Matrix Algebra

⇒ What is matrix ?

A matrix is a rectangular array of numbers (real or complex) enclosed by pair of brackets (or double vertical ~~lin~~ rolls and the numbers in the array are called the entries or elements of the matrix and which can be written ~~as~~ of the form

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix}$$

Here the numbers $a_{11}, a_{12}, \cdots, a_{mn}$ are known as the elements of the matrix.

The above matrix has m rows and n columns and is called an $(m \times n)$ matrix (read "m by n matrix")

Order of matrix :

~~The~~ No. of rows and columns in a matrix ~~together~~ is called order of a matrix.

The above matrix has m rows and n colums.
so order of the matrix is $(m \times n)$.

Note:

- The above matrix is also written as

$$\left[(a_{ij}) \right], \text{ where } i = 1, 2, \ldots, m \text{ (rows)}$$
$$j = 1, 2, \ldots, n \text{ (columns)}$$

- Matrices are generally denoted by capital letters A, B, X, Y etc.
- [ ] or, ( ) or, || || are used for the notations of matrices.

Example:

01. $A = \begin{bmatrix} 1 & 0 & -5 \\ 2 & -3 & 7 \end{bmatrix}$ is a matrix which has 2 rows and 3 columns. So its order is (2×3).

02. $B = \begin{pmatrix} 2 & 0 & i \\ -i & 1 & 4 \\ 1+i & -5 & 3 \end{pmatrix}$ is a matrix of order (3×3).

⇒ Addition and subtraction of matrices :

Let $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{(m\times n)}$ and

$B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{(m\times n)}$ are two equal matrices

Then $A+B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{pmatrix}_{(m\times n)}$

and $A-B = \begin{pmatrix} a_{11}-b_{11} & a_{12}-b_{12} & \cdots & a_{1n}-b_{1n} \\ a_{21}-b_{21} & a_{22}-b_{22} & \cdots & a_{2n}-b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1}-b_{m1} & a_{m2}-b_{m2} & \cdots & a_{mn}-b_{mn} \end{pmatrix}_{(m\times n)}$.

Examples:

If $A = \begin{pmatrix} 1 & -2 \\ 3 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 7 \\ -5 & 4 \end{pmatrix}$ are two matrices,

then $A+B = \begin{pmatrix} 1+2 & -2+7 \\ 3+(-5) & 5+4 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ -2 & 9 \end{pmatrix}$

and $A-B = \begin{pmatrix} 1-2 & -2-7 \\ 3-(-5) & 5-4 \end{pmatrix} = \begin{pmatrix} -1 & -9 \\ 8 & 1 \end{pmatrix}$

H.W.

Let $A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$

are two matrices,

Determin $A+B$ and $A-B$?

Matrix Multiplication:

Two matrices A and B are possible for multiplication if no. of columns in A = no. of rows in B

Let $A = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix}_{(1\times 3)}$ and $B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}_{(3\times 1)}$

are two matrices

Then $AB = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$

$= \begin{pmatrix} a_1 b_1 + a_2 b_2 + \cdots + a_n b_n \end{pmatrix}_{(1\times 1)}$

Again, let $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{(m\times n)}$ and

$B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & & & \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}_{(n\times p)}$

$\therefore$ Then $AB = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & & & \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$

$$= \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21}+a\cdots+a_{1n}b_{n1} & a_{11}b_{12}+a_{12}b_{22}+\cdots+a_{1n}b_{n2} & \cdots & a_{11}b_{1p}+a_{12}b_{2p}+\cdots+a_{1n}b_{np} \\ a_{21}b_{11}+a_{22}b_{21}+\cdots+a_{2n}b_{n1} & a_{21}b_{12}+a_{22}b_{22}+\cdots+a_{2n}b_{n2} & \cdots & a_{21}b_{1p}+\cdots+a_{2n}b_{np} \\ \vdots & & \vdots & \\ a_{m1}b_{11}+a_{m2}b_{21}+\cdots+a_{mn}b_{n1} & a_{m1}b_{12}+a_{m2}b_{22}+\cdots+a_{mn}b_{n2} & \cdots & a_{m1}b_{1p}+a_{m2}b_{2p}+\cdots+a_{mn}b_{np} \end{pmatrix}_{(m\times p)}$$

Example:- 01:

Let $A = \begin{pmatrix} 1 & -3 & 5 \\ 2 & 0 & -1 \end{pmatrix}_{(2\times 3)}$ and $B = \begin{pmatrix} 1 & -1 \\ -2 & 4 \\ 3 & 0 \end{pmatrix}_{(3\times 2)}$

Then show that $AB \neq BA$.

