Slide-01

- Numerical Methods: Algorithm that are used to obtain numerical solutions of a mathematical problem.

- Why do we need numerical methods?

1. No analytical solution exists.
2. An analytical solution is difficult to obtain or not practical.

Numerical analysis:

- It is a branch of mathematics that solves continous problem using numeric approximation.

- It involves designing methods that give approximat but accurate numeric solutions, which is useful in cases where the exact solution is impossible or probbitively expensive. to ~~accurea~~ calculate.

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- Solution of Nonlinear Equations

- some simple equations can be solved analytically:

$x^2+4x+3=0$

$\cancel{\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}}$

analytic solution roots

$$=\frac{-4\pm\sqrt{4^2-4(1)(3)}}{2(1)} \qquad \boxed{\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$x=-1$, and $x=-3$

- Many others equation have no analytical solution

$$\left.\begin{array}{l} x^9-2x^2+5=0 \\ x=e^{-x} \end{array}\right\} \text{No analytical solution.}$$

- Errors

- one of the most important aspects of numerical analysis is the error analysis. Errors may occur at any stage of the process of solving a problem.

- By the error we mean the difference between the true value and the approximate value.

Error = True value − Approximate value.

Types of Errors

- **Inherent Errors:** These are the errors involved in the statement of a problem. If the data or parameters are in some way determined by physical measurement, they will probably differ from the exact values.

- **Analytic Errors:** These are the errors introduced due to transforming a physical or mathematical problem into a computational problem.

For example, If we compute sin(x) by the formula then it leads to an analytical error.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

1. If π = 22/ 7 is approximated as 3.14, find the absolute error, relative error and relative percentage error.
2. Define error and write the different types of error in numerical analysis.
3. Find the root of tan (x)+ x = 0 up to two decimal places, which lies between 2 and 2.1 using bisection method.

- Round-off errors : Roundoff error is the difference between an approximation of a number used in computation and it's exact value. Rounding multiple times can cause error to accumulate.

For example,

9.945309 is rounded to two decimal places is 9.95, then rounded again to one decimal place (10.0), the total error is

$$(10.0 - 9.945309 = 0.54691.$$

Calculation of Errors

$\boxed{1.12}$ If $\Delta x = 0.005$, $\Delta y = 0.001$ be the absolute errors in $x = 2.11$ and $y = 4.15$, find the relative error in the computation of $x+y$.

Solution:

$$x = 2.11 \quad , \quad y = 4.15$$

$$\therefore x+y = 2.11 + 4.15 = 6.26$$

and $\Delta x = 0.005$, $\Delta y = 0.001$

$$\therefore \Delta x + \Delta y = 0.005 + 0.001 = 0.006$$

The relative error in $(x+y)$ is

$$E_R = \left| \frac{\Delta x + \Delta y}{(x+y)} \right| = \frac{0.006}{6.26} = 0.000958$$

The relative error in $(x+y) = 0.001$ approximately

$\boxed{1.14}$ If $x = 2.536$, find the absolute error and relative error when –

i) x is rounded and

ii) x is truncated to two decimal digits.

(i) Here $X = 2.536$

Rounded-off value of X is $x = 2.54$

The absolute Error in x is.

$$E_A = |2.536 - 2.54|$$

$$= |-0.004| = 0.004$$

Relative error, $E_R = \frac{0.004}{2.536} = 0.0015772$

$$= 1.5772 \times 10^{-3}$$

(ii) Truncated value of X is $x = 2.53$

Absolute Error, $E_A = |2.536 - 2.53| = 0.006$

$\therefore$ Relative error $= \frac{0.006}{2.536} = 0.0023659$

$$= 2.3659 \times 10^{-3}$$

1.16 The number $x = 37.46235$ is rounded off to four significant figures, compute the absolute error, relatative error and the percentage error.

