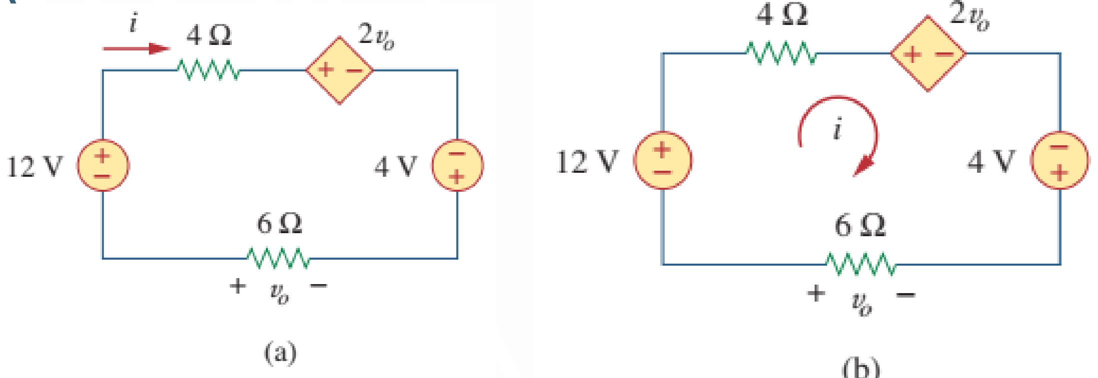


Figure 2.23
For Example 2.6.

Solution:
We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \tag{2.6.1}$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \tag{2.6.2}$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

Example 2.7

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

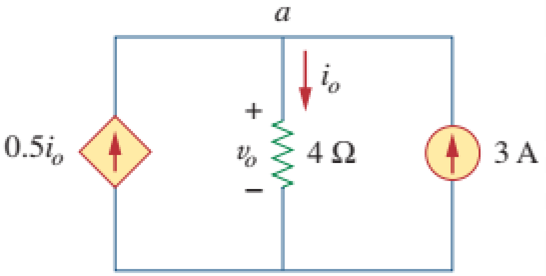


Figure 2.25
For Example 2.7.

Solution:
Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \quad \Rightarrow \quad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

Example 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

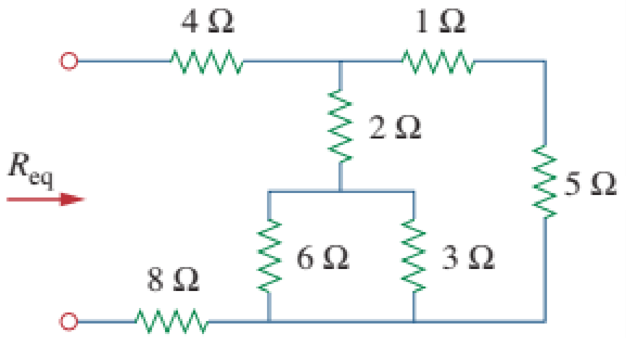


Figure 2.34
For Example 2.9.

Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

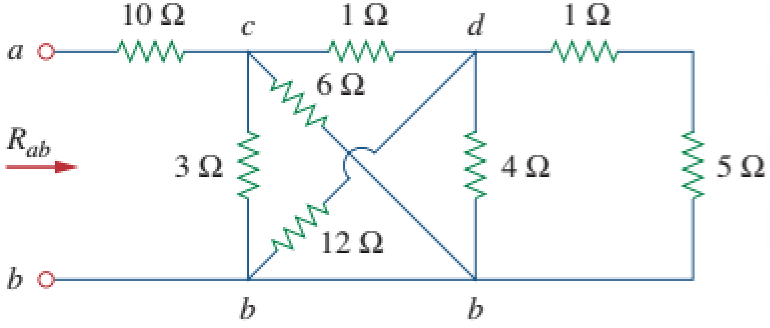


Figure 2.37
For Example 2.10.

Problem 2.34:

2.34 Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit of Fig. 2.98. Find the overall absorbed power by the resistor network.

