

# Algorithms & Data Structure

Kiran Waghmare

# Program 2

HighArray
public HighArray()//Constructor
public boolean find (int key) public void insert(int value) public boolean delete(int long) public void display()

HighArrayApp
main() create object
insert()// all elements
display() find() delete()

# Performance of Algorithms

Algorithm: set of rules required to perform calculations.

- complexity

  - Time complexity

  - Space complexity

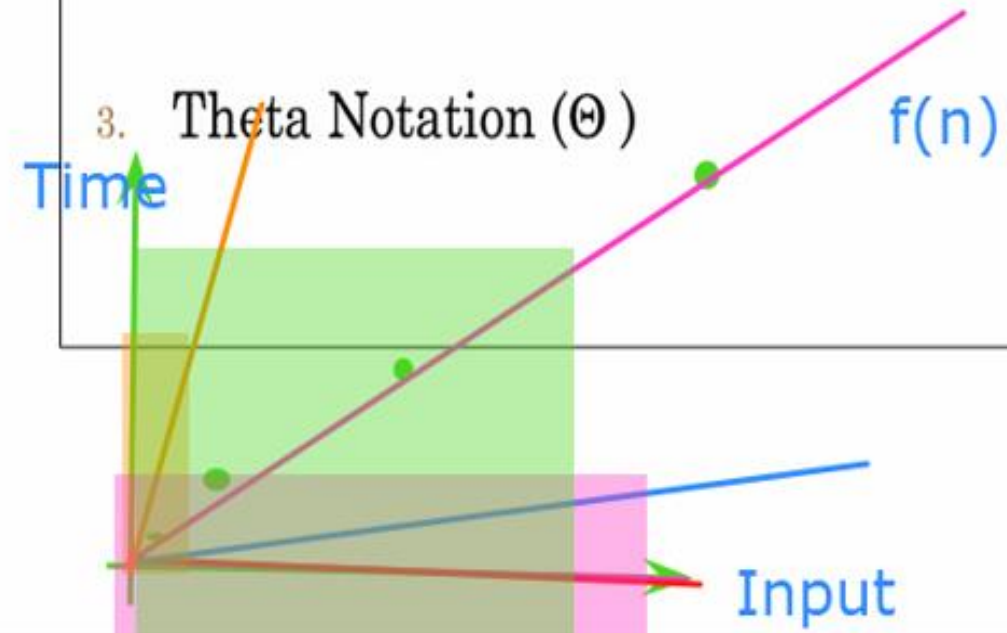
## WHY IMPORTANT ??

- Give a simple characterization of an algorithm's efficiency.
- Allow comparison of performances of various algorithms

1-1sec  
10-10sec  
1000-1000sec  
1lac-1lacsec

## ASYMPTOTIC NOTATIONS

1. Big-oh Notation ( $O$ )
2. Big-Omega Notation ( $\Omega$ )
3. Theta Notation ( $\Theta$ )



## WHY IMPORTANT ??

- Give a simple characterization of an algorithm's efficiency.
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## ASYMPTOTIC NOTATIONS

1. Big-oh Notation ( $O$ ) **Worst case**
2. Big-Omega Notation ( $\Omega$ ) **Best case**
3. Theta Notation ( $\Theta$ ) **Average case**

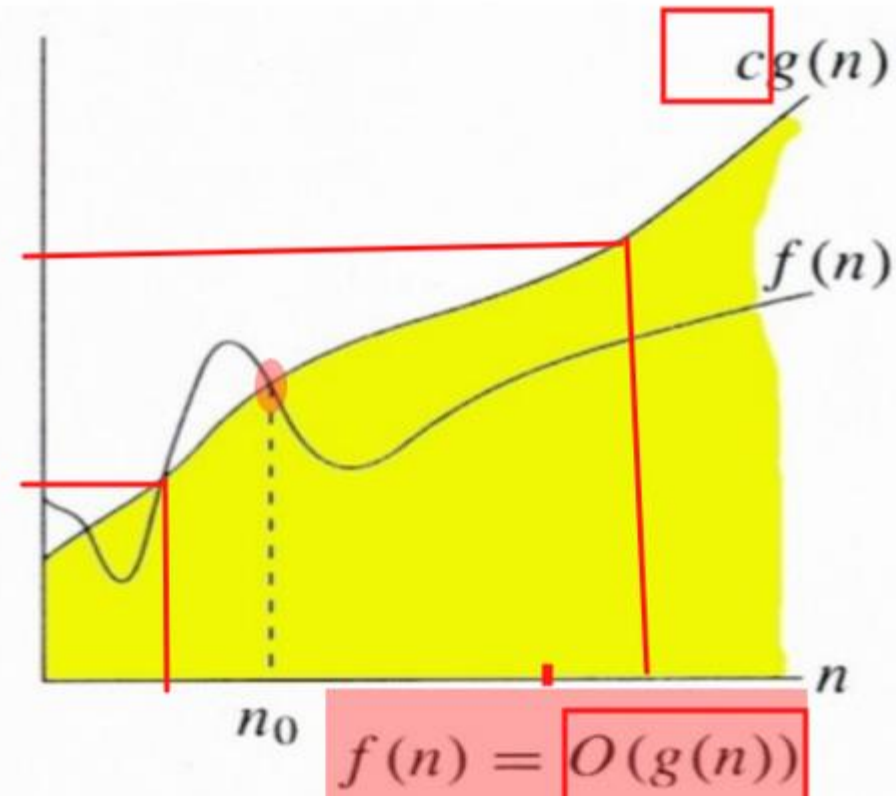
# BIG-OH NOTATION (O)

