Determinants

⇒ Define determinants ?

A determinant of order n is a square array of n^2 quantities a_{ij} $(i, j = 1, 2, ---, n)$ enclosed between two vertical bars and is generally written in the form given below:

$$D = \begin{vmatrix} a_{11} & a_{12} & ---- & a_{1n} \\ a_{21} & a_{22} & --- & a_{2n} \\ - & - & - & - \\ a_{n1} & a_{n2} & --- & a_{nn} \end{vmatrix}$$

which is an ordinary number.

The n^2 quantities a_{ij} $(i, j = 1, 2, ---, n)$ which are numbers (or sometimes functions.) are called the elements of the determinant.

⇒ Sarrus diagrams for determinants of order 2

Determinant: $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

Sarrus diagram:
$$\begin{tikzcd} a_1 \arrow[dr] & b_1 \\ a_2 \arrow[ur] & b_2 \end{tikzcd}$$

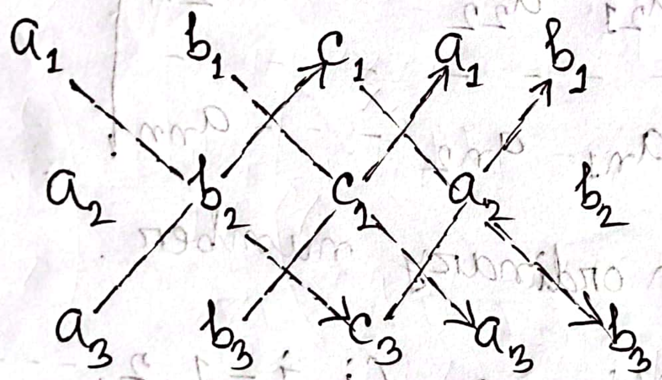
Value: $a_1 b_2 - a_2 b_1$

⇒ Sarrus diagram for determinant of order 3

Determinant:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Sarrus diagram:



Value:

$$a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3 - a_3 b_2 c_1 - b_3 c_2 a_1 - c_3$$

$$= a_1 (b_2 c_3 - b_3 c_2) - b_1 (a_2 c_3 - a_3 c_2) + c_1 (a_2 b_3 -$$

⇒ **Example:**

Find the values of the following determinants by using the Sarrus diagram.

(i) $\begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \\ 3 & 2 & 7 \end{vmatrix}$

Solⁿ:

(i) $\begin{matrix} 1 & 3 \\ 5 & 7 \end{matrix}$ $= 1\times 7 - 5\times 3 = 7 - 15 = -8$

$\therefore \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix} = -8$

(ii) $\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 0 & 4 & 5 \\ 3 & 2 & 7 & 3 & 2 \end{matrix}$

$= (1\cdot5\cdot7) + (2\cdot0\cdot3) + (3\cdot4\cdot2) - (3\cdot5\cdot3) - (2\cdot0\cdot2) - (7\cdot4\cdot2)$

$= 35 + 0 + 24 - 45 - 0 - 56$

$= 59 - 101 = -42$

$\therefore \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \\ 3 & 2 & 7 \end{vmatrix} = -42$ Ans

H.W. Find the values of the following determinant using Sarrus diagram

$$\begin{vmatrix} 5 & 10 & -15 \\ 3 & 2 & 0 \\ 2 & 4 & 6 \end{vmatrix}$$

⇒ Minors and Co-factors:

Let us consider a determinant

$$D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

If we delete the i-th row and j-th column and then form a determinant from all the elements remaining, we shall have a new determinant of $(n-1)$ rows and $(n-1)$ columns, which is called the minor of the element a_{ij} and is denoted by M_{ij}

The minor M_{ij} multiplied by $(-1)^{i+j}$ is called the co-factor of a_{ij} in D and will be denoted by A_{ij}

$$\therefore A_{ij} = (-1)^{i+j} M_{ij}$$

⇒ Example:

Find the minors and co-factors of

$$\begin{vmatrix} 5 & 9 & 0 \\ 1 & 3 & -2 \\ 4 & 8 & 12 \end{vmatrix}$$

Solⁿ:

Minor of the element $5 = \begin{vmatrix} 3 & -2 \\ 8 & 12 \end{vmatrix} = M_{11}$

" " " " $9 = \begin{vmatrix} 1 & -2 \\ 4 & 12 \end{vmatrix} = M_{12}$

" " " " $0 = \begin{vmatrix} 1 & 3 \\ 4 & 8 \end{vmatrix} = M_{13}$.

