بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

Simulation and Modeling-Lecture: 13

Runs up and Runs down Test

We do Runs up and Runs down Test the hypothesis of numbers. Determining the numbers are independently distributed or not. We are going to test independency of numbers distribution.

Algorithm:

Step-1: Define the hypothesis for testing as-

$$H_0 : R_i \sim independently$$

$$H_1 : R_i \nsim \; independently$$

Step-2: Write down the sequence of runs up and runs down.

Step-3: Count the total number of runs (a), present in the sequence.

Step-4: Count the mean and variance of a as,

$$\mu_a = \frac{2N-1}{3} \;\;\&\;\; {\sigma_a}^2 = \frac{16N-29}{90}$$

Step-5: Standard normal test statistics,

$$Z_0 = \frac{a-\mu_a}{\sigma_a} \;\; ; \;\; Z_0 \sim N\,[0,1]$$

Step-6: Determine the critical value $Z\alpha/_2$ & $-Z\alpha/_2$

Step-7: If,

$$-Z\alpha/_2 \le Z_0 \le Z\alpha/_2$$

Then,

$$H_0 \; is \; not \; rejected$$

Else

$$H_0 \; is \; rejected$$

Created by
Shahadat Hossain
Lecturer, Dept of CSE
City University

Problem: Test the following numbers by Runs up and Runs down test.

$$0.12, 0.01, 0.23, 0.28, 0.89, 0.31, 0.64, 0.28, 0.33, 0.93$$

$$where\ \ \alpha = 0.05\ \&\ Z\alpha/_2 = \ Z_{0.05}/_2 = Z_{0.025} = 1.96$$

Solution:

The hypothesis for the given test is-

$$H_0 : R_i \sim independently$$

$$H_1 : R_i \nsim \ independently$$

The sequence of Runs up and Runs down test is-

$$0.01 - 0.12 = negative\ value\ so\ take\ the\ \ negetive\ sign = -$$

$$0.23 - 0.01 = positive\ value\ so\ take\ the\ \ positive\ sign = +$$

$$0.28 - 0.23 = positive\ value\ so\ take\ the\ \ positive\ sign = +$$

$$0.89 - 0.28 = positive\ value\ so\ take\ the\ \ positive\ sign = +$$

$$0.31 - 0.89 = negative\ value\ so\ take\ the\ \ negetive\ sign = -$$

$$0.64 - 0.31 = positive\ value\ so\ take\ the\ \ positive\ sign = +$$

$$0.28 - 0.64 = negative\ value\ so\ take\ the\ \ negetive\ sign = -$$

$$0.33 - 0.28 = positive\ value\ so\ take\ the\ \ positive\ sign = +$$

$$0.93 - 0.33 = positive\ value\ so\ take\ the\ \ positive\ sign = +$$

So, the sequence of the Runs up and Runs down test: $- + + + - + - + +$

$$\underset{1}{-}\quad \underset{2}{\underbrace{+++}}\quad \underset{3}{-}\quad \underset{4}{+}\quad \underset{5}{-}\quad \underset{6}{\underbrace{++}}$$

Total number of runs $(a) = 6$

We know the mean and variance of a as,

$$\mu_6 = \frac{2 \times 10 - 1}{3} = 6.33$$

$$\sigma_6{}^2 = \frac{16 \times 10 - 29}{90} = 1.45$$

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Shahadat Hossain
Lecturer, Dept of CSE
City University

And, standard normal test statistics,

$$Z_0 = \frac{6 - 6.33}{\sqrt{1.45}} = -0.27$$

Now we have $Z_0 = -0.27$ & $Z_{0.025} = 1.96 \ \therefore \ -Z_{0.025} = -1.96$

$$-Z_{0.025} = -1.96 \leq Z_0 = -0.27 \leq Z_{0.025} = 1.96 \ ; True$$

$\therefore \ H_0 \ is \ accepted. \ Given \ random \ number \ is \ independently \ distributed \ from \ each \ other.$

