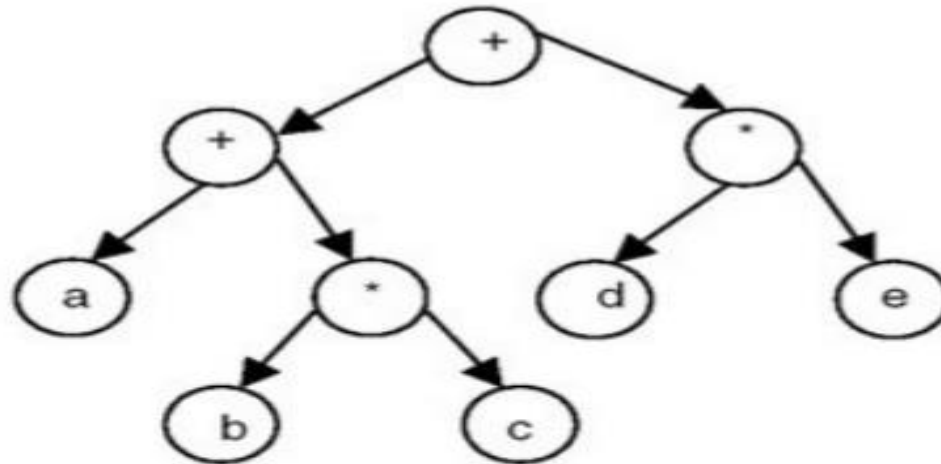


# Algorithms & Data Structure

Kiran Waghmare

# Expression Binary Tree Traversal

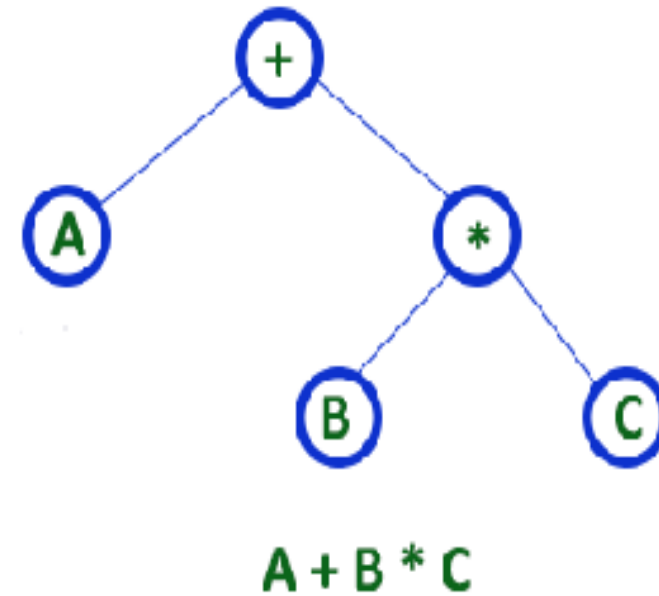
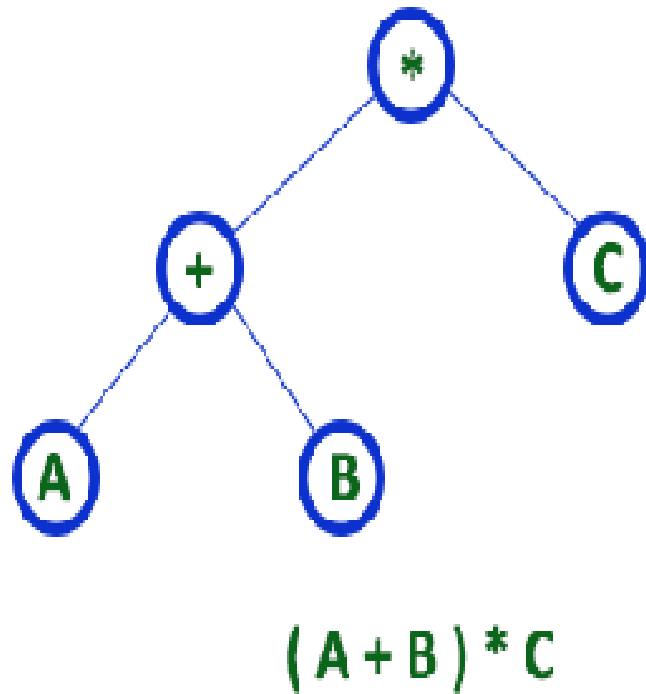
If an expression is represented as a binary tree, the inorder traversal of the tree gives us an infix expression, whereas the postorder traversal gives us a postfix expression as shown in Figure.



