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Note: 1(i)
(Change of axes)

Course Title: Analytical & vector Geometry

Course code: MAT-103

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program: CSE

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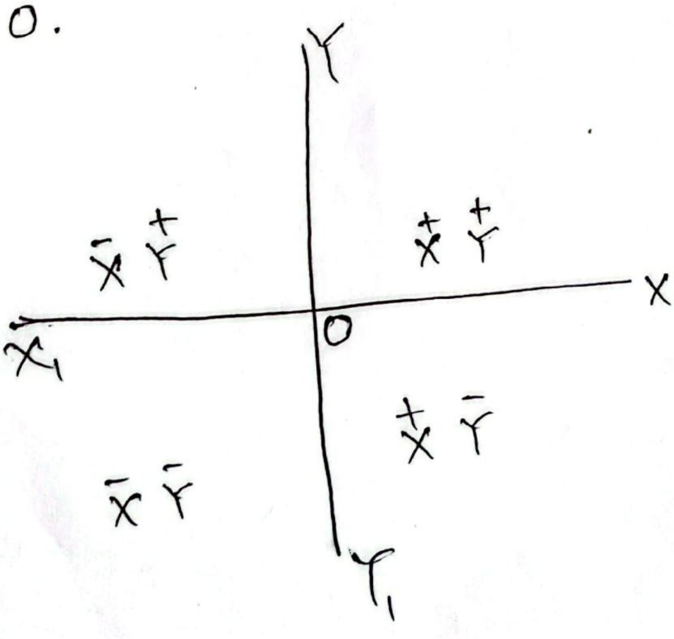
① Co-ordinates

Definition: (Co-ordinate):
Plane co-ordinate geometry is that branch of geometry in which we use two numbers, called co-ordinates to indicate the position of a point in a plane. Co-ordinate geometry is more powerful than ordinary, because we can use the methods of Algebra in it.

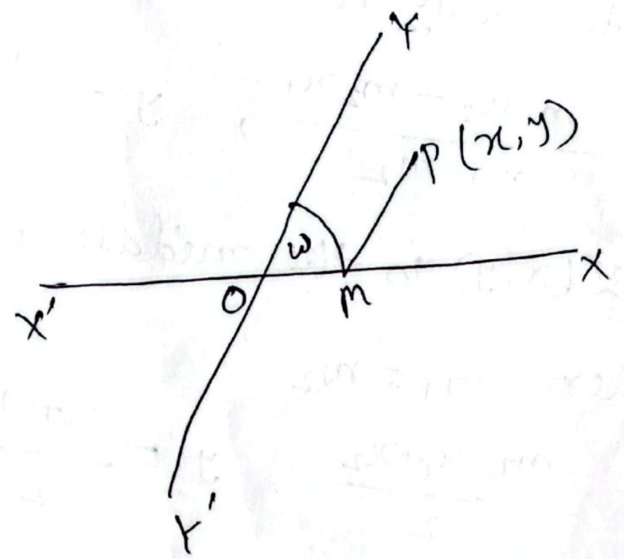
Rectangular co-ordinates: In this system the plane is divided into four quadrants by two perpendicular lines XOX and YOY intersecting at a point O.



{ abscissa = ভুজ
  ordinate = কোটি

Oblique axes: (তির্যক, বক্র, neither perpendicular nor parallel, not at right angles):

When the axes of co-ordinate axes not at right angles, they are said to be obliques axes, sometimes axes inclined (বাঁকা, বাঁকান) to one another at an acute or obtuse (সূক্ষ্ম বা less than one right angle ~~or~~ স্থূল বা greater than one right angle) angle are more convenient (suitable উপযোগী or commodious সুবিধাজনক).



Distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

is $P_1P_2 = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

The distance of the point $P(x, y)$ from the origin (O)

$op = \sqrt{x^2+y^2}$

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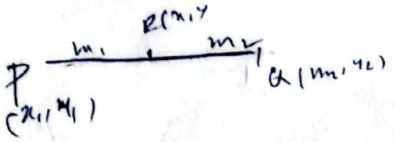


Section Ratio:

The co-ordinate of a point $R(x, y)$ which divides the join of two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in a given ratio $m_1 : m_2$

