

CSE-213

(Data Structure)

Lecture on

Recursion, Pointers and Records

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Recursion

Recursion: Basic idea

- ❑ We have a bigger problem whose solution is difficult to find
- ❑ We divide/decompose the problem into smaller (sub) problems
 - Keep on decomposing until we reach to the smallest sub-problem (base case) for which a solution is known or easy to find
 - Then go back in reverse order and build upon the solutions of the sub-problems
- ❑ Recursion is applied when the solution of a problem depends on the solutions to smaller instances of the same problem

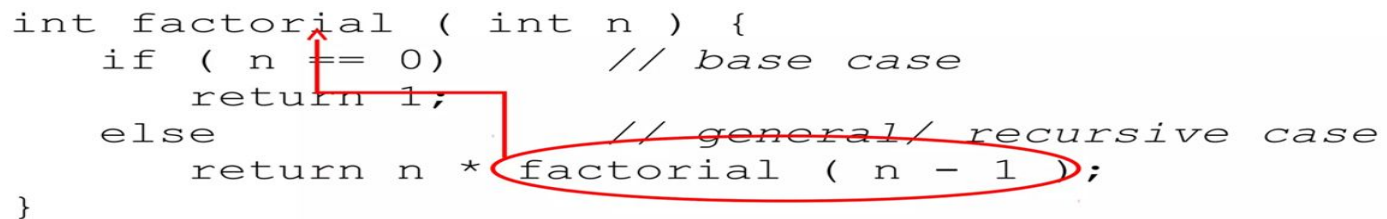
Recursion

- Recursion in data structure is when a function calls itself indirectly or directly, and the function calling itself is known as a recursive function.
- It's generally used when the answer to a larger issue could be depicted in terms of smaller problems.

Recursive Function

❑ A function which calls itself

```
int factorial ( int n ) {  
    if ( n == 0 )          // base case  
        return 1;  
    else                   // general/ recursive case  
        return n * factorial ( n - 1 );  
}
```



Recursion: Function

- ✓ A function is said to be *Recursively defined* if the function definition refers to itself.
- ✓ A *Recursive Function* must have the following two properties:
 - There must be certain arguments, called **BASE VALUE**, for which the function does not refer to itself.
 - Each time the function does refer to itself, the argument of the function must be closer to a **BASE VALUE**.
- ✓ A *Recursive Function* with two properties is also said to be *well-defined*.

Recursion: Factorial Function

- ✓ In some problems, it may be natural to define the problem in terms of the problem itself.
- ✓ Recursion is useful for problems that can be represented by a **SIMPLER VERSION** of the same problem.
- ✓ Example: the factorial function

$$6! = 6 * 5 * 4 * 3 * 2 * 1$$

We could write:

$$6! = 6 * 5!$$

Recursion: Factorial Function

- ✓ In general, we can express the factorial function as follows:

$$n! = n * (n-1)!$$

Is this correct? Well... almost.

- ✓ The factorial function is **ONLY DEFINED** for *positive* integers. So we should be a bit more precise:

i) $n! = 1$ (if n is equal to 1)

ii) $n! = n * (n-1)!$ (if n is larger than 1)

- ✓ Observe that, this definition of $n!$ is recursive, since it refers to itself when it uses $(n-1)!$, However,

- ✓ i) the value of $n!$ is explicitly given when $n=0$ (**BASE VALUE**)

- ✓ ii) the value of $n!$ for arbitrary n is defined in terms of a smaller value of n which is closer to the **BASE VALUE**

Recursion: Factorial Function

EXAMPLE: Let's calculate **3!** Using the recursive definition.

1) $3! = 3 \cdot 2!$

2) $2! = 2 \cdot 1!$

3) $1! = 1 \cdot 0!$

4) $0! = 1$ (BASE VALUE)

5) $1! = 1 \cdot 1 = 1$

6) $2! = 2 \cdot 1 = 2$

7) $3! = 3 \cdot 2 = 6$

- ✓ Observe that we back track in the reverse order of the original postponed evaluations.
- ✓ Recall that this type of postponed processing tends itself to the use of STACKS.

Recursion: Function

Assume the number typed is 3, that is, numb=3.

fac(3) :

3 <= 1 ? **No.**

fac(3) = 3 * fac(2)

fac(2) :

2 <= 1 ? **No.**

fac(2) = 2 * fac(1)

fac(1) :

1 <= 1 ?

