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Part-A

Q1) One light day $= 24\times60\times60$ s

(a) No. of meters in a light day $= c\times 24\times60\times60$

$= 86400c$

$\approx \boxed{2.592\times10^{13}\,m}$

(b) No. of km in a light microsecond $= c\times10^{-6}$

$= 3\times10^{8}\times10^{-6}$

$= \boxed{300m}$

(c) light parsec $= 3.09\times10^{16}\,m$

No. of sec $= \dfrac{3.09\times10^{16}}{3\times10^{8}}$

$= 1.03\times10^{8}\,s$

No. of years $= \dfrac{1.03\times10^{8}}{24\times60\times60}$

$= \boxed{3.23 \text{ years}}$

(d) No. of seconds in a light fermi, $10^{-15} = 3\times10^{8}\times t(s)$

$\Rightarrow t(s) = \dfrac{10^{-15}}{3\times10^{8}}$

$\approx \boxed{3.33\times10^{-24}\,s}$

Q(2) (x,y,z,t) frame O: $(x,y,z,t) = (100km, 10km, 5km, 2ms)$

(a) $v_x' = 0.95c \Rightarrow v = 0.95c$

$x' = \gamma(x-vt)$ $y'=y$ $z'=z$ $t' = \gamma\left(t-\frac{vx}{c^2}\right)$

$\gamma = \dfrac{1}{\sqrt{1-(0.95)^2}} = 3.2025$

$\Rightarrow x' = \gamma\,(100\times10^{3} - 0.95\times c\times 2\times10^{-3})$

$= \gamma\,(10^{5} - 3\times0.95\times2\times10^{5})$

$= -\gamma\times4.7\times10^{5} \approx \boxed{-1505.175\ Km}$

Now, $t' = \gamma\left(2\times10^{-3} - \frac{0.95\times10^{-3}}{3}\right)$

$= \gamma(1.6833)\times10^{-3}$

$\simeq 5.39\,ms$

$\therefore \boxed{\begin{array}{l} x' = -1505.175\ km \\ y' = y = 10\,km \\ z' = z = 55\,km \\ t' = 5.39\ ms \end{array}}$

(b) Inverse L.T.

$x = \gamma(x' + vt')$; $y = y'$, $z = z'$, $t = \gamma\left(\frac{vx'}{c^2} + t'\right)$

$\Rightarrow x = \gamma(-1.505175\times10^6 + 5.31\times0.95\times3\times10^5)$

$\simeq 99.19\,km \approx 100\,km$

$y = y' = 10\,km$ $z = z' = 55\,km$

$t = \gamma\left(5.39\times10^{-3} - \frac{0.95\times1505.175\times10^3}{3\times10^8}\right)$

$\simeq 1.99\,ms \simeq 2ms$

Thus, we have verified our answers using inverse LT

Q6. $v'_x = 0.6c = v$

(a) $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{0.8} = 1.25$

(b) $t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$x = 240m$, $t = x/v$

$t' = \gamma\left(\frac{x}{v} - \frac{xv}{c^2}\right)$

$= \frac{x\gamma}{v}\left(1 - \frac{v^2}{c^2}\right)$

$= \frac{x\gamma}{v}\times\frac{1}{\gamma^2} = \frac{x}{v\gamma} = \frac{240}{\frac{5}{4}\times0.6\times3\times10^8}$ $\therefore \boxed{t' \simeq 1.07\mu s}$

will have passed according to the clock frame.

7. $v = 630\,ms^{-1}$

$L = L_0\sqrt{1-\frac{v^2}{c^2}}$

(a) Fractional contraction : $\frac{\Delta L}{L_0}$

$= \frac{L_0 - L}{L_0} = 1 - \frac{L}{L_0}$

$= 1 - \sqrt{1-\frac{v^2}{c^2}}$

$\approx 1 - \left(1 - \frac{v^2}{2c}\right)$ [using Binomial expansion)

$= \frac{v^2}{2c^2}$

$\approx \left(\frac{630}{3\times10^8}\right)^2 \times \frac{1}{2}$

$\approx 2.205\times10^{-12}$

$\approx \boxed{2.21\times10^{-12}}$

(b) $\underbrace{\Delta t}_{\text{earth clock}} = \gamma \underbrace{\Delta \tau}_{\text{mover's clock}}$ (time dilation)

$\Delta t - \Delta\tau = 1\mu s = 10^{-6}s$

$\Delta t - \Delta\tau = (\gamma - 1)\Delta\tau = 10^{-6}s$ — (i)

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$ (Binomial approx.)

