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Chapter-17

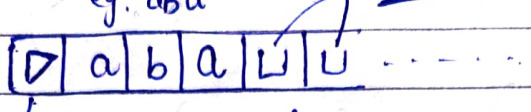
Turing Machine

↳ Invented by Alan Turing (1912-1954)

Turing Machine

① Input Tape

eg: aba



↳ left end symbol
(It cannot allow the string to make a step to the leftmost cell).

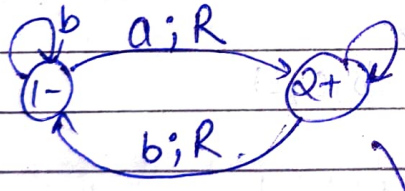
② Read Write Head

Read ↳ move it one position towards the right tape square
(R, →)

Read ↳ move it one position towards the left tape square
(←, L).

write ↳ write the content to the current tape square.

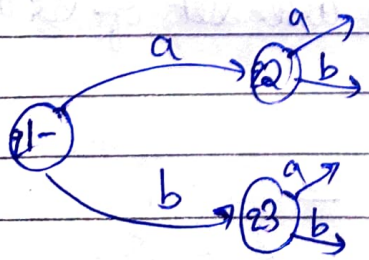
eg



③ Control logic unit

current state	Current Symbol	Next state	Action
1	a	2	R
2	b	1	L

eg:



When ▷ is encountered, tape must move right to prevent head of M from falling off to the left end.

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control logic

$$\delta\begin{pmatrix}\text{current}\\\text{state}\end{pmatrix},\begin{pmatrix}\text{symbol}\\\text{read}\end{pmatrix}) = \begin{pmatrix}\text{Next}\\\text{state}\end{pmatrix},\text{Action}) \rightarrow \rightarrow \text{ or } \leftarrow$$

$\Sigma - \{\triangleright\}$.

$$\delta(q_1, a) = (q_2, \rightarrow)$$

Note $\delta(q_i, \triangleright) = (q_j, \rightarrow)$

When u read ▷ (left end) then you explicitly perform the action right side (→).

eg. | ▷ | a | b | c | ⊔ | ⊔ | $\Sigma = \{a, b, c, \sqcup, \triangleright\}$

$\delta(q_0, b) \rightarrow (q_1, \rightarrow)$ | ▷ | a | b | c |

$(q_1, \leftarrow)$ | ▷ | a | b | c |

(q_1, a) | ▷ | a | b | c | ⊔ |

$a, b, c, \sqcup$

→ Action performs can be right hand or left hand or replace with all sets of alphabets except left end. $\Sigma - \{\triangleright\}$

→ Current states and Next states are set of states of K.

→ Symbol that can be read: Σ

→ Start state will be one i.e. $s \in K$.

a) $\delta(q, \triangleright) = (p, \rightarrow)$

b) $\delta(q, \varepsilon) = (p, b)$ → where $b \in \{\Sigma \cup \{\leftarrow, \rightarrow\} - \{\triangleright\}\}$
↳ machine never writes $\triangleright$

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Symbols

→ Halting state is the accept (+) or DeadEnd (reject) state ↓ i.e. **H**

* Turing Machine Symbols : quintuple $(K, \Sigma, \delta, s, H)$

K: finite set of states

Σ: alphabet set containing blank symbol $\sqcup$, left end symbol $\triangleright$, but not $\leftarrow$ or $\rightarrow$ eg: $\Sigma = \{a, b, \triangleright, \sqcup\}$

$s \in K$ is the start state

$H \subseteq K$ is the set of halting states (when m/c reaches halting state, operation stops)

δ: Transition function.

