

Since D is diagonal with descending singular values, then $\|Dy\|$ is minimized for $y = [0, \dots, 1]^T$

Then $x = Vy$ is the last column of matrix V .

Alternate proof based on eigenvalue

The solution to $\|Ax\|^2 = (Ax)^T(Ax) = x^T A^T A x$ s.t. $x^T x = 1$ is the eigenvector corresponding to the least eigenvalue of $A^T A$.

From Lagrange Multiplier

$$L(x) = x^T A^T A x - \lambda(x^T x - 1)$$

Take der w.r.t x and set it to zero give

$$A^T A x - \lambda x = 0.$$

or λ is an eigenvalue of $A^T A$ and $x = e_i$ is the eigenvector corresponding to $A^T A$.

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correspond
of $A^T A$.

- 1) Why is
- 2) What are
- 3) Given
- 4) Problem
- 4) Assl. t
- 5) What
- 6) What
- the so
- R?
- 7) Show