Sol:

we have, $X = 37.46235$, $x = 37.46000$

Absolute error $= |37.46236 - 37.46000|$

$\therefore E_A = 0.00235$

$$E_R = \frac{0.00235}{37.46236} = 6.27 \times 10^{-5}$$

$$E_P = E_R \times 100 = 6.27 \times 10^{-5} \times 100 = 6.27 \times 10^{-3}$$

• Accuracy: Accuracy is the degree of closeness between a measurement and it's true value. The ability of an instrument to measure the accurate value is known as accuracy.

• Precision

Precision is the degree to which repeated measurements under the same conditions show the same results. The closeness of two or more measurements to each other is known as the precision of a substance.

- Algebraic Equations

The equation $f(x) = 0$ is said to be algebraic if $f(x)$ is purely a polynomial in x.

Example: $x^3 - 7x + 8 = 0$

$$x^4 + 4x^3 + 7x^2 + 6x + 3 = 0$$

- Transcendental Equation

If $f(x)$ contains some other functions, mainely, Trigonmetric, logarithmic, Exponential etc then the equation $f(x) = 0$ is called a Transcendental Equation.

$$3 \tan 3x = 3x + 1,$$

$$x - 2 \sin x = 0$$

$$e^x = 4x$$

- Nonlinear Equation : Roots

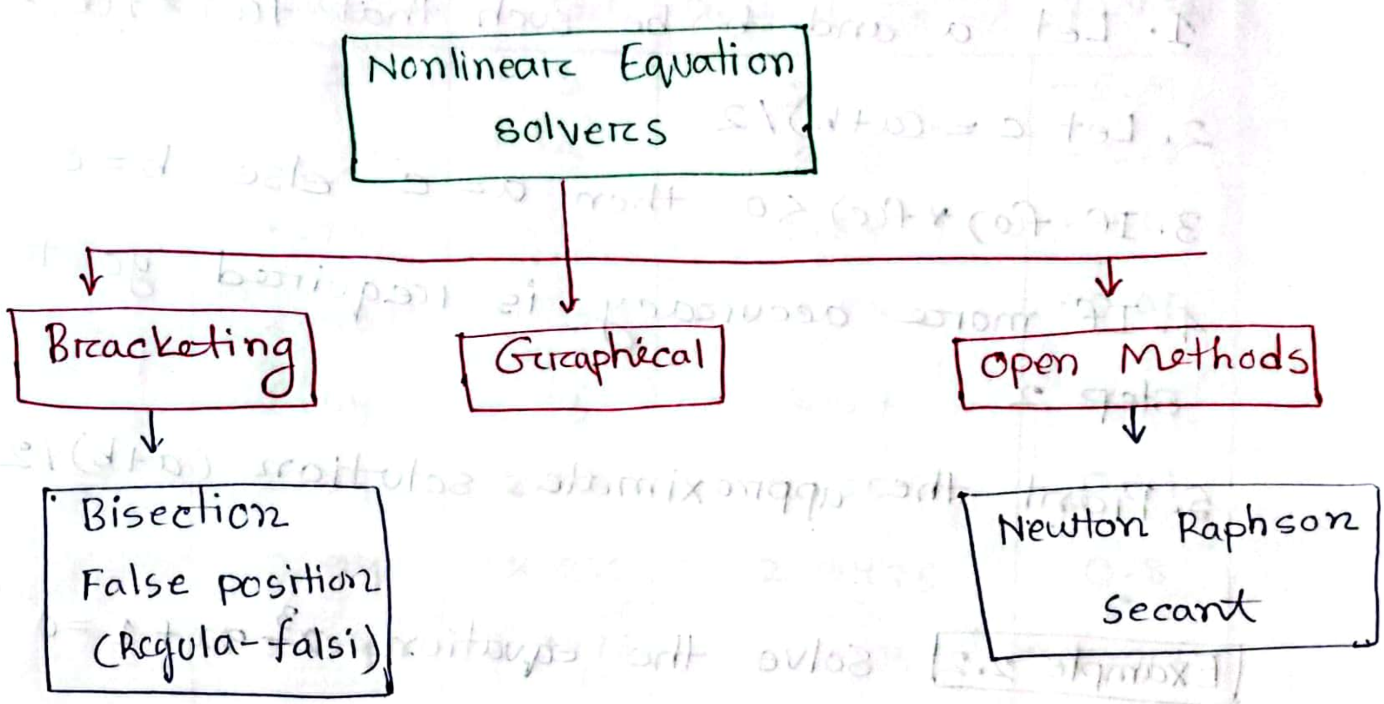
→ objective is to find a solution of

$$f(x) = 0$$

where F is a polynomial or a transcendental function, given explicitly.

