EC5.201 - Signal Processing - Final exam

Date: 05 August, 2021 | Maximum marks: 40
Instructor: Santosh Nannuru | Exam duration: 2 hours

Instructions:

a) The exam has 3 pages

b) There are 8 questions for a total of 40 marks

c) Write your name and roll number on the first page

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1. **[4 Marks]** $H(z)$ is a digital band-pass filter that has pass-band region $[-\frac{\pi}{2}, -\frac{\pi}{4}] \cup [\frac{\pi}{4}, \frac{\pi}{2}]$.

   (a) Explain what will be the nature of the filter given by $H(-z)$.

   (b) Clearly identify its pass-band and stop-band regions.

2. **[6 Marks]** A student is investigating whether an alternate transform to DFT can be constructed which gives real-valued transform for real-valued signals. She proposes to use the following:

$$X[k] = \sum_{n=0}^{N-1} x[n]\left(\cos\left(\frac{2\pi nk}{N}\right) + \sin\left(\frac{2\pi nk}{N}\right)\right), \quad k = 0, 1, \ldots, N-1$$

   where $X[k]$ is the new N-length transform of the N-length signal $x[n]$.

   (a) An important requirement of any transform is that it should be invertible. Show that the above proposed transform is in fact invertible with the inverse given by

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\left(\cos\left(\frac{2\pi nk}{N}\right) + \sin\left(\frac{2\pi nk}{N}\right)\right), \quad n = 0, 1, \ldots, N-1$$

   (b) How is the above transform related to the DFT of $x[n]$? Derive expressions.

3. **[4 Marks]** An audio compact disc (CD) stores audio signals sampled at 44.1 kHz and quantized using 32 bits. Assume there are two audio channels. Answer the following with explanations:

   (a) If the CD contains 5 songs of duration 5 mins each, what is the amount of memory used up in storing the files in the CD?

   (b) If it is known that the stored audio signals have a maximum frequency of 10 kHz, could you suggest a way to reduce the amount of memory used in the CD without any loss in audio quality using concepts from the course? How much memory can you save?

4. **[6 Marks]** A discrete-time LTI system is described by the difference equation

$$y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

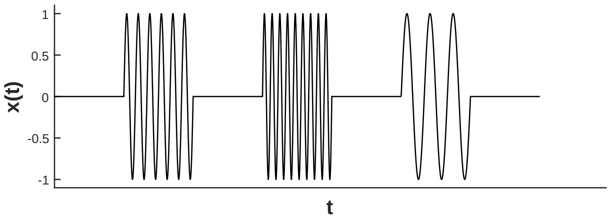
where $x[n]$ and $y[n]$ are input and output signals respectively. Assume that this system is stable.

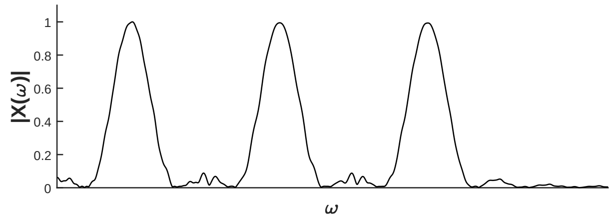
(a) Find expression for its frequency response.

(b) Given the constants a_1 and a_2, identify the constraints on b_0, b_1 and b_2 such that this system has constant magnitude response.

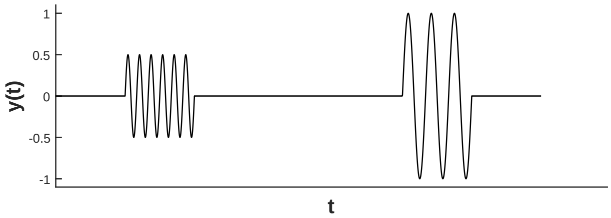
(c) Identify conditions on the constants a_i and b_i for this system to have linear phase.

5. **[4 Marks]** A real signal $x(t)$ contains three sinusoidal pulses as shown in the figure. The magnitude of its Fourier transform $X(\omega)$ is also drawn (only positive frequencies are shown).





When this signal is passed through an LTI system, the output signal has the following waveform.



(a) Give a rough sketch of the magnitude of the frequency response of this LTI system.

(b) Give clear explanation for your answer.

6. **[6 Marks]** An alternate bilinear transformation is proposed to convert analog filters into digital filters given by the following transformation

$$s = \frac{2}{T}\frac{1+z^{-1}}{1-z^{-1}}.$$

(a) Identify how do the points in s-plane map to the points in z-plane using this transformation.

(b) If the analog filter is stable and causal, what can be said about the corresponding digital filter obtained by this method?

7. **[5 Marks]** An LTI system with impulse response $h(t)$ produces output $y(t) = h(t) \star x(t)$ when the input is $x(t)$, here $\star$ denotes continuous-time convolution. The bandlimited signal $x(t)$ is sampled at rate $f_s = \frac{1}{T_s}$ (which is above its Nyquist rate) to obtain the discrete-time signal $x[n] = x(nT_s)$.

(a) Show that $y(t)$ can also be sampled at rate f_s without any loss of information.

Let the samples be $y[n] = y(nT_s)$.

(b) Find the impulse response $h[n]$ of a discrete-time LTI system such that $y[n] = h[n] \star x[n]$, here $\star$ denotes discrete-time convolution. Justify whether this impulse response is unique.

8. **[5 Marks]** The N-length signals $x_1[n], x_2[n]$ and $x_3[n]$ have N-point DFT denoted by $X_1[k], X_2[k]$ and $X_3[k]$ respectively. A 3N-length signal $f[n]$ is defined as follows

$$f[n] = \begin{cases} x_1[n/3] & \text{, if } \quad n \bmod 3 = 0 \\ x_2[(n-1)/3], & \text{if } \quad n \bmod 3 = 1 \\ x_3[(n-2)/3], & \text{if } \quad n \bmod 3 = 2 \end{cases}$$

(a) Find the 3N-point DFT of $f[n]$ i.e. $F[k]$ in terms of $X_1[k], X_2[k]$ and $X_3[k]$.

(b) If $x_1[n] = x_2[n] = x_3[n]$, find $F[k]$ in terms of $X_1[k]$. Simplify your answer and write down $F[k]$ if $X_1[k] = [1, 2, 3, 4]$.

Best of Luck!