Problem: Test the following numbers by Runs up and Runs down test.

$$0.08, 0.18, 0.23, 0.36, 0.42, 0.55, 0.63, 0.72, 0.89, 0.91$$

$$where \ \ \alpha = 0.05 \ \& \ Z_{\alpha/2} = \ Z_{0.05/2} = Z_{0.025} = 1.96$$

Solution:

The hypothesis for the given test is-

$$H_0 : R_i \sim independently$$

$$H_1 : R_i \not\approx \ \ independently$$

The sequence of Runs up and Runs down test is-

$$0.18 - 0.08 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.23 - 0.18 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.36 - 0.23 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.42 - 0.36 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.55 - 0.42 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.63 - 0.55 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.72 - 0.63 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.89 - 0.72 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.91 - 0.89 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

So, the sequence of the Runs up and Runs down test: $+ + + + + + + + +$

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Shahadat Hossain
Lecturer, Dept of CSE
City University

$$+ + + + + + + + + +$$

$$1$$

Total number of runs $(a) = 1$

We know the mean and variance of a as,

$\mu_6 = \dfrac{2 \times 10 - 1}{3} = 6.33$

$\sigma_6{}^2 = \dfrac{16 \times 10 - 29}{90} = 1.45$

And, standard normal test statistics,

$$Z_0 = \frac{1 - 6.33}{\sqrt{1.45}} = -4.43$$

Now we have $Z_0 = -4.43$ & $Z_{0.025} = 1.96 \quad \therefore \; -Z_{0.025} = -1.96$

$$-Z_{0.025} = -1.96 \leq Z_0 = -4.43 \leq Z_{0.025} = 1.96 \;;False$$

$\therefore \; H_0 \; is \; rejected. \; Given \; random \; number \; is \; not \; independently \; distributed \; from \; each \; other.$

Created by
Shahadat Hossain
Lecturer, Dept of CSE
City University

Runs above and below Mean Test

Algorithm:

Step-1: Define the hypothesis for testing as-

$$H_0 : R_i \sim independently$$

$$H_1 : R_i \nsim \quad independently$$

Step-2: Write down the sequence of runs above and below mean. [0.495 & 49.5]

Step-3: Count the total number of observations $above\ mean\ (n_1)$ and number of observations $below\ mean\ (n_2)$ & $b = total\ number\ of\ runs.$

Step-4: Compare mean & variance of b,

$$\mu_b = \frac{2n_1n_2}{N} + \frac{1}{2}$$

$$\sigma_b{}^2 = \frac{2n_1n_2(2n_1n_2 - N)}{N^2(N-1)}$$

Step-5: Compute the standard normal test statistics,

$$Z_0 = \frac{b - \mu_b}{\sigma_b} \ ; \ Z_0 \sim N\ [0,1]$$

Step-6: Determine the critical value $Z\alpha/_2$ & $-Z\alpha/_2$

Step-7: If,

$$-Z\alpha/_2 \leq Z_0 \leq Z\alpha/_2$$

Then,

$$H_0\ is\ not\ rejected$$

Else

$$H_0\ is\ rejected$$

Created by
Shahadat Hossain
Lecturer, Dept of CSE
City University

Problem: Test the following numbers by Runs above and below mean test.

$$0.11, 0.23, 0.45, 0.08, 0.11, 0.50, 0.09, 0.60, 0.81$$

$$where \ \ \alpha = 0.05 \ \& \ \ Z\alpha_{/2} = \ \ Z_{0.05}{}_{/2} = Z_{0.025} = 1.96$$

Solution:

The hypothesis for the given test is-

$$H_0 : R_i \sim independently$$

$$H_1 : R_i \nsim \ \ independently$$

As, all the given value we have, is decimal point value. so, our mean will be 0.495. We will compare given value with 0.495 for generating the sequence for the test.

