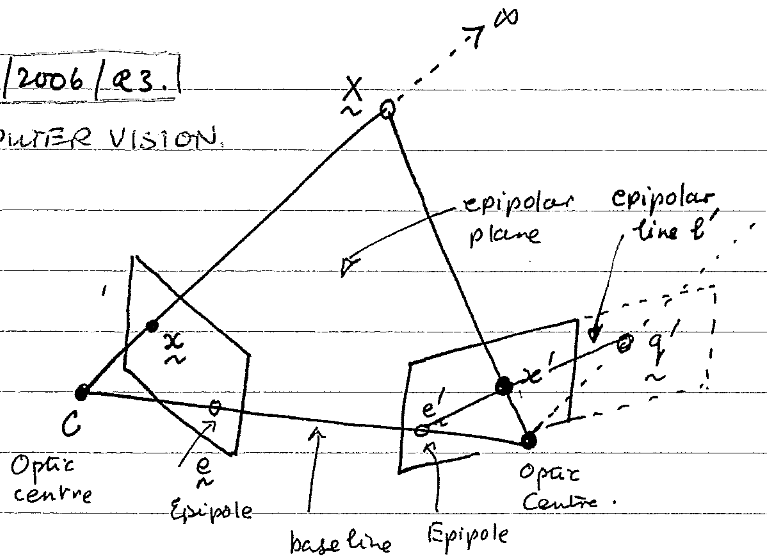
C4B/2006/Q3.

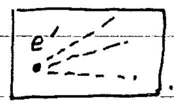
DWM.

COMPUTER VISION.



(a) Ray backprojected from $\underset{\sim}{x}$. The scene point $\underset{\sim}{X}$ lies somewhere on it. $\underset{\sim}{X}$ projects into second camera at x', which must lie on line from e', the epipole, to $\underset{\sim}{q}'$. This epipolar line l' is the intersect of the plane $CC'\underset{\sim}{X}$ with the image plane. (+4).

(ii) As $\underset{\sim}{x}$ (and $\underset{\sim}{X}$) vary a new epipolar plane is formed. All the planes passed through the epipolar points $\underset{\sim}{e}$ and $\underset{\sim}{e}'$. Epipolar lines will radiate from the epipole (+2).



(b) Projection of point at infinity Q is

$$\underset{\sim}{q}' = K'[R|t]\begin{bmatrix} K^{-1}\underset{\sim}{x} \\ 0 \end{bmatrix} = K'RK^{-1}\underset{\sim}{x} \quad (+2)$$

Epipole is projection of the optic centre $\underset{\sim}{e}' = K'[R|t]\begin{bmatrix} \underset{\sim}{0} \\ 1 \end{bmatrix} = K'\underset{\sim}{t}$ (+2)

Hence epipolar line $l' = \underset{\sim}{e}' \times \underset{\sim}{q}' = \underline{\underline{K'\underset{\sim}{t} \times K'RK^{-1}\underset{\sim}{x}}}$ (+1).

(c) By definition $\underset{\sim}{l}' = F\underset{\sim}{x} \Rightarrow F\underset{\sim}{x} = K't \times K'RK^{-1}\underset{\sim}{x}$

$$\Rightarrow \underset{\sim}{F} = \underline{[K'^{-T}][t_\times][R][K^{-1}]} \quad (+2)$$

Part (c)/ctd.

Any point $\underset{\sim}{x}'$ on line ℓ' has $(\underset{\sim}{x}'^T)\,\underset{\sim}{\ell}' = 0$.

Hence $\underset{\sim}{x}'^T [F] \underset{\sim}{x} = 0$. (+2).

(d). Single camera, $[K]=[\underline{K}'] = I$. and $t = [\,0\;0\;t_z\,]^T$.

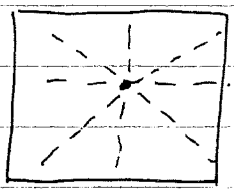
$$[x'\;y'\;1]\,[I]\begin{bmatrix} 0 & -t_z & 0 \\ t_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}[I][I]\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0.$$

$$[x'\;y'\;1]\begin{bmatrix} -t_z y \\ t_z x \\ 0 \end{bmatrix} = 0.$$

$$-t_z x' y + t_z x y' = 0.$$

So $\dfrac{y}{x} = \dfrac{y'}{x'}$ (+3).

(ii)



We expect the epipolar lines to radiate from the optic centre. $\dfrac{y}{x} = \dfrac{y'}{x'}$ expresses exactly this

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C4B Computer Vision

2007/Q5

(a) $$\lambda_i \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{pmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{pmatrix}$$

$$\lambda_i = p_{31} X_i + p_{32} Y_i + p_{33} Z_i + p_{34}$$

For $\lambda_i x_i \Rightarrow p_{31} X_i x_i + p_{32} Y_i x_i + p_{33} Z_i x_i + p_{34} x_i$

$$= p_{11} X_i + p_{12} Y_i + p_{13} Z_i + p_{14} \quad \ldots (A)$$

For $\lambda_i y_i \Rightarrow p_{31} X_i y_i + p_{32} Y_i y_i + p_{33} Z_i y_i + p_{34} y_i$

$$= p_{21} X_i + p_{22} Y_i + p_{23} Z_i + p_{24} \quad \ldots (B)$$

These can be rewritten as

$$\begin{array}{l} \text{Eq (A)} \rightarrow \\ \text{B} \rightarrow \end{array} \begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -X_i x_i & -Y_i x_i & -Z_i x_i & -x_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -X_i y_i & -Y_i y_i & -Z_i y_i & -y_i \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{34} \end{bmatrix} = \underset{\sim}{0}$$

which is identical in operation to the matrix given. $[A][p] = \underline{0}$

Alg: (1) Use 6 or more points in $[A]$, solving by least squares.
Then wrap $\underset{\sim}{p}$ into matrix $[P]$ ← known up to scale.