(i) Internally are

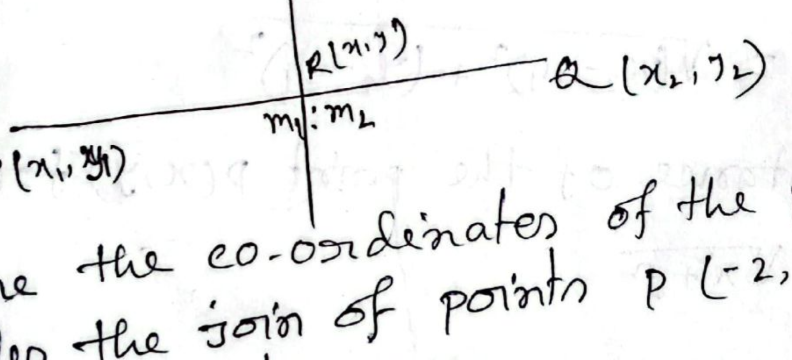
$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

(ii) Externally are

$$x = \frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \quad y = \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}$$

(iii) If $R(x, y)$ is the middle point of the line PQ: then $m_1 = m_2$

$$\therefore x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$



Determine the co-ordinates of the points $R(x, y)$ which divides the join of points $P(-2, 1)$ and $Q(3, -4)$ externally in the ratio $8 : 3$

Soln: $R(x, y)$ must be outside segment PQ, then by formula we have

$$x = \frac{8 \cdot 3 - 3 \cdot (-2)}{8 - 3} = 6 \quad \text{and} \quad y = \frac{8 \cdot (-4) - 3 \cdot 1}{8 - 3} = -7$$

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Signature of the invigilator .... Loose Sheet Sl No. 1058

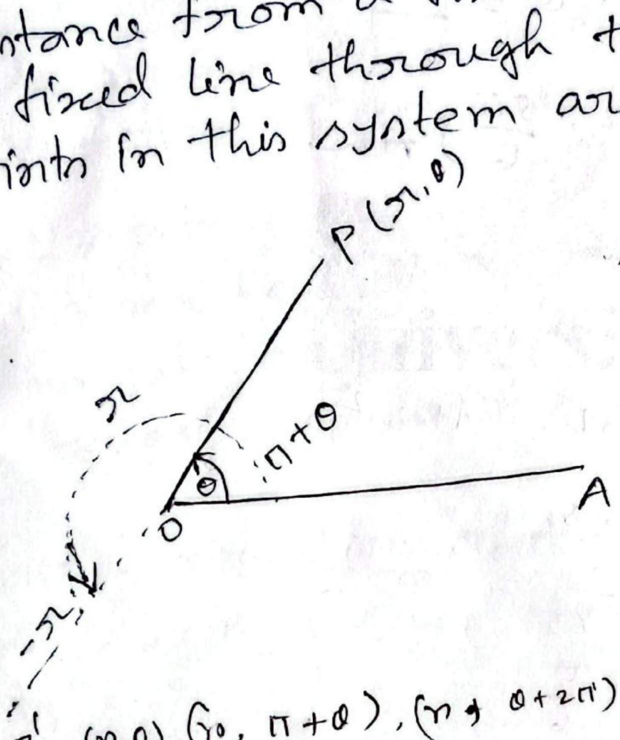
Batch.......... ID .... Name ....

polar co-ordinates

TE

Polar co-ordinates:

In using the cartesian co-ordinates x, y, we indicate the position of a point in a plane in terms of its distances from two lines. It is sometimes preferable to show its location in terms of its distance from a fixed point and its direction from a fixed line through this point. The co-ordinates of points in this system are called polar-co-ordinates.



Here O is called pole.

OA = polar axis

r = radius

θ = vectorial angle.

$(r,\theta), (r, \pi+\theta), (r, \theta+2\pi), \ldots$ anticlockwise

$(r, \theta-\pi), (r, \theta-2\pi)$ clock wise.

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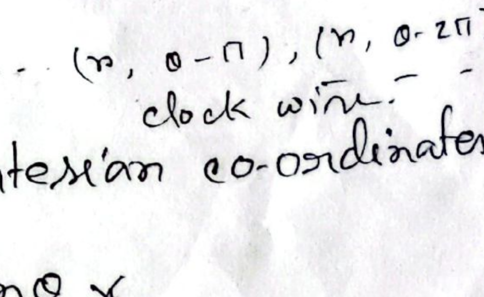
Relation between the polar and cartesian co-ordinates:

TE

(i) $x = r\cos\theta$ and $y = r\sin\theta$

(ii) $x^2+y^2=r^2$

(iii) $\frac{y}{x} = \tan\theta$



Find the polar co-ordinates of the point whose cartesian co-ordinates are $(-\sqrt{3}, 1)$, $(5, 12)$

(i) from $x^2 + y^2 = r^2$ $\quad (\sqrt{3}, 1)$

$$r = \sqrt{x^2+y^2} = \sqrt{(-\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

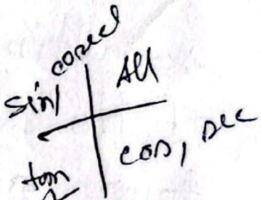
(ii) and from $\frac{y}{x} = \tan\theta$

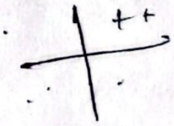
$$\Rightarrow \tan\theta = \frac{\cancel{-\sqrt{3}}\ -1}{\sqrt{3}} = \tan\left(\pi - \frac{\pi}{6}\right)$$

$$\therefore \tan\alpha = \tan\frac{5\pi}{6}$$

$$\therefore \theta = \frac{5\pi}{6}$$

$$\therefore (r, \theta) = \left(2, \frac{5\pi}{6}\right) \text{ Ans.}$$





$\tan 30 = \frac{1}{\sqrt{3}}$

$\frac{5\pi}{6} = \pi - \frac{\pi}{6}$

=

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$$

$$\frac{y}{x} = \tan\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right)$$

$$\theta = \frac{\pi}{6}$$

$$(r, \theta) = \left(2, \frac{\pi}{6}\right)$$

⑥ The straight line

The equation of straight line:

(a) parallel to the x axis is $y = \text{constant}$

(b) Parallel to the y axis is $x = \text{constant}$.

Slope intersect form:

x	y
0	3
	5
2	7

$\Delta y = 2$, $\Delta y = 2$, Δx

The equation of a straight line which cuts of a intercept on the axis of y and is inclined at a given angle θ to the positive direction of x axis is

$m = -5$, intercept $= 6$

$y = -5x + 6$

$m = -1$, points $= \frac{4}{5}$

$y = -x + \frac{4}{5}$

$P_1(2,6)$, $P_2 = (5,0)$

$m = \frac{\Delta y}{\Delta x} = \frac{-}{2}$

$y = 2x + 3$

$$y = x\tan\theta + c \qquad \text{where } m = \tan\theta$$

$$\Rightarrow y = mx + c$$

Note: If the straight line passes through the origin $(0,0)$ then the equation of the line is $y = mx$.

The general equation of the first degree is of the form $Ax + By + C = 0$ where A, B, C constants.

Perpendicular distance:

The length of the perpendicular distance d from a point $P(x_1, y_1)$ on the line $ax + by + c = 0$ is

$$d = \pm \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$

Bisector of angles between two lines:

The equation of the bisectors of the angle between two straight lines $ax+by+c=0$ and $a_1x+b_1y+c_1=0$ are

$$\frac{ax+by+c}{\sqrt{a^2+b^2}} = \pm \frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}$$

Angle between two lines:

The angle ϕ between two lines $y=m_1x+c_1$ —(i) and $y=m_2x+c_2$ (ii) is $\tan\phi = \pm \frac{m_1-m_2}{1+m_1m_2}$

Note: If the two straight lines are $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, then

$$\tan\phi = \pm \frac{a_2b_1-a_1b_2}{a_1a_2+b_1b_2}$$

Condition of parallelism of these (i) and (ii)

is $m_1=m_2$

or $a_2b_1-a_1b_2=0$ or $a_1/a_2 = \frac{b_1}{b_2}$

Condition of perpendicularity of these lines

(i) and (ii) is $m_1m_2=-1$

$\therefore a_1a_2+b_1b_2=0$

TE # Determine the co-ordinates of the points R(x, y) which divides the join of points P(-2, 1) and Q(3, -4) externally in the ratio 8:3.

Solution: R(x, y) must be outside segment PQ, then we know

R(x, y) 8:3 P(-2,1) Q(3,-4)

$$x = \frac{8.3 - 3(-2)}{8-3} = 6$$

and
$$y = \frac{8(-4) - 3(1)}{8-3} = -7$$

$\therefore$ R(6, -7) Ans.

