TOC paper 2013

1. (a) $S = \{ab, bb\}$ $T = \{ab, bb, bbbb\}$

$S^* = \{ab, bb, abbb, abab \ldots\}$

$T^* = \{ab, bb, bbbb, abab \ldots\}$

So, the words in S^* is equal to the words in T^* and S is the subset of T.

Therefore $S^* = T^*$

(b) FA is the computing machine that accepts words in lg. It has finite set of states, one of which is designated as Initial state and some of which are designated as final state.

(c) $S \rightarrow XaXaX$
$X \rightarrow bX \mid aX \mid \lambda$

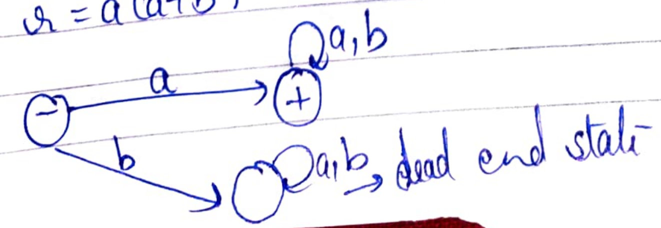
$L = \{aa, aaa, baa, aab, aba, aaaa, abab, \ldots\}$

This CFG generates a language having 'aa' somewhere.

$L_g = (a+b)^n a (a+b)^n a (a+b)^n$ where $n \geq 0$

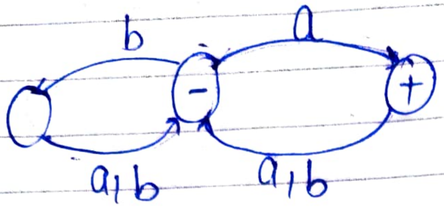
(d) Dead-end state → It is a rejecting state that is essentially a dead end. Once the machine enters a dead state, there is no way for it to reach an accepting state.

eg. $r = a(a+b)^*$



(e) $(aaa+b)^*$ or $(b+aaa)^*$ or $((aaa)^*b^*)^*$ or $(b^*(aaa)^*)^*$

(f)



$L = \{a, aaa, aba, aabaa,$

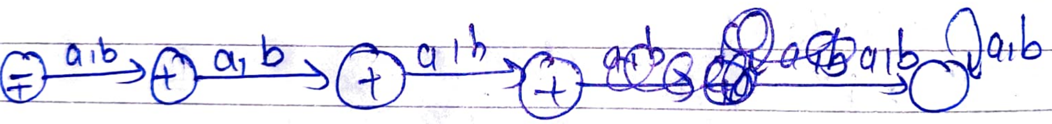
odd length words ending with a

$r = ((a+b)(a+b))^* a$

(g) words that have length fewer than four letters.

~~$r = (a+b)(a+b)(a+b)$~~

$r = \lambda + (a+b) + (a+b)(a+b) + (a+b)(a+b)(a+b) + \cancel{(a+b)(a+b)(a+b)(a+b)}$



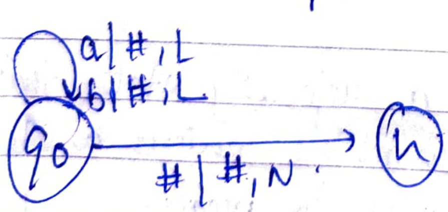
(h)

$q_0, a \rightarrow q_0 \#, L)$

$q_0, b \rightarrow q_0, \#, L$

$q_0, \# \rightarrow h, \#$

$h, \# \rightarrow$ accept



(f) (a) bba* (a+b)

L = { bba, bbb, bbaa, bbab, bbaaa, bbaab, ... }

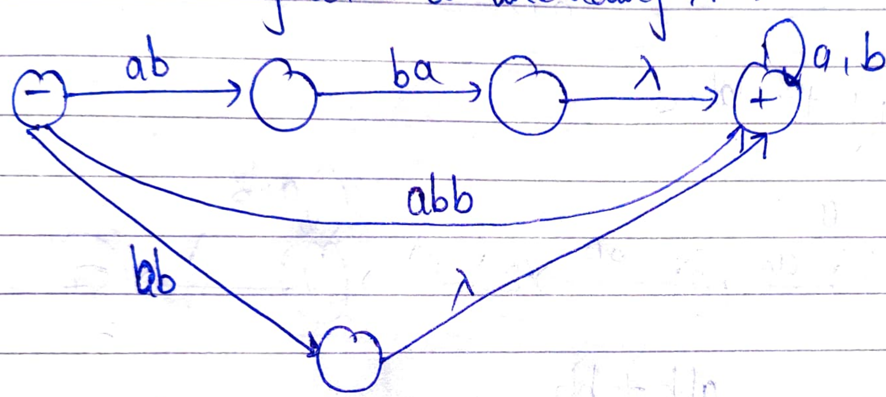
all words starting with bb.

(b) ((a+b)a)*

L = { λ, aa, ba, aaa,

all word ending with a including λ.

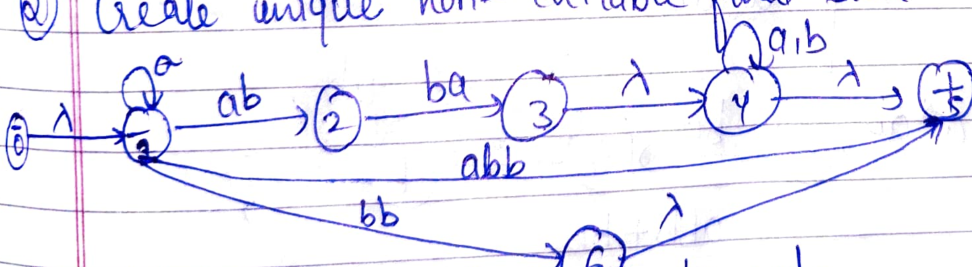
g)



* Steps for mapping TG to RE

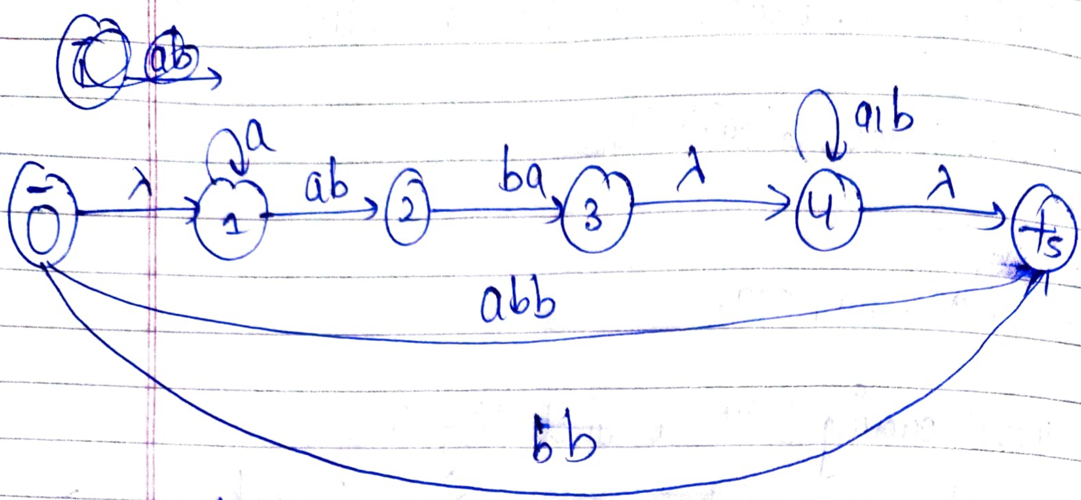
① Create unique non-enterable initial state

② Create unique non-exitable final state.

