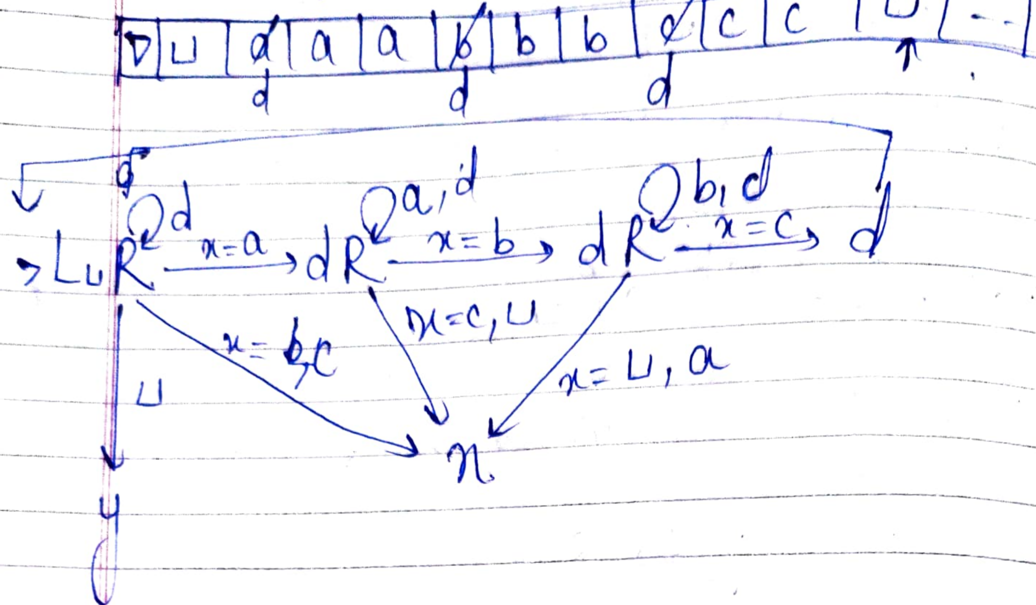
4 (a) T/M for $a^n b^n c^n$.



(b) Universal Turing Machine

It is like single TM that has a solution to all problems that is computable.
It contains TM description as input along with input string, runs the TM on input and return as result.

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TOC-1

1 a $(S^+)^* = (S^*)^+$

S^* = all concatenation of words S, excluding λ

$(S^+)^*$ (λ including) = S^*

S^* = {all words including λ}

$(S^*)^+$ = all concatentations of words in S^* but not λ

S^* already contain λ, so $(S^*)^+$ contain λ too,

No new words are added with + operator.

$\therefore (S^+)^* = (S^*)^+$

b CFG $a^* b^*$.

~~S → aS | bS | λ~~

S → aSb

S → λ

~~S → aXb~~

~~X → aX | bX | λ~~

c $(a+b)^* (ab+ba) + a + b + λ$

d DFA

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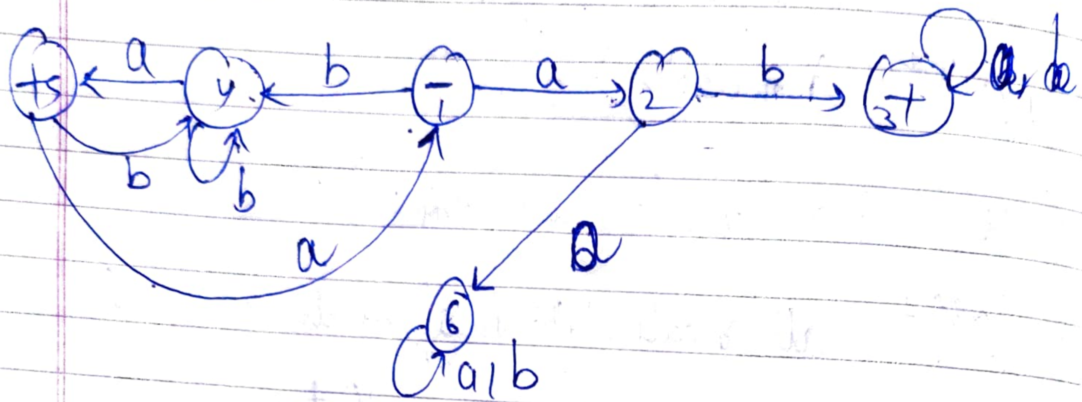
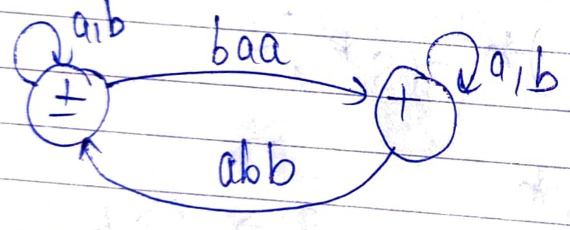
(e) either starts with ab or ends with ba

$r = ab(a+b)^* + (a+b)^* ba$

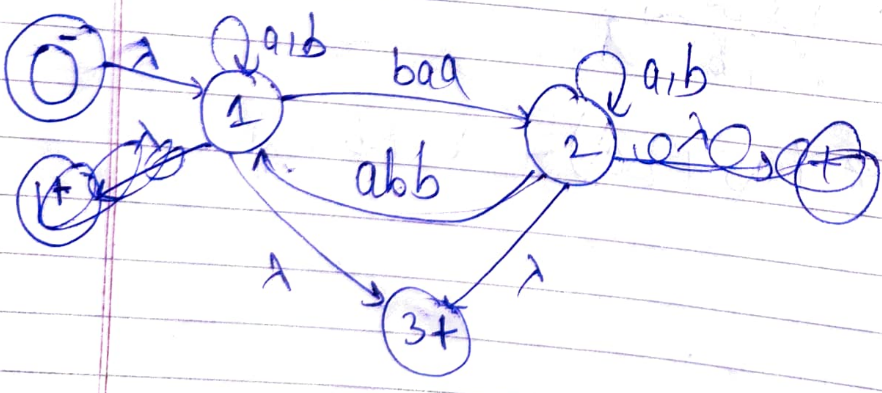


(f) >L L and >L $\xrightarrow{\sqcup}$ R

(g)



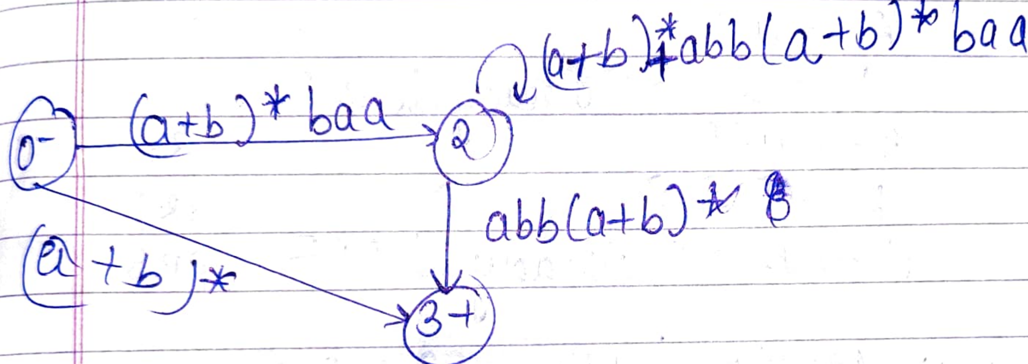
(i) non-enterable inital & non exitable final state



eliminate & bypass rule

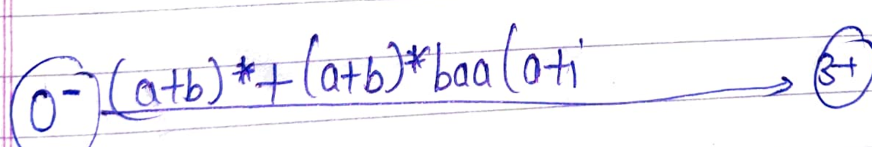
state 1

I	O	label
0	2	$(a+b)^*baa$
2	3	$abb(a+b)^*$ ~~baa~~
0	3	$(a+b)^*$
2	2	~~$(a+b)^*baa$~~ $abb(a+b)^*baa$.