Sol^n:

$$AB = \begin{pmatrix} 1 & -3 & 5 \\ 2 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 4 \\ 3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times 1+(-3)(-2)+5\times 3 & 1\times(-1)+(-3)\times 4+5\times 0 \\ 2\times 1+0\times(-2)+(-1)\times 3 & 2\times(-1)+0\times 4+(-1)\times 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+6+15 & -1-12+0 \\ 2+0-3 & -2+0+0 \end{pmatrix}$$

$$\therefore AB = \begin{pmatrix} 22 & -13 \\ -1 & -2 \end{pmatrix}$$

Again, $BA = \begin{pmatrix} 1 & -1 \\ -2 & 4 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 1 & -3 & 5 \\ 2 & 0 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 1\times1+(-1)\times2 & 1\times(-3)+(-1)\times0 & 1\times5+(-1)\times(-1) \\ (-2)\times1+4\times2 & (-2)\times(-3)+4\times0 & (-2)\times5+4\times(-1) \\ 3\times1+0\times2 & 3\times(-3)+0\times0 & 3\times5+0\times(-1) \end{pmatrix}$$

$$= \begin{pmatrix} 1-2 & -3+0 & 5+1 \\ -2+8 & 6+0 & -10-4 \\ 3+0 & -9+0 & 15+0 \end{pmatrix}$$

$$\therefore BA = \begin{pmatrix} -1 & -3 & 6 \\ 6 & 6 & -14 \\ 3 & -9 & 15 \end{pmatrix}$$

$\therefore AB \neq BA$ (showed)

⇒ Example – 02 :

If $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$, then determine the value of $A^3 + A^2 - 21A$.

Sol^n:

Given, $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

Then $A^2 = A \cdot A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix} \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

$$= \begin{pmatrix} (-2)(-2)+2\cdot 2+(-3)(-1) & (-2)2+2\cdot 1+(-3)(-2) & (-2)(-3)+2(-6)+(-3)\times 0 \\ 2(-2)+1\cdot 2+(-6)(-1) & 2\cdot 2+1\cdot 1+(-6)(-2) & 2(-3)+1(-6)+0(- \\ (-1)(-2)+(-2)\cdot 2+0(-1) & (-1)\cdot 2+(-2)\cdot 1+0\cdot(-2) & (-1)(-3)+(-2)(-6)+0\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4+4+3 & -4+2+6 & 6-12+0 \\ -4+2+6 & 4+1+12 & -6-6+0 \\ 2-4+0 & -2-2+0 & 3+12+0 \end{pmatrix}.$$

$$\therefore A^2 = \begin{pmatrix} 11 & 4 & -6 \\ 4 & 19 & -12 \\ -2 & -4 & 15 \end{pmatrix}$$

Again,

$$A^3 = A^2 \cdot A = \begin{pmatrix} 11 & 4 & -6 \\ 4 & 17 & -12 \\ -2 & -4 & 15 \end{pmatrix} \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

$$\Rightarrow A^3 = \begin{pmatrix} 11(-2)+4\cdot 2+(-6)(-1) & 11\cdot 2+4\cdot 1+(-6)(-2) & (-6) \\ 4(-2)+17\cdot 2+(-12)(-1) & 4\cdot 2+17\cdot 1+(-12)(-2) & \\ (-2)(-2)+(-4)2+15(-1) & (-2)\cdot 2+(-4)\cdot 1+15(-2) & \end{pmatrix}$$

