LAB-6

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6.1 b:

Given, **x(t)=e^{jt}**
On calculating its FT, we get
X(jw) = $\frac{4\sin(2w)}{2w}$

- As the input signal is rectangular pulse of unit amplitude, therefore its continuous time FT results in **sinc function.**
- The sinc is completely **real**.
- Hence, its **imaginary** part is **0**.
- The **phase** plot continuously shifts from $\pi/2$ to -$\pi/2$ due to shift in X(jw) values from positive to negative values in **oscillatory** fashion.

6.1 c:

Observations:

- On changing T from 2 to 1, the **signal is compressed** in time by factor of 2 while its **Fourier transform** is **expanded** by the same factor.
- Also there is **change in amplitude.**
- On changing T from 2 to 4, the **signal is expanded** in time by factor of 2 while its **Fourier transform is compressed** by the same factor.
- Also there is **change in amplitude.**

FT Property:

Frequency Scaling Property: If a function is expanded in time by a quantity a, the Fourier Transform is compressed in frequency by same amount and vice-versa.

Hence, if **x(t)**------FT------->**X(jw)**

Then **x(at)**------FT------> $\frac{1}{|a|}X(\frac{jw}{|a|})$

6.1 d:

(a).Given, **x(t)=**e^{jt}

On calculating its FT, we get

X(jw) = $\frac{2\sin(\pi(1-w))}{1-w}$

Observations:

- The shape **is sinc function**
- Img(X(jw)) **is 0** because X(jw) is purely real as calculated.

(b).Given, **x(t) =cos t**

$=(e^{jt}+e^{-jt})/2$

On calculating its FT, we get

X(jw) = $\frac{2w\sin(\pi w)}{1-w^2}$

Observations:

- The shape is symmetric **sinc function**.
- From the symmetry, we observe that **X(jw) is even** which is evident from the expression.
- **Img(X(jw)) is 0** because X(jw) is purely real as calculated.

6.1 e:

(a).Given, **x(t)= 1+t for t=[-1,0)**
= 1-t for t=[0,1]

On calculating its FT, we get

X(jw) = $\left(\frac{\sin\left(\frac{w}{2}\right)}{\frac{w}{2}}\right)^2$

Observations:

- The shape is **sinc function**
- **Img(X(jw)) is 0** because X(jw) is purely real as calculated.

6.2 : A signal and its samples

The signal has been plotted.

6.3 : Fast Fourier Transform(Radix-2)

When N=2, X(k)=$\sum_{n=0}^{1} x(n)e^{\frac{-j\pi kn}{2}}$

where x(n) is the signal obtained on decomposing the input signal into odd and even components until its length is 2.

6.4 : Filtering of periodic signals with LTI systems

6.4 a:

- The relationship between input and output signal can be given as:

$$b_k = a_k(H(jw))$$

- From the relation, we observe that there is **no change in periodicity** of output signal.

6.4 b:

(b).Ideal Low Pass Filter(LPF)
Given, **x(t)=cos t**

$$= (e^{jt} + e^{-jt})/2$$

$$F\{\cos(t)\} = \pi(\delta(w+1) + \delta(w-1))$$

On comparing with the standard equation:

$$x(t) = \sum_{k=-N}^{N} a_k e^{jkt}$$

$$a_{-1} = a_1 = \frac{1}{2}$$

$$a_0 = 0$$

0 otherwise

$\rightarrow$ **A=$[\frac{1}{2}, 0, \frac{1}{2}]$**

Given, $w_0 = 1$

- The filter will only allow frequencies range such that $kw_0<|w_c|$
- Also k ranges between -1 to 1
- Hence, allowed frequencies=**$w_c<-1$ or $w_c>1$**

Case I: $w_c=2$

- Hence **$b_k=a_k$** for **$k=-1,0,1$**

Case II: $w_c=0.5$

- Hence **$b_k=0$** for **$k=-1,0,1$**

(c).Ideal High Pass Filter(HPF)

Given**, x(t)=cos t**

$$=(e^{jt}+e^{-jt})/2$$

As calculated above,

→**A=$[\frac{1}{2},0,\frac{1}{2}]$**

Given, **$w_0=1$**

- The filter will only allow frequencies range such that $kw_0>|w_c|$
- Also k ranges between -1 to 1
- Hence, allowed frequencies=**$-1<w_c<1$**

Case I: $w_c=2$

- Hence **$b_k=0$** for **$k=-1,0,1$**

Case II: w_c=0.5

- Hence $b_k=a_k$ for **k=-1,0,1**

(d).Non-Ideal Filter(LPF)

Given, frequency response, **H(w)=**$\frac{G}{a+w}$ where a>o,G>0

Observations:

- The output signal has a **phase shift**
- The **amplitude** of output signal is **decreased.**

Complex valued nature of LTI system

- Complex valued nature of LTI system corresponds to **phase shift in frequency domain.**
- On taking inverse FT, this corresponds **to time scale shift in time domain.**

6.5 : Reconstruction methods

Comparison between the quality of three interpolations

- The quality of interpolated signal is **not uniform** throughout.
- The signal is **better** interpolated in **middle** than at ends.
- If we ignore ends,
  Quality of interpolation: **sinc>Linear>Zero Order**
- End error occurs due to **unavailability of sufficient samples at the end.**

Maximum Absolute Error(MAE) between original and reconstructed signal in interval [0.25,1.75]

- **Zero order hold:** 3
- **Linear hold:** 0.3827
- **Sinc:** 0.6350

This is evident from above explanation.

6.6 : Sampling of non-band limited signal

t_samples vector: -1 : T_s : 1 where T_s is sampling interval.

Observations:

- **On decreasing sampling interval, sampling frequency increases.**
- Hence, the **number of samples increases** and **signal is better reconstructed.**

6.7 : Audio signals

CompressionM...	'Uncompressed'
NumChannels	2
SampleRate	8000
TotalSamples	268237
Duration	33.5296
Title	'Impact Moderato'
Comment	[ ]
Artist	'Kevin MacLeod'
BitsPerSample	16

Rest explanation is given in code.

6.8 : Aliasing

Given**, x(t)=cos(5πt)**

Nyquist rate: 2*5π=10π rad/sec

w_s=2π/T_s

Observations:

- On **increasing sampling interval**, the **error** or deviation between original and reconstructed signal **increases**,

Reason:

- When **T_s<=0.2 s** or **w_s>=10π rad/sec , the sampling frequency is above the Nyquist rate**. Hence the signal is **properly reconstructed**.
- As the **sampling frequency decreases**, the signal is **imperfectly reconstructed** which is evident from the plot.