state 2

I	O	
0	3	$(a+b)^*baa\left((a+b)^*+abb(a+b)^*baa\right)^*abb(a+b)^*$



$r = (a+b)^* + (a+b)^*baa.(a+b)^*abb\left((a+b)^*(\lambda+abb.baa)\right)$.

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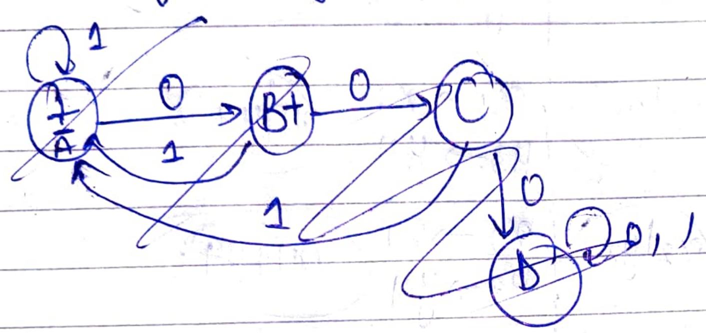
(i) $L = \{a^n b a^{n+1}$ where $n = 1, 2, 3 \ldots\}$.

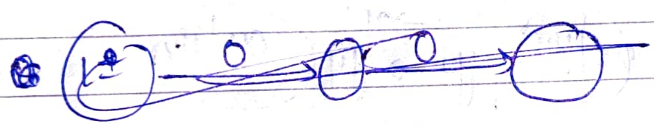
$a^4 b a^5 \in L$

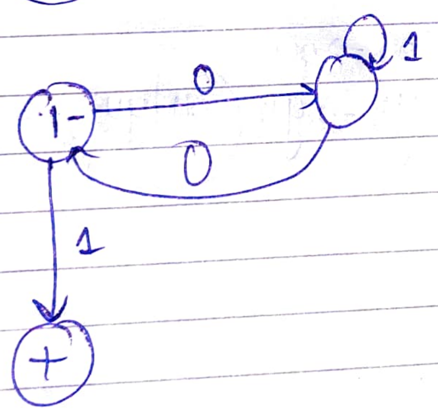
pump y one more time

$a^4 b^2 a^5 \notin L$.

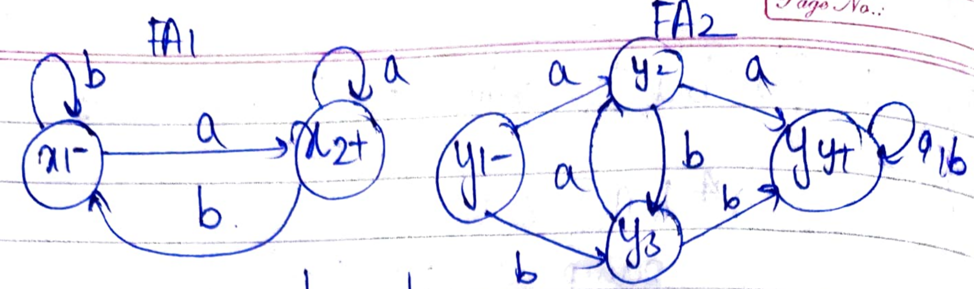
(j) DFA for every 00 followed by 1





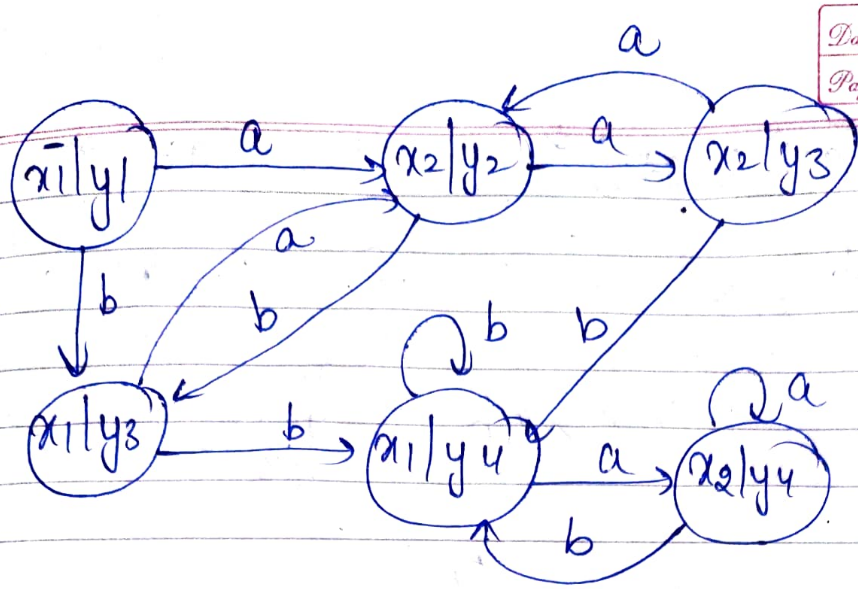


Q2.(a)

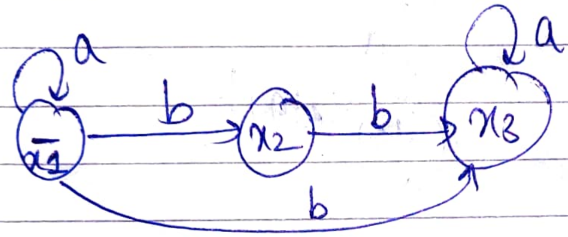


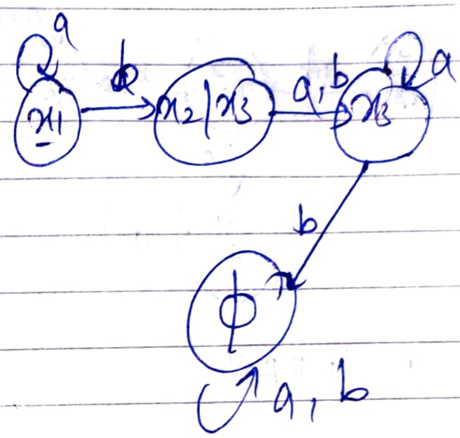
State	a	b
x_1/y_1	$x_1 \to x_2$ $y_1 \to y_2$ x_2/y_2	$x_1 \to x_1$ $y_1 \to y_3$ x_1/y_3
x_2/y_2	$x_2 \to x_2$ $y_2 \to y_3$ x_2/y_3	$x_2 \to x_1$ $y_2 \to y_3$ x_1/y_3
x_1/y_3	$x_1 \to x_2$ $y_3 \to y_2$ x_2/y_2	$x_1 \to x_1$ $y_3 \to y_4$ x_1/y_4
x_2/y_3	$x_2 \to x_2$ $y_3 \to y_2$ x_2/y_2	$x_2 \to x_1$ $y_3 \to y_4$ x_1/y_4
x_1/y_4	$x_1 \to x_2$ $y_4 \to y_4$ x_2/y_4	$x_1 \to x_1$ $y_4 \to y_4$ x_1/y_4
x_2/y_4	$x_2 \to x_2$ $y_4 \to y_4$ x_2/y_4	$x_2 \to x_1$ $y_4 \to y_4$ x_1/y_4

