Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

   Ans- We have a normal distribution with Mu = 45 and Sigma = 8.0. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have $X \leq 50$ so the question is to find $P(X > 50)$.
   $P(X > 50) = 1 - P(X \leq 50)$.
   $Z = (X - Mu)/ Sigma = (X - 45)/8.0$
   Thus the question can be answered by using the normal table to find
   $P(X \leq 50) = P(Z \leq (50 - 45)/8.0) = P(Z \leq 0.625) = 73.4\%$
   Probability that the service manager will not meet his demand will be
   100-73.4 = 26.6% or 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   - True
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
   - True

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans.

$2X1 \sim N(2\,u, 4\,\sigma^2)$ and

$X1+X2 \sim N(\mu + \mu, \sigma^2 + \sigma^2) \sim N(2\,u, 2\sigma^2)$

$2X1-(X1+X2) = N(4\mu, 6\,\sigma^2)$

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

    Ans- Option D

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

        Ans-Rs 603.68

    B. Specify the 5^{th} percentile of profit (in Rupees) for the company
        Ans-Rs 476.33

    C. Which of the two divisions has a larger probability of making a loss in a given year?
        Ans-First division has a larger probability.