讨论三角复合映射

积分特性

# 前景

我们已然讨论了对称性和周期性的复合映射继承性.

即 $f(x)$ 为周期或对称

$$\begin{cases} f(x\pm T) = f(x) & \text{：周期性} \\ f(x) = f(2s-x) & \text{：对称性} \end{cases}$$

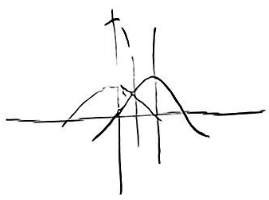
复合继承 Let $\psi(x) = \varphi(f(x))$

则

$$\begin{cases} \psi(x\pm T) = \varphi[f(x\pm T)] = \varphi(f(x)) = \psi(x) \\ \psi(2s-x) = \varphi[f(2s-x)] = \varphi[f(x)] = \psi(x) \end{cases}$$

故而因 $\underline{\sin(x)}$，$\underline{\cos(x)}$ 以及 $\underline{\sin(x)与\cos x}$ 具有对称性和周期性，从而有一系列推论

周期性 $\begin{cases} \sin x : \sin(x+2\pi) = \sin(x) \\ \cos x : \cos(x+2\pi) = \cos(x) \end{cases}$



对称性 $\begin{cases} \sin x : \sin(x) = \sin(\pi-2x) \\ \cos x : \cos(x) = \cos(-x) \end{cases}$

$\sin x \& \cos x$： $\sin(x) = \cos(\frac{\pi}{2}-x)$ 关于 $\frac{\pi}{4}$

或 $\cos(x) = \sin(\frac{\pi}{2}-x)$

亦可用诱导公式进行验证.

现结合上之前的周期性与对称性积分特性.

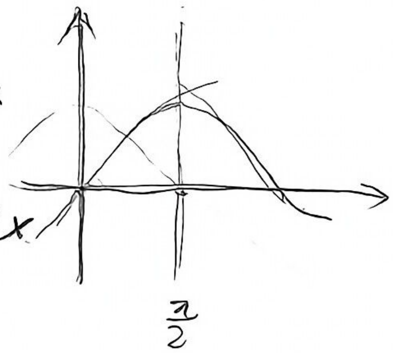
积分特性

$$\begin{cases} \int_a^{a+T} f(x)dx = \int_0^T f(x)dx \\ \int_a^b f(x)dx = \int_{2s-b}^{2s-a} f(x)dx \end{cases}$$

分别运用在 $f(\sin x)$ $f(\cos x)$, $f(\sin x)$ 与 $f(\cos x)$ 中.

① $\int_0^{\frac{\pi}{2}} f(\sin x)dx = \int_0^{\frac{\pi}{2}} f(\cos x)dx$

Proof: 因 $f(\sin x)$ 与 $f(\cos x)$ 关于 $\frac{\pi}{4}$ 对称



则 $\int_0^{\frac{\pi}{2}} f(\sin x) = \int_{\frac{\pi}{2}-\frac{\pi}{2}}^{\frac{\pi}{2}-0} f(\cos x)dx$

$= \int_0^{\frac{\pi}{2}} f(\cos x)dx$

② $\int_0^{\pi} f(\sin x)dx = 2\int_0^{\frac{\pi}{2}} f(\sin x)dx$

proof: 因 $\sin(x)$ 关于 $\frac{\pi}{2}$ 对称

$$\begin{aligned}\int_0^{\pi} f(\sin x)dx &= \int_0^{\frac{\pi}{2}} f(\sin x)dx + \int_{\frac{\pi}{2}}^{\pi} f(\sin x)dx \\ &= \int_0^{\frac{\pi}{2}} f(\sin x)dx + \int_{\pi-\pi}^{\pi-\frac{\pi}{2}} f(\sin x)dx \\ &= \int_0^{\frac{\pi}{2}} f(\sin x)dx + \int_0^{\frac{\pi}{2}} f(\sin x)dx \\ &= 2\int_0^{\frac{\pi}{2}} f(\sin x)dx\end{aligned}$$

等层约束 $\int_0^{\frac{\pi}{2}} f(\sin x)dx = \int_{\frac{\pi}{2}}^{\pi} f(\sin x)dx$