Part-B

Simpson 1/3 rules + Simpson 3/8 rules

N= Even

sub interval

Multiple of 3

Formula Simpson 1/3:

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(Y_0+Y_n) + 4(Y_1+Y_3+\dots) + 2(Y_2+Y_4+\dots)\right]$$

where, a, b is the interval of integration

$$h = \frac{b-a}{n}$$

Y_0 = First term, $Y_n \rightarrow$ Last term

~~Using~~ Simpson 1/3 Formula:

x	F(x)	
x_0 0.0	1.0000	
x_1 0.1	0.9975	Y_1
x_2 0.2	0.9900	Y_2
x_3 0.3	0.9776	Y_3
x_4 0.4	0.8604	

Using Simpson 1/3 Formula:

$$\int_a^b F(x)\,dx \to \frac{h}{3}\left[(Y_0 + Y_n) + 4(Y_1 + Y_3 - \cdots) + 2(Y_2 + Y_4 + \cdots)\right]$$

$$\to \frac{0.1}{3}\left[(1 + .8604) + 4(0.9975 + 0.9776 \ldots) + 2(0.9900 + 0.8604 \ldots\right.$$

$$\to 0.39136$$

Find the value of $\int_1^2 \frac{1}{x}$ using simpsons 1/3 rule.

where, $n=10$

Here, $a=1$, $b=2$, $h=\frac{b-a}{n}=\frac{2-1}{10}=0.1$

$$Y=F(x)=\frac{1}{x}$$

$$\boxed{x_i=a+ih} \rightarrow 0\ldots N$$

$X_0=a+ih=1+0*0.1=1;\ Y_0=\frac{1}{x_0}=\frac{1}{1}=1$

$x_1=a+ih=1+1*0.1=1.1;\ Y_1=\frac{1}{x_1}=\frac{1}{1.1}=0.909$

$x_2=a+ih=1+2\times0.1=1.2;\ Y_2=\frac{1}{x_2}=\frac{1}{1.2}=0.83$

$x_3=a+ih=1+3*0.1=1.3;\ Y_3=\frac{1}{x_3}=\frac{1}{1.3}=0.76$

$x_4=a+ih=1+4*0.1=1.4;\ Y_4=\frac{1}{x_4}=\frac{1}{1.4}=0.71$

$x_5=a+ih=1+5*0.1=1.5;\ Y_5=\frac{1}{x_5}=\frac{1}{1.5}=0.66$

$x_6=a+ih=1+6*0.1=1.6;\ Y_6=\frac{1}{x_6}=\frac{1}{1.6}=0.62$

$x_7=a+ih=1+7*0.1=1.7;\ Y_7=\frac{1}{x_7}=\frac{1}{1.7}=0.58$

$x_8 = a + ih = 1 + 8 * 0.1$

$x_9 = a + ih = 1 + 9 * .1$

$x_{10} = a + ih = 1 + 9 \times$

$$\int_1^2 F(x)\,dx \rightarrow \frac{h}{3}\left[(Y_0 + Y_n) + 4(Y_1 + Y_3 + \dots) + 2(Y_2 + Y_4 \dots)\right]$$

7/05/24

Simpson $\frac{3}{8}$ rules:

$$\int_a^b F(x)\,dx = \frac{3h}{8}\left[(Y_0+Y_n)+3(Y_1+Y_2)+2(Y_3+Y_6+Y_9+\dots)\right.$$

where a, b is the meterval of Integration

$$h=\frac{b-a}{n}$$

$Y_1+Y_2+Y_4 \longrightarrow$ (rest of the terms)

$Y_3+Y_6+\dots \longrightarrow$ (3 multiple) ৩ দ্বারা ভাগ করলে ভাগশেষ ০ হবে।

দুই ধরনের math আসতে পারে।

Find the value of $\int_0^3 \frac{1}{1+x}$ using simpson $\frac{3}{8}$ rule $n=6$

Here, $a=0$ $\quad h=\frac{b-a}{n}=\frac{3-0}{6}=0.5$

$b=3$

$n=6$

$\boxed{x = a + ih}$

$x_0 = a + ih = 0 + 0 * 0.5 = 0$; $Y_0 = \frac{1}{1+x_0} = \frac{1}{1+0} = 1$

$x_1 = a + ih = 0 + 1 * 0.5 = 0.5$; $Y_1 = \frac{1}{1+x_1} = \frac{1}{1+0.5} = 0.67$

$x_2 = a + ih = 0 + 2 * 0.5 = 1$; $Y_2 = \frac{1}{1+Y_2} = \frac{1}{1+1} = 0.5$

$x_3 = a + ih = 0 + 3 * 0.5 = 1.5$; $Y_3 = \frac{1}{1+X_3} = \frac{1}{1+1.5} = 0.4$

