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CST
29
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(1)

```
void func( int n)
{   int j=1 , i=0;
    while ( i<n)
    {   i+=j;
        j++;
    }
}
```

$\Rightarrow$ for $j=1$ $\quad i=1;$
$j=2$ $\quad i=1+2;$
$j=3$ $\quad i=1+2+3;$ $\Big]$ m levels

$\therefore$ $1+2+3+ \cdots < n$

$1+2+3+m < n$

$\frac{m(m+1)}{2} < n$

$m \approx \sqrt{n}$

by summation method:

$\Rightarrow \sum_{i=1}^{m} 1 \Rightarrow 1+1+ \cdots \sqrt{n}$ times

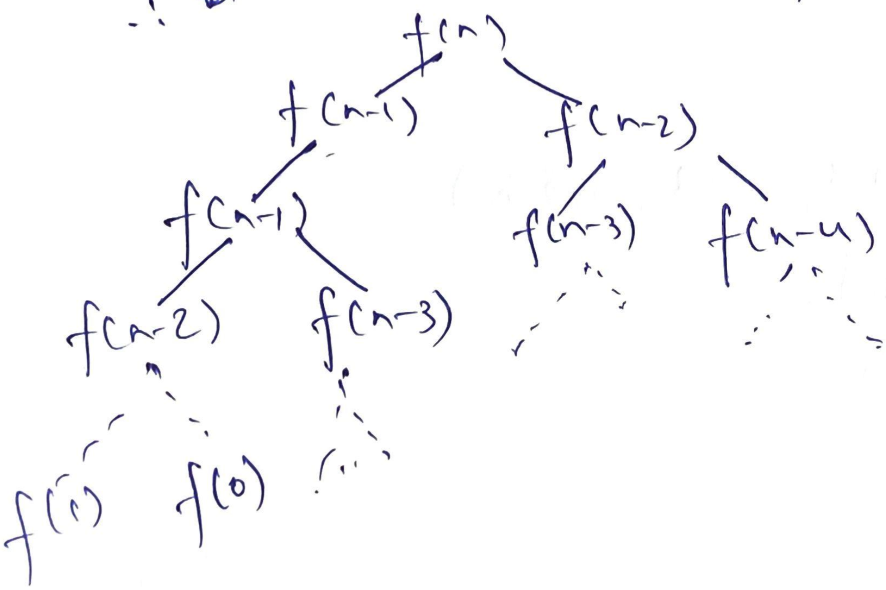
$\therefore T(n) = \sqrt{n}$

(2) For fibonacci series:-

$f(n) = f(n-1) + f(n-2)$

$f(0) = 0 \qquad f(1) = 1$

$\therefore$



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∵ at every function call we get two function calls for n levels:-

we have ⇒ 2×2 ..... n times

$\therefore T(n) = 2^n$

Maximum Space ÷ considering recursive stack ÷

no of calls max. = n

For each call we have space complexity O(1)

$\therefore T(n) = O(n)$

3)

a) n logn:-

<u>quick sort</u>

```
void func( int arr[], int l, int h)
{   if( l < h)
    { int pi = partion(arr, lo, h);
      func( arr, l, pi-1);
      func( arr, pi+1, h);
    }
}

int partion( int arr[], int l, int h)
{  int pi = arr[h];
   int i = (l-1);
```

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```
for( int j = l ; j <= h ; j++ )
{  if ( arr[i] < pi )
     {   i++;
         swap( arr[i) , arr[j] );
     }
}
  swap( arr[i+1], arr[h] );
  return (i+1);
}
```

(b) n^3 :-

Multiplication of two square matrix

```
for (i=0 ; i< n ; i++)
  {  for (j=0 ; j< c2 ; j++)
     {   for (k=0 ; k < c1 ; k++)
         { res[i][j] += a[i][k] * b[k][j];
```

(c) log(log n)

