Fuzzy Systems
Lecture 5

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Fuzzy Set and Logic

- ✓ Fuzzy logic was first introduced by **Lotfi A. Zadeh** in 1965, where the term Fuzzy means something not clear or vague or hazy.

- ✓ In Boolean system the result is either 1 (true) or 0 (false) but Fuzzy logic provides the truth value between 0 and 1.

- ✓ Fuzzy logic actually deals with *linguistic variables*, which is words or sentences in artificial or natural language. For example, 'Temperature' is a linguistic variable and to qualify the temperature, we use the terms such as 'very cold', 'cold, 'cool', 'warm', 'hot' 'very hot' etc. These are the *linguistic values* (not numerical values) of the temperature. The terms such as 'very cold', 'cold, 'cool', 'warm', 'hot' 'very hot' etc. are linguistic values of temperature rather than numerical values -20^0C, 15^0C or 20^0C.

- ✓ The degree of truth of input or output linguistic values of a Fuzzy system is defined by *membership function* (MF). For example the numerical value of linguistic variable -25^0C provides the *degree of MF* of the corresponding linguistic values as: [very-cold cold cool warm hot very-hot] = [0.7 0.3 0 0 0 0].

Crisp Set and Fuzzy Set

- ✓ In a *crisp set*, an element is either a member of the set or not; but Fuzzy sets, allow elements to be partially member of a set. The *degree of membership* can be in range of 0 to 1 determined by MFs.

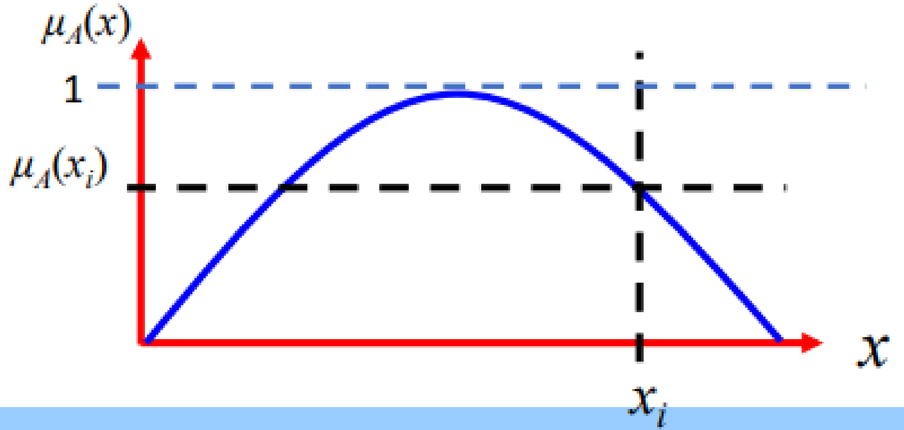
- ✓ In classical set also called crispy set $A = \{x_1, x_2, x_3, \ldots, x_n\}$ is defined by a function $f_A(.)$ called *characteristics function*, to declare whether an element belongs to set A or not expressed as,

$$f_A(x_i) = \begin{cases} 1, x_i \in A \\ 0, x_i \notin A \end{cases}$$

There is not concept of partial membership of an element in crisp set.

- ✓ If U is a universal set or *universe of discourse*, then the Fuzzy set A associated with members of U is written as a collection of ordered pairs as, $A = \{(x_i, \mu_A(x_i)) \mid x_i \in U\}$

- ✓ where $\mu_A(x)$ is called membership function (MF) and for a particular element $x_i \in U$, $\mu_A(x_i)$ is called *grade of membership function* has the value in the range of [0, 1]. Actually, MF quantifies the degree of belongingness of x_i to A.



The Fuzzy membership function has the following properties on Fuzzy set A and B:

$$\mu_{A \cup B}(x_i) = max(\mu_A(x_i),\ \mu_B(x_i))$$

$$\mu_{A \cap B}(x_i) = min(\mu_A(x_i),\ \mu_B(x_i))$$

$$\mu_{A^c}(x_i) = 1 - \mu_A(x_i)$$

Example-1

Given, U = $\{p, q, r, s\}$,where A and B are the Fuzzy sets associated with U. Here $\mu_A(p) = 0.2$, $\mu_A(q) = 0.7$, $\mu_A(r) = 0.18$, $\mu_A(s) = 0.15$, $\mu_B(p) = 0.3$, $\mu_B(q) = 0.6$, $\mu_B(r) = 0.15$, $\mu_B(s) = 0.1$. Write the Fuzzy sets $A \cup B$, $A \cap B$, A^C and B^C.

Ans.

$A \cup B = \{(p, \max(0.2, 0.3)), (q, \max(0.7, 0.6)), (r, \max(0.18, 0.15)), (s, \max(0.15, 0.1))\}$

$= \{(p, 0.3), (q, 0.7), (r, 0.18), (s, 0.15)\}$

$A \cap B = \{(p, \min(0.2, 0.3)), (q, \min(0.7, 0.6)), (r, \min(0.18, 0.15)), (s, \min(0.15, 0.1))\}$

$= \{(p, 0.2), (q, 0.6), (r, 0.15), (s, 0.1)\}$

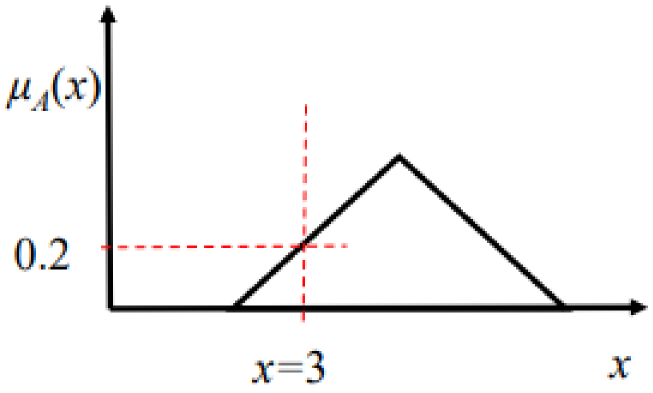
$A^C = \{(p, 1\text{-}0.2), (q, 1\text{-}0.7), (r, 1\text{-}0.18), (s, 1\text{-}0.15)\}$

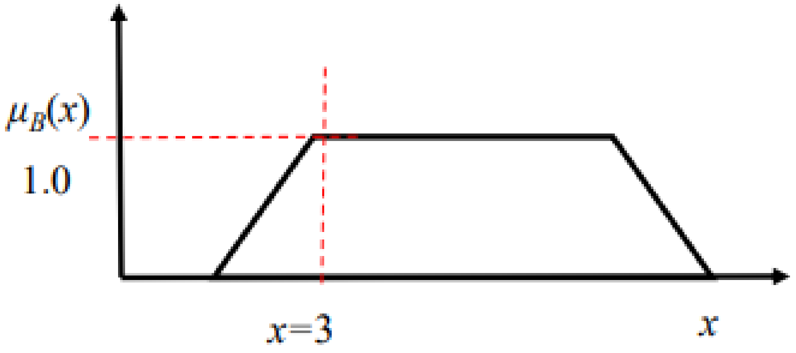
$= \{(p, 0.8), (q, 0.3), (r, 0.82), (s, 0.85)\}$

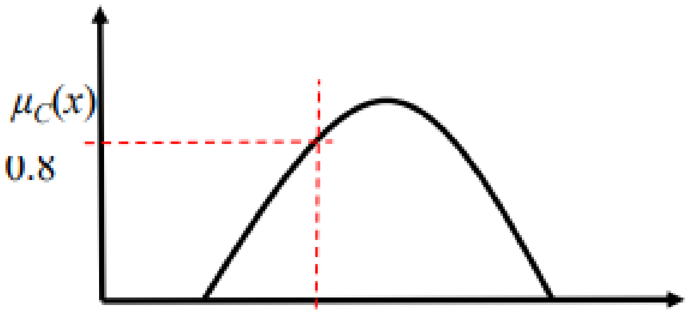
$B^C = \{(p, 0.7), (q, 0.4), (r, 0.85), (s, 0.9)\}$

Example-2

Draw triangular, trapezoidal and Gaussian MF against Fuzzy sets A, B and C then determine $\mu_{A \cup B \cup C}(x_i)$ and $\mu_{A \cap B \cap C}(x_i)$graphically taking x_i =3.







Ans. $\mu_{A \cup B \cup C}(x_i = 3) = max(0.2,\ 1.0,\ 0.8) = 1.0$

$\mu_{A \cap B \cap C}(x_i = 3) = min(0.2,\ 1.0,\ 0.8) = 0.2$

- ✓ Another notation widely used in defining Fuzzy set A is expressed as,

$$A = \frac{a_1}{x_1} + \frac{a_2}{x_2} + \frac{a_3}{x_3} + \ldots \ \ldots \ \ldots + \frac{a_1}{x_n} = \sum_{i=1}^{n} \frac{a_i}{x_i}$$

, where X is the universe of discourse and $x_1, x_2, x_3, \ldots \ \ldots \ \ldots, x_n$ are the elements associated with the Fuzzy set A.

- ✓ Here the division is not the mathematical division, a_i indicates the *grade of membership* of x_i in A.

- ✓ More generalized form is: $A = \{(x_i, \mu_A(x_i) \mid x_i \in U\}$ or $A = \sum_{i=1}^{n} \frac{\mu_A(x_i)}{x_i}$, where $\mu_A(x_i) = a_i$

- ✓ The summation is not the mathematical summation, it provides collection of (x_i, a_i) pairs. Above definition is applicable for discrete elements of a set.

Similarly, when X is an interval of continuous real number then Fuzzy set A is defined as,

$A = \int_X \frac{\mu_A(x)}{x}$, where the integration does not have usual meaning used in mathematics, its indicates the collection of pairs $(x, \mu_A(x))$.

For example the Fuzzy set $A=\{(x_1, 0.1), (x_2, 0.7), (x_2, 0.65), (x_4, 0.2)\}$ can be written in another notation like, $A = \frac{0.1}{x_1} + \frac{0.7}{x_2} + \frac{0.65}{x_3} + \frac{0.2}{x_4}$

Linguistic Variable

The name of a Fuzzy set is a linguistic variable (called Fuzzy variable) may have a set of values called linguistic values or states of a variable. For example, a Fuzzy variable ‘Speed’ can be further divided based on requirement in real life or common sense into ‘Fast’, ‘Medium’ and ‘Slow’ called states of ‘Speed’. Few examples of Fuzzy variable and states ae shown in table below.

Linguistic variable (Fuzzy variable)	**Linguistic values (States of a variable)**
Temperature	Very low, Low, Medium, High
Height	Tall, Medium, Short
Speed	Fast, Medium, Slow
Age	Child, Young, Old

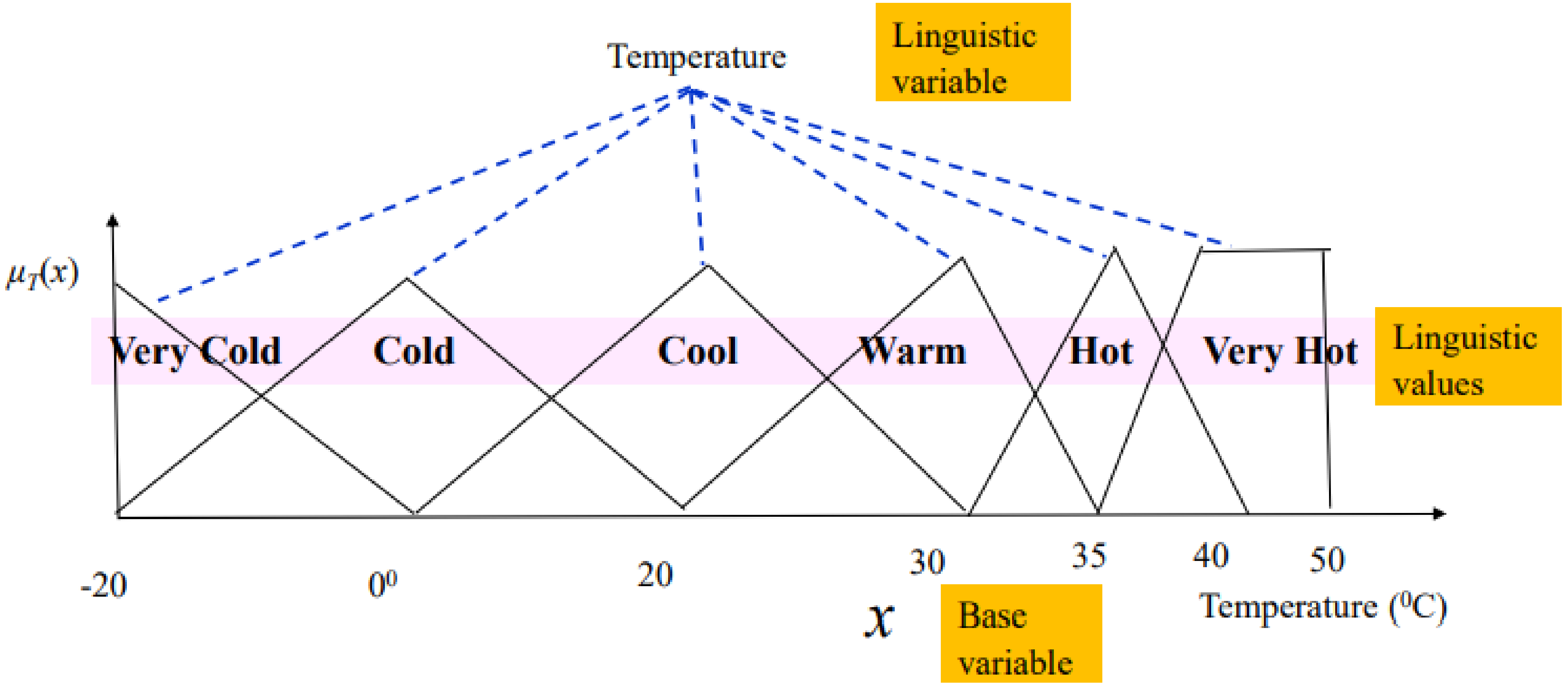
- Each **linguistic variable** is defined in terms of a **base variable**, whose values are assumed to be real numbers within a specific interval of real numbers. A base variable is a variable in the classical sense.