$x_4 = a + ih = 0 + 4 * 0.5 = 2$; $Y_4 = \frac{1}{1+X_4} = \frac{1}{1+2} = 0.33$

$x_5 = a + ih = 0 + 5 * 0.5 = 2.5$; $Y_5 = \frac{1}{1+X_5} = \frac{1}{1+2.5} = 0.286$

$x_6 = a + ih = 0 + 6 * 0.5 = 3$; $Y_6 = \frac{1}{1+x_6} = \frac{1}{1+3} = .25$

using simpson $\frac{3}{8}$ rule,

$$\int_a^b F(x)\,dx = \frac{3h}{8}\left[(Y_0 + Y_n) + 3(Y_1 + Y_2 + Y_4) + 2(Y_3 + Y_6 + Y_9)\right]$$

$$= \frac{3 \times .5}{8}\left[(1 + 0.25) + 3(0.67 + 0.5) + 2(0.4)\right]$$

$$= 0.1875\left[1.25 + 3.51 + 0.8\right]$$

$= 1.38$

H.W

1. Simpson's $\frac{1}{3}$ rule: $\int_0^1 \frac{1}{1+x^2}\,dx$, when $n=4$

2. Simpson's $\frac{3}{8}$ rules: $\int_{-3}^{3} x^4\,dx$, where $n=6$

[Tabular - Graph value বের করে দেখাতে হবে]

Trapezoidal rule

1. Two Assumption
2. Both Limit should be Finite
3. Function $F(x)$ will be continued between Limit a, b

(4) Steps:- N: Number of intervals

- $h = \frac{b-a}{n}$

- $x_i = a + ih$ [where $i = 0, 1, 2, \ldots n$]

- Find the value of $F(x)$, $Y \ldots Y_n$

5. $\int_a^b F(x)\,dx = \frac{h}{2}\{Y_0 + 2(Y_1 + Y_2 + Y_3 + \ldots Y_{n-1}) + Y_n\}$

h

Find the value of $\int_0^{12} \frac{1}{1+x}\,dx$, where $h = 2$

Here,

$a = 0$, $b = 12$, $h = 2$

$h = \frac{b-a}{n}$ $\Rightarrow n = \frac{b-a}{h} = \frac{12-0}{2}$

$= 6$

$N = 6$

$h = 2$

$x_i = a + ih$

$x_0 = a + ih = 0 + 0 * 2 = 0$; $\quad Y_0 = \frac{1}{1+x_0} = \frac{1}{1+0} = 1$

$x_1 = a + ih = 0 + 1 * 2 = 2$; $\quad Y_1 = \frac{1}{1+2} = \frac{1}{3} = 0.333$

$x_2 = a + ih = 0 + 2 * 2 = 4$; $\quad Y_2 = \frac{1}{1+4} = \frac{1}{5} = 0.20$

$x_3 = a + ih = 0 + 3 * 2 = 6$; $\quad Y_3 = \frac{1}{1+6} = \frac{1}{7} = 0.142$

$x_4 = a + ih = 0 + 4 * 2 = 8$; $\quad Y_4 = \frac{1}{1+8} = \frac{1}{9} = 0.111$

$x_5 = a + ih = 0 + 5 * 2 = 10$; $\quad Y_5 = \frac{1}{1+10} = \frac{1}{11} = 0.090$

$x_6 = a + ih = 0 + 6 * 2 = 12$; $\quad Y_6 = \frac{1}{1+12} = \frac{1}{13} = 0.076$

using Trapezoidal rule:

$$\int_a^b F(x)\,dx = \frac{h}{2}\left\{ Y_0 + 2(Y_1 + Y_2 + Y_3 + Y_4 + Y_5) + Y_n \right\}$$

$$= \frac{2}{2}\left\{ 1 + 2(0.33 + .20 + 0.142 + 0.111 + 0.090) + 0.076 \right\}$$

$$= 2.82$$

* Euler Method

$$\frac{dy}{dx} = 1 + xy, \text{ where } Y(0) = 2$$

Find the value $f(0.1)$
$f(0.2)$
$f(0.3)$

Here, $X_0 = 0$, $Y_0 = 2$

$$\boxed{X_{i+1} = X_i + h; \quad Y_{i+1} = Y_i + h\,F(x_i, Y_i)}$$

$X_{0+1} = 0 + 0.1 = 0.1$ // $Y_{0+1} = 2 + 0.1 * 1 = 2.1$

$X_{1+1} = 0.1 + 0.1 = 0.2$ // $Y_{1+1} = 2.1 + 0.1 * 0.21 = 2.121$

H.W

Find the value of $\frac{dy}{dx} = -xy^2$ using Euler method where $Y(2) = 1$, $n = 4$

Find out $Y(2.2) = ?$

14/09/24

* Tailor series:

$$f(x) = y(x) = y_0 + \frac{(x-x_0)}{1!} y_0' + \frac{(x-x_0)^2}{2!} y_0'' + \frac{(x-x_0)^3}{3!} y_0''' + \ldots$$

Find the value of $F(0.1)$ using the Tailor Series where $\cancel{y' = x^2}$

$y = x^2 y - 1$, $Y(0) = 1$.