Inorder : a + b \* c + d \* e

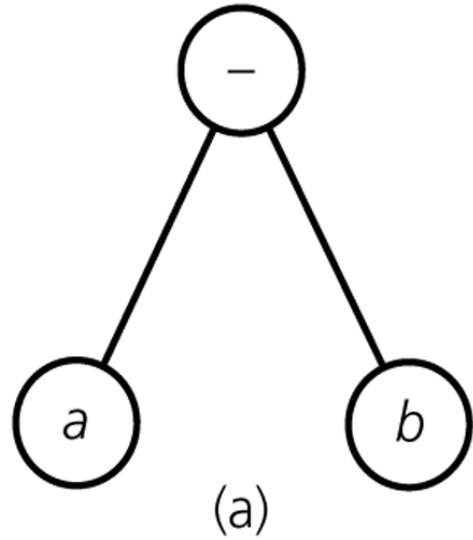
postorder : abc\*+de\*+

**Strictly binary tree data structure is used to represent mathematical expressions.**

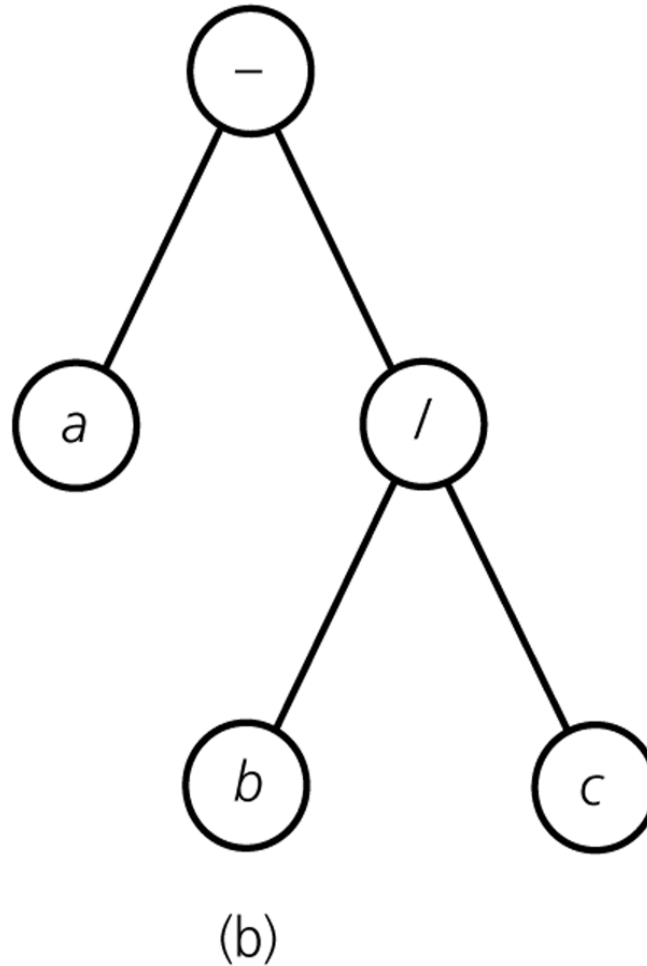


# Binary Tree – Representing Algebraic Expressions

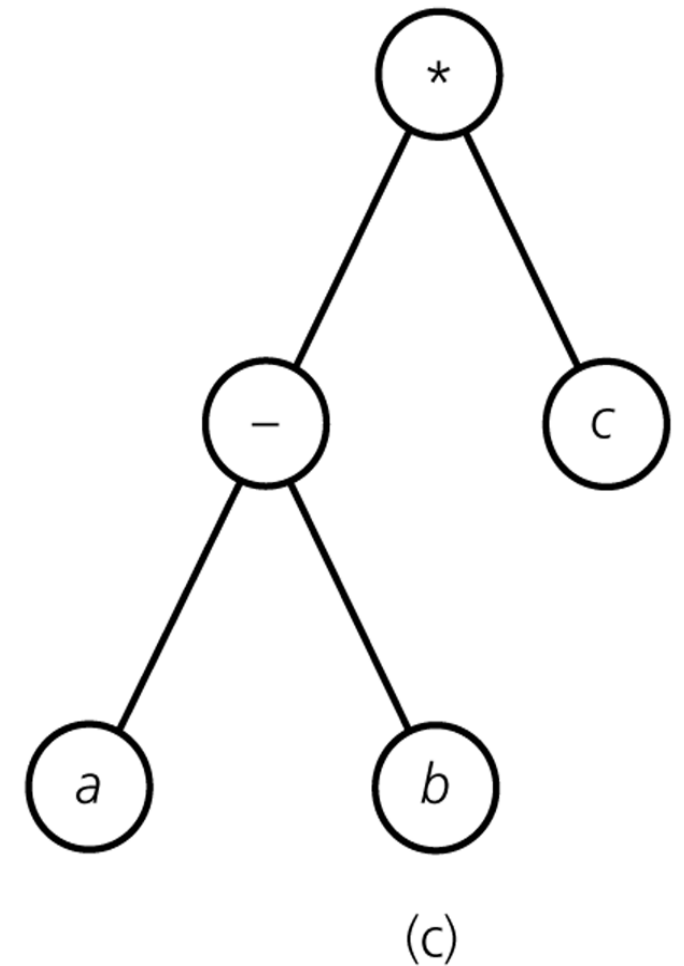
$$a - b$$



$$a - b / c$$

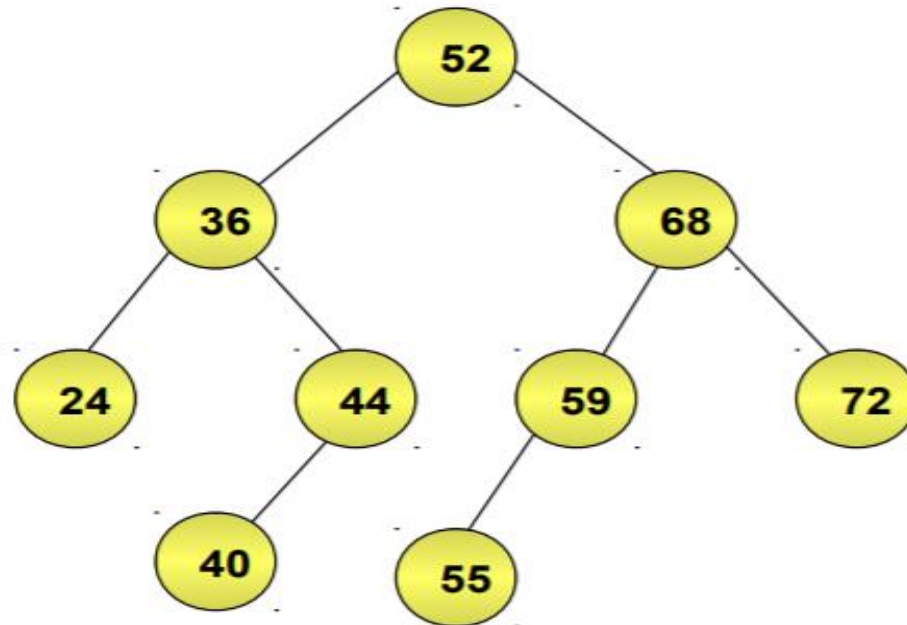


$$(a - b) * c$$



# Binary Search Tree

- ◆ Binary search tree is a binary tree in which every node satisfies the following conditions:
  - ◆ All values in the left subtree of a node are less than the value of the node.
  - ◆ All values in the right subtree of a node are greater than the value of the node.
- ◆ The following is an example of a binary search tree.

