Divisibility

Q1) which one of the following number is divisible by 99?

(a) 3572404 (b) 135792
(c) 913464 (d) 114345

Sol)

For divisibility 99, the number should be divisible by 11 & 9

(a) 3572404.

$3+5+7+2+4+0+4 = 25$

Hence, it is not divisible by 9

$\therefore$ it is not divisible by 99.

(b) 135792

$1+3+5+7+9+2 = 27$

it is divisible by 9

$(1+5+9) - (3+7+2)$

$15 - 12$

3

it is not divisible by 11

$\therefore$ it is not divisible by 99

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(c) 913464

9 + 1 + 3 + 4 + 6 + 4 = 27
it is divisible by 9

(9 + 3 + 6) - (1 + 4 + 4)
18 - 9
9
it is not divisible by 11
Hence not divisible by 99

(d) 114345

1 + 1 + 4 + 3 + 4 + 5 = 18
it is divisible by 9

(1 + 4 + 4) - (1 + 3 + 5)
9 - 9
0
it is divisible by 11
Hence, no. is divisible by 99

∴ option (d) is correct.

Q2)

$$\frac{(2n+2)^2}{4}$$

dimplify:

$\therefore (2n+2)^2$

$4n^2+4n+4$

$4(n^2+n+1)$

Now $4(n^2+n+1)$ is clearly divisible by 4

$\therefore$ remainder is 0 //

Q3)

$19506 \div 9$

$$\begin{array}{r} 2167 \\ 9\overline{)19506} \\ 18 \\ \hline 15 \\ 9 \\ \hline 60 \\ 54 \\ \hline 66 \\ 63 \\ \hline 3 \end{array}$$

The 1st no. $= 2167 \times 9$

$= 19503$

2n no $= 19503 + 9$

$= 19512$

$\therefore$ The Two nearest numbers To 19506 that are divisible by 9 are

19503, 19512 //

Q4)

M39048458N

Since no. is divisible by 11

$\therefore$ $(M+9+4+4+8) - (3+0+8+5+N)$

$(M+25) - (16+N)$

$M+25-16-N$

$M-N+9$ —(1)

Since no. is also divisible 8

$\therefore$ 58N ~~is~~ is divisible by 8

$$\begin{array}{r} 7 \\ 8\overline{)58N} \\ \underline{56} \\ 2N \end{array}$$

$\therefore$ $\boxed{N=4}$

Putting $N=4$ in eqn (i)

$M-N+9$

$M-4+9$

$M+5$

Since, to get divisible by 11, the ~~no~~ sum should be multiple of 11

$\therefore$ $\boxed{M=6}$

Hence, $M = 6$, $N = 4$

Q5)

763X4Y2

Since, the number is divisible by 9

$\therefore$ $7+6+3+x+4+y+2$

$x+y+22$

(0, 5), (1, 4), (2, 3) (3, 2)
(4, 1), (5, 0)

Q6)

$n = 8q + 3$

$6n = 8p + \_\_$

$6(8q+3)$
$48q + 18$

$18 \div 8$

remainder is 2

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Q7)

for divisibility of 12 the no. should be divisible by both 4 & 3.

Let SPR

9 P 2

for 3   ~~for 4~~

9 + P + 2   9

11 + P

$\boxed{P = 1}$ //

So, our no. will be

912

Q8)

7 x 8 6038

(7 + 8 + 0 + 8) - (x + 6 + 3)

23 - x - 9

14 - x

Hence, $\boxed{x = 3}$ //

14 - 3 = 11 divisible by 11

15 12 60

(Q9)

Hence, common multiple of 3, 5, 12 is 60.

