

SVD  $\rightarrow$  the relevant case:

Let  $A$  be an  $m \times n$  matrix,  $m \geq n$ . Then  $A$  can be factored as  $A = UDV^T$ .

$U = m \times n$  matrix with orthogonal columns.

$D = n \times n$  diagonal matrix of singular values in descending order, all  $\geq 0$ .

$V = n \times n$  orthogonal matrix, whose columns are singular vectors corresponding to  $D$ .  $V$  has orthonormal columns, means  $V^T V = I_{nn}$ .

and  $U$  is norm preserving in that  $\|Ux\| = \|x\|$  for any vector  $x$ .

SVD for homogeneous least squares solution:

Minimize  $\|Ax\|^2$  such that  $\|x\| = 1$ .

or minimize  $[A^T A] x$  such that  $x^T x = 1$ .

Let  $A = UDV^T$ .

Then  $\|UDV^T x\| = \|DV^T x\|$ .

[Note that  $U$  is norm preserving and hence  $\|Ux\|^2 = x^T U^T U x = x^T x = \|x\|^2$ .

or  $\|Ux\| = \|x\|$ . Hence in above

$\|UDV^T x\| = \|DV^T x\|$ .

also  $\|V^T x\| = \|x\|$  (since  $V$  is orthonormal)

sub  $y = V^T x =$

Then  $\|y\| = \|V^T x\| = \|x\|$ .

Minimize  $\|Dy\|$  subject to  $\|y\| = 1$ .