Q9,b



(b) NFA → DFA

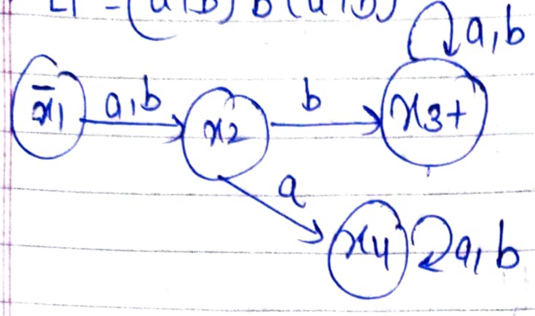


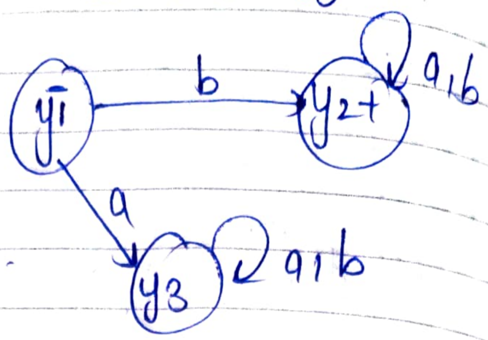


Now

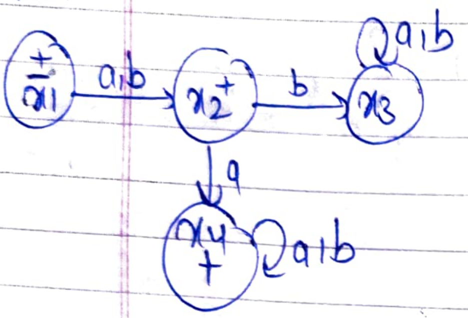
	a	b
x_1	x_1	$x_2\|x_3$
$x_2\|x_3$	$x_2 \to \phi$ $x_3 \to x_3$ x_3	$x_2 \to x_3$ $x_3 \to \phi$ x_3
x_3	x_3	ϕ
ϕ	ϕ	ϕ

Q3 (a) $L_1 = (a+b)\,b\,(a+b)^*$ $\qquad$ $L_2 = b\,(a+b)^*$

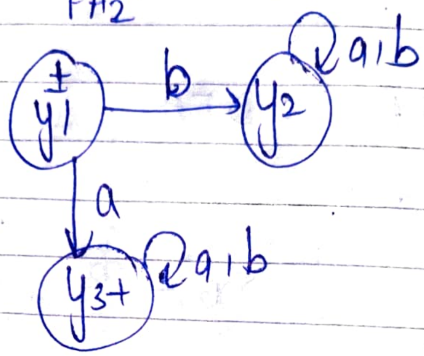




FA_1'

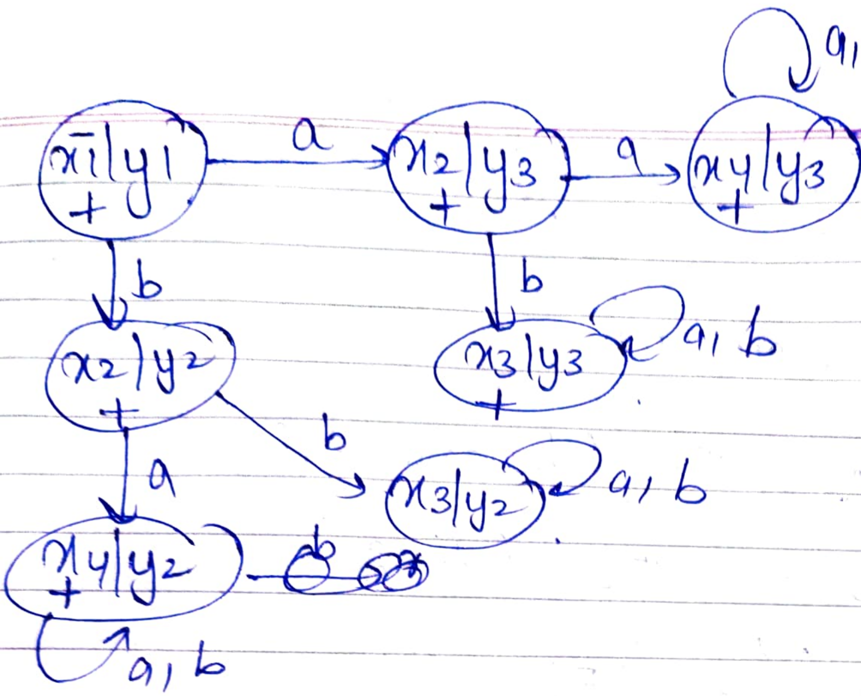


FA_2'

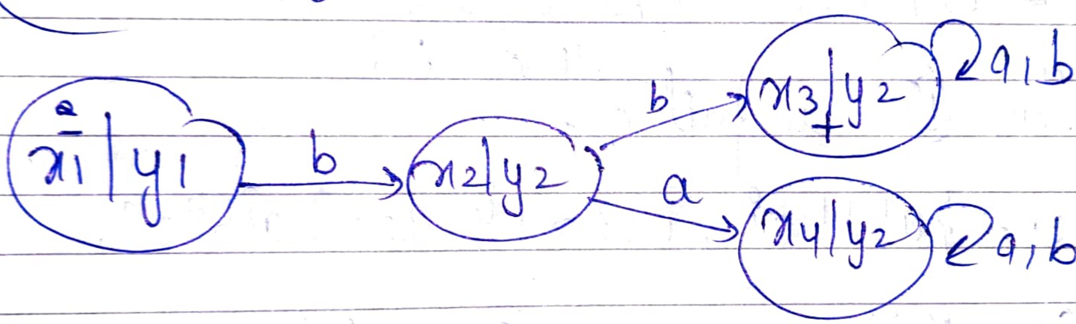


$FA_1' + FA_2'$

States	a	b
x_1^-/y_1	$x_1 \to x_2$, $y_1 \to y_3$ $\quad x_2/y_3$	$x_1 \to x_2$, $y_1 \to y_2$ $\quad x_2/y_2$
x_2/y_3	$x_2 \to x_4$, $y_3 \to y_3$ $\quad x_4/y_3$	$x_2 \to x_3$, $y_3 \to y_3$ $\quad x_3/y_3$
x_2/y_2	$x_2 \to x_4$, $y_2 \to y_2$ $\quad x_4/y_2$	$x_2 \to x_3$, $y_2 \to y_2$ $\quad x_3/y_2$
x_4/y_3	$x_4 \to x_4$, $y_3 \to y_3$ $\quad x_4/y_3$	x_4/y_3
x_3/y_3	x_3/y_3	x_3/y_3
x_4/y_2	x_4/y_2	x_4/y_2
x_3/y_2	x_3/y_2	x_3/y_2



$(FA_1' + FA_2')'$



$r = bb(a+b)^*$

$a^n b^n a^n b^n a^n$

$a^4 b^4 a^4 b^4 a^4$

$\underbrace{a}_{u}\underbrace{a}_{v}\underbrace{a}_{w}\underbrace{a}_{x}\underbrace{b}_{y}$

$a^6 b^4 a^4 b^4 a^4 \notin L$.