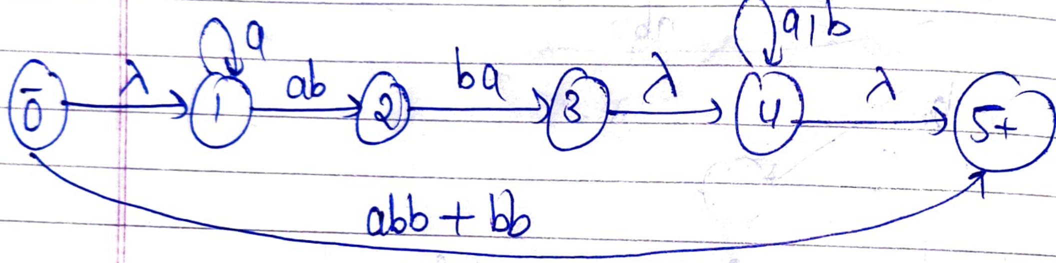


③ Apply bypass & eliminate rule

state 6	Incoming	Outgoing	label
	1	5	bb

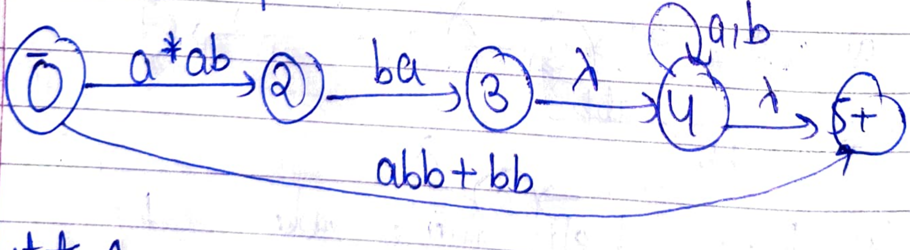


unify the labels

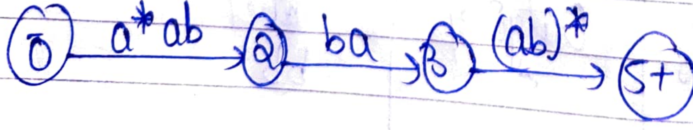


→ ~~bypass~~ state 1

I	O	label
0	2	a*ab

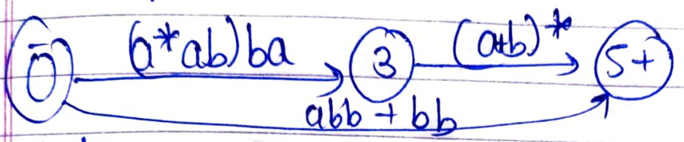


state 4



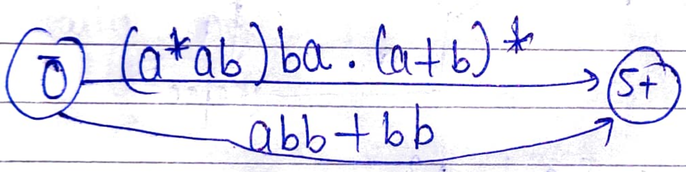
state 2

I	O	label
0	3	(a* ab) ba

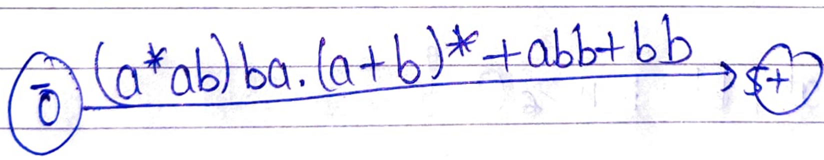


state3

I	O	label
0	5	(a* ab) ba . (a+b)*



unify labels.



r.e = (a* ab) ba (a+b)* + abb + bb

(k) $L = \{a^n S$, where S starts with b and length(S) = n$\}$

$L = \{a^n b^n\}$

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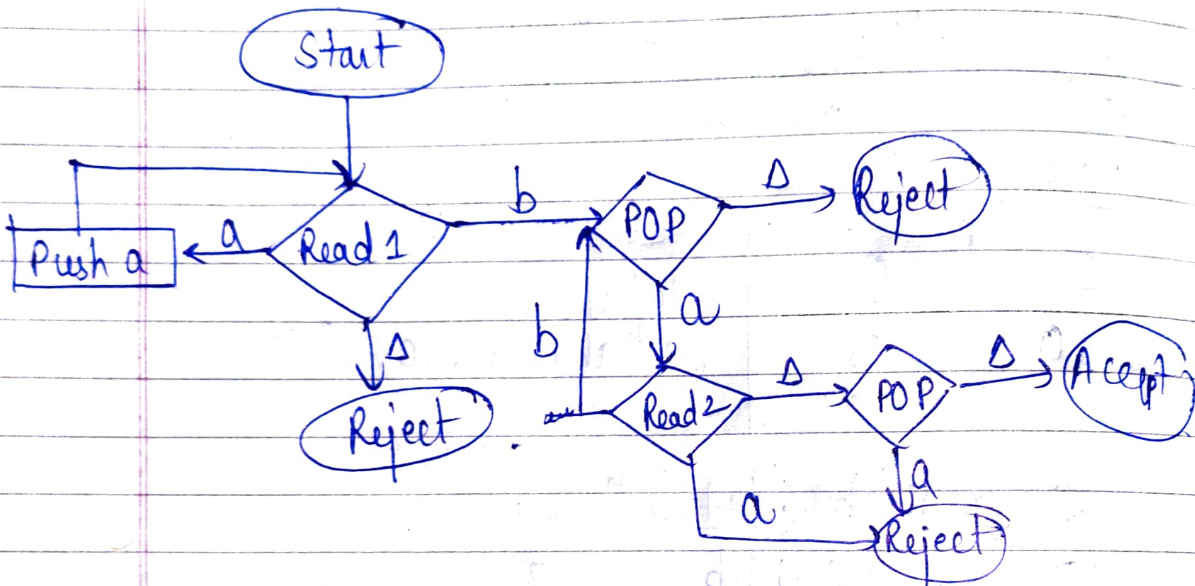
Assume, $n \geq 1$

$a^n b^n$

a	b	Δ

smallest word



Read : I/P Tape, $\Sigma = \{a, b, \Delta\}$
pop : Stack, $\Gamma = \{\Delta, a\}$

(4) $L = \{a^n, b^n : n \geq 0\}$

If L is an infinite reg. lg accepted by FA with N states then all words belonging to L having length at least N, can be divided into subcategories x, y & z such that:

① $w = xyz$ & $y \neq \lambda$
② $|w| \geq N$ where N is no. of states in FA
③ $|x| + |y| \leq N$.

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Then, following is true,

$w = xyz \in L$

then $xy^k z \in L$ $k = 0, 1, 2 \ldots$

Consider FA with $N = 8$ states and word

$w = a^4 b^4 \in L$

clearly $|w| \geq 8$ and it can broken into 3 parts, x, y, z s.t. $|x| + |y| \leq 8$

$w = a^4 b^4$

clearly, x & y will be entirely from a's and pumping y one more time will yield more no. of a's before b ~~as compared to~~

$$\underbrace{a^3}_{x} \underbrace{a}_{y} \underbrace{b^4}_{z}$$

pump y one more time,

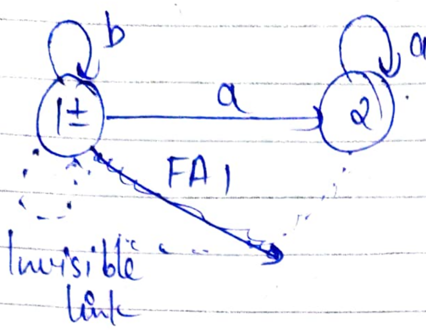
$a^3 a^2 b^4 = a^5 b^4 \notin L$

Thus, we arrive at a contradiction.

