EXTRAS: Unit Root Tests: ADF, KPSS, and PP

Augmented Dickey-Fuller (ADF) Test

Purpose: Tests for the presence of a unit root (non-stationarity) in a time series.

Statement of Hypothesis:

- **Null Hypothesis ($\mathbf{H_0}$):** The time series has a unit root (non-stationary).
- **Alternative Hypothesis ($\mathbf{H_1}$):** The time series is stationary (no unit root).

Test Statistic:

The ADF regression model is:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \epsilon_t$$

where:

- α = constant, β = trend coefficient, γ = coefficient on lagged level
- Δy_{t-i} = lagged differences (to correct autocorrelation).

The test statistic is the *t-statistic* on γ.

Critical Values:

- Compare the ADF test statistic with **MacKinnon critical values** (different for models with constant/trend).
- If test statistic $<$ critical value $\rightarrow$ **Reject $\mathbf{H_0}$** (stationary).
- If test statistic $>$ critical value $\rightarrow$ **Fail to reject $\mathbf{H_0}$** (non-stationary).

Conclusion:

- Rejecting H_0 implies stationarity.
- Failing to reject suggests differencing is needed.

Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test

Purpose: Tests for stationarity around a deterministic trend (opposite of ADF).

Statement of Hypothesis:

- **Null Hypothesis ($\mathbf{H_0}$):** The time series is stationary (no unit root).
- **Alternative Hypothesis ($\mathbf{H_1}$):** The time series has a unit root (non-stationary).

Test Statistic:

Based on residuals from a regression of y_t on a constant (or constant + trend).
Computes a **LM statistic**:

$$\text{KPSS} = \frac{1}{T^2} \sum_{t=1}^{T} \frac{S_t^2}{s^2}$$

where S_t is the partial sum of residuals, s^2 is the error variance estimator.

Critical Values:

- Compare KPSS statistic with critical values (different for level/trend stationarity).
- If test statistic > critical value → **Reject H_0** (non-stationary).
- If test statistic < critical value → **Fail to reject H_0** (stationary).

Conclusion:

- Rejecting H_0 suggests differencing is needed.
- Failing to reject implies stationarity.

Phillips-Perron (PP) Test

Purpose: Tests for a unit root while accounting for autocorrelation and heteroskedasticity without adding lagged differences.

Statement of Hypothesis:

- **Null Hypothesis (H_0):** The time series has a unit root (non-stationary).
- **Alternative Hypothesis (H_1):** The time series is stationary.

Test Statistic:

Modifies the Dickey-Fuller *t-statistic* to account for serial correlation using non-parametric adjustments.
Regression model:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \epsilon_t$$

The PP test corrects for autocorrelation using Newey-West standard errors.

Critical Values:

- Uses the same **MacKinnon critical values** as ADF.
- If test statistic < critical value → **Reject H_0** (stationary).
- If test statistic > critical value → **Fail to reject H_0** (non-stationary).

Conclusion:

Similar to ADF but more robust to autocorrelation.

Summary

Test	**H_0 (Null)**	**H_1 (Alt.)**	**Test Statistic**	**Critical Values**	**Implication**
ADF	Unit root (non-st)	Stationary	t-statistic on γ	MacKinnon	Differencing if non-stat
KPSS	Stationary	Non-stati	LM statistic	KPSS crit values	Differencing if non-stat
PP	Unit root (non-st)	Stationary	Modified t-stat	MacKinnon	Differencing if non-stat