Here $x_0 = 0$

$y_0 = 1$

$$\frac{dy}{dx}$$

$$y' = x^2 y - 1 \quad \text{——— (i)}$$

$$= x^2 \frac{d}{dx}(y) + y \frac{d}{dx}(x^2)$$

$$y'' = x^2 y' + y.2x = x^2 y' + 2xy \quad \text{——— (ii)}$$

$$y''' = x^2 y' + 2xy$$

$$= x^2 \frac{d}{dx}(y') + y' \frac{d}{dx}(x^2) +$$

$$= x^2 y'' + y' 2x +$$

$$y''' = x^2 y'' + 4xy' + 2y \quad \text{——— (iii)}$$

$$y^{iv} = x^2 y''' + 6xy'' + 6y' \quad \text{——— (iv)}$$

$$y(x) = y_0 + \frac{(x-x_0)}{1!} y_0' + \frac{(x-x_0)^2}{2!} y_0'' + \frac{(x-x_0)^3}{3!} y_0''' + \frac{(x-x_0)^4}{4!} y_0^{iv} + \ldots\ldots$$

$$y_0' = x^2 y - 1 \Rightarrow 0^2.1 - 1 = -1.$$

$y'' = x^2y' + 2xy \Rightarrow \quad = 0$

$= 2$

$y''' = x^2y'' + 4xy' + 2y \Rightarrow$

$y^{iv} = x^2y''' + 6xy'' + 6y' \Rightarrow \quad = -6$

$$\therefore y(x) = y_0 + \frac{(x-x_0)}{1!}y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \frac{(x-x_0)^4}{4!}y_0^{iv} + \cdots$$

$$= 1 + \frac{(x-0)}{1}(-1) + \frac{(x-0)^2}{2} \times 0 + \frac{(x-0)^3}{6}(2) + \frac{(x-0)^4}{24} \times (-6)$$

$$y(x) = 1 - x + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

H.W

(i) $\sin x$

$\cos x$

(ii) $\frac{dy}{dx} = x - y^2$

where $y(0) = 1$

* Picard's iteration:

$$y(t) = y_0 + \int_{t_0}^{t} F(x, y(x))\, dx$$

$$\rightarrow y - y_0 = \int_{t_0}^{t} f(x, y(x))\, dx$$

<u>Example:</u> Given that $\frac{dy}{dx} = x + y^2$, and that $y = 0$ when $x = 0$, determine the value of y when $x = 0.3$, [Correct to Fourth places of decimals]

<u>Solution:</u> Given,

$$y - y_0 = \int_{x_0}^{x} (x + y^2)\, dx$$

Here, $x_0 = 0$

[y = constant]

$$y = y_0 + \frac{x^2}{2} = 0 + \frac{(0.3)^2}{2} = 0.0450$$

← আগের পেইজ

$$y_2 = x + y_1^2$$

$$= x + \frac{x^4}{4}$$

$$= \frac{x^2}{2} + \frac{x^5}{5\times4} \Rightarrow \frac{x^2}{2} + \frac{x^5}{20}$$

$$\Rightarrow \frac{(0.3)^2}{2} + \frac{(0.3)^5}{20}$$

$$= 0.0451$$

$$y_3 = x + y_2^2$$

$$= x + \left(\frac{x^2}{2} + \frac{x^5}{20}\right)^2$$

$$= x + \frac{x^4}{4} + 2\cdot\frac{x^2}{2}\cdot\frac{x^5}{20} + \frac{x^{10}}{400}$$

$$= x + \frac{x^4}{4} + \frac{x^7}{20} + \frac{x^{10}}{400}$$

$$= \frac{x^2}{2} + \frac{x^5}{20} + \frac{x^8}{160} + \frac{x^{11}}{4400}$$

$$= 0.0451$$

* Romberg: method:

