1.1 F.S coefficients

(a).

Given,

$$x(t) = 2cos(2\pi t) + cos(6\pi t)$$

$$\omega = \frac{2\pi}{T} = 2\pi$$

Now, using

$$cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \ and \ sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}$$

$$x(t) = e^{j\,2\pi t} + e^{-j\,2\pi t} + \frac{1}{2}e^{j\,6\pi t} + \frac{1}{2}e^{-j\,6\pi t}$$

Type equation here.

Now, replacing 2pi by w

$$x(t) = e^{j\,\omega t} + e^{-j\,\omega t} + \frac{1}{2}e^{j\,3\omega t} + \frac{1}{2}e^{-j\,3\omega t}$$

On comparing with the standard equation

$$d_{-1} = 1, d_1 = 1, d_{-3} = 0.5,\ d_3 = 0.5$$

1.3 Gibbs Phenomenon

(a).

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} 1 \times e^{-jk\omega_0 t}dt$$

$$a_k = \frac{1}{T}\left[\frac{e^{-jk\omega_0 T_1}}{-jk\omega_0} - \frac{e^{jk\omega_0 T_1}}{-jk\omega_0}\right]$$

$$a_k = \frac{1}{-jk\omega_0 T}\left[\frac{1}{e^{jk\omega_0 T_1}} - e^{jk\omega_0 T_1}\right]$$

$$a_k = \frac{1}{-jk\omega_0 T}\left[\frac{1 - e^{2jk\omega_0 T_1}}{e^{jk\omega_0 T_1}}\right]$$

$$\omega_0 T = 2\pi$$

On further solving, we get

$$\boldsymbol{a_k} = \left[\frac{\sin k\omega_0}{k\pi}\right] \quad \textbf{when k!=0}$$

$$\boldsymbol{a_0} = \frac{2T1}{T}$$

Hence, it is real.

(b). On increasing T, number of sample increases.
Range of k values increases.
Amplitude decreases

(c). As N increases, there is increase in oscillation near the point of discontinuity of the signal. Although there is reduction in overshoot area between original and reconstructed signal, there is always some overshoot which can be seen on increasing N. This phenomenon is known as Gibbs phenomenon.

1.4 Fourier series

(c).

If x(t) is an even function, then:

We can use this property for even function: $\int_{-T/2}^{T/2} x(t)dt = 2\int_0^{T/2} x(t)dt$

$$a_0 = \frac{2}{T}\int_0^{T/2} x(t)dt$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos\left(\frac{2n\pi t}{T}\right)dt$$

As an even multiplied an even signal is even.

$$b_n = 0, as\ b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin\left(\frac{2n\pi t}{T}\right)dt = 0$$

As an even multiplied by an odd signal is odd.

$$d_k = \frac{a_k - jb_k}{2}, \qquad k = 1,2,\dots$$

$$d_k = \frac{a_k + jb_k}{2}, \qquad k = -1,-2,\dots..$$

$$d_k = a_0, \qquad k = 0$$

$$d_k = \frac{2}{T}\int_0^{T/2} x(t)dt\,, \qquad k = 0$$

$$d_k = \frac{a_n}{2} = \frac{2}{T}\int_0^{T/2} x(t)\cos\left(\frac{2n\pi t}{T}\right)dt$$

As the function x(t) is even and symmetric, the complex part of FS Coefficient is 0, leaving behind only real part.

If x(t) is an odd function, then:

We can use the property of odd function: $\int_{-T/2}^{T/2} x(t)dt = 0$.

$$a_n = 0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, cos\left(\frac{2n\pi t}{T}\right) dt = 0$$

As an odd signal multiplied by an even signal is odd.

$$b_n = \frac{4}{T}\int_{0}^{T/2} x(t)\, sin\left(\frac{2n\pi t}{T}\right) dt$$

Product of an odd and odd signal is an even signal.

$$d_k = \frac{a_k - jb_k}{2}, \qquad k = 1,2, \dots$$

$$d_k = \frac{a_k + jb_k}{2}, \qquad k = -1, -2, \dots ..$$

$$d_k = a_0, \qquad k = 0$$

$$d_k = 0, \qquad k = 0$$

$$d_k = \frac{-jb_n}{2} = \frac{-2j}{T}\int_{0}^{T/2} x(t)\, sin\left(\frac{2n\pi t}{T}\right) dt\,, \qquad k = 1,2, \dots ..$$

$$d_k = \frac{jb_n}{2} = \frac{2j}{T}\int_{0}^{T/2} x(t)\, sin\left(\frac{2n\pi t}{T}\right) dt\,, \qquad k = -1, -2, \dots ..$$

As the function x(t) is odd, the real part of FS Coefficient is 0, leaving behind only part