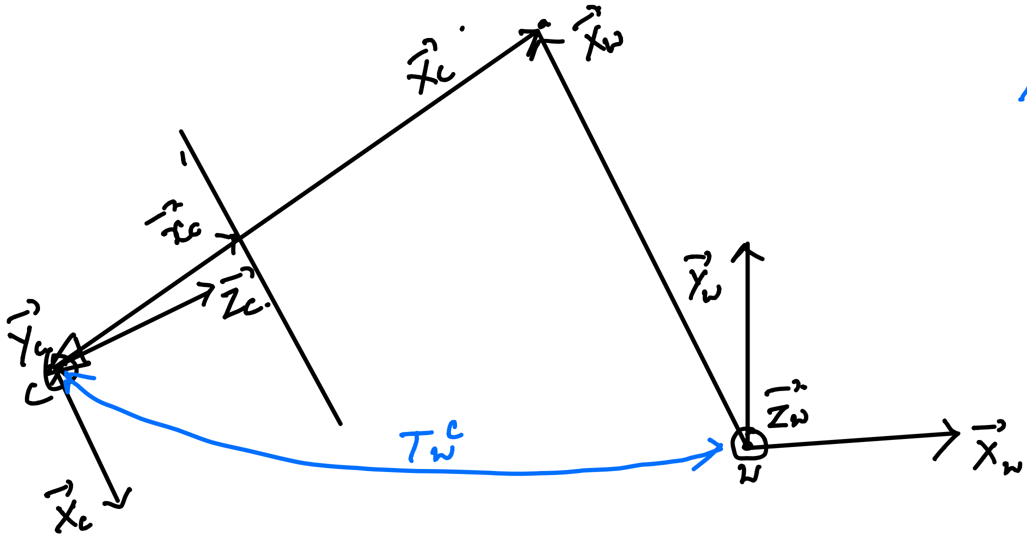

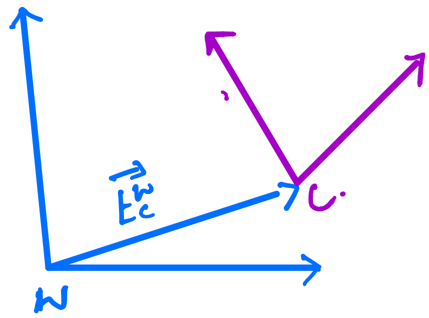

t_c^w in $\{C\}$ is $R_w^c \vec{t}_c^w$ and t_w^c is simply $-R_w^c t_c^w$

$$\underset{(3\times1)}{\vec{X}_c} = \underset{(3\times4)}{\begin{bmatrix} R_w^c & t_w^c \end{bmatrix}} \vec{X}_{w\,(4\times1)} \longrightarrow (1)$$

$$\vec{x}_{c\,(3\times1)} = K_{3\times3} \vec{X}_{c\,(3\times1)} \longrightarrow (2).$$

$$= K \begin{bmatrix} R_w^c & t_w^c \end{bmatrix} \vec{X}_w \longrightarrow (3) = P_{3\times4} \vec{X}_w \longrightarrow (4).$$

$P_{3\times4} \longrightarrow$ Camera Projection Matrix.

Camera Calibration: Process of estimating P.

Often (1) is rewritten as:

$$\vec{X}_c = R_w^c \vec{X}_w + t_w^c \longrightarrow (5) = R_w^c \vec{X}_w - R_w^c t_c^w \longrightarrow (6)$$

$$= R_w^c \begin{bmatrix} I & -t_c^w \end{bmatrix} \vec{X}_{w\,(4\times1)} \longrightarrow (7)$$

$$\text{or} \quad \vec{x}_c = K R_w^c \begin{bmatrix} I & -t_c^w \end{bmatrix} \vec{X}_{w\,(4\times1)} \longrightarrow (8)$$

$$\text{or} \quad \boxed{\vec{x}_c = P_{3\times4} \vec{X}_w \longrightarrow 9}$$

or rather $\lambda \vec{x}_c = P \vec{X}_w$ or $\vec{x}_c \approx P \vec{X}_w$

or simply $\boxed{\vec{x} \simeq P\vec{X}}$

$\vec{x}_c \simeq P\vec{X}_w$ or
$\vec{x}_c = PX_w$
$\vec{x}_c \not\simeq K\vec{x}_c$
$\vec{x}_c = K\vec{x}_c$

To estimate P:

↳ as a first step towards estimating K.

$P_{3\times4}$ = 12 parameters.

Every pair of correspondence $X_i \leftrightarrow x_i$ gives two equations (Why?).

Hence 6 pairs of correspondences are needed to solve for P.

As a matter of fact one needs to solve for only 11 parameters of P as $\vec{x} \simeq P\vec{X}$ is a homogeneous equation, which means

any $\underline{\gamma P\vec{X}}$ also projects to same $\vec{x}$.

Hence every P_{ij} can be divided by P_{34} for example and P_{34} made 1.

Hence 6 pairs of correspondences $X_i \leftrightarrow x_i$ are still required to solve for the 11 parameters of P.