" " " " $8 = \begin{vmatrix} 5 & 0 \\ 1 & -2 \end{vmatrix} = M_{32}$

" " " " $12 = \begin{vmatrix} 5 & 9 \\ 1 & 3 \end{vmatrix} = M_{33}$

and

$\therefore$ Co-factors of 5, $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 3 & -2 \\ 8 & 12 \end{vmatrix}$

" " " 9, $A_{12} = (-1)^{1+2} M_{12} = -M_{12}$

" " " 0, $A_{13} = (-1)^{1+3} M_{13} = M_{13}$

" " " 8, $A_{32} = (-1)^{3+2} M_{32} = -M_{32}$

" " " 12, $A_{33} = (-1)^{3+3} M_{33} = M_{33}$

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Fundamental properties of determinants:

(i) If all the elements in a row (or in a column) of a determinant are zero, the determinant is equal to zero.

ii) If two rows (or two columns) of a determinant are identical, the determinant is equal to zero.

(iii) The determinant of any two rows (or any two columns) of a determinant changes the sign of the determinant.

(iv) The value of the determinant is not changed when the rows are changed to columns and the columns to rows.

(v) If every element in a row or in a column is multiplied by any scalar k, the determinant is multiplied by that scalar k.

⇒ **Example - 01 :**

Evaluate $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-1 & 1 \\ a^2-b^2 & b^2-c^2 & c^2 \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix} \quad [C_1' = C_1 - C_2 \,;\, C_2' = C_2 - C_3]$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a^2-b^2 & b^2-c^2 & c^2 \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ (a+b)(a-b) & (b+c)(b-c) & c^2 \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) & c^3 \end{vmatrix}$$

$$= \cancel{(a-b)\,(b-c)} \qquad = 1 \cdot \begin{vmatrix} (a+b)(a-b) & (b+c)(b-c) \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) \end{vmatrix}$$

$$= (a-b)\,(b-c) \begin{vmatrix} a+b & b+c \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix}$$

$$=(a-b)(b-c)\{(a+b)(b^2+bc+c^2)-(b+c)(a^2+ab+b^2)\}$$

$$=(a-b)(b-c)\{ab^2+abc+ac^2+b^3+b^2c+bc^2-(a^2b+ab^2+b^3+a^2c+abc+b^2c)\}$$

$$=(a-b)(b-c)\{\cancel{ab^2}+\cancel{abc}+ac^2+\cancel{b^3}+\cancel{b^2c}+bc^2-a^2b-\cancel{ab^2}-\cancel{b^3}-a^2c-\cancel{abc}-\cancel{b^2c}\}$$

$$=(a-b)(b-c)(ac^2-a^2c+bc^2-a^2b)$$

$$=(a-b)(b-c)\{ac(c-a)+b(c^2-a^2)\}$$

$$=(a-b)(b-c)\{ac(c-a)+b(c+a)(c-a)\}$$

$$=(a-b)(b-c)(c-a)\{ac+b(c+a)\}$$

$$=(a-b)(b-c)(c-a)(ac+bc+ab)$$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

