Exploring Fractal Analysis as a Technique to Calculate Fractal Dimension of Human Face

Aditya Takkar, Anuj Kumar, Ashish Sharma, S.Sahu, Prabhat Munshi[1]

Indian Institute of Technology Jammu, India

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Abstract: Fractal Dimension analysis of square images(64 by 64) of human faces is a mathematical measure of the complexity and irregularity of its structure.

Introduction

Fractal dimension is a mathematical concept used to describe the complexity and irregularity of objects in nature and mathematics. It is a measure of how much space a fractal object occupies in a mathematical or physical space. Unlike traditional Euclidean geometry, where the dimension of an object is a whole number, fractal dimension can be a fractional number. This is because fractals possess self-similarity, which means that they have similar patterns at different scales. Therefore, their dimensionality is not limited to whole numbers.

Fractals are mathematical objects that are self-similar at different scales. This means that they have similar patterns that repeat themselves when you zoom in or out of the object. Examples of fractals include the Mandelbrot set, the Sierpinski triangle, and the Koch snowflake.

The fractal dimension is a measure of the degree of self-similarity of a fractal object. It is calculated using different methods, such as box counting, which involves dividing the fractal into smaller and smaller boxes and measuring how many boxes are needed to cover the fractal at different scales. The fractal dimension can be used to classify and compare different fractals based on their level of complexity.

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Fractals

Objects in the real world do not exhibit any regular geometrical shapes. Mandelbrot showed that this problem of absence of geometrical representation can be resolved by using fractals. Ever Since then, fractals have been extensively used for the study of various physical processes.Mandelbrot defines "fractal" as a set for which Hausdorff–Besicovitch dimension strictly exceeds the topological dimension.Following Chen et al.[2], we provide here another illustration of the definition of fractal dimension for an object X in an m-dimensional space. If m is an integer, and it is the minimum integer dimensional space among all possible integer dimensional space which can develop X, and $N(\epsilon)$ is the number of m-dimensional spheres of diameter ε needed to cover X, then

$$N(\in) \propto \left(\frac{1}{\in}\right)^{D}$$

where D is fractal dimension.

$$D = 3 - H$$

where H is Hurst coefficient.

1. Estimation of fractal dimension

For a $M \times M$ pixel block image, let $I(x, y)$ be the pixel intensity and $x, y = 0, 1, 2 \ldots\ldots M-1$. The following items are defined:

(1) Scale: distance Δr between two pixels. The unit of distance is a pixel.

(2) NSR = normalized scale range vector. It consists of reference integer scale and is monotonic increasing. $NSR = [ndr(1), ndr(2), \ldots, ndr(k) \ \ldots ndr(n)]$ where k is an integer, $ndr(k) = k$ and the n value depends on practical considerations. In general, for an $M \times M$ image, we can select it to be ,then $NSR = [1, 2, 3, 4 \ldots, M]$.

[2] FD=3-H

(3) NPN: normalized pixel pair number vector. It consists of elements which are the numbers of pixel pairs with scale (distance) values whose integer parts are the same integer reference scale.

$$NPN = \left[npn(1), npn(2), \ldots, npn(k), \ldots, npn(n) \right]$$

where $npn(k)$ is the total number of pixel pairs with distance Δr and k <Δr< k + 1.

(4) $NMSID$: normalized multi-scale intensity difference vector. It consists of different absolute intensity difference averages around each normalized reference scale (NSR).

$$NMSID = \left[ndi(1), ndi(2), \ldots ndi(k), \ldots, ndi(n) \right]$$

where $ndi(k)$ is the average of absolute intensity difference of all pixel pairs with distance values whose integer parts are $ndr(k)$.

Thus

$$ndi(k) = \frac{\sum_{x1=0}^{M-1} \sum_{y1=0}^{M-1} \sum_{x2=0}^{M-1} \sum_{y2=0}^{M-1} \left| I(x2, y2) - I(x1, y1) \right|}{npn(k)}$$

where x1, y1, x2 and y2 must satisfy

$$ndr(k) \leq \sqrt{(x2 - x1)^2 + (y2 - y1)^2} < ndr(k + 1)$$

or

$$k \leq \sqrt{(x2 - x1)^2 + (y2 - y1)^2} < (k + 1)$$

ln($NMSID$) vs. ln(NSR) graph is plotted. A least square linear regression provides the slope H of the resultant curve. Fractal Dimension (FD) is given by the relation:

$$FD = 3 - H$$

A true mathematical fractal exhibits fractal behavior (self-similar-ity) over all ranges of scale. However, real objects, physical processes and images cannot be true mathematical fractals because they do not exhibit self-similarity over all ranges of scales. Moreover, a certain degree of inherent randomness is always associated with them.Therefore, when we evaluate the fractal dimension of an image we have to consider a certain range of scales. Upper limit of this range is decided by the overall size of the image and the lower limit is fixed by the pixel size.