∴ n is divisible by 3, 5, 12, the next no. that is divisible by 3, 5, 12 will be

$\boxed{n+60}$

(Q10)

5M83M4M1

5+M+8+3+M+4+M+1
21+3M

minimum value m=2

21+3×2
21+6 = 27. divisible by 9

maximum value m= 8

21+3×8
21+24 = 45 divisible by 9

The product:

8×2
→ $\boxed{16}$

Unit digits

Q1)

$$3^{65} \times 6^{59} \times 7^{71}$$

$3^{65} = \frac{65}{4}$ rem = 1

$= 3^{1}$

$\therefore$ unit digit = 3

$6^{59} = \frac{59}{1} =$

unit digit = 6

$7^{71} = \frac{71}{4}$ rem = 3

unit digit = 3

now, $3 \times 6 \times 3$

$\Rightarrow 54$

Q2) $(173)^{45} \times (152)^{77} \times (777)^{999}$

this can be resolve into

$(3)^{45} \times (2)^{77} \times (7)^{99}$

$3^{45} = \frac{45}{4} \Rightarrow rem = 1$

$3^1 = 3$

unit ~~go~~ digit = 3

$2^{77} = \frac{77}{4} \Rightarrow 1$

$2^1 = 2$

unit digit 2

$7^{99} = \frac{99}{4} \Rightarrow rem\ 3$

$7^3 = 3$

unit digit 3

$\therefore$ now, $3 \times 2 \times 3$

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Q3) $6^{256} - 4^{256}$

$\cancel{6}$

$4^{56} = \frac{\cancel{56}}{2} \Rightarrow$ rem 0

$\cancel{4^1}\ 4^2$ 5 6

4^1 4^2

9

4^{55} 4^{56}

now

$6 - 6 = 0$

Q4) $264^{102} + 264^{103}$

This can be resolue into

$(4)^2 + (4)^3$

$6 + \cancel{8}4$

$1\cancel{0}$

unit diget is 0

Q5)

$(316)^{3n} + 1$

$6^{3n} + 1$

$6 + 1$

unitdigit $= 7$

Q6)

$(7^{95} - 3^{58})$

$7^{95} = \frac{95}{4} \Rightarrow$ rem $= 3$

$7^3 = 3$

$3^{58} = \frac{58}{4} \Rightarrow$ rem $= 2$

$3^2 = 9$

now,

$3 - 9$

6

Q7)

30^{2720}

$(3 \times 10)^{2720}$

$3^{2720} \times 10^{2720}$

the rightmost non-zero digit of the no. 30^{2720} is same as the unit digit of the no. 3^{2720}

3^{7720}

$3^{20} \Rightarrow 20$

3 9 7 1

3^1 3^2 3^3 3^4

3^5 3^6 3^7 3^8

⋮

$\underline{3^{20}}$

unit digit = 1

$\therefore$ rightmost nonzero digit is 1

Q8)

81 × 82 × 83 × 84 × 86 × 87 × 88 × 89

now, for last digit

1 × 2 × 3 × 4 × 6 × 7 × 8 × 9

7257$\underline{6}$

Hence, last digit is 6

Q9)

12345 × 54321

for last 3 digits we take

345 × 321

110745

last 3 digits = 745

Q10).

$1^5 + 2^5 + 3^5 \dots 9^5$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$

45

$\boxed{5}$ is the last digit

Factors

Q1)

$$6^4 \times 8^6 \times 10^8 \times 14^{10} \times 22^{12}$$

Hence, it is ~~a~~ ~~p~~ the prime factrization of a no. therefor

~~Number of prime factorizatn =~~

Number of prime factor $= 5$

Q2)

$$N = a^4 \times b^3 \times c^7$$

$2^0\, c^2\, 3^4\, c^6$

$PS = 3 \times 2 \times 4$

$= 24$

$4 \times 5 \times 3$

60

$1 \times 1 \times 3$

Q3)

$$12^3 \times 30^4 \times 35^2$$

Even numbers $= 4 \times 5 \times 3$

$= 60$

Q4)

$N = 2^7 \times 3^4$
$M = 2^4 \times 3^2 \times 5$

Hence, the no. of factors of N that are common with the factors of M are

$CF = 2^4 \times 3^2$
$= 5 \times 3$
$CF = 15$

Q5)