→ Exact solutions are not possible for most equation.

- Nonlinear Equation Solvers



Bisection Method

- Bisection is a straight-forward technique to find numerical solutions.

- It is used to find the roots of polynomial equation.

- This method is also called a interval halving method, the binary method or the dichotomy method.

Bisection Method Algorithm

1. Let a and b be such that $f(a) * f(b) < 0$

2. Let $c = (a+b)/2$

3. If $f(a) * f(c) < 0$ then $a = c$ else $b = c$

4. If more accuracy is required go to step 2.

5. Print the approximate solution $(a+b)/2$.

Example 2.2 Solve the equation $x^3 - 9x + 1 = 0$ for the root lying between 2 and 3, correct to three significant figures.

Solution: we have,

$$f(x) = x^3 - 9x + 1$$

$$f(2) = 2^3 - 9 \times 2 + 1 = 8 - 18 + 1 = -9$$

$$f(3) = 3^3 - 9 \times 3 + 1 = 27 - 27 + 1 = 1$$

$$\therefore f(2) f(3) = (-9) \times 1 = -9 < 0$$

Let, $a_0 = 2$; $b_0 = 3$

n	a_n	b_n	$x_{n+1} = \left(\frac{a_n+b_n}{2}\right)$	$f(x_{n+1})$
0	2	3	2.5	-5.8
1	2.5	3	2.75	-2.9
2	2.75	3	2.88	-1.03
3	2.88	3	2.94	-0.05
4	2.94	3	2.97	0.47
5	2.94	2.97	2.955	0.21
6	2.94	2.955	2.9475	0.8
7	2.94	2.9475	2.9438	0.017
8	2.94	2.9438	2.9419	0.016
check 9	2.9419	2.9438	2.9428	0.003

In the 8th step a_n, b_n and x_{n+1} are equal up to three significant figures. we can take 2.94 as a root up to three significant figures.

$\therefore$ The root of $x^3 - 9x + 1 = 0$ is 2.94.

2.4 Find the root of $\tan x + x = 0$ up to two decimal places, which lies between 2 and 2.1

<u>Solution:</u> Let,

$$f(x) = \tan x + x$$

$$f(2) = \tan 2 + 2 = -0.18$$

$$f(2.1) = \tan 2.1 + 2.1 = 0.39$$

The root lies between 2.0 and 2.1

$$a_0 = 2 \;, b_0 = 2.1$$

n	a_n	b_n	$c_n = \left(\frac{a_n + b_n}{2}\right)$	$f(c_n)$
0	2	2.1	2.05	0.12
1	2	2.05	2.025	-0.023
2	2.025	2.05	2.0375	0.053
3	2.025	2.0375	2.03125	0.0152
4	2.025	2.03125	2.02812	-0.0039
5	2.02812	2.03125	2.02968	0.0056
check 6	2.02812	2.02968	2.02889	0.00081

In the 5th step a_n, b_n and c_n are equal up to two decimal places.

$\therefore x = 2.03$ is a root of $f(x) = 0$, currect up to two decimal places.

- Bracketing method vs open method

Bracketing methods:

i. These methods require an initial interval ([a,b]) where the function ~~changes~~ (f(x)) changes sign, meaning f(a) and f(b) have opposite signs.

ii. They work by narrowing down this interval step by step.

iii. Braketing methods are more reliable but slower.

open methods:

i. open methods start with one or two initial guess.

ii. They can be much faster than bracketing methods.

iii. Open methods may diverge if the initial guesses are not close to the actul root.