$$\Rightarrow A^3 = \begin{pmatrix} 11(-2)+4\cdot 2+(-6)(-1) & 11\cdot 2+4\cdot 1+(-6)(-2) & 11(-3)+4(-6)+(-6)\cdot 0 \\ 4(-2)+17\cdot 2+(-12)(-1) & 4\cdot 2+17\cdot 1+(-12)(-2) & 4(-3)+17(-6)+(-12)\cdot 0 \\ (-2)(-2)+(-4)2+15(-1) & (-2)\cdot 2+(-4)\cdot 1+15(-2) & (-2)(-3)+(-4)(-6)+15\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} -22+8+6 & 22+4+12 & -33-24+0 \\ -8+34+12 & 8+17+24 & -12-102+0 \\ 4-8-15 & -4-4-30 & 6+24+0 \end{pmatrix}$$

$$= \begin{pmatrix} -8 & 38 & -57 \\ 38 & 49 & -114 \\ -19 & -38 & 30 \end{pmatrix}$$

And $21A = 21\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

$$= \begin{pmatrix} -42 & 42 & -63 \\ 42 & 21 & -126 \\ -21 & -42 & 0 \end{pmatrix}$$

$$\therefore A^3 + A^2 - 21A = \begin{pmatrix} -8 & 38 & -57 \\ 38 & 49 & -114 \\ -19 & -38 & 30 \end{pmatrix} + \begin{pmatrix} 11 & 4 & -6 \\ 4 & 17 & -12 \\ -2 & -4 & 15 \end{pmatrix} - \begin{pmatrix} -42 & 42 & -63 \\ 42 & 21 & -126 \\ -21 & -42 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -8+11+42 & 38+4-42 & -57-6+63 \\ 38+4-42 & 49+17-21 & -114-12+126 \\ -19-2+21 & -38-4+42 & 30+15-0 \end{pmatrix}$$

$$= \begin{pmatrix} 45 & 0 & 0 \\ 0 & 45 & 0 \\ 0 & 0 & 45 \end{pmatrix} = 45\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ Ans

H.W.

1. If $A = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$ ~~and~~, then prove that

$$A^2 - 3A + 2I = 0, \text{ where } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2. If $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}$, prove that

$$A^3 - 4A^2 - A + 4I = 0 \; ; \; I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. If $A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix}$

then show that $AB \neq BA$.

⇒ Transpose of a matrix:

Let A is an $m \times n$ matrix over the real field $\mathbb{R}$. Then the matrix $n \times n$ obtained from the matrix A by writing its rows as columns and its columns as rows is called the transpose of A and is denoted by the symbol A^T or A'.

For example:

1. Let $A = \begin{pmatrix} 1 & 0 & 5 & -7 \\ 2 & 3 & -1 & 6 \end{pmatrix}_{(2\times4)}$, then

$$A^T = \begin{pmatrix} 1 & 2 \\ 0 & 3 \\ 5 & -1 \\ -7 & 6 \end{pmatrix}_{(4\times2)}$$

2. Let $A = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}$, then $A^T = \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix}$.

⇒ Theorems on Transpose matrix:

If A and B are comparable matrices and A^T and B^T are the transpose matrices of A and B respectively, then

(i) $(A^T)^T = A$

e.g. Let $A = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}$, then $A^T = \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix}$.

$$\therefore (A^T)^T = \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix}^T = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix} = A$$

(ii) $(A+B)^T = A^T + B^T$.

e.g. Let $A = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 4 \\ -1 & 0 \end{pmatrix}$

$\therefore A^T = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix}$ and $B^T = \begin{pmatrix} 5 & 4 \\ -1 & 0 \end{pmatrix}^T = \begin{pmatrix} 5 & -1 \\ 4 & 0 \end{pmatrix}$

$\therefore A^T + B^T = \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} 6 & -2 \\ 7 & 5 \end{pmatrix}$

Again, $A+B = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix} + \begin{pmatrix} 5 & 4 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 7 \\ -1 & 5 \end{pmatrix}$

$\therefore (A+B)^T = \begin{pmatrix} 6 & 7 \\ -1 & 5 \end{pmatrix}^T = \begin{pmatrix} 6 & -1 \\ 7 & 5 \end{pmatrix}$.

