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Small-Step Semantics Assignment

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1 Small-Step Semantics (25 points)

Let us define the calculator language below that performs arithmetic and boolean operations. It has three different syntactic classes: (1) arithmetic expressions a (2) boolean expressions b, and (3) final values v.

The following questions require you define small-step semantics. That is, first you need to define the configuration. Then, you need to write inference rules that show one configuration small-steps to another configuration.

$$
\begin{aligned}
n &\in \mathbb{Z} \\
a &::= n \mid a_1 + a_2 \mid a_1 \times a_2 \\
b &::= \text{true} \mid \text{false} \mid a = a \mid a \neq a \\
&\quad \mid a \leq a \mid a > a \mid \neg b \mid b \&\& b \\
v &::= n \mid \text{true} \mid \text{false}
\end{aligned}
$$

(a) Write small-step semantics for the syntactic class of arithmetic expressions generated by a.

(b) Write small-step semantics for the syntactic class of boolean expressions generated by b.

(a) Small-step semantics for arithmetic expressions

Configuration: $\langle a, \sigma \rangle$, where a is an arithmetic expression and σ is the store.

Inference rules:

$$\frac{}{\langle n, \sigma \rangle \to_a n}$$

$$\frac{\langle a_1, \sigma \rangle \to_a a_1'}{\langle a_1 + a_2, \sigma \rangle \to_a a_1' + a_2}$$

$$\frac{\langle a_2, \sigma \rangle \to_a a_2'}{\langle n_1 + a_2, \sigma \rangle \to_a n_1 + a_2'}$$

$$\frac{}{\langle n_1 + n_2, \sigma \rangle \to_a n_3}$$

where n_3 is the sum of n_1 and n_2

$$\frac{\langle a_1, \sigma \rangle \to_a a_1'}{\langle a_1 \times a_2, \sigma \rangle \to_a a_1' \times a_2}$$

$$\frac{\langle a_2, \sigma \rangle \to_a a_2'}{\langle n_1 \times a_2, \sigma \rangle \to_a n_1 \times a_2'}$$

$$\frac{}{\langle n_1 \times n_2, \sigma \rangle \to_a n_3}$$

where n_3 is the product of n_1 and n_2

(b) Small-step semantics for boolean expressions

Configuration: $\langle b, \sigma \rangle$, where b is a boolean expression and σ is the store.

Inference rules:

$$\frac{}{\langle \text{true}, \sigma \rangle \to_b \text{true}}$$

$$\frac{}{\langle \text{false}, \sigma \rangle \to_b \text{false}}$$

$$\frac{\langle a_1, \sigma \rangle \to_a a_1'}{\langle a_1 = a_2, \sigma \rangle \to_b a_1' = a_2}$$

$$\frac{\langle a_2, \sigma \rangle \to_a a_2'}{\langle n_1 = a_2, \sigma \rangle \to_b n_1 = a_2'}$$

$$\frac{}{\langle n_1 = n_2, \sigma \rangle \to_b \text{true}}$$

if $n_1 = n_2$

$$\frac{}{\langle n_1 = n_2, \sigma \rangle \to_b \text{false}}$$

if $n_1 \neq n_2$

$$\frac{\langle a_1, \sigma \rangle \to_a a_1'}{\langle a_1 \neq a_2, \sigma \rangle \to_b a_1' \neq a_2}$$

$$\frac{\langle a_2, \sigma \rangle \to_a a_2'}{\langle n_1 \neq a_2, \sigma \rangle \to_b n_1 \neq a_2'}$$

$$\frac{}{\langle n_1 \neq n_2, \sigma \rangle \to_b \text{true}}$$

if $n_1 \neq n_2$

$$\frac{}{\langle n_1 \neq n_2, \sigma \rangle \to_b \text{false}}$$

if $n_1 = n_2$

$$\frac{\langle a_1, \sigma \rangle \to_a a_1'}{\langle a_1 \leq a_2, \sigma \rangle \to_b a_1' \leq a_2}$$

$$\frac{\langle a_2, \sigma \rangle \to_a a_2'}{\langle n_1 \leq a_2, \sigma \rangle \to_b n_1 \leq a_2'}$$

$$\frac{}{\langle n_1 \leq n_2, \sigma \rangle \to_b \text{true}}$$

if $n_1 \leq n_2$

$$\frac{}{\langle n_1 \leq n_2, \sigma \rangle \rightarrow_b \text{false}}$$

if $n_1 > n_2$

$$\frac{\langle a_1, \sigma \rangle \rightarrow_a a_1'}{\langle a_1 > a_2, \sigma \rangle \rightarrow_b a_1' > a_2}$$

$$\frac{\langle a_2, \sigma \rangle \rightarrow_a a_2'}{\langle n_1 > a_2, \sigma \rangle \rightarrow_b n_1 > a_2'}$$

$$\frac{}{\langle n_1 > n_2, \sigma \rangle \rightarrow_b \text{true}}$$

if $n_1 > n_2$

$$\frac{}{\langle n_1 > n_2, \sigma \rangle \rightarrow_b \text{false}}$$

if $n_1 \leq n_2$

$$\frac{\langle b, \sigma \rangle \rightarrow_b b'}{\langle \neg b, \sigma \rangle \rightarrow_b \neg b'}$$

$$\frac{}{\langle \neg \text{true}, \sigma \rangle \rightarrow_b \text{false}}$$

$$\frac{}{\langle \neg \text{false}, \sigma \rangle \rightarrow_b \text{true}}$$

$$\frac{\langle b_1, \sigma \rangle \rightarrow_b b_1'}{\langle b_1 \&\& b_2, \sigma \rangle \rightarrow_b b_1' \&\& b_2}$$

$$\frac{}{\langle \text{true} \&\& b, \sigma \rangle \rightarrow_b b}$$

$$\frac{}{\langle \text{false} \&\& b, \sigma \rangle \rightarrow_b \text{false}}$$