- Applications of numerical method in CSE

Numerical analysis play a crucial role in solving complex problems and making accurate predictions in various fields and also have numerous applications in CSE.

1. Simulation and modeling:
Numerical analysis significantly used in CSE for simulating and modeling real world system.

2. Algorithm optimization
Numerical analysis are employed to optimize algorithm for efficiency and speed.

3. Machine learning and data analysis
Numerical methods are fundamental for machine learning and data analysis.

4. Numerical solutions of differential equations:
Many engineering problems involve differential equations that can not be solved analytically. But it can be solved by numerical methods.

5. optimization problems

Numerical optimization methods are applied to solve complex engineering problems such as resource allocation, network optimization and logistics.

- Slide 4

- Bisection method vs Regula falsi method

Bisection Method	Regula Falsi method
The bisection method is a root finding method that applies to continuous function.	The false position method is a very old method to solve equation.
It is simple to use and easy to implement	Simple to use compared to Bisection method.
The bisection method is slow and linear	This method is faster than the bisection method
Formula is: $x_3 = (x_1+x_2)/2$	Formula is: $x_3 = \dfrac{x_1(fx_2) - x_2(fx_1)}{f(x_2) - f(x_1)}$
It is also known as the Bolzano method.	It is also known as the false position method.

• False position method

2.25. Find an approximate value of the root of the equation x^3+x-1 near $x=1$, by the method of falsi using the formula twice.

Sol: Here,

$$f(x) = x^3+x-1$$

$$f(0.5) = 0.5^3+0.5-1 = -0.375$$

$$f(x)^1 = 1$$

Hence the root lies between 0.5 and 1

Let, $a = 0.5$

$b = 1$

$$x_0 = \frac{a.f(b) - b.f(a)}{f(b) - f(a)}$$

$$= \frac{0.5(1) - 1.(-0.375)}{1 - (-0.375)}$$

$$= 0.64$$

$$f(x_0) = f(0.64) = -0.0978 < 0$$

Now, the root lies between 0.64 and 1

applying the formula again,

$$x_1 = \frac{0.64 \times 1 - 1 \times (-0.0979)}{1 - (-0.0979)}$$

$$= 0.672$$

2.27. Solve the equation $x \tan x = -1$ by regula falsi method, starting with 2.5 and 3.0 as the initial approximations to the root.

<u>Sol:</u> We have,

$$f(x) = x \tan x + 1$$

$$f(2.5) = 2.5 \tan 2.5 + 1 = -0.8675$$

$$f(3) = 3 \tan 3 + 1 = 0.5724$$

Let, $a = 2.5$, $b = 3$

First approximation,

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2.5 \times (0.5724) - 3 \times (-0.8675)}{0.5724 - (-0.8675)}$$

$$= 2.8012$$

$\therefore f(x_1) = f(2.8012) = 0.00787$

$f(a) \cdot f(x_1) < 0$

therefore, the root lies between 2.5 and 2.8012

Second approximation,

$$x_2 = \frac{a \cdot f(x_1) - x_1 f(a)}{f(x_1) - f(a)}$$

$$= \frac{2.5 \times (0.00787) - \overset{2.8012}{\cancel{(0.00787)}} \times (-0.86}{0.00787 - (-0.8675)}$$

$$= 2.7984$$

$\therefore f(a) \cdot f(x_1) < 0$

The root lies between 2.5 and 2.7984

Third approximation,

$$x_3 = \frac{a f(x_2) - x_2 \cdot f(a)}{f(x_2) - f(a)}$$

$$= \frac{2.5 \times 0.000039 - 2.7984 \times (-0.8675)}{0.000039 - (-0.8675)}$$

$$= 2.7982$$

$\therefore$ The required root is 2.798.

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- **Advantages of secant method**

1. Faster convergence: The secant method converges more quickly than the bisection method.

2. No derivative required: unlike newton's method, which relies on evaluating the derivative of the function, the secant method does not require the functions derivative.