(2) The left 3×3 of $[P]$ is $[K][R]$

Invert: $[P]_{3\times3}^{-1} = \underset{\text{orthogonal}}{[R]^{-1}} \underset{\text{upper } \Delta\text{'ular.}}{[K]^{-1}}$

(3) Perform QR decomp $[R] \equiv$ "Q^{-1}" and $[K] \equiv$ "R^{-1}"

(4) Rescale $[K] \leftarrow [K]/k_{33}$ and $[P] \leftarrow [P]/k_{33}$.

(5) Now $Kt = \begin{pmatrix} p_{14} \\ p_{24} \\ p_{34} \end{pmatrix}$ $\qquad t = K^{-1} \begin{pmatrix} p_{14} \\ p_{24} \\ p_{34} \end{pmatrix}$ //

C4B/2007

Q5/ctd

(b) We know $\begin{pmatrix} R_1 & t_1 \\ 0^T & 1 \end{pmatrix}\begin{pmatrix} X \\ 1 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0^T & 1 \end{pmatrix} E \begin{pmatrix} X \\ 1 \end{pmatrix}$

$$\Rightarrow \quad E \equiv \begin{pmatrix} R_1 & t_1 \\ 0^T & 1 \end{pmatrix}$$

Also $\begin{pmatrix} R_2 & t_2 \\ 0^T & 1 \end{pmatrix}\begin{pmatrix} X \\ 1 \end{pmatrix} = \begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix} E \begin{pmatrix} X \\ 1 \end{pmatrix}$

Hence $\begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix} = \begin{pmatrix} R_2 & t_2 \\ 0^T & 1 \end{pmatrix}\begin{pmatrix} R_1 & t_1 \\ 0^T & 1 \end{pmatrix}^{-1} = \begin{pmatrix} R_2 & t_2 \\ 0^T & 1 \end{pmatrix}\begin{pmatrix} R_1^T & -R_1^T t_1 \\ 0^T & 1 \end{pmatrix}$

$$\Rightarrow (R \;\; t) = \left(R_2 R_1^T \,\middle|\, \left(-R_2 R_1^T t_1 + t_2\right)\right)$$

(c) The usual.

Useful in matching because search is along a line — 1D rather than 2D search.

(d) $K_2^{-T} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad [R\,|\,t] = \left[I \,\middle|\, \left(\begin{smallmatrix} 0 \\ 0 \\ -1 \end{smallmatrix} + \begin{smallmatrix} 0 \\ 0 \\ 2 \end{smallmatrix}\right)\right] = \left[I \,\middle|\, \begin{smallmatrix} 0 \\ 0 \\ 1 \end{smallmatrix}\right]$

$$\Rightarrow F = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} I \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\simeq \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -4 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[x_2 \;\; y_2 \;\; 1]\begin{bmatrix} 0 & -4 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = [x_2 \, y_2 \, 1]\begin{bmatrix} -4y_1 \\ 4x_1 \\ 0 \end{bmatrix}$$

$$\Rightarrow -4x_2 y_1 + 4x_1 y_2 = 0.$$

Hence $\dfrac{y_2}{x_2} = \dfrac{y_1}{x_1}$ radial — consistent with translation along optic axis.

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C4B Computer Vision 2008

(a) (i) Point along back projected ray is $\begin{pmatrix} Z K^{-1}\underset{\sim}{x} \\ 1 \end{pmatrix}$

and hence the point at ∞ is $\underset{\sim}{e} \stackrel{e}{=} \begin{pmatrix} K^{-1}\underset{\sim}{x} \\ 0 \end{pmatrix}$

$$\underset{\sim}{q}' = K'[R|\underset{\sim}{t}]\begin{pmatrix} K^{-1}\underset{\sim}{x} \\ 0 \end{pmatrix} = K' R K^{-1} \underset{\sim}{x}$$

(ii) $\underset{\sim}{e}' = K'[R|\underset{\sim}{t}]\begin{pmatrix} \underset{\sim}{0} \\ 1 \end{pmatrix} = K'\underset{\sim}{t}$.

(iii) l' runs from $\underset{\sim}{e}'$ and q' $\Rightarrow$ $\underset{\sim}{l}' = \underline{e}' \times \underline{q}' = K'\underset{\sim}{t} \times K'RK^{-1}\underset{\sim}{x}$

[5]

(b) $\underset{\sim}{l}' = F\underset{\sim}{x} \Rightarrow F = K'\underset{\sim}{t} \times K'RK^{-1}$

$$= (K'^{-T})(t \times RK^{-1})$$

$$= K'^{-T}[t_\times] R K^{-1}$$

Any point on $\underset{\sim}{l}'$ is $\underset{\sim}{x}'^{T}\underset{\sim}{l}' = 0 \Rightarrow \underset{\sim}{x}'^{T} F \underset{\sim}{x} = 0$ [5]

(c) $\lambda\underset{\sim}{x} = K\underset{\sim}{X}$; $\lambda'\underset{\sim}{x}' = K' R \underset{\sim}{X}$ when $\underset{\sim}{t} = 0$ (pure rotation)

$\Rightarrow \underset{\sim}{X} = K^{-1}\lambda\underset{\sim}{x} \Rightarrow \lambda'\underset{\sim}{x}' = K' R K^{-1} \lambda \underset{\sim}{x}$

$\Rightarrow \underset{\sim}{x}' \sim H\underset{\sim}{x}$ where $H \sim K'RK^{-1}$ [4]

(d). Rotates, zooms and translates $\Rightarrow \lambda'x' = K'(R\underset{\sim}{X} + \underset{\sim}{t})$ but $\hat{\underset{\sim}{n}}^T \underset{\sim}{X} = d$.