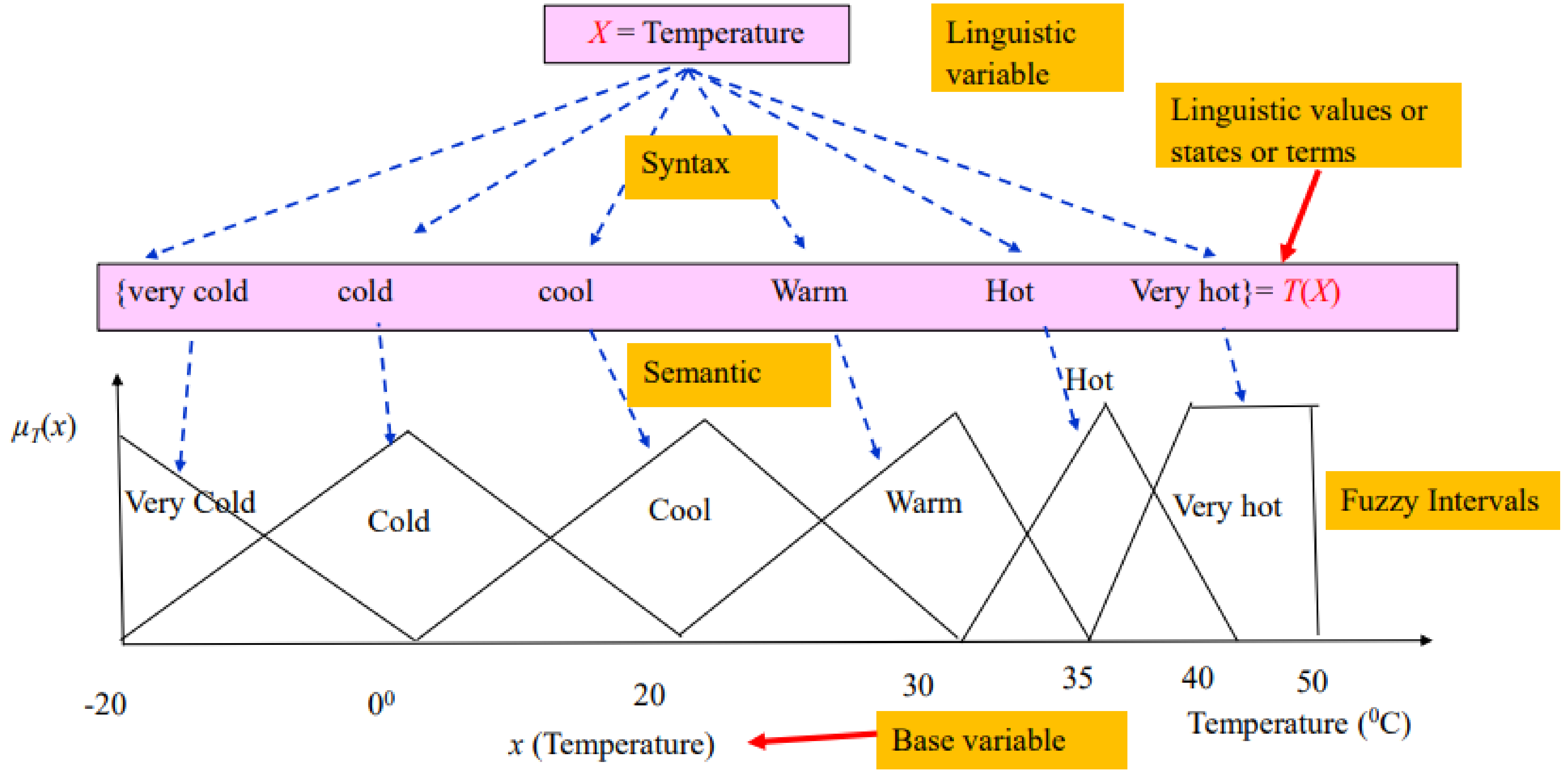
- For example **Age** is a **linguistic variable** and its **linguistic values/states** are: *baby*, *child*, *young*, *mid-age* and *old*.

- The range of base variable for **linguistic variable** Age is 0 to 100 years.

- Each Fuzzy value/state under a Fuzzy variable covers an interval/range of base variable. For example the range of base variable for *baby* is 0 to 5 years, for *child* 3 to 19 years, for *young* 15 to 40 years etc.
- There may be some overlap of above sub-range of base variable against **linguistic values**.

- ✓ The linguistic variable, Temperature, is expressed by another variable x, where the value of x is real number within specific range (universe of real numbers) called base variable for example the range of x is [-20^0C to 50^0C] shown as the independent variable (or argument of MF) of MF.
- ✓ The linguistic values of the variable Temperature is: 'very cold', 'cold, 'cool', 'warm', 'hot' and 'very hot', each has a specific range of real number under the universe of base variable (indicated by x). The linguistic variable, linguistic values, MFs of linguistic values and base variable is shown in a single figure in fig.2.





A linguistic variable is characterized by a quintuple: (X, $T(X)$, U, G, M)

Where, X is the name of the variable (usually word)

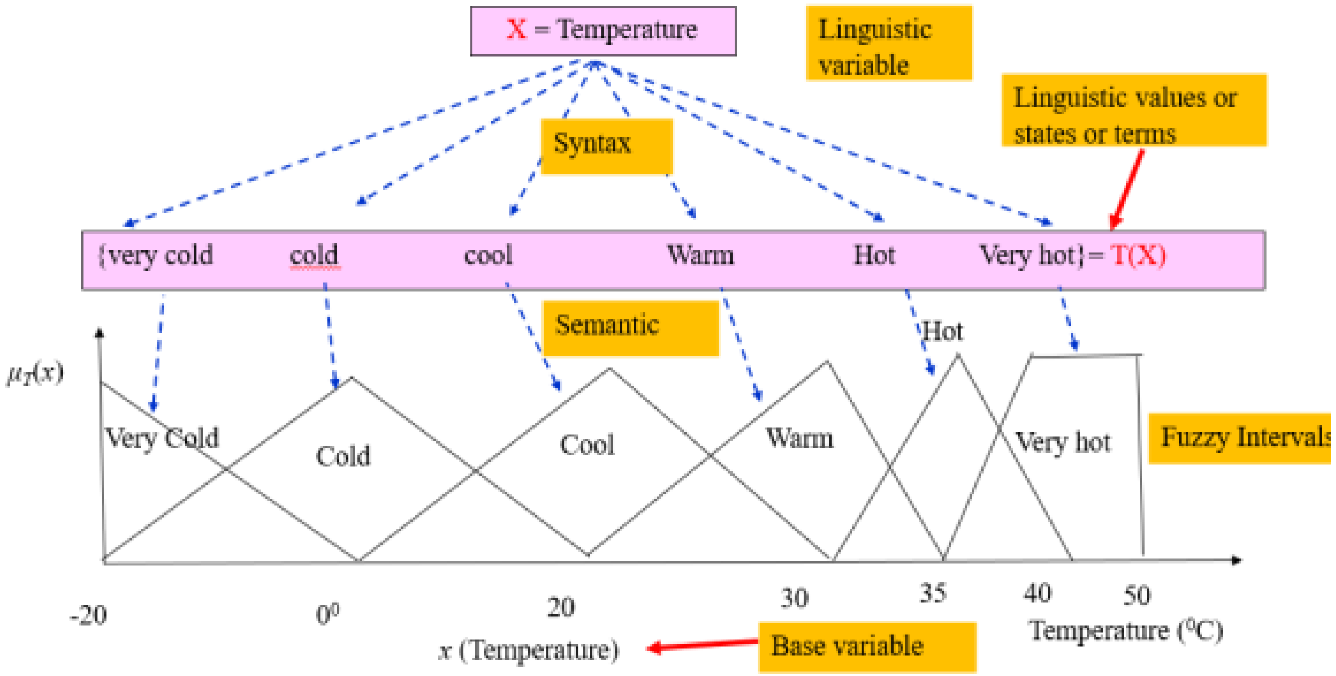
$T(X)$ is the set of name of linguistic values of the variable X. Each linguistic value covers a range of base variable u under U.

U is the universe of discourse (range of real number)

G is a collection of **syntax rules** (a grammar) that produces correct expressions in $T(X)$ or generates names of linguistic values.

M is a set of **semantic rules** that maps $T(X)$ into MF under U so that we find the meaning of linguistic values.

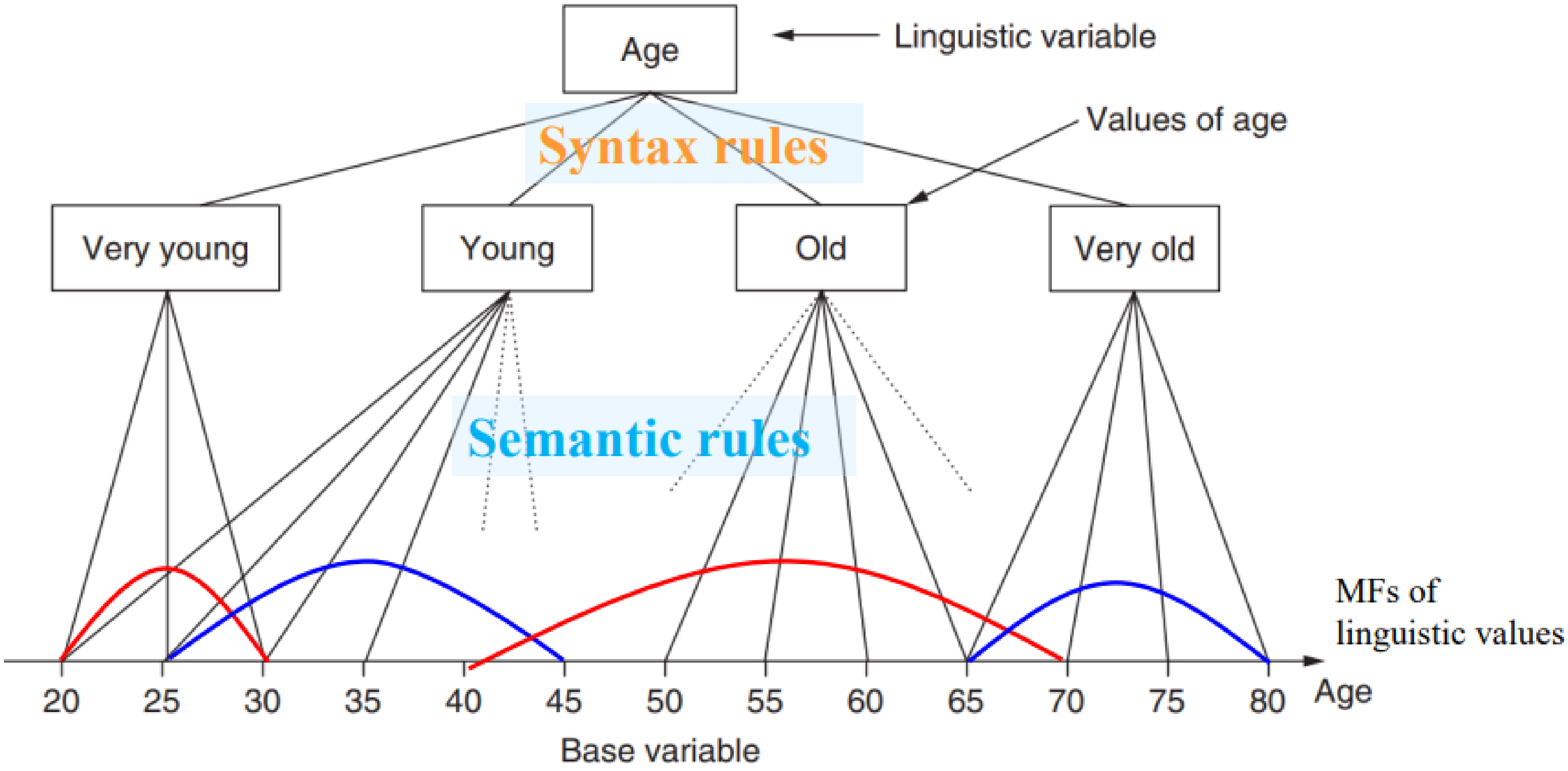
- ✓ To assign the linguistic variable (the name of the variable is Temperature) to its linguistic values (very cold, cold, cool, Warm, Hot, Very hot), we need some syntax rules. Based on the syntax rule we can say which part of temperature now I feel.