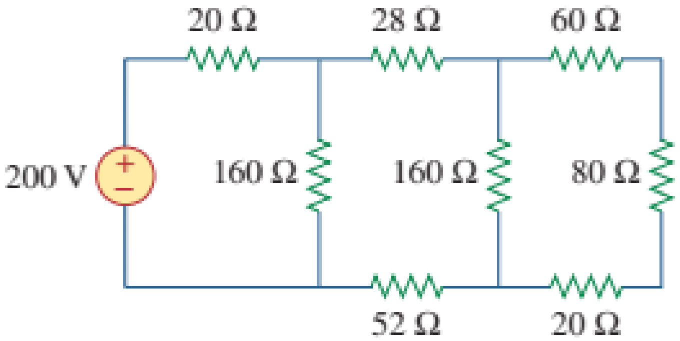


Figure 2.98
For Prob. 2.34.

Problem 2.34: Solution

40//(10 + 20 + 10)= 20 Ω,
40//(8+12 + 20) = 20 Ω

$$R_{eq} = 20 + 20 = \underline{40\ \Omega}$$

$$I = \frac{V}{R_{eq}} = 12/40, \qquad P = VI = \frac{12^2}{40} = \underline{3.6\ \text{W}}$$

Problem 2.61:

2.61 As a design engineer, you are asked to design a lighting system consisting of a 70-W power supply and two light bulbs as shown in Fig. 2.124. You must select the two bulbs from the following three available bulbs.

$R_1 = 80\ \Omega$, cost $= \$0.60$ (standard size)
$R_2 = 90\ \Omega$, cost $= \$0.90$ (standard size)
$R_3 = 100\ \Omega$, cost $= \$0.75$ (nonstandard size)

The system should be designed for minimum cost such that lies within the range $I = 1.2\text{ A} \pm 5$ percent.

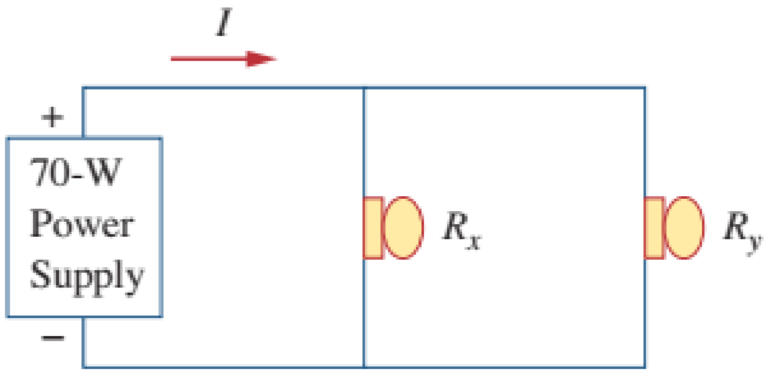
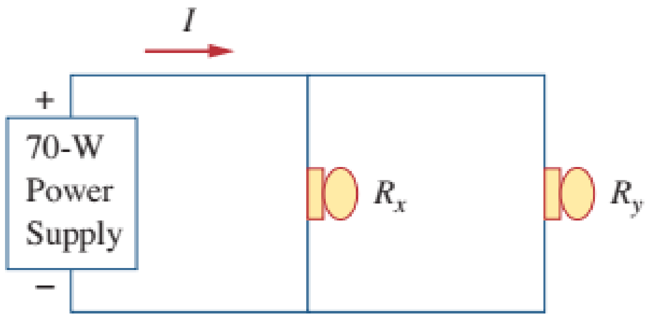


Figure 2.124
For Prob. 2.61.

Problem 2.61: Solution

There are three possibilities, but they must also satisfy the current range of 1.2 + 0.06 = 1.26 and 1.2 – 0.06 = 1.14.

$R_1 = 80\ \Omega$, cost $= \$0.60$ (standard size)
$R_2 = 90\ \Omega$, cost $= \$0.90$ (standard size)
$R_3 = 100\ \Omega$, cost $= \$0.75$ (nonstandard size)



(a) Use R_1 and R_2:

$R = R_1 \| R_2 = 80 \| 90 = 42.35\Omega$

$p = i^2R = 70W$

$i^2 = 70/42.35 = 1.6529$ or $i = 1.2857$ (which is outside our range)

cost = \$0.60 + \$0.90 = \$1.50

(b) Use R_1 and R_3:

$R = R_1 \| R_3 = 80 \| 100 = 44.44\ \Omega$

$i^2 = 70/44.44 = 1.5752$ or $i = 1.2551$ (which is within our range), cost = \$1.35