- **Secant method Math**

Example 1:
Compute the root of the equation $xe^{-\frac{x}{2}} = 1$ in the interval $[0, 2]$ using the secant method. The root should be correct to three decimal places.

<u>Sol:</u> Let, $\therefore f(x) = xe^{-\frac{x}{2}} - 1$

$x_0 = 1.42 \quad \therefore f(x_0) = -0.0086$

$x_1 = 1.43 \quad \therefore f(x_1) = 0.00034$

apply secant method,

First approximation,

$$x_2 = x_1 - \frac{x_0 - x_1}{f(x_0) - f(x_1)} \times f(x_1)$$

$$= 1.43 - \frac{1.42 - 1.43}{(-0.0086) - 0.00034} \times 0.00034$$

$$= 1.4296$$

$\therefore f(x_2) = -0.000011$

The second approximation,

$$x_3 = x_2 - \frac{x_1 - x_2}{f(x_1) - f(x_2)} \times f(x_2)$$

$$= 1.4296 - \frac{1.43 - 1.4296}{0.00034 - (-0.000011)} \times (-0.000011)$$

$$= 1.4292.$$

Since x_2, x_3 matching up to three decimal places, the required root is 1.429.

- Slide 5

- Newton Raphson's method

- Find the root of the equation $f(x) = x^3 - 3 = 0$, if the initial value is 2.

Sol: Given,

$$x_0 = 2 \; ; \; f(x) = x^3 - 3$$

$$f'(x) = 3x^2$$

$$f'(x_0) = 3 \times 2^2 = 3 \times 4 = 12$$

$$f(x_0) = 8 - 3 = 5$$

using newton raphson method-

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{5}{12}$$

$$\therefore x_1 = 1.583$$

$$f(x_1) = 1.583^3 - 3 = 0.967$$
$$f'(x_1) = 3\times(1.583)^2 = 7.52$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.583 - \frac{0.967}{7.52} = 1.4544$$

$$\therefore f(x_2) = 1.4544^3 - 3 = 0.0765$$
$$f'(x_2) = 3\times 1.4544^2 = 6.346$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.4544 - \frac{0.0765}{6.346}$$
$$= 1.4423$$

Therefore, thei root of the equation is approximately $x = 1.442$

- Find the root of the equation $\sin x = 1 + x^3$, between -2 and -1 correct to 3 decimal place by newton raphson method.

Sol. Let, $f(x) = x^3 - \sin x + 1$.

$f'(x) = 3x^2 - \cos x$

$\therefore f(-1) = (-1)^3 - \sin(-1) + 1 = 0.8415$.

$f(-2) = (-2)^3 - \sin(-2) + 1 = -6.0907$.

$\therefore f(-1) f(-2) < 0$

$f(x) = 0$ has a root between -2 and -1.

$x_0 = -1$, be the initial approximation

First approximation,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1 - \frac{0.8415}{2.4597}$$

$$= -1.34211$$

Second approximation,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \cancel{}$$

$$= -1.34211 - \frac{-0.4438}{5.177}$$

$$= -1.2564$$

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$\therefore$ The root is -1.249 Ans

* Fixed point iteration method

- Find the first approximate root of the equation $2x^3 - 2x - 5 = 0$ up to 4 decimal places.

Sol:

Given, $2x^3 - 2x - 5 = 0$

To find the value of x_0, we have to choose a and b such that $f(a) <$ and $f(b) > 0$.