$\lambda\underset{\sim}{x} = K\underset{\sim}{X} \Rightarrow \dfrac{\underset{\sim}{X}}{\lambda} = K^{-1}\underset{\sim}{x}$

$\hat{n}^T\underset{\sim}{X} = d \Rightarrow \dfrac{\hat{n}^T X}{\lambda} = \hat{\underset{\sim}{n}}^T K^{-1}\underset{\sim}{x}$

$\Rightarrow \dfrac{d}{\lambda} = \hat{n}^T K^{-1} \underset{\sim}{x} \Rightarrow \dfrac{\underset{\sim}{t}}{\lambda} = \dfrac{\underset{\sim}{t}\,\hat{n}^T K^{-1}\underset{\sim}{x}}{d}$

[3]

$$\frac{\lambda' x'}{\lambda} = K'\left(R\frac{\underset{\sim}{X}}{\lambda} + \frac{\underset{\sim}{t}}{\lambda}\right) = K'\left(RK^{-1}\underset{\sim}{x} + \frac{\underset{\sim}{t}\,\hat{\underset{\sim}{n}}^T K^{-1}\underset{\sim}{x}}{d}\right)$$

$$\Rightarrow \underset{\sim}{x}' \sim \underbrace{K'\left(RK^{-1} + \frac{\underset{\sim}{t}\,\hat{\underset{\sim}{n}}^T K^{-1}}{d}\right)}_{\sim J \text{ a } 3\times3 \text{ matrix}} \underset{\sim}{x}$$

[3]

2009

C4B Qu 5

(a) Points in the world are first transferred into the camera's frame $\underset{\sim}{X}^c = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \underset{\sim}{X}^w$

The points undergo projection into the ideal image

$$\underset{\sim}{x}^I = [I \mid \underset{\sim}{0}]\underset{\sim}{X}^c$$

The intrinsics transform the ideal point into the actual image

$$\underset{\sim}{x} = \begin{pmatrix} f & s & u_0 \\ 0 & \alpha f & v_0 \\ 0 & 0 & 1 \end{pmatrix} \underset{\sim}{x}^I = K\underset{\sim}{x}^I$$

Combining these $\underset{\sim}{x} = K[R \mid \underset{\sim}{t}]\underset{\sim}{X}^w$ // (+3)

(ii) Take the left hand 3x3 of P as P_{Left}.

Invert it to give $P_{LEFT}^{-1} = R^T K^{-1}$

R^T: rotation matrix; K^{-1}: upper triangular.

Perform "QR" decomposition on P_{LEFT}^{-1}

Then $R \leftarrow \text{"}Q\text{"}^T$ // and $K \leftarrow \text{"}R\text{"}^{-1}$

The scale uncertainty will enter K

Hence $temp = K_{33}$

All $K_{ij} \leftarrow K_{ij}/temp$ //

Also $P_{ij} \leftarrow P_{ij}/temp$

Now $Kt = \begin{pmatrix} p_{14} \\ p_{24} \\ p_{34} \end{pmatrix}$ and $t = K^{-1}\begin{pmatrix} p_{14} \\ p_{24} \\ p_{34} \end{pmatrix}$ // (+4)

(b) $X^B = \begin{bmatrix} R_1^T & -R_1^T t \\ 0^T & 1 \end{bmatrix} X^1 \Rightarrow \underset{\sim}{X}^2 = \begin{bmatrix} R_2 & t_2 \\ 0^T & 1 \end{bmatrix}\begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0^T & 1 \end{bmatrix}\underset{\sim}{X}^1$

$$= \begin{bmatrix} R_2 R_1^T & -R_2 R_1^T t_1 + t_2 \\ 0 & 1 \end{bmatrix}\underset{\sim}{X}^1$$

But $\underset{\sim}{X}^2 = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}\underset{\sim}{X}^1$

Hence $R = R_2 R_1^T$ and $t = -R_2 R_1^T \underset{\sim}{t}_1 + \underset{\sim}{t}_2$ // (+5)

(c) Optic centre is $\begin{bmatrix} \underset{\sim}{0} \\ 1 \end{bmatrix}$. Pt at ∞ is $\begin{bmatrix} K_1^{-1} x_1 \\ 0 \end{bmatrix}$ — both in camera 1 frame.

Project into camera 2:

$$e_2 \sim K_2 [R|t] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = K_2 t \qquad q_2 \sim K_2 [R|t] \begin{bmatrix} K_1^{-1} x_1 \\ 0 \end{bmatrix} = K_2 R K_1^{-1} x_1$$

(+2)

Hence $\ell_2 \sim K_2 \underset{\sim}{t} \times K_2 R K_1^{-1} \underset{\sim}{x_1}$

and, as $\ell_2 \sim F x_1$, $\Rightarrow F \sim K_2 \underset{\sim}{t} \times K_2 R K_1^{-1}$

$$= K_2^{-T} (\underset{\sim}{t} \times R K_1^{-1})$$

Intro skew-symmetric $[t_\times]$ .... $= K_2^{-T} [t_\times] R K_1^{-1}$ (+2)

(d) $K_1 = I$, but $K_2 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow K_2^{-T} = \begin{pmatrix} \frac{1}{2} & & \\ & \frac{1}{2} & \\ & & 1 \end{pmatrix}$

$R = I$ as parallel

$$t_\times = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

So $$F \sim \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1/2 \\ 0 & 1 & 0 \end{pmatrix}$$ (+2)

$$\ell_2 \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1/2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1/2 \\ 1 \end{pmatrix}$$

This is the line $0x + -\frac{1}{2} y + 1 = 0$

$$\Rightarrow y = 2$$ (+2)

Agrees with expectation parallel epipolar lines along rows but y value doubled because of doubled focal length.

