Bisection Method

Algorithm:

1. For any continous value of $F(x)$ with interaval $[a, b]$ such that $f(a) * f(b) < 0$

2. Find the midpoint of a,b where,

$$c = \frac{a+b}{2}$$

3. If $F(c) = 0$; then c is the root.

4. If $F(c) \neq 0$; $f(a) * f(c) < 0$ then set,

$b = c$

else $a = c$ [Positive]

Math: Find out the root from the following function using Bisection method:

① $x^3 - 4x - 9$

② $x^3 - 4$

③ $x^3 - 3x - 5$

Solve:

(1.) Let,

$$F(x) = x^3 - 4x - 9$$

and,

$a = 2$, then $F(2) = 2^3 - 4\times2 - 9 = 8 - 8 - 9 = -9$

$a = 3$, then $F(3) = 3^3 - 4\times3 - 9 = 27 - 12 - 9 = 6$

So,

$F(a) * F(b) < 0$

$(-9) * 6 = -54$ which is vaild

Now,

mid point, $c = \frac{a+b}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5$

and,

$F(c) = (2.5)^3 - 4\times2.5 - 9 = 15.625 - 10 - 9 = -3.375$

a	b	$F(a)$	$F(b)$	$c = \frac{a+b}{2}$	$F(c)$
2	3	-9	6	2.5	-3.375
2.5 ~~3.375~~	3	-3.375	6	2.75	0.797
2.5	2.75	-3.37	0.797	2.625	-1.41
2.63	2.75	-1.41	0.797	2.69	-0.29
2.69	2.75	-0.29	0.797	2.72	0.24
2.69	2.72	-0.29	0.24	2.70	-0.117
2.7	2.72	-0.117	0.24	2.71	0.06
2.7	2.71	-0.117	0.06	2.705	-0.117
2.7	2.71	-0.117	0.06	2.705	-0.117

So, the root is 2.705

② Let,

$$f(x) = x^3 - 4$$

and

$a = 1$, then $f(1) = 1^3 - 4 = 1 - 4 = -3$

$b = 2$, then $f(2) = 2^3 - 4 = 8 - 4 = 4$

$$f(a) * f(b) < 0$$

Here, $(-3) * 4 = -12$ which is valid

Now, midpoint, $c = \frac{a+b}{2} = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$\therefore f(c) = (1.5)^3 - 4 = 3.375 - 4 = -0.625$

a	b	f(a)	f(b)	$c = \frac{a+b}{2}$	f(c)
1	2	-3	4	1.5	-0.625
1.5	2	-0.625	4	1.75	1.36
1.5	1.75	-0.625	1.36	1.625	0.29
1.5	1.625	-0.625	0.29	1.56	-0.20
1.56	1.625	-0.20	0.29	1.59	0.0196
1.56	1.59	-0.20	0.0196	1.57	-0.13
1.57	1.59	-0.13	0.0196	1.58	-0.055
1.58	1.59	-0.055	0.0196	1.58	-0.055

$\therefore$ The root is 1.58

(3) Let, $f(x) = x^3 - 3x - 5$

with in terval value $a=2, b=3$

$\therefore f(a) = 8-6-5 = -3$

$\therefore f(b) = 27-9-5 = 13$

$f(a) * f(b) < 0$

$\therefore$ The root lies between 2 and 3

Now,

mid point, $c = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$

$\therefore f(c) = (2.5)^3 - 7.5 - 5 = 3.125$

iteration table:

a	b	f(a)	f(b)	$c=\frac{a+b}{2}$	f(c)
2	3	-3	13	2.5	3.125
2	2.5	-3	3.125	2.25	-0.359
2.25	2.5	-0.359	3.125	2.375	1.2714
2.25	2.375	-0.359	1.2714	2.3125	0.4289
2.25	2.3125	-0.359	0.4289	2.2812	0.0281
2.25	2.2812	-0.359	0.0281	2.2656	-0.1676
2.26	2.2812	-0.1676	0.0281	2.2734	-0.0704
2.27	2.2812	-0.070	0.0281	2.2773	-0.0216
2.27	2.2812	-0.0216	0.0281	2.2792	-0.0022

$\therefore$ The root is 2.27

Regular Falsi Method

Algorithm:

① Choose two real number a, b such that $f(a) * f(b) < 0$

② Define root, $c = \dfrac{a * f(b) - f(a) * b}{f(b) - f(a)}$

③ Find $f(c)$

④ If $f(a) * f(b) \leq 0$ then set $b = c$, else $a = c$ and return to step 1 until the root matched.