Fractal dimension for Sierpinski Carpet

The Sierpinski Carpet is a well-known fractal pattern that is named after Polish mathematician Waclaw Sierpinski. It is a self-similar pattern that can be constructed by starting with a square and repeatedly removing the middle third of each side, resulting in a pattern that becomes more and more intricate as the process is repeated.

The fractal dimension of the Sierpinski Carpet is a measure of its complexity, and can be calculated using a formula known as the Hausdorff dimension. The Hausdorff dimension is a way of measuring the "fractional dimension" of an object, and is defined as follows:

$$D = \frac{\log(N)}{\log\left(\frac{1}{R}\right)}$$

where D is the Hausdorff dimension, N is the number of smaller copies of the object that are required to cover it at a given scale, and r is the scaling factor used to generate the smaller copies.

For the Sierpinski Carpet, the scaling factor is 3, since each side is divided into 3 equal parts, and N is also 8, since each iteration creates 8 smaller copies of the pattern. Therefore, we can calculate the Hausdorff dimension as follows:

D = log(8) / log(1/3) = 1.8928

This means that the Sierpinski Carpet has a Hausdorff dimension of approximately 1.89, which is a non-integer dimension that reflects its self-similar and highly intricate nature.

The fractal dimension of the Sierpinski Carpet has important implications in various areas of science and mathematics, including chaos theory, dynamical systems, and fractal geometry. It is also a popular subject of study and artistic exploration, and has inspired many other fractal patterns and structures.

Graphs of Lg(NMSID) vs Lg(NSR)

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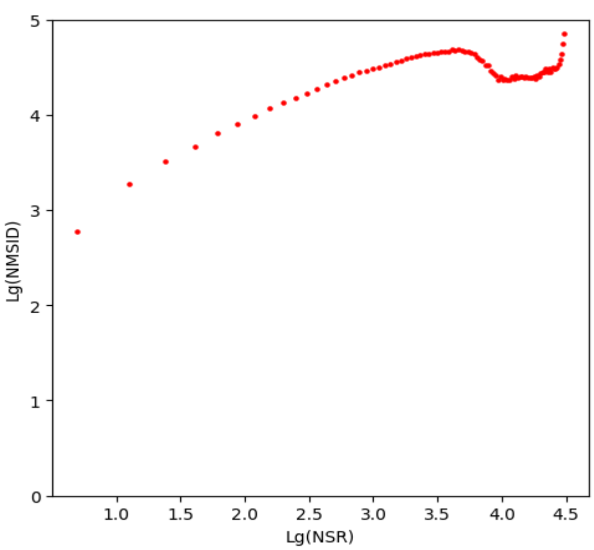
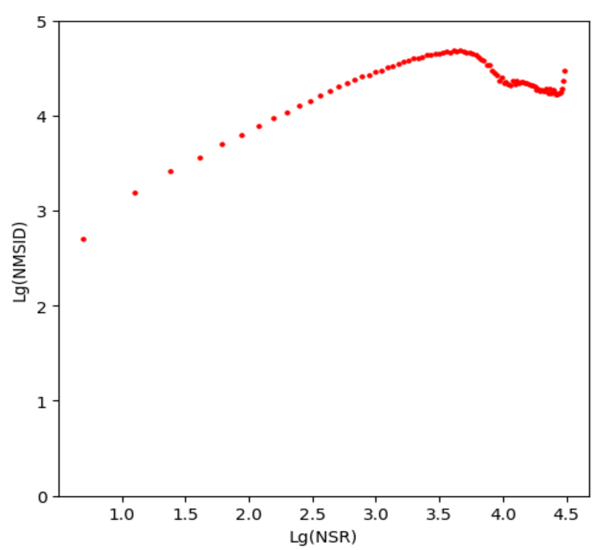
The Fractal Dimension of these faces were calculated using the Fractal Dimension Analysis Algorithm . The Code produces these Graphs which upon further analysis give us the Fractal Dimension of these faces.