(c) Use R_2 and R_3:

$R = R_2 \| R_3 = 90 \| 100 = 47.37\Omega$

$i^2 = 70/47.37 = 1.4777$ or $i = 1.2156$ (which is within our range), cost = \$1.65

Note that cases (b) and (c) satisfy the current range criteria and (b) is the cheaper of the two, hence the correct choice is:

$\underline{R_1 \text{ and } R_3}$

Problem 2.74:

2.74 The circuit in Fig. 2.134 is to control the speed of a motor such that the motor draws currents 5 A, 3 A, and 1 A when the switch is at high, medium, and low positions, respectively. The motor can be modeled as a load resistance of 20 mΩ. Determine the series dropping resistances R_1, R_2, and R_3.

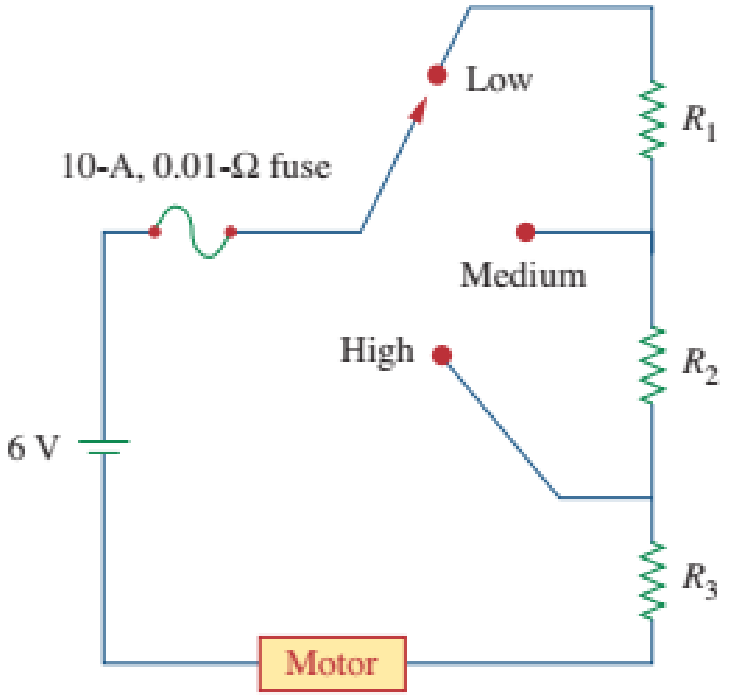


Figure 2.134
For Prob. 2.74.

Problem 2.74: Solution

With the switch in high position,

$$6 = (0.01 + R_3 + 0.02) \times 5 \longrightarrow R_3 = \underline{\mathbf{1.17\ \Omega}}$$

At the medium position,

$$6 = (0.01 + R_2 + R_3 + 0.02) \times 3 \longrightarrow R_2 + R_3 = 1.97$$

$$\text{or } R_2 = 1.97 - 1.17 = \underline{\mathbf{0.8\ \Omega}}$$

At the low position,

$$6 = (0.01 + R_1 + R_2 + R_3 + 0.02) \times 1 \longrightarrow R_1 + R_2 + R_3 = 5.97$$

$$R_1 = 5.97 - 1.97 = \underline{\mathbf{4\ \Omega}}$$

Problem 2.77:

2.77 Suppose your circuit laboratory has the following standard commercially available resistors in large quantities:

1.8 Ω   20 Ω   300 Ω   24 kΩ   56 kΩ

Using series and parallel combinations and a minimum number of available resistors, how would you obtain the following resistances for an electronic circuit design?

(a) 5 Ω   (b) 311.8 Ω

(c) 40 kΩ   (d) 52.32 kΩ

Problem 2.77: Solution

(a) $5\ \Omega = 10 \| 10 = 20 \| 20 \| 20 \| 20$

i.e., **four 20 Ω resistors in parallel**.

(b) $311.8 = 300 + 10 + 1.8 = 300 + 20 \| 20 + 1.8$

i.e., **one 300Ω resistor in series with 1.8Ω resistor and a parallel combination of two 20Ω resistors**.

(c) $40\text{k}\Omega = 12\text{k}\Omega + 28\text{k}\Omega = 24 \| 24\text{k} + 56\text{k} \| 56\text{k}$

i.e., **Two 24kΩ resistors in parallel connected in series with two 56kΩ resistors in parallel**.

