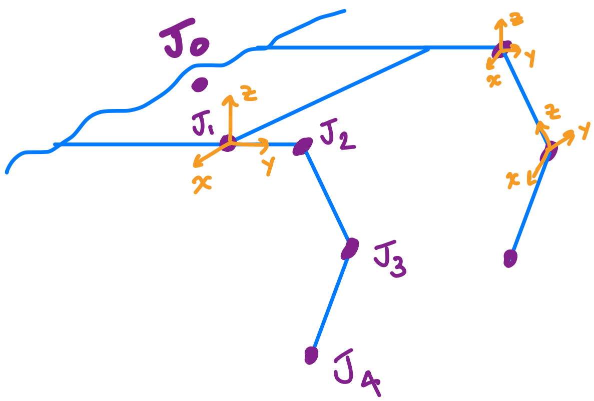
Co-ordinate System:



J_0: center of Dog [CoM]

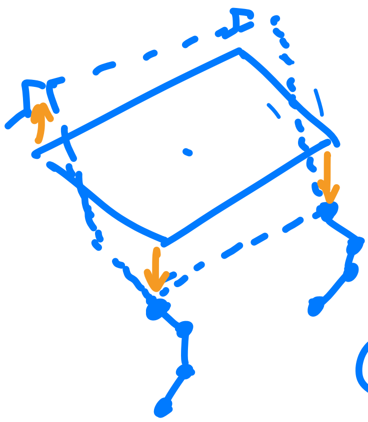
J_1: Leg origin

J_2: Femur origin

J_3: Tibia origin

J_4: foot

Orientation Control / Rotating Main Body:



① New co-ord of feet relative to J_0, center of Robot:

$$\overrightarrow{XYZ}_0 = R^{-1}\Big(\underbrace{[\text{CoM origin}]}_{\text{location of } J_1 \text{ relative to } J_0} + \underbrace{[x, y, z]}_{\text{location of foot, relative to } J_1} - [\text{Center of Rotation}]\Big)^T$$

$R = R_{yaw} R_{pitch} R_{roll}$, desired rotation matrix

J_R, point to rotate about.

$$= [x\ y\ z] \leftarrow \text{column vector}$$

② New co-ord of feet relative to leg's origin:

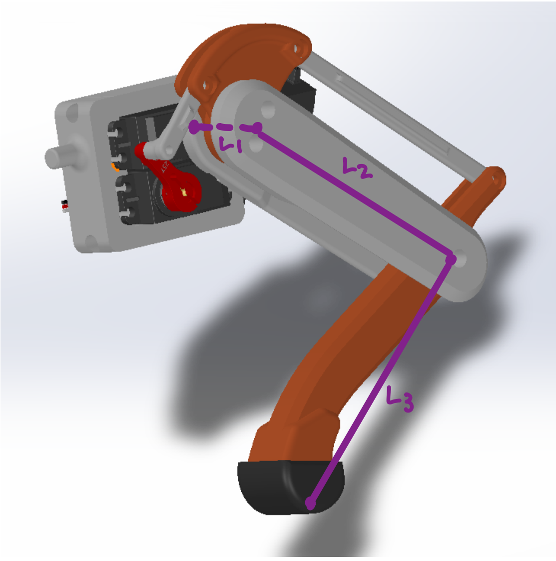
$$\overrightarrow{XYZ}_1 = \overrightarrow{XYZ}_0 - [\text{CoM origin}] + [\text{Center of Rotation}]$$

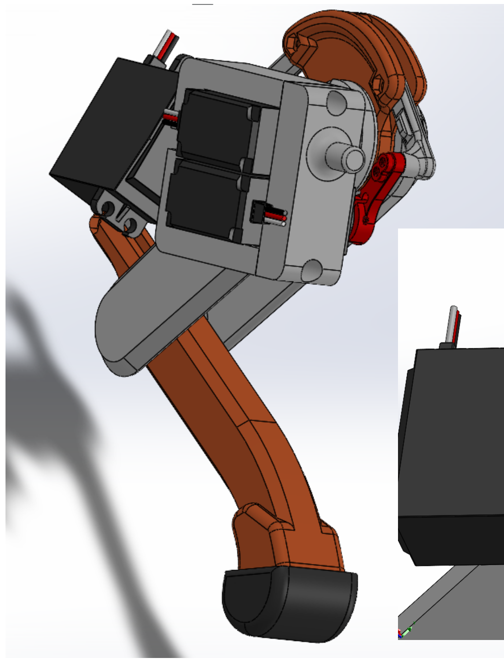
$$= [x', y', z'] \leftarrow \text{Use Inverse Kinematics on this.}$$

