Independent University, Bangladesh

School of Engineering, Technology and Sciences

Department of Computer Science & Engineering

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CSC501, CSC301, CSE437, CEN437: Finite Automata and Computability

Assignment – 02

Submitted to: A.R Azimul Haque

Submitted By: Group- K

Name: Nafisa Nawal

ID: 1810064

Name: Md. Ashikur Rahman

ID: 1831110

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Answer to the que : 1

Prob Definition: let "P" be a program of length n. Then a snapshot or instantaneous description of a program "P" is a pair (i, σ) where $1 \leq i \leq n+1$ & "σ" is a state of "P".

let (i, σ) be a nonterminal snapshot of program "P" then its successor (j, τ) will depend on the i_{th} instruction of "P".

So, the computation of "P" ~~beginnig~~ beginning with the snapshot $(1, \sigma)$, where σ consists of the equation $X=3$, $Y=0$, $Z=0$.

Snapshot (i, σ)	Instruction	Successor (j, τ)
1. $(X=3, Y=0, Z=0)$	[A] If $X \neq 0$ GOTO B	3. $(X=3, Y=0, Z=0)$
3. $(X=3, Y=0, Z=0)$	[B] $X \leftarrow X-1$	4. $(X=2, Y=0, Z=0)$
4. $(X=2, Y=0, Z=0)$	$Y \leftarrow Y+1$	5. $(X=2, Y=1, Z=0)$
5. $(X=2, Y=1, Z=0$	$Z \leftarrow Z+1$	6. $(X=2, Y=1, Z=1)$
6. $(X=2, Y=1, Z=1$	GOTO A	1. $(X=2, Y=1, Z=1)$
1. $(X=2, Y=1, Z=1)$	[A] If $X \neq 0$ GOTO B	3. $(X=2, Y=1, Z=1)$
3. $(X=2, Y=1, Z=1)$	[B] $X \leftarrow X-1$	4. $(X=1, Y=1, Z=1)$
4. $(X=1, Y=1, Z=1)$	$Y \leftarrow Y+1$	5. $(X=1, Y=2, Z=1)$
5. $(X=1, Y=2, Z=1)$	$Z \leftarrow Z+1$	6. $(X=1, Y=2, Z=2)$
6. $(X=1, Y=2, Z=2)$	GOTO A	1. $(X=1, Y=2, Z=2)$

Snapshot (i, σ)	Instruction	Successor (j, τ)
1. $(X=1, Y=2, Z=2)$	[A] If $x \neq 0$ GOTO B	3. $(X=1, Y=2, Z=2)$
3. $(X=1, Y=2, Z=2)$	[B] $x \leftarrow x-1$	4. $(X=0, Y=2, Z=2)$
4. $(X=0, Y=2, Z=2)$	$Y \leftarrow Y+1$	5. $(X=0, Y=3, Z=2)$
5. $(X=0, Y=3, Z=2)$	$Z \leftarrow Z+1$	6. $(X=0, Y=3, Z=3)$
6. $(X=0, Y=3, Z=3)$	GOTO A	1. $(X=0, Y=3, Z=3)$
1. $(X=0, Y=3, Z=3)$	[A] If $X \neq 0$ GOTO B	2. $(X=0, Y=3, Z=3)$
2. $(X=0, Y=3, Z=3)$	GOTO C	7. $(X=0, Y=3, Z=3)$
7. $(X=0, Y=3, Z=3)$	[C] If $Z \neq 0$ GOTO D	9. $(X=0, Y=3, Z=3)$
9. $(X=0, Y=3, Z=3)$	[D] $Z \leftarrow Z-1$	10. $(X=0, Y=3, Z=3)$
10. $(X=0, Y=3, Z=2)$	$X \leftarrow X+1$	11. $(X=1, Y=3, Z=2)$
11. $(X=1, Y=3, Z=2)$	GOTO C	7. $(X=1, Y=3, Z=2)$