N has 5 factors

case 1

$N = P^4$

The smallest N of this form is $2^4 = 16$

No. of factor of 16

$2^4 = 5$ factor

case - 2

$N = P_1^2 \times P_2$

The smallest N of this form is $2^2 \times 3 = 12$

factors of 12

$2^2 \times 3 = \cancel{3 \times 2 = 12}\ 3 \times 2 = 6$

which donot matched

$\therefore$ $N = 16$ is the smallest number that has exactly 5 factors

$\therefore$ $N - 1 = 16 - 1$

$= 15$

$15 = 3 \times 5$

No. of factors $= 2 \times 2$

$= 4$

Q6)

$\frac{720}{7}$

Q7) 10!

$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$
$= \cancel{3740240}\ 3628800$

~~Prim~~ Now,
$10! = 328800 = 2^8 \times 3^4 \times 5^2 \times 7$

$TF = 9 \times 5 \times 3 \times 2$
$= 270$

Q8) $2^7 \times 3^6 \times 5^4 \times 7^3$

$P.S = 4 \times 4 \times 3 \times 2$
$= 96$

Q9) $480 = 2^5 \times 3 \times 5$

$\therefore$ 480 can be written in $\boxed{24}$ ways

Q10) $2^5 \times 3^4 \times 5^3$ & $2^3 \times 5^4 \times 7^5$

Common Factor $= 2^3 \times 5^4$
$= 4 \times 5 = 20$

~~Fa~~ D Factors that are not common $= 120 - 20$
$= 100$

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Remainders

Q1) $\frac{7^{25}}{6}$

$\Rightarrow \frac{7^{25}}{6} = 1$ $\frac{7^{1}}{6}$ 1

Q2) $\frac{3^{45}}{8}$

$\frac{3^{45}}{8} = 3$ $\frac{3^{1}}{8}$ 3 $\frac{3^{2}}{8}$ 1

$\frac{3^{3}}{8}$ $\frac{3^{4}}{8}$

$\vdots$

$\frac{3^{45}}{8}$

Q3) $\frac{4^{96}}{8}$

$\frac{4^{96}}{8} = 4$ $\frac{4^{1}}{8}$ 4 $\cancel{\frac{4^{2}}{8}}$ 0

Q4) $\frac{1415^{16}}{95}$

$\left(\frac{1415 \times 1415 \times 1415 \cdots 16 \text{ times}}{5}\right)$

$\frac{0^{16}}{5} = 0$

Q5) $\frac{67^{99}}{7}$

$\left(\frac{67 \times 67 \times 67 \times \cdots 99 \text{ times}}{7}\right)$

$\frac{4^{99}}{7} =$

$\frac{4^1}{7}$ (4) $\frac{4^2}{7}$ (2) $\frac{4^3}{7}$ (1)

$\frac{4^4}{7}$ $\frac{4^5}{7}$ $\frac{4^6}{7}$

$\vdots$

$\frac{4^{99}}{7}$

$\frac{67^{99}}{7} = \frac{4^{99}}{7} = 1$

Q6)
$$\frac{73 \times 75 \times 78 \times 57 \times 197 \times 37}{34}$$

$$\Rightarrow \frac{73}{34} \times \frac{75}{34} \times \frac{78}{34} \times \frac{57}{34} \times \frac{197}{34} \times \frac{37}{34}$$

$$\frac{5 \times \cancel{7} \times 10 \times 23 \times 2\cancel{5} \times 3}{34}$$

~~279450~~

$$\frac{35 \times 10 \times 23 \times 2\cancel{5} \times 3}{34}$$

$$\frac{1 \times 10 \times 23 \times 2\cancel{5} \times 3}{34}$$

$$\frac{10 \times 23 \times 2\cancel{5} \times 3}{34}$$

$$\frac{22 \times 2\cancel{5} \times 3}{34}$$

$$\frac{24 \times 3}{34} = 4$$

Q7)
$N = 1421 * 1423 * 1425 \div 12$

$$\frac{10}{12} = \frac{1421 \times 1423 \times 1425}{12}$$

$$= \frac{5 \times 7 \times 9}{12}$$

$$= \frac{315}{12}$$

$$= \frac{3}{12}$$

remainder $= 3$

(Q5)