$\therefore$ L is not a reg. lg.

Section

2 a



$(FA_1)^*$?

$L = \{\lambda, b, bb \dots\}$

$(FA_1)^*$.

states	a	b
x_1	x_2	x_1
x_2	x_2	x_2

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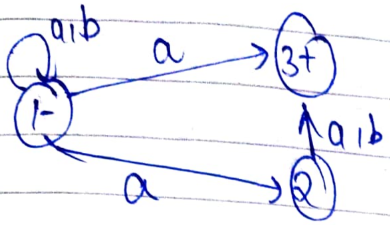
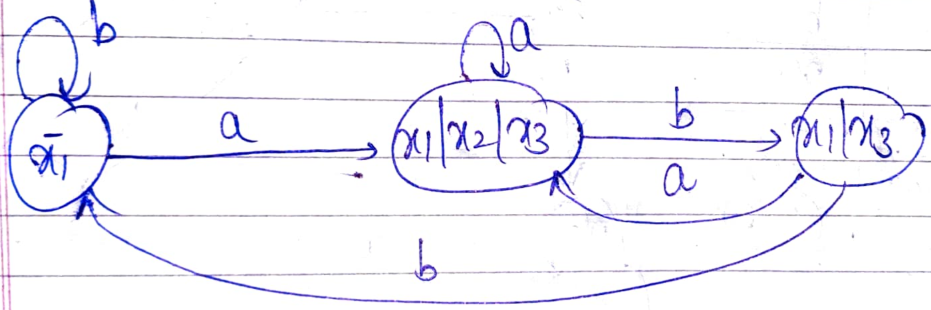
(b) NFA → DFA



states	a	b
$\bar{x}_1$	$x_1 \mid x_3 \mid x_2$	x_1
~~x_2~~	~~x_3~~	~~x_3~~
~~x_3~~	~~ϕ~~	~~ϕ~~
ϕ	ϕ	ϕ
$x_1 \mid x_3 \mid x_2$	$x_1 \to x_1 \mid x_2 \mid x_3$ $x_2 \to x_3$ $x_3 \to \phi$ $x_1 \mid x_2 \mid x_3$	$x_1 \to x_1$ $x_2 \to x_3$ $x_3 \to \phi$ $x_1 \mid x_3$
$x_1 \mid x_3$	$x_1 \to x_1 \mid x_2 \mid x_3$ $x_3 \to \phi$ $x_1 \mid x_2 \mid x_3$	$x_1 \to x_1$ $x_3 \to \phi$ x_1



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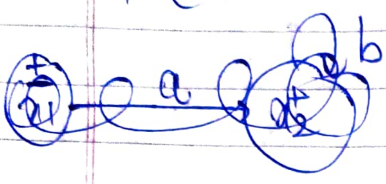
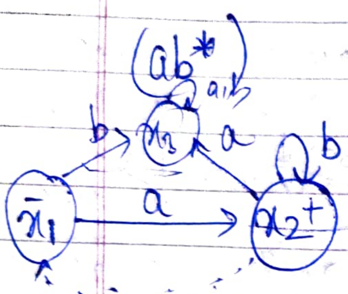
Q3. $L_1: (ab^*)^*$ $L_2: a(a+b)^*$

FA1

$r_1 = (ab^*)^*$



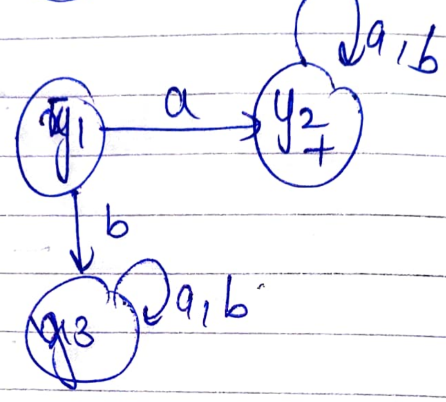


$(FA_1)^* = (ab^*)^*$

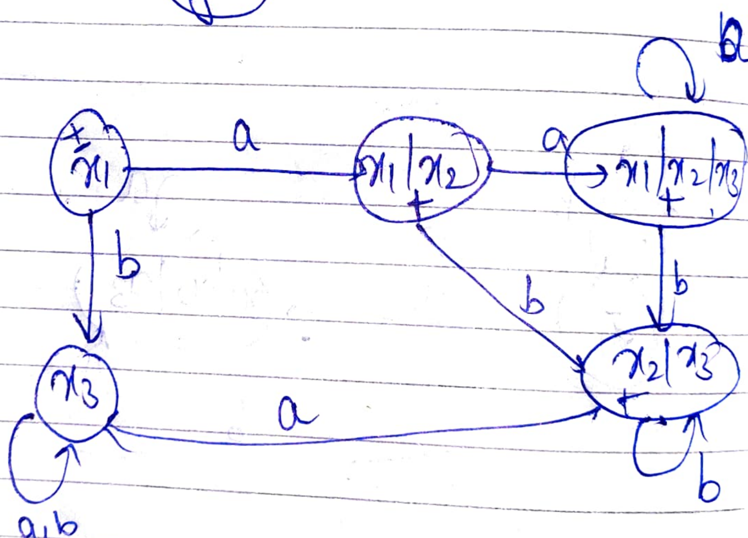
FA2

$r_2 = a(a+b)^*$

(FA_2).

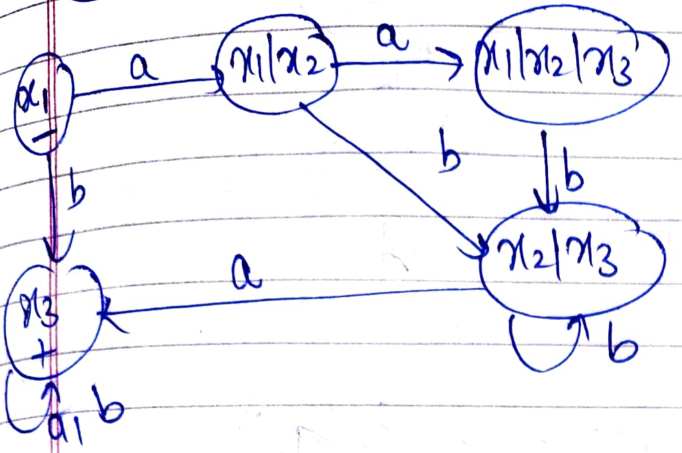


State	a	b
x_1	$x_2 \mid x_1$	x_3
$x_2 \mid x_1$	$x_1 \to x_1 \mid x_2$ $x_2 \to x_3$ $x_1 \mid x_2 \mid x_3$	$x_1 \to x_3$ $x_2 \to x_2$ $x_2 \mid x_3$
x_3	x_3	x_3
$x_2 \mid x_3$	$x_2 \to x_3$ $x_3 \to x_3$ x_3	$x_2 \to x_2$ $x_3 \to x_3$ $x_2 \mid x_3$
$x_1 \mid x_2 \mid x_3$	$x_1 \to x_1 \mid x_2$ $x_2 \to x_3$ $x_3 \to x_3$ $x_1 \mid x_2 \mid x_3$	$x_1 \to x_3$ $x_2 \to x_2$ $x_3 \to x_3$ $x_2 \mid x_3$