$$T(n) = T(n/2) + n$$

- Gives the **upper bound** of algorithm's running time.
- Let  $f: \mathbb{N} \rightarrow \mathbb{R}$  be a function. Then  $O(f)$  is the set of functions

$O(f) = \{ g: \mathbb{N} \rightarrow \mathbb{R} \mid \text{there exists a constant } c \text{ and a natural number } n_0 \text{ such that}$

$$|g(n)| \leq c |f(n)| \text{ for all } n \geq n_0 \}$$



# BIG-OMEGA NOTATION ( $\Omega$ )

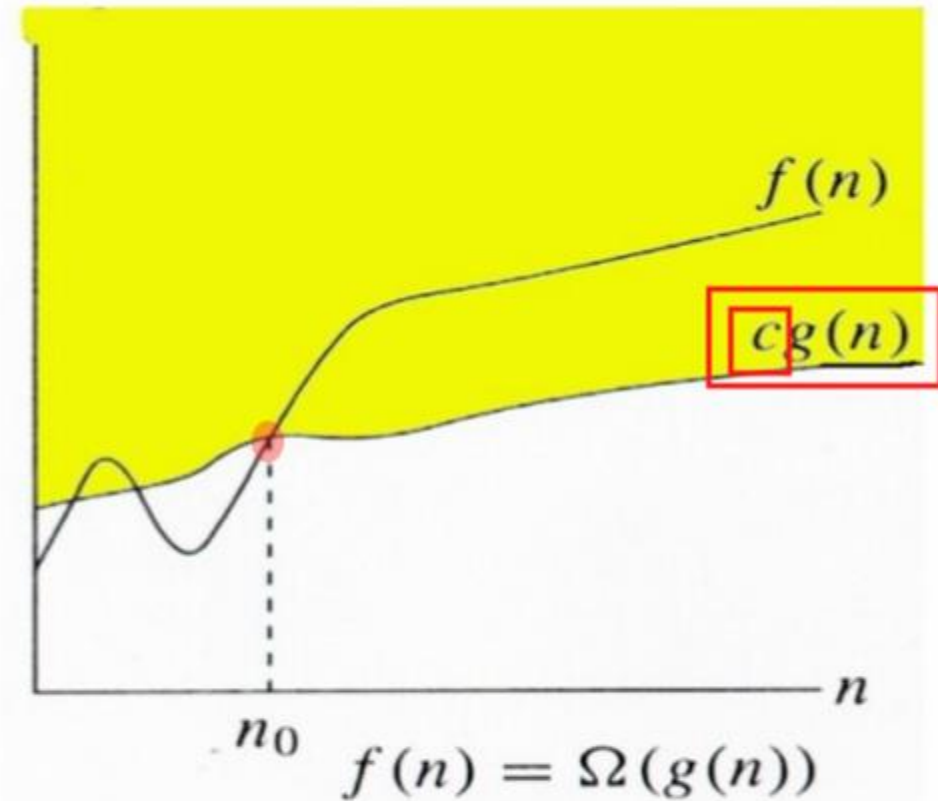
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- Gives the **lower bound** of algorithm's running time.
- Let  $f, g: \mathbb{N} \rightarrow \mathbb{R}$  be functions from the set of natural numbers to the set of real numbers.

We write  $g \in \Omega(f)$  if and only if there exists some real number  $n_0$  and a positive real constant  $c$  such that

$$g(n) \geq c f(n)$$

for all  $n$  in  $\mathbb{N}$  satisfying  $n \geq n_0$ .