(iii) $(AB)^T = B^T A^T$

e.g. $A = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 4 \\ -1 & 0 \end{pmatrix}$

$\therefore AB = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 5 & 4 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 5-3 & 4+0 \\ 0-5 & 0+0 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ -5 & 0 \end{pmatrix}$

$\therefore (AB)^T = \begin{pmatrix} 2 & 4 \\ -5 & 0 \end{pmatrix}^T = \begin{pmatrix} 2 & -5 \\ 4 & 0 \end{pmatrix}$

Again. $A^T = \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix}$ and $B^T = \begin{pmatrix} 5 & -1 \\ 4 & 0 \end{pmatrix}$

$\therefore B^T A^T = \begin{pmatrix} 5 & -1 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 5-3 & 0-5 \\ 4+0 & 0+0 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ 4 & 0 \end{pmatrix}$

(iv)// $(\alpha A)^T = \alpha A^T$, where α is a scalar

Let $A = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}$, $\Rightarrow A^T = \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix}$ and $\alpha = 5$

$\therefore \alpha A^T = 5 \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 15 & 25 \end{pmatrix}$

Again $(\alpha A) = 5 \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 5 & 15 \\ 0 & 25 \end{pmatrix}$

$\therefore (\alpha A)^T = \begin{pmatrix} 5 & 15 \\ 0 & 25 \end{pmatrix}^T = \begin{pmatrix} 5 & 0 \\ 15 & 25 \end{pmatrix}$

H.W.

If $A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 5 & -1 \\ 2 & 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 5 & 3 \\ 7 & -2 & 1 \\ 2 & 0 & -3 \end{pmatrix}$

then show that $(AB)^T = B^T A^T$

⇒ Diagonal Matrix:

A square matrix whose elements $a_{ij} = 0$, when $i \neq j$ is called a diagonal matrix.

Example:

$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are diagonal matrices.

⇒ Identity matrix / Unit matrix:

A square matrix whose elements $a_{ij} = 0$ when $i \neq j$ and $a_{ij} = 1$ when $i = j$ is called the identity matrix,

For examples, $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

⇒ Singular and Non-Singular matrices:

A square matrix whose determinants is zero is called singular matrix.

If determinant $\neq 0$, then the matrix is non-singular.

Example:

1. Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$. Then

$$\det(A) = |A| = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 1\times 4 - 2\times 2 = 0$$

$\therefore$ A is singular matrix.

2. If $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$.

$$\therefore \det(A) = |A| = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= 2\{(-1)\times 1 - 0\times(-2)\} - 1\{1\times 1 - 0\times 1\} + 3\{1\times(-2) - 1\times(-1)\}$$

$$= 2(-1+0) - 1(1-0) + 3(-2+1)$$

$$= -2 - 1 - \cancel{6}\,3$$

$$= \cancel{-9}\ -6 \neq 0$$

$\therefore$ A is non-singular.

⇒ Inverse Matrix:

A square matrix A is said to be invertible if there exists a unique matrix B such that

$$AB = BA = I \quad , \text{where } I \text{ is identity matrix.}$$

then we call such a matrix B is the inverse of A and denoted by $A^{-1} = B$.

Example:

Let $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$

Then $AB = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

and $BA = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 4-3 & 2-2 \\ -6+6 & -3+4 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$\therefore AB = BA = I$

$\therefore A^{-1} = B$ and $B^{-1} = A$

⇒ Adjoint / Adjugate Matrix:

Let $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$ be a $(m \times n)$ matrix.

Let D be the determinant of A. Then

$$D = |A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix}$$

Let A_{ij} $(i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n)$ be the co-factors of the determinant D.

Form the matrix (A_{ij}). Then the transpose of the matrix (A_{ij}) is called the adjoint matrix of A and is generally denoted by Adj A

$$\therefore \text{Adj} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & & \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{pmatrix}^T = \begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & & & \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}$$

$$A_{32} = -\begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} = -(1\times(-1) - 3\times 1) = -(-1-3) = 4$$

$$A_{33} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1\times 3 - 1\times 2 = 3-2 = 1$$

$$\therefore \text{Adj } A = \begin{pmatrix} 1 & -2 & -5 \\ 3 & -6 & 3 \\ -11 & 4 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 3 & -11 \\ -2 & -6 & 4 \\ -5 & 3 & 1 \end{pmatrix}$$

Ans

H.W.
Find the adjoint matrix of $A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

⇒ Singular and non-singular matrices:

Let D be the determinant of a square matrix A.