Ques. Consider the Turing Machine $M = (K, \Sigma, \delta, s, H)$

$K = \{q_0, q_1, h\}$

$\Sigma = \{\triangleright, \sqcup, a\}$

$s = \{q_0\}$

$h = \{h\}$

and δ is given,

q	σ	$\delta(q, \sigma)$
q_0	a	$(q_1, \sqcup)$
q_0	$\sqcup$	$(h, \sqcup)$
q_0	$\triangleright$	$(q_0, \rightarrow)$
q_1	a	(q_0, a)
q_1	$\sqcup$	$(q_0, \rightarrow)$
q_1	$\triangleright$	$(q_1, \rightarrow)$

δ (current state, symbol read) = (Next state, Action)

Now, word = aaa

Turing Machine

Initial Configuration $(q_0, \triangleright aaa\sqcup) \vdash_M (q_1, \triangleright \sqcup aa\sqcup)$

$\vdash_M (q_0, \triangleright \sqcup a a \sqcup)$

$\vdash_M (q_1, \triangleright \sqcup \sqcup a \sqcup)$

$\vdash_M (q_0, \triangleright \sqcup \sqcup \sqcup a \sqcup)$

$\vdash_M (q_1, \triangleright \sqcup \sqcup \sqcup \sqcup \sqcup)$

$\vdash_M (q_0, \triangleright \sqcup \sqcup \sqcup \sqcup \sqcup)$

$\vdash_M (h, \triangleright \sqcup \sqcup \sqcup \sqcup \sqcup)$

~~Ques.~~ ~~$(q_0, \triangleright aaa\sqcup) \vdash_M (q_1, \triangleright aa\sqcup\sqcup)$~~

~~$\vdash_M$~~

Ques Consider the TM , $M = (k, \Sigma, \delta, s, H)$

$k = \{q_0, h\}$

$\Sigma = \{a, \sqcup, \triangleright\}$

$s = \{q_0\}$

$H = \{h\}$

q	σ	$\delta(q, \sigma)$
q_0	a	$(q_0, \leftarrow)$
q_0	$\sqcup$	$(h, \sqcup)$
q_0	$\triangleright$	$(q_0, \rightarrow)$

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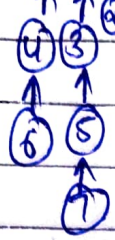
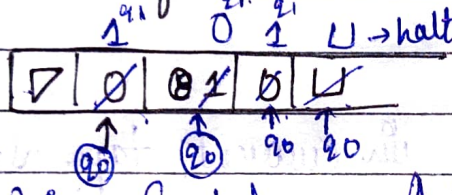
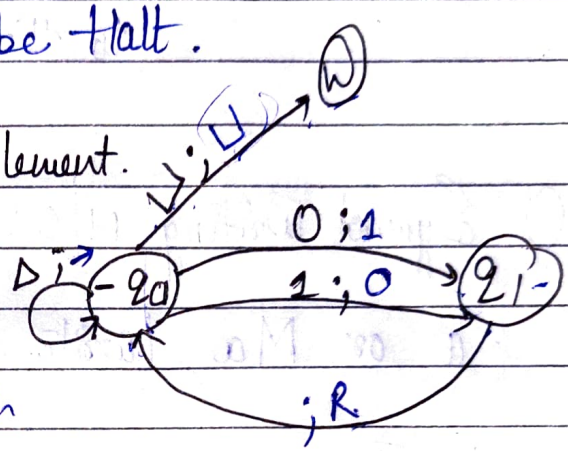
Initial configuration $(q_0, \triangleright aaa \sqcup)$



→ This machine can never be Halt.

Problem: To find 1's complement.





Logic:

Symbol Read	Action
0	1
1	0
▽	→
⊔	(Halt, ⊔)

Transition Table

states	δ(,)
$(q_0, 0)$	$(q_1, 1)$
$(q_0, 1)$	$(q_1, 0)$
$(q_0, \triangleright)$	$(q_0, \rightarrow)$
$(q_0, \sqcup)$	$(h, \sqcup)$
$(q_1, 0)$	$(q_0, \rightarrow)$
$(q_1, 1)$	$(q_0, \rightarrow)$
$(q_1, \triangleright)$	$(q_0, \rightarrow)$
$(q_1, \sqcup)$	$(q_0, \rightarrow)$