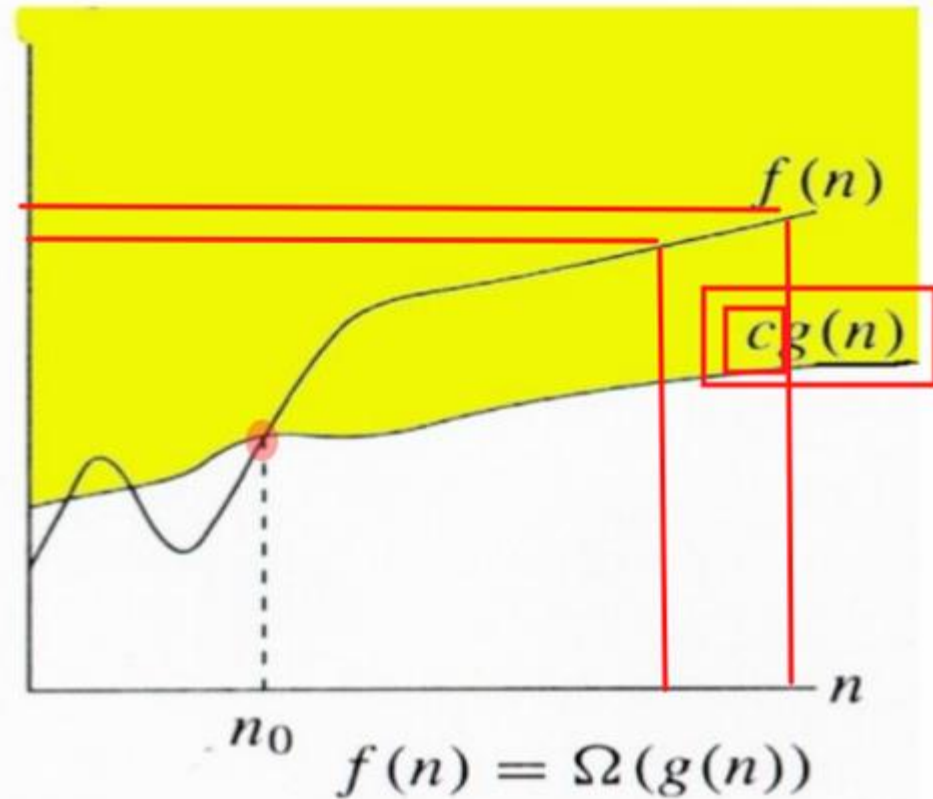
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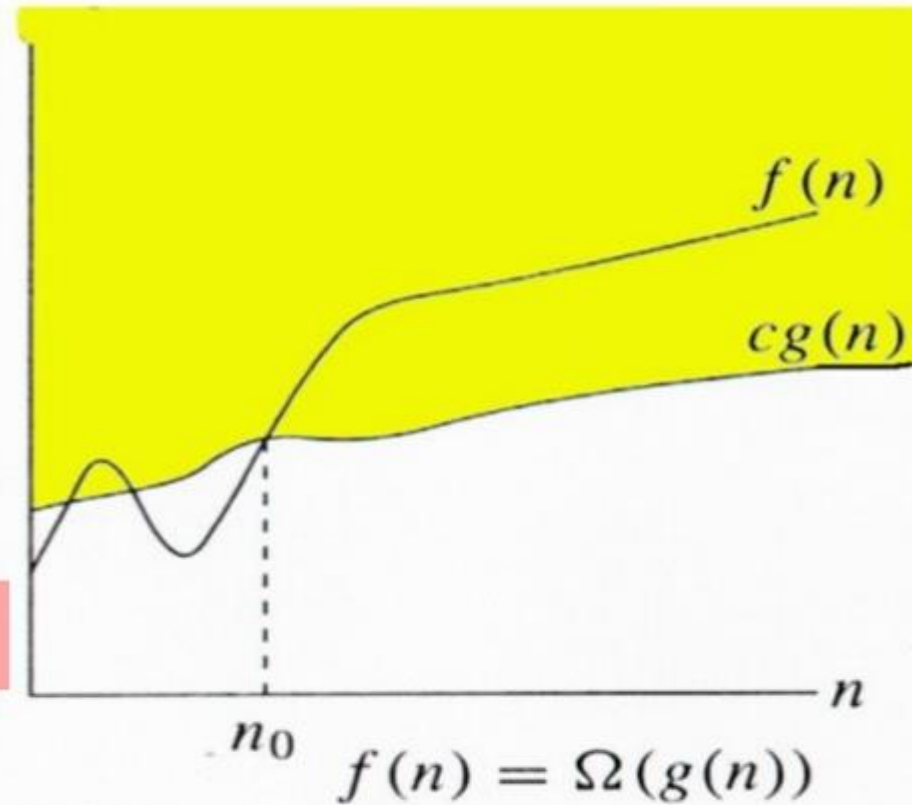
# THETA NOTATION (Θ)

○ If  $f$  and  $g$  are functions from  $S$  to the real numbers, then we write  $g \in \Theta(f)$  if and only if there exists some real number  $n_0$  and positive real constants  $C$  and  $C'$  such that

$$C |f(n)| \leq |g(n)| \leq C' |f(n)|$$

for all  $n$  in  $S$  satisfying  $n \geq n_0$ .