Ans

⇒ **Example-02:**

Prove that
$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Solⁿ:

$$L.H.S. = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & bc & c(a+c) \\ a(a+b) & b^2 & ac \\ ab & b(b+c) & c^2 \end{vmatrix}$$

$$= abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$= abc \begin{vmatrix} a-(a+b)-b & c-b-(b+c) & (a+c)-a-c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}; \quad r_1' = r_1 - r_2 - r_3$$

$$= abc \begin{vmatrix} \cancel{a}-\cancel{a}-b-b & \cancel{c}-b-b-\cancel{c} & \cancel{a}+\cancel{c}-\cancel{a}-\cancel{c} \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$=abc\begin{vmatrix} -2b & -2b & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$=abc\cdot(-2b)\begin{vmatrix} 1 & 1 & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$=-2ab^2c\begin{vmatrix} 1-1 & 1 & 0 \\ a+\cancel{b}-\cancel{b} & b & a \\ b-(b+c) & b+c & c \end{vmatrix} \quad [\,;\, c_1' = c_1 - c_2$$

$$=-2ab^2c\begin{vmatrix} 0 & 1 & 0 \\ a & b & a \\ -c & b+c & c \end{vmatrix}$$

$$=\cancel{-2ab^2c\cdot 1}\begin{vmatrix} a & a \\ -c & c \end{vmatrix}$$

$$=-2ab^2c\left\{0\times\begin{vmatrix} b & a \\ b+c & c \end{vmatrix} - 1\times\begin{vmatrix} a & a \\ -c & c \end{vmatrix} + 0\times\begin{vmatrix} a & b \\ -c & b+c \end{vmatrix}\right\}$$

$$=-2ab^2c\left\{0-\begin{vmatrix} a & a \\ -c & c \end{vmatrix}+0\right\}$$

$$=-2ab^2c\{-(ac+ac)\}$$

$$=-2ab^2c\times -2ac$$

$$=4a^2b^2c^2$$

$=R.H.S.$ (proved)

City University

Additional Script

Main Script Serial No: ______________________ Serial No: 231- 24734

Student Name : ______________________ ID No: ______________________

Signature of the Invigilator : ______________________

Example - 03:

Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

Sol^n:

$$L.H.S. = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (c+a)^2-b^2 & b^2-b^2 \\ c^2 & c^2-c^2 & (a+b)^2-c^2 \end{vmatrix} \quad \begin{array}{l} C_2' = C_2 - C_1 \text{ and} \\ C_3' = C_3 - C_1 \end{array}$$

$$= \begin{vmatrix} (b+c)^2 & (a+b+c)(a-b-c) & (a+b+c)(a-b-c) \\ b^2 & (c+a+b)(c+a-b) & 0 \\ c^2 & 0 & (a+b+c)(a+b-c) \end{vmatrix}$$

$$=(a+b+c)\cdot(a+b+c)\begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$=(a+b+c)^2\begin{vmatrix} b^2+2bc+c^2 & a-b-c & a-b-c \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$=(a+b+c)^2\begin{vmatrix} \cancel{b^2}+2bc+\cancel{c^2}-\cancel{b^2}-\cancel{c^2} & a-b-c-(c+a-b)-0 & a-b-c-0-(a+b- \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$R_1' = R_1 - R_2 - R_3$$

$$=(a+b+c)^2\begin{vmatrix} 2bc & \cancel{a}-\cancel{b}-c-c-\cancel{a}+\cancel{b} & \cancel{a}-b-\cancel{c}-\cancel{a}-b+\cancel{c} \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$=(a+b+c)^2\begin{vmatrix} 2bc & -2c & -2b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$= (a+b+c)^2 \cdot 2 \begin{vmatrix} bc & -c & -b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$= 2(a+b+c)^2 \cdot \frac{1}{bc} \begin{vmatrix} bc & -bc & -bc \\ b^2 & b(c+a-b) & 0 \cdot c \\ c^2 & b \cdot 0 & c(a+b-c) \end{vmatrix}$$

$$= \frac{2}{bc} (a+b+c)^2 \begin{vmatrix} bc & -bc & -bc \\ b^2 & bc+ab-b^2 & 0 \\ c^2 & 0 & ac+bc-c^2 \end{vmatrix}$$