Yes.

return 1

fac(2) = 2 * 1 = 2

return fac(2)

fac(3) = 3 * 2 = 6

return fac(3)

- i) **n! = 1** (if n is equal to 1)
ii) **n! = n * (n-1)!** (if n is larger than 1)



```
int fac(int numb) {  
    if (numb <= 1)  
        return 1;  
    else  
        return numb * fac(numb-1);  
}
```

fac(3) has the value 6

Recursion: Function

For certain problems (such as the factorial function), a recursive solution often leads to short and elegant code. Compare the recursive solution with the **iterative solution**:

Recursive solution

```
int fac(int numb) {  
    if (numb <= 1)  
        return 1;  
    else  
        return numb * fac(numb - 1);  
}
```

Iterative solution

```
int fac(int numb) {  
    int product = 1;  
    while (numb > 1) {  
        product *= numb;  
        numb--;  
    }  
    return product;  
}
```

Iteration Vs Recursion

Two approaches to writing repetitive algorithms:

1. Iteration
2. Recursion

Recursion vs. Iteration: Computing N!

□ The factorial of a positive integer n , denoted $n!$, is defined as the product of the integers from 1 to n . For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.

- Iterative Solution

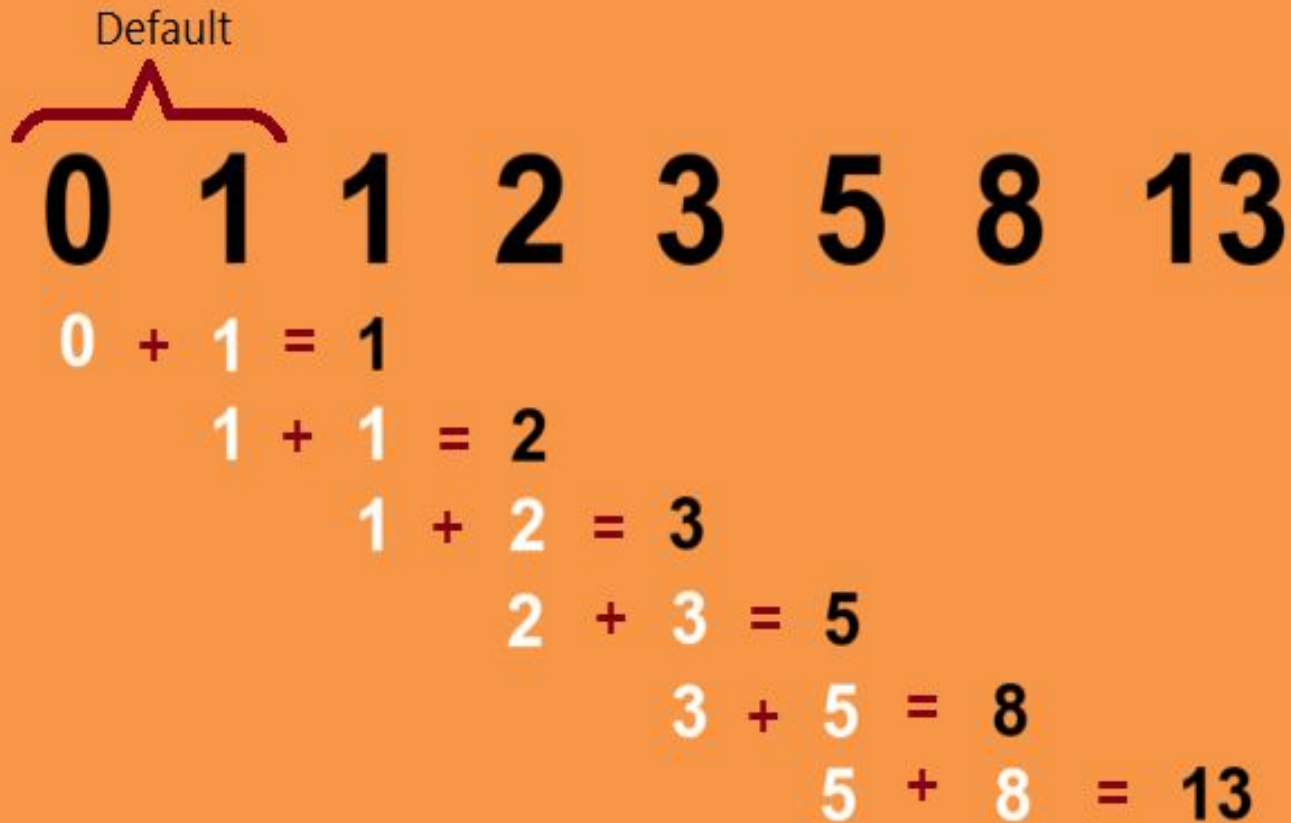
$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 3 \cdot 2 \cdot 1 & \text{if } n \geq 1 \end{cases}$$