Thus,  $\Theta(f) = O(f) \cap \Omega(f)$



# INTUITION ABOUT THE NOTATIONS

notation

$O$  (Big-Oh)

$\Omega$  (Big-Omega)

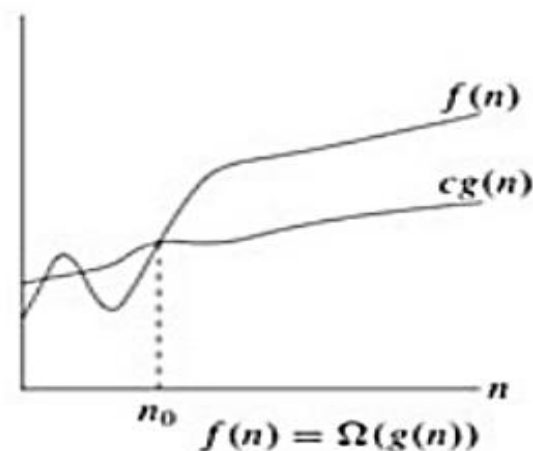
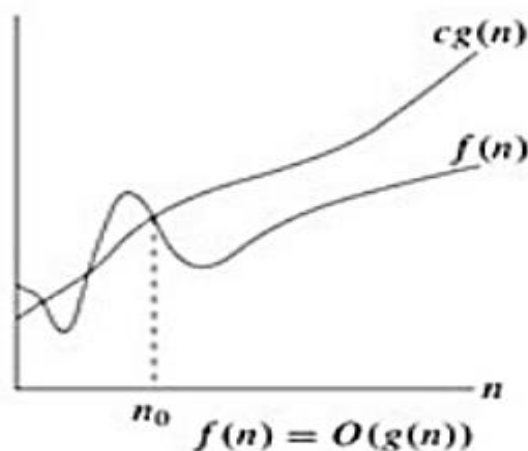
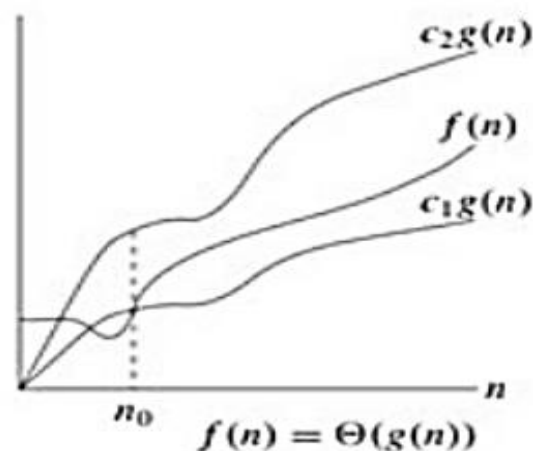
$\Theta$  (Theta)

intuition

$$f(n) \leq g(n)$$

