

**The Universe Across Scales**  
**IIIT-H, Semester: Winter 24, Module 2, Assignment 1**

*Submission deadline: April 1, 2025*

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1. Starting from the fluid equation (with  $c = 1$ )

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0,$$

show that for cold matter, for which  $p = 0$ , the energy density scales as  $\rho \propto a^{-3}$ .

2. Show that in a matter dominated universe (*i.e.*,  $K = 0$ ,  $\Lambda = 0$ , and radiation energy density = 0) the scale factor varies with time as

$$a(t) = a_0 \frac{t^{2/3}}{t_0^{2/3}},$$

where  $a_0$  is the value of the scale factor at present time  $t = 0$ .

3. Starting from the Friedmann equation show that in the  $\Lambda$ CDM model the age of the universe is given as

$$t = \int_0^1 \frac{dx}{H_0 x \sqrt{\Omega_\Lambda^0 + \Omega_K^0 x^{-2} + \Omega_M^0 x^{-3} + \Omega_R^0 x^{-4}}}.$$

Perform this integration numerically and find the value of  $t$  in years. Submit the code. *Hint: Last lecture*