DWM Computer Vision (1) C4B [2010] v1 17/1/10

(a)

$$\underline{X}^1 = \begin{bmatrix} R_1 & t_1 \\ 0^T & 1 \end{bmatrix} \underline{X}^B \Rightarrow \underline{X}^B = \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0^T & 1 \end{bmatrix} \underline{X}^1$$

and

$$X^2 = \begin{bmatrix} R_2 & t_2 \\ 0^T & 1 \end{bmatrix} X^B = \begin{bmatrix} R_2 & t_2 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0^T & 1 \end{bmatrix} X^1$$

$$= \begin{bmatrix} R_2 R_1^T & -R_2 R_1^T t_1 + t_2 \\ 0^T & 1 \end{bmatrix} X^1$$

$$\Rightarrow R = R_2 R_1^T \quad \text{and} \quad \underline{t} = -R_2 R_1^T \underline{t}_1 + \underline{t}_2$$ [3]

(b) Projection of C_1's optic centre is $\underline{e}' = K_2 [R|t] \begin{pmatrix} \underline{0} \\ 1 \end{pmatrix} = K_2 \underline{t}$

Back projection of x_1 to point at ∞ is $\begin{pmatrix} K_1^{-1} x_1 \\ 0 \end{pmatrix}$

Projection of this into camera 2 is $\underline{q}' = K_2 [R|t] \begin{pmatrix} K_1^{-1} x_1 \\ 0 \end{pmatrix} = K_2 R K_1^{-1} x_1$

Epipolar line

$$\underline{l}' \sim \underline{e}' \times \underline{q}' = K_2 \underline{t} \times K_2 R K_1^{-1} \underline{x}_1$$

$$= K_2^{-T} (\underline{t} \times R K_1^{-1} x_1)$$

$$= \underbrace{K_2^{-T} [t_\times] R K_1^{-1}}_{= F_{21}} \underline{x}_1$$ [5]

(c) (i)

(1) Invert the left hand 3×3 of P_n

(2) As $P_n^{left} = KR$, $[P_n^{left}]^{-1} = R^{-1} K^{-1} \Rightarrow$ Performed QR decomposition

"Q" ← R^{-T} and "R" ← K^{-1}

So R ← "Q^T" and K ← "R^{-1}"

(3)* R is properly scaled, but K will not be. Correct this using

$P \leftarrow P/K_{33}$, $K \leftarrow K/K_{33}$.

(4) Right hand column of $\begin{pmatrix} p_{14} \\ p_{24} \\ p_{34} \end{pmatrix} = K t \Rightarrow t = K^{-1} \begin{pmatrix} p_{14} \\ p_{24} \\ p_{34} \end{pmatrix}$

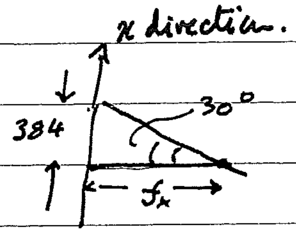
[3]

* There are certain sign ambiguities that require correction, but these do not need elucidating here.

(c) (ii) Square pixels suggest $K_{11} = K_{22}$

X-field of view is $60°$

$$f_x = \frac{384}{\tan 30°} = 665$$



Expect tiny skew, and u_0, v_0 in centre of image — although they won't be! Hence

$$K = \begin{bmatrix} 665 & \approx 0 & \approx 384 \\ 0 & 665 & \approx 288 \\ 0 & 0 & 1 \end{bmatrix}$$

[4]

(d) x_3 must lie on $\ell_3^{(1)} \sim F_{31} \underset{\sim}{x}_1$

and on $\ell_3^{(2)} \sim F_{32} \underset{\sim}{x}_2$

$$\Rightarrow \quad \underset{\sim}{x}_3 \sim \underbrace{\underset{\sim}{\ell}_3^{(1)} \times \underset{\sim}{\ell}_3^{(2)}}_{\text{Homogeneous coordinate property.}} = F_{31} \underset{\sim}{x}_1 \times F_{32} \underset{\sim}{x}_2$$

[5]

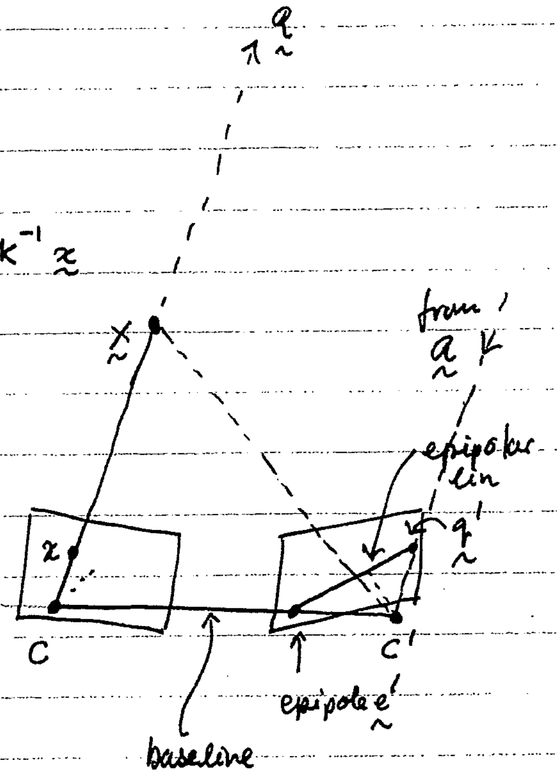
—#—

C4B Computer Vision (DWM) $\boxed{2011}$

[+1] (a) $\tilde{q} = \begin{bmatrix} K^{-1}\tilde{x} \\ 0 \end{bmatrix}$

[+1] $\tilde{q}' = K'[R|\tilde{t}] \begin{bmatrix} K^{-1}\tilde{x} \\ 0 \end{bmatrix} = K'RK^{-1}\tilde{x}$

[+1] Epipole $\tilde{e}' = K'[R|\tilde{t}]\begin{bmatrix} \tilde{0} \\ 1 \end{bmatrix} = K'\tilde{t}$



[+2]

(b) $\tilde{l}' = \tilde{e} \times \tilde{q}' = K'\tilde{t} \times K'RK^{-1}\tilde{x}$

$$= K'^{-T}(\tilde{t} \times RK^{-1}\tilde{x})$$

$$= K'^{-T}[t_\times]RK^{-1}\tilde{x}$$

$$t_\times = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

But $l' = F\hat{x}$

Hence $\underline{\underline{[F] = K'^{-T}[t_\times]RK^{-1}}}$ [+5]

(c) Convergent cameras have epipolar lines which radiate from the epipoles (cat's whiskers). By rectifying the images so that they appear to arise from parallel cameras with the same intrinsic calibrations, the epipoles become pts at ∞ (in 2D) and thus the epipolar lines are parallel and follow the image rows. Hence matching easier / quicker. [+2].