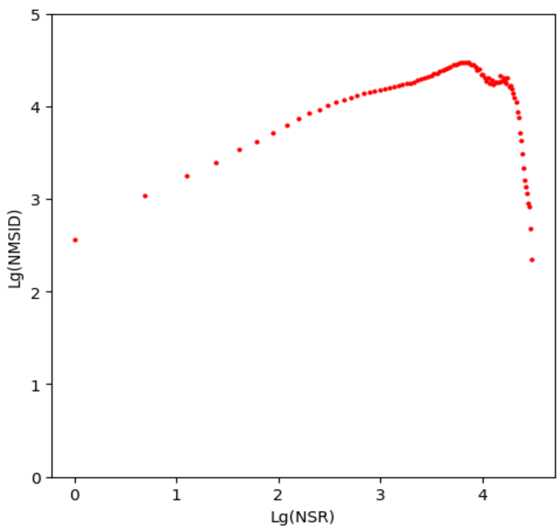
H

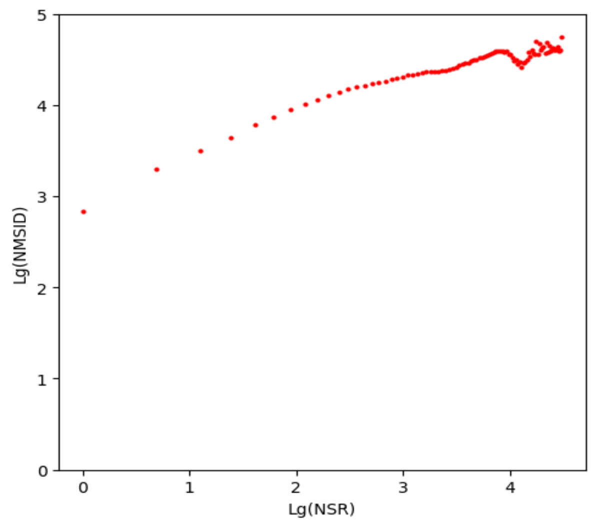
H

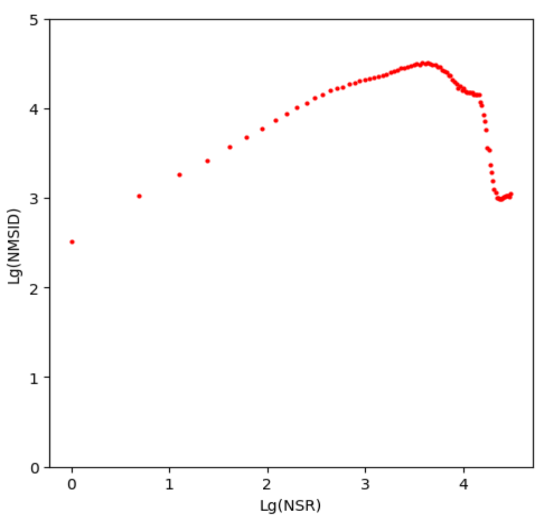
h

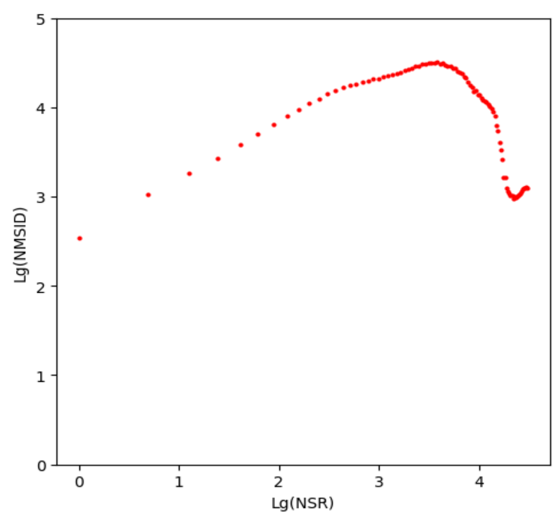





Lg(NMSID)
Lg(NSR)


Lg(NMSID)
Lg(NSR)


Lg(NMSID)
Lg(NSR)


Lg(NMSID)
Lg(NSR)