- Recursive Solution

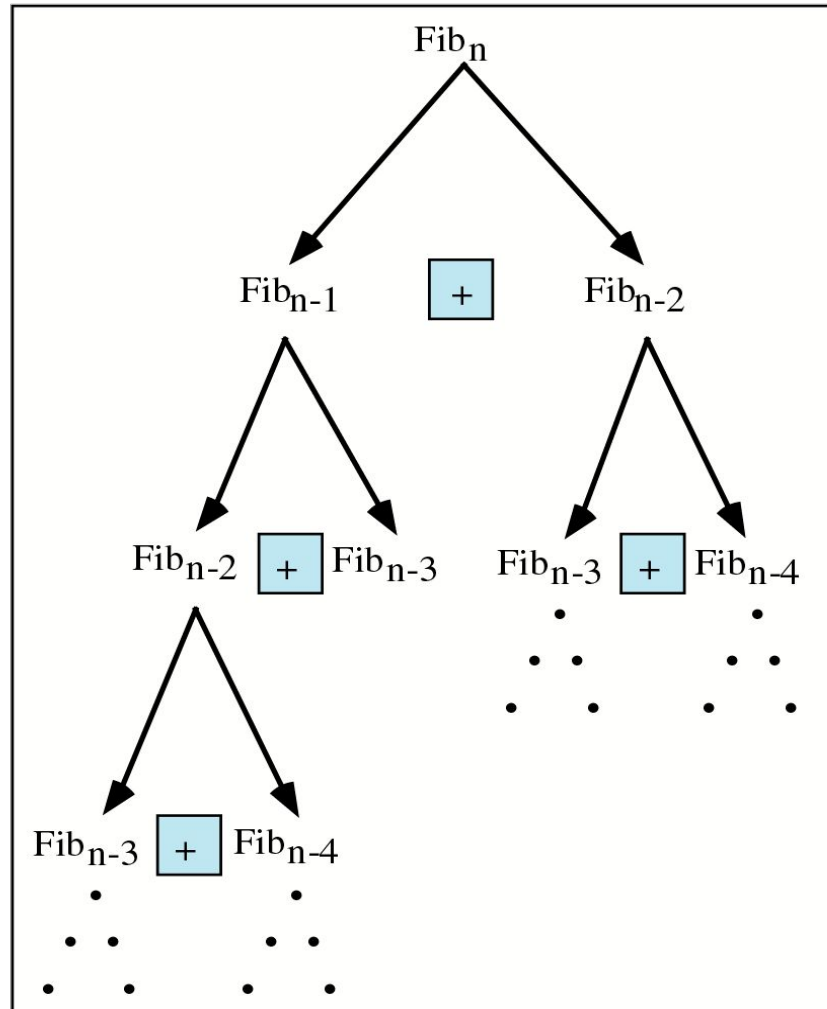
$$\text{factorial}(n) = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot \text{factorial}(n - 1) & \text{if } n \geq 1 \end{cases}$$