(d)

C4B/2011

(d) $$\underset{\sim}{x} = K[I|0]\underset{\sim}{X}_{4\times1} = K\underline{X}_{3\times1}$$

$$\underset{\sim}{x}_{rect} = K'[R|0]X_{4\times1} = K'R\underset{\sim}{X}_{3\times1} = K'R\,K^{-1}\underset{\sim}{x}$$

ie. $$\underline{\underline{H = K'RK^{-1}}}$$ [+4]

(e) We know $\underset{\sim}{x}' = K'[R|t]\underset{\sim}{X}_{4\times1}$

The sting here is to realize that the require projection matrix is $$\underset{\sim}{x}'_{rect} = \underbrace{K[I|R^{-1}\underset{\sim}{t}]}_{P_{rect}}X_{4\times1}$$

Then $x' = K'(RX+t)$ (a)

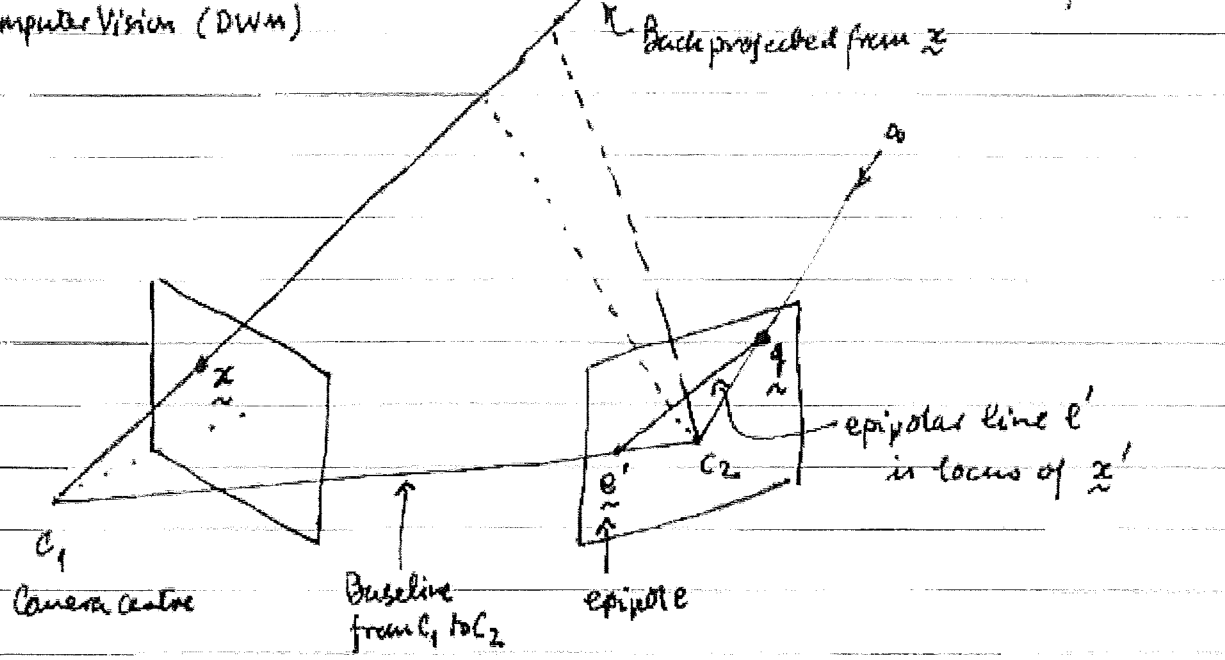
$$x'_{rect} = K(X+R^{-1}t)$$
$$= KR^{-1}(RX+t) = KR^{-1}K'^{-1}\underset{\sim}{x}' \quad \text{from (a)}$$

[+4]

C18 Solutions 2012

C18 Computer Vision (DWM)

(a)



+2

Point in first image is $\underset{\sim}{x}$, back-projected to ∞ at $\begin{pmatrix} K^{-1}\underset{\sim}{x} \\ 0 \end{pmatrix}$ +1

Optical centre C_1 projects to epipole $\underset{\sim}{e}' = K'[R|t]\begin{bmatrix} \underset{\sim}{0} \\ 1 \end{bmatrix} = K'\underset{\sim}{t}$ +1

Point at ∞ projects to $\underset{\sim}{q} = K'[R|t]\begin{bmatrix} K^{-1}x \\ 0 \end{bmatrix} = K'RK^{-1}x$ +1

$\Rightarrow$ Epipolar line $\underset{\sim}{\ell}' = K'\underset{\sim}{t} \times K'RK^{-1}\underset{\sim}{x}$ +1

$= K'^{-T}(t \times RK^{-1}\underset{\sim}{x}) = K'^{-T}[t_\times]RK^{-1}\underset{\sim}{x}$

But by definition $\underset{\sim}{\ell}' = F\underset{\sim}{x}$ +1 $\Rightarrow$ $\underline{\underline{F = K'^{-T}[t_\times]RK^{-1}}}$

(As homogeneous, all equalities above are merely up to scale.)