Math:

① $x^3 - 4x - 9$

② $x^3 - 4$

③ $x^3 - 3x - 5$

(1) Let, $f(x) = x^3 - 4x - 9$ with interval value

$a = 2, \quad b = 3$

$\therefore f(a) = 2^3 - 4\times2 - 9 = -9$

$\therefore f(b) = 3^3 - 4\times3 - 9 = 6$

$f(a) * f(b) \leq 0$

So, the root lies between 2 and 3

$$\therefore \quad c = \frac{a*f(b) - f(a)*b}{f(b) - f(a)} = \frac{2*6 + 9*3}{6+9} = 2.6$$

$\therefore f(c) = (2.6)^3 - 4\times2.6 - 9 = -1.824$

iteraction table:

a	b	$f(a)$	$f(b)$	$c = \frac{a*f(b) - b*f(a)}{f(b) - f(a)}$	$f(c)$
2	3	-9	6	2.6	-1.824
2.6	3	-1.824	6	2.69	-0.295
2.69	3	-0.295	6	2.705	-0.02795
2.705	3	-0.02745	6	2.7063	-0.0094
2.7063	3	-0.0094	6	2.7064	-0.0029

$\therefore$ The root is 2.70

(2) Let, $f(n) = n^3 - 4$ with interval value

$a = 1, \quad b = 2$

$\therefore f(a) = 1^3 - 4 = -3$

$\therefore f(b) = 2^3 - 4 = 4$

$\therefore f(a) * f(b) \leq 0$

So, the root lies between 1 and 2

$$\therefore c = \frac{a*f(b) - b*f(a)}{f(b) - f(a)} = \frac{1*4 - 2*(-3)}{4+3} = 1.42$$

$\therefore f(c) = (1.42)^3 - 4 = -1.13$

iteration table:

a	b	f(a)	f(b)	$c = \frac{a*f(b) - b*f(a)}{f(b) - f(a)}$	f(c)
1	2	-3	4	1.42	-1.13
1.42	2	-1.13	4	1.54	-0.34
1.54	2	-0.34	4	1.57	-0.13
1.57	2	-0.13	4	1.58	-0.055
1.58	2	-0.055	4	1.58	-0.055

$\therefore$ The root is 1.58

(3) Let, $f(x) = x^3 - 3x - 5$ with interval value $a = 2$, $b = 3$

$\therefore f(a) = 8 - 6 - 5 = -3$

$\therefore f(b) = 27 - 9 - 5 = 13$

Now, $f(a) * f(b) \le 0$

So, the root lies between 2 & 3.

$$\therefore c = \frac{a * f(b) - b * f(a)}{f(b) - f(a)} = \frac{2*13 - 3*(-3)}{13+3} = 2.18$$

$\therefore f(c) = (2.18)^3 - 3 \times 2.18 - 5 = -1.179$

a	b	$f(a)$	$f(b)$	$c = \frac{a*f(b) - b*f(a)}{f(b) - f(a)}$	$f(c)$
2	3	-3	13	2.18	-1.179
2.18	3	-1.179	13	2.24	-0.43
2.24	3	-0.43	13	2.26	-0.23
2.26	3	-0.23	13	2.27	-0.11
2.27	3	-0.11	13	2.27	-0.11

So, the root is 2.27

One point iteration method

Algorithm:

i) Given an equation $f(x)=0$

ii) Convert $f(x)=0$ into the form of

$$x=g(x)$$

iii) Let the initial gues value be 0.5/3.2

[Note: Chose 3.2 when there are log, sinc, cosc, e^x... exponential operators, otherwise choose 0.5]

iv) Do

$$x_{i+1}=g(x_i)$$

while (none of the convergence iteration is matched)

Math:

① x^3-4x-9

② x^3-4

③ x^3-3x-5

(1)