- ✓ Again, we need to measure the linguistic values: very cold, cold, cool, Warm, Hot, Very hot etc. numerically i.e. we have assign the range of temperature in ^{0}C for each of them. For example, very cold means the range [-20^{0}C to 0^{0}C].

- ✓ Therefore, we need some semantic rules to assign each linguistic value to its numerical range (range of real number) based on MF so that we can measure it. The mapping of linguistic value to its MF is considered as the semantic rule as shown in fig. above.

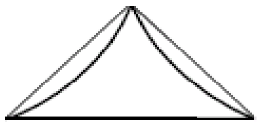
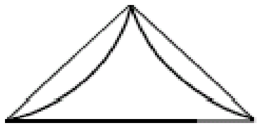
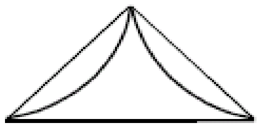
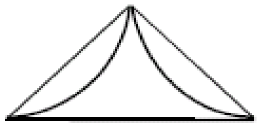
Example of characterization of linguistic variable by quintuple.



A set of **semantic rules**, which assign to each linguistic term/value its meaning in terms of an appropriate fuzzy interval defined on the range of the **base variable**.

Hedge

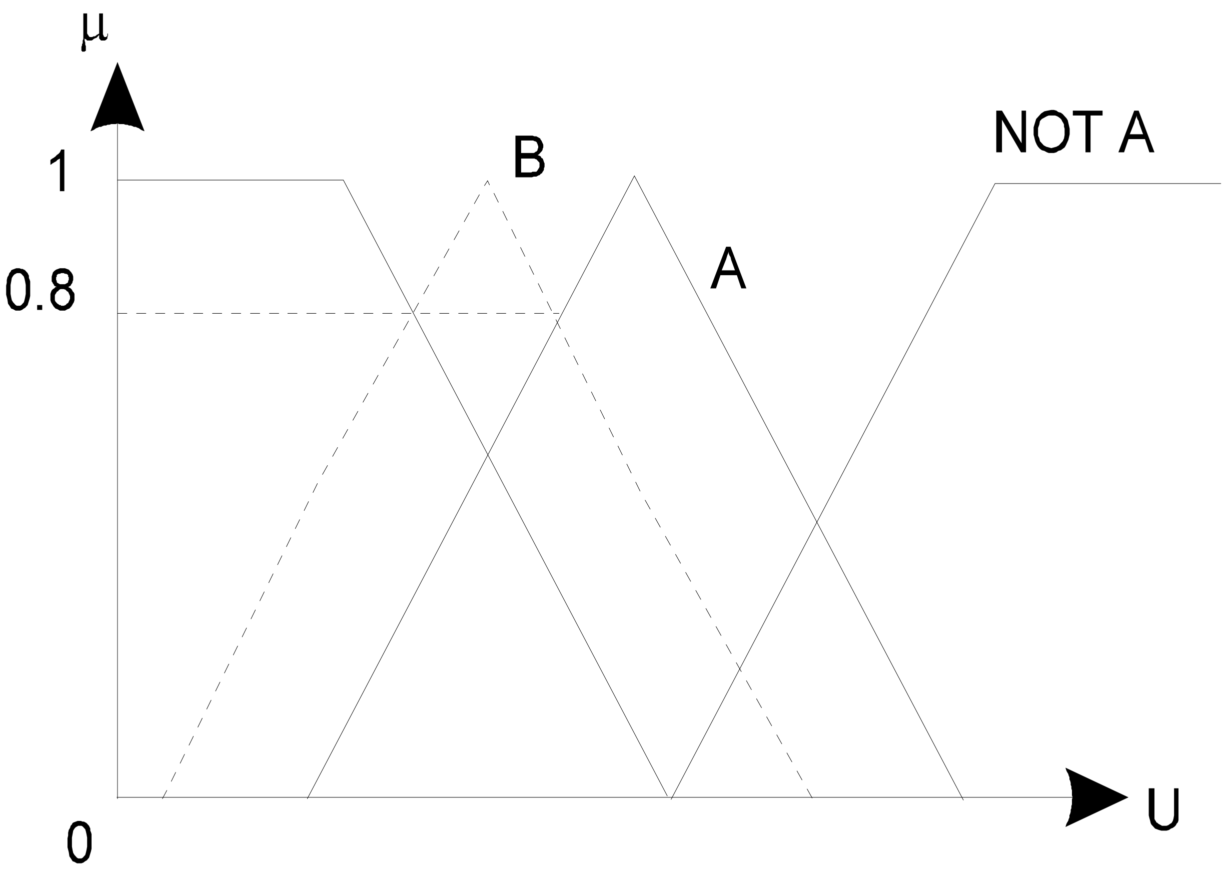
- A linguistic variable carries with it the concept of fuzzy set qualifiers, called hedges.
- Hedges are terms that modify the shape of fuzzy sets. They include adverbs such as very, somewhat, quite, more or less and slightly.

Hedge	*Mathematical Expression*	*Graphical Representation*
A little	$[\mu_A(x)]^{1.3}$	
Slightly	$[\mu_A(x)]^{1.7}$	
Very	$[\mu_A(x)]^{2}$	
Extremely	$[\mu_A(x)]^{3}$	

Fuzzy Modifier

- A linguistic modifier is an operation that modifies the meaning of a term.
- For example, in the sentence, very close to 0, the word *very* modifies *Close to 0* which is a fuzzy set. Examples of other modifiers are a little, more or less, possibly, definitely.
- Very a=a^2 Somewhat a=Morl a=a^1/2
- Extremely a =a^3 Little bit a = a^1.3 Slightly a=a^1/3
- Very Very a= a^4 Slightly=(a)^1.7
- Young=[1 0.36 0.01 0 0]
- Very very young = young^4 =[1 0.13 0 0 0]

Operations with fuzzy sets...



- A graphical representation of similarity S (the black area) between two fuzzy sets B and A based on possibility P and necessity N measures (see the formulas in the text).

Operation with fuzzy sets

- Dilation: Increases the degree of a membership.

$DIL(A)= [\mu_A(x)]^{1/2}$

- Concentration: decreases the degree of a membership function.

$CON(A)=[\mu_A(x)]^2$

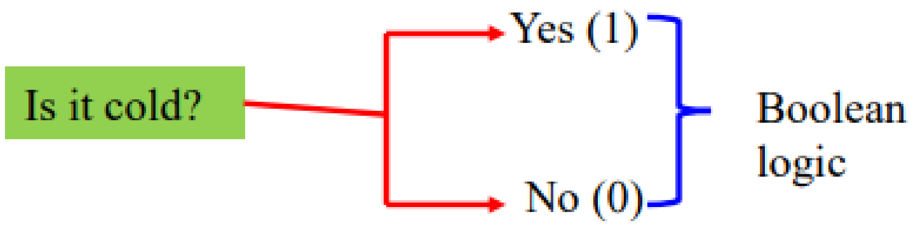
- Normalization: normalize the membership function.

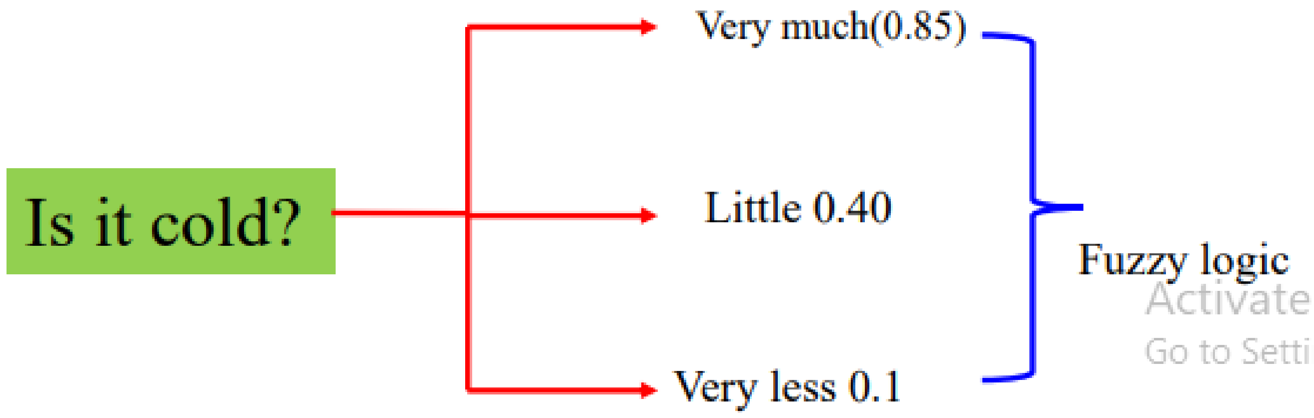
$NORM(A)= [\mu_A(x)]/ \max[\mu_A(x)]$

Fuzzy Logic

- ✓ Crisp logic (crisp) is the same as Boolean logic, which deals with absolute true (logic 1) or absolute false (logic 0). The membership function of crisp logic is rectangular pulse with amplitude of 1 and absent of pulse i.e. amplitude of 0.

- ✓ The Fuzzy logic deals with logical reasoning with vague or imprecise statements, where the truth of the statement is not binary (true or false), but the degree of truth lies between 0 and 1. For example: ‘Karim is Young’. He may be ‘very young’ or ‘Moderately Young’ or ‘Partially Young’ or ‘Not Young’.





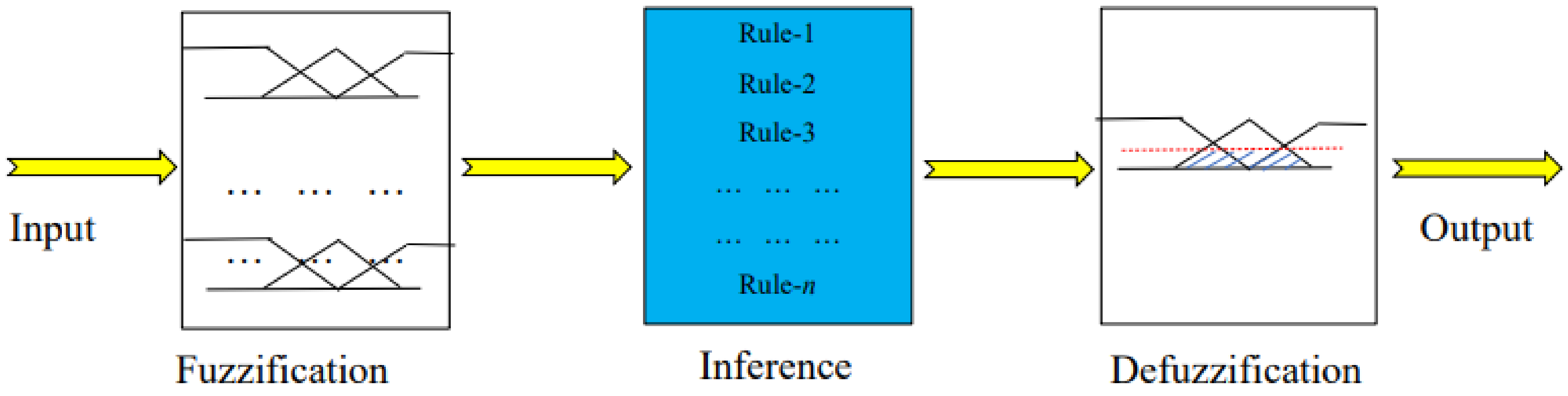
- ✓ Fuzzy logic is also called many-valued logics since variables having truth value in the range of 0 to 1 and in linguistic form it may be written 'Fully true', 'Moderately true', 'Partially true', 'Not true' etc.

- ✓ Let us consider a 'If-then statements' logical reasoning: '*If Kabir is young and Kabir is rich then he will get the loan*'. The truth of above reasoning is determined by the truth values of its component sentences.