Recursion: Fibbonacc

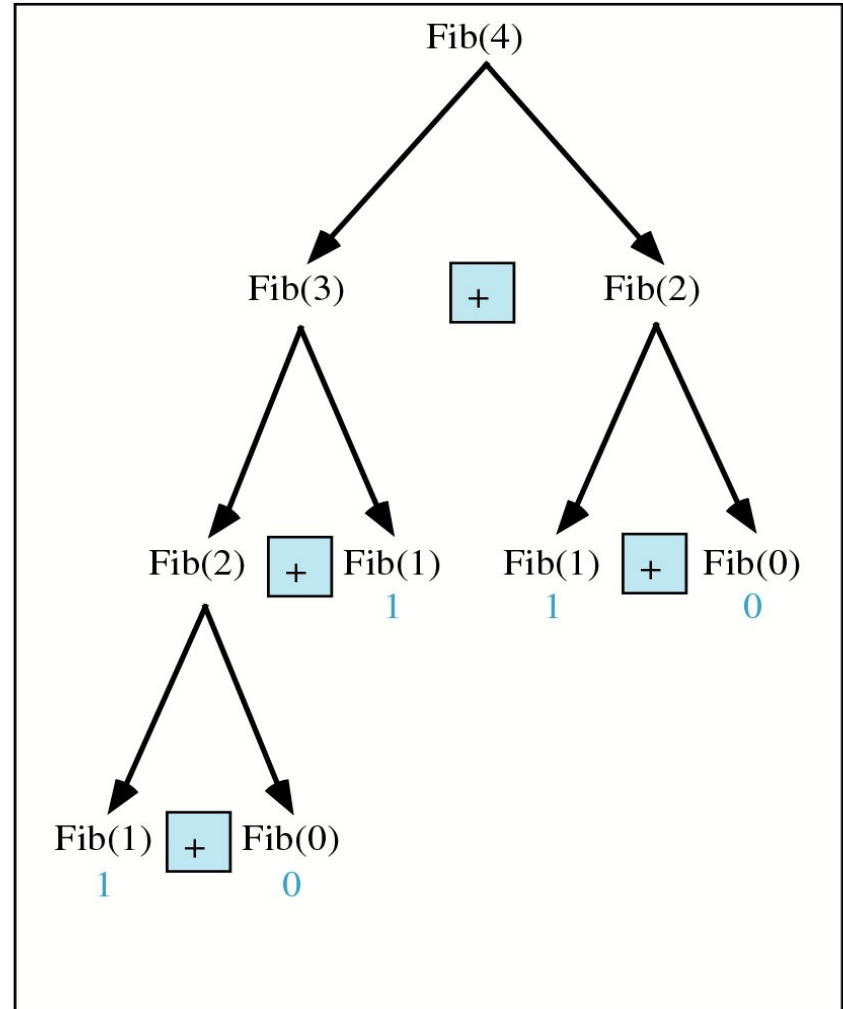
Fibonacci Sequence



Recursion: Fibonacci



(a) $\text{Fib}(n)$



(b) $\text{Fib}(4)$

Recursion: Fibbonacc

// Calculate Fibonacci numbers using recursive function.

// A very inefficient way, but illustrates recursion well

```
int fib(int number)
{
    if (number == 0) return 0;
    if (number == 1) return 1;
    return (fib(number-1) + fib(number-2));
}
```

Recursive definition:

$F(0) = 0;$

$F(1) = 1;$

$F(\text{number}) = F(\text{number}-1) + F(\text{number}-2);$

Pointers

Let DATA be any array. A variable **P** is called a *pointer* if **P** “points” to an element in DATA, i.e., if **P** contains the address of an element in DATA.

Pointer Arrays

An array **PTR** is called a *pointer array* if each element of **PTR** is a pointer

Pointer and Pointer array are used to facilitate the processing the information in DATA

| Group 1 | Group 2 | Group 3 | Group 4 |
|----------------------------------|---|----------------|--|
| Evans Harris Lewis Shaw | Conrad Felt Glass Hill King Penn Silver Troy Wagner | Davis Segal | Baker Cooper Ford Gray Jones Reed |

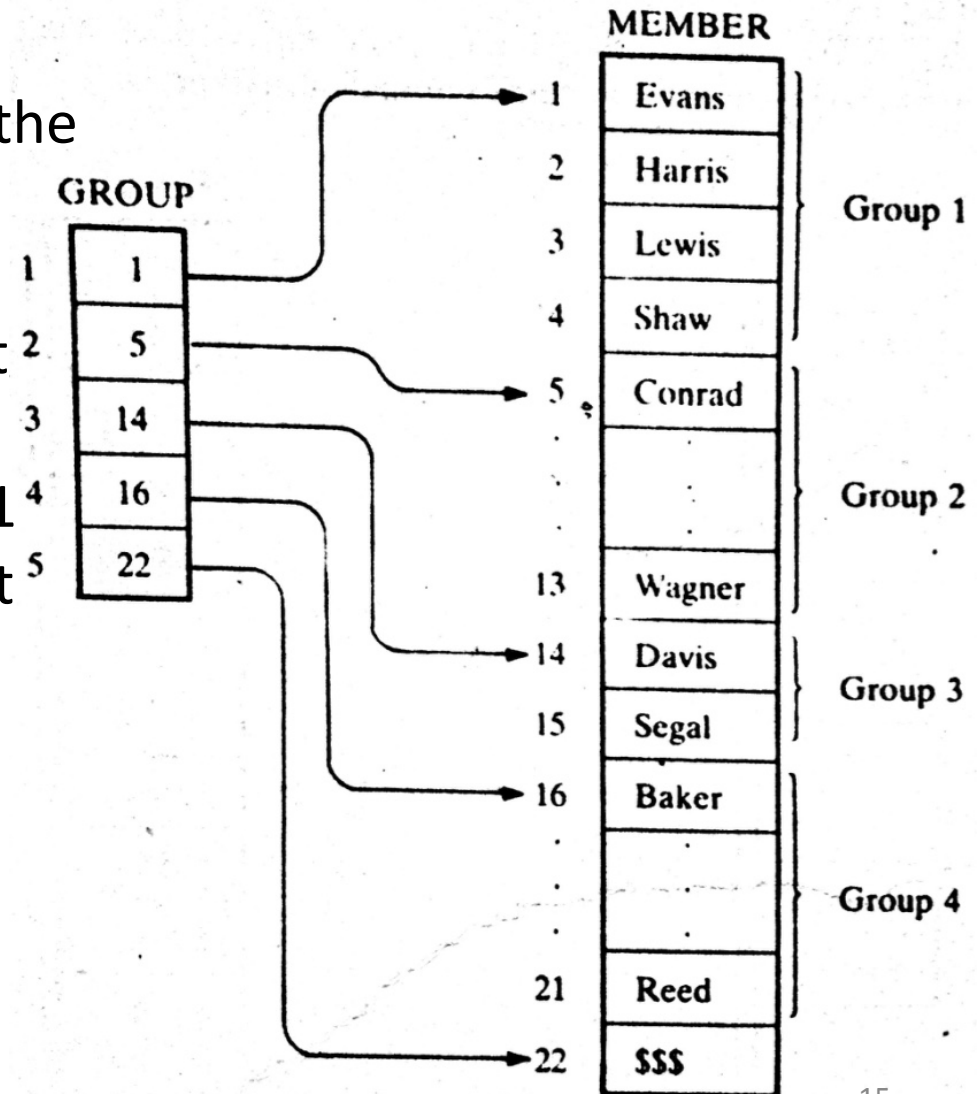
How the membership list can be stored in memory keeping track of the different groups?