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* Notation of Turing M/C

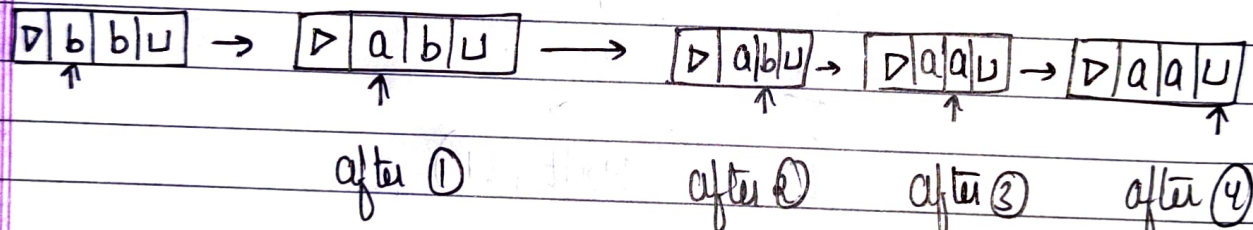
① Head Moving M/C :

L or ←
R or → (move rightwards 'RW' head one type square to the right)

② Symbol Writing M/C :

a or Ma (write a in current tape square)

eg. I/P Tape



M → >aRaR

① ② ③ ④

This m/c replacing the first few ~~correct~~ characters with 'a'.

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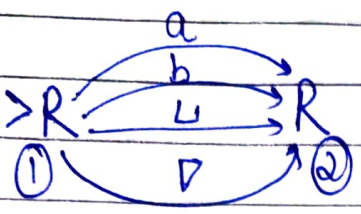
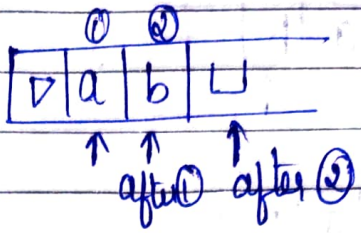
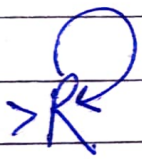
$\Sigma = \{a, b, \sqcup, \triangleright\}$





$>R \xrightarrow{\triangleright, \sqcup, a, b} R \quad = \quad >RR \quad = \quad >R^2$

↳ If edge contain all symbols in Σ

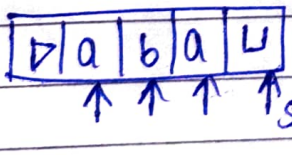


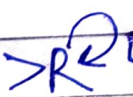
$x \neq \sqcup$

⋮

Symbol on current tape square.

do
{ R
} while ($x \neq \sqcup$)



or 

or $>R_{\sqcup}$ → read-write head.

(Move towards the right until you reach the first blank square).

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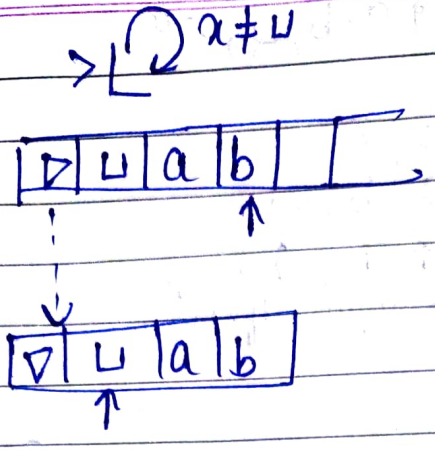
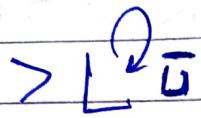
③ 

or 

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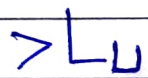
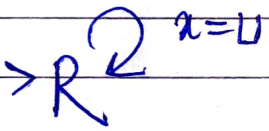
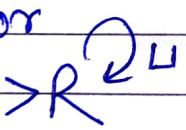
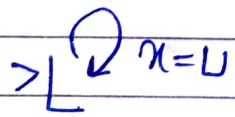
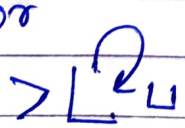


④ 

or 

~~or~~ or 

⑤ 

or 

or