Let,
$$f(x) = 2x^3 - 2x - 5$$
$$f(1) = 2\cdot1^3 - 2\cdot1 - 5 = -5$$
$$f(2) = 2\cdot2^3 - 2\cdot2 - 5 = 7$$

Thus $a = 1$ and $b = 2$

Therefore, $x_0 = \frac{a+2}{2} = \frac{1+2}{2} = 1.5$

$$\therefore f(x) = 2x^3 - 2x - 5 = 0$$
$$\Rightarrow 2x^3 = 2x + 5$$
$$\Rightarrow x^3 = \frac{2x+5}{2}$$
$$\therefore g(x) = \left(\frac{2x+5}{2}\right)^{\frac{1}{3}}$$

Now applying iternative method-

$$x_n = g(x_{n-1}) \text{ for } n = 1, 2, 3, 4, 5, 6 \ldots$$

for $n=1$, $x_1 = g(x_0) = \left(\frac{2 \times 1.5 + 5}{2}\right)^{\frac{1}{3}} = 1.5874$

for $n=2$, $x_2 = g(x_1) = \left(\frac{2 \times 1.587 + 5}{2}\right)^{\frac{1}{3}} = 1.59891$

for $n=3$, $x_3 = g(x_2) = \left(\frac{2 \times 1.5989 + 5}{2}\right)^{\frac{1}{3}} = 1.60037$

for $n=4$, $x_4 = g(x_3) = \left(\frac{2 \times 1.60037 + 5}{2}\right)^{\frac{1}{3}} = 1.60057$

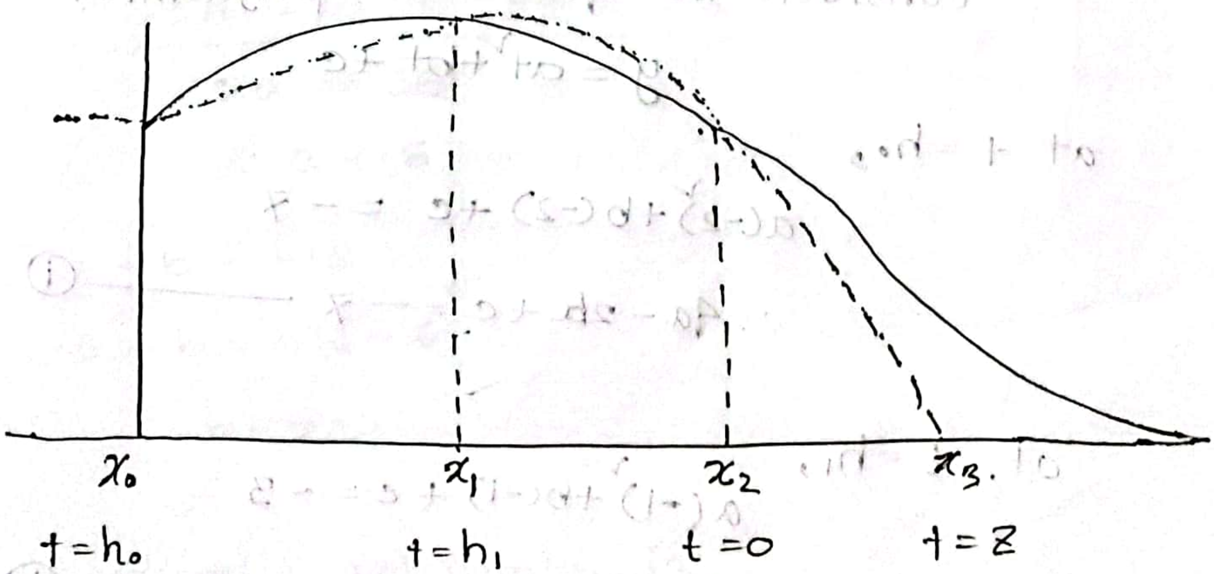
for $n=5$, $x_5 = g(x_4) = \left(\frac{2 \times 1.60057 + 5}{2}\right)^{\frac{1}{3}} = 1.60059$

The approximate root of $2x^3 - 2x - 5 = 0$ by the fixed point iteration method is 1.60006 .