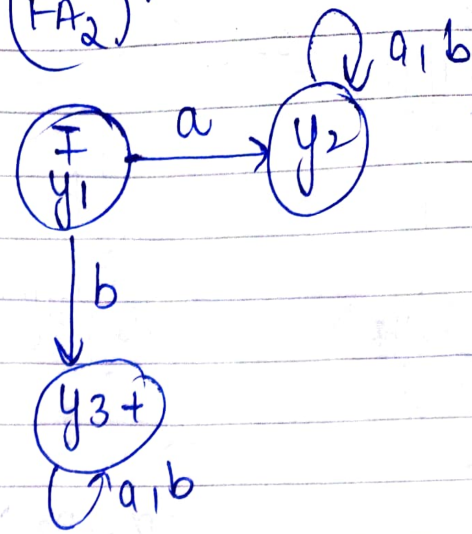


$x_2 \mid y_2$ x

$(FA_1)'$

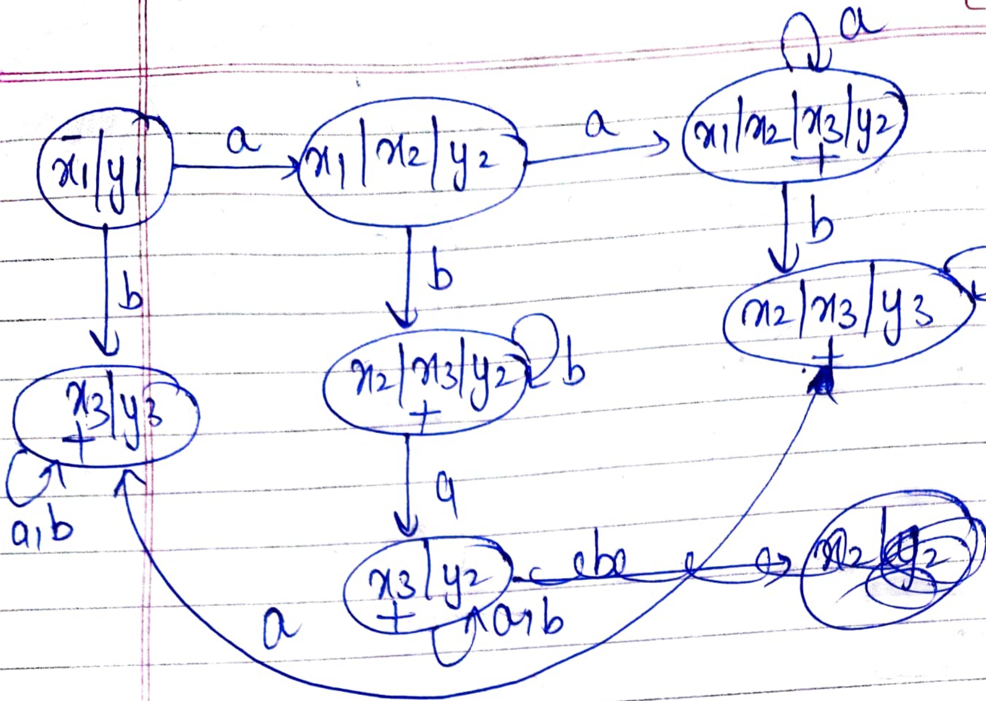


$(FA_2)'$



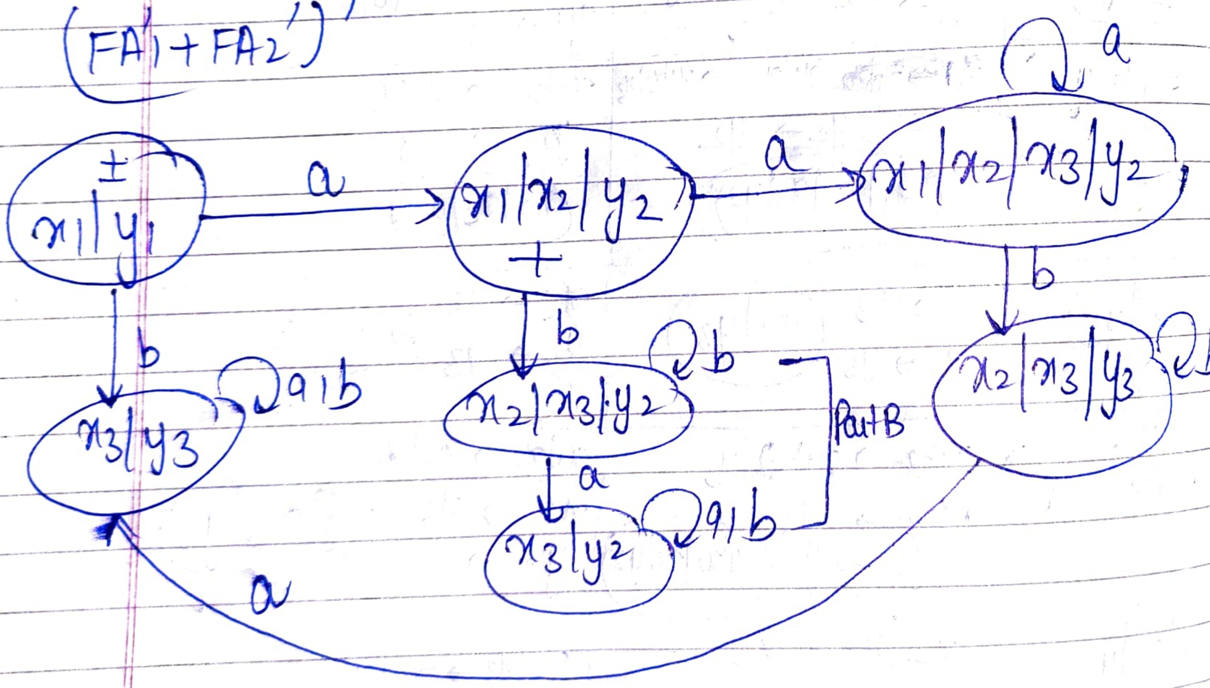
$FA_1' + FA_2'$

states	a	b
$x_1 \mid y_1$ −	$x_1 \to x_1 \mid x_2$, $y_1 \to y_2$; $x_1 \mid x_2 \mid y_2$	$x_1 \to x_3$, $y_1 \to y_3$; $x_3 \mid y_3$
$x_1 \mid x_2 \mid y_2$	$x_1 \mid x_2 \to x_1 \mid x_2 \mid x_3$, $y_2 \to y_2$; $x_1 \mid x_2 \mid x_3 \mid y_2$	$x_1 \mid x_2 \to x_2 \mid x_3$, $y_2 \to y_2$; $x_2 \mid x_3 \mid y_2$
$x_3 \mid y_3$	$x_3 \to x_3$, $y_3 \to y_3$; $x_3 \mid y_3$	$x_3 \to x_3$, $y_3 \to y_3$; $x_3 \mid y_3$
$x_1 \mid x_2 \mid x_3 \mid y_2$	$x_1 \mid x_2 \to x_1 \mid x_2 \mid x_3$, $x_3 \to x_3$, $y_2 \to y_2$; $x_1 \mid x_2 \mid x_3 \mid y_2$	$x_1 \mid x_2 \to x_2 \mid x_3$, $x_3 \to x_3$, $y_2 \to y_3$; $x_2 \mid x_3 \mid y_3$
$x_2 \mid x_3 \mid y_2$	$x_2 \mid x_3 \to x_3$, $y_2 \to y_2$; $x_3 \mid y_2$	$x_2 \mid x_3 \to x_2 \mid x_3$, $y_2 \to y_2$; $x_2 \mid x_3 \mid y_2$
$x_2 \mid x_3 \mid y_3$	$x_2 \mid x_3 \to x_3$, $y_3 \to y_3$; $x_3 \mid y_3$	$x_2 \mid x_3 \to x_2 \mid x_3$, $y_3 \to y_3$; $x_2 \mid x_3 \mid y_3$
$x_3 \mid y_2$	$x_3 \to x_3$, $y_2 \to y_2$; $x_3 \mid y_2$	$x_3 \to x_3$, $y_2 \to y_2$. $x_3 \mid y_2$
$x_2 \mid x_3 \mid y_2$	$x_2 \mid x_3 \to x_3$, $y_2 \to y_2$; $x_3 \mid y_2$	$x_2 \mid x_3 \to x_2 \mid x_3$, $y_2 \to y_2$; $x_2 \mid x_3 \mid y_2$