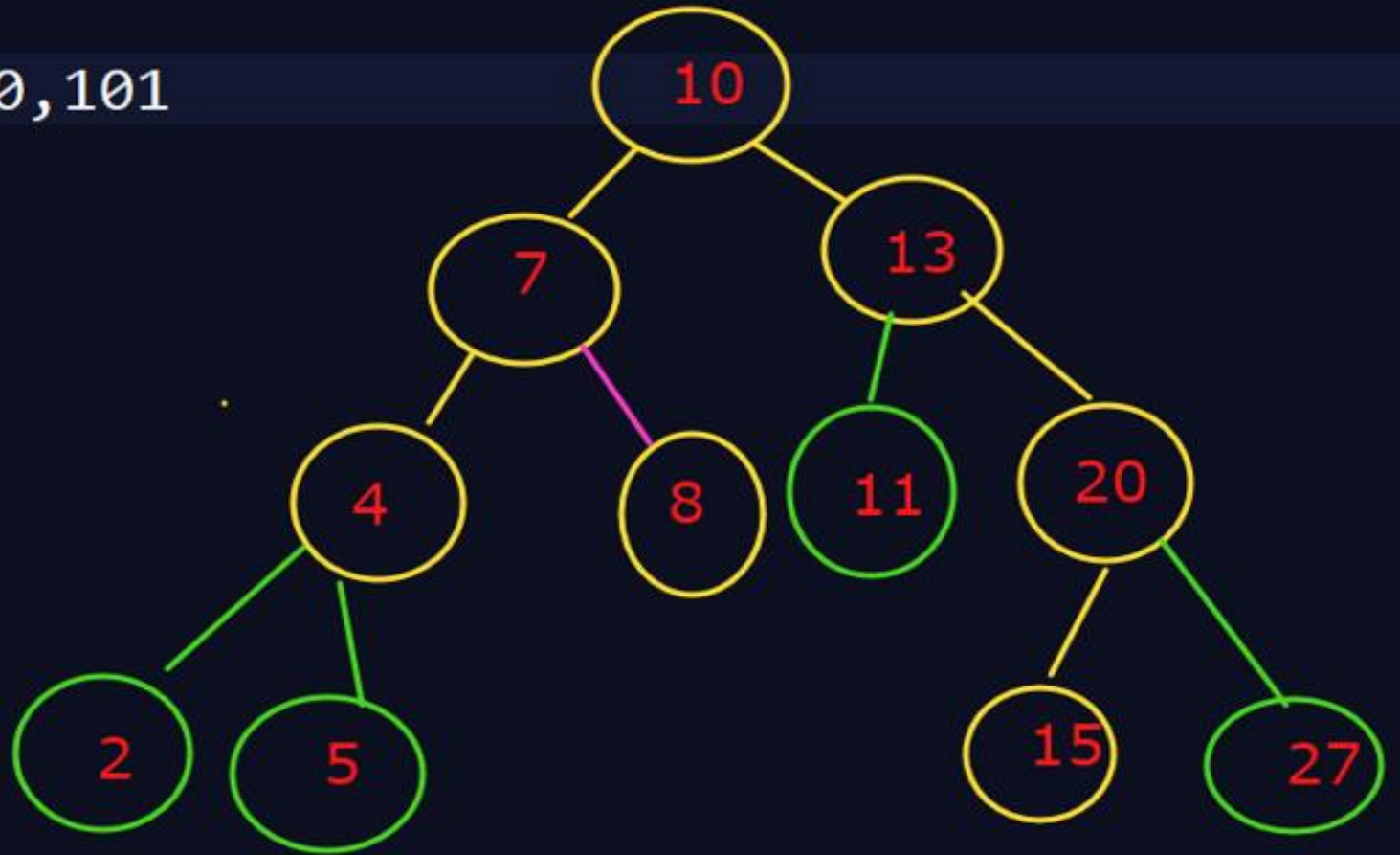


# Operations on a Binary Search Tree

- The following operations are performed on a binary search tree...
  - Search
  - Insertion
  - Deletion
  - Traversal