$\frac{2^{256}}{17}$

$$\frac{2^{1}}{17}\ \frac{2^{2}}{17}\ \frac{2^{3}}{17}\ \frac{2^{4}}{17}\ \frac{2^{5}}{17}\ \frac{2^{6}}{17}\ \frac{2^{7}}{17}\ \frac{2^{8}}{17}$$

(2) (4) (8) (16) (15) (13) (9) (1)

$$\frac{2^{9}}{17}\ \frac{2^{10}}{17}\ \frac{2^{11}}{17}\ \frac{2^{12}}{17}\ \frac{2^{13}}{17}\ \frac{2^{14}}{17}\ \frac{2^{15}}{17}\ \frac{2^{16}}{17}$$

$\vdots$

$\frac{2^{256}}{17}$

$\frac{2^{256}}{17} = 1$ //

(1a)

$\frac{39^{97!}}{40}$

$\frac{39^{97}}{40}$

Factorials

Q1)

$$\frac{20!}{21}$$

$$\frac{20!}{3\times7} \Rightarrow \frac{20!}{7^n} = \frac{20}{7} + \cancel{\frac{20}{49}}$$

$$= 2$$

Q2)

$$\frac{31!}{32}$$

$$\frac{31!}{32\cancel{4}\cancel{8}} \Rightarrow \frac{31!}{2^5} = \frac{31}{2} + \frac{31}{4} + \frac{31}{8} + \frac{31}{16}$$

$$= 15 + 7 + 3 + 1$$

$$= 26$$

Now, $\frac{26}{5} = 5$

Q3) $\cancel{\frac{28!}{28}}$

$$\cancel{\frac{28!}{28} \Rightarrow \frac{28!}{4\times7} = \frac{28!}{7^n} = \frac{28}{7}}$$

$$= \cancel{4}$$

Q3) Prime factorizatⁿ of 28! includes all the number from 1 to 28. So
List of prime factors less than 28 are
2, 3, 5, 7, 11, 13, 17, 19, 23

Hence the largest prime no. and less than 28 is 23

Q4)

$\frac{97!}{10^n}$

$\frac{97!}{2 \times 5} \Rightarrow \frac{97}{5^n} = \frac{97}{5} + \frac{97}{25}$

$= 19 + 3$

$= 22$

Q5) $\frac{54!}{12^n}$

$\frac{54!}{4 \times 3} \Rightarrow \frac{54!}{4^n} = \frac{54}{2} + \frac{54}{4} + \frac{54}{8} + \frac{54}{16} + \frac{54}{32}$

2 2

$= 27 + 13 + 6 + 3 + 1$

$= 50$

Hence $4 = 2 \times 2$

$\therefore 4^n = \frac{50}{2} = 25$

Q6)

$$\frac{60!}{2^x}$$

$$\Rightarrow \frac{60!}{2^x} = \frac{60}{2} + \frac{60}{4} + \frac{60}{8} + \frac{60}{16} + \frac{60}{32}$$

$$= 30 + 15 + 7 + 3 + 1$$

$$= 56$$

Q8)

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 5040$$

2	5040
2	2520
2	1260
2	630
3	315
3	105
5	35
7	7
	1

$$5040 = 2^4 \times 3^2 \times 5 \times 7$$

now,

$$\frac{50!}{2^n} = \frac{50}{2} + \frac{50}{4} + \frac{50}{8} + \frac{50}{16} + \frac{50}{32}$$