POINTER ARRAYS

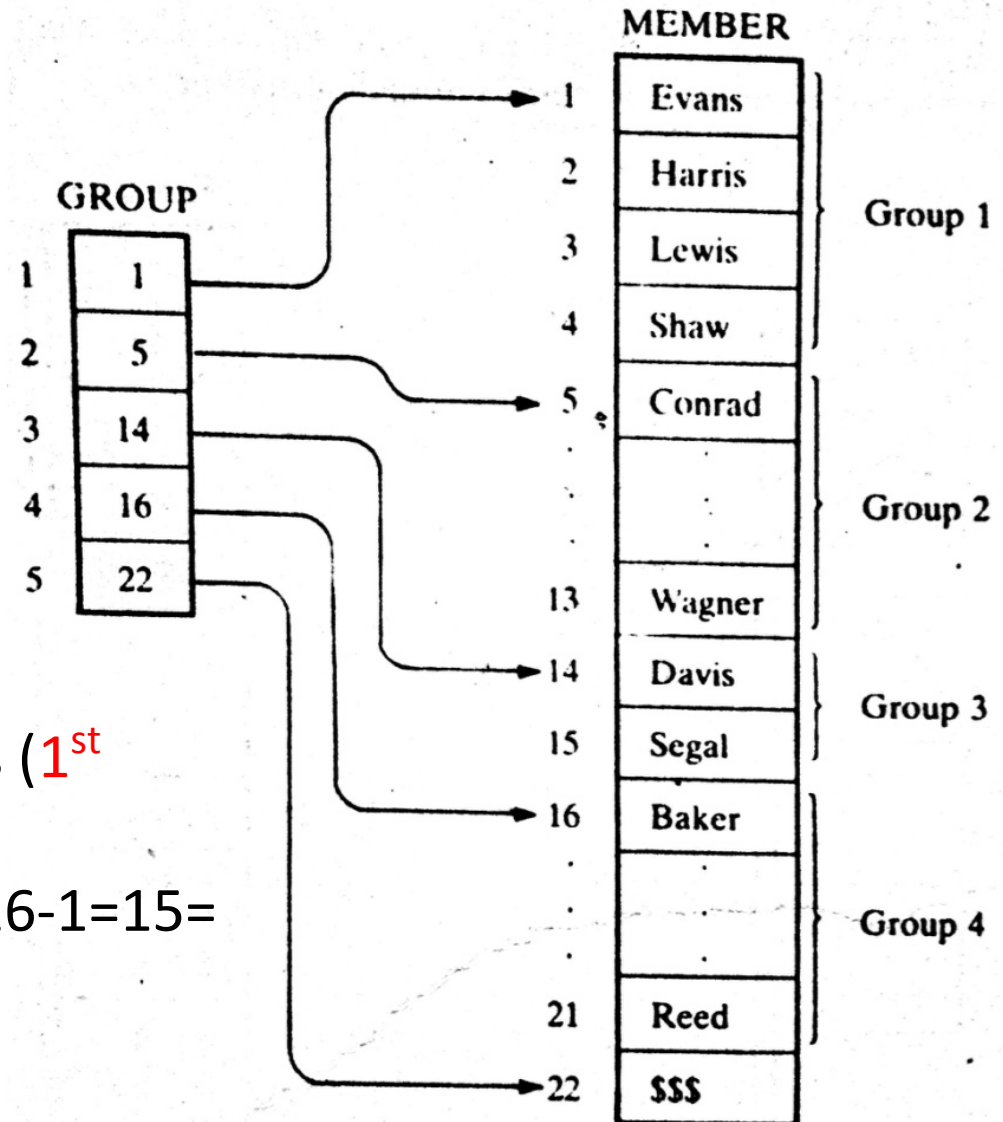
Pointer arrays is introduced in the last two space-efficient data structure.

