LAB-8

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8.1a.

Given, $H_{LPF}(e^{jw})$ =1 for $|w| <= \frac{\pi}{6}$

= 0 for $\frac{\pi}{6} <= |w| <= \pi$

⇨ $h_{LPF}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{HLPF}(e^{jw})e^{jwn}\,dw$

⇨ $= \frac{1}{2\pi}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} e^{jwn}\,dw$

⇨ $h_{LPF}[n] = \frac{\sin(\frac{\pi}{6}n)}{\pi n}$ for n!=0

⇨ and $= \frac{1}{6}$ for n=0

Given, $H_D(e^{jw})$ $=e^{-jwnc}$ for $|w| <= \frac{\pi}{6}$

= 0 for $\frac{\pi}{6} <= |w| <= \pi$

⇨ $h_D[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{HD}(e^{jw})e^{jwn}\,dw$

⇨ $=\frac{1}{2\pi}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} e^{jw(n-nc)}\, dw$

⇨ $h_{LPF}[n] = \frac{\sin(\frac{\pi}{6}(n-nc))}{\pi(n-nc)}$ for $n!=n_c$

⇨ and $= \frac{1}{6}$ for $n=n_c$

w[n]=1 for 0<=n<=50
=0 Otherwise

⇨ $h[n]=h_D[n]$ for 0<=n<=50
=0 Otherwise

8.1b.

From the plot, we see that the phase is linear.

8.1c.

Given, $H_{LPF}(e^{jw})$ =1 for $|w|<=\frac{\pi}{6}$
= 0 for $\frac{\pi}{6}<=|w|<=\pi$

⇨ $h_{LPF}[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{HLPF}\left(e^{jw}\right)e^{jwn}\, dw$

⇨ $=\frac{1}{2\pi}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} e^{jwn}\, dw$

⇨ $h_{LPF}[n] = \frac{\sin(\frac{\pi}{6}n)}{\pi n}$ for n!=0

⇨ and $= \frac{1}{6}$ for n=0

Given, $H_D(e^{jw}) = e^{-jwnc}$ for $|w| <= \frac{\pi}{6}$

$= 0$ for $\frac{\pi}{6} <= |w| <= \pi$

⇨ $h_D[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{HD}\left(e^{jw}\right)e^{jwn}\, dw$

⇨ $= \frac{1}{2\pi}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} e^{jw(n-nc)}\, dw$

⇨ $h_{LPF}[n] = \frac{\sin(\frac{\pi}{6}(\mathrm{n}-\mathrm{nc}))}{\pi(\mathrm{n}-\mathrm{nc})}$ for $n != n_c$

⇨ and $= \frac{1}{6}$ for $n = n_c$

$w[n] = 0.42 - 0.5\cos(\frac{\pi(n+50)}{50}) + 0.08\cos(\frac{2\pi(n+50)}{50}$

for $-50 <= n <= 50$

$= 0$ Otherwise

⇨ $h[n] = h_D[n]w[n]$ for $-50 <= n <= 50$

$= 0$ Otherwise

8.1d.

As N increases for each fixed window, the width of the main lobe decreases, which results in a decrease in the transition width between passbands and stopbands. But the side bands area remains roughly with the change in N.

For all the window types and N values, we get Linear-phase systems as seen in the plots. The also change more rapidly in the main lobe as N increases. Also, the side lobe phases change more frequently compared to the main lobe.

	Rectangular	Bartlett	Hamming	Hanning	Blackman
Main Lobe Width in terms of samples	16	38	37	34	58
Main Lobe Width in terms of frequency	0.064*PI	0.152*PI	0.148*PI	0.136*PI	0.232*PI

As N Increases, width of main lobe decreases and amplitude of the side lobes remain roughly constant for a fixed window.

8.1f.

- From the plot, we see that the phase is linear.
- Also, from magnitude response, we see that it is **high pass filter.**

8.2a.

$$H(z) = b_0\left(1 - e^{j\omega_0}z^{-1}\right)\left(1 - e^{-j\omega_0}z^{-1}\right)$$

$$since, H(1) = 1$$

$$Hence, b_0 = \frac{1}{(1 - e^{j\omega_0})(1 - e^{-j\omega_0})}$$

$$On\ further\ solving$$

$$b_0 = \frac{1}{(2(1 - cos\omega_0)}$$

8.2b.

$$H(z) = \frac{b_0\left(1 - e^{j\omega_0}z^{-1}\right)\left(1 - e^{-j\omega_0}z^{-1}\right)}{(1 - r_0e^{j\omega_0}z^{-1})(1 - r_0e^{-j\omega_0}z^{-1})}$$

$$H(1) = \frac{b_0\left(1 - e^{j\omega_0}\right)\left(1 - e^{-j\omega_0}\right)}{(1 - r_0e^{j\omega_0})(1 - r_0e^{-j\omega_0})} = 1$$

$$On\ calculating\ we\ get,$$

$$b_0 = \frac{(1 - 2r_0cos\omega_0 + r_0^2)}{(2 - 2cos\omega_0)}$$

8.2c.

- The FIR Filter is causal as it only exists for n>=0.
- The ROC of H(z) is outside the outermost circle $|z| = r_0$, and includes $z = \infty$.
- The IIR Filter is stable as all the poles lies inside the unit circle in the ROC of H(z).

8.2d.

- On increasing r_0, the magnitude response becomes steeper near the pole.
- On increasing r_0, there is increase in deviation of phase response from linear characteristics.
- On increasing r_0, the length of impulse response also increases.

8.2e,f.

From the plot and sound, we observe that we were successful in removing noise to a great extent.

8.3d

Differences obtained in Magnitude and Phase Response:
N = 51 Difference = 13.25
N = 75 Difference = 13.26
N = 101 Difference = 13.27