Snapshot (i, σ)	Instruction	Successor (j, τ)
7. (X=1, Y=3, Z=2)	[C] If Z ≠ 0 GOTO D	9. (X=1, Y=3, Z=2)
9. (X=1, Y=3, Z=2)	[D] Z ← Z-1	10. (X=1, Y=3, Z=1)
10. (X=1, Y=3, Z=1)	X ← X+1	11. (X=2, Y=3, Z=1)
11. (X=2, Y=3, Z=1)	GOTO C	7. (X=2, Y=3, Z=1)
7. (X=2, Y=3, Z=1)	[C] If Z≠0 GOTO D	9. (X=2, Y=3, Z=1)
9. (X=2, Y=3, Z=1)	[D] Z ← Z-1	10. (X=2, Y=3, Z=0)
10. (X=2, Y=3, Z=0)	X ← X+1	11. (X=3, Y=3, Z=0)
11. (X=3, Y=3, Z=0)	GOTO C	7. (X=3, Y=3, Z=0)
7. (X=3, Y=3, Z=0)	[C] If Z ≠0 GOTO D	8. (X=3, Y=3, Z=0)
8. (X=3, Y=3, Z=0)	GOTO E	12. (X=3, Y=3, Z=0) This is terminal snapshot

Answer to the question no: 2

Prob Definition: let "P" be a program and let $r_1, r_2, \ldots r_m$ be m given numbers. The state σ of "P" is defined to consist of equations.

$$X_1 = r_1,\ X_2 = r_2 \ \ldots\ X_m = r_m,\ Y = 0$$

together with the equation $V = 0$ for each variable V in P other than $X_1, X_2, \ldots X_m, Y$.

Solution:

[A] If $X \neq 0$ GOTO M

GOTO E

[M] $X \leftarrow X - 1$

GOTO N

[N] $Z \leftarrow 4$

[K] $Z \leftarrow Z - 1$

$Y \leftarrow Y + 1$

If $Z \neq 0$ GOTO K

GOTO A

Answer to the question no: 3

Preob definition:

Euclidean GCD

Input : Two positive integers a, b

Output : $d = \gcd(a,b)$, and integers u, v such that $a.u + b.v = d$

The following is preogream P that computes $\gcd(x_1, x_2)$ using the Euclidean Algorithm.

$Y \leftarrow X_1$

$Z_6 \leftarrow X_2$

If $X_1 \neq 0$ GOTO A

$Y \leftarrow X_2$

GOTO E

[A] If $X_2 \neq 0$ GOTO B

$Y \leftarrow X_1$

GOTO E

[B] If $Z_6 \neq 0$ GOTO C
GOTO E

~~[B] If $Z_6 \neq$~~

[C] $Z_1 \leftarrow Y$
$Y \leftarrow Z_6$
$Z_3 \leftarrow 0$
$Z_4 \leftarrow Z_1$
$Z_4 \leftarrow Z_4 - Z_6$
$Z_5 \leftarrow 0$

[D_1] If $Z_4 \neq 0$ GOTO D_2
GOTO D_3

[D_2] $Z_4 \leftarrow Z_4 - Z_6$
$Z_3 \leftarrow Z_3 + 1$
GOTO D_1

[D_3] $Z_4 \leftarrow Z_3$

[D_4] If $Z_4 \neq 0$ GOTO D_5
GOTO D_6

[D_5] $Z_5 \leftarrow Z_5 + Z_6$
$Z_4 \leftarrow Z_4 - 1$

GOTO D4

[D6] $Z_6 \leftarrow Z_1 - Z_5$

GOTO B

[Note: The instructions 13 to 28 in the above program compute the remainder when Z_1 is divided by Z_6]

Answer to the que:4

Prob definition: Here the function $\langle x, y \rangle$

$= 2^x (2y+1) \dot{-} 1$

We have to find the Godel number of the sequence [2, 0, 5, 6, 0, 0, 2, 3] and [3, 0, 4, 2]

(a) The Godel number of the sequence [2, 0, 5, 6, 0, 0, 2, 3] is:

$[2, 0, 5, 6, 0, 0, 2, 3] = 2^2 \cdot 3^0 \cdot 5^5 \cdot 7^6 \cdot 11^0 \cdot 13^0 \cdot 17^2 \cdot 19^3$

$= 2.915123099 \times 10^{15}$

(b) The Godel Number of the sequence [3, 0, 4, 2] is:

$[3, 0, 4, 2] = 2^3 \cdot 3^0 \cdot 5^4 \cdot 7^2 = 245000$

Conclusion:

1. P beginning with the snapshot $(1, \sigma)$ where σ consists of the equation $X = 3, Y = 0, Z = 0$.
Then we find all the snapshots (i, σ) and successors (j, τ)

2. We are using a programming language in P (without using Macros) that computes the function $f(x) = 4x$

3. $gcd(x_1, x_2)$ is the greatest common division of x_1 & x_2. We are using a programming language in P that computes gcd.

4. We are using function $\langle x, y \rangle = 2^x (2y+1) \dot{-} 1$ then to find the Godel number of the sequence.