$$0.11 - 0.495 = negative \ value \ so \ take \ the \ \ negetive \ sign = -$$

$$0.23 - 0.495 = negative \ value \ so \ take \ the \ \ negetive \ sign = -$$

$$0.45 - 0.495 = negative \ value \ so \ take \ the \ \ negetive \ sign = -$$

$$0.08 - 0.495 = negative \ value \ so \ take \ the \ \ negetive \ sign = -$$

$$0.11 - 0.495 = negative \ value \ so \ take \ the \ \ negetive \ sign = -$$

$$0.50 - 0.495 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.09 - 0.495 = negative \ value \ so \ take \ the \ \ negetive \ sign = -$$

$$0.60 - 0.495 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

$$0.81 - 0.495 = positive \ value \ so \ take \ the \ \ positive \ sign = +$$

So, the sequence of the Runs up and Runs down test: $- \ - \ - \ - \ - \ + \ - \ + \ +$

$$\underbrace{- \ - \ - \ - \ -}_{1} \ \underbrace{+}_{2} \ \underbrace{-}_{3} \ \underbrace{+ \ +}_{4}$$

$\therefore b = 4, \quad n_1 = \ 3 \quad , n_2 = 6$

Now,

$$\mu_4 = \frac{2 \times 3 \times 6}{9} + \frac{1}{2} = 4.5$$

$$\sigma_4{}^2 = \frac{2 \times 3 \times 6(2 \times 3 \times 6 - 9)}{9^2(9 - 1)} = 1.5$$

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Shahadat Hossain
Lecturer, Dept of CSE
City University

$$\therefore Z_0 = \frac{4 - 4.5}{\sqrt{1.5}} = -0.4$$

Now we have $Z_0 = -0.4$ & $Z_{0.025} = 1.96 \quad \therefore -Z_{0.025} = -1.96$

$$-Z_{0.025} = -1.96 \leq Z_0 = -0.4 \leq Z_{0.025} = 1.96 \ ; True$$

$\therefore$ H_0 *is accepted. Given random number is independently distributed from each other.*

Problem: Test the following numbers by Runs above and below mean test.

$$44, 40, 33, 26, 27, 44, 46, 54, 50, 51, 62, 33, 18, 28, 45, 36, 22$$

$$where \ \alpha = 0.05 \ \& \ Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$$

Solution:

The hypothesis for the given test is-

$$H_0 : R_i \sim independently$$

$$H_1 : R_i \nsim independently$$

As, all the given value we have, is integer value. so, our mean will be 49.5. We will compare given value with 49.5 for generating the sequence for the test.

$$44 - 49.5 = negative\ value\ so\ take\ the\ negetive\ sign = -$$

$$40 - 49.5 = negative\ value\ so\ take\ the\ negetive\ sign = -$$

$$33 - 49.5 = negative\ value\ so\ take\ the\ negetive\ sign = -$$

$$26 - 49.5 = negative\ value\ so\ take\ the\ negetive\ sign = -$$

$$27 - 49.5 = negative\ value\ so\ take\ the\ negetive\ sign = -$$

$$44 - 49.5 = negative\ value\ so\ take\ the\ negetive\ sign = -$$

$$46 - 49.5 = negative\ value\ so\ take\ the\ negetive\ sign = -$$

$$54 - 49.5 = positive\ value\ so\ take\ the\ positive\ sign = +$$

$$50 - 49.5 = positive\ value\ so\ take\ the\ positive\ sign = +$$

$$51 - 49.5 = positive\ value\ so\ take\ the\ positive\ sign = +$$

$$62 - 49.5 = positive\ value\ so\ take\ the\ positive\ sign = +$$

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Shahadat Hossain
Lecturer, Dept of CSE
City University

$$33 - 49.5 = negative\ value\ so\ take\ the\ \ negetive\ sign = -$$

$$18 - 49.5 = negative\ value\ so\ take\ the\ \ negetive\ sign = -$$

$$28 - 49.5 = negative\ value\ so\ take\ the\ \ negetive\ sign = -$$

$$45 - 49.5 = negative\ value\ so\ take\ the\ \ negetive\ sign = -$$

$$36 - 49.5 = negative\ value\ so\ take\ the\ \ negetive\ sign = -$$

$$22 - 49.5 = negative\ value\ so\ take\ the\ \ negetive\ sign = -$$

So, the sequence of the Runs up and Runs down test: $- - - - - - - - + + + + - - - - - - -$

