LAB 4 – Discrete-Time FT And LTI Systems

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4.1 Discrete-time Fourier transform (DTFT)

(c). Given, x[n]=a^nu[n]

Clearly, on increasing the value of b, the peaks in DTFT become more prominent and the rate of decay of signal decreases.

4.2 Discrete-time filters

(a). Impulse response

Since an order–M moving average Filter is a discrete time LTI system with input x[n] and output y[n], the relation between them is given by:

$$y[n] = \frac{1}{M}\sum_{m=0}^{M-1} x[n-m]$$

For Impulse response of the system, the input x[n] on substituting as δ[n] , we get

$$h[n] = \begin{cases} \frac{1}{M}, & for\, n = 0, 1, \ldots, M-1 \\ 0, & otherwise \end{cases}$$

(e). Since ,as calculated above,

$$h[n] = \begin{cases} \frac{1}{M}, & for\ n = 0, 1, \ldots, M-1 \\ 0, & otherwise \end{cases}$$

Converting it into frequency domain for analysis,

$$H(\omega) = \frac{1}{M} \sum_{m=0}^{M-1} e^{-j\omega m}$$

On solving, we get

$$H(\omega) = \frac{1}{M} \frac{\left(1 - e^{-j\omega M}\right)}{\left(1 - e^{-j\omega}\right)}$$

Observation:

Noise Reduction

Clearly, on increasing the number of points in filter i.e M, there is significant reduction in noise and we get smoother signal as output.

Trade-off

- The disadvantage of Moving Average system is that edges become less sharper.
- On increasing the value of M, the attenuation for the higher frequency gets reduced.

(f). Clearly, on increasing the value of M, the distortion at the bottom of Magnitude spectrum gets significantly reduced and correspondingly, peaks become more sharper.

(g).Impulse response

For Impulse response of the system, the input x[n] on substituting as δ[n] , we get

$$h[n] = \begin{cases} 1 \;\; for\, n = 0 \\ -1\, for\, n = 1 \\ 0, \;\; otherwise \end{cases}$$

Observation

As the circuit acts as a simple differentiator, which has input relation given by given by:

y[n] = x[n] – x[n-1]

On increasing the value of M, there is no clear change in magnitude spectrum and it is very noisy as the y[n] acts as a differentiator,

(h).

The nature of all above filters has a lowpass characteristic.

It allows low frequency components to pass through it while attenuates higher frequency components.

4.3 Inverse DTFT (script)

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$As\ X(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases}$$

$$x[n] = \frac{1}{2\pi jn}\left[e^{j\omega_c n} - e^{-j\omega_c n}\right]$$

On solving, we get

$$x[n] = \frac{1}{\pi n}\sin(\omega_c n) = \frac{\omega_c}{\pi}\frac{\sin(\omega_c n)}{\omega_c n} = \frac{\omega_c}{\pi} sinc(\omega_c n)$$

(c).

$$As\ X(e^{j\omega}) = \begin{cases} 1, & \omega_1 \leq |\omega| \leq \omega_2 \\ 0, & |\omega| < \omega_1\ and\ \omega_2 < |\omega| < \pi \end{cases}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$x[n] = \frac{1}{2\pi}\int_{\omega_1}^{\omega_2} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{-\omega_2}^{-\omega_1} e^{j\omega n} d\omega$$

$$x[n] = \frac{1}{2\pi j n}\left[e^{j\omega_2 n} - e^{j\omega_1 n} + e^{-j\omega_1 n} - e^{-j\omega_2 n}\right]$$

$$x[n] = \frac{1}{\pi n}[\sin(\omega_2 n) - \sin(\omega_1 n)] = \frac{\omega_2}{\pi}\frac{\sin(\omega_2 n)}{\omega_2 n} - \frac{\omega_1}{\pi}\frac{\sin(\omega_1 n)}{\omega_1 n}$$

$$= \frac{\omega_2}{\pi}[sinc(\omega_2 n)] - \frac{\omega_1}{\pi}[sinc(\omega_1 n)]$$