$$= \frac{2(a+b+c)^2}{bc} \begin{vmatrix} bc & \cancel{-bc}+\cancel{bc} & \cancel{-bc}+\cancel{bc} \\ b^2 & bc+ab\cancel{-b^2}+\cancel{b^2} & 0+b^2 \\ c^2 & 0+c^2 & ac+bc\cancel{-c^2}+\cancel{c^2} \end{vmatrix} \begin{array}{l} C_2' = C_2 + C_1 \\ C_3' = C_3 + C_1 \end{array}$$

$$= \frac{2(a+b+c)^2}{bc} \begin{vmatrix} bc & 0 & 0 \\ b^2 & bc+ab & b^2 \\ c^2 & c^2 & ac+bc \end{vmatrix}$$

$$= \frac{2(a+b+c)^2}{bc} \begin{vmatrix} bc & 0 & 0 \\ b^2 & b(c+a) & b^2 \\ c^2 & c^2 & c(a+b) \end{vmatrix}$$

$$= \frac{2(a+b+c)^2}{bc} \cdot bc \begin{vmatrix} bc & 0 & 0 \\ b^2 & c+a & \\ c^2 & c & \end{vmatrix}$$

$$= \frac{2(a+b+c)^2}{\cancel{bc}} \cdot \cancel{bc} \begin{vmatrix} b(c+a) & b^2 \\ c^2 & c(a+b) \end{vmatrix}$$

$$= 2(a+b+c)^2 \, b \cdot c \cdot \begin{vmatrix} c+a & b \\ c & a+b \end{vmatrix}$$

$$= 2bc(a+b+c)^2 \left\{(c+a)(a+b) - bc\right\}$$

$$= 2bc(a+b+c)^2 \left\{ca + \cancel{cb} + a^2 + ab - \cancel{bc}\right\}$$

$$= 2bc(a+b+c)^2 \cdot a(c+a+b)$$

$$= 2abc(a+b+c)^3$$

$$= R \cdot H \cdot S.$$

$$=(a+b+c)^2\begin{vmatrix} 2b & 2(a-b) & 2ab \\ a-b-c & b+c-a & a^2 \\ 0 & b-c-a & (c-a)^2 \end{vmatrix}$$

$$=2(a+b+c)^2\begin{vmatrix} b & a-b & ab \\ a-b-c & b+c-a & a^2 \\ 0 & b-c-a & (c-a)^2 \end{vmatrix}$$

$$=2(a+b+c)^2$$

⇒ Example – 03:

Prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}=(1+a^2+b^2)^3$$

Sol^n:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}=\begin{vmatrix} 1+a^2-b^2 & & \\ & & \\ & & \end{vmatrix}$$

⟹ Example-08:

Show that

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

Solⁿ:

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \frac{abc}{abc}\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} abc & a^2 & a^3 \\ bca & b^2 & b^3 \\ cab & c^2 & c^3 \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \quad \text{——— (1)}$$

$$= \begin{vmatrix} 1-1 & a^2-b^2 & a^3-b^3 \\ 1-1 & b^2-c^2 & b^3-c^3 \\ 1 & c^2 & c^3 \end{vmatrix} \quad r_1' = r_1 - r_2 ;\ r_2' = r_2 - r_3$$

$$= \begin{vmatrix} 0 & (a-b)(a+b) & (a-b)(a^2+ab+b^2) \\ 0 & (b+c)(b-c) & (b-c)(b^2+bc+c^2) \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & a+b & a^2+ab+b^2 \\ 0 & b+c & b^2+bc+c^2 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= (a-b)(b-c) \left\{ 0 \times \begin{vmatrix} b+c & b^2+bc+c^2 \\ c^2 & c^3 \end{vmatrix} - 0 \times \begin{vmatrix} a+b & a^2+ab+b^2 \\ c^2 & c^3 \end{vmatrix} + 1 \times \begin{vmatrix} a+b & a^2+ab+b^2 \\ b+c & b^2+bc+c^2 \end{vmatrix} \right\}$$