Inverse Kinematics:

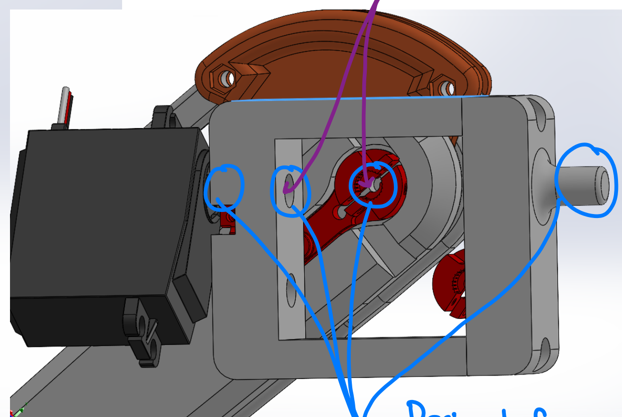
Known: Foot (end effector) position
↳ X, Y, Z {given} relative to J_1

Desired: Joint angles: θ_{hip}, θ_{femur}, θ_{tibia}





Intersection of these points
⇒ leg origin J_1



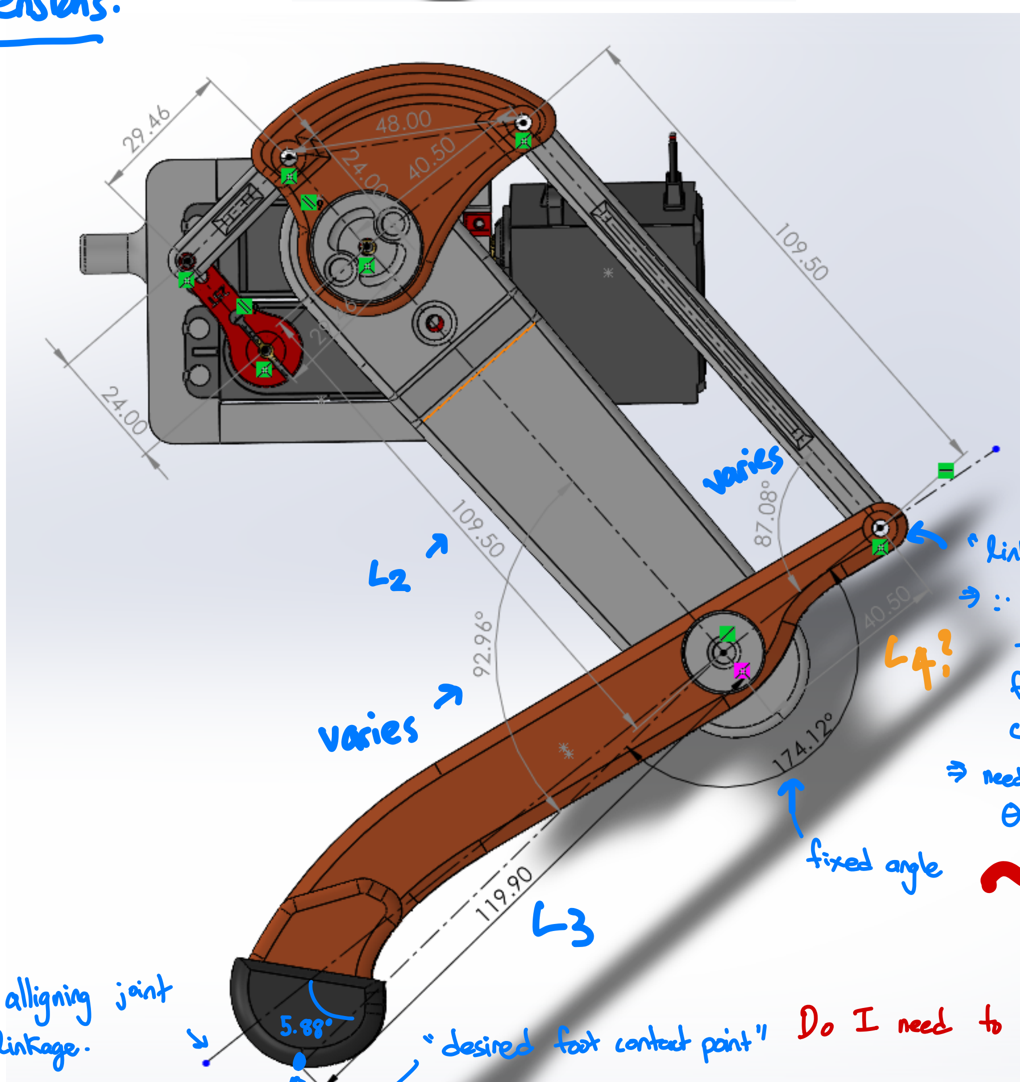
Designed for all to be collinear