(d)
$$\begin{aligned} 42.32\text{k}\Omega &= 42l + 320 \\ &= 24\text{k} + 28\text{k} = 320 \\ &= 24\text{k} = 56\text{k} \| 56\text{k} + 300 + 20 \end{aligned}$$

i.e., **A series combination of a 20Ω resistor, 300Ω resistor, 24kΩ resistor, and a parallel combination of two 56kΩ resistors.**

Problem 2.79:

2.79 An electric pencil sharpener rated 240 mW, 6 V is connected to a 9-V battery as shown in Fig. 2.138. Calculate the value of the series-dropping resistor R_x needed to power the sharpener.

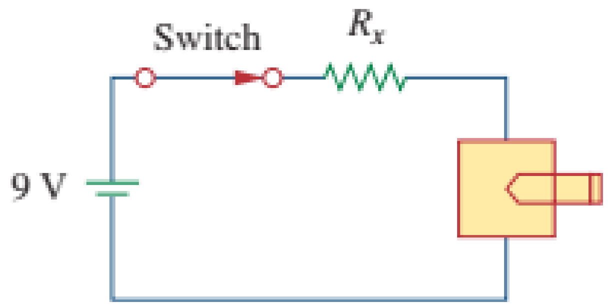


Figure 2.138
For Prob. 2.79.

Problem 2.77: Solution

Since $p = v^2/R$, the resistance of the sharpener is

$$R = v^2/(p) = 6^2/(240 \text{ x } 10^{-3}) = 150\Omega$$
$$I = p/(v) = 240 \text{ mW}/(6\text{V}) = 40 \text{ mA}$$

Since R and R_x are in series, I flows through both.

$$IR_x = V_x = 9 - 6 = 3 \text{ V}$$
$$R_x = 3/(I) = 3/(40 \text{ mA}) = 3000/(40) = \underline{75\ \Omega}$$

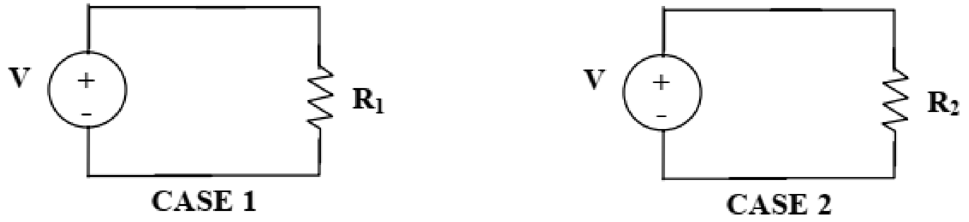
Problem 2.80:

2.80 A loudspeaker is connected to an amplifier as shown in Fig. 2.139. If a 10-Ω loudspeaker draws the maximum power of 12 W from the amplifier, determine the maximum power a 4-Ω loudspeaker will draw.

Figure 2.139
For Prob. 2.80.

Problem 2.77: Solution

The amplifier can be modeled as a voltage source and the loudspeaker as a resistor:



Hence $p = \frac{V^2}{R}, \quad \frac{p_2}{p_1} = \frac{R_1}{R_2} \longrightarrow p_2 = \frac{R_1}{R_2}p_1 = \frac{10}{4}(12) = \underline{\mathbf{30\ W}}$

Example 2.15

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i.

Wye-Delta Transformations

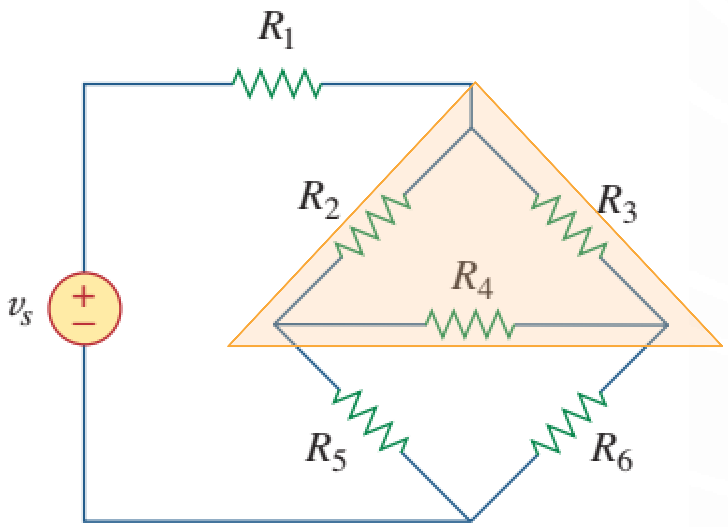


Figure 2.46
The bridge network.