How does one solve?

$$x_i = P_{11}X_i + P_{12}Y_i + P_{13}Z_i + P_{14} \longrightarrow (9)$$

$$y_i = P_{21}X_i + P_{22}Y_i + P_{23}Z_i + P_{24} \longrightarrow (10)$$

$$Z_i = P_{31} X_i + P_{32} Y_i + P_{33} Z_i + P_{34} \longrightarrow (11).$$

$$x_i = \frac{x_i}{z_i}, \quad y_i = \frac{y_i}{z_i} \longrightarrow (12).$$

as $\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = Z_i \begin{bmatrix} x_i/z_i \\ y_i/z_i \\ 1 \end{bmatrix}$

Then $$x_i = \frac{P_{11} X_i + P_{12} Y_i + P_{13} Z_i + P_{14}}{P_{31} X_i + P_{32} Y_i + P_{33} Z_i + P_{34}}.$$

or. $$X_i P_{11} + Y_i P_{12} + Z_i P_{13} + P_{14} - P_{31} X_i x_i - P_{32} Y_i x_i - P_{33} Z_i x_i - P_{34} x_i = 0 \longrightarrow (13)$$

Illy $$X_i P_{21} + Y_i P_{22} + Z_i P_{23} + P_{24} - P_{31} X_i y_i - P_{32} Y_i y_i - P_{33} Z_i y_i - P_{34} y_i = 0 \longrightarrow (14)$$

or $$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -X_i x_i & -Y_i x_i & -Z_i x_i & -x_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -X_i y_i & -Y_i y_i & -Z_i y_i & -y_i \end{bmatrix}_{2\times 12} \begin{bmatrix} P_{11} \\ P_{12} \\ \vdots \\ \vdots \\ P_{34} \end{bmatrix} = 0_{12\times 1} \longrightarrow (15)$$

For every $\vec{X_i} \leftrightarrow \vec{x_i}$ we get 2 eqns of the form (15)

If there are $M >> 6$ correspondences

we have $A_{2M \times 12} P = 0 \longrightarrow (16)$

How to solve for P:?

Overdetermined set of equations

Avoid the trivial solution $P_{3\times4} = 0_{3\times4}$.

$SVD(A) = U D V^T$.

Last column of $V_{12\times12}$, a 12×1 column vector is the solution for P.

Divide each v_i by v_{12}. or $P_{34} = 1$ and $P_{ij} = \frac{P_{ij}}{P_{34}}$ is the final solution for P.

How to get K from P?

Before that let us look at $AP = 0_{12\times1}$.

Since it is overdetermined and the observations are noisy $\nexists$ NO P for which $AP = 0$.

Hence find the best P for which $\|AP\|_2^2$ is minimized or $P^T A^T A P$ is minimized such that $\|P\| = 1$.

The solution is by SVD.

Consider $h_1^T B h_2 = 0 \longrightarrow (1)$

$h_1^T = [h_{11} \; h_{21} \; h_{31}]$; $h_2 = [h_{12} \; h_{22} \; h_{32}]^T$

$$\begin{bmatrix} h_{11} & h_{21} & h_{31} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0 \longrightarrow (2)$$

$$\begin{bmatrix} h_{11} & h_{21} & h_{31} \end{bmatrix} \begin{bmatrix} b_{11} h_{12} + b_{12} h_{22} + b_{13} h_{32} \\ b_{12} h_{12} + b_{22} h_{22} + b_{23} h_{32} \\ b_{13} h_{12} + b_{23} h_{22} + b_{33} h_{32} \end{bmatrix} = 0 \longrightarrow (3)$$

$$= h_{11}(b_{11} h_{12} + b_{12} h_{22} + b_{13} h_{32}) + h_{21}(b_{12} h_{12} + b_{22} h_{22} + b_{23} h_{32})$$
$$+ h_{31}(b_{13} h_{12} + b_{23} h_{22} + b_{33} h_{32}) = 0 \longrightarrow (4)$$

$$= [h_{11} h_{12} \quad h_{11} h_{22} + h_{21} h_{12}. \quad h_{11} h_{32} + h_{31} h_{12} \quad h_{21} h_{22}. \quad h_{21} h_{32} + h_{31} h_{22} \quad h_{31} h_{32}] b = 0. \longrightarrow (5).$$

$$b = [b_{11} \; b_{12} \; b_{13} \; b_{22} \; b_{23} \; b_{33}]^T.$$

Rewrite (5) as $v_{12}^T b = 0$

where
$$v_{12} = \begin{bmatrix} h_{11} h_{12}. \\ h_{11} h_{22} + h_{21} h_{12} \\ h_{11} h_{32} + h_{31} h_{12} \\ h_{21} h_{22} \\ h_{21} h_{32} + h_{31} h_{22} \\ h_{31} h_{32} \end{bmatrix} \; \& \; v_{ij} = \begin{bmatrix} h_{1i} h_{1j} \\ h_{1i} h_{2j} + h_{2i} h_{1j} \\ h_{1i} h_{3j} + h_{3i} h_{1j} \\ h_{2i} h_{2j} \\ h_{2i} h_{3j} + h_{3i} h_{2j} \\ h_{3i} h_{3j} \end{bmatrix}$$

Then for each image we obtain

$$\begin{bmatrix} v_{12}^T \\ v_{11}^T - v_{22}^T \end{bmatrix} b = 0.$$