$\therefore b = 3, \quad n_1 = 4 \quad , n_2 = 13$

Now,

$$\mu_4 = \frac{2 \times 4 \times 13}{17} + \frac{1}{2} = 6.61$$

$$\sigma_4{}^2 = \frac{2 \times 4 \times 13(2 \times 4 \times 13 - 17)}{17^2(17 - 1)} = \frac{9048}{4624} = 1.96$$

$$\therefore Z_0 = \frac{3 - 6.61}{\sqrt{1.96}} = -2.58$$

Now we have $Z_0 = -2.58$ & $Z_{0.025} = 1.96 \ \therefore \ -Z_{0.025} = -1.96$

$$-Z_{0.025} = -1.96 \leq Z_0 = -2.58 \leq Z_{0.025} = 1.96 \ ; True$$

$\therefore \ H_0\ is\ rejected.\ Given\ random\ number\ is\ not\ independently\ distributed\ from\ each\ other.$

Created by
Shahadat Hossain
Lecturer, Dept of CSE
City University

Length of Runs by Runs up / down or Runs above / below Mean

Algorithm:

Step-1: Define the hypothesis for testing as-

$$H_0 : R_i \sim independently$$

$$H_1 : R_i \nsim \ independently$$

Step-2: Write down the sequence of runs up and runs down/runs above and below mean [$as\ mentioned\ in\ the\ question. For\ runs\ above\ and\ below\ mean\ consider\ this\ value$ 0.495 & 49.5]

Step-3: find the total length of runs in sequence.

Step-4: Prepare table of observations of runs,

$Runs\ length(i)$	1	2	……………………..	n
$Observation\ of\ runs\ (O_i)$	$-$	$-$	………………………….	$-$

Step-5: Compute the expected value of y_i,

$i. For$ runs up and runs down,

$$E(y_i) = \begin{cases} \frac{2}{(i+3)!}[N^2(i^2+3i+1)-(i^3+3i^2-i-4)]\ ; applicable\ if\ i \leq N-2 \\ \frac{2}{N!}\ ; applicable\ if\ i = N-1 \end{cases}$$

$ii. For$ runs above and below mean,

$$E(y_i) = \frac{NW_i}{E(I)}\ ; N > 20$$

$$Where, W_i = approximate\ probability\ of\ particular\ length$$

$$E(I) = approximate\ expected\ number\ of\ length$$

$$W_i = \left(\frac{n_1}{N}\right)^i \left(\frac{n_2}{N}\right) + \left(\frac{n_1}{N}\right)\left(\frac{n_2}{N}\right)^i$$

$$E(I) = \frac{n_1}{n_2} + \frac{n_2}{n_1}$$

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Shahadat Hossain
Lecturer, Dept of CSE
City University

Step-6: Mean for runs up and runs down,

$$\mu_a = \frac{2N-1}{3}$$

expected total number of runs for runs above and below mean,

$$E(A) = \frac{N}{E(I)}$$

Step-7: Compute expected number of $runs \geq \ max \ length$

$$i. For \text{ runs up and runs down} = \mu_a - \sum_{i=1}^{m} E(y_m)$$

$$ii. For \text{ runs above and below mean} = E(A) - \sum_{i=1}^{m} E(y_m)$$

Step-8: Apply the Chi-square test

Step-9: Compute,

$$X^2_{\alpha, \ n-1}$$

Step-10: Compare,

$$if, \qquad X_0^2 < X^2_{\alpha, \ n-1} \ is \ true$$

$$then, \qquad H_0 \ is \ accepted$$

Created by
Shahadat Hossain
Lecturer, Dept of CSE
City University