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① sol: Given - that,

let  $x = (\lambda x. \text{ref } 42) 1$  in ! $x$

Context (E)

$\Rightarrow \boxed{E = \text{let } x = e \text{ in } !x}$   $\rightarrow$  one step Evaluation.

Where  $\boxed{e = (\lambda x. \text{ref } 42) 1}$

now evaluating  $e$ ,

$\rightarrow$  Applying the function  $(\lambda x. \text{ref } 42) 1$

replace  $x$  in  $\text{ref } 42$  with 1. Since  $\text{ref } 42$  doesn't depend on  $x$

$e \rightarrow \text{ref } 42$ , by substituting  $\text{ref } 42$  in context

let  $x = \text{ref } 42$  in ! $x$ .

result = 42  $\Rightarrow e = (\lambda x. \text{ref } 42) 1$ .

② sol: Given STLC term,  $(\lambda x. \text{int } ()) (5+3)$

the type derivation is as follows

$\Rightarrow \lambda x : \text{int}. ()$  is a lambda function.

$\therefore$  The function type is  $\Rightarrow \text{int} \rightarrow \text{unit}$ .

Type derivation:-

$\frac{}{\vdash 5 : \text{int}} \text{ (T-Int)} \quad \frac{}{\vdash 3 : \text{int}} \text{ (T-Int)}$

$\frac{}{\vdash (5+3) : \text{int}} \text{ (T-Add)}$

$\frac{}{\vdash () : \text{unit}} \text{ (T-unit)}$

$\frac{}{\vdash (\lambda x : \text{int}. ()) : \text{int} \rightarrow \text{unit}} \text{ (T-Abs)}$

$\frac{}{\vdash (\lambda x : \text{int}. ()) (5+3) : \text{unit}} \text{ (T-App)}$

③ sol: Given that.

$$\Omega = (\lambda x. x x) (\lambda x. x x)$$

- ~~Attempt to~~ The term  $\Omega$  cannot be well-typed in STLC.
- To type  $x x$ ,  $x$  would need to have a type of the form  $T \rightarrow T$  for some type  $T$ .
- However this leads to an infinite type  $T = T \rightarrow T$ .
- $T$ , which is not allowed in STLC's finite types.

④ sol: Given that,

$$\vdash \lambda a : x. \lambda b : y. (a (b + 3))$$

- The constraint for the above term is as follows.
- $a (b + 3)$ , Here  $a$  is applied to the result  $(b + 3)$  which is an int. This means " $a$ " must be a function.
- Therefore, the constraint set is  $\{ y \equiv \text{int}, x \equiv \text{int} \rightarrow z, \text{constraint } z \equiv \text{type of } (a (b + 3)) \}$
- $\{ y \equiv \text{int}, x \equiv \text{int} \rightarrow z, z \equiv \text{type of } (a (b + 3)) \}$

⑤ sol: The given constraint set is not unifiable.

$$\{ z \equiv \text{int}, x \equiv x \rightarrow z, y \equiv \text{int}, \text{int} \equiv \text{int} \}$$

- The constraint  $x \equiv x \rightarrow z$  leads to an infinite type when we try to substitute  $z$  with  $\text{int}$ .
- This would result in  $x \equiv x \rightarrow \text{int}$ , which further expands to  $x \equiv (x \rightarrow \text{int}) \rightarrow \text{int}$ , and so on indefinitely.
- "STLC" does not allow infinite types.