Find out the value of $\int_0^1 \frac{1}{1+x} dx$ using Romberg Method. where

$h = 0.5$

Case : 1. Trapizoidal rule

$x_0 = a + ih = 0 + 0 * 0.5 = 0$; $y_0 = \frac{1}{1+0} = 1$

$x_1 = a + ih = 0 + 1 * 0.5 = 0.5$; $y_1 = \frac{1}{1+0.5} = 2$

$x_2 = a + ih = 0 + 2 * 0.5 = 1$; $y_2 = \frac{}{} = 0.5$

$$I_1 = \frac{h}{2}\left[y_0 + 2(y_1 + \dots + y_{n-1}) + y_n\right]$$

$$= \frac{0.5}{2}\left[\right.$$

$= 0.7084$

Case: 2:

$$h = \frac{h}{2} = \frac{0.5}{2} = 0.25$$

$X_0 = a+ih = 0+0*0.25 = 0$; $Y_0 = \frac{1}{1+x} = 1$

$x_1 = a+ih = 0+1*.25 = 0.25$; $y_1 = \frac{1}{1+0.25} =$

$x_2 = a+ih = 0+2*0.25 = 0.5$; $y_2 = \frac{1}{1+0.5} = 0.67$

$x_3 = a+ih = 0+3*0.25 =$; $y_3 = \frac{1}{1+} =$

$x_4 = a+ih = 0+4*0.25 =$;

$$I_2 = \frac{h}{2}\left[Y_0 + 2(y_1 + \dots y_{n-1}) + y_n\right]$$

$=$

$= 0.6970$

Case 3: $h = \frac{h}{4} = \frac{0.5}{4} = 0.125$

x_0, y_0

$$\left.\begin{array}{l} x_8 \to 1 \\ x_8 \to 0.5 \end{array}\right\} - I_3 = 0.6941$$

$$I_1' = I_2 + \frac{1}{3}(I_2 - I_1)$$

$$I_2' = I_3 + \frac{1}{3}(I_3 - I_2)$$

$$I_1'' = I_2' + \frac{1}{3}(I_2' - I_1')$$

$\boxed{\text{Boys — 2nd}}$

$\boxed{\text{Stitchusion}}$ $\to$ ~~[illegible]~~

N.A

20/05/24

* Runge - Kutta method:

→ 2nd order

→ 4th order

$$\frac{d}{dx}(y) = f(x,y)$$

with intial condition $y(x_0) = y_0$,

For this we can Formulate

Runge. Kutta methods as follows →

1st order RK:

$$y_1 = y_0 + hF(x_0, y_0)$$

$$= y_0 + hy_0' \quad [\text{since } y' = f(x,y)]$$

2nd order RK:

$$y_1 = y_0 + \frac{1}{2}(K_1 + K_2)$$

Here, $k_1 = hF(x_0, y_0)$

$k_2 = hF(x_0 + h, y_0 + k_1)$

3rd order RK:

$$y_1 = y_0 + \frac{1}{6}(k_1 + 4k_2 + k_3)$$

Here, $k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$k_3 = hf(x_0 + h, y_0 + k_1)$

4th order RK method:

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Here, $k_1 = hF(x_0, y_0)$

$k_2 = hf[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}]$

$k_3 = hf[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}]$

$K_4 = hf(x_0 + h,\ y_0 + k_3)$

[২য় and ৪th অর্ডার থেকে আসবে]

Given,

$\frac{d}{dx}(y) = x^2 + y^2$, $y(1) = 1.2$, Find the value $y(1.05)$ using RK-4 method.

Solⁿ: $x_0 = 1;\ y_0 = 1.2$

also, $h = 0.05$

$K_1 = h\,F(x_0, y_0)$

$= 0.05\left[1^2 + (1.2)^2\right]$

$= 0.122$

$$K_2 = hF\left[x_0 + \frac{h}{2},\ y_0 + \frac{K_1}{2}\right]$$

$$= 0.05\left[F\left(1 + \frac{0.05}{2},\ 1.2 + \frac{0.122}{2}\right)\right]$$

$$= 0.05\left[F(1.025,\ 1.261)\right]$$

$$= 0.05\left[(1.025)^2 + (1.261)^2\right]$$

$$\cancel{= 0.05[1.05 +}$$

$$= 0.1320$$

$$K_3 = hF\left[\left(x_0 + \frac{h}{2},\ y_0 + \frac{K_2}{2}\right)\right]$$

$$= 0.05 + \left[F\left(1 + \frac{0.05}{2},\ 1.2 + \frac{0.1320}{2}\right)\right]$$

$$= 0.1326$$

$$K_4 = hF(x_0 + h,\ y_0 + K_3)$$

$$= 0.05 \times F(1 + 0.05,\ 1.2 + 0.1326)$$

$$= 0.1439$$

Empa

$$y(1.05) = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$= 1.2 + \frac{1}{6}(0.122 + 2(0.1320) + 2(0.1326) + 0.1439)$$

$$= 1.333$$

Part-B

1. Simpson $\frac{1}{3}$

    1.1 Simpson $\frac{3}{8}$

2. Trapezoidal rule
3. Euler method
4. Tailor Series
5. Picard's
6. Rumberg
7. RK method (1 .... 4)

    (2-4)