2, 4, 5, 5, 7, 8, 10, 11, 12, 13, 15, 20, 27

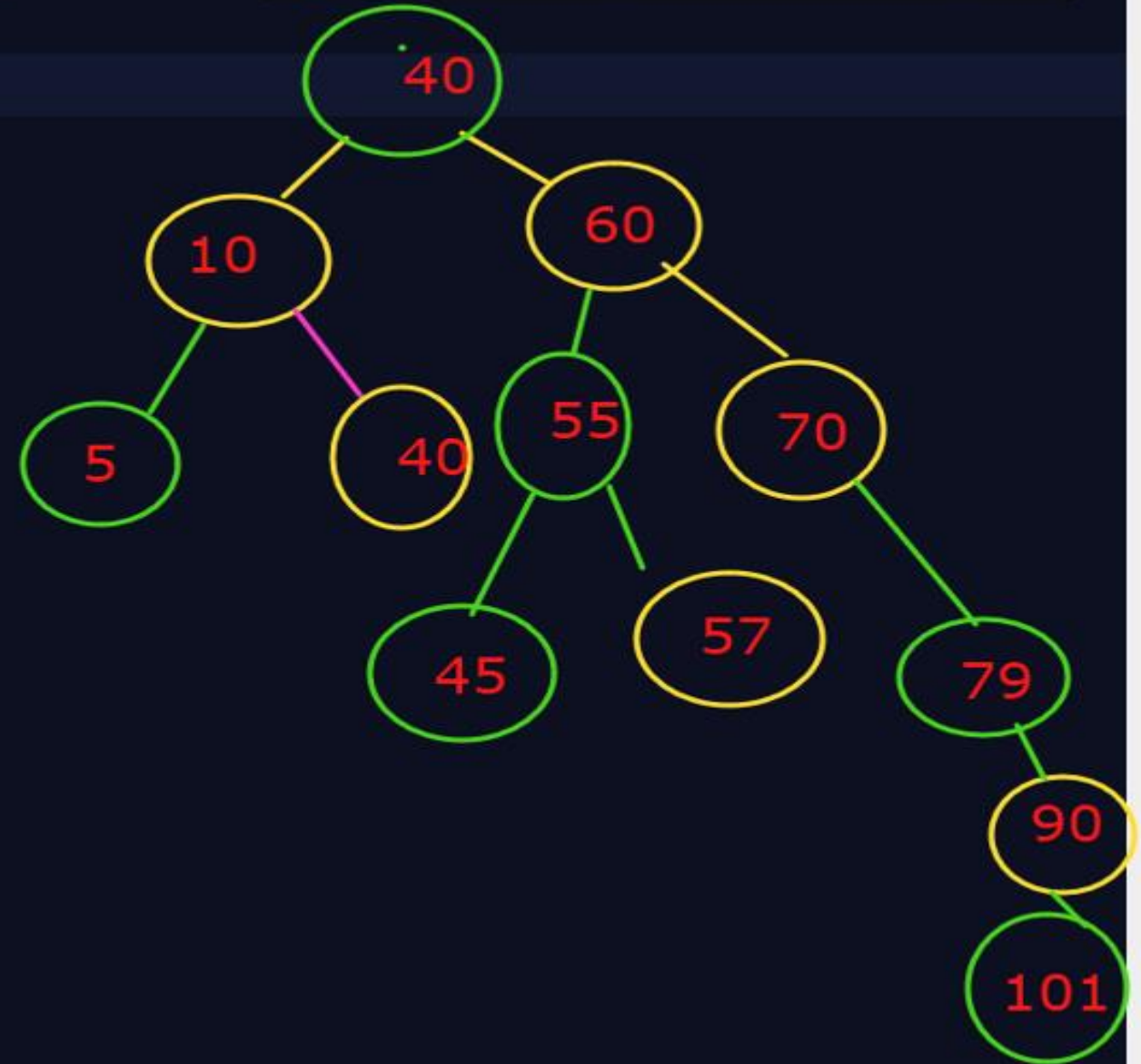
50, 60, 70, 10, 40, 45, 79, 90, 101





2, 4, 5, 5, 7, 8, 10, 11, 12, 13, 15, 20, 27

50, 60, 70, 10, 40, 45, 79, 90, 101





```

if(key <= root.data)
    root.left = deletedata(root.left,key);
else if(key > root.data)
    root.right = deletedata(root.right,key);