$$= 25 + 12 + 6 + 3 + 1$$

$$= 47$$

$$\frac{50!}{3^n} - 2^n = 2^4 = \frac{47}{4} = 11$$

$$\frac{50!}{3^n} = \frac{50}{3} + \frac{50}{9} + \frac{50}{27}$$

$$= 16 + 5 + 1$$

$$= 22$$

$$3^n = 3^2 = \frac{22}{2} = 11$$

$$\frac{50!}{5^n} = \frac{50}{5} + \frac{50}{25}$$

$$= 10 + 2$$

$$= 12$$

$$5^n = 5 = 12$$

$$\frac{50!}{7^n} = \frac{50}{7} + \frac{50}{49}$$

$$= 7 + 1$$

$$= 8$$

$$7^n = 7 = 8$$

Thus the highest power of 7! that divides 50! is 8 //

Qa 4)

$$\frac{625!}{5} = \frac{625}{5} + \frac{625}{25} + \frac{625}{125} + \frac{625}{625}$$

$$= 125 + 25 + 5 + 1$$

$$= 156$$

$$\frac{624!}{5} = \frac{624}{5} + \frac{624}{25} + \frac{624}{125}$$

$$= 124 + 24 + 04$$

$$= 152$$

difference in trailing zeros

$$= 156 - 152 = 4 //$$

Q10)

$10 = 2 \times 5$

$i=5, \; 5^5$ = 5 factors
$i=10, \; 10^{10}$ = 10 factors
$i=15, \; 15^{15}$ = 15 f
$i=20 \quad 20^{20} = 20$
$i=25 \quad 25^{25} = 50$
$i=30 \quad 30^{30} = 30$
$i=35 \quad 35^{35} = 35$
$i=40 \quad 40^{40} = 40$
$i=45 \quad 45^{45} = 45$
$i=50 \quad 50^{50} = 100$
$i=55 \quad 55^{55} = 55$
$i=60 \quad 60^{60} = 60$
$i=65 \quad 65^{65} = 65$
$i=70 \quad 70^{70} = 70$
$i=75 \quad 75^{\frac{75}{2}} = 150$
$i=80 \quad 80^{80} = 80$
$i=85 \quad 85^{85} = 85$
$i=90 \quad 90^{90} = 90$
$i=95 \quad 95^{95} = 95$
$i=100 \quad 100^{100} = 200$

$5 + 10 + 15 + 20 + 50 + 30 + 35 + 40 + 45 + 100 + 55 + 60 + 65 + 70 +$
$150 + 80 + 85 + 90 + 95 + 200 = 1{,}065$ //

HCF/LCM

Q1) 15, 25, 40 & 75

LCM = ~~5×3×5~~ 5×3×5×8
= 600

5	15, 25, 40, 75
3	3, 5, 8, 15
5	1, 5, 8, 5
	1, 1, 8, 1

greatest four digit no. = 9999

9999 ÷ 600 = 16 remainder 399

∴ To get the largest four digit no. divisible by 600

9999 − 399 = $\boxed{9600}$

So, the greatest four digit no. divisible by 15, 25, 40 & 75 is $\boxed{9600}$ //

Q2)

HCF = 11
LCM = 7700
1st no. = 275

∴ LCM × HCF = a × b

7700 × 11 = 275 × b

$b = \frac{7700 \times 11}{275} = 308$ //

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Q3) Let the bells be A, B, C, D, E, F they toll at interval 2, 4, 6, 8, 10, 12

LCM = 2×2×3×2×5
= 120

2	2, 4, 6, 8, 10, 12
2	1, 2, 3, 4, 5, 6
3	1, 1, 3, 2, 5, 3
	1, 1, 1, 2, 5, 1

Hence after 120 sec all the bell rang at once

~~1 2 mi~~ that means after ~~ee~~ every 2 min

~~e~~ So, in 30 min = $\frac{30}{2}$
= 15

Total tolls together = 15+1
= 16

Q4)

4665 − 1305 = 3360
6905 − 4665 = 2240
6905 − 1305 = 5600

HCF of

3360 = 2×2×2×2×2×3×5×7
2240 = 2×2×2×2×2×2×5×7
5600 = 2×2×2×2×2×5×5×7

HCF = 2×2×2×2×2×5×7
= 1120

The sum of the digit = 1+1+2+0 = 4

Q5)