Non-context free

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Q4(a) Let us assume that L_1 & L_2 are closed under complement. —— (1)

If L_1 & L_2 are CFL

L_1', L_2' — CFL (by assumption (1))

$L_1' + L_2'$ —— CFL (union)

$(L_1' + L_2')'$ — CFL ((1))

$= L_1 \cap L_2$ (As CFLs are not closed under $\cap$]

Thus (1) is failed.

eg. $L_1 = \{a^n b^n c^i, \; n, i \geq 1\}$ $L_2 = \{a^i b^n c^n, \; n, i \geq 1\}$

$S_1 \rightarrow X_1 Y_1$
$X_1 \rightarrow a X_1 b \mid ab$
$Y_1 \rightarrow c Y_1 \mid c$

$S_2 \rightarrow X_2 Y_2$
$X_2 \rightarrow a X_2 \mid a$
$Y_2 \rightarrow b Y_2 c \mid bc$

$L_1 \cap L_2 = \{a^n b^n c^n\} \neq$ CFL

So, CFL are not in intersection as well as in complementation.

λaaX $\quad$ λaaX

$aXabX$ $\quad$ $\lambda aabX$

aab

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$S \to XaX$ $\quad$ PR1

$X \to aX \mid bX \mid \lambda$ $\quad$ PR2

let

$S \Rightarrow XaX$

$\Rightarrow \lambda aaX$

$\Rightarrow \lambda aabX$

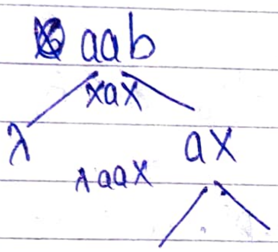
$\Rightarrow aab$

$S \Rightarrow XaX$

$\Rightarrow XabX$

$\Rightarrow aXabX$

$\Rightarrow aab$



(a)

(b) Halting Problem → Problem where we don't know when m/c will stop or not.

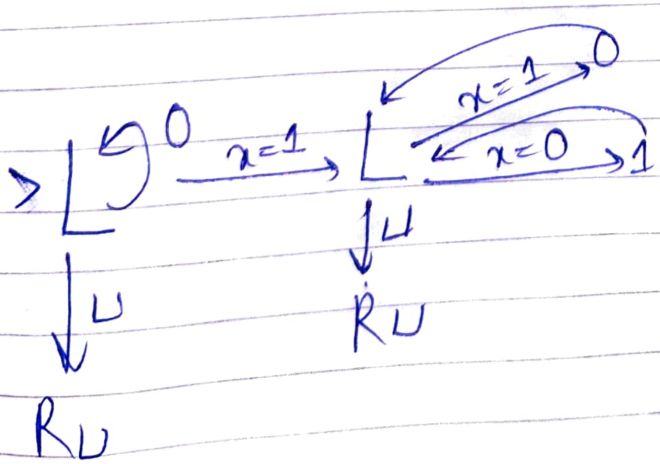
(c).

$\triangleright \sqcup w \underline{\sqcup} \; \vdash^*_M \; \triangleright \sqcup - w \sqcup$

▷	⊔	1	0	1	0	0	⊔	⊔

$\vdash^*_M$

▷	⊔	0	1	1	0	0	⊔	⊔



Q7 (a) (i) $(a+b)^* ab (a+b)^*$

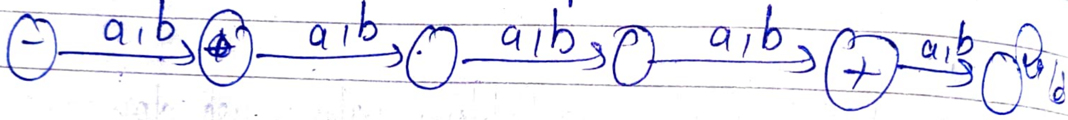
$L = \{ab, aab, bab, aab, \ldots\}$

all word having ab somewhere

(ii) $(a(a+bb)^*)^*$

$\{L = \lambda, a, aa, aaa, abb, \ldots\}$

words starting with a followed by ~~even no. of b's and~~ b's in clumps. including λ.

(b) 

(c) Start → Read 1 → Read2 → Pop

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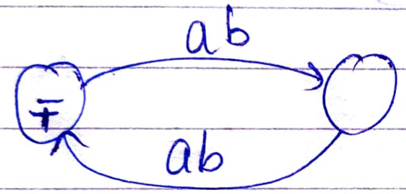
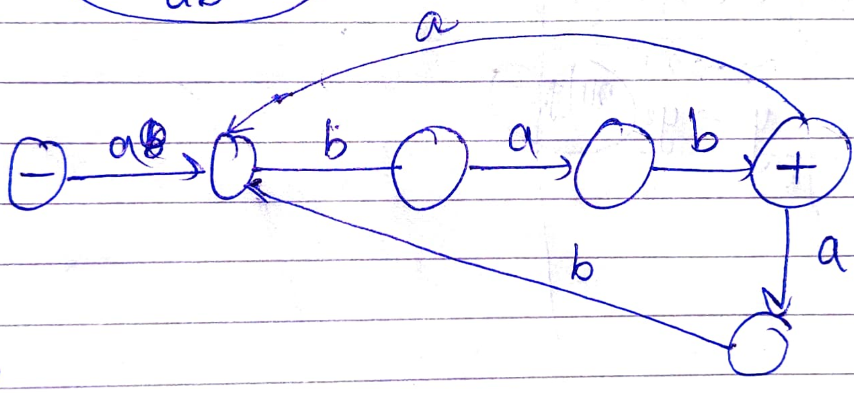
(a) $S^* = \{ab, ba, abab, baba, abba, baab \ldots\}$

(b) a/o.

(iii) λ

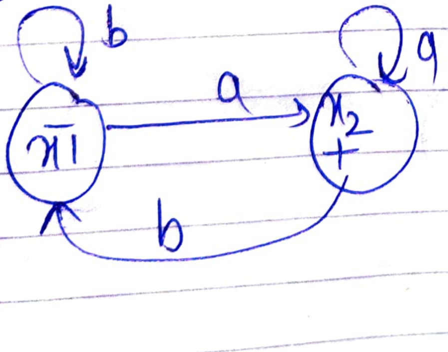
~~$b^*ab^*ab^*$~~ $(a+\lambda)(b+ba)^*$.