$$f(n) \geq g(n)$$

$$f(n) = g(n)$$



# Analogy to Arithmetic Operators

$$f(n) = O(g(n)) \approx a \leq b$$

$$f(n) = \Omega(g(n)) \approx a \geq b$$

$$f(n) = \Theta(g(n)) \approx a = b$$

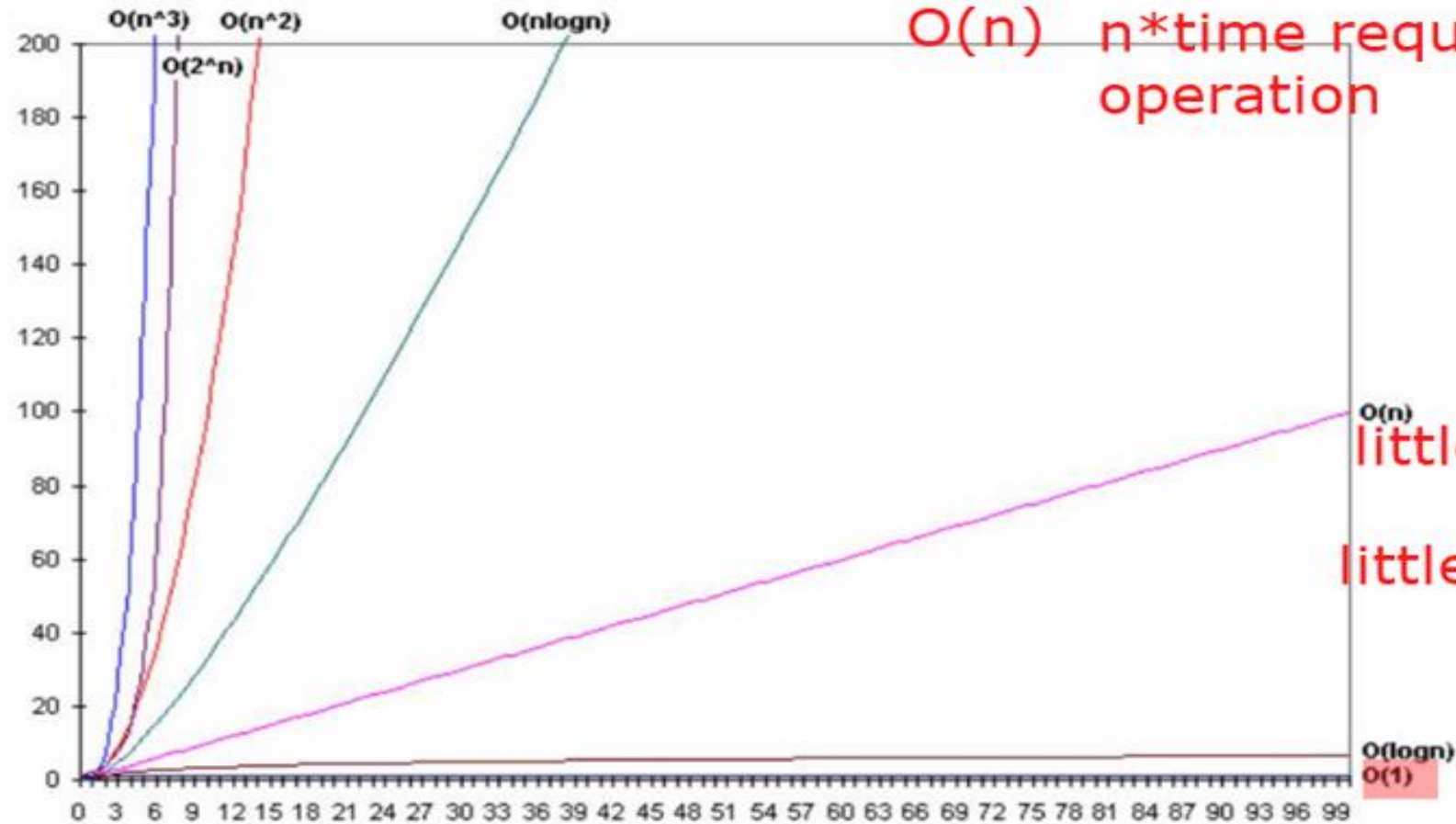
$$f(n) = o(g(n)) \approx a < b \text{ little-Oh}$$