Reference Dimensions:

L_1: Hip

L_2: Femur

L_3: Tibia



"linkage point"
⇒ ∵ tibia curves, this point, the joint, and desired foot contact point are not collinear
⇒ need to account for offset
$\theta_{tibia\ offset} = 5.88°$

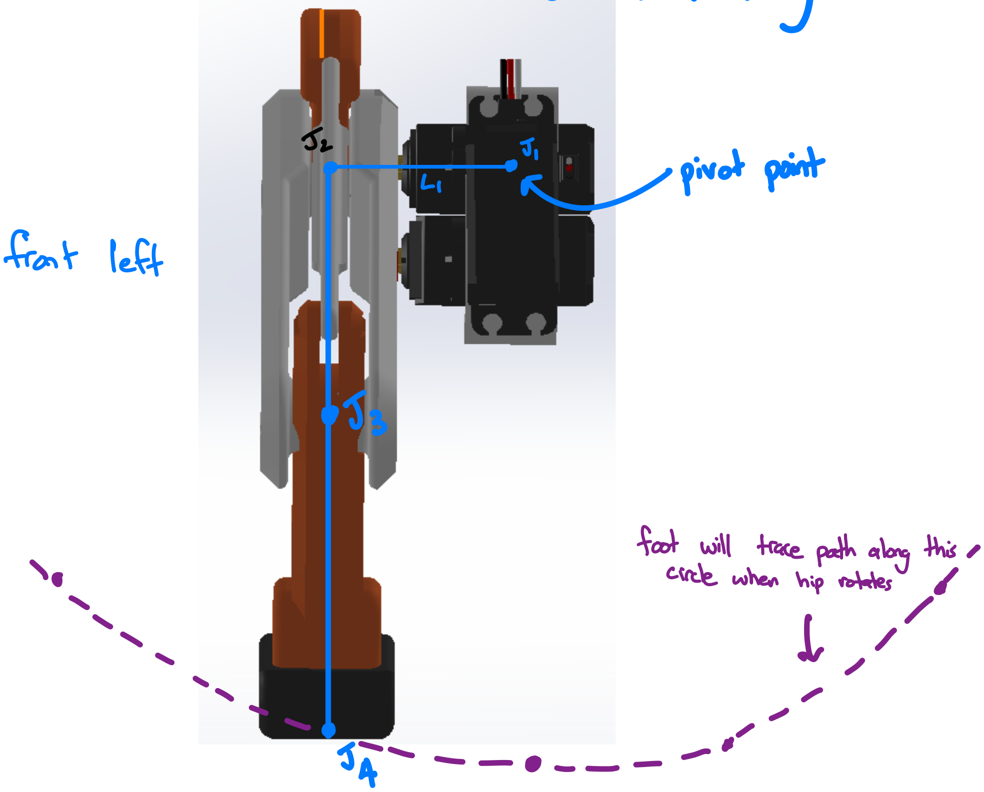
axis alligning joint and linkage.