- Slide-6
- Muller's method (theory)

Muller's method is an iterative method. It requires three starting points. $(x_0, f(x_0))$, $(x_1, f(x_1))$, and $(x_2, f(x_2))$ A parabola is constructed that passes through these points then the quadratic formula is used to find ~~a~~ root of the quadratic for the next approximation. consider the quadratic polynomial involving it -

$$y = at^2 + bt + c$$



The quadratic formula is used to find the roots $t = z_1, z_2$

$$z = \frac{-2c}{b \pm (b^2 - 4ac)^{\frac{1}{2}}}$$

• Find the root of the equation $f(x) = x^3 - 3x - 5 = 0$ which lies between 2 and 3 by using muller's method.

Sol: Let, $x_0 = 1$, $x_1 = 2$, $x_2 = 3$

$$f(x_0) = 1^3 - 3 \cdot 1 - 5 = 1 - 3 - 5 = -7$$

$$f(x_1) = 2^3 - 3 \cdot 2 - 5 = 8 - 6 - 5 = -3$$

$$f(x_2) = 3^3 - 3 \cdot 3 - 5 = 27 - 9 - 5 = 13$$

and,

$$h_0 = x_0 - x_2 = 1 - 3 = -2$$

$$h_1 = x_1 - x_2 = 2 - 3 = -1$$

consider the quadratic polynomial

$$y = at^2 + bt + c$$

at $t = h_0$,

$$a(-2)^2 + b(-2) + c = -7$$

$$\therefore 4a - 2b + c = -7 \quad \text{———— (i)}$$

at $t = h_1$,

$$a(-1)^2 + b(-1) + c = -3$$

$$\therefore a - b + c = -3 \quad \text{———— (ii)}$$

at $t = 0$,

$$a(0)^2 + b \cdot 0 + c = 13$$

$$\therefore c = 13 \quad \text{———— (iii)}$$

from i, ii, iii we get,

$$4a - 2b + c = -7$$

$$\Rightarrow 4a - 2b + 13 = -7$$

$$\Rightarrow \therefore 4a - 2b = -7 - 13$$

$$\therefore 4a - 2b = -20 \quad \text{—— (iv)}$$

$$a - b + c = -3$$

$$\Rightarrow a - b + 13 = -3$$

$$\therefore a - b = -16 \quad \text{—— (v)}$$

from (iv) and ((v)×2) we get

$$\begin{array}{r} 4a - 2b = -20 \\ 2a - 2b = -32 \\ \hline 2a = 12 \end{array}$$

$$\therefore a = 6$$

$$\therefore a - b = -16$$

$$\Rightarrow -b = -16 - 6$$

$$\therefore b = 22$$

the quadratic polynomial is,

$$y = at^2 + bt + c = 6t^2 + 22t + 13$$

$$\therefore z = \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$= \frac{-2 \times 13}{22 \pm \sqrt{22^2 - 4 \times 6 \times 13}}$$

$$= \frac{-26}{22 \pm 13.1148}$$

Since $b>0$, we use postive sign for the square root and obtain

$$z = \frac{-26}{35.1148} = -0.7404$$

$x_3 = x_2 + z = 3 - 0.7404 = 2.2596$ is the next approximation.

2.2596 is the required root of the given equation.

- slide 8

- Newton's forward interpolation

Given that,

$$\sqrt{12500} = 111.8034 , \sqrt{12510} = 111.8481$$

$$\sqrt{12520} = 111.8928 , \sqrt{12530} = 111.9375$$

Find the value of $\sqrt{12516}$.

<u>Sol:</u> The difference table is –

x	$y = \sqrt{x}$	Δy	$\Delta^2 y$
12500 x_0	111.8034 y_0		
		0.0447 Δy_0	
12510	111.8481		0 $\Delta^2 y_0$
		0.0447	
12520	111.8922		0
		0.0447	
12530	111.9375		

we have $x_0 = 12500$, $h = 10$ and $x = 12516$

$$u = \frac{x - x_0}{h} = \frac{12516 - 12510}{10} = 1.6$$

from newton's forward interpolation formula.

$$f(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots$$

$$\Rightarrow f(12516) = 111.8034 + 1.6 \times 0.0447 + 0 + 0$$

$$\therefore \sqrt{12516} = 111.87492$$

• Evaluate $y = e^{2x}$ for $x = 0.05$ using the following table.