$$f(n) = \omega(g(n)) \approx a > b \text{ little-Omega}$$



# Is there a “real” difference?

- Growth of functions



$O(1)$  const time for each operation  
 $O(n)$   $n$ \*time required for that operation

$O(n)$   
little-Oh

little-Omega

$O(\log n)$   
 $O(1)$

# Best, Worst, Average Cases

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Not all inputs of a given size take the same time to run.

Sequential search for  $K$  in an array of  $n$  integers:

- Begin at first element in array and look at each element in turn until  $K$  is found

Best case:

Worst case:

Average case:



# Asymptotic Notations

Following are the commonly used asymptotic notations to calculate the running time complexity of an algorithm.

- **O Notation**
- **$\Omega$  Notation**
- **$\theta$  Notation**

int a1[10];

Access  $O(1)$

Insert  $O(1) \rightarrow O(n):n \text{ elements}$

Search  $O(1) \rightarrow O(n):n \text{ elements}$

Delete  $O(1) \rightarrow O(n):n \text{ elements}$

Worst Case



# Linear Search

- Find 37?

Key=37

0	1	2	3	4	5	6	7	8
20	35	37	40	45	50	51	55	67

↑  
≠

↑  
≠

↑  
=

key

**Return 2**

10 20 30 40 50 60 70 1 5 8

# Binary Search

$$\text{mid} = (0 + 8) / 2 = 4$$

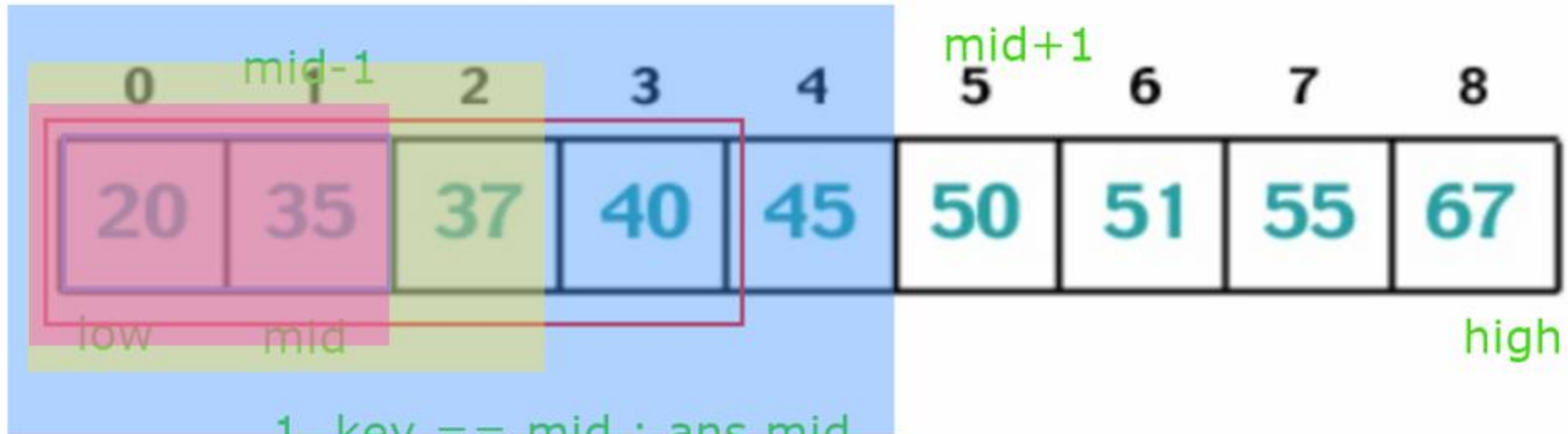
Key=37

Key=35

$O(\log n)$

- Find 37?

1. Sort Array.



1. key == mid ; ans mid

2. key < mid ; Left

3. key > mid ; Right

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class BS

{

static int bsearch(int a1[], int key)

{

int l=0, h=a1.length-1;

while(l <= h)

{

int m=l+(h-1)/2;

if(a1[m] == key)

return m;

if(a1[m] < key)

l=m+1;

else

h=m-1;

}

return -1;

}

public static void main(String args[])

{

Mouse

Select

Text

Draw

Stamp

Spotlight

Eraser

Format

Undo

Redo

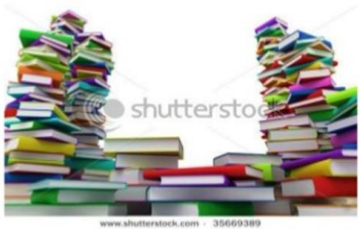


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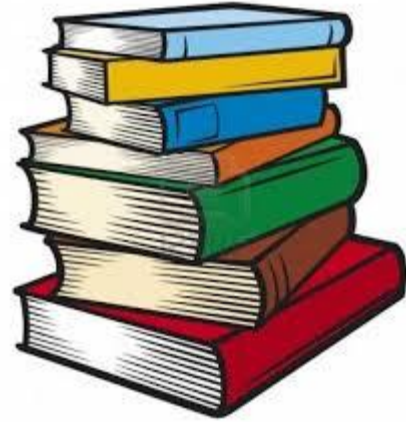
$$h = (7 - 4) / 2 \\ = 1.5$$

## Examples of stack



# Stacks

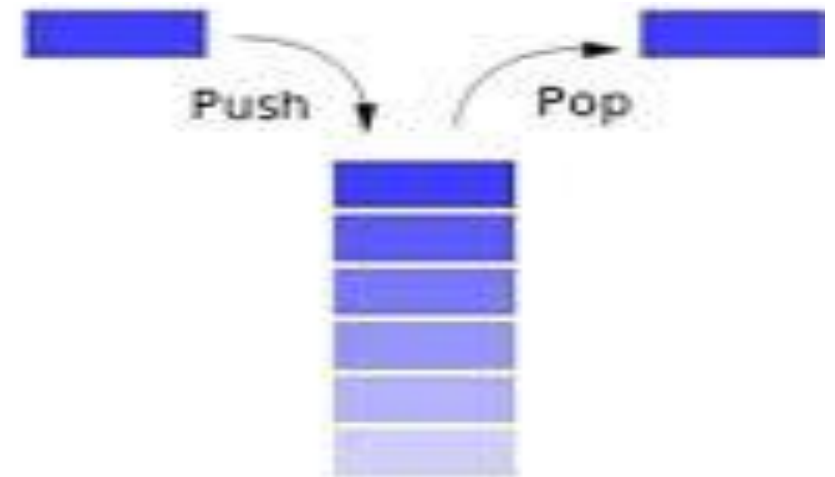
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**Stack of books**