Let, $f(x) = x^3 - 4x - 9$

Now, $f(x) = 0$

$\Rightarrow x^3 - 4x - 9 = 0$

$\Rightarrow x^3 = 4x + 9$

$\Rightarrow x = \sqrt[3]{4x+9}$

$\therefore g(x) = \sqrt[3]{4x+9}$

Initial one point is 0.5

$\therefore x_0 = g(0.5) = \sqrt[3]{4\times 0.5+9} = 2.224$

$x_1 = g(x_0) = \sqrt[3]{4\times 2.224+9} = 2.6157$

$x_2 = g(x_1) = \sqrt[3]{4\times 2.224+9} = 2.6899$

$x_3 = g(x_2) = \sqrt[3]{4\times 2.6899+9} = 2.7035$

$x_4 = g(x_3) = \sqrt[3]{4\times 2.7035+9} = 2.7059$,

$\therefore$ The root is 2.7059

② x^3-4

Let, $f(x) = x^3 - 4$

Now, $f(x) = 0$

$\Rightarrow x^3 - 4 = 0$

$\Rightarrow x^3 = 4$

$\Rightarrow x = \sqrt[3]{4}$

$\therefore x = 1.587$

So, the root is 1.587

③ $x^3 - 3x - 5$

Let, $f(x) = x^3 - 3x - 5$

Now, $f(x) = 0$

$\Rightarrow x^3 - 3x - 5 = 0$

$\Rightarrow x^3 = 3x + 5$

$\therefore x = \sqrt[3]{3x+5}$

$\therefore g(x) = \sqrt[3]{3x+5}$

initial one point is 0.5

$\therefore x_0 = g(0.5) = \sqrt[3]{3\times0.5+5} = 1.8662$

$x_1 = g(x_0) = \sqrt[3]{3\times1.8662+5} = 2.1966$

$x_2 = g(x_1) = \sqrt[3]{3\times2.1966+5} = 2.26304$

$x_3 = g(x_2) = \sqrt[3]{3\times2.26304+5} = 2.27594$

P.T.O

$x_4 = g(x_3) = \sqrt[3]{3\times 2.27594+5} = 2.2784$

So, the root is 2.2784

Newton Raphson Method

Algorithm:

1. find $f'(x)$
2. find a, b so that $f(a) * f(b) < 0$
3. Assume $x_0 = a$
4. find out, $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
5. find the values $x_1, x_2, x_3 \ldots x_n$ untill any two successive values are equal.

Math:

1. x^3-4x-9
2. x^3-4
3. x^3-3x-5

①

Let, $f(x) = x^3 - 4x - 9$

$\therefore f'(x) = 3x^2 - 4$

Let, $a = 2$, $b = 3$

$\therefore f(a) = 2^3 - 4\times2 - 9 = -9$

$\therefore f(b) = 3^3 - 4\times3 - 9 = 6$

$f(a) * f(b) = -54 < 0$

So, the root lies between 2 and 3

Assuming,

$x_0 = a$

$\therefore x_0 = 2$

now, $f(x_0) = 2^3 - 4\times2 - 9 = -9$

$f'(x_0) = 3\times2^2 - 4 = 8$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \left(\frac{-9}{8}\right) = 3.125$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.125 - \frac{3.125^3 - 4\times3.125 - 9}{3\times3.125^2 - 4} = 2.768$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.768 - \frac{2.768^3 - 4\times2.768 - 9}{3\times2.768^2 - 4} = 2.70$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.70 - \frac{2.70^3 - 4\times2.70 - 9}{3\times2.70^2 - 4} = 2.70$$

$\therefore$ So, the root is 2.70.

② Let, ~~$f(x)=x^3-$~~

$$f(x) = x^3 - 4$$

$$\therefore f'(x) = 3x^2$$

Let, $a=1$, $b=2$

$\therefore f(a) = 1^3 - 4 = -3$

$\therefore f(b) = 2^3 - 4 = 4$

Now, $f(a) * f(b) = -3 \times 4 = -12 < 0$

So the root lies between 1 and 2

Assuming,

$$x_0 = a$$

$$\therefore x_0 = 1$$

$\therefore f(x_0) = 1^3 - 4 = -3$

$\therefore f'(x_0) = 3 \times 1^2 = 3$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{-3}{3} = 2$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2^3-4}{3\times 2^2} = 1.66$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.66 - \frac{1.66^3-4}{3\times 1.66^2} = 1.59$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.59 - \frac{1.59^3-4}{3\times 1.59^2} = 1.58$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1.58 - \frac{1.58^3-4}{3\times 1.58^3} = 1.58$$

So, the root is 1.58

(3)

Let, $f(x) = x^3 - 3x - 5$

$\therefore f'(x) = 3x^2 - 3$

Let, $a = 2$, $b = 3$

$\therefore f(a) = 8 - 6 - 5 = -3$

$f(b) = 27 - 9 - 5 = 13$

$f(a) * f(b) = -3 \times 13 = -39 < 0$

So, the root lies between 2 and 3.

Assuming,

$x_0 = a$

$x_0 = 2$

$\therefore f(x_0) = -3$

$f'(x_0) = 3 \times 2^2 - 3 = 9$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{-3}{9} = 2.33$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.33 - \frac{2.33^3 - 3 \times 2.33 - 5}{3 \times 2.33^2 - 3} = 2.28$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.28 - \frac{2.28^3 - 3 \times 2.28 - 5}{3 \times 2.28^2 - 3} = 2.27$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.27 - \frac{2.27^3 - 3 \times 2.27 - 5}{3 \times 2.27^2 - 3} = 2.27$

So, the root is 2.27