"desired foot contact point"

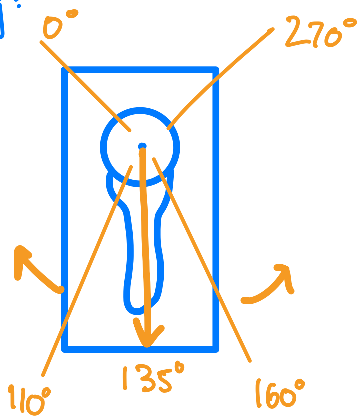
Do I need to get this complex?!

Hip Joint :

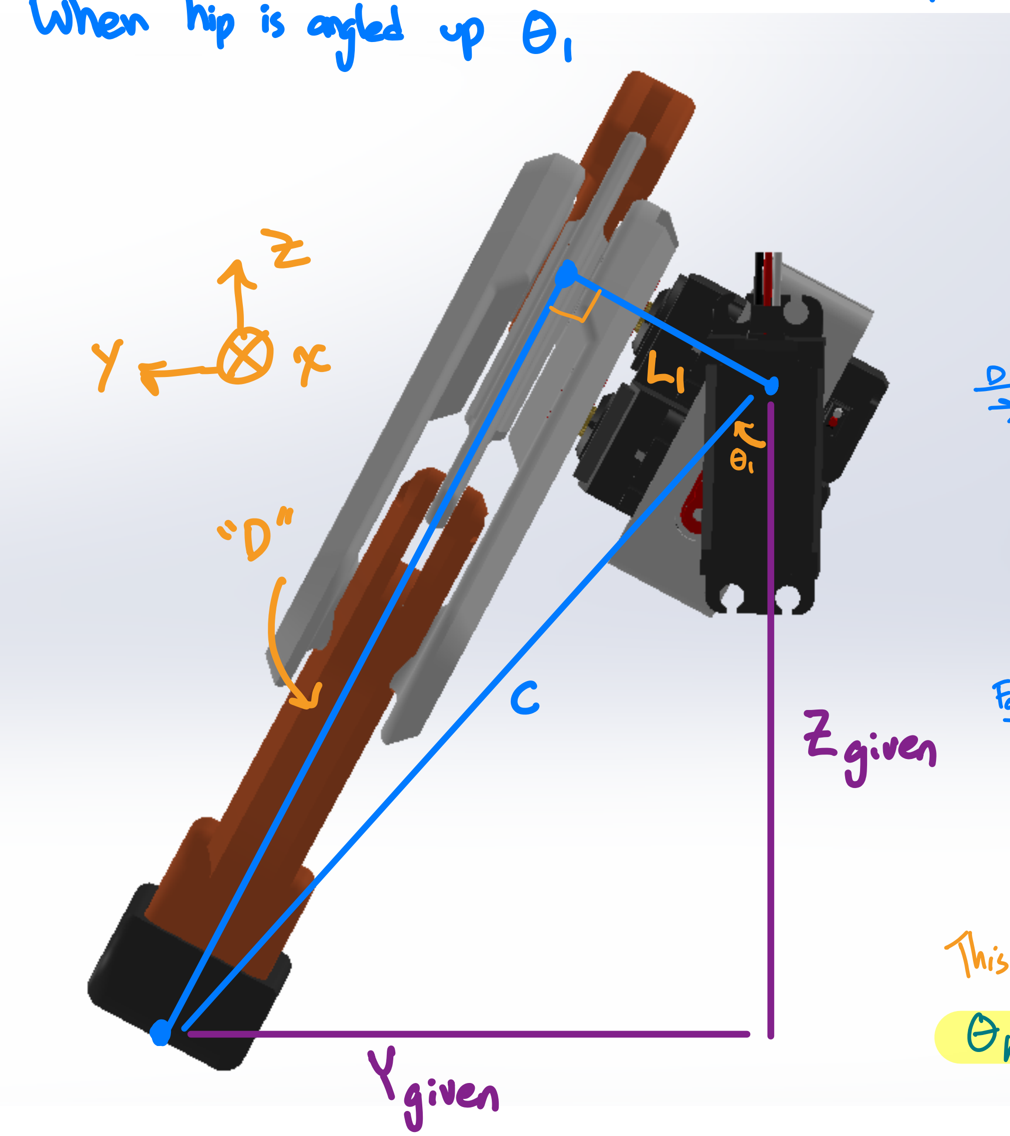
Back view of front left leg



Front view of hip servo on front left leg:



When hip is angled up θ_1



"D" → leg length desired in the leg plane

Goal: Find θ_1, D.

D →

$$C = \sqrt{Y_{given}^2 + Z_{given}^2}$$

$$D = \sqrt{C^2 - L_1^2}$$

$$\Rightarrow \boxed{D = \sqrt{Y_{given}^2 + Z_{given}^2 - L_1^2}}$$

→ for vertical leg ⇒ $Y_{given} = L_1$

For θ_1

$$\boxed{\theta_1 = \tan^{-1}\left(Y_{given} / Z_{given} \right)}$$

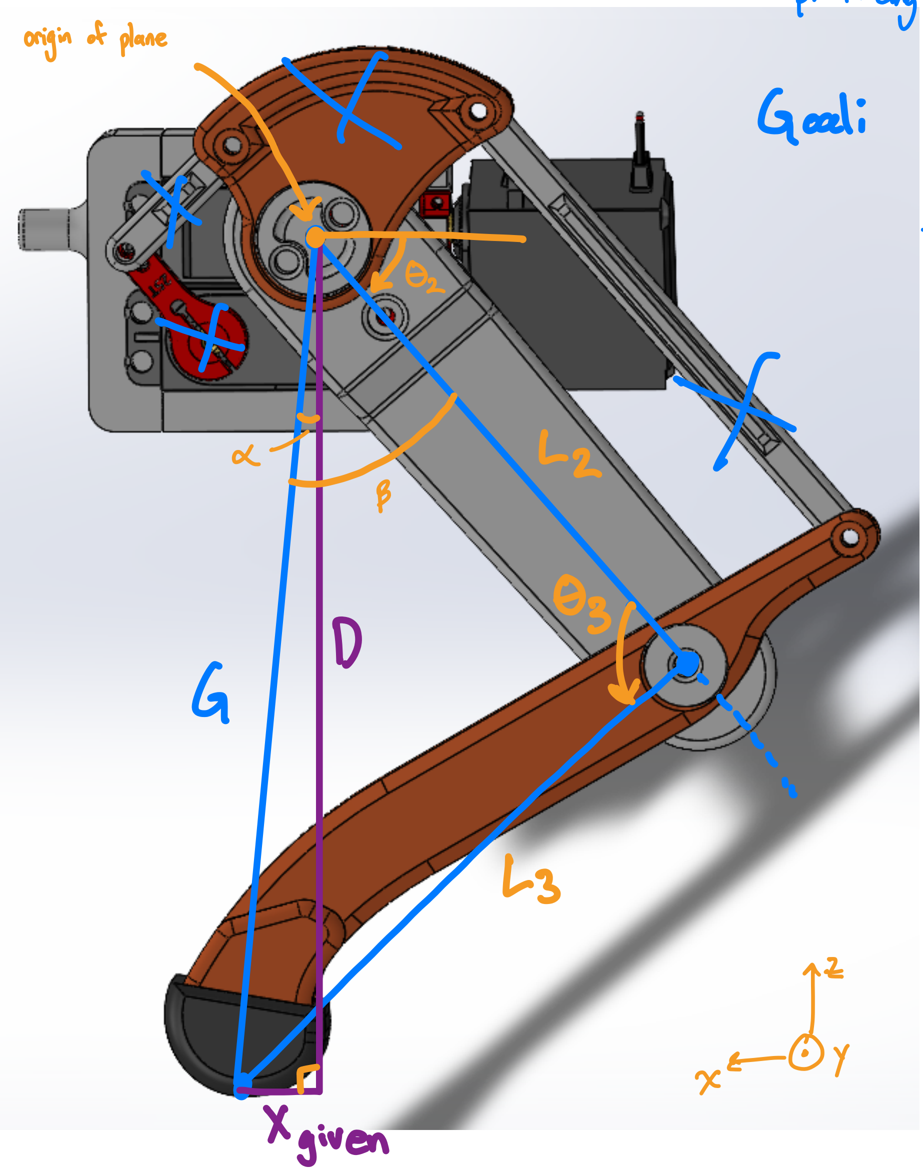
For right leg, Y_{given} flips ⇒ $\theta_1 = -\tan\ldots$