x	0.00	0.10	0.20	0.30	0.40
$y = e^{2x}$	1.000	1.2214	1.4918	1.8221	2.255

<u>Sol:</u> The difference table is

x	$y=e^{2x}$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.00	1.000				
		0.2214			
			0.0490		
0.10	1.2214			0.0109	
		0.2704			0.0023
			0.0599		
0.20	1.4918			0.0132	
		0.3303			
0.30	1.8221		0.0731		
		0.4034			
0.40	2.255				

We have, $x_0 = 0.00$, $x = 0.05$, $h = 0.1$

$$\therefore u = \frac{x - x_0}{h} = \frac{0.05 - 0.00}{0.1} = 0.5$$

using newton's forward formula,

$$f(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$\Rightarrow f(0.05) = 1.000 + 0.1107 - 0.006125 + 0.000681 - 0.000090 = 1.105166$$

$$\therefore f(0.05) \approx 1.052$$

- Newtons backward Interpolation.

- calculate the value of $f(7.5)$ for the table.

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

<u>Sol:</u> 7.5 is near to the end of the table, we use newtons backword formula to find $f(7.5)$.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1	1					
		7				
2	8		12			
		19		6		
3	27		18		0	
		37		6		0
4	64		24		0	
		61		6		0
5	125		30		0	
		91		6		0
6	216		36			
		127			6	
7	343		42	6		
		169				
8	512					

we have,

$$x_n = 8,\ x = 7.5,\ h = 1$$

$$u = \frac{x - x_n}{h} = \frac{7.5-8}{1} = -0.5$$

$\therefore$ we get,

$$f(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n +$$

$$\Rightarrow f(7.5) = 512 + (-0.5)(165) + \frac{(-0.5)(-0.5+1)}{2}(42) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{6} 6 + \cdots$$

$$= 512 - 84.5 - 5.25 - 0.375$$

$$= 421.87$$

- slide - 9

- Use Newton divided difference formula and evaluate $f(6)$ given,

x	5	7	11	13	21
$f(x)$	150	392	1452	2316	9702

Sol:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
5	150				
		121			
7	392		24		
		265		1	
11	1452		32		0
		457		1	
13	2316		46		
		917			
21	9702				

Note: $f(x, x_0) = \frac{f(x) - f(x_0)}{x - x_0} = \frac{392 - 150}{7 - 5} = 121$

we have,

$$f(x_0) = 150,\ f(x_0,x_1) = 121,\ f(x_0,x_1,x_2) = 24,\ f(x_0,x_1,x_2,x_3) = 1$$

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0,x_1,x_2,x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4) f(x_0,x_1,x_2,x_3) + \cdots$$

$$f(6) = 150 + (6-5)(121) + (6-5)(6-7)(24) + (6-5)(6-7)(6-11) \cdot 1 + 0 + \cdots$$

$$f(6) = 150 + 121 - 24 + 5$$

$$\therefore f(x) = 252$$

- Find the form of the function $f(x)$ under suitable assumption from the following data.

x	0	1	2	5
$f(x)$	2	3	12	147

Sol: The divided difference table is given as under:

x	$f(x)$	Δ	Δ^2	Δ^3
0	2			
		1		
1	3		4	1
		9		
2	12		9	
		45		
5	147			

we have,

$$x_0 = 0, f(x_0) = 2, f(x_0, x_1) = 1, f(x_0, x_1, x_2) = 4,$$

$$f(x_0, x_1, x_2, x_3) = 1$$

The Newton's divided difference intepolation formula is

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) +$$
$$(x - x_0)(x - x_1)(x - x_2) \cdot f(x_0, x_1, x_2, x_3)$$

Substituting we get,

$$f(x) = 2 + (x-0)1 + (x-0)(x-1)4 + (x-0)(x-1)(x-2)1$$

$$\therefore f(x) = x^3 + x^2 \pm x + 2$$