$$= (a-b)(b-c)\{0 - 0 + (a+b)(b^2+bc+c^2) - (b+c)(a^2+ab+b^2)\}$$

$$= (a-b)(b-c)\{\cancel{ab^2} + \cancel{abc} + ac^2 + \cancel{b^3} + \cancel{b^2c} + bc^2 - a^2b - \cancel{ab^2} - \cancel{b^3} - a^2c - \cancel{abc} - \cancel{b^2c}\}$$

$$= (a-b)(b-c)(ac^2 - a^2c + bc^2 - a^2b)$$

$$= (a-b)(b-c)\{ac(c-a) + b(c^2-a^2)\}$$

$$= (a-b)(b-c)\{ac(c-a) + b(c+a)(c-a)\}$$

$$= (a-b)(b-c)(c-a)\{ac+b(c+a)\}$$

$$= (a-b)(b-c)(c-a)(ab+bc+ca) \quad \text{--- (2)}$$

$$\therefore \begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+$$

(Proved)

H.W.

1. Show that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

2. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b-$

$\Rightarrow$ Application of determinants to linear equations (Cramer rule):

Consider the system of n- linear equations in n- unknowns $x_1, x_2, ---, x_n$ be

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + --- + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + --- + a_{2n}x_n = b_2 \\ ------------------ \\ a_{n1}x_1 + a_{n2}x_2 + --- + a_{nn}x_n = b_n \end{array}\right\} \longleftarrow (1)$$

such that $D = \begin{vmatrix} a_{11} & a_{12} & ---- & a_{1n} \\ a_{21} & a_{22} & ---- & a_{2n} \\ - & - & - & - \\ a_{n1} & a_{n2} & ---- & a_{nn} \end{vmatrix} \neq 0$

Then the system (1) has a unique solution

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}, \quad ----, \quad x_n = \frac{D_n}{D}$$ where

$D_1 = \begin{vmatrix} b_1 & a_{12} & --- & a_{1n} \\ b_2 & a_{22} & --- & a_{2n} \\ - & - & - & - \\ b_n & a_{n2} & --- & a_{nn} \end{vmatrix}$, $D_2 = \begin{vmatrix} a_{11} & b_1 & --- & a_{1n} \\ a_{21} & b_2 & --- & a_{2n} \\ - & - & - & - \\ a_{n1} & b_n & --- & a_{nn} \end{vmatrix}$, ..., $D_n = \begin{vmatrix} a_{11} & a_{12} & --- & b_1 \\ a_{21} & a_{22} & --- & b_2 \\ - & - & - & - \\ a_{n1} & a_{n2} & --- & b_n \end{vmatrix}$,

⇒ Example-04:

Apply Cramer's rule to solve the following system of linear equations:

$$3x+y+2z=3$$
$$2x-3y-z=-3$$
$$x+2y+z=4$$

Soln:

Here, $D=\begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix}$

$$=3\cdot\begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix}-1\cdot\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}+2\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$$

$$=3\{(-3)\times1-2(-1)\}-1\{2\times1-1\times(-1)\}+2\{2\times2-1\times(-3)\}$$

$$=3(-3+2)-1(2+1)+2(4+3)$$

$$=-3-3+14$$

$$=8\neq0$$

So the given system exists solution.

$$\cancel{\because D_1 = \begin{vmatrix} 11 \\ 10 \\ 9 \end{vmatrix}} \quad \therefore D_1 = \begin{vmatrix} 3 & 1 & 2 \\ -3 & -3 & -2 \\ 4 & 2 & 1 \end{vmatrix}$$

$$= 3\cdot\begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} - 1\cdot\begin{vmatrix} -3 & -1 \\ 4 & 1 \end{vmatrix} + 2\cdot\begin{vmatrix} -3 & -3 \\ 4 & 2 \end{vmatrix}$$

$$= 3\{(-3)\times 1 - 2\times(-1)\} - 1\{(-3)\times 1 - 4\times(-1)\} + 2\{(-3)\times 2 - 4\times(-3)\}$$