If $D=0$, then A is called the singular matrix

If $D\neq 0$, " A " " " non-singular ".

Example:
Examine the matrices $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$ singular or non singular?

Example:

Illustrate the adjoint matrix of $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & -1 \\ 2 & 1 & 0 \end{pmatrix}$

Solⁿ:

$$\text{Det}(A),\ D = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & -1 \\ 2 & 1 & 0 \end{vmatrix} = 1\{3\times 0 - 1(-1)\} - 2\{1\times 0 - 2(-1)\} + 3(1\times 1 - 2\times 3)$$

$$= 1\times(0+1) - 2(0+2) + 3(1-6)$$

$$= 1 - 4 - 15 = -18$$

Now the cofactors of D are

$$A_{11} = \begin{vmatrix} 3 & -1 \\ 1 & 0 \end{vmatrix} = 0\times 3 - 1(-1) = 0+1 = 1$$

$$A_{12} = -\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = -\{1\times 0 - 2(-1)\} = -(0+2) = -2$$

$$A_{13} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1\times 1 - 2\times 3 = 1 - 6 = -5$$

$$A_{21} = -\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = -(0\times 2 - 3\times 1) = -(0-3) = 3$$

$$A_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} = 1\times 0 - 3\times 2 = 0 - 6 = -6$$

$$A_{23} = -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -(1\times 1 - 2\times 2) = -(1-4) = 3$$

$$A_{31} = \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = 2(-1) - 3\times 3 = -2 - 9 = -11$$

Sol^n:

Det (A), $D_1 = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 1\times4 - 2\times2 = 4-4=0$

$\therefore$ A is singular matrix.

Again, det (B), $D_2 = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{vmatrix}$

$$= 2\{(-1)\times1 - 0\times(-2)\} - 1(1\times1 - 0\times1) + 3\{1\times(-2) - 1\times(-1)\}$$

$$= 2(-1+0) - 1(1-0) + 3(-2+1)$$

$$= -2-1-3$$

$$= -6 \neq 0$$

$\therefore$ B is non-singular.

$\Rightarrow$ Process of finding the inverse of a square matrix:

Let A be a square matrix and D be the determinant of A.

If $D=0$, then A is singular and has no inverse.

If $D\neq0$, " A is non-singular and A^{-1} exists.

Find the adjoint of A, then

$$A^{-1} = \frac{1}{D}\text{Adj } A = \frac{\text{Adj } A}{|A|}$$

Example: Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -1 \\ 3 & 3 & 2 \end{bmatrix}$$

Solⁿ:

$$\text{Det}(A),\ |A| = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 0 & -1 \\ 3 & 3 & 2 \end{vmatrix} = 2\{0\times2-3(-1)\}+1\{4\times2-3(-1)\} +3(4\times3-3\times0)$$

$$= 2(0+3)+1(8+3)+3(12-0)$$

$$= 6+11+36 = 53 \neq 0$$

Since $|A| \neq 0$, so A is non-singular and A^{-1} exists.

Now the co-factors of the determinant of A are

$$A_{11} = \begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix} = 0+3 = 3,\quad A_{12} = -\begin{vmatrix} 4 & -1 \\ 3 & 2 \end{vmatrix} = -(8+3) = -11$$

$$A_{13} = \begin{vmatrix} 4 & 0 \\ 3 & 3 \end{vmatrix} = 12-0 = 12,\quad A_{21} = -\begin{vmatrix} -1 & 3 \\ 3 & 2 \end{vmatrix} = -(-2-9) = 11$$

$$A_{22} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = 4-9 = -5,\quad A_{23} = -\begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} = -(6+3) = -9$$