(b) $\underset{\sim}{x}'$ must lie on epipolar line $\Rightarrow$ $\underset{\sim}{x}'^T\underset{\sim}{\ell}' = 0$ $\Rightarrow$ $\underset{\sim}{x}'^T F \underset{\sim}{x} = 0$

+2

(c) $R = I$ and $K' = K = I$ $\Rightarrow$ $F = [t_\times]$ +1 $\Rightarrow$ $\underset{\sim}{x}'^T[t_\times]\underset{\sim}{x} = 0$

(i) So $[x' \; y' \; 1]\begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = [x'y'1]\begin{bmatrix} -t_z y + t_y \\ t_z x - t_x \\ -t_y x + t_x y \end{bmatrix}$

$= -t_z y x' + t_y x' + t_z x y' - t_x y' - t_y x + t_x y$

$= \begin{bmatrix} (y-y') & (x'-x) & (xy'-yx') \end{bmatrix}\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$

+2

C18 2012 [illegible].

d(ii) For three correspondences:

$$A\underset{\sim}{t} = \begin{bmatrix} (y_1 - y_1') & (x_1' - x_1) & (x_1 y_1' - y_1 x_1') \\ (y_2 - y_2') & (x_2' - x_2) & (x_2 y_2' - y_2 x_2') \\ (y_3 - y_3') & (x_3' - x_3) & (x_3 y_3' - y_3 x_3') \end{bmatrix} \underset{\sim}{t} = 0$$

$\underset{\sim}{t}$ is in the nullspace of $A_{3\times3}$.

(d) (i)
$$A = \begin{bmatrix} (2 - 3/2) & (1 - 1) & (1 \cdot 3/2 - 2 \cdot 1) \\ (-1 + 1/3) & (2/3 - 1/2) & (1/2 \cdot 1/3 + 1 \cdot 2/3) \\ (1/2 - 2/3) & (1/3 - 0) & (0 \cdot 2/3 - 1/2 \cdot 1/3) \end{bmatrix}$$

a) $$At = \begin{bmatrix} 1/2 & 0 & -1/2 \\ -2/3 & 1/6 & 1/2 \\ -1/6 & 1/3 & -1/6 \end{bmatrix} \underset{\sim}{t}$$

Use $t = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A\underset{\sim}{t} = \begin{bmatrix} 1/2 + 0 - 1/2 \\ -2/3 + 1/6 + 1/2 \\ -1/6 + 1/3 - 1/6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ as required

DWM $\boxed{C18/2013}$

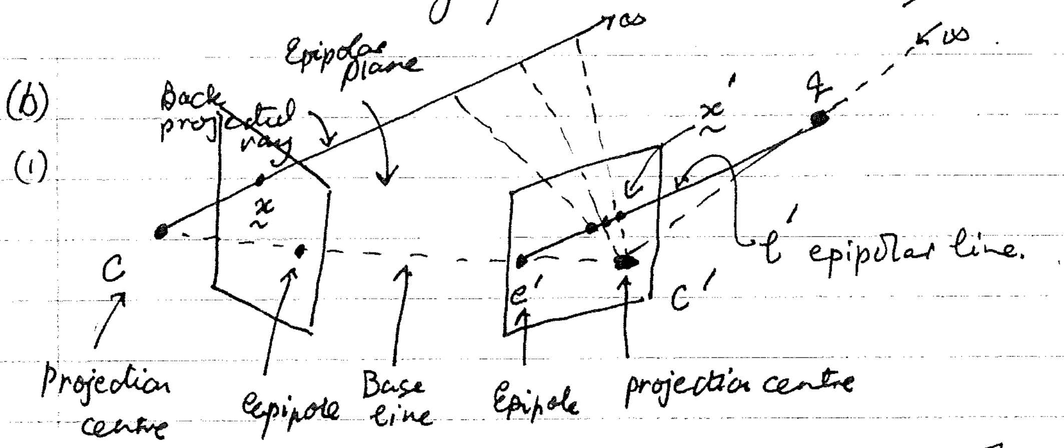
(a) When the world coordinates and camera coordinates coincide, $[I|\underset{\sim}{0}]$ projects a homogeneous scene point $\underset{\sim}{X} = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$ into the "ideal" image. The intrinsic matrix K maps positions in the ideal image into those in the actual image

$$K = \begin{pmatrix} f_x & s & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

f_x, f_y are the focal lengths in the x and y directions, $s \simeq 0$ is the skew and u_0, v_0 the principal point. If world and camera coordinates do not coincide a rotation and translation is required – the extrinsic parameters. $\underset{\sim}{X}^c = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \underset{\sim}{X}$

So the projection is $[I/0]\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}\underset{\sim}{X} = [\tilde{R}|t]\underset{\sim}{X}$. (Then K' transforms into the actual image of this second camera.) [3]

(b)
(i)



Given a point $\underset{\sim}{x}$, the scene point lies somewhere on the back-projected ray. The correspondence lies somewhere on the intersection of the epipolar plane with the second image plane. This is the epipolar line $\underline{\ell'}$. The limits are given as $\underset{\sim}{e}'$ and $\underset{\sim}{q}$. $\underset{\sim}{e}'$ is the epipole: the intersection of the base line with the 2nd image plane. $\underset{\sim}{q}$ is point where a line parallel to the back projected ray intersects the image. [3]

(b)
(ii) $e' = K'[R|\underset{\sim}{t}]\begin{bmatrix} \underset{\sim}{0} \\ 1 \end{bmatrix} = K'\underset{\sim}{t}$ and $\underset{\sim}{q} = K'[R|t]\begin{bmatrix} K^{-1}x \\ 0 \end{bmatrix} = K'RK^{-1}\underset{\sim}{x}$

$\underset{\sim}{\ell}' = \underset{\sim}{e}' \times \underset{\sim}{q} = K'\underset{\sim}{t} \times K'RK^{-1}x = K'^{-T}(\underset{\sim}{t} \times RK^{-1})\underset{\sim}{x}$

using relationship given.

