One way to understand why Projection results in Homogenous Eqns.

Suppose we have set of eqns in non homogenous equation:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \longrightarrow (1).$$

$\mathbb{R}^2$ 2×1 $\quad$ 2×3 $\quad$ $\mathbb{R}^3$ 3×1.

Rewrite it as

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}_{2\times1} = \underbrace{\begin{bmatrix} X_1 & Y_1 & Z_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_1 & Y_1 & Z_1 \end{bmatrix}_{2\times6}}_{M} \underbrace{\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix}_{6\times1}}_{A.} \longrightarrow (2).$$

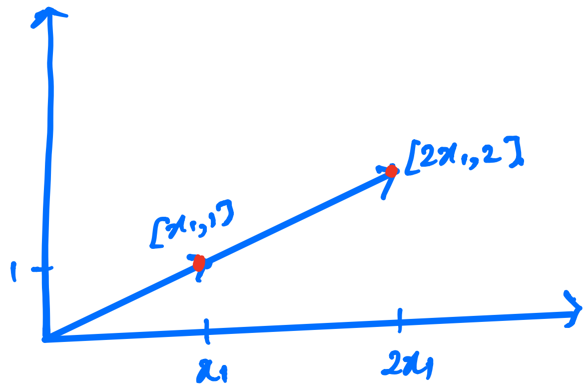
$$b_{2\times1} = M_{2\times6} A_{6\times1} \longrightarrow (3).$$

If we have more observations, say n observation

$$b_{2n\times1} = M_{2n\times6} A_{6\times1} \longrightarrow (4).$$

$A = [M^T M]^{-1} M^T b$ is the pseudoinverse least squares solution.

However if $x \in \mathbb{P}^1$, $x = \begin{bmatrix} x \\ 1 \end{bmatrix}$ is a projective line or a ray in 2D



$\lambda \vec{x} = x$. $\in \mathbb{R}^1$

A point x_1 in 1D becomes a ray passind through origin in $\mathbb{P}^1$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \approx \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \longrightarrow (5)$$

$$y_1 \begin{bmatrix} \frac{x_1}{y_1} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{a_{11}X_1 + a_{12}X_1 + a_{13}Z_1}{a_{21}X_1 + a_{22}Y_1 + a_{23}Z_1} \\ 1 \end{bmatrix} \longrightarrow (6)$$

$$\text{or } y_1 \begin{bmatrix} x_1' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{a_{11}X_1 + a_{12}Y_1 + a_{13}Z_1}{a_{21}X_1 + a_{22}Y_1 + a_{23}Z_1} \\ 1 \end{bmatrix} \longrightarrow (7).$$

write x_1' as x_1 for simplicity

$$a_{21}X_1x_1 + a_{22}Y_1x_1 + a_{23}Z_1x_1 - a_{11}X_1 - a_{12}Y_1 - a_{13}Z_1 = 0 \longrightarrow (8)$$

or $\begin{bmatrix} -X_1 & -Y_1 & -Z_1 & X_1 x_1 & Y_1 x_1 & Z_1 x_1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} \longrightarrow (9).$

or $M_{n\times 6} A_{6\times 1} = 0 \longrightarrow (10)$ for n observations or a overdetermined system

For non-trivial null space there should be at-least one free variable, which is say $a_{23} = 1$ and represent all others in terms of a_{23}

For example if $2x+3y=0$, then $x = \frac{-3y}{2}$

then $\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$ is a solution to the above.

If we have one more eqn it necessarily ought to be $\lambda(2x+3y)=0$ to maintain non-triviality of the Null Space.

i.e $2x+3y=0$

$2\lambda x + 3\lambda y = 0$