STA 2422: GAME THEORY

Lecture 4: Week Four

Instructor: Mwelu Susan *Topic: Mixed Strategy*

4.24 MIXED STRATEGY

Constant sum games are games of pure conflict; one player's gain is the other player's loss. In the penalty-taking game left, centre and right are the pure strategies of the striker and the goalkeeper. If the striker decides that he is going to kick the ball to the left this would imply that he had chosen one of his pure strategies. Alternatively he might prefer to randomise between his pure strategies by, for instance, throwing a dice before he runs up to kick the ball. He could kick to the left if the dice showed 1 or 2, to the right if it showed a 3 or 4 and to the centre of the goal otherwise. If he did this the probability of him choosing any one of his three pure strategies would be $\frac{1}{3}$. This can be written as $P(left) = \frac{1}{3}, P(center) = \frac{1}{3}, P(right) = \frac{1}{3}$. Strategies that mix up a player's pure strategies in this way are called mixed strategies. Mixed strategies like these can be useful in games of pure conflict like penalty taking, where one player doesn't want the other to be able to predict their move. A game in strategic form does not always have a Nash equilibrium in which each player deterministically chooses one of his strategies. However, players may instead randomly select from among these pure strategies with certain probabilities. Randomizing one's own choice in this way is called a mixed strategy. When players randomise they are using mixed strategies and the related solution concept is that of a mixed strategy Nash equilibrium. Any finite strategic-form game has an equilibrium if mixed strategies are allowed.

4.24.1 Nash equilibrium in mixed strategies

Choosing a mixed strategy might be a rational way of dealing with uncertainty about what you think the other player is likely to be doing, for example if neither of you have a dominant strategy or if there are multiple pure strategy Nash equilibria. However, even when it is rational for players to randomise, mixed strategies will only be optimal if they are best responses to each other.

4.24.1.1 Chicken game

Consider the chicken game whose pay off matrix is shown in table 4.8. The two players are Tony and Daniel who are choosing between challenging eacg other to a fight or not. Neither really wants to fight nor do they want to lose face by backingdown.

		Daniel	
		Backdown	Challenge
	Backdown	1,1	<u>0</u>,<u>2</u>
Tony	Challenge	<u>2</u>,<u>0</u>	-3,-3

Table 4.8: International trade pay-off matrix

In this game the two Nash equilibria are {backdown, challenge} and {challenge, backdown}. Each player prefers a different Nash equilibrium. Here, Tony prefers the equilibrium in which Daniel backs down but Daniel prefers Tony to backdown. In these circumstances it is not clear how or if the players will manage to coordinate their strategy choices. One solution to this coordination problem might be for the players to choose mixed strategies. A mixed strategy is essentially a rule that prescribes certain action choices according to a probability distribution. For the players' mixed strategies to be optimal they need to be best responses to each other. If Tony chooses to backdown with probability p and challenge with probability $1-p$ and Daniel chooses to backdown with probability q and challenge with probability $1-q$, then for p and q to constitute a mixed strategy Nash equilibrium they must be best responses to each other.

If either player chooses a mixed strategy then they must be indifferent between playing either of their pure strategies. If not, then one pure strategy would be preferred and they would choose that rather than randomising. For a mixed strategy to be part of a Nash equilibrium for Tony, he must be indifferent between choosing challenge or backdown. If this is the case his expected pay-off from choosing challenge must be the same as his expected pay-off from choosing backdown. Similarly, if Daniel chooses a mixed strategy, his expected pay-off from choosing challenge must be the same as his expected pay-off from backdown.

Tony's perspective

Tony's expected pay-off from choosing backdown depends on Daniel's strategy. Suppose Daniel chooses a mixed strategy of backdown with probability q and challenge with probability $1-q$. In this case; Tony's expected pay-off from choosing backdown is

$$1(q) + 0(1-q) \tag{4.1}$$

Tony's expected pay-off from choosing challenge is

$$2(q) + (-3)(1-q) \tag{4.2}$$

Equating these two expected pay offs yields

$$
\begin{aligned}
1(q) + 0(1-q) &= 2(q) + (-3)(1-q) \\
4q &= 3 \\
q &= \frac{3}{4}
\end{aligned}
\tag{4.3}
$$

In other words it is rational for Tony to choose a mixed strategy if Daniel chooses backdown with probability $\frac{3}{4}$ (or is choosing backdown three-fourths of the time).

Daniel's perspective

Daniel's expected pay-off from choosing backdown depends on Tony's strategy. Suppose Tony chooses a mixed strategy of backdown with probability p and challenge with probability $1-p$. In this case;

Daniel's expected pay-off from choosing backdown is

$$
1(p) + 0(1-p) \tag{4.4}
$$

Daniel's expected pay-off from choosing challenge is

$$
2(p) + (-3)(1-p) \tag{4.5}
$$

Equating these two expected pay offs yields

$$
\begin{aligned}
1(p) + 0(1-p) &= 2(p) + (-3)(1-p) \\
4p &= 3 \\
p &= \frac{3}{4}
\end{aligned}
\tag{4.6}
$$

This meand that if $p = 3/4$ and $q = 3/4$ then it makes sense for Daniel and Tony to choose a mixed strategy. Then Daniel and Tony's mixed strategies will be best responses to each other since they can do no better by choosing something else. Therefore the mixed strategies are Nash equilibrium strategies which can be written as $\{$(Tony: backdown $\frac{3}{4}$, challenge $\frac{1}{4}$) (Daniel: backdown $\frac{3}{4}$, challenge $\frac{1}{4}$)$\}$

Mixed strategies have a certain appeal when players are trying to be unpredictable as in games of pure conflict like penalty taking. They also make sense when one player is trying to deter some action by another, for instance in quality control exercises in which auditors make random checks to discourage malpractice (or encourage good practice). The random checks made by officials at customs controls are another example of this kind. Mixed strategies may also be rational when a player just doesn't know what else to do. Even if mixed strategies appeal to the players in a game one problem with the mixed strategy Nash equilibrium concept in practice is that players somehow need to work out each other's mixed strategies in order to make it operational.