a
a
a
b
b
b
b
b
a
a,b
a,b
a
$x_1|y_1$
$x_1|x_2|y_2$
$x_1|x_2|x_3|y_2$
$x_3|y_3$
$x_2|x_3|y_2$
$x_2|x_3|y_3$
$x_3|y_2$
$x_2|y_2$

Now

$(FA_1' + FA_2')'$


a
a
a
b
b
b
a,b
b
b
a
a,b
a
Part B
$x_1|y_1$
$x_1|x_2|y_2$
$x_1|x_2|x_3|y_2$
$x_3|y_3$
$x_2|x_3|y_2$
$x_2|x_3|y_3$
$x_3|y_2$

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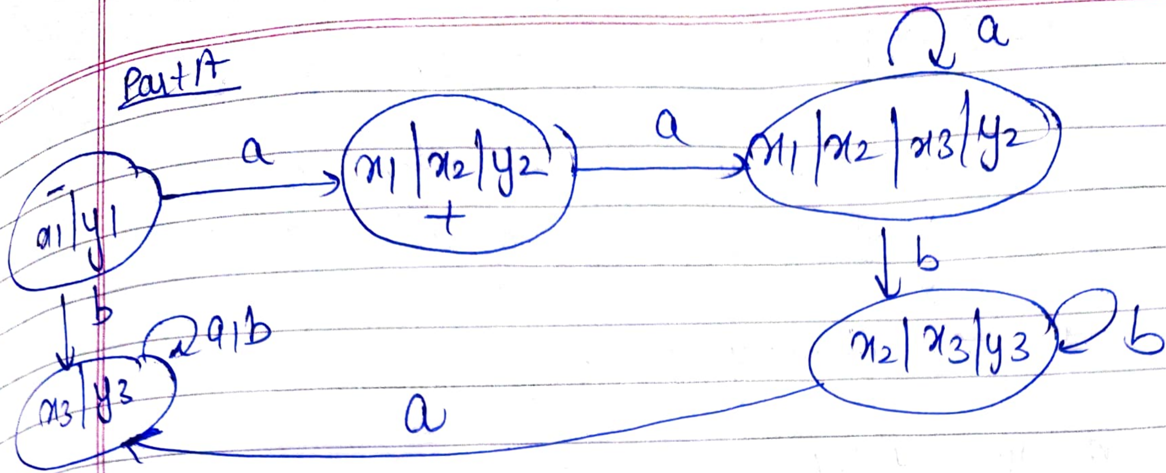
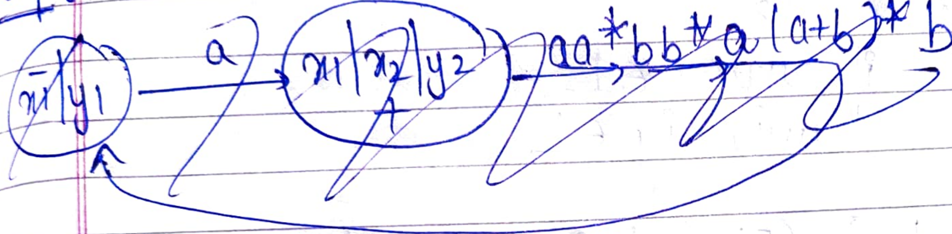
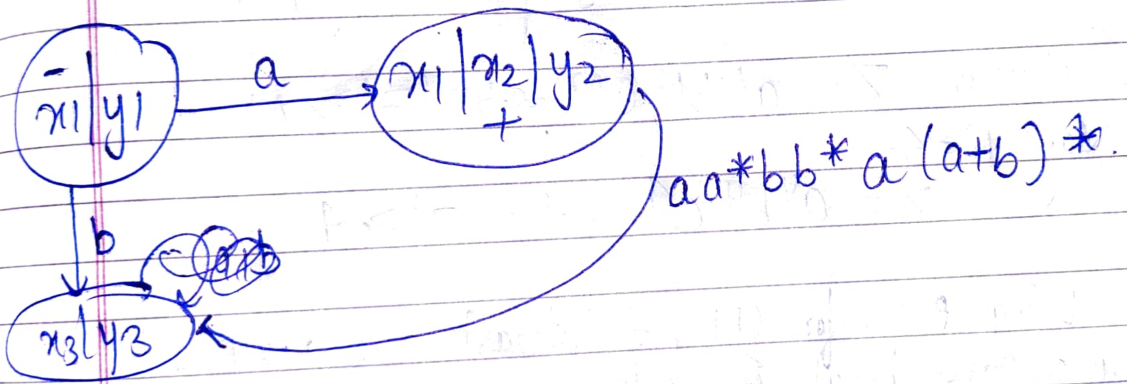
Part A



Step 1:





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(b) $L = \{a^n b^n c^n$ where $n = 1, 2, \dots\}$ is non-CFG

for every CFG, there is an integer N, s.t.
for every string Z in L of length $\geq N$

$\exists$ $Z = uvwxy$ s.t.

① $|uvwxy| \geq N$
② $|vwx| \leq N$
③ $|vx| > 0$
④ for all $i > 0$,
if $uvwxy \in L$
$uv^iwx^iy \in L$

Step 1: assume L is CFL and $\exists$ positive integer N

Step 2: choose $Z = a^N b^N c^N$ s.t. $|z| \geq N$
say $N = 4$
$Z = a^4 b^4 c^4$ s.t. $|z| \geq 4$

Step 3: Using P.L. for CFL, $\exists$ atleast one way s.t.
we can break z into $uvwxy$
s.t. $uv^iwx^iy \in L$

Obs 1: v \$ x contain exactly 1 symbol.

1.1. v \$ x are exactly same symbol

$\underbrace{a}_{u}\underbrace{a}_{v}\underbrace{a}_{w}\underbrace{a}_{x}\underbrace{bbbb\,cccc}_{y}$

clearly $|vwx| \leq \cancel{4}\ 2$

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pumping v \$ x one more time,
$uv^2wx^2y = a^6b^4c^4 \notin L$
we arrive at contradiction.
Similarly, v \$ x can be from only b's or c's.

1.2. v \$ x are diff. symbols.

$\underbrace{aaa}_{u}\underbrace{a}_{v}\underbrace{b}_{w}\underbrace{b}_{x}\underbrace{bb\ cccc}_{z}$

clearly, $|vwx| \leq 4$
pumping v \$ x one more time,
$uv^2wx^2y = a^5b^5c^4 \notin L$
we arrive at contradiction.
Similarly, v \$ x can be from b \$ c respectively.

Obs 2 either v or x contain more than one type of symbol.