The pointer array contains the locations of the.....

- ✓ Different groups, or
- ✓ First element in the different groups.
- ✓ $\text{GROUP}[L]$ and $\text{GROUP}[L+1]-1$ contain respectively, the first and last element in group L .



POINTER ARRAYS: Example



Suppose $L=3$

$\text{GROUP}[L]=\text{GROUP}[3]=14=\text{Davis}$ (**1st Element of grp 3**)

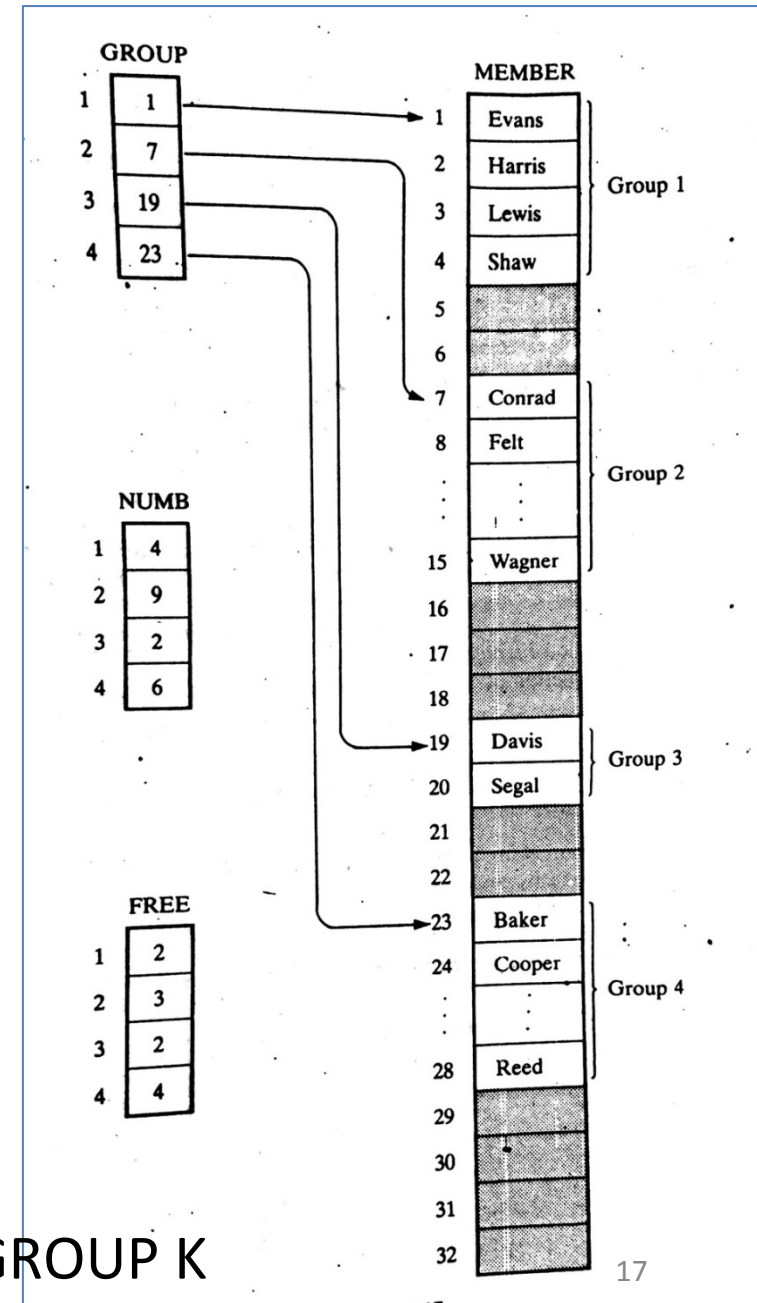
$\text{GROUP}[L+1]-1=\text{GROUP}[3+1]-1=16-1=15=\text{Segel}$ (**last element of grp 3**)

