

$$P_i = \vec{p}_i$$

$P_i: i = \{1, \dots, m\}$ set of m points in frame, I .

$Q_j: j = \{1, \dots, m\}$ set of m point in frame, J .

$$\vec{P}_i = R_J^I \vec{Q}_j + \vec{t}_J^I \longrightarrow (1)$$

Dropping suffixes.

$$\vec{P}_i = R_J^I \vec{Q}_j + \vec{t} \quad \text{or} \quad \vec{P}_i = R \vec{Q}_j + \vec{t} \longrightarrow (2)$$

To solve for R and t .

Use Z-Y-X Euler angles to represent

$$R_J^I \text{ or } R \text{ as: } \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix} \longrightarrow (3)$$

$$\text{or } R = \begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix} \longrightarrow (4)$$

Using small angle approximations to get (4)
 $\boxed{\cos\alpha=1, \sin\alpha=\alpha, \sin\beta\cos\gamma=0 \text{ etc}}$

Then.

$$\begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix} \begin{bmatrix} Q_{x1} \\ Q_{y1} \\ Q_{z1} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} P_{x1} \\ P_{y1} \\ P_{z1} \end{bmatrix} \longrightarrow (5)$$

1st correspondence.

Be careful to about which frames are these vectors represented in.