```

case 1: No child

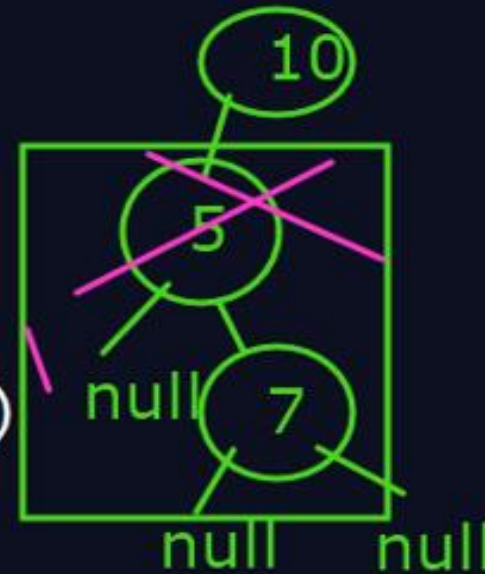


//Code for case 1 & 2

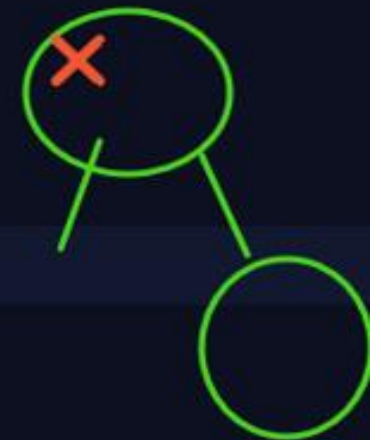
```

if(root.left == null)
    return root.right;
else if(root.right == null)
    return root.left;

```



case 2: 1 child



//code

}

Node deletedata(Node root, int key)

{

//Empty tree

if(root == null)

return root;

if(key <= root.data)

root.left = deletedata(root.left, key);

else if(key > root.data)

root.right = deletedata(root.right, key);

else{

//Code for case 1 & 2

if(root.left == null)

return root.right;

else if(root.right == null)

return root.left;

```
root.data = minvalue(root.right)
```

```
//deleting an element in Inorder and  
//replace it with successor node
```

```
root.right = deletedata(root.right, root.data);
```

```
}
```

```
return root;
```

```
}
```

```
int minvalue(Node root)
```

```
{
```

```
int x = root.data;
```

```
while(root.left != null)
```

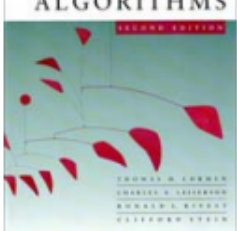
```
{
```

```
    x = root.left.data;
```

```
    root = root.left;
```

```
}
```

```
return x;
```



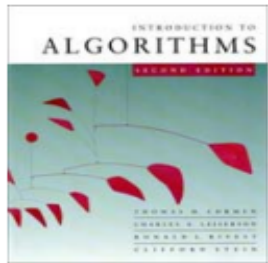
# Balanced search trees

***Balanced search tree:*** A search-tree data structure for which a height of  $O(\lg n)$  is guaranteed when implementing a dynamic set of  $n$  items.