$z = \underbrace{aaa}_{u}\underbrace{ab}_{v}\underbrace{b}_{w}\underbrace{b}_{x}\underbrace{b\ cccc}_{y}$

clearly, $|vwx| \leq 4$
pumping v \$ x one more time,
$uv^2wx^2y = a^3ababbb^3c^4 \notin L$
we arrive at contradiction.
Similarly, v can be bc.

For none of the breakup, pumping lemma hold,
Thus we arrive at a contradiction to our assumption.
Therefore, $a^nb^nc^n$ is not CFL.

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Q4 (a) $S \rightarrow aXb \mid bXa \mid \lambda$

$X \rightarrow aX \mid bX \mid \lambda$

(b) $S \rightarrow XaX$

$X \rightarrow aX \mid bX \mid \lambda$

$S \rightarrow XaX$

$\Rightarrow (a+b)^m a (a+b)^n$

$= (a+b)^m a (a+b)^n$

$CFL(X) = \{(a+b)^n \mid n = 0, 1, 2 \ldots\}$

$CFL(S) = \{(a+b)^m a (a+b)^n \mid m, n = 0, 1, \ldots\}$

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TOC Ques paper

(1) (a) To prove, $(S^+)^* = S^*$.

Let $\Rightarrow \Sigma = \{a, b\}$

$S^+ = \{a, b, aa, ab, ba, bb \ldots\}$

$(S^+)^* = \{\lambda, a, b \ldots\}$

$S^* = \{\lambda, a, b \ldots\}$

$\therefore (S^+)^* = S^*$.

(b) L = do not end with double letter.

$r = (a+b)^* (ab + ba) + a + b + \lambda$

(c) $(ab)^* a = a(ba)^*$

~~$L_1 = \{a, aa, ba, aab, bba \ldots\}$~~
~~$L_2 = \{a, aa, ab, \ldots\}$~~

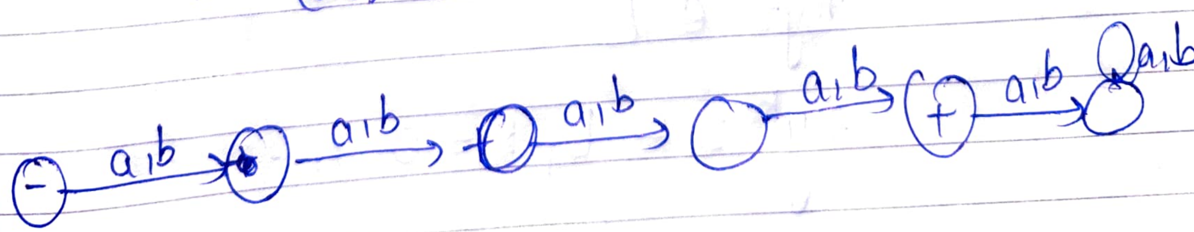
~~$L_1 \neq L_2$~~

$L_1 = \{a, aba, ababa, \ldots\}$

$L_2 = \{a, aba, ababa, \ldots\}$

$\therefore (ab)^* a = a(ba)^*$

(d)



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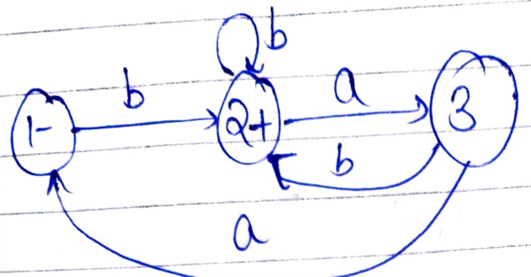
(e) $L = \{\lambda, a, b, ab \ldots\}$



(f) $L = \{ S a^{length(s)}$

$= \{(a+b)^* a^{(a+b)^*}$

(g) $L = \{ a^n, n = 1, 4, 9, \ldots \}$

If L is inf. reg. lg accepted by FA with N states then all words belonging to L having length at least N, can be divided into subcategories x, y, z, s.t.

① $w = x, yz$ & $y \neq \lambda$

② $|w| \geq N$ where N is no states

③ $|x| + |y| \leq N$

Then following is true,

$xyz \in L$

$xy^k z \in L, k = 0, 1, \ldots$

Consider FA with $N = 4$ states,

word $= a^4 \in L$

clearly $|w| \geq N$ and broke into 3 parts

x, y, z s.t. $|x| + |y| \leq 8$

$w = a^4$

$w = \underbrace{a}_{x}\ \underbrace{a}_{y}\ \underbrace{a\ a}_{z}$

also, pump y one more time.

$w = a\ aaa\ aa = a^5 \notin L$

Contradiction, a^n is not a reg. lng.

(h).

(i)

(j) $\triangleright \sqcup w \underline{\sqcup} \vdash_M^* \triangleright \sqcup \sqcup w \underline{\sqcup}$

$\boxed{\triangleright | \sqcup | a | b | \sqcup | \underline{\sqcup}} \vdash_M^* \boxed{\triangleright | \sqcup | \sqcup | a | b | \underline{\sqcup}}$

$$\begin{tikzcd} >L \arrow[r, "x \neq \sqcup"] \arrow[d, "\sqcup"] & \sqcup R x L \arrow[l, bend right] \\ R_\sqcup^2 & \end{tikzcd}$$

Part B

Q2 (a) lg accepted by FA can be easily described by simple exp. called R.E.

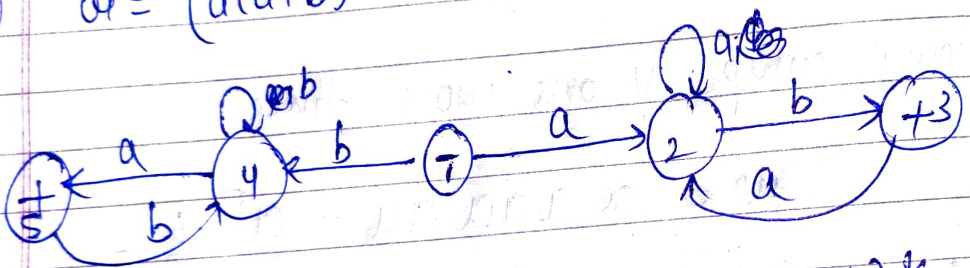
→ all symbols of alphabet set Σ can be treated as r.e

→ empty 'λ' is also a r.e.