$\gamma = 1 + \frac{1}{2}\left(\frac{630}{3\times10^8}\right)^2$

$\approx 1 + \frac{1}{2}(4.41\times10^{-12})$

$= 1 + 2.205\times10^{-12}$

Now from eq^n (i)

$(\gamma-1)\Delta\tau = 10^{-6}$

$(1 + 2.205\times10^{-12} - 1)\Delta\tau = 10^{-6}$

$\Rightarrow \Delta\tau = \frac{10^{-6}}{2.205\times10^{-12}} \approx 4.535\times10^5 s$

$\Rightarrow$ Earth time, $\Delta t = \Delta\tau + 1\mu s$

$\approx 4.535\times10^5$

≈ 5.26 days $\Rightarrow$ It would take $\boxed{5.26 \text{ days}}$

9. velocity of O' relative to O : $v = 0.6c$
velocity of particle in O's frame : $u' = 0.4c$

$$u = \frac{u'+v}{1+\frac{u'v}{c^2}}$$ (Relativistic velocity addition formula)

$$= \frac{0.4c+0.6c}{1+\frac{0.4c\times0.6c}{c^2}} = \frac{c}{1.24} \approx 0.806c$$

$\Rightarrow \boxed{u \approx 0.806c}$

Part B

16. $V(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$

Time Independent Schrodinger Eqn:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + V\psi = E\psi$$

Inside the well:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} = E\psi \quad (\because V(x)=0)$$

If $E=0$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} = 0 \Rightarrow \phi(x) = Ax+B$$

Boundary conditions: $\psi(0) = 0 \Rightarrow B=0$ — (i)
$\psi(a)=0 \Rightarrow A=0$ — (ii)

From (i) & (ii)
$\boxed{\phi(x) = 0 \quad \forall x}$

The only soln is $\phi(x)=0$, which corresponds to no-particle (zero probability everywhere)
This is not valid physical state as wave fn must be normalized & describe a non-zero probability of finding particle somewhere.

$\Rightarrow$ $\boxed{\text{No acceptable soln to TISE for infinite square well with } E=0}$

18. Inside the well, we have wave f^n:

$$\psi(x,t) = \sqrt{\frac{2}{a}} \sum_{n=1}^{\infty} c_n e^{\frac{-i\hbar n^2\pi^2 t}{2ma^2}} \sin\left(\frac{n\pi x}{a}\right)$$

We want $\psi(x,0) = \psi(x,T)$

$$\Rightarrow \sqrt{\frac{2}{a}} \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{a}\right) = \sqrt{\frac{2}{a}} \sum c_n \sin\left(\frac{n\pi x}{a}\right) e^{\frac{-in^2\pi^2\hbar T}{2ma^2}}$$

$$\Rightarrow e^{\frac{-in^2\pi^2\hbar T}{2ma^2}} = 1$$

$$e^{-i\theta} = 1, \quad \theta = \frac{n^2\pi^2\hbar T}{2ma^2}$$

$$\Rightarrow \theta = 2k\pi, \quad k \in \mathbb{Z}$$

$$\frac{n^2\pi^2\hbar T}{2ma^2} = 2k\pi \Rightarrow \boxed{T = \frac{4kma^2}{n^2\hbar\pi}}$$

$n=1, k=1$ for T to be periodic

$$\therefore \boxed{T = \frac{4ma^2}{\pi\hbar}}$$

20. We have a spin 1/2 particle that passes through Stern-Gerlach magnets and second magnet is rotated at θ relative to field.
The probability of detecting spin-up relative to the second magnet is twice the probability of detecting spin down.

$$\Rightarrow P\uparrow = 2P\downarrow$$

$$P\uparrow = \cos^2\left(\frac{\theta}{2}\right), \quad P\downarrow = \sin^2\left(\frac{\theta}{2}\right)$$

Hence, $\cos^2\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}$

or $\tan^2\frac{\theta}{2} = \frac{1}{2}$

$$\Rightarrow \theta = 2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \boxed{\theta \approx 70.52^\circ}$$

21. To ensure no causal influence, we need to make sure that events are space-like i.e. the minimum distance b/w stations must be greater than the distance light can travel i.e. the time it takes to perform one measure

Measurement time: $\Delta t = 1ms = 10^{-3} s$

Distance required $= c \times \Delta t$

$= 3\times10^{8} \times 10^{-3}$

$= 3\times10^{5} m$

$= 300 km$

$\Rightarrow$ The stations must be atleast $\boxed{300km}$ apart

22. Heisenberg uncertainity principle:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x (m\Delta v) \geq \frac{\hbar}{2}$$

$$\Rightarrow \Delta x \geq \frac{\hbar}{2m\Delta v}$$

$$\hbar = \frac{h}{2\pi} \simeq 1.055\times10^{-34}$$

(a) $v = 10^{6} ms^{-1}$

Mass of neutrons $= 1.675\times10^{-27} kg$

$$\Delta x \geq \frac{1.055\times10^{-34}}{2\times1.675\times10^{-27}\times10^{6}} \simeq 3.15\times10^{-14} m$$

or $\boxed{\Delta x \gtrsim 3.15\times10^{-14} m}$

(b) $v = 2ms^{-1}$

$m = 50 kg$

$$\Delta x \geq \frac{1.055\times10^{-34}}{2\times50\times2} \simeq 5.28\times10^{-37} m$$

or $\boxed{\Delta x \geq 5.28\times10^{-37} m}$