C18/2013

(b)(ii) /ctd. We can write $\underset{\sim}{t}\times$ as a matrix $[t_\times] = \begin{pmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{pmatrix}$

So $\underline{F = K'^{-T} [t_\times] R K^{-1}}$ [+3]

(c) The only requirement for a homogeneous $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ is that $x = \frac{x_1}{x_3}$ $y = \frac{x_2}{x_3}$

Obviously $\frac{xZ}{Z} = x$, $\frac{yZ}{Z} = y$ — so good rep.

$$\underset{\sim}{x} = \begin{pmatrix} xZ \\ yZ \\ Z \end{pmatrix} = K\left[I \mid 0\right]\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = K \underset{\sim}{X_3}$$

$$\Rightarrow \underset{\sim}{X_3} = K^{-1}\underset{\sim}{x}$$

[+3]

(d) $$\underset{\sim}{x}' = K'[R \mid \underset{\sim}{t}]\begin{pmatrix} \underset{\sim}{X_3} \\ 1 \end{pmatrix} = K' R K^{-1} \underset{\sim}{x} + K' \underset{\sim}{t}$$
$$= A\underset{\sim}{x} + \underset{\sim}{b}$$

$$\underset{\sim}{x}^T F^T \underset{\sim}{x}' = \underset{\sim}{x}^T \left[K^{-T} R^T [t_\times]^T K'^{-1}\right]\left[K' R K^{-1} x + K' t\right]$$

$$= \underset{\sim}{x}^T \left[K^{-T} R^T [-t_\times] R K^{-1} \underset{\sim}{x}\right]$$
$$+ x^T K^{-T} R^T [-t_\times] K'^{-1} K' t$$

$$= \underbrace{(RK^{-1}x)^T [-t_\times] (RK^{-1}x)} + 0 \quad \text{as } [t_\times]\underset{\sim}{t} \equiv 0.$$

This is a scalar α $\Rightarrow \alpha = \alpha^T$

$$\Rightarrow \alpha = (RK^{-1}x)^T [-t_\times]^T (RK^{-1}x)$$

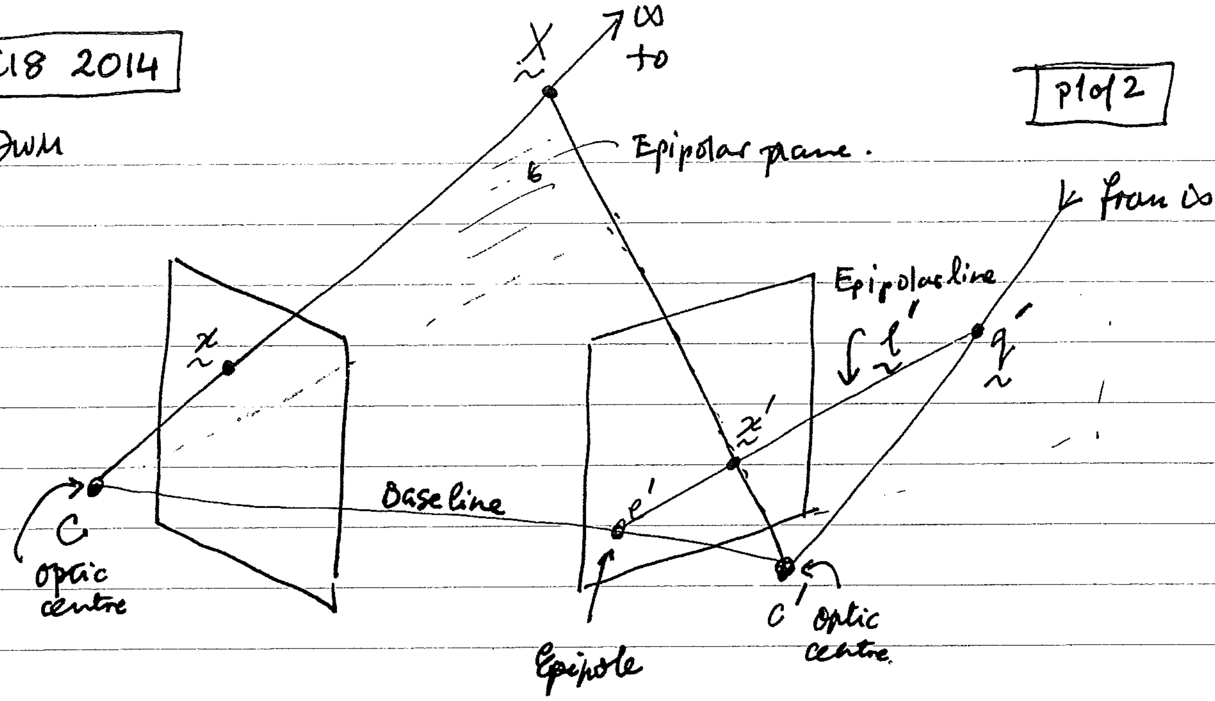
But $t_\times^T = -t_\times$ $\Rightarrow$ $\alpha = -\alpha$ $\Rightarrow$ $\underline{\alpha = 0}$

Hence $\underset{\sim}{x}^T F^T \underset{\sim}{x}' = 0$ is satisfied. [+4]

C18 2014

Q1 DWM

(a)



- Back projection of $\tilde{x}$ is $\tilde{X} = \begin{pmatrix} \alpha K^{-1} x \\ 1 \end{pmatrix}$
- When $\alpha = 0$: projection of optic centre in C' is $\tilde{e}' = K'[R|t]\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1\end{pmatrix} = K'\tilde{t}$.
- When $\alpha = \infty$: projection of pt at ∞ into C' is $\tilde{q}' = K'[R|t]\begin{bmatrix} K^{-1}x \\ 0 \end{bmatrix}$
$$= K'RK^{-1}\tilde{x}.$$
- Epipolar line is $\tilde{l}' = \tilde{e}' \times \tilde{q}' = K't \times K'RK^{-1}x$
$$= K'^{-T}\, t \times RK^{-1}\tilde{x}$$
$$= K'^{-T}[t_\times]RK^{-1}\tilde{x}$$

But $l' = F\tilde{x}$ defines F $\Rightarrow$ $\underline{\underline{F = K'^{-T}[t_\times]RK^{-1}}}$. [6]

(b) • Given $\tilde{x}$, the search for correspondence is constrained to a (subsection of a) line $\tilde{l}'$, rather than to the whole image.
• When searching along l', could use similarity of features, feature ordering, similarity of disparity with neighbouring regions. [3]

(c) (i) The vanishing point is the projection of a point at ∞
⇒ the depth is known, not uncertain ⇒ no need to consider projection of the entire back-projected ray from C.