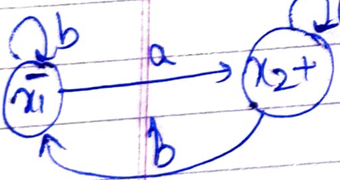
(b) ~~$r = (a^* b\ a^* b\ a^* b)$~~

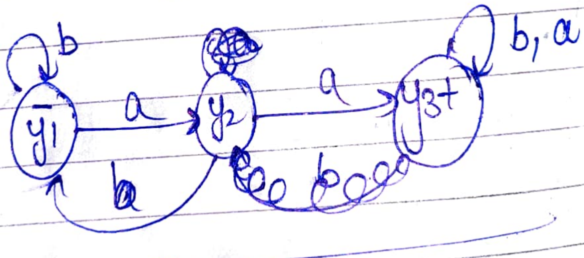
$r = a(aa)^*$

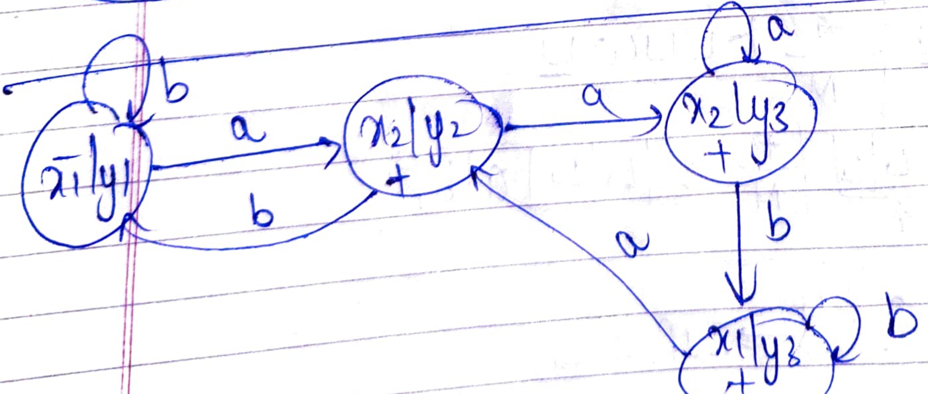
(c) $r = (a(a+b)^*b + b(a+b)^*a)$



Q3.(a) $L_1 = (a+b)^*a$ $L_2 = (a+b)^*aa(a+b)^*$







Dated aSb aSbb aabb

Page No. aSb aSab aSbab aabab

	a		b	
$x_1 \| y_1$	$x_1 \to x_2$, $y_1 \to y_2$	$x_2 \| y_2$	$x_1 \to x_1$, $y_1 \to y_1$	$x_1 \| y_1$
$x_2 \| y_2$	$x_2 \to x_2$, $y_2 \to y_3$	$x_2 \| y_3$	$x_2 \to x_1$, $y_2 \to y_1$	$x_1 \| y_1$
$x_2 \| y_3$	$x_2 \to x_2$, $y_3 \to y_3$	$x_2 \| y_3$	$x_2 \to x_1$, $y_3 \to y_3$	$x_1 \| y_3$
$x_1 \| y_3$	$x_1 \to x_2$, $y_3 \to y_3$	$x_2 \| y_3$	$x_1 \to x_1$, $y_3 \to y_3$	$x_1 \| y_3$

(b) $S \to aSb \mid Sb \mid Sa \mid a$

Ans (a). $a^n b a^n$, $n = 1, 2, 3 \ldots$

If L is inf. reg. lg accepted by FA with N states then all words belonging to L having length at least N, can be divided into subcategories x, y, z, s.t

1. $w = xyz$ & $y \neq \lambda$
2. $|w| \geq N$
3. $|x| + |y| \leq N$

Then, true,

$xyz \in L$

$xy^k z \in L \quad k = 0, 1, \ldots$

Consider FA with $N = 8$ states

$w = a^4 b a^4 \in L$

clearly $|w| \geq N$

x, y, z s.t. $|x| + |y| \leq 8$

$\underbrace{aaaa}_{x}\underbrace{b}_{y}\underbrace{aaa}_{z}a$

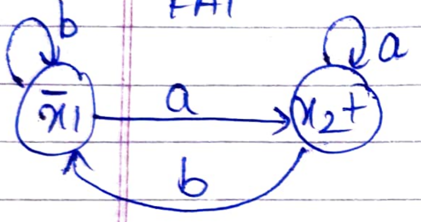
pump y one more time.

$xyyz = a^4 b^2 a^4 \notin L$

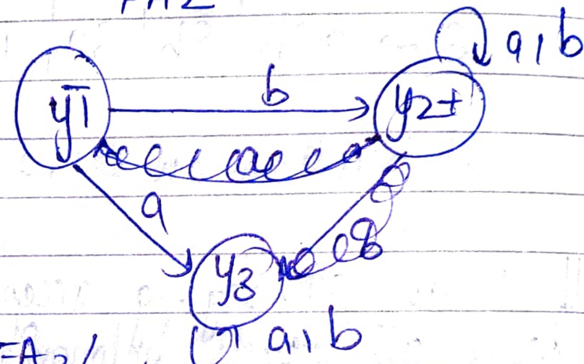
contradiction,
$a^n b a^n \neq$ R.L.

(b) $L_1 = (a+b)^* a$ $\quad$ $L_2 = b(a+b)^*$

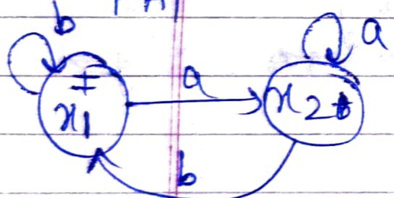
FA1



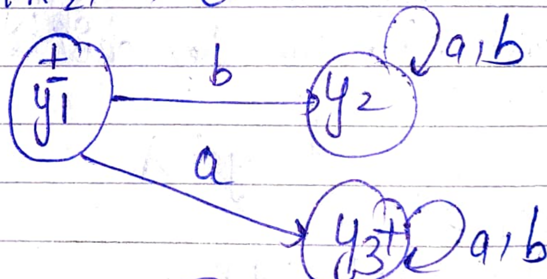
FA2



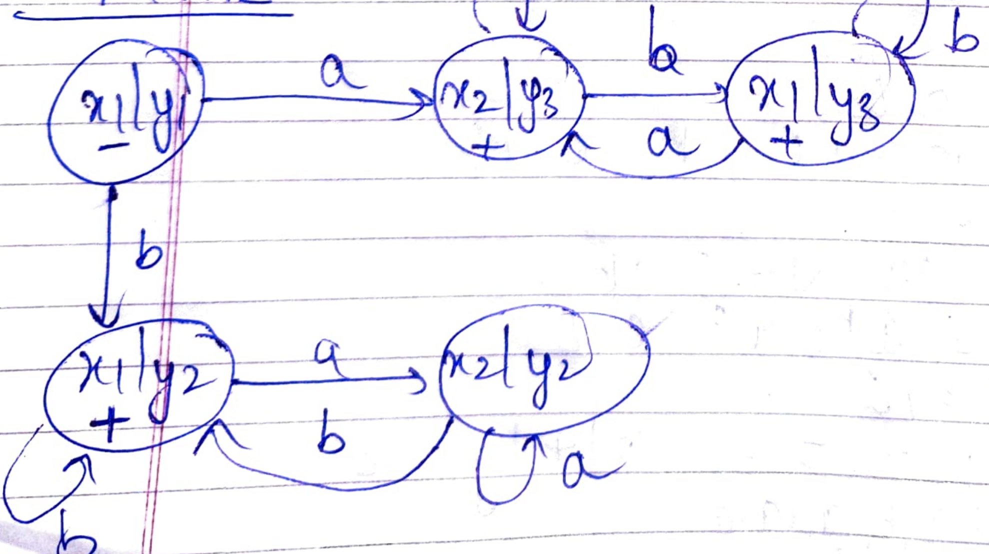
FA1'



FA2'