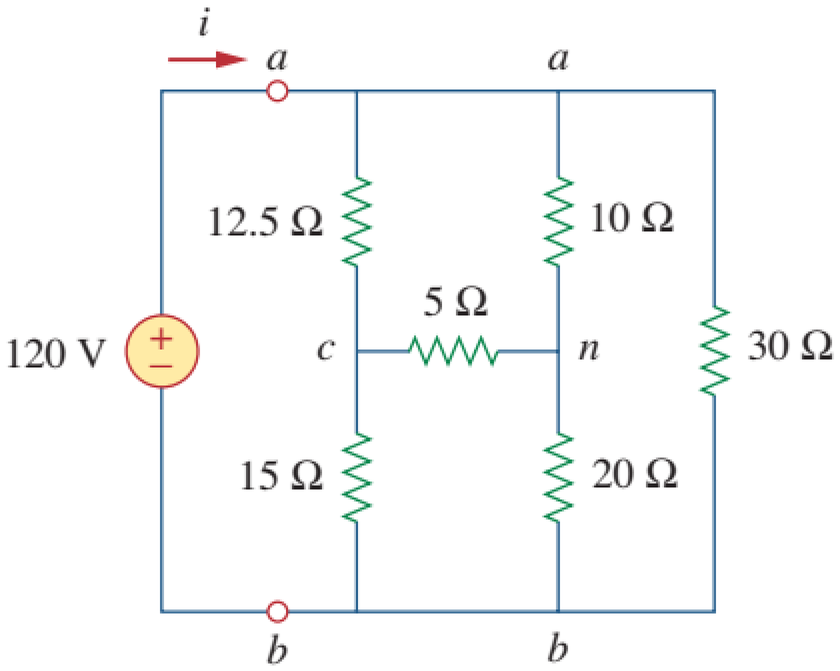


Figure 2.52
For Example 2.15.

Wye-Delta Transformations

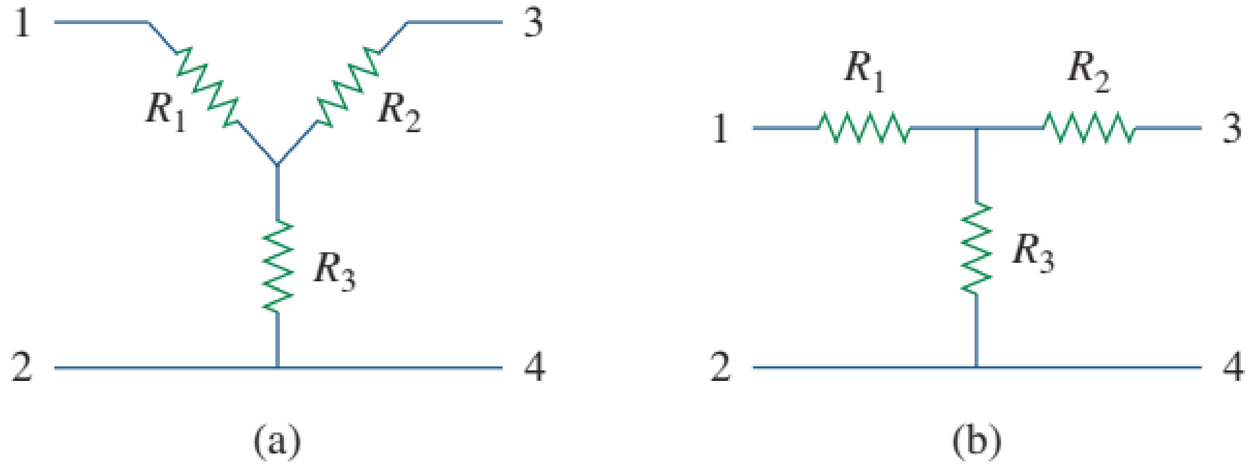


Figure 2.47
Two forms of the same network: (a) Y, (b) T.