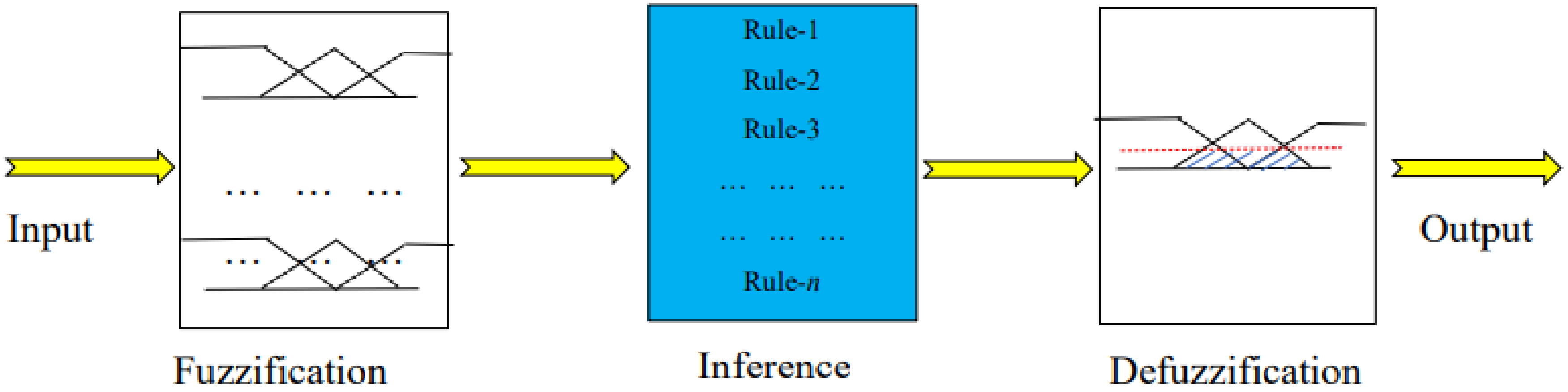
Fuzzy Inference System (FIS)

- ✓ To solve Engineering problem we use Fuzzy Inference System (FIS) consists of three parts: Fuzzification, Inference (rule based) and De-fuzzification like fig.below.

- ✓ *Fuzzification* is the process of converting a crisp input value or conventional numerical data to a fuzzy value based on our knowledge or using grade of MF. For example, crisp value of -20^{0}C to converted to linguistic value 'very low temperature' with grade of MF of 0.8.



✓ *Defuzzification* is the reversed way i.e. Fuzzy to crisp conversion. For example linguistic value: 'very low speed' with grade of MF 0.25 is converted to 120 rpm.



✓ *Fuzzy inference* is the process of formulating the mapping from a given input to an output using fuzzy logic. Here a combination of 'input linguistic values' is converted to a 'output linguistic value' based on *if-then* rule. For example 'if *temperature* is low and *humidity* is high then *electrical load* will be moderate'.

Input Fuzzy variables or linguistic variables

Output Fuzzy variable or linguistic variable

Here *high*, *low, moderate* etc. are linguistic values of linguistic variables.

An *argument* in propositional logic is a sequence of propositions. All but the final (other than the final) proposition in the argument are called *premises* and the final proposition is called the *conclusion*.

Also called antecedent

Also called Consequence

If *premises* then *conclusion*.

Combination of statements (or propositions) based on Fuzzy input variables and values.

Statements (or proposition) based on Fuzzy output variables and values.

Fuzzy rules follow the format of above argument i.e. each rule makes an inference about outcome (or conclusion) based on logical combinations (and, or, not etc.) premises.

Example-3

In electrical load forecasting we have the following Fuzzy inputs, Fuzzy output and states of Fuzzy variables as: Fuzzy input variables are: **Temperature, Humidity and Season**

States of Temperature→ Low, Medium and High

States of Humidity→ Dry and Wet

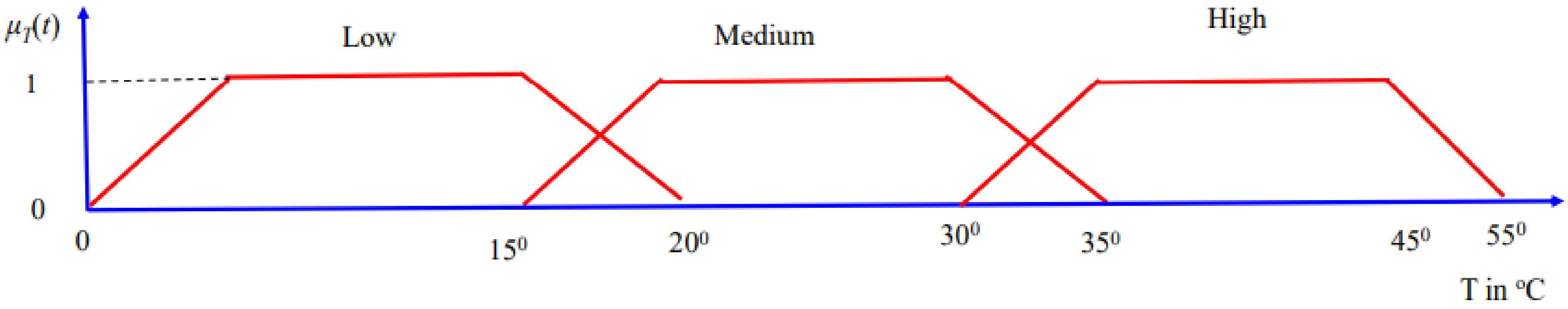
States of Season→ Summer, Raining, Winter and Spring

Fuzzy output variable is: Electrical Load

States of Load→ Low, Medium, High

Each state of input and output variable has a range of numerical value with some strength or amplitude in the range of [0, 1]. The phenomenon is shown by membership function (MF) like below.

States of Temperature→ Low, Medium and High



The MFs can be triangular, trapezoidal, gaussian etc.

To relate input and output we apply Fuzzy rules. Example of four Fuzzy rules applicable for electrical load forecasting is given below:

Premises consist of input Fuzzy valuables and values

Conclusion consist of output Fuzzy valuables and value

If 'Temperature is Low' and 'Humidity is Dry' and 'the Season is Winter' then 'Electrical Load will be Low'.

If 'Temperature is Low' and 'Humidity is Wet' and 'the Season is Spring' then 'Electrical Load will be Medium'

If 'Temperature is High' and 'Humidity is Wet' and 'the Season is Summer' then 'Electrical Load will be High'

If 'Temperature is Medium' and 'Humidity is Wet' and 'the Season is Rainy' then 'Electrical Load will be Medium'

Classification of Fuzzy Rule

Three main types of Fuzzy rule: Mamdani, Sugeno and Tsukamoto Fuzzy rules are used in in different application, where they depend on types of MFs and Defuzzification techniques. Here we only consider Mamdani and Sugeno rules.

Mamdani Rule

IF x is A and y is B THEN z is C

Here, x and y are the input variable and z is output variable. The linguistic values A, B and C are associated with linguistic variables x, y and z. Here both antecedents and consequent have Fuzzy number. Above rule is in the form of *multiple input single output* (MISO).

The Mamdani rule needs both input and output MF and defuzzification is done in several ways like centroid method, center of sum method, height method, weighted average method, center of largest area method, middle of maxima etc.

Takagi-Sugeno Fuzzy Rule

This model is known as **Sugeno Fuzzy rule,** was proposed by Takagi, Sugeno and Kang in 1985. Format of this rule is given as:

$$IF\ x\ is\ A\ and\ y\ is\ B\ THEN\ z = f(x,y)$$

Here, *A* and *B* are Fuzzy values or linguistic value or Fuzzy numbers in antecedents like Mamdani rule and $z = f(x,y)$ is a crisp function in the consequent. For example

Rule-1: If x is A_1 and y is B_1 then $z_1 = a_1x+b_1y+c_1 = f_1(x, y)$

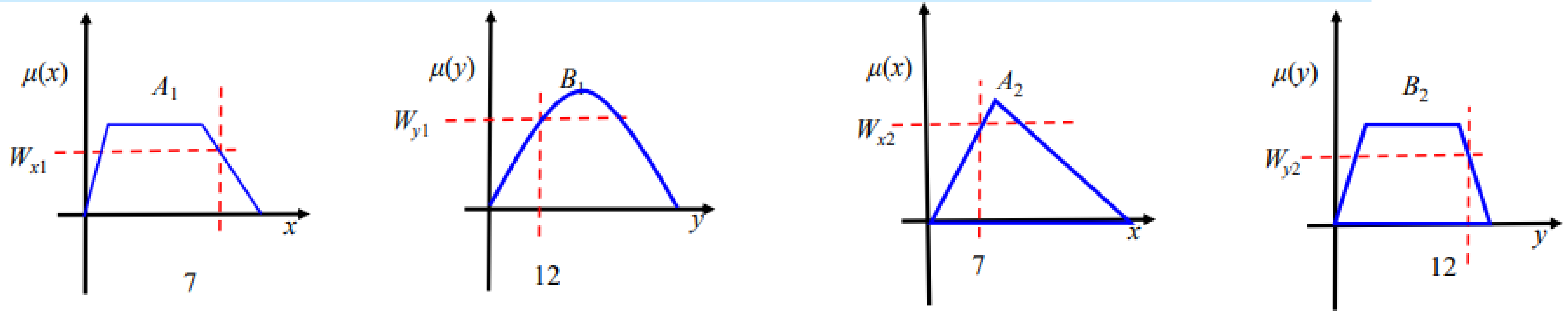
Rule-2: If x is A_2 and y is B_2 then $z_2 = a_2x+b_2y+c_2 = f_2(x, y)$

Mamdani rule can accept numeric as well as Fuzzy inputs but **Sugeno Fuzzy** rule accepts only numeric inputs.

Q. Determine overall output taking $x = 7$ and $y = 12$ on the MFs of fig. below.

Ans. Here $z_1 = a_1x+b_1y+c_1 = 7a_1+ 12b_1+c_1$

$z_2 = a_2x+b_2y+c_2 = 7a_2+ 12b_2+c_2$



Rule-1: If x is A_1 and y is B_1 then $z_1 = a_1x+b_1y+c_1 = f_1(x, y)$

Rule-2: If x is A_2 and y is B_2 then $z_2 = a_2x+b_2y+c_2= f_2(x, y)$

For the rule 1, 'If x is A_1 and y is B_1' provides $\min(W_{x1}, W_{y1}) = W_{x1}$

For the rule 2, If x is A_2 and y is B_2' provides $\min(W_{x2}, W_{y2}) = W_{y2}$

The overall output will be,

$z = (z_1 * W_{x1}+z_2 * W_{y2})/(W_{x1}+ W_{y2})$

$= \{(7a_1+ 12b_1+c_1)* W_{x1}+(7a_2+ 125b_2+c_2)* W_{y2}\}/(W_{x1}+ W_{y2})$

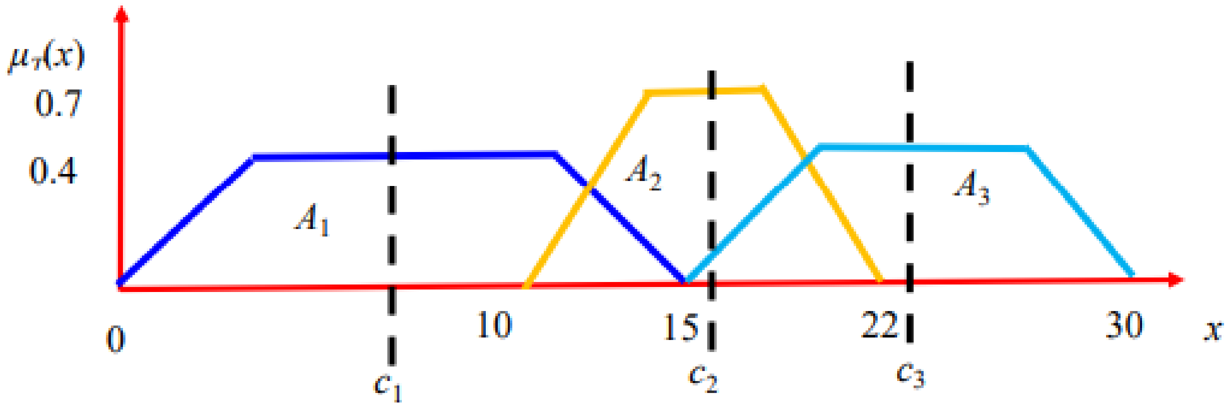
If A_1, A_2, B_1, B_2, W_{x1}, W_{x2}, W_{y1} and W_{y2} are given we can evaluate z, where we don't need output MF hence Defuzzification is less time consuming compared to Mamdani case.

De-fuzzification

In FIS the Fuzzy value is finally converted to crisp value (numerical value), called defuzzification, which is the reversed process of Fuzzification. This section provides four widely used De-fuzzification techniques: center of sum (COS) method, Mean Max membership (Middle of Maxima), weighted average method and centroid method.

Center of sum (COS) method

In center of sum (COS) method individual area $A_1, A_2, A_3, \ldots, A_n$ is weighted by its center $c_1, c_2, c_3, \ldots, c_n$ on the base. The weighted sum $\sum_{i=1}^{n} c_i A_i$ is divided by sum of area $\sum_{i=1}^{n} A_i$ to get the De-fuzzified value x^*. The overlapping area is considered twice in COS whereas centroid method considers the overlapped area only once.