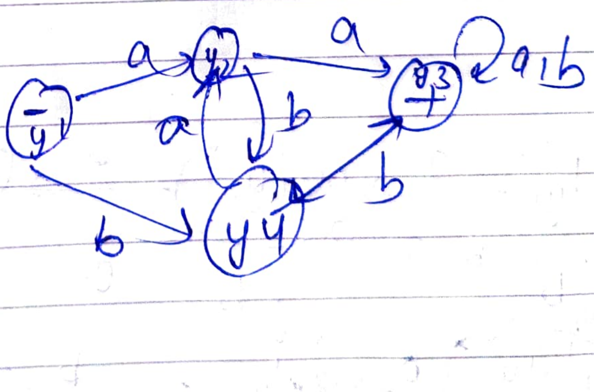


FA1

(b) FA1



FA2



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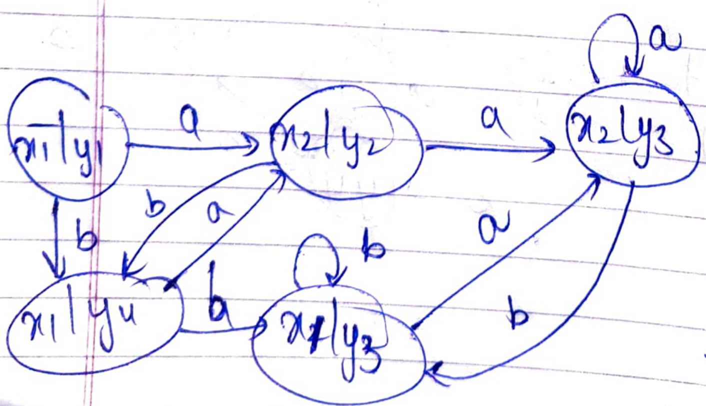
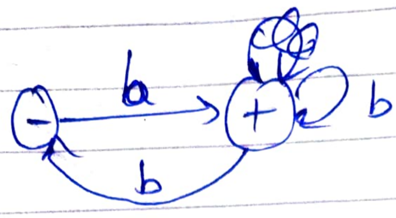
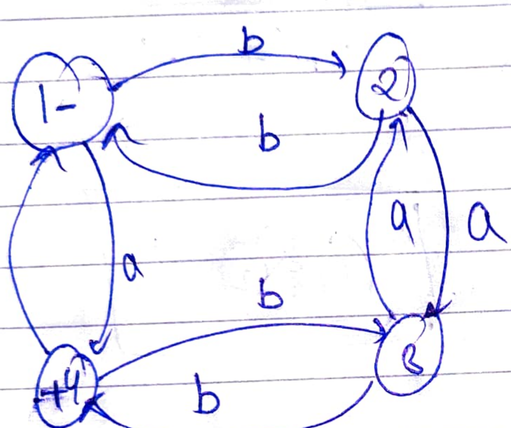
$FA_1 + FA_2$

	a	b
$x_1\|y_1$	$x_1 \to x_2$ $y_1 \to y_2$ $x_2\|y_2$	$x_1 \to x_1$ $y_1 \to y_4$ $x_1\|y_4$
$x_1\|y_4$	$x_1 \to x_2$ $y_4 \to y_2$ $x_2\|y_2$	$x_1 \to x_1$ $y_4 \to y_3$ $x_1\|y_3$
$x_2\|y_2$	$x_2 \to x_2$ $y_2 \to y_3$ $x_2\|y_3$	$x_2 \to x_1$ $y_2 \to y_4$ $x_1\|y_4$
$x_1\|y_3$	$x_1 \to x_2$ $y_3 \to y_3$ $x_2\|y_3$	$x_1 \to x_1$ $y_3 \to y_3$ $x_1\|y_3$
$x_2\|y_3$	$x_2 \to x_2$ $y_3 \to y_3$	$x_2 \to x_1$ $y_3 \to y_3$ $x_1\|y_3$



(c) $r = a(aa)^* + (bb)^*$





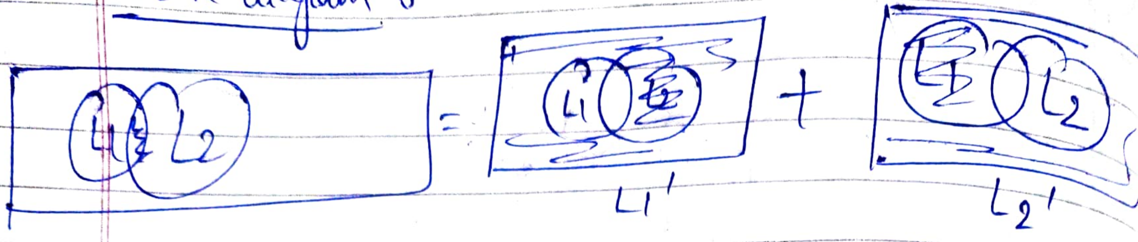
Q3 (a) If L_1 & L_2 are R.L, then $L_1 \cap L_2$ is a R.L.

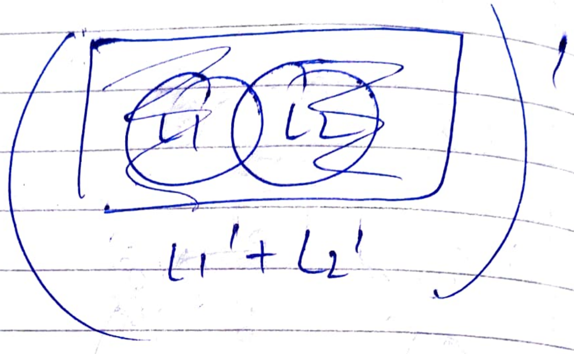
In other words, Reg lg closed under $\cap$.

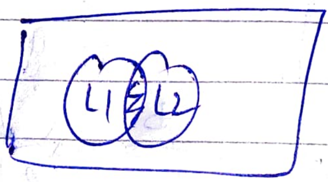
<u>By demorgan's law</u>

$$L_1 \cap L_2 = (L_1' + L_2')'$$

Venn diagram :







As, L_1 & L_2 are reg.

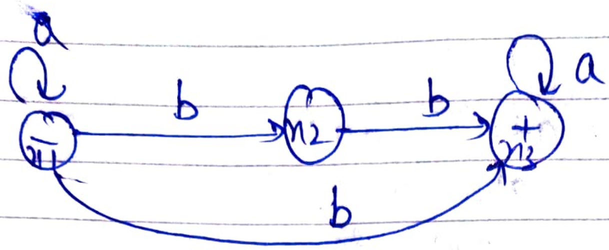
So, L_1' & L_2' are also reg.

$L_1' + L_2'$ are also reg. (closed under union)

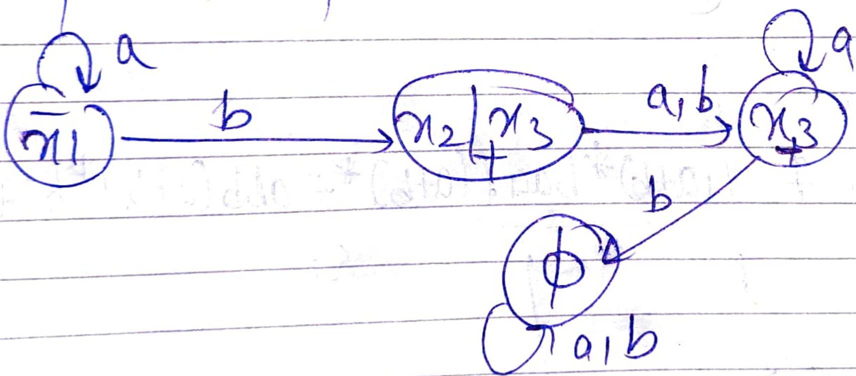
$(L_1' + L_2')'$ is also reg. (closed under comp)

So, $L_1 \cap L_2$ is reg. lg.