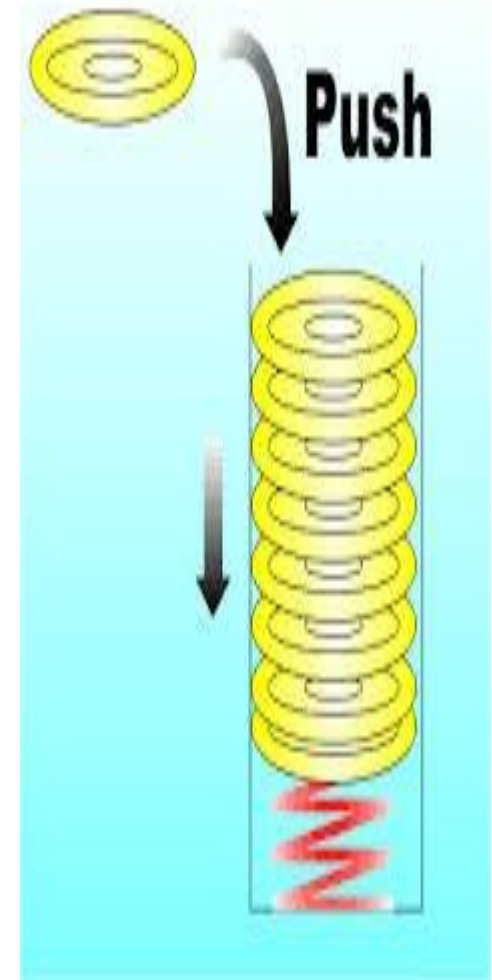
**Stack of Coins**



**Memory stack**

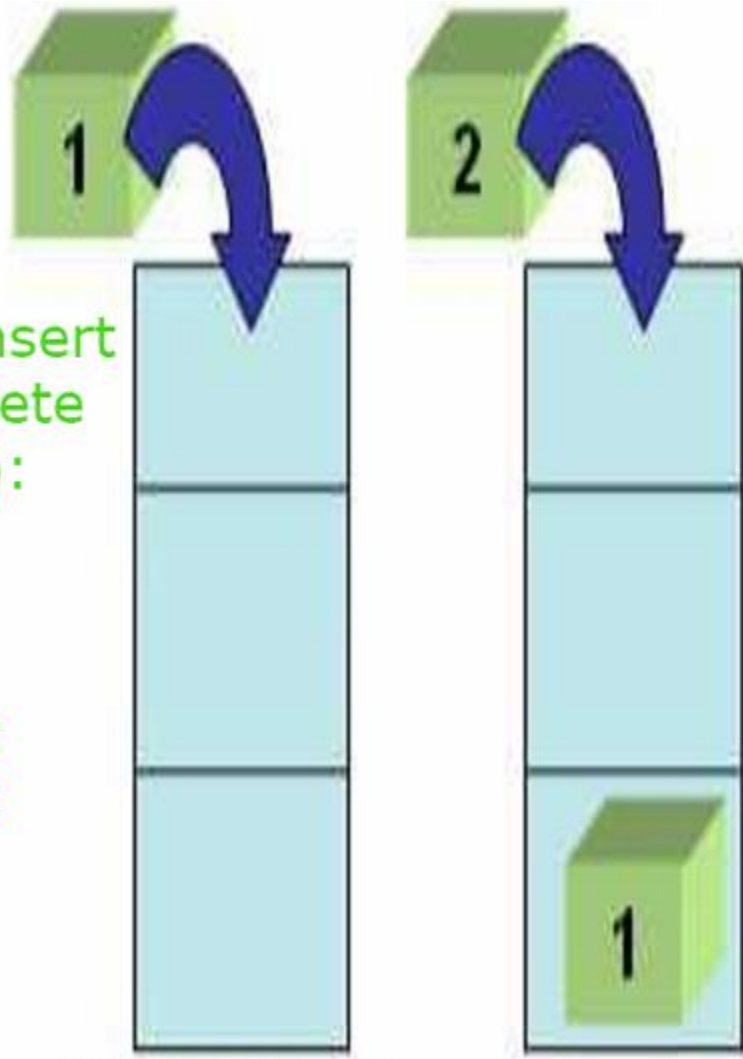
# Stack

- Stack is an ordered list of similar data type.
- Stack is a LIFO structure. (Last in First out).
- push() function is used to insert new elements into the Stack and pop() is used to delete an element from the stack. Both insertion and deletion are allowed at only one end of Stack called Top.
- Stack is said to be in Overflow state when it is completely full and is said to be in Underflow state if it is completely empty.