## Examples:

- AVL trees
- 2-3 trees
- 2-3-4 trees
- B-trees
- Red-black trees



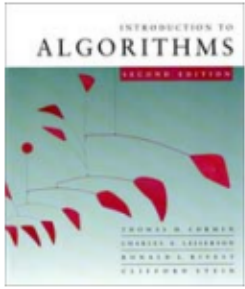


# Red-black trees

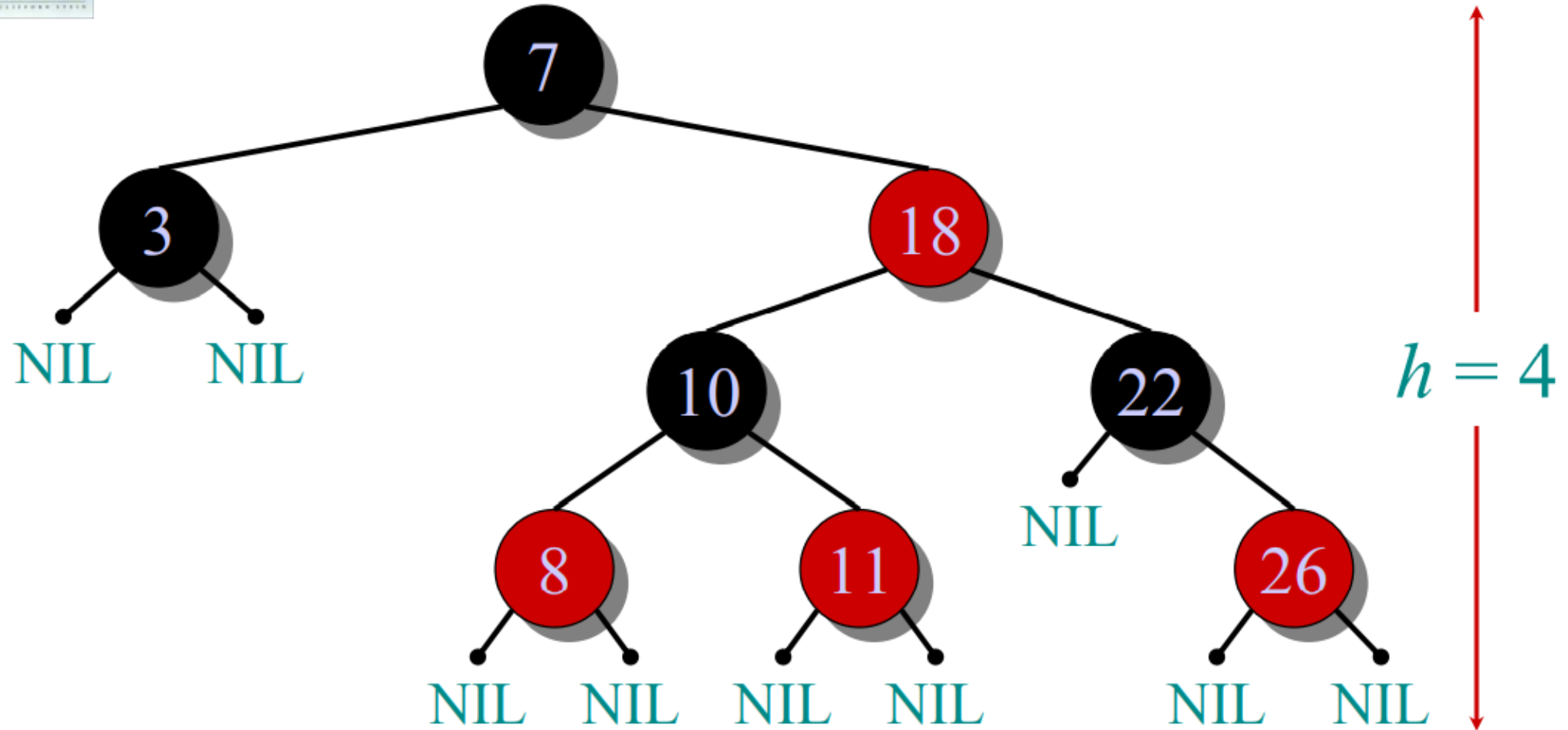
This data structure requires an extra one-bit **color** field in each node.

## *Red-black properties:*

1. Every node is either red or black.
2. The root and leaves (**NIL**'s) are black.
3. If a node is red, then its parent is black.
4. All simple paths from any node **x** to a descendant leaf have the same number of black nodes = **black-height(x)**.



# Example of a red-black tree



}  
Application of Trees:  
m-ways tree

B-Tree

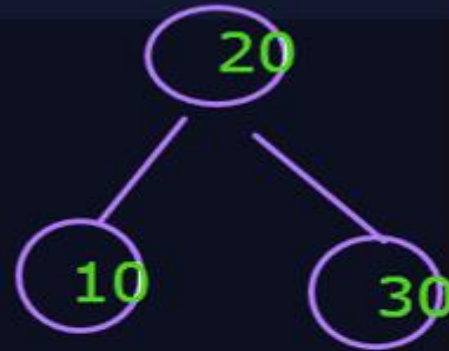
2-3-4 Tree



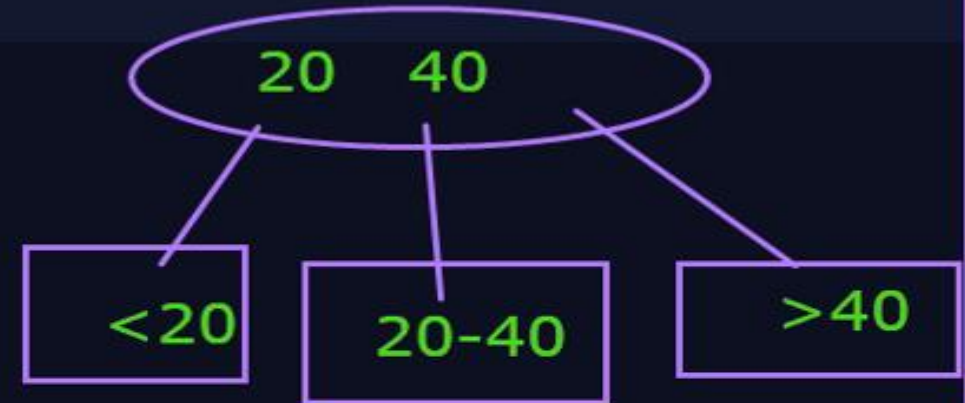
BT : 2 child

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2 Tree



2-3 Tree



2-3 Tree : B-Tree of order 3  
2-3-4 Tree : B- Tree of Order 4





# AVL Tree

# AVL Tree (Adelson – Velskii – Landis)

- **Binary Tree:**

- A binary search tree (BST) is a tree in which all nodes follow the below mentioned properties –

- The left sub-tree of a node has key less than or equal to its parent node's key.