$183 - 91 - 43 = 48$

$183 - 91 = 92$

$183 - 43 = 140$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$92 = 2 \times 2 \times 23$

$140 = 2 \times 2 \times 5 \times 7$

$HCF = 2 \times 2$

$= 4$

Q6)

$a \times b = 4107$

$HCF = 37$

$L \times H = a \times b$

$L \times 37 = 4107$

$L = \frac{4107}{37}$

$\cancel{\frac{4107}{37} \times 37} = \cancel{\frac{4107 \times b}{b}}$

Let the two no. be

$37a$ & $37b$

$37a \times 37b = 4107$

$1369 \times ab = 4107$

$ab = \frac{4107}{1369}$

$ab = 3$

So, the pairs will be $(3,1)$ & $(1,3)$

The numbers are:

$37a = 37 \times 1$
$= 37$

and

$37b = 37 \times 3$
$= 111$

Q7)

Let the three no. be $3x$, $4x$ & $5x$

$LCM = 3x \times 4x \times 5x$
$= 60x$

given LCM

$2400 = 60x$

$x = \cancel{2400}\ 40$

The no. are $3x = 120$
$4x = 160$
$5x = 200$

HCF of 120, 160 & 200

$120 = 2 \times 2 \times 2 \times 3 \times 5$
$160 = 2 \times 2 \times 2 \times 2 \times 2 \times 5$
$200 = 2 \times 2 \times 2 \times 5$

$HCF = 2 \times 2 \times 2 \times 5 = 40$

Q8)

1.08 , 0.36 , 0.9

$1.08 \times 100 = 108$

$0.36 \times 100 = 36$

$0.9 \times 100 = 90$

$$\begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 2 & 27 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 2 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$108 = 2 \times 2 \times 3 \times 3 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$90 = 2 \times 3 \times 3 \times 5$

~~HCF~~ GCD $= 2 \times 3 \times 3$

$= 18$

GCD $= \frac{18}{100} = 0.18$

Q9)

$ab = 2028$

HCF $= 13$

Let the no. be $13a$ & $13b$

$13a \times 13b = 2028$

$169ab = 2028$

$ab = \frac{2028}{169}$

$ab = 12$

The possible pairs are $(1, 12)$ ~~(12, 1)~~ $(3, 4)$

~~Q9)~~

Q10)

6, 9, 15 & 18

2	6, 9, 15, 18
3	3, 9, 15, 9
3	1, 3, 5, 3
	1, 1, 5, 1

LCM = 2 × 3 × 3 × 5
= 90

So, 90 + 4 = 94

94 is not divisible by 7
184 is not divisible by 7
274 is not divisible by 7
364 is divisible by 7

Hence the number is 364

Q11)

2	5, 6, 4, 3
3	5, 3, 2, 3
	5, 1, 2, 1

LCM = 60

2497 ÷ 60 gives remainder 37

The number should be added is (60 - 37 = 23)

∴ The no. is 23.

$\frac{90}{18}$

12)

LCM = 2 × 5 × 3 × 7 × 4
= 840

2	5, 6, 7, 8
	5, 3, 7, 4

~~840~~ 843 is not disible by 9
1683 is disidble by 9

∴ The no. is 1683 //

13)

A = 252
B = 308
C = 198

LCM = 2 × 2 × 3 × 3 × 7 × 11
= 2772

2772 ~~min~~ sec
46 min 12 sec //

14)

HCF = 23

LCM = HCF × p × q (where p & q are uncommon factors)