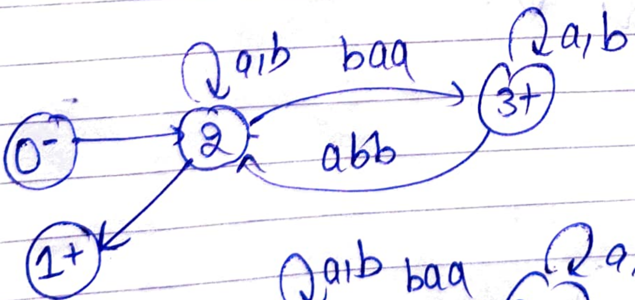
4 (a) NFA → DFA

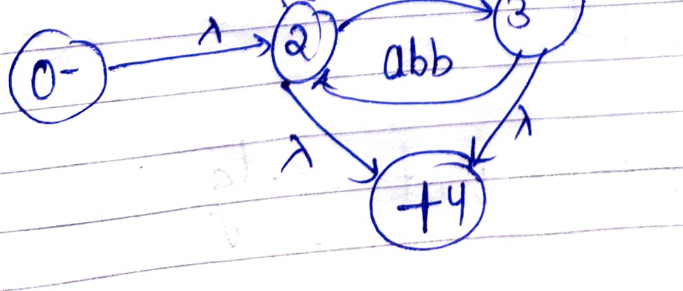


State	a	b
x_1	x_1	$x_2 \| x_3$
$x_2 \| x_3$	x_3	x_3
x_3	x_3	ϕ
ϕ	ϕ	ϕ



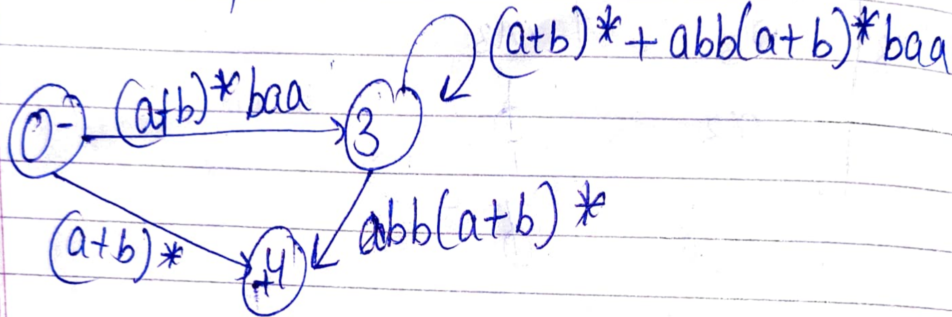
Q5





state 2

I	O	label
0	3	$(a+b)^*baa$
3	4	$abb(a+b)^*$
0	4	$(a+b)^*$
3	3	$abb(a+b)^*baa$



State 3

0	4	$\underbrace{(a+b)^*baa\left((a+b)^*+abb(a+b)^*baa\right)^* abb(a+b)^*}_{r_1}$

$r = (a+b)^* + r_1$

(b) $a^n b^{2n}$

$a^4 b^8$

$\underset{x}{a}\ \underset{y}{a}\ \underset{z}{aa}\ b^8$

$a^5 b^8 \notin L$ not reg. lg.

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6 (a) $S \rightarrow XYX$

$X \rightarrow aX \mid bX \mid \lambda$

$Y \rightarrow bbb$

$L = \{bbb, abbb, bbbb \dots\}$

$(a+b)^* bbb (a+b)^*$

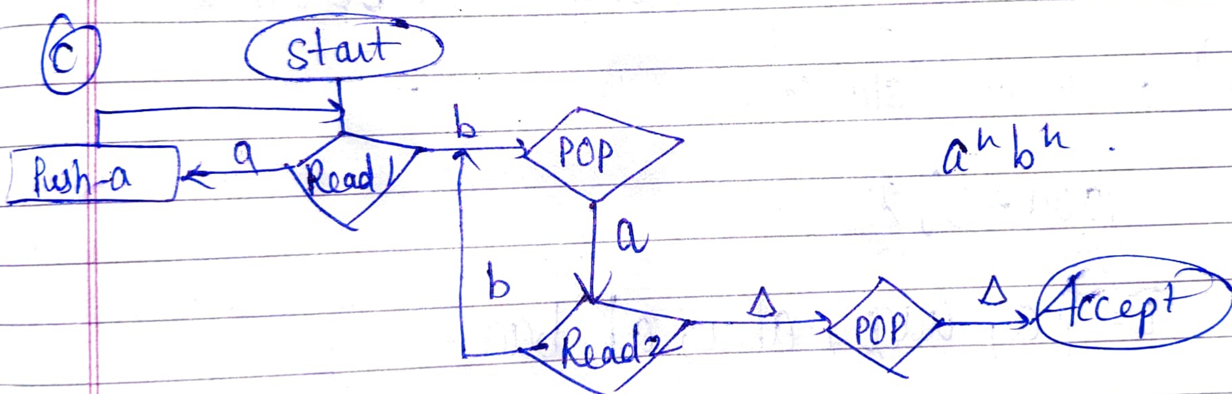
(b) all non-palindromes.

$L = \{ab, abaabaab, ababaaaba \dots\}$

$S \rightarrow aXb \mid bXa \mid aSa \mid aSb$

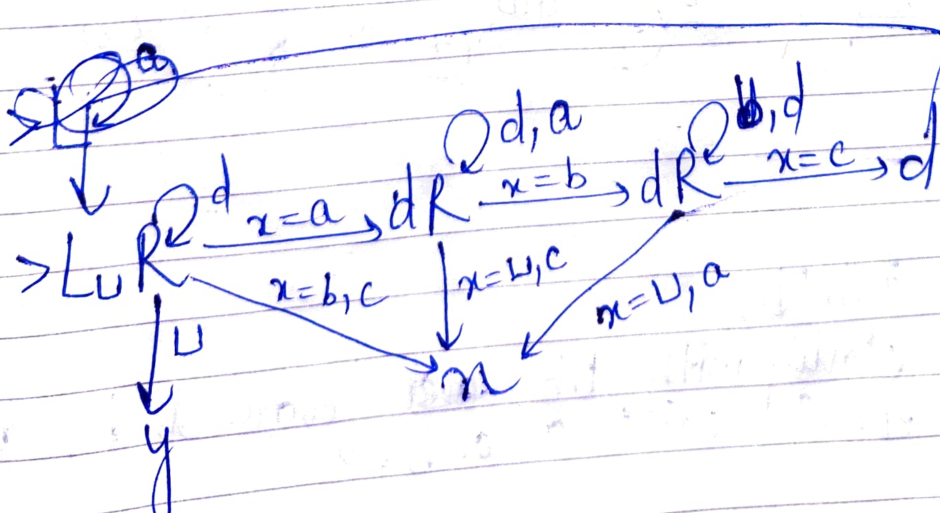
$X \rightarrow aX \mid bX \mid \lambda$

(c)



$a^n b^n$.