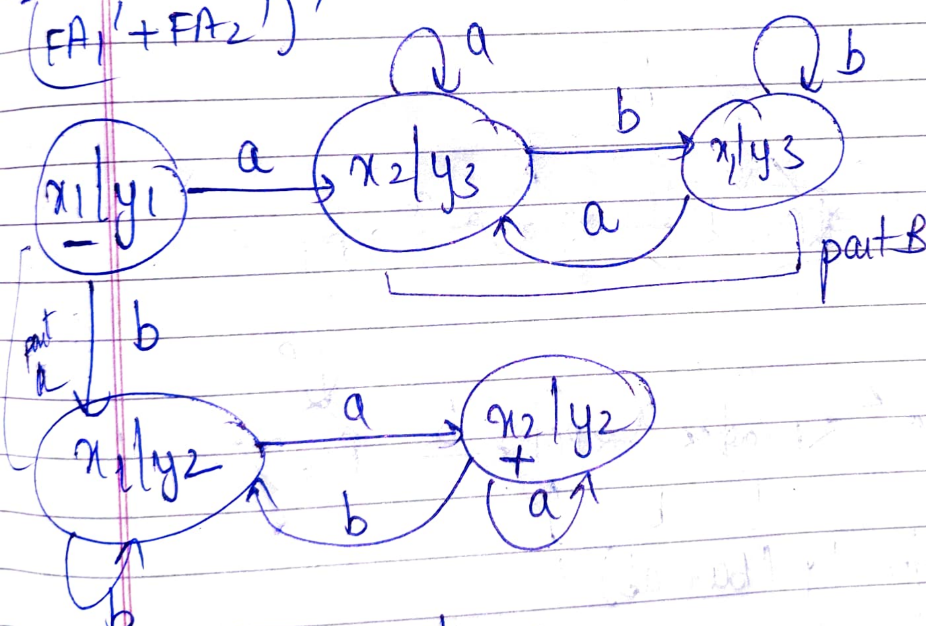
FA1' + FA2'



states	a		b	
$x_1 \mid y_1$ $-$	$x_1 \to x_2$, $y_1 \to y_3$	$x_2 \mid y_3$	$x_1 \to x_1$, $y_1 \to y_2$	$x_1 \mid y_2$
$x_2 \mid y_3$	$x_2 \to x_2$, $y_3 \to y_3$	$x_2 \mid y_3$	$x_2 \to x_1$, $y_3 \to y_3$	$x_1 \mid y_3$
$x_1 \mid y_2$	$x_1 \to x_2$, $y_2 \to y_2$	$x_2 \mid y_2$	$x_1 \to x_1$, $y_2 \to y_2$	$x_1 \mid y_2$
$x_1 \mid y_3$	$x_1 \to x_2$, $y_3 \to y_3$	$x_2 \mid y_3$	$x_1 \to x_1$, $y_3 \to y_3$	$x_1 \mid y_3$
$x_2 \mid y_2$	$x_2 \to x_2$, $y_2 \to y_2$	$x_2 \mid y_2$	$x_2 \to x_1$, $y_2 \to y_2$	$x_1 \mid y_2$

Now

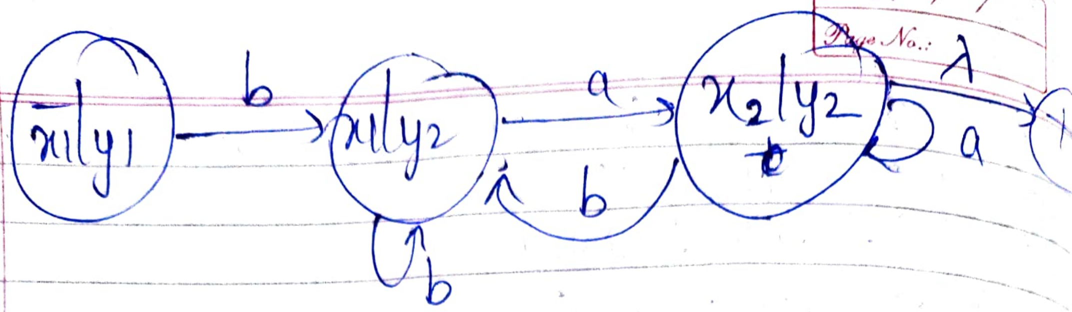
$(FA_1' + FA_2')'$



Mapping to r.e.

bb^* e aa^*b

non-enteriable final state



aa^*b

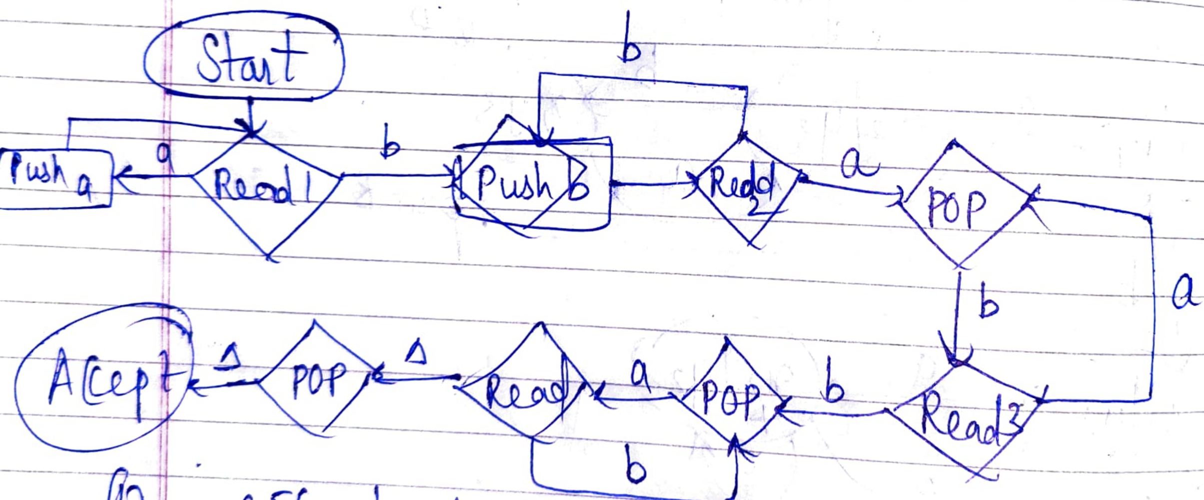
~~bb*~~

R.e = $bb^*.aa^*b$

Q5. (a) PDA for $a^n b^m a^m b^n$, $m,n \geq 1$.

Read: I/p tape: $\Sigma = \{a, b, \Delta\}$

pop: stack = $\Gamma = \{a, b, \Delta\}$



(b) CFG for lg $(ba+ab)^*$

$S \rightarrow aSb \mid bSa \mid \lambda$

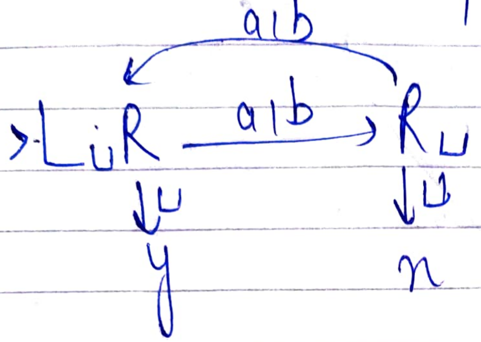
Dated: / /

Page No.:

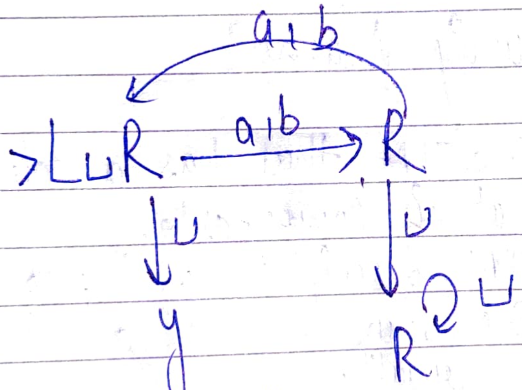
All recursive lg is also recursively enumerable

Let take an example;

$w \in \{a|b\}$ having even length



↓ for R.E.



(b) $a^n b^n a^n b^n \notin CFG$ for $n = 1, 2, 3$

$a^4 b^4 a^4 b^4$

(1) select v & x from same symbol

$\underset{u\,v\,w\,x\,y}{aaaabbbb}a^4c^4$

$a^6 b^4 a^4 c^4 \in L$