University of Massachusetts Lowell — Comp 3010: Organization of Programming Languages

Assignment 5

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Make sure that the remaining pages of this assignment do not contain any identifying information.

1 Substitution (20 points)

(a) Write an inductive definition of the function FV.

$$\begin{aligned} FV(x) &= \{x\} \quad \text{(for a variable } x\text{)} \\ FV(\lambda x.\, e) &= FV(e) \setminus \{x\} \quad \text{(for an abstraction)} \\ FV(e_1\ e_2) &= FV(e_1) \cup FV(e_2) \quad \text{(for an application)} \end{aligned}$$

(b) Show the result of the following substitutions.

i. $(\lambda z.\, y\ \lambda y.\, y\ z\ w)\{(\lambda x.\, x)/y\}$

$$\begin{aligned} &= \lambda z.\, (\lambda x.\, x)\ \lambda y.\, y\ z\ w \\ &= \lambda z.\, \lambda x.\, x\ \lambda y.\, y\ z\ w \end{aligned}$$

ii. $((\lambda x.\, x\ y)\ (\lambda z.\, x\ z))\{(\lambda w.\, w\ w)/x\}$

$$\begin{aligned} &= ((\lambda w.\, w\ w)\ y)\ (\lambda z.\, (\lambda w.\, w\ w)\ z) \\ &= (\lambda w.\, w\ w\ y)\ (\lambda z.\, \lambda w.\, w\ w\ z) \end{aligned}$$

iii. $(\lambda y.\, x\ y)\{(\lambda z.\, y\ z)/x\}$

$$\begin{aligned} &= \lambda y.\, (\lambda z.\, y\ z)\ y \\ &= \lambda y.\, \lambda z.\, y\ z\ y \end{aligned}$$

iv. $((\lambda x.\, \lambda w.\, w\ x)\ \lambda y.\, x\ y)\{(w\ w)/x\}$

$$\begin{aligned} &= (\lambda w.\, w\ (w\ w))\ \lambda y.\, (w\ w)\ y \\ &= \lambda w.\, w\ (w\ w)\ \lambda y.\, w\ w\ y \end{aligned}$$

(c) For each alternate definition, give an example substitution in which the original and alternate definitions produce different results.

(i) Alternate definition:

$$(\lambda y.\, e')\{e/x\} = \begin{cases} \lambda y.\, e' & \text{if } x = y \\ \lambda y.\, e'\{e/x\} & \text{if } x \neq y \end{cases}$$

Example: $(\lambda y.\, x)\{y/x\}$
- **Original Definition:** $\lambda z.\, z$ - **Alternate Definition:** $\lambda y.\, y$

(ii) Alternate definition:

$$(\lambda y.\, e')\{e/x\} = \begin{cases} \lambda y.\, e' & \text{if } x = y \\ \lambda y.\, e'\{e/x\} & \text{if } x \neq y \text{ and } y \notin FV(e) \\ \lambda z.\, (e'\{z/y\})\{e/x\} & \text{if } x \neq y \text{ and } y \in FV(e),\ \text{where } z \notin FV(e) \cup \{x\} \end{cases}$$

Example: $(\lambda y.\, x\ y)\{y/x\}$
- **Original Definition:** $\lambda z.\, z\ y$ - **Alternate Definition:** $\lambda y.\, y\ y$