8



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1) (a) S is a subset of T
∴ $S^* = T^*$

(b) DFA

(c) $S \to XaXaX$
$X \to bX \mid aX \mid \lambda$

L = { aa, aaa, aab, aba, baa, aaaa, abab,

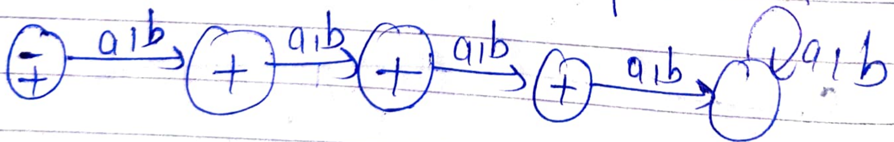
all words having aa somewhere ~~or all words having~~

(d) dead-end state.

(e) $(aaa+b)^*$

(f) L = { a, aaa, aba, ab baa - - }

odd words ~~having~~ starting with a

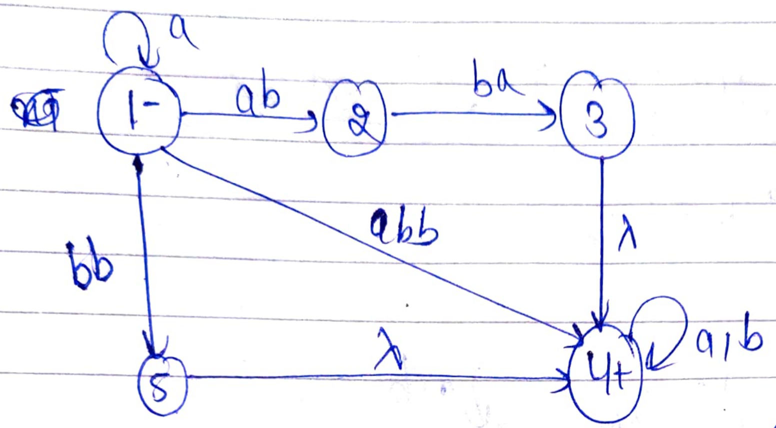
(g) 

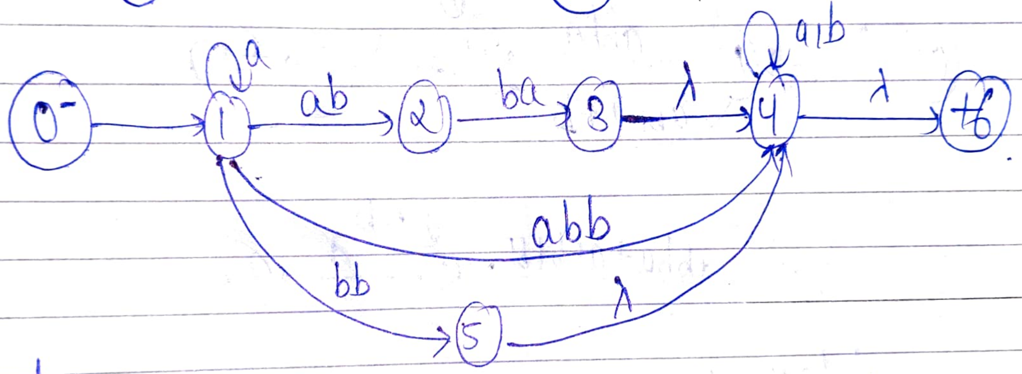
(h)

(i) (a) $bba^*(a+b)$
words starting with bb ~~and~~ followed by a and ending with a or b.

(b) $((a+b)a)^*$

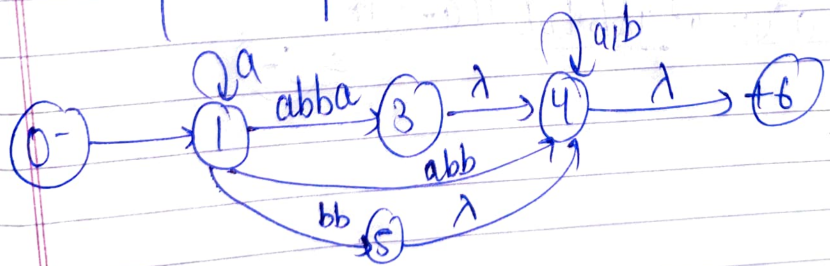
words starting with a or b and ending with a including λ.