```
for (i=2 ; i<n ; i = i * i)
   {  c++;
   }
```

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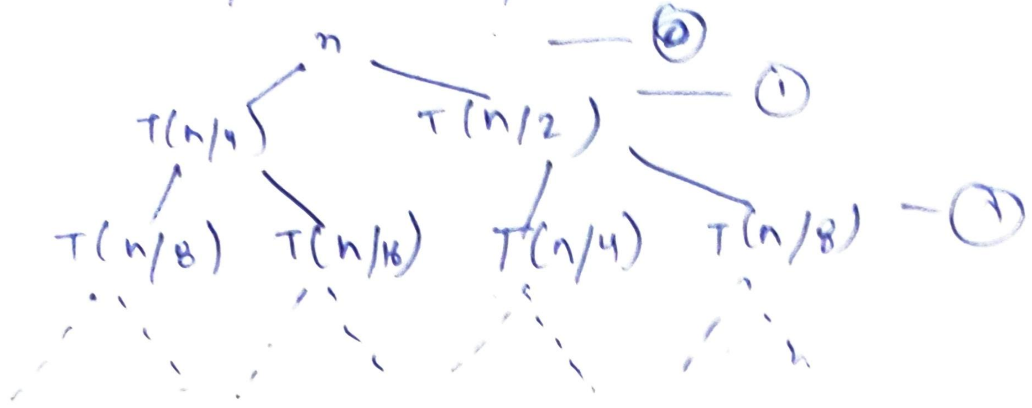
(4) $T(n) = T(n/4) + T(n/2) + cn^2$



At level →

$0 \rightarrow cn^2$

$1 \rightarrow \frac{n^2}{4^1} + \frac{n^2}{2^2} = \frac{c\,5n^2}{16}$

$2 \rightarrow \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 c$

$\vdots$

$\text{max level} = \frac{n}{2^k} = 1$

$\Rightarrow k = \log_2 n$

$\therefore T(n) = \left(cn^2 + \left(\frac{5}{16}\right)n^2 + \left(\frac{5}{16}\right)^2 \cdots + \left(\frac{5}{16}\right)^{\log n} n^2\right)$

$T(n) = cn^2 \cdot \left[1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \cdots \left(\frac{5}{16}\right)^{\log n}\right]$

$T(n) = cn^2 \times 1 \times \left(\frac{1 - \left(\frac{5}{16}\right)^{\log n}}{1 - 5/16}\right)$

$= cn^2 \times \frac{11}{5}\left(1 - \left(\frac{5}{16}\right)^{\log n}\right)$

$\therefore T(n) = O(n^2 c) \Rightarrow O(cn^2)$

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(5) int fun (int n)
{
for (i=1; i<=n; i++)
{ for (j=1; j<n; j+=i)
{
}
}
}

for → i j

i	j
1	1
2	1 + 3 + 5
3	1 + 4 + 7
4	1 + 5 + 9
⋮	⋮
n	

$$\sum_{i=1}^{n} \frac{(n-1)}{i}$$

$$\because T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \frac{(n-1)}{3} \cdots + \frac{(n-1)}{n}$$

$$T(n) = n\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right] - 1 \times n\left[1 + \frac{1}{2} + \frac{1}{3} \cdots \frac{1}{n}\right]$$

$$= n\log n - \log n$$

$$\therefore T(n) = O(n \log n)$$

(6) for (i=2; i<=n; i = pow(i,k))
{ O(1)
}

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fulfil the go... in Marks (CO...

for → i

2^1

2^k

2^{k^2}

2^{k^3}

$\vdots$

2^{k^m}

when, $2^{k^m} <= n$

$k^m = \log_2 n$

$m = \log_k \log_2 n$

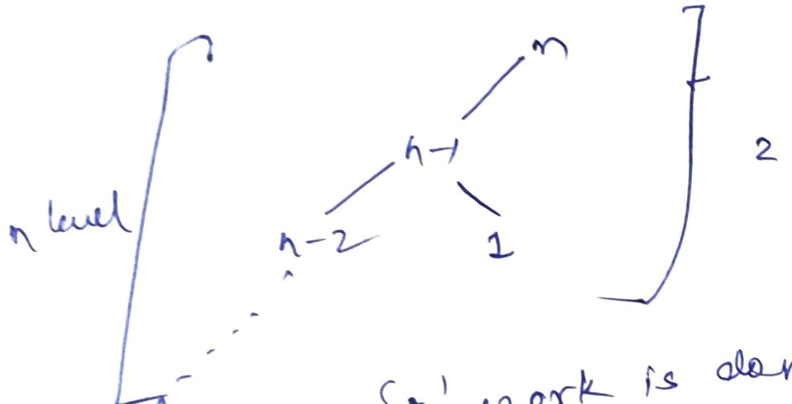
$\therefore \sum_{i=1}^{m} 1$

$\Rightarrow 1 + + + \ldots m$ times

$\Rightarrow T(n) = O(\log_k \log n)$

(7) Given algo divides array in 99% & 1% part

$\therefore T(n) = T(n-1) + O(1)$



'n' work is done at each level for merging

$T(n) = (T(n-1) + T(n-2) + \ldots T(1) + O(1)) \times n$

$= n \times n$

$T(n) = O(n^2)$

lowest higher = 2

height higher = n

$\therefore$ diff $= n-2 \quad \because (n > 1)$

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(8) considering for large values of 'n'.

a) $100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

b) $1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < n\log n < 2n < 4n < \log(n!) < n^2 < n < 2^{\log 2n} < 5n$

c) $96 < \log_8 n < \log 2n < 5n < n\log_6 n < n\log_2 n < \log(n!) < 8n^2 < 7n^3 < n! < 8^{2n}$

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