This θ_1 is a virtual angle. For servo,

$$\theta_{hip\,servo} = 135° + \theta_1$$

↑ to align servo 0° w/ math

Leg Plane (X-Z plane): Simplify first, by not accounting for $\theta_{offset} = 5.88°$ and pretending tibia servo is mounted on knee joint.



Goal: Find θ_2, θ_3:

$\rightarrow$ $G = \sqrt{D^2 + X_{given}^2}$

For θ_3, cos law:

$$G^2 = L_2^2 + L_3^2 - 2L_2L_3\cos\theta_3$$

$$\Rightarrow \boxed{\theta_3 = \cos^{-1}\left(\frac{G^2 - L_2^2 - L_3^2}{-2L_2L_3}\right)}$$

For θ_2:

$$\boxed{\alpha = \tan^{-1}\left(\frac{X_{given}}{D}\right)}$$

β: Sine law

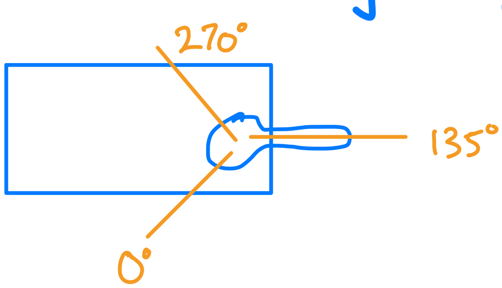
$$\frac{\sin\beta}{L_3} = \frac{\sin\theta_3}{G}$$

$$\Rightarrow \boxed{\beta = \sin^{-1}\left(\frac{L_3\sin\theta_3}{G}\right)}$$

$$\Rightarrow \boxed{\theta_2 = \frac{\pi}{2} - (\beta - \alpha)}$$

θ_2, θ_3 are "virtual angles" $\rightarrow$ for servo,

Femur Servo Mounting / Range of Motion:

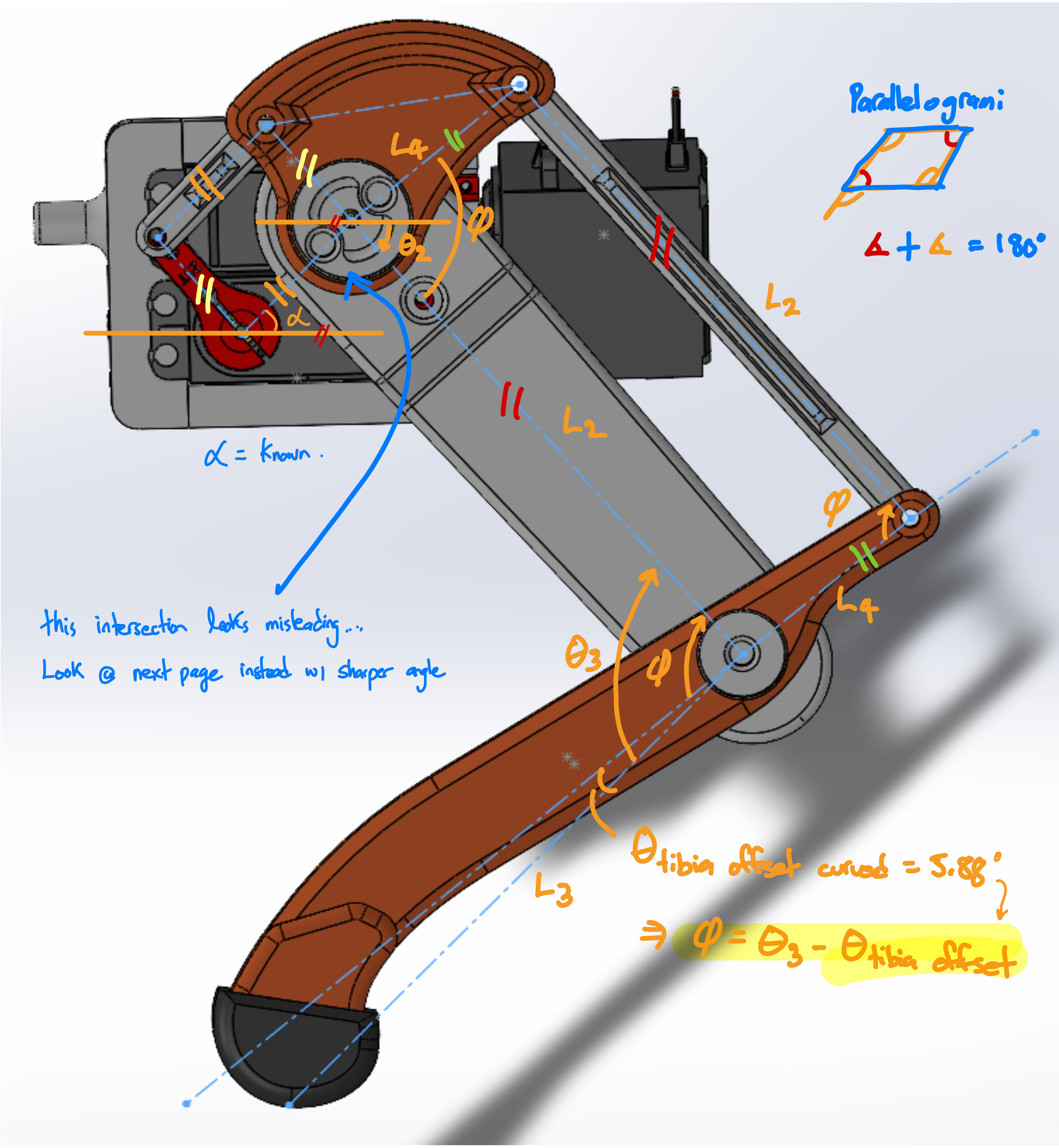


$$\Rightarrow \theta_{femur\ servo} = 135° - \theta_2$$

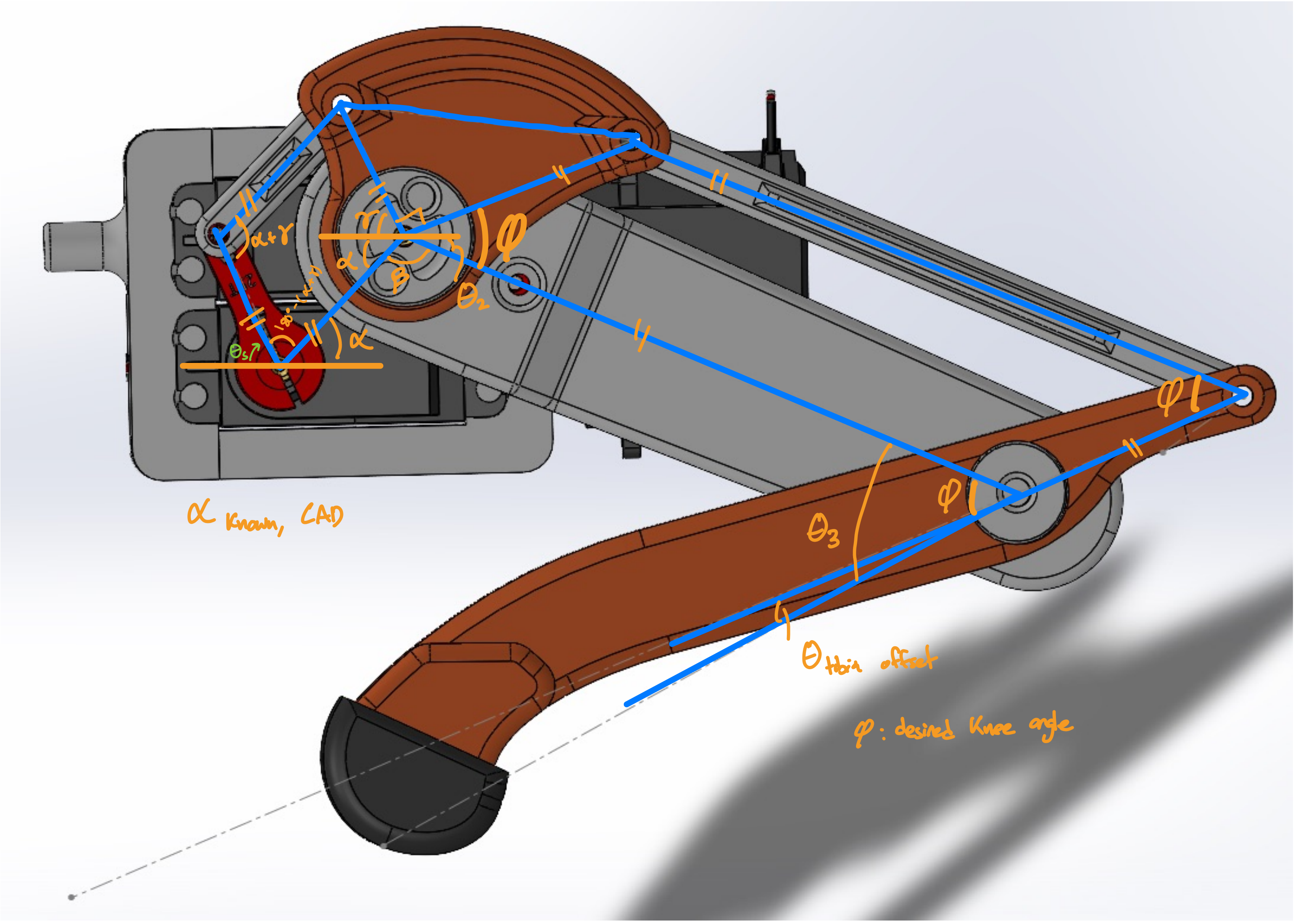
↑ offset alignment, varies w/ each servo

For θ_{tibia} servo, need to draw linkages....

// = parallel pairs ⇒ linkages are designed to be 2 consecutive parallelograms



Analyzing sharper angle for $\theta_{servo\ tibia}$



$\rightarrow \beta = 180 - \alpha - \theta_2$

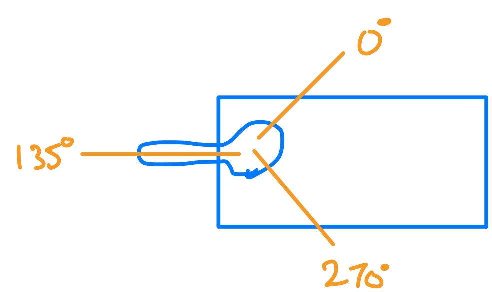
$\rightarrow \gamma = 360 - 90 - \varphi - \beta - \alpha$

$$\therefore \theta_s = 180^\circ - \alpha - [180^\circ - (\alpha + \gamma)]$$
$$= -\alpha + \alpha + \gamma$$
$$= \gamma$$
$$= 360^\circ - 90 - \varphi - \beta - \alpha \quad [\text{Subst. } \gamma$$
$$= 360^\circ - 90^\circ - \varphi - \alpha - (180^\circ - \alpha - \theta_2) \quad [\text{Subst. } \beta$$
$$= 360^\circ - 90^\circ - \varphi - \alpha - 180^\circ + \alpha + \theta_2$$
$$= 90^\circ - \varphi + \theta_2$$

Now, to account for servo alignment:

$$\theta_{servo,\,tibia} = 135^\circ - \theta_s$$

offset alignment



or possibly $= \varphi + \theta_2$ based on other sources??