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The area of a quadrilateral whose vertices

TE are $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$

X

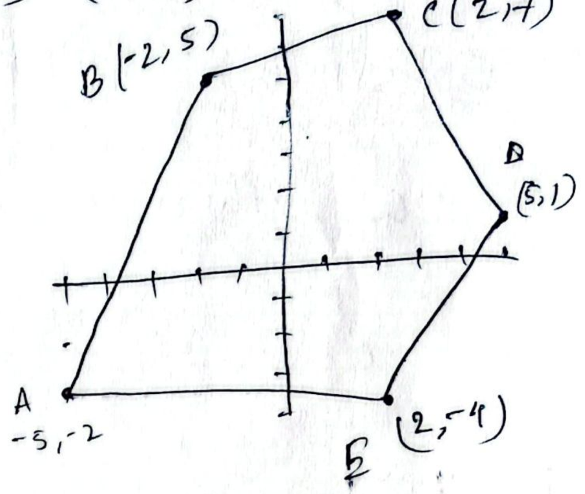
$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}\Big[(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_4 - x_4 y_3) + (x_4 y_1 - x_1 y_4)\Big]$$

(9)

\# Find the area of ~~the triangle~~ a pentagon whose vertices are $(-5,-2), (-2,5), (2,7) (5,1), (2,-4)$

Solution: The area of a pentagon whose vertices are $(-5,-2), (-2,5), (2,7), (5,1), (2,-4)$



$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} -5 & -2 \\ -2 & 5 \\ 2 & 7 \\ 5 & 1 \\ 2 & -4 \\ -5 & -2 \end{vmatrix}$$

$$= \frac{1}{2}\Big[(-5(5)-(-2)(-2)) + (-2(7) - 2(5)) + (2(1) - 5(7)) + (5(-4) - 2(1)) + (2(-2) - (-5)(-4))\Big]$$

$$= \frac{1}{2}\big[(-25-4) + (-14-10) + (2-35) + (-20-2) + (-4-20)\big]$$

$$= \frac{1}{2}\big[-29-24-33-22-24\big]$$

$$= -\frac{132}{2} = -66$$

$= 66$ (since area is always positive)

$= 66$ square units.

Find the area of the triangle whose vertices are (2, 3), (5, 7), (-3, 4)

Solution:

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 3 \\ 5 & 7 \\ -3 & 4 \\ 2 & 3 \end{vmatrix}$$

$$= \frac{1}{2}\left[2.7 + 5.4 + (-3)(3) - 2.4 - (-3).7 - 5.3\right]$$

$= 11.5$ square units.

Try to yourself

Find the area of the quadrilateral whose angular points are

H/W

(a) (1, 1), (3, 4), (5, -2), (4, 7) — Ans: 7.5

(b) (1, 2), (6, 2), (5, 3), (3, 4) — Ans: 5.5

(c) (2, 4), (3, 2), (8, 4), (7, 6) — Ans: 12

(d) (-3, 4), (6, 2), (4, -3). — Ans: 24.5

\# Find the cartesian co-ordinates of the poi[nts] whose polar co-ordinates are ~~(i) 5, (-π~~ :

(i) $(5, -\pi/4)$ (ii) $(2, 330^\circ)$, (iii) $(3, 585^\circ)$, (iv) $(6, 780^\circ)$ (v) $(2, 1395^\circ)$, (vi) $(7, 870^\circ)$ (vii) $(9, 1560^\circ)$

TW

Solution: (i) we know,

$$x = r\cos\theta = 5\cos(-\pi/4) = 5\cos\left(\frac{\pi}{4}\right) = 5\cos 45^\circ$$

$$= 5 \cdot \frac{1}{\sqrt{2}} = \frac{5 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{5\sqrt{2}}{2}$$

$$\therefore x = \frac{5\sqrt{2}}{2}$$

and $y = r\sin\theta = 5\sin(-\pi/4) = -5\sin 45^\circ$

$$= -5 \cdot \frac{1}{\sqrt{2}} = -\frac{5\sqrt{2}}{2}$$

$\therefore$ The cartesian co-ordinates are $\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$.

A

(ii) $x = 2\cos 330^\circ = 2\cos(360^\circ - 30^\circ)$

$$= 2\cos 30^\circ$$

$$= \cancel{2 \cdot \frac{2}{\sqrt{3}}}$$

$$= 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

$y = 2\sin 330^\circ = 2\sin(360^\circ - 30^\circ) = -2\sin 30^\circ$

$$= -2 \cdot \frac{1}{2} = -1$$

$\therefore$ The cartesian co-ordinates are $(\sqrt{3}, -1)$