- The right sub-tree of a node has key greater than or equal to its parent node's key.

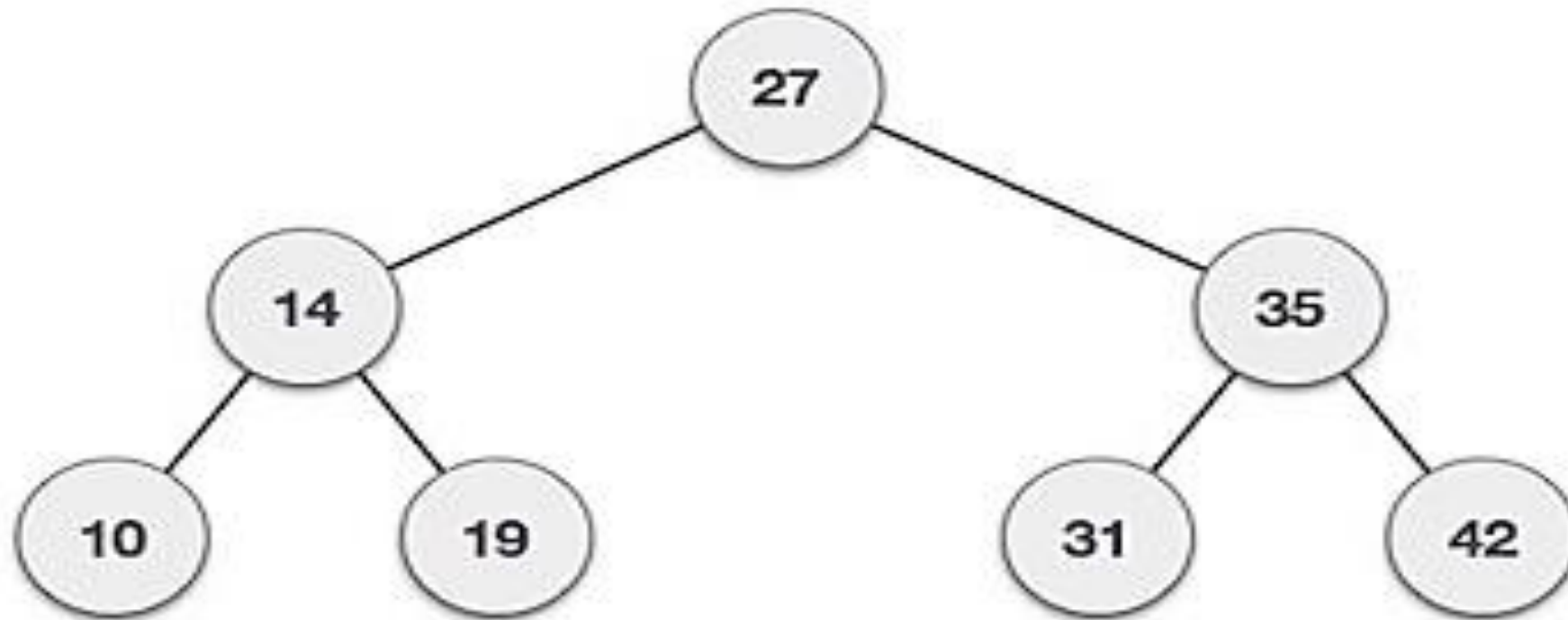
- **Thus, a binary search tree (BST) divides all its sub-trees into two segments;**

- ***left* sub-tree and *right* sub-tree and can be defined as –**

- **$\text{left\_subtree (keys)} \leq \text{node (key)} \leq \text{right\_subtree (keys)}$**

Example:

# Example



# AVL Rotations

- To make itself balanced, an AVL tree may perform four kinds of rotations –

1. Left rotation
2. Right rotation
3. Left-Right rotation
4. Right-Left rotation

- 

- First two rotations are single rotations and next two rotations are double rotations. Two have an unbalanced tree we at least need a tree of height 2