Consider again:

$$P = RQ + t$$

It can be rewriten in the form

$$A_{3\times 12}\begin{bmatrix} R_{9\times 1} \\ t_{3\times 1} \end{bmatrix} = b_{12\times 1}. \longrightarrow (1)$$

Why cannot we solve it as a least squares solution in terms of $[r_{11}, \dots, r_{33}]$ and $[t_x \; t_y \; t_z]^T$.

The above formulation is non-convex because it is subject to the non-convex constraint:

$$RR^T = R^T R = I_{3\times 3} \longrightarrow (2).$$

$$\|R_1\| = \|R_2\| = \|R_3\| = 1 \longrightarrow (3)$$

$$R_i . R_j = 0 \longrightarrow (4).$$

$$P_i = RQ_i + t$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} Q_{1x} \\ Q_{1y} \\ Q_{1z} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{1z} \end{bmatrix} \longrightarrow (5).$$

$$r_{11} Q_{1x} + r_{12} Q_{1y} + r_{13} Q_{1z} + t_x = P_{1x}.$$

$$r_{21} Q_{1x} + r_{22} Q_{1y} + r_{23} Q_{1z} + t_y = P_{1y}$$

$$r_{31} Q_{1x} + r_{32} Q_{1y} + r_{33} Q_{1z} + t_z = P_{1z}.$$

$$\begin{bmatrix} Q_{1x} & Q_{1y} & Q_{1z} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & Q_{1x} & Q_{1y} & Q_{1z} & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q_{1x} & Q_{1y} & Q_{1z} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} \\ r_{12} \\ \vdots \\ r_{33} \\ t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{1z} \end{bmatrix}$$

Posed as least squares. but this is really NOT convex due to constraint on R.

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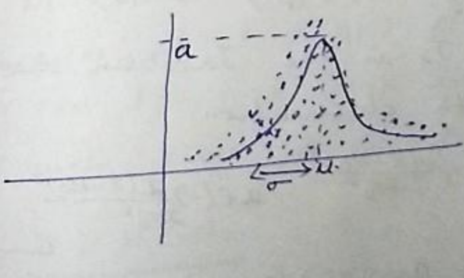
Let $y_i = a \exp\left( \frac{x_i - u}{2\sigma^2} \right)$. $\longrightarrow (1)$.

given a set of $m = 50$ observations evaluate or find the best estimate for $a$, $u$ and $\sigma$.



$$~~y = a e^{-(\frac{x-u}{2\sigma^2})^2}~~$$

$$y = a e^{-\frac{(x-u)^2}{2\sigma^2}} \longrightarrow (1)$$

Linearize $y$ about $[a_0, u_0, \sigma_0]^T = \beta_0$

then ~~$y = y_0 + f'(x)|$~~

$$~~y = y_0 + f'(\beta)\big|_{\beta=\beta_0} (\delta\beta) \longrightarrow (2)~~$$

Let $y_0 = f(\beta)|_{\beta=\beta_0} = f(a_0, u_0, \sigma_0) \longrightarrow (2)$.

Then first order approximation of Taylor's series gives

$$y = y_0 + f'(\beta)_{\beta=\beta_0} \, \delta(\beta). \longrightarrow (3)$$

where $\delta(\beta) = [\delta a_0 \; \delta u_0 \; \delta\sigma_0]^T \longrightarrow (4)$.

or for the $i$th observation

$$y_i = y_0 + f'(\beta i)_{\beta i=\beta_0} \cdot \delta(\beta i) \longrightarrow (5).$$

$$= y_0 + J_i \, \delta(\beta i) \longrightarrow (6)$$

where $J_i$ is the Jacobian due to the $i$th observation.

$$J_i = \left[ e^{-\frac{(x-u_0)^2}{2\sigma_0^2}} \quad a_0 e(\cdot) \frac{2(x-u_0)}{2\sigma_0^2} \quad J_{i3} \right] \longrightarrow (7).$$

where $J_{i3} = \dfrac{a_0 e(\cdot)(x-u_0)}{\sigma_0^3}$

or $J_i = \begin{bmatrix} J_{i1} & J_{i2} & J_{i3} \end{bmatrix} \longrightarrow (8).$

$= \left[\frac{\partial y_i}{\partial a} \quad \frac{\partial y_i}{\partial \mu} \quad \frac{\partial y_i}{\partial \sigma}\right]$

$J_i \delta\beta_i = y_i - y_0 \longrightarrow (9).$

since $\delta\beta_1 = \delta\beta_2 = \cdots = \delta\beta_m = \delta\beta$

$J_i \delta\beta = y_i - f(x_i, \beta_0). \longrightarrow (10).$

where $y_0 = f(x_i, \beta_0)$. or simply $f(\beta_0)$

Then
$$\begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_m \end{bmatrix}_{m\times 3} \delta\beta_{3\times 1} = \begin{bmatrix} y_1 - f(x_1, \beta_0) \\ y_2 - f(x_2, \beta_0) \\ \vdots \\ y_m - f(x_m, \beta_0) \end{bmatrix}_{m\times 1} \longrightarrow (11).$$

or $A_{m\times 3}\, \delta\beta_{3\times 1} = Y_{m\times 1}.$

or $J_{m\times 3}\, \delta\beta_{3\times 1} = Y_{m\times 1} \longrightarrow (12)$

$$\boxed{\delta\beta = [J^T J]^{-1} J^T Y \longrightarrow (13).}$$

Now $\beta_{t+1} = \beta_t \mp \delta\beta \longrightarrow (14).$

or $\beta(n+1) = \beta(n) \mp \delta\beta$

Linearize about $\beta(n+1)$ to solve for new $\delta\beta$ and keep doing this till such time $\underline{\beta(n+1) - \beta(n) < \epsilon}$.

or $\underline{n > maxitr}$.

The LM algorithm:

$$\left[J^T J + \lambda I\right] \delta\beta = J^T [Y].$$

$$\text{or} \quad \boxed{\delta\beta = \left[J^T J + \lambda I\right]^{-1} J^T Y.}$$