$$A_{31} = \begin{vmatrix} -1 & 3 \\ 0 & -1 \end{vmatrix} = 1-0 = 1,\quad A_{32} = -\begin{vmatrix} 2 & 3 \\ 4 & -1 \end{vmatrix} = -(-2-12) = 14$$

$$A_{33} = \begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix} = 0+4 = 4$$

$$\therefore \text{Adj } A = \begin{pmatrix} 3 & -11 & 12 \\ 11 & -5 & -9 \\ 1 & 14 & 4 \end{pmatrix}^T = \begin{pmatrix} 3 & 11 & 1 \\ -11 & -5 & 14 \\ 12 & -9 & 4 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{Adj\, A}{|A|} = \frac{1}{53}\begin{pmatrix} 3 & 11 & 1 \\ -11 & -5 & 14 \\ 12 & -9 & 4 \end{pmatrix}$$ Ans

⇒ Example:

If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, then prove that $(AB)^{-1} = B^{-1}A^{-1}$.

Soln:

Now, ~~L.H.S~~ $AB = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}\begin{pmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 2+3+1 & 5+1+2 & 3+2+1 \\ 2+6+3 & 5+2+6 & 3+4+3 \\ 2+12+9 & 5+4+18 & 3+8+9 \end{pmatrix}$$

$$\therefore AB = \begin{pmatrix} 6 & 8 & 6 \\ 11 & 13 & 10 \\ 23 & 27 & 20 \end{pmatrix}$$

$$\therefore Det(AB),\ D_1 = \begin{vmatrix} 6 & 8 & 6 \\ 11 & 13 & 10 \\ 23 & 27 & 20 \end{vmatrix}$$

$$= 6(13\times20 - 27\times10) - 8(11\times20 - 23\times10) + 6(11\times27 - 23\times13)$$

$$= 6(-10) - 8(-10) + 6(-2)$$

$$= -60 + 80 - 12 = 8 \neq 0$$

Now the co factors of AB are

$$(AB)_{11} = \begin{vmatrix} 13 & 10 \\ 27 & 20 \end{vmatrix} = 13\times20 - 27\times10 = 260 - 270 = -10$$

$$(AB)_{12} = -\begin{vmatrix} 11 & 10 \\ 23 & 20 \end{vmatrix} = -(11\times20 - 23\times10) = -(220-230) = 10$$

$$(AB)_{13} = \begin{vmatrix} 11 & 13 \\ 23 & 27 \end{vmatrix} = 11\times27 - 23\times13 = -2$$

$$(AB)_{21} = -\begin{vmatrix} 8 & 6 \\ 27 & 20 \end{vmatrix} = -(8\times20 - 6\times27) = 2$$

$$(AB)_{22} = \begin{vmatrix} 6 & 6 \\ 23 & 20 \end{vmatrix} = 6\times20 - 23\times6 = -18$$

$$(AB)_{23} = -\begin{vmatrix} 6 & 8 \\ 23 & 27 \end{vmatrix} = -(6\times27 - 23\times8) = 22$$

$$(AB)_{31} = \begin{vmatrix} 8 & 6 \\ 13 & 10 \end{vmatrix} = 8\times10 - 13\times6 = 2$$

$$(AB)_{32} = -\begin{vmatrix} 6 & 6 \\ 11 & 10 \end{vmatrix} = -(6\times10 - 11\times6) = 6$$

$$(AB)_{33} = \begin{vmatrix} 6 & 8 \\ 11 & 13 \end{vmatrix} = 6\times13 - 8\times11 = -10$$

$$\therefore Adj(AB) = \begin{pmatrix} -10 & 10 & -2 \\ 2 & -18 & 22 \\ 2 & 6 & -10 \end{pmatrix}^T = \begin{pmatrix} -10 & 2 & 2 \\ 10 & -18 & 6 \\ -2 & 22 & -10 \end{pmatrix}$$

$$\therefore (AB)^{-1} = \frac{Adj(AB)}{D_1} = \frac{1}{8}\begin{pmatrix} -10 & 2 & 2 \\ 10 & -18 & 6 \\ -2 & 22 & -10 \end{pmatrix}$$