POINTER ARRAYS: Extended

- Here unused memory cells are indicated by the shading.
- Observe that now there are some empty cells between the groups.
- Accordingly, a new element may be inserted in a new group without necessarily moving the elements in any other group.
- Using the data structure, one requires an array NUMB which gives the number of elements in each group.
- Observe that $\text{GROUP}[K+1] - \text{GROUP}[K]$ is the total number of space available for group K. Hence

$$\text{FREE}[K] = \text{GROUP}[K+1] - \text{GROUP}[K] - \text{NUMB}[K]$$

Gives the number of empty cells following GROUP K



POINTER ARRAYS: Extended, Example

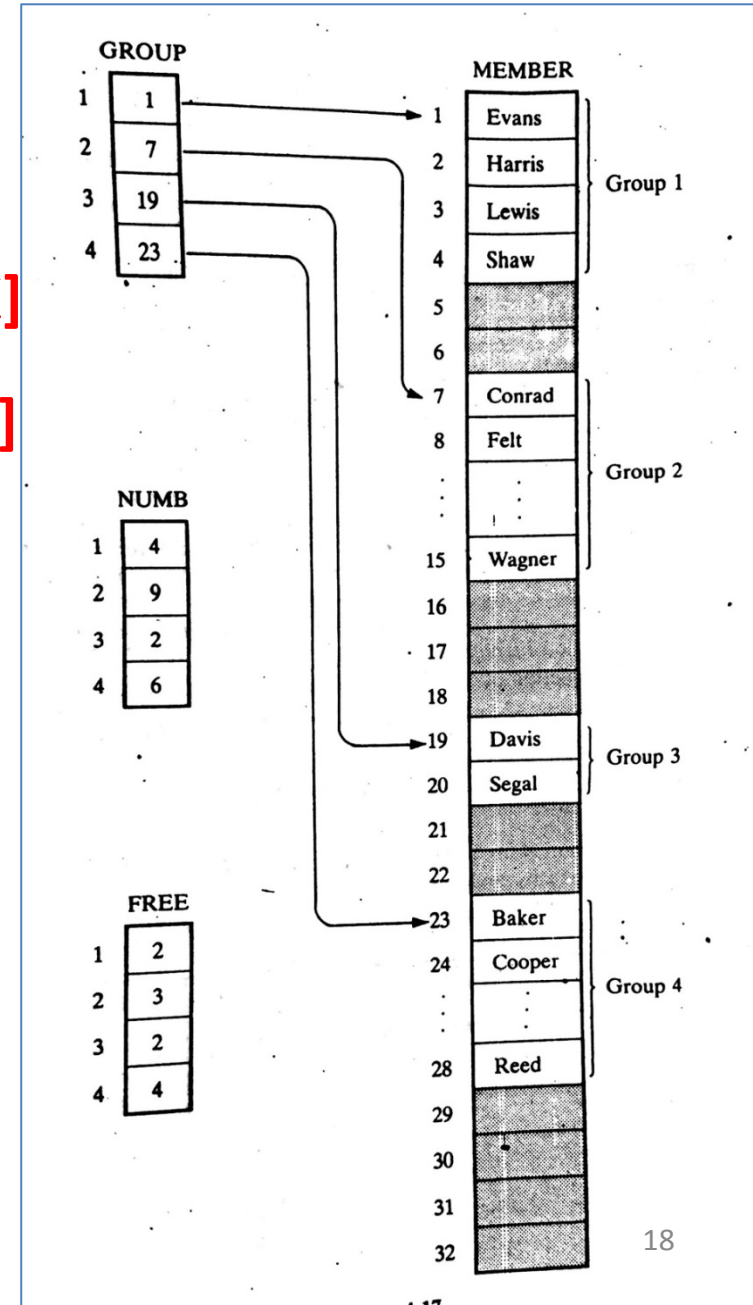
Suppose, we want to print only the number of FREE cells of GROUP 2. Then

$$\text{FREE}[K] = \text{GROUP}[K+1] - \text{GROUP}[K] - \text{NUMB}[K]$$

$$\begin{aligned}\text{FREE}[2] &= \text{GROUP}[2+1] - \text{GROUP}[2] - \text{NUMB}[2] \\ &= 19 - 7 - 9 \\ &= 3\end{aligned}$$

For GROUP 4?

Try now



RECORDS

- ✓ A **record** is a collection of related data items, each of which is called a field or attribute, and
- ✓ a **file** is a collection of similar records.
- ✓ Although, a record is a collection of data items, it differs from a linear array in the following ways.....
 - A record may be a collection of nonhomogeneous data;
 - The data items in a record are indexed by attribute names, so there may not be a natural ordering of its elements.

RECORDS: Structure Example

- 1. Newborn
 - 2. Name
 - 2. Sex
 - 2. Birthday
 - 3. Month
 - 3. Day
 - 3. Year
 - 2. Father
 - 3. Name
 - 3. Age
 - 2. Mother
 - 3. Name
 - 3. Age

Under the relationship of group item to sub- item, the data items in a record form a hierarchical structure which can be described by mean of “Level” numbers


| Name | | Se | Birthday | | Father | | Mother | |
|------|--|----|----------|--|--------|-----|--------|-----|
| | | x | | | Name | Age | Name | Age |

Indexing Items in a Record

- ✓ Suppose we want to access some data item in a record.
- ✓ We can not simply write the data name of the item since the same may appear in different places in the record. For example.....