Example-4

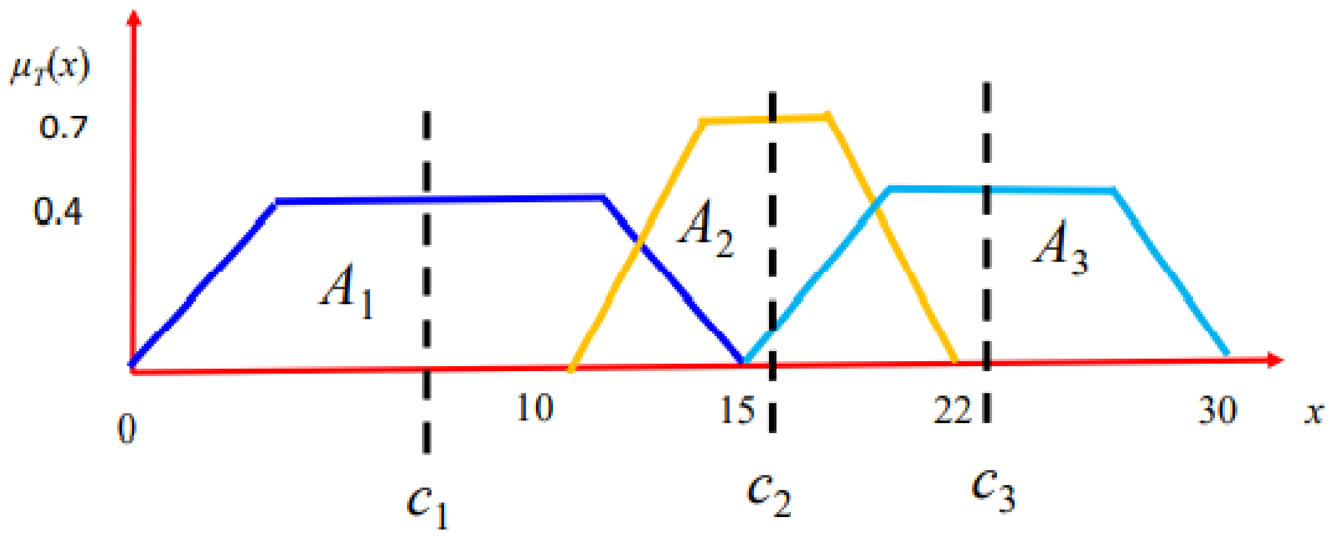
Determine De-fuzzified value x^* of MFs of fig. below using COS, considering overlapped area at most twice.

Here, $A_1 = 0.5*0.4*(15+7) = 4.4$, $c_1 = (15+0)/2 = 7.5$;

$A_2 = 0.5*0.7*(12+4) = 5.6$, $c_2 = (10+22)/2 = 16$;

$A_3 = 0.5*0.4*(15+5) = 4$, $c_1 = (15+30)/2 = 22.5$.

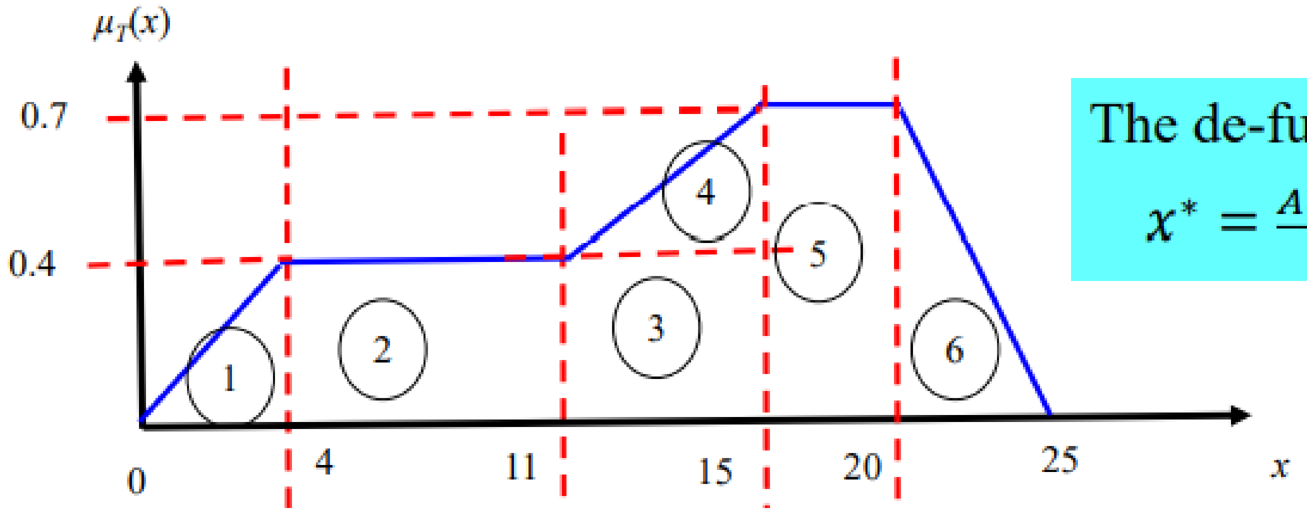
$$x^* = \frac{\sum_{i=1}^{n} c_i A_i}{\sum_{i=1}^{n} A_i} = (4.4*7.5+5.6*16+4*22.5)/(4.4+5.6+4) = 212.6/14 = 15.18$$



Centroid Method:

In this method we have to take the aggregate area, where overlapped area is considered only once. The aggregate area is divided into areas of standard shape of geometry (triangle, trapezium, rectangle etc). If the aggregate area has the segments: A_1, A_2, A_3, ..., A_n and the corresponding centroid (intersection of x-axis and vertical line through the centroid of an area) on the base are found as G_1, G_2, G_3, … , G_n.

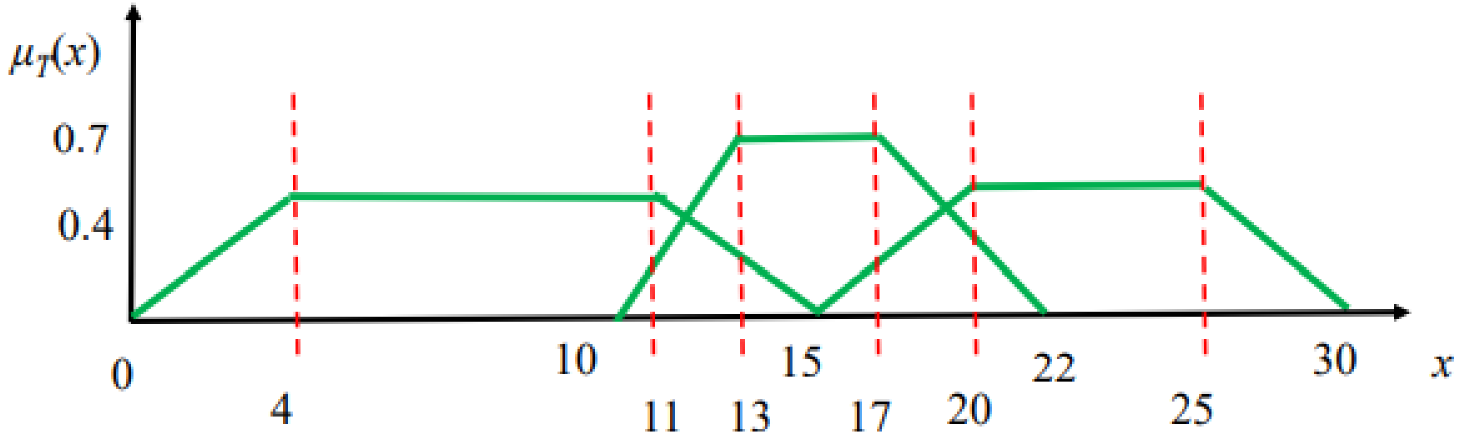
The De-fuzzified value x^* is expressed as, $x^* = \frac{\sum_{i=1}^{n} G_i A_i}{\sum_{i=1}^{n} A_i}$



The de-fuzzified value will be,

$$x^* = \frac{A_1G_1+A_2G_2+A_3G_3+A_4G_4+A_5G_5+A_6G_6}{A_1+A_2+A_3+A_4+A_5+A_6}$$

Mean Max membership (Middle of Maxima):
It is also called mean of maxima (MOM), where mean value of points of highest degree of MFs are taken. Let us determine the De-fuzzified value x* of fig below. Here x* = (13+17)/2 = 15.



Weighted average method:
In weighted average method of defuzzification using,
$x^* = \frac{\sum_{i=1}^{n} w_i c_i}{\sum_{i=1}^{n} w_i}$, where w_i is the degree of ith MF and c_i is corresponding center.
In above figure the De-fuzzified value x^* will be,
= (0.4*7.5+0.7*16+0.4*22.5)/(0.4+0.7+0.4) = 23.2/1.5=15.47

Fuzzification of numerical data

Consider the relation between temperature and speed of fan of a controller with numerical data like table-1. Here temperature is Fuzzy input and speed of fan is Fuzzy output.

SL	Temperature (^{0}C)	Fan speed (rpm)
1	43	100
2	45	86
3	54	70
4	57	66
5	65	38
6	68	25
7	72	12
8	79	0

The first step of FIS is Fuzzification of data i.e. convert the numerical data into Fuzzy variable.

From the table the highest and lowest value of temperature are 43^0 F and 79^0 F respectively. Let us divide temperature into 4 ranges, where each range is named with a Fuzzy variable. States of Temperature or name of Fuzzy variables are→ Cold, Cool, Warm and Hot
Ranges of input variable or temperature are:

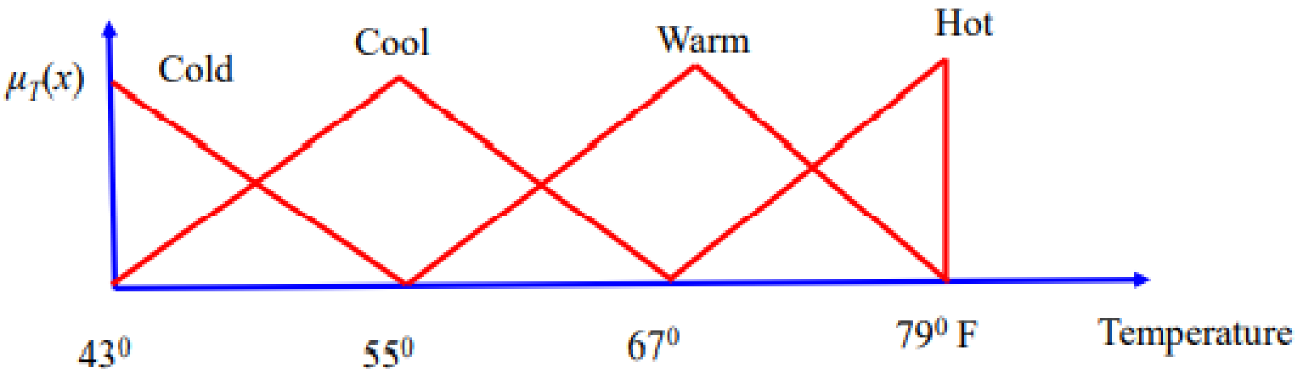
Cold→43^0 F to 55^0 F,

Cool→43^0 F to 67^0 F,

Warm→(43^0 F+ 67^0 F)/2 to 79^0 F → 55^0 F to 79^0 F

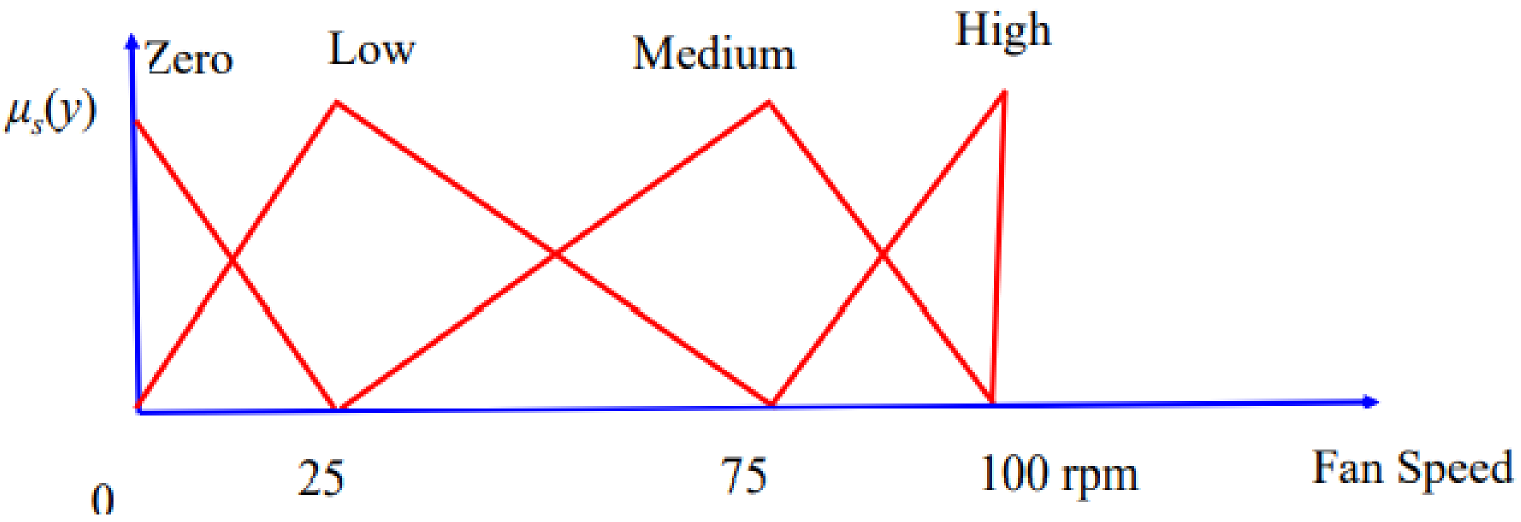
Hot→ (55^0 F+ 79^0 F)/2 to 79^0 F → 67^0 F to 79^0 F

Let us choose triangular MF like the fig. below.