$\tilde{v}_1$: The point at ∞ is $\begin{bmatrix} K^{-1}v_1 \\ 0 \end{bmatrix} \Rightarrow v_1' = K'[R|t]\begin{bmatrix} K^{-1}v_1 \\ 0 \end{bmatrix}$
$$= K'RK^{-1}\tilde{v}_1.$$

Obviously, $v_2' = K'RK^{-1}\tilde{v}_2$. [3].

C18 2015 DWM.

Q1(a) • The projection equation is $\underset{\sim}{x} \stackrel{P}{=} K[R|\underset{\sim}{t}]\underset{\sim}{X} = K[I|0]\begin{bmatrix} R & \underset{\sim}{t} \\ 0^T & 1 \end{bmatrix}\underset{\sim}{X}$.

The 3D homogeneous point $\underset{\sim}{X}$ is given in the world frame, and rotation R and translation $\underset{\sim}{t}$ define its Euclidean transformation in to camera frame.

$[I|0]$ is the vanilla projection matrix, projecting a point in the camera frame into an idealized image. K, the camera intrinsic matrix, is an affine transformation moving the ideal projection into the actual pixelated image.

$K = \begin{pmatrix} f & sf & u_0 \\ 0 & \gamma f & v_0 \\ 0 & 0 & 1 \end{pmatrix}$, where f is the focal length, γ the aspect ratio, s the skew, and u_0, v_0 the position of the principal point in the actual image.

• Once $P = K[R|t]$ is recovered up to scale:- As $P_{left} = KR$, performing QR decomposition on $P_{left}^{-1} = R^{-1}K^{-1}$ yields $R^{-1} = "Q"$ and $K^{-1} = "R"$.

Hence $R = "Q"^{-1}$ and $K = "R"^{-1}$, upto scale in the case of K.

Adjust scale $K \leftarrow K/K_{33}$, and similarly for $P \leftarrow P/K_{33}$.

Then as $P_4 = Kt$, $t = K^{-1}P_4$. [5]

(b) Points on the back-projected ray are $\begin{bmatrix} \alpha K^{-1}\underset{\sim}{x} \\ 1 \end{bmatrix}$. Choose optic centre ($\alpha = 0$) and point at infinity ($\alpha = \infty$), $\begin{pmatrix} \underset{\sim}{0} \\ 1 \end{pmatrix}$ and $\begin{pmatrix} K^{-1}\underset{\sim}{x} \\ 0 \end{pmatrix}$ respectively.

Project both into C' as $e' = K'[R|t]\begin{bmatrix} \underset{\sim}{0} \\ 1 \end{bmatrix} = K'\underset{\sim}{t}$ and $q' = K'[R|t]\begin{bmatrix} K^{-1}\underset{\sim}{x} \\ 0 \end{bmatrix}$
$= K'RK^{-1}x$

Epipolar line $\ell' = e' \times q' = K'\underset{\sim}{t} \times K'[R]K^{-1}x = K'^{-T}[t_\times][R]K^{-1}\underset{\sim}{x} = F\underset{\sim}{x}$

So $\underline{\underline{F = [K'^{-T}][\underset{\sim}{t}_\times]RK^{-1}}}$ where $[t_\times]$ is the skew symmetric "vector product" matrix. [4]

(c) We are told $\underset{\sim}{n}^T X_3 = 1$. We know the exact point on the back projected ray, which in turn projects to a known point in C'.

Point is $\begin{bmatrix} \alpha K^{-1}x \\ 1 \end{bmatrix} \Rightarrow \underset{\sim}{n}^T\alpha K^{-1}\underset{\sim}{x} = 1 \Rightarrow \frac{1}{\alpha} = \underset{\sim}{n}^T K^{-1}\underset{\sim}{x}$. Point is $\stackrel{P}{=} \begin{bmatrix} K^{-1}x \\ 1/\alpha \end{bmatrix} = \begin{bmatrix} K^{-1}x \\ n^T K^{-1}\underset{\sim}{x} \end{bmatrix}$ [3]

(d) Obvious that $\underset{\sim}{X} = \begin{bmatrix} K^{-1}x \\ n^T K^{-1}x \end{bmatrix} = \begin{bmatrix} I \\ n^T \end{bmatrix} K^{-1}\underset{\sim}{x} \Rightarrow x' = K'[R|t]\begin{bmatrix} I \\ n^T \end{bmatrix} K^{-1}\underset{\sim}{x}$

But, as $\underset{\sim}{x}' = H\underset{\sim}{x}$, $\underline{\underline{H = K'[R|t]\begin{bmatrix} I \\ \underset{\sim}{n}^T \end{bmatrix} K^{-1}}}$

The fundamental matrix requires $\underset{\sim}{x}'^T F\underset{\sim}{x} = 0 \Rightarrow \underset{\sim}{x}^T[H^T F]\underset{\sim}{x} = 0$ for all $\underset{\sim}{x}$

Now $[\underset{\sim}{x}^T A\underset{\sim}{x}]^T = -[x^T A x]$. But this requires $\underset{\sim}{x}^T A\underset{\sim}{x} = 0 \Rightarrow$ $\underline{\underline{H^T F \text{ is antisymmetric}}}$ for antisymmetric A