The 1st no. = 23 × 13 = 299
2nd no. = 23 × 14 = 322 //

15) 12, 18, 21 & 30

LCM = 2×3×2×3×7×5
= 1260

2	12, 18, 21, 30
3	6, 9, 21, 15
	2, 3, 7, 5

ie we need a no. that is double divisible by 1260

$\frac{1260}{2} = 630$ //

16) l = 3.78 m = 378 cm
b = 5.75 m = 575 cm

2	378
3	189
3	63
7	21
	1

3	575
5	175
5	35
7	7
	1

~~HCF~~ =
378 = 2×3×3×7
575 = 5×5×3×7
HCM = 3×7
= 21 cm

17) a, b, c
ab = 551
bc = 1073

2	550
5	275
5	55
11	11
	1

29	1073
31	37
	1

19	551
29	29
	1

HCF
550 = 19 × 29
1073 = 31 × 29
HCF = 29 //

$b = 29$

$a = \frac{551}{29} = 19$

$c = \frac{1073}{29} = 37$ //

The sum $= a + b + c$

$\Rightarrow 29 + 19 + 37 = 85$ //

Q18)

for, 1657 $\Rightarrow$ $(1657 - 6 = 1651)$

for 2037 $\Rightarrow$ $(2037 - 5 = 2032)$

$1651 = 13 \times 127$

$2032 = 2 \times 2 \times 2 \times 2 \times 127$

$HCF = 127$

13	1651
	127

2	2032
2	1016
2	508
2	254
	127

So, the greatest number $= 127$ //

Q19)

$LCM = 48$

The no. are in the ratio $2:3$

So, Let the no. be $2x, 3x$

LCM of $2x$ & $3x$ is $6x$

$6x = 48$

$x = 8$

The no. ~~be~~ will be $= 2x = 16$
The ~~...~~ 2^{nd} no. will be $= 3x = 24$

The sum $= 16+24$
$= 40$ //

20)

~~7~~ $7m = 700 cm$
$3m\ 85cm = 385 cm$
$12m\ 95cm = 1295 cm$

$$\begin{array}{r|l} 2 & 700 \\ \hline 2 & 350 \\ \hline 5 & 175 \\ \hline 3 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 5 & 385 \\ \hline 7 & 77 \\ \hline 11 & 11 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 5 & 1295 \\ \hline 7 & 259 \\ \hline 37 & 37 \\ \hline & 1 \end{array}$$

$700 = 2\times2\times5\times3\times7$
$385 = 5\times7\times11$
$1295 = 5\times7\times31$

HCF $= 35$ //

The greatest possible length is $\boxed{35 cm}$

21) Let x & y be the no.
$161 = 7\times23$

$$\begin{array}{r|l} 7 & 161 \\ \hline & 23 \end{array}$$

So, $x = 23$
$y = 7$

Putting the value of x & y in $3y - x$

$3y - x = 3\times7 - 23$
$= 21 - 23$
$= -2$ //

22)

HCF = 11

LCM = 385

$L \times H = ab$

$11 \times 385 = 11a \times 11b$

$ab = 35$

~~$11ab = 4235$~~

~~$ab = \frac{4235}{11}$~~

~~$ab = 385$~~

co-prime pairs

(1, 35) (5, 7)

The possible pairs

(11, 385) & (55, 77)

Since one number lies between 75 & 125

$\therefore$ 77

23)

$a+b = 55 \Rightarrow a = 55-b$

HCF = 5

LCM = 120

LCM $\times$ HCF = ab

$120 \times 5 = (55-b)b$

$600 = 55b - b^2$

$b^2 - 55b + 600 = 0$

$b^2 - 40b - 15b + 600 = 0$

$$b(b-40) - 15(b-40) = 0$$

$$(b-15)(b-40) = 0$$

$$b = 15, 40$$

if $b = 15$; $b = 40$

$a = 40$; $a = 15$

sum of reciprocals:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{15} + \frac{1}{40}$$

$$= \frac{40+15}{600}$$

$$= \frac{\cancel{55}\,11}{\cancel{600}\,120}$$

$$= \frac{11}{120}$$

24)

Pens = 1001

Pencils = 910

7	1001
11	143
13	13
	1

2	910
5	455
7	91
13	13
	1

$1001 = 7 \times 11 \times 13$

$910 = 2 \times 5 \times 7 \times 13$

HCF $= 13 \times 7$

$= 91$

Q.