From the table the highest and lowest value of fan speed are 0 and 100 rpm respectively.
Let us divide fan speed into 4 ranges, where each range is named with a Fuzzy variable.
States of Temperature or name of Fuzzy variables are→ Zero, Low, Medium and High.

The MF of output variable is given below.



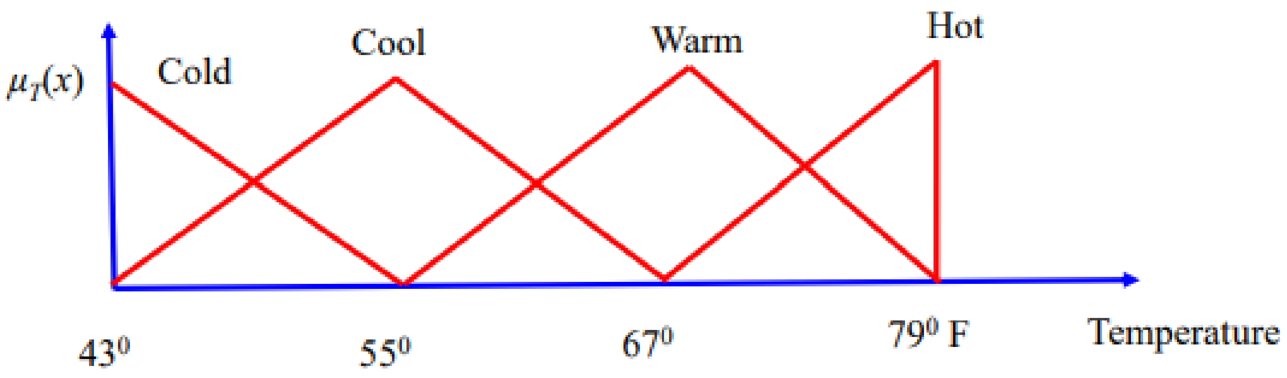
Algorithm

- Define linguistic variables and terms.
- Construct membership functions for them.
- Construct knowledge base of rules.
- Convert crisp data into fuzzy data sets using membership functions. *Fuzzification*‘
- Evaluate rules in the rule base. *Interfaceengine* Combine results from each rule. Composition
- Convert output data into non-fuzzy values. *Defuzzification*

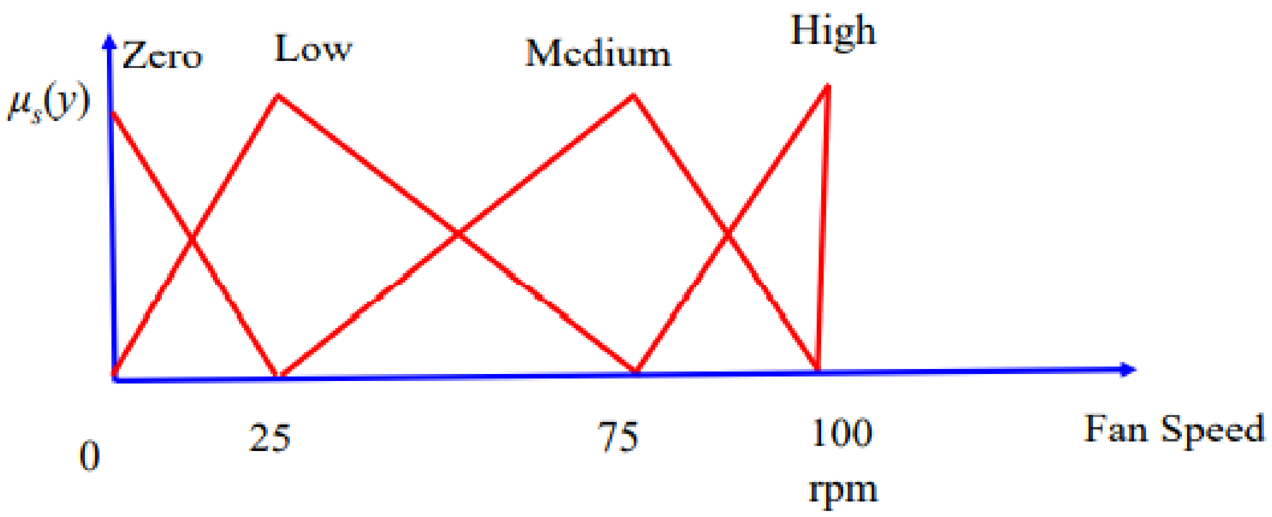
Exercise

SL	Temperature (^0C)	Fan speed (rpm)
1	43	100
2	45	86
3	54	70
4	57	66
5	65	38
6	68	25
7	72	12
8	79	0

Let us convert the table with numerical data into table with Fuzzy variable using MFs.



SL	Temperature (^0C)	Fan speed (rpm)
1	Cold	High
2	Cold	High
3	Cool	Medium
4	Cool	Medium
5	Warm	Low
6	Warm	Low
7	Hot	Zero
8	Hot	Zero



From the table with Fuzzy variable, let us determine Fuzzy Rules.

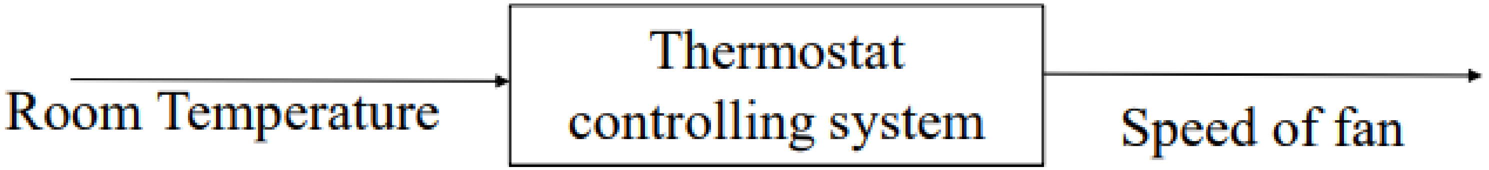
SL	Temperature (^{0}C)	Fan speed (rpm)
1	Cold	High
2	Cold	High
3	Cool	Medium
4	Cool	Medium
5	Warm	Low
6	Warm	Low
7	Hot	Zero
8	Hot	Zero

The Fuzzy rules are:

1. If temp is cold then fan speed is high
2. If temp is cool then fan speed is medium
3. If temp is warm then fan speed is low
4. If temp is hot then fan speed is zero

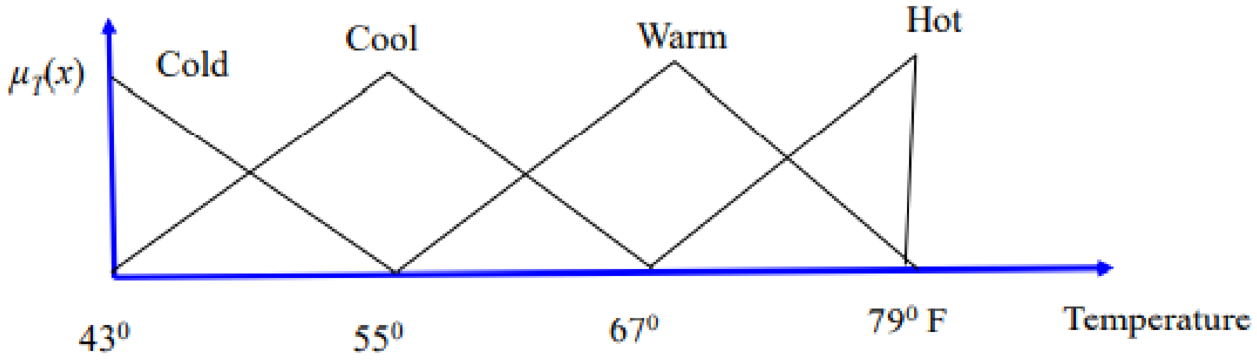
Using Fuzzy rules and MFs of input and output variable we can determine speed of fan for any temperature.

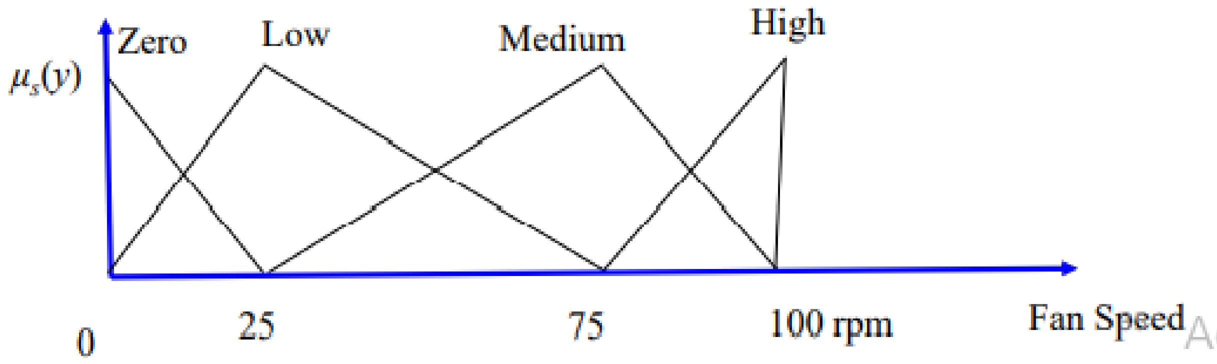
Example-5 Consider a thermostat controlling a fan of heater has the Fuzzy input: room temperature and Fuzzy output: speed of the heater fan. The room temperature is sensed by the sensor and the output of the senor is used to control the speed of the fan.



The Fuzzy rules are:

1. If temp is cold then fan speed is high
2. If temp is cool then fan speed is medium
3. If temp is warm then fan speed is low
4. If temp is hot then fan speed is zero





Determine speed of fan at 70^0F.

Using the concept of equivalent triangle, the degree of MF of Hot at 70^0F is,

$1/h_1 = (79\text{-}67)/(70\text{-}67)$

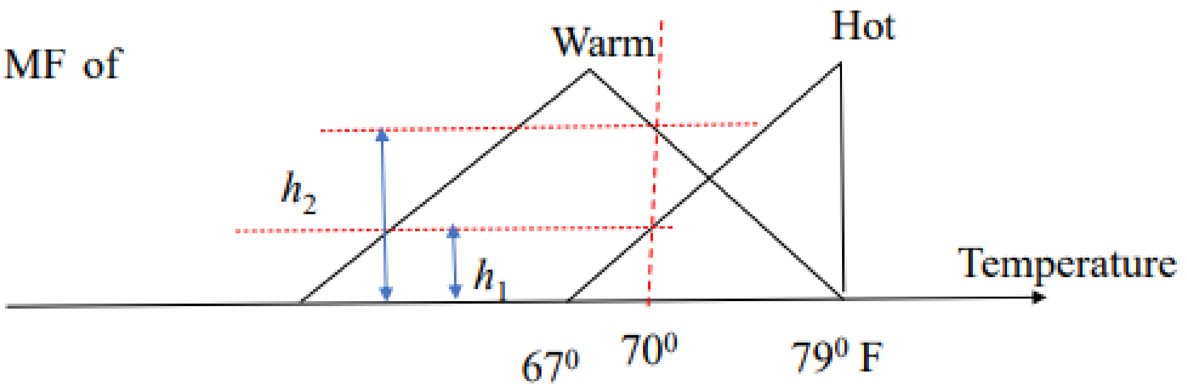
Or, $h_1 = 0.25$ (grade of MF of Hot)