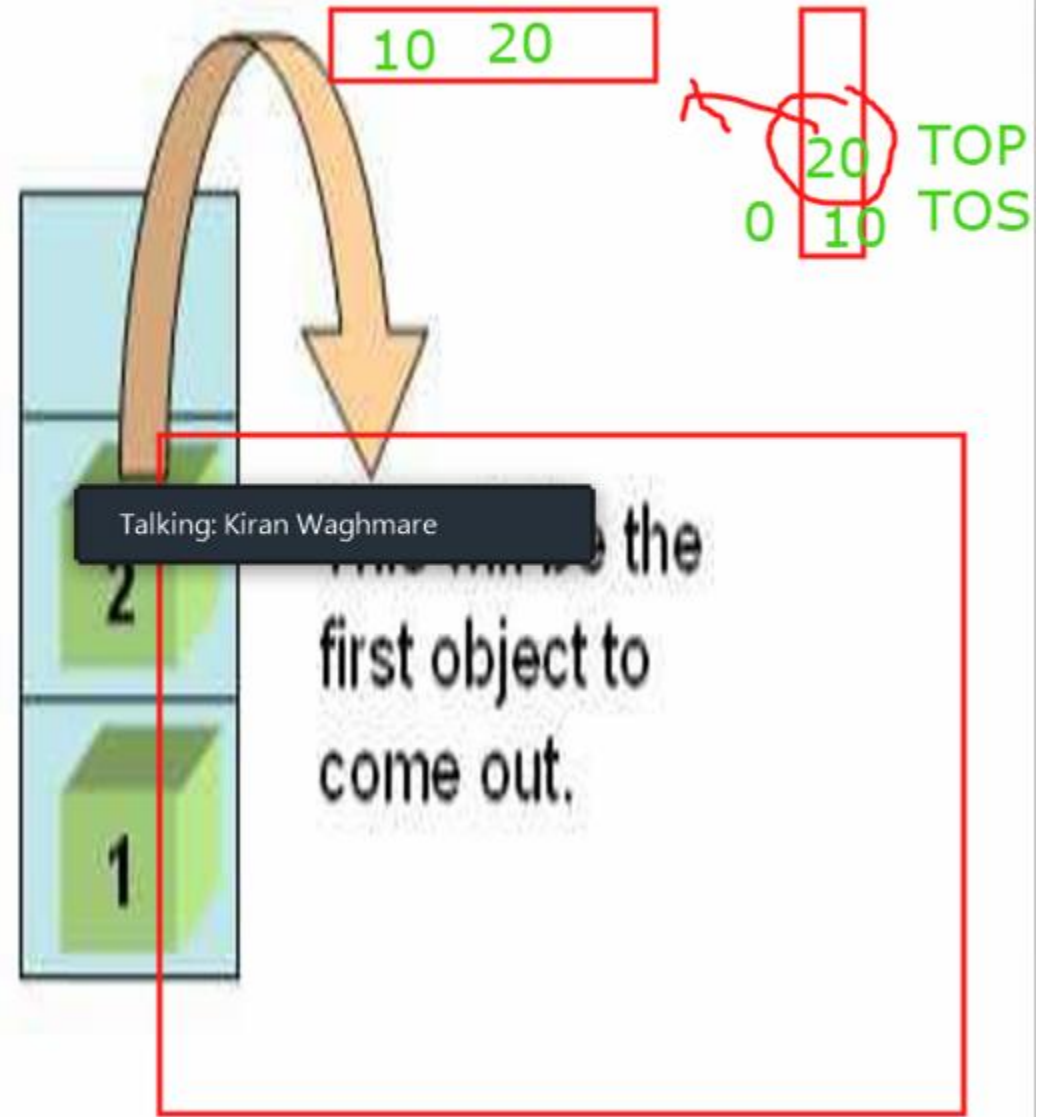




PUSH : Insert  
POP : Delete  
isEmpty():  
isFull():  
peek():  
count():  
change():  
display():

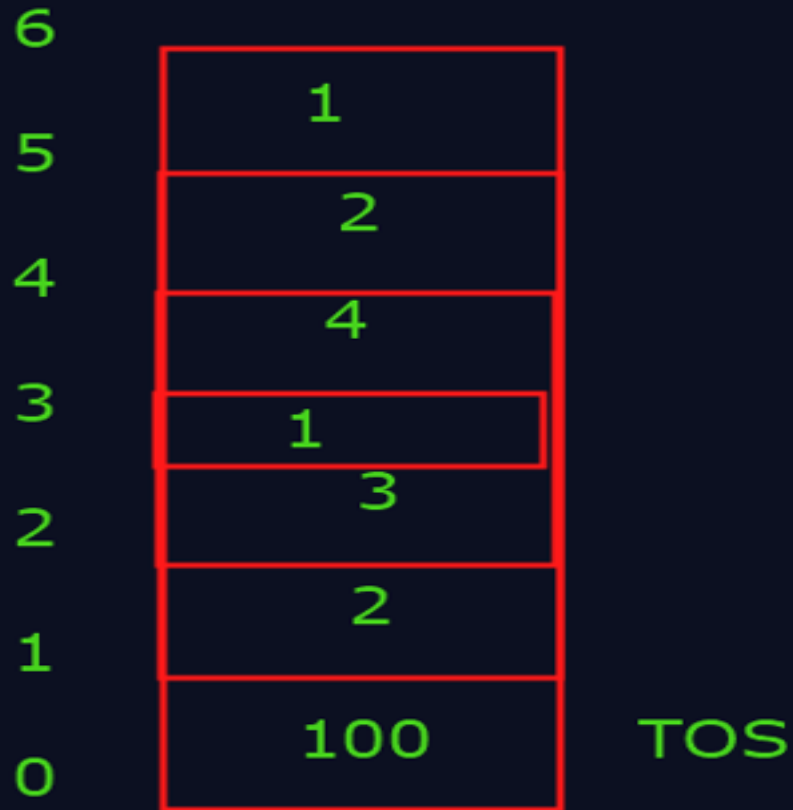


Empty Stack





## Array Representation of Stack



```
int S[7];  
push(10);  
push(20);  
Push(30);  
Push(40);  
Pop();  
pop(20);  
pop();  
pop();
```

TOS=30  
TOS=-1;

**Underflow:** stack is empty &  
we are trying to delete element

**Overflow:** Stack is full &  
we are trying to insert an element