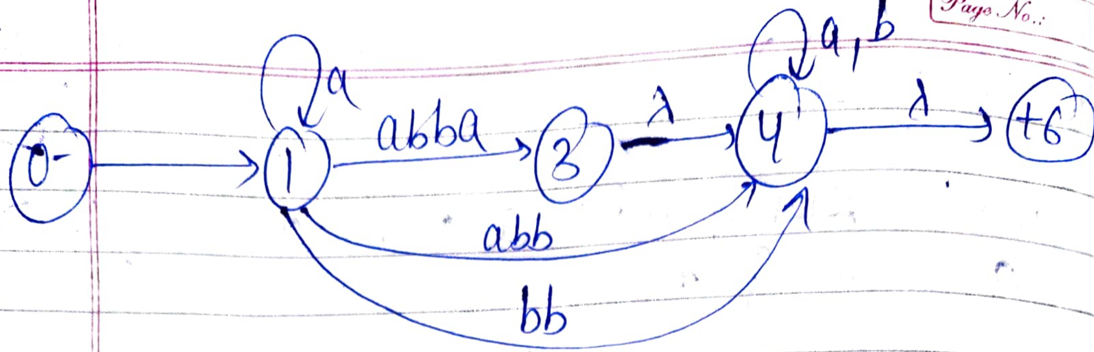
Now,
state 2

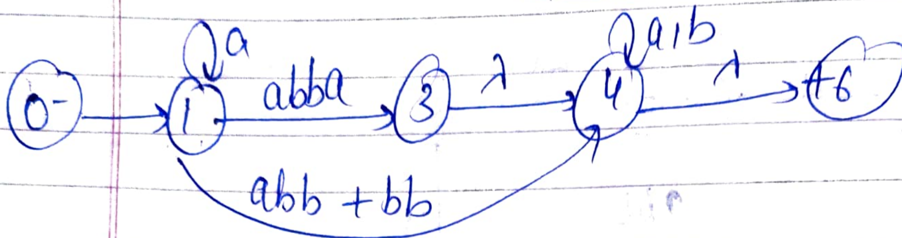
I	O	
1	3	ab.ba



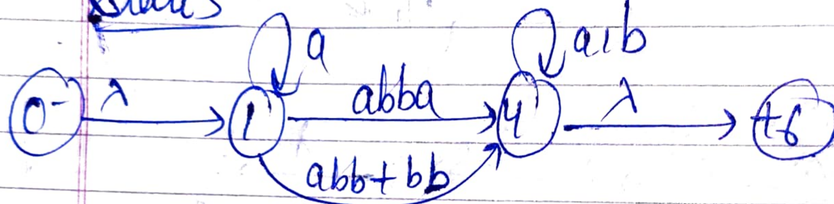
states

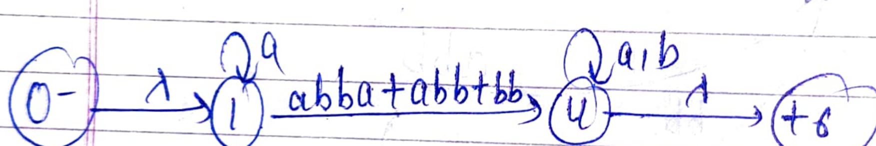
I	O	lable
1	4	bb



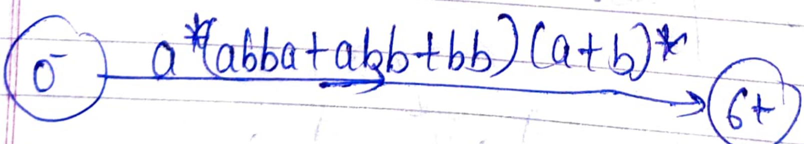


state 3

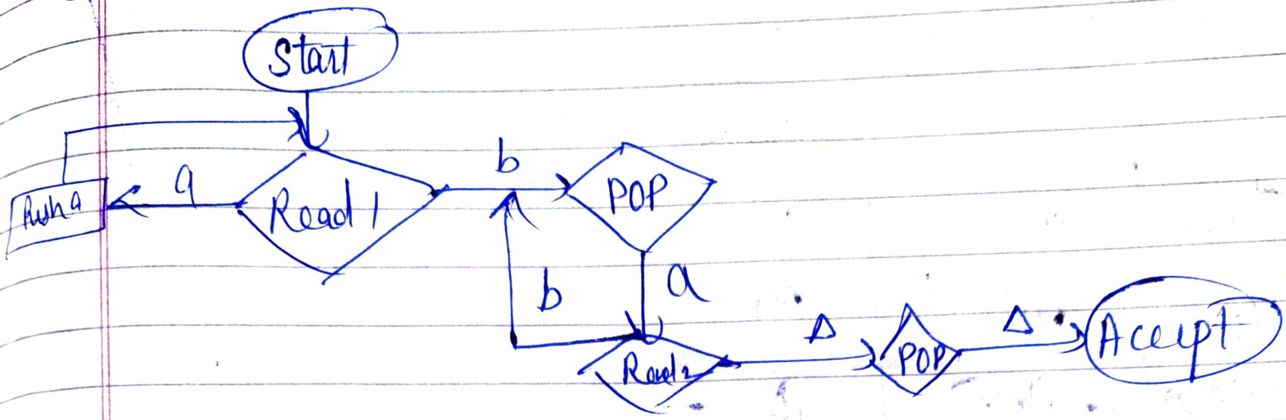




state 4, 1



(*) $a^n b^n$



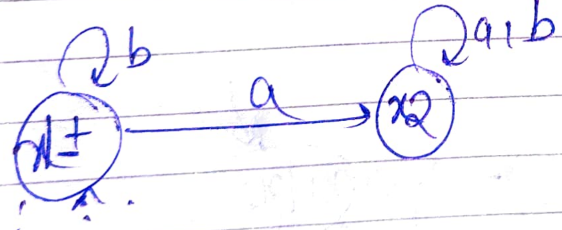
(*) $a^n b^n$

$\underbrace{aa}_{x}\underbrace{aa}_{y}\underbrace{bbbb}_{z}$

aaaaaabbbb, $a^6 a^4 \notin L$

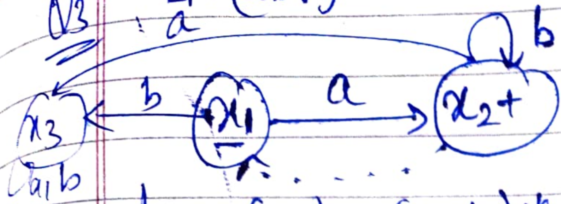
Section B

(2) (a)



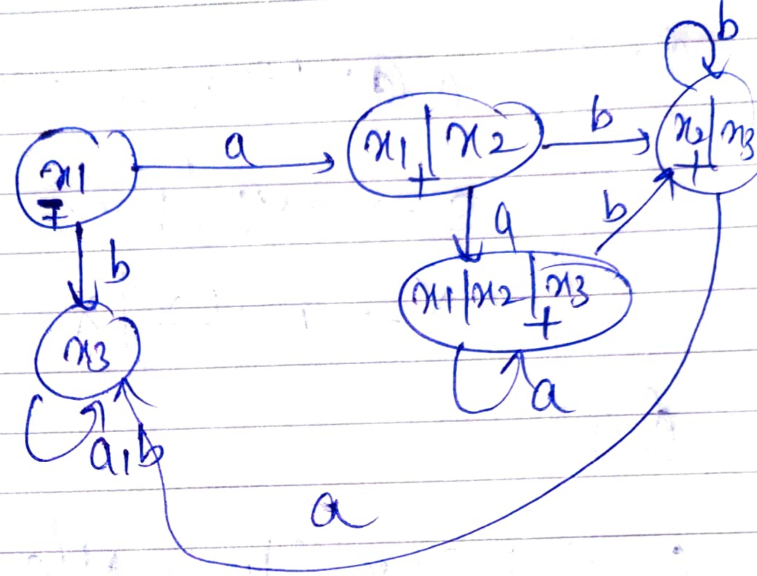
	a	b
x_1	x_2	x_1
x_2	x_2	x_2

Q3 $L_1 : (ab^*)^*$

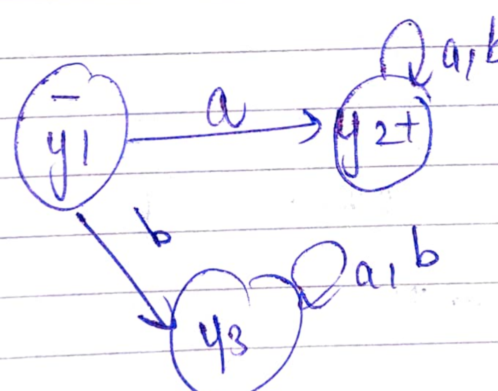


for $(L_1) = (ab^*)^*$

state	a	b
x_1	x_2/x_1	x_3
x_3	x_3	x_3
x_1/x_2	$x_1 \to x_1/x_2$ $x_2 \to x_3$ $x_1/x_2/x_3$	$x_1 \to x_3$ $x_2 \to x_2$ x_2/x_3
$x_1/x_2/x_3$	$x_1 \to x_1/x_2$ $x_2 \to x_3$ $x_3 \to x_3$	$x_1 \to x_3$ $x_2 \to x_2$ $x_3 \to x_3$
x_2/x_3	$x_2 \to x_3$ $x_3 \to x_3$	$x_2 \to x_2$ $x_3 \to x_3$



$L_2 = a(a+b)^*$



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Q4. (a) $S \to AA$
$A \to AAA$
$A \to bA \mid Ab \mid a$

aa
aaa
baa
abaa

strings ending with aa.

(b) $S \to aSb$
$S \to \lambda$

$S \Rightarrow aSb$ (PR1)
$\Rightarrow aaSbb$ (PR1)
$\Rightarrow a^n S b^n$ (applying PR1 no. of times)
$= a^n b^n$ (PR2)

$CFL(S) = \{a^n b^n \mid n = 0, 1, 2 \ldots\}$

$a^* b^*$

(5) $a^n b^m a^m b^n$ $\quad m, n \geq 1$