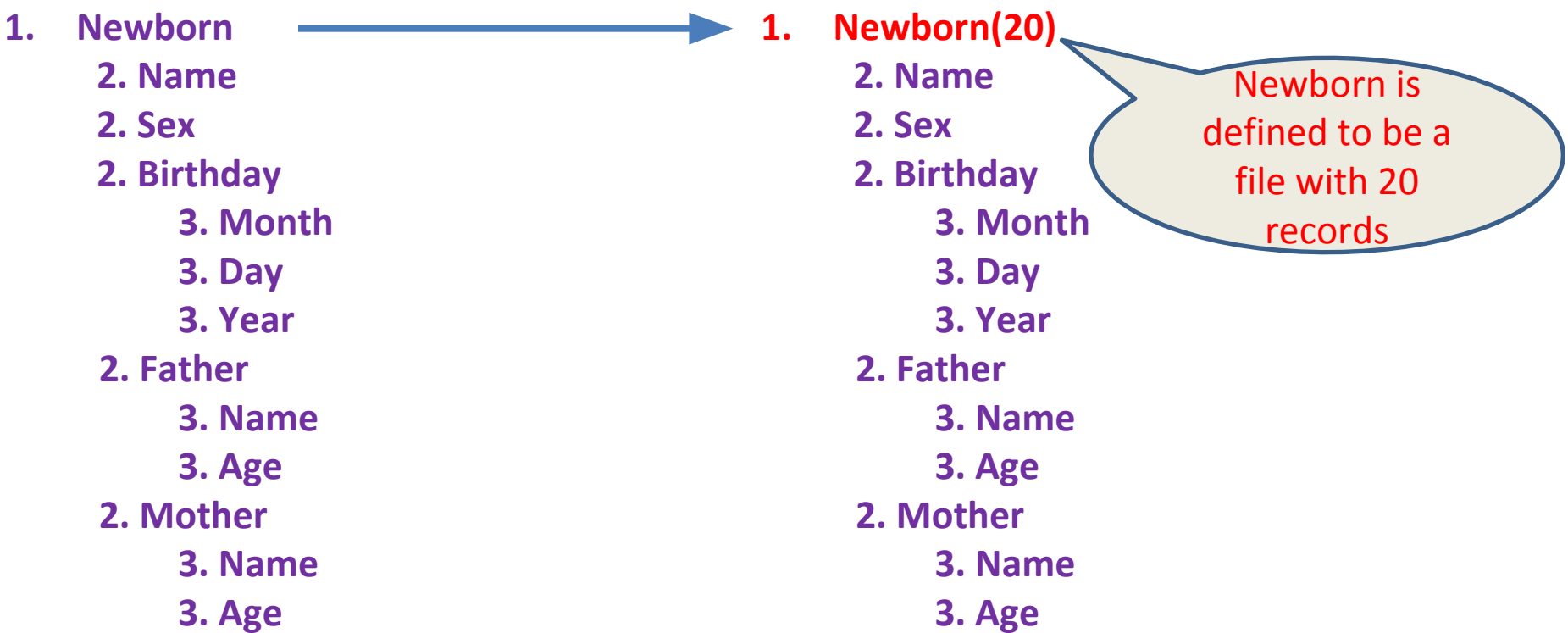
- 1. Newborn
 - 2. Name
 - 2. Sex
 - 2. Birthday
 - 3. Month
 - 3. Day
 - 3. Year
- 2. Father
 - 3. Name
 - 3. Age
- 2. Mother
 - 3. Name
 - 3. Age

- In order to specify a particular item,
 - ❖ we may have to *qualify* the name by using appropriate group item names in the structure.
 - ❖ This *qualification* is indicated by using decimal points (periods) to separate group items from subitems.
 - ❖ Example: Newborn.Father.Age or Father.Age



Fully qualified
reference

Indexing Items in a Record



- ✓ The Name of the sixth newborn to be referenced by writing
Newborn.Name[6]
- ✓ The age of the father of the 6th newborn may be referenced by writing.....
Newborn.Father.Age[6]

Representation of RECORDS in memory

Since records may contain nonhomogeneous data, the element of a record can not be stored in an array.

See Example: 4.18, 4.20, 4.21

MATRICES and SPARSE MATRICES

Teach Yourself with example

Try to understand the SOLVED Problems