~~R.H.S :~~ Again, $\det(A), D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix}$

$$= 1(18-12) - 1(9-3) + 1(4-2)$$

$$= 6 - 6 + 2 = 2 \neq 0$$

Co-factors of det(A) are

$A_{11} = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 18-12 = 6$, $A_{12} = -\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -(9-3) = -6$

$A_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 4-2 = 2$, $A_{21} = -\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -(9-4) = -5$

$A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 9-1 = 8$, $A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -(4-1) = -3$

$A_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3-2 = 1$, $A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2$

$A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 = 1$

$$\therefore Adj(A) = \begin{pmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{pmatrix}^T = \begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{Adj(A)}{D_2} = \frac{1}{2}\begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}$$

Also, $\det(B) = D_3 = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$

$$= 2(1-4) - 5(3-2) + 3(6-1)$$

$$= -6 - 5 + 15$$

$$= 4 \neq 0$$

co-factors of $\det(B)$ are

$B_{11} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = \overset{1-4}{\cancel{4-1}} = -3$, $B_{12} = -\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -(3-2) = -1$

$B_{13} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 6-1 = 5$, $B_{21} = -\begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} = -(5-6) = 1$

$B_{22} = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 2-3 = -1$, $B_{23} = -\begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} = -(4-5) =$

$B_{31} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10-3 = 7$, $B_{32} = -\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -(4-9) = 5$

$B_{33} = \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} = 2-15 = -13$

$$\therefore \operatorname{Adj}(B) = \begin{pmatrix} -3 & -1 & 5 \\ 1 & -1 & 1 \\ 7 & 5 & -13 \end{pmatrix}^T = \begin{pmatrix} -3 & 1 & 7 \\ -1 & -1 & 5 \\ 5 & 1 & -13 \end{pmatrix}$$

$$\therefore B^{-1} = \frac{\operatorname{Adj}(B)}{D_3} = \frac{1}{4}\begin{pmatrix} -3 & 1 & 7 \\ -1 & -1 & 5 \\ 5 & 1 & -13 \end{pmatrix}$$

$$\therefore B^{-1}A^{-1} = \frac{1}{4}\begin{pmatrix} -3 & 1 & 7 \\ -1 & -1 & 5 \\ 5 & 1 & -13 \end{pmatrix} \times \frac{1}{2}\begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}$$

$$= \frac{1}{8}\begin{pmatrix} -3 & 1 & 7 \\ -1 & -1 & 5 \\ 5 & 1 & -13 \end{pmatrix}\begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}$$

$$= \frac{1}{8}\begin{pmatrix} -18-6+14 & 15+8-21 & -3-2+7 \\ -6+6+10 & 5-8-15 & -1+2+5 \\ 30-6-26 & -25+8+39 & 5-2-13 \end{pmatrix}$$

$$= \frac{1}{8}\begin{pmatrix} -10 & 2 & 2 \\ 10 & -18 & 6 \\ -2 & 22 & -10 \end{pmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1} \quad \text{(proved)}$$

H.W. 1. If $A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & 3 \\ 1 & -1 & 1 \end{pmatrix}$, then find A^{-1}.

2. If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, prove that $A^3 = A^{-1}$.

⇒ Solution of linear equations by applying matrices:

⇒ Example-01:

Solve the following linear equⁿs with the help of matrices

$$2x + y = 1$$
$$x - 2y = 3$$

Solⁿ:

The system of linear equⁿs can be written in matrix form as

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\Rightarrow AX = B \quad \text{--- (1)} \qquad \Rightarrow X = A^{-1}B \quad \text{--- (2)}$$

where $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Now det(A), ~~to~~ $D = \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = -4 - 1 = -5 \neq 0$

The co-factors of A are

$A_{11} = |-2| = -2$, $A_{12} = -|1| = -1$, $A_{21} = -|1| = -1$, $A_{22} = |2| = -2$

$$\therefore \operatorname{Adj}(A) = \begin{pmatrix} -2 & -1 \\ 1 & -2 \end{pmatrix}^T = \begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{Adj(A)}{D} = \frac{1}{-5}\begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}$$