$$= 3(-3+2) - 1(-3+4) + 2(-6+12)$$

$$= -3 - 1 + 12$$

$$= 8$$

$$D_2 = \begin{vmatrix} 3 & 3 & 2 \\ 2 & -3 & -1 \\ 1 & 4 & 1 \end{vmatrix} = 3\begin{vmatrix} -3 & -1 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$$

$$= 3(-3+4) - 3(2+1) + 2(8+3)$$

$$= 3 - 9 + 22$$

$$= 16$$

and $$D_3 = \begin{vmatrix} 3 & 1 & 3 \\ 2 & -3 & -3 \\ 1 & 2 & 4 \end{vmatrix} = 3\begin{vmatrix} -3 & -3 \\ 2 & 4 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$$

$$= 3(-12+6) - 1(8+3) + 3(4+3)$$

$= -18 - 11 + 21$

$= -8$

$\therefore x = \frac{D_1}{D} = \frac{8}{8} = 1$, $y = \frac{D_2}{D} = \frac{16}{8} = 2$ and $z = \frac{D_3}{D} = \frac{-8}{8} = -1$

$\therefore x = 1$; $y = 2$ and $z = -1$ are the solⁿ of the given system.

⇒ Example-05:

Solve the following linear equations with the help of determinant

$\cancel{x - 2y + 3z = 11}$

$2x - y + 2z = 2$

$x + 10y - 3z = 5$

$-x + y + z = -3$

Solⁿ:

$$\text{Here, } D = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 10 & -3 \\ -1 & 1 & 1 \end{vmatrix} = 2\begin{vmatrix} 10 & -3 \\ 1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -3 \\ -1 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 10 \\ -1 & 1 \end{vmatrix}$$

$= 2(10+3) + 1(1-3) + 2(1+10)$

$= 26 - 2 + 22$

$= 46 \neq 0$

$\therefore$ The system exists solution.

$$\text{Now } D_1 = \begin{vmatrix} 2 & -1 & 2 \\ 5 & 10 & -3 \\ -3 & 1 & 1 \end{vmatrix} = 2\begin{vmatrix} 10 & -3 \\ 1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 5 & -3 \\ -3 & 1 \end{vmatrix} + 2\begin{vmatrix} 5 & 10 \\ -3 & 1 \end{vmatrix}$$

$$= 2(10+3) + 1(5-9) + 2(5+30)$$

$$= 26 - 4 + 70$$

$$= 92$$

$$\therefore x = \frac{D_1}{D} = \frac{92}{46} = 2$$

$$\text{Again, } D_2 = \begin{vmatrix} 2 & 2 & 2 \\ 1 & 5 & -3 \\ -1 & -3 & 1 \end{vmatrix} = 2\begin{vmatrix} 5 & -3 \\ -3 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & -3 \\ -1 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 5 \\ -1 & -3 \end{vmatrix}$$

$$= 2(5-9) - 2(1-3) + 2(-3+5)$$

$$= -8 + 4 + 4$$

$$= 0$$

$$\therefore y = \frac{D_2}{D} = \frac{0}{46} = 0$$

$$\text{Also, } D_3 = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 10 & 5 \\ -1 & 1 & -3 \end{vmatrix} = 2\begin{vmatrix} 10 & 5 \\ 1 & -3 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 5 \\ -1 & -3 \end{vmatrix} + 2\begin{vmatrix} 1 & 10 \\ -1 & 1 \end{vmatrix}$$

$$= 2(-30-5) + 1(-3+5) + 2(1+10)$$

$= -70 + 2 + 22$

$= -46$

$\therefore z = \frac{D_3}{D} = \frac{-46}{46} = -1$

$\therefore$ The sol^n of the given system is

$x = 2,\ y = 0$ and $z = -1$ Ans

H.W.

Solve the following system of linear equations with the help of determinant.

(i) $x - 2y + 3z = 11$
$2x + y + 2z = 10$
$3x + 2y + z = 9$

$[x = 2, y = 0, z = 3]$

(ii) $x + y + z = 1$
$x + y - 2z = 0$
$2x - y - z = 0$

$[x = y = z = 1]$

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