The degree of MF of Warm at 70^0F is,

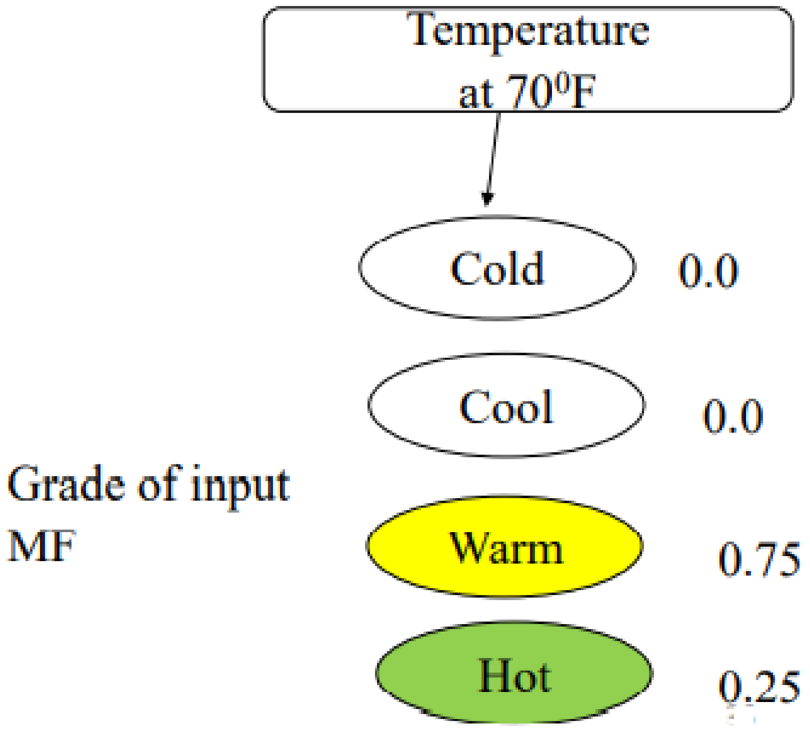
$1/h_2 = (79\text{-}67)/(79\text{-}70)$

Or, $h_2 = 0.75$ (grade of MF of warm)

The degree/grade of MFs for cold and cool are both 0

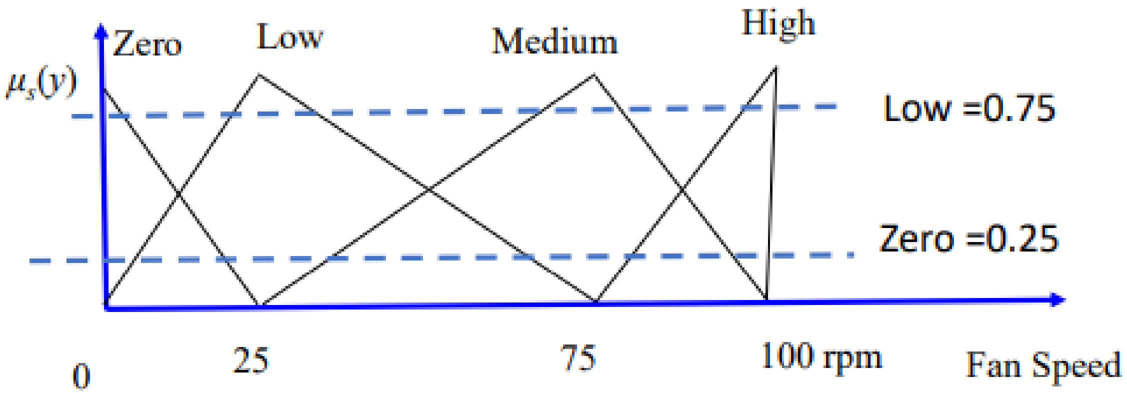


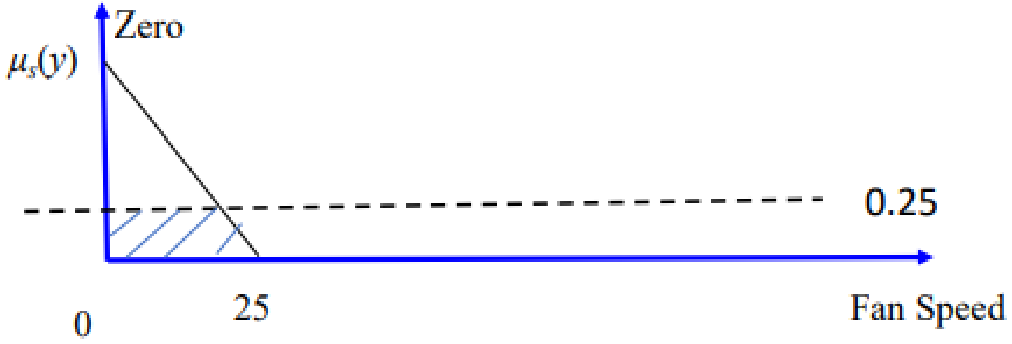
From the input MF at 70^0F,

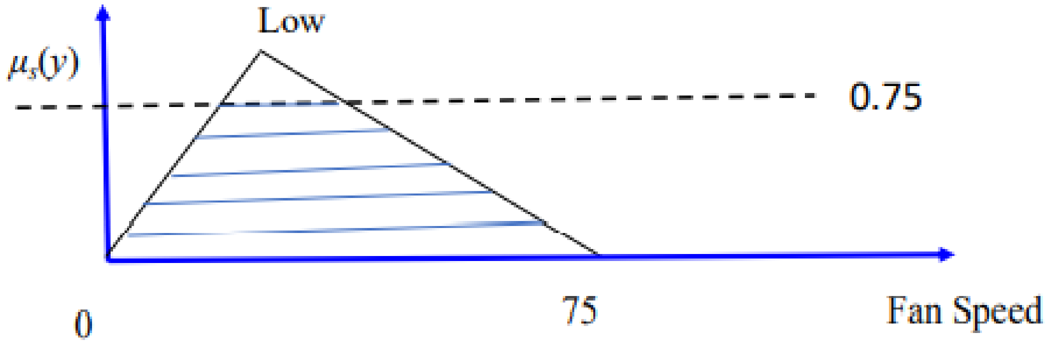


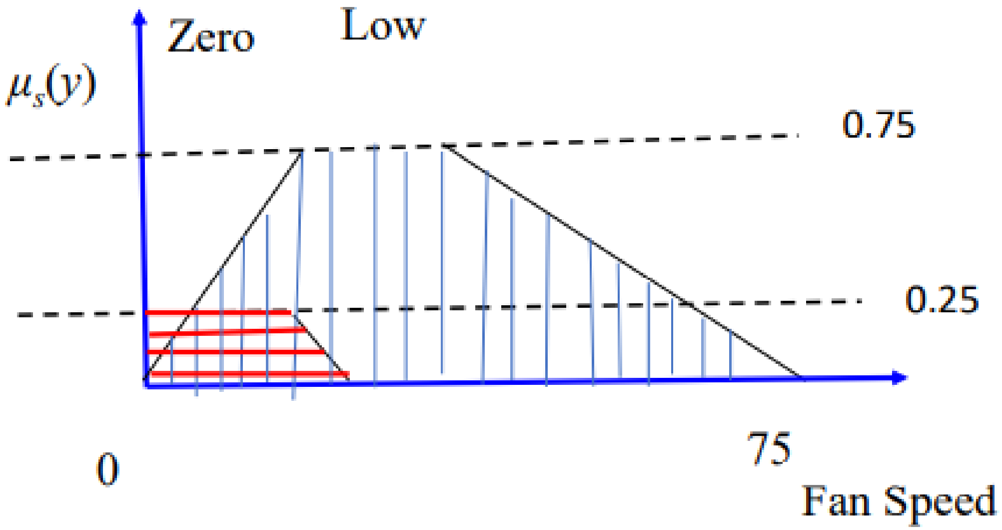
The Fuzzy rules are:

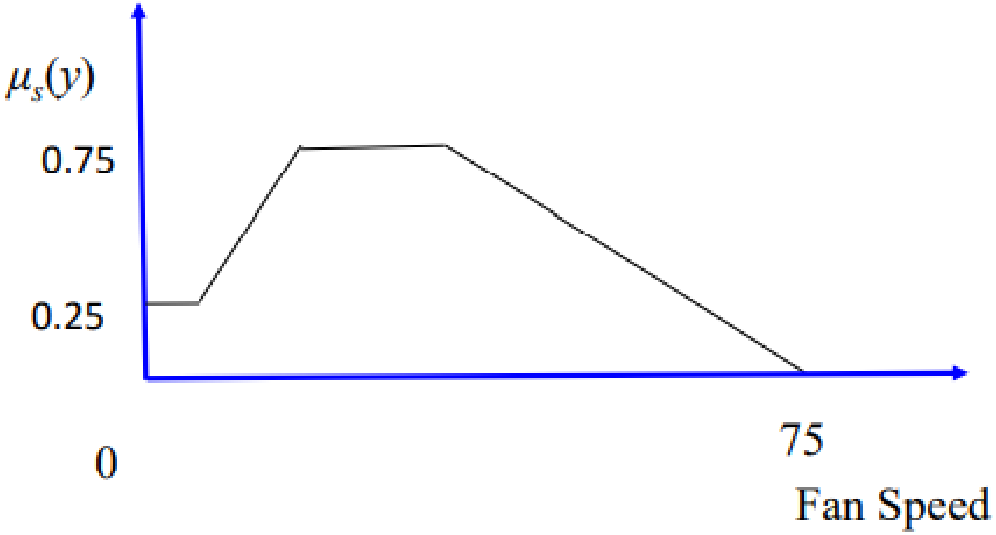
1. If temp is cold then fan speed is high $\rightarrow$ truth value is 0
2. If temp is cool then fan speed is medium $\rightarrow$ truth value is 0
3. If temp is warm then fan speed is low $\rightarrow$ truth value is 0.75
4. If temp is hot then fan speed is zero $\rightarrow$ truth value is 0.25

