

$$\alpha x_1 - \alpha y_1 + \beta z_1 + t_x = p_{x1} \rightarrow (6)$$

$$\alpha x_1 + \alpha y_1 - \gamma z_1 + t_y = p_{y1} \rightarrow (7)$$

$$-\beta x_1 + \gamma y_1 + z_1 + t_z = p_{z1} \rightarrow (8)$$

~~$$\alpha y_1 - \alpha z_1 = \alpha z_1 + t_x$$~~

$$\begin{bmatrix} -\alpha y_1 & \alpha z_1 & 0 & 1 & 0 & 0 \\ \alpha x_1 & 0 & -\alpha z_1 & 0 & 1 & 0 \\ 0 & -\alpha x_1 & \alpha y_1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} p_{x1} - \alpha x_1 \\ p_{y1} - \alpha y_1 \\ p_{z1} - \alpha z_1 \end{bmatrix}$$

$A_{3 \times 6} \quad X_{6 \times 1} = b_{3 \times 1} \rightarrow (9)$

If there are n such correspondences stack up the A matrix such that

$$\begin{bmatrix} A^1_{3 \times 6} \\ A^2_{3 \times 6} \\ \vdots \\ A^n_{3 \times 6} \end{bmatrix} X_{6 \times 1} = \begin{bmatrix} b^1_{3 \times 1} \\ b^2_{3 \times 1} \\ \vdots \\ b^n_{3 \times 1} \end{bmatrix} \rightarrow (10)$$

$(3n \times 6) \times (6 \times 1) = (3n \times 1)$

$$\text{or } \boxed{A X = b}$$

$A_{3n \times 6} \quad X_{6 \times 1} = b_{3n \times 1}$

Pseudo inverse / SVD to solve for X .