Now ~~multiply both sides of ①~~ ② $\Rightarrow$

$$X = -\frac{1}{5}\begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -2+3 \\ -1-6 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} 1 \\ -7 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{1}{5} \\ \frac{7}{5} \end{pmatrix}$$

$\Rightarrow$ Solve the following linear equations with the help of matrices

$$3x+5y-7z=13$$
$$4x+y-12z=6$$
$$2x+9y-3z=20$$

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Sol^n: The given linear equ^{n}s can be written in matrix as

$$\begin{pmatrix} 3 & 5 & -7 \\ 4 & 1 & -12 \\ 2 & 9 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 \\ 6 \\ 20 \end{pmatrix}$$

$$\Rightarrow AX = B \Rightarrow X = A^{-1}B \quad \text{—— (1)}$$

where $A = \begin{pmatrix} 3 & 5 & -7 \\ 4 & 1 & -12 \\ 2 & 9 & -3 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $B = \begin{pmatrix} 13 \\ 6 \\ 20 \end{pmatrix}$

Now $\det(A)$, $D = \begin{vmatrix} 3 & 5 & -7 \\ 4 & 1 & -12 \\ 2 & 9 & -3 \end{vmatrix}$

$$= 3(-3+108) - 5(-12+24) - 7(36-2)$$

$$= 315 - 60 - 238$$

$$= 17 \neq 0$$

Now the cofactors of D are

$$A_{11} = \begin{vmatrix} 1 & -12 \\ 9 & -3 \end{vmatrix} = -3+108 = 105$$

$$A_{12} = -\begin{vmatrix} 4 & -12 \\ 2 & -3 \end{vmatrix} = -(-12+24) = -12$$

$$A_{13} = \begin{vmatrix} 4 & 1 \\ 2 & 9 \end{vmatrix} = 36-2 = 34$$

$$A_{21} = -\begin{vmatrix} 5 & -7 \\ 9 & -3 \end{vmatrix} = -(-15+63) = -48$$

$$A_{22} = \begin{vmatrix} 3 & -7 \\ 2 & -3 \end{vmatrix} = -9+14 = 5$$

$$A_{23} = -\begin{vmatrix} 3 & 5 \\ 2 & 9 \end{vmatrix} = -(27-10) = -17$$

$$A_{31} = \begin{vmatrix} 5 & -7 \\ 1 & -12 \end{vmatrix} = -60+7 = -53$$

$$A_{32} = -\begin{vmatrix} 3 & -7 \\ 4 & -12 \end{vmatrix} = -(-36+28) = 8$$

$$A_{33} = \begin{vmatrix} 3 & 5 \\ 4 & 1 \end{vmatrix} = 3-20 = -17$$

$$\therefore Adj(A) = \begin{pmatrix} 105 & -12 & 34 \\ -48 & 5 & -17 \\ -53 & 8 & -17 \end{pmatrix}^T = \begin{pmatrix} 105 & -48 & -53 \\ -12 & 5 & 8 \\ 34 & -17 & -17 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{Adj(A)}{D} = \frac{1}{17}\begin{pmatrix} 105 & -48 & -53 \\ -12 & 5 & 8 \\ 34 & -17 & -17 \end{pmatrix}$$

Now from ①, we get

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{17}\begin{pmatrix} 105 & -48 & -53 \\ -12 & 5 & 8 \\ 34 & -17 & -17 \end{pmatrix}\begin{pmatrix} 13 \\ 6 \\ 20 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{17}\begin{pmatrix} 105\times 13 - 48\times 6 - 53\times 20 \\ -12\times 13 + 5\times 6 + 8\times 20 \\ 34\times 13 - 17\times 6 - 17\times 20 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{17}\begin{pmatrix} 17 \\ 34 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$\therefore x = 1,\ y = 2$ and $z = 0$.

Ans

H-W. Solve the following system of equ's using matrices

(i)
$5x - 6y + 4z = 15$
$7x + 4y - 3z = 19$
$2x + y + 6z = 46$
(ans: $x = 3, y = 4, z = 6$)

(ii)
$2x - 3y + 4z = 1$
$3x + 4y - 5z = 10$
$5x - 7y + 2z = 3$
(ans: $x = 2, y = 1, z = 0$)