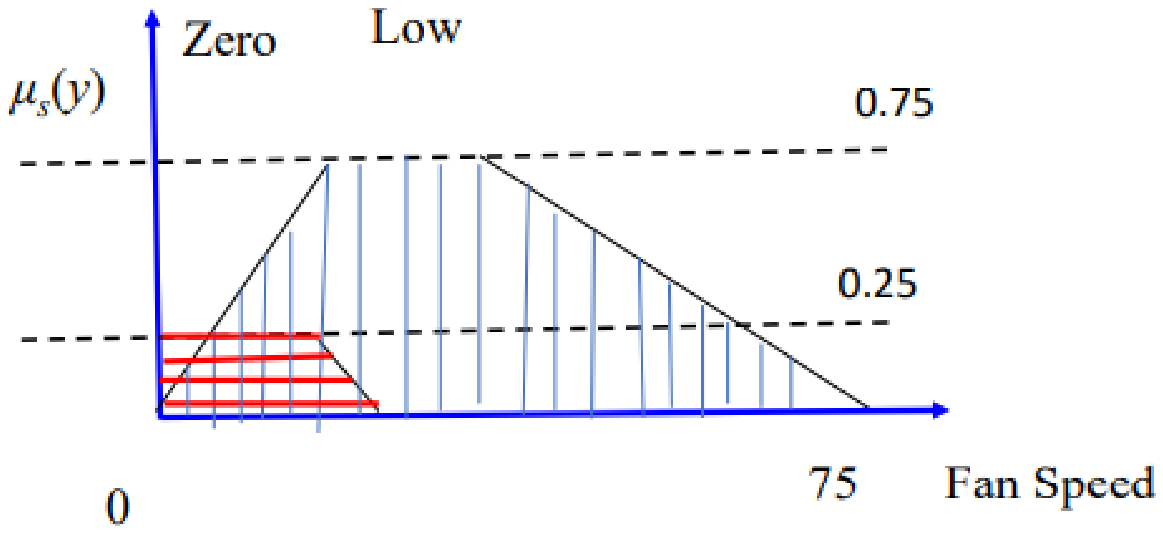

Zero
Low
Medium
High
$\mu_s(y)$
Low =0.75
Zero =0.25
0
25
75
100 rpm
Fan Speed

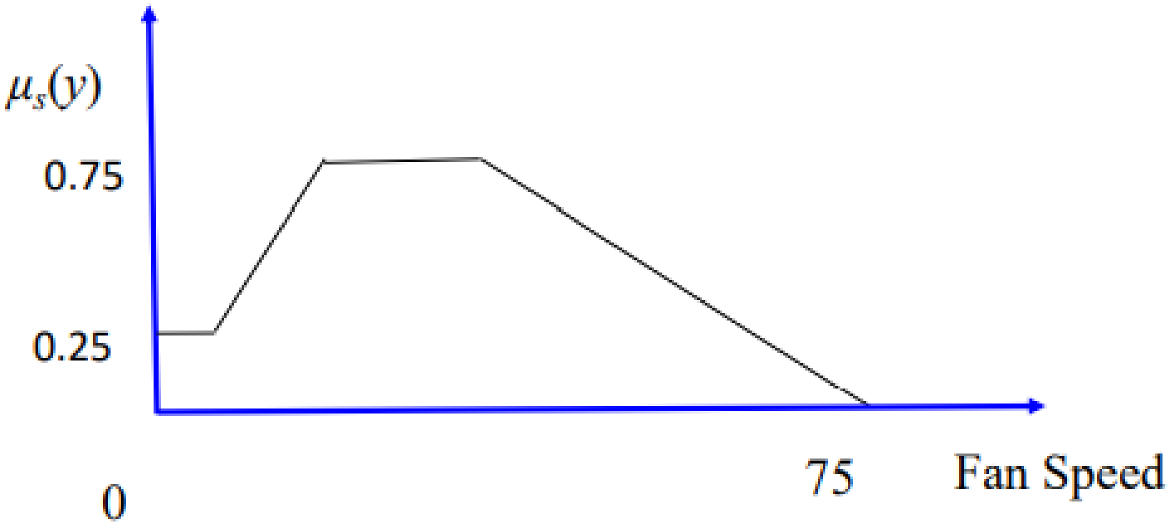

Zero
$\mu_s(y)$
0.25
0
25
Fan Speed


Low
$\mu_s(y)$
0.75
0
75
Fan Speed


Zero
Low
$\mu_s(y)$
0.75
0.25
0
75
Fan Speed

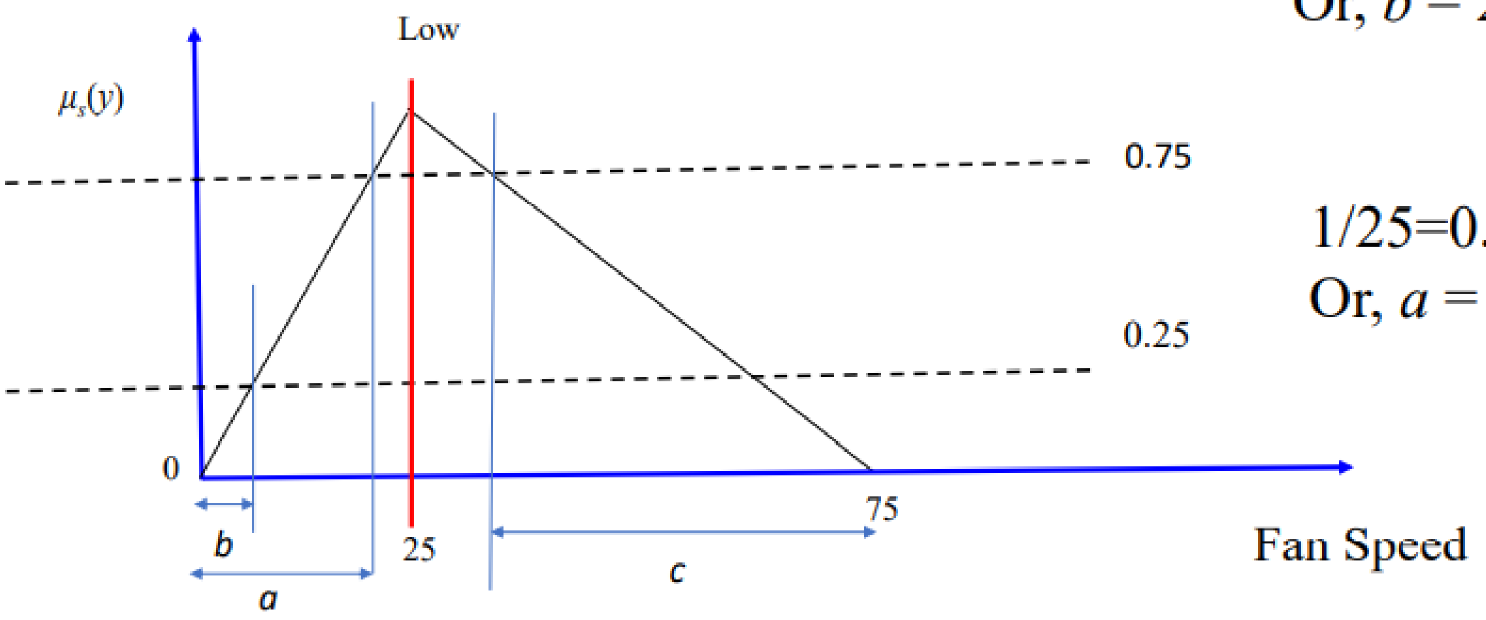

$\mu_s(y)$
0.75
0.25
0
75
Fan Speed


Zero
Low
$\mu_s(y)$
0.75
0.25
0
75
Fan Speed


$\mu_s(y)$
0.75
0.25
0
75
Fan Speed

$1/25 = 0.25/b$

Or, $b = 25*0.25 = 6.25$

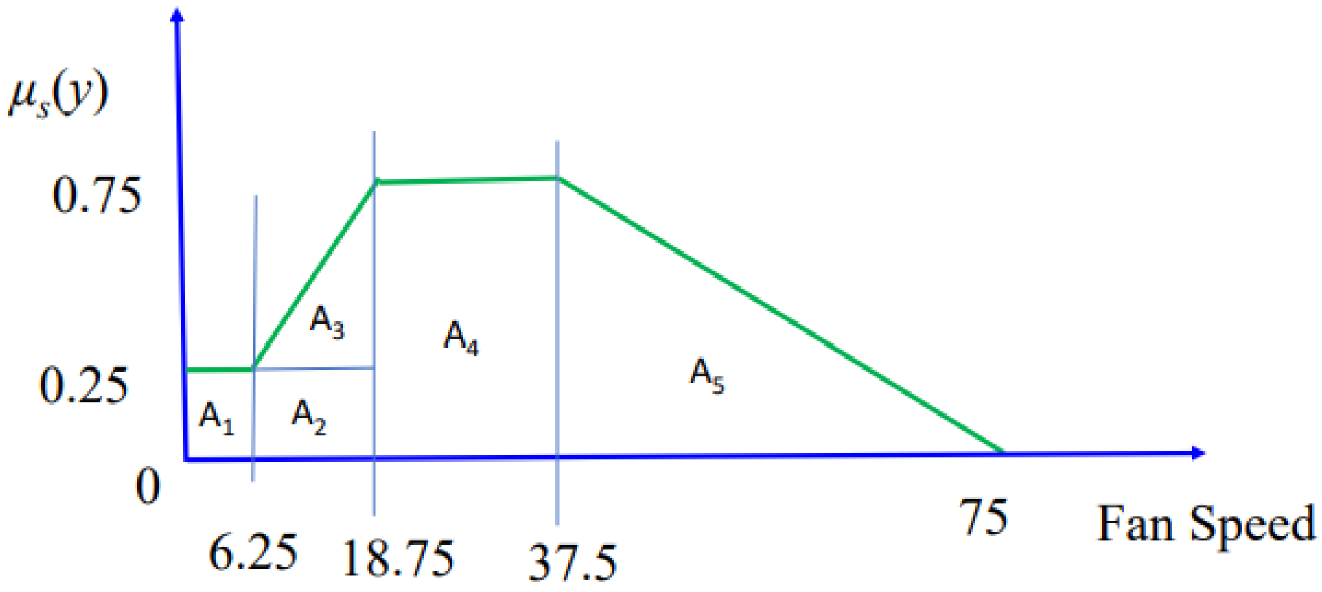


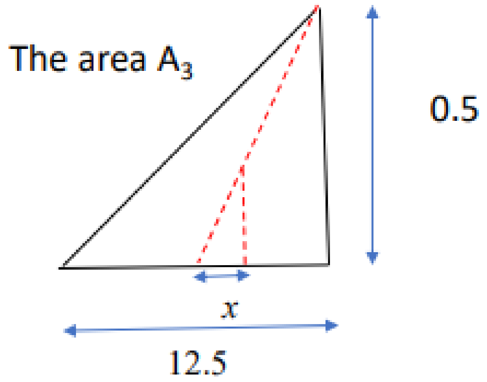
$1/25 = 0.75/a$

Or, $a = 25*0.75 = 18.75$

$1/(75-25) = 0.75/c$

Or, $c = 50*0.75 = 37.5$

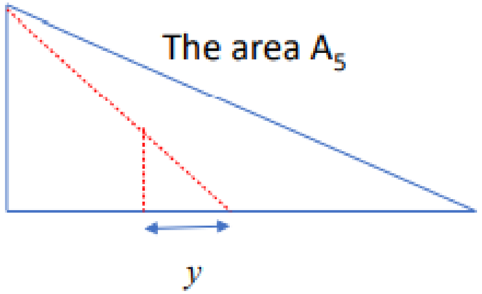




3L/L = (12.5/2)/x
Or, x = 2.083

A1 = 6.25*0.25 = 1.5625
C1 = (0+6.25)/2= 3.125
A2 = (18.75-6.25)*0.25 = 3.125
C2 = (18.75+6.25)/2 = 12.5

A3 = 0.5*12.5*0.5 = 3.125
C3 = (18.75+6.25)/2+ 2.083 = 14.583
A4–(37.5-18.75)*0.75 – 14.0625
C4 = (37.5+18.75)/2 = 28.125
A5 = 0.5*(75-37.5)*0.75 = 14.0625
C5 = (75+37.5)/2-6.25 = 50



3L/L = (37.5/2)/y
Or, y = 6.25

The defuzzified value,

$X^* = (A_1C_1+A_2C_2+A_3C_3+ A_4C_4+ A_5C_5)/(A_1+A_2+A_3+ A_4+ A_5)$

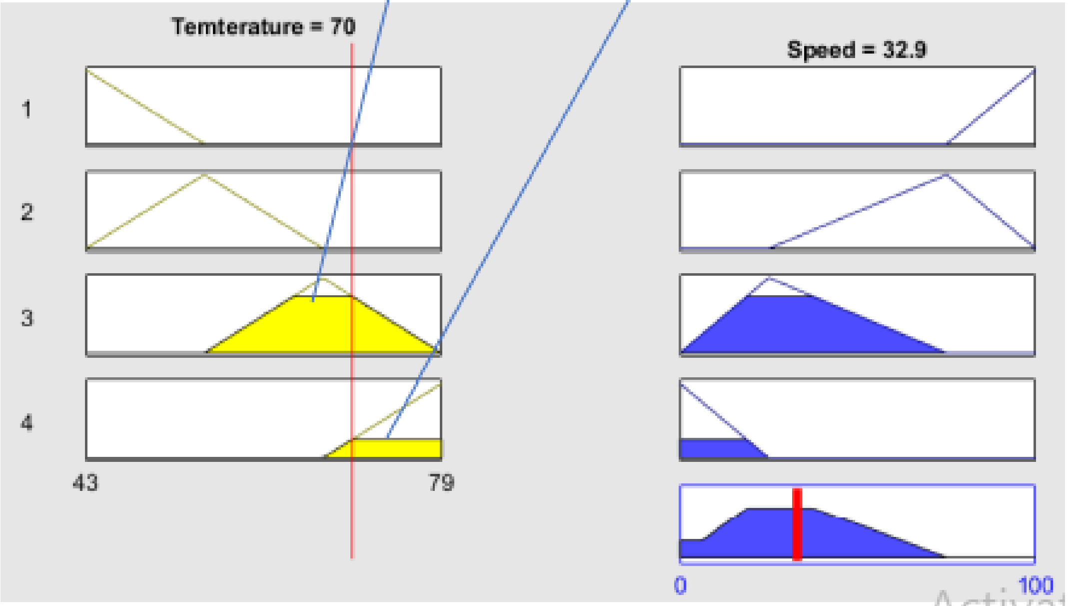
$= (1.5625 * 3.125 + 3.125 * 12.5 + 3.125 * 14.583+ 14.0625* 28.125+ 14.0625*50)/(1.5625 + 3.125 + 3.125+ 14.0625+ 14.00625)$

$= \quad 1188.15/35.9375$

$= 33.06$ rpm.

MF of speed of Low having grade of 0.75

MF of speed of Zero having grade of 0.25



The Fuzzy rules are:

1. If temp is cold then fan speed is high $\rightarrow$ truth value is 0
2. If temp is cool then fan speed is medium $\rightarrow$ truth value is 0
3. If temp is warm then fan speed is low $\rightarrow$ truth value is 0.75
4. If temp is hot then fan speed is zero $\rightarrow$ truth value is 0.25

Application of fuzzy System

- **Domestic Goods**
- Microwave Ovens
- Refrigerators
- Toasters
- Vacuum Cleaners
- Washing Machines
- **Environment Control**
- Air Conditioners/Dryers/Heaters
- Humidifiers

Advantages of FLSs

- Mathematical concepts within fuzzy reasoning are very simple.
- You can modify a FLS by just adding or deleting rules due to flexibility of fuzzy logic.
- Fuzzy logic Systems can take imprecise, distorted, noisy input information.
- FLSs are easy to construct and understand.
- Fuzzy logic is a solution to complex problems in all fields of life, including medicine, as it resembles human reasoning and decision making.

Disadvantages of FLSs

- There is no systematic approach to fuzzy system designing.
- They are understandable only when simple.
- They are suitable for the problems which do not need high accuracy.

End of Class