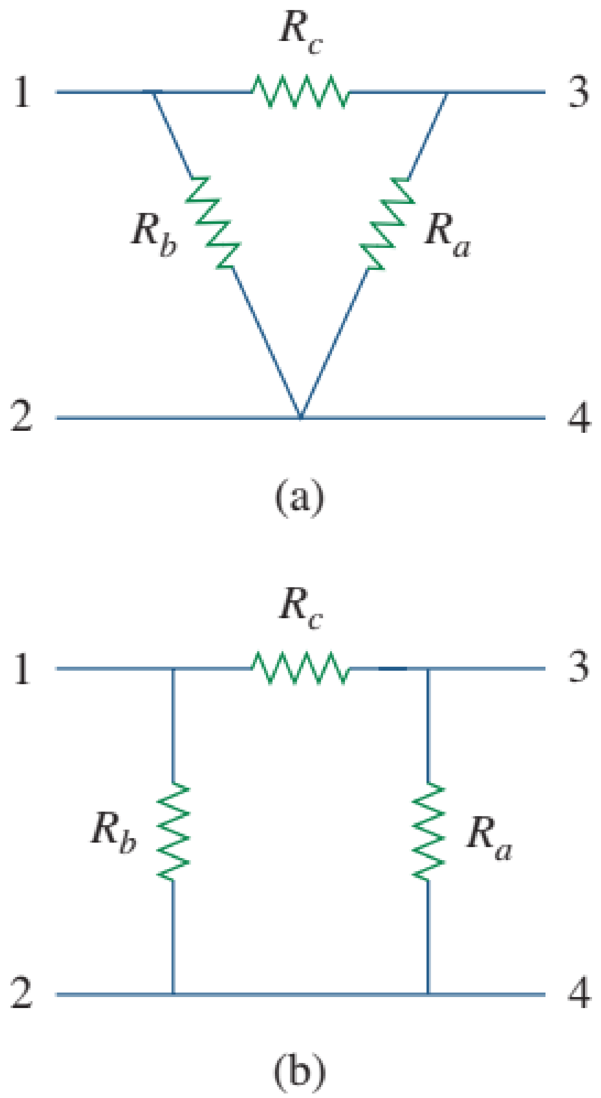


Figure 2.48
Two forms of the same network: (a) Δ, (b) Π.

Wye-Delta Transformations

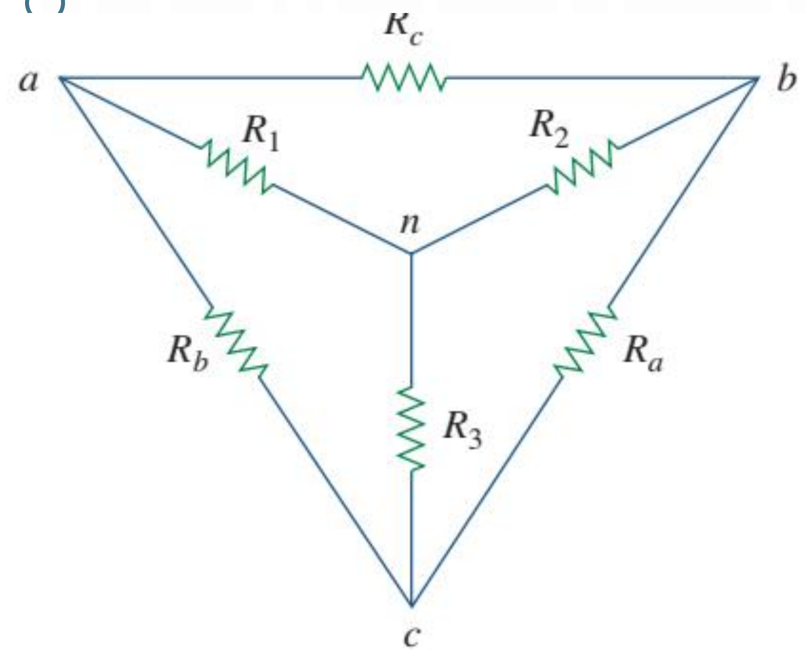


Figure 2.49
Superposition of Y and Δ networks as an aid in transforming one to the other.

Delta to Star

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Star to Delta

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Example 2.15

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i.

Solution:

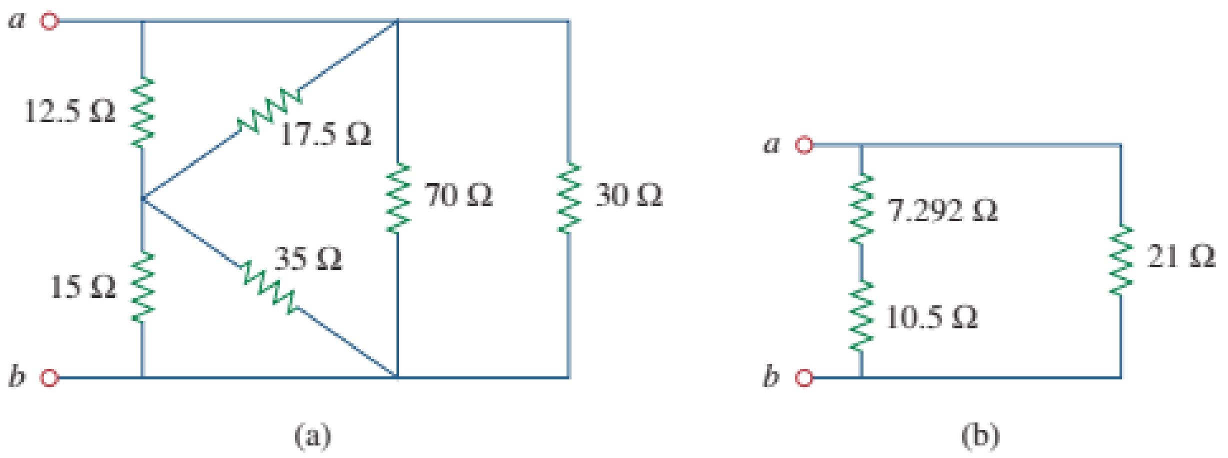


Figure 2.53
Equivalent circuits to Fig. 2.52, with the voltage source removed.

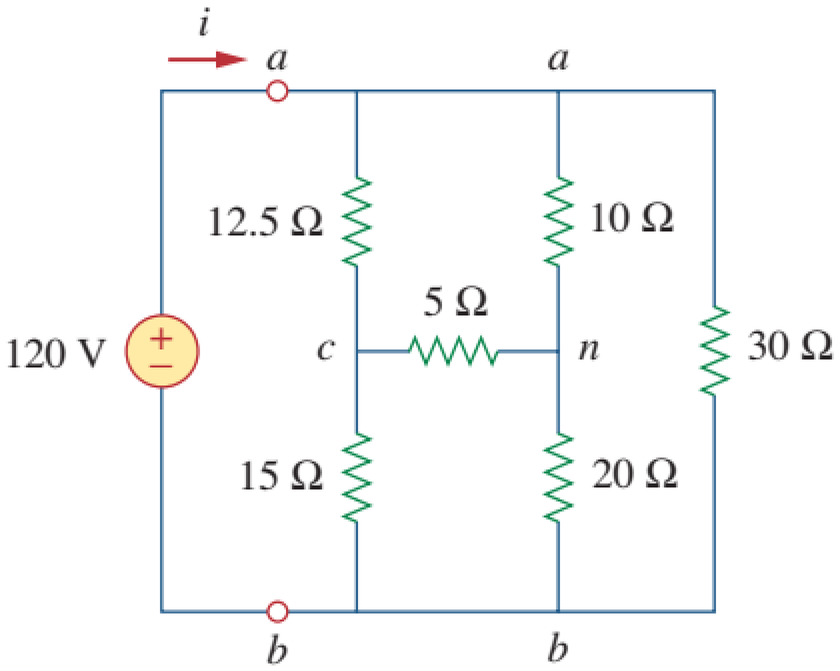


Figure 2.52
For Example 2.15.

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

Assignment 02

Text Book: ‘Fundamentals of Electric Circuits’ – Charles K Alexander and Mathew N O Sadiku. (5th Ed)

Problems: 2.18, 2.22, 2.23, 2.30, 2.32, 2.38, 2.47, 2.57, 2.75, 2.72

END LESSON 2: LAWS OF CIRCUIT ANALYSIS

- Next Lesson…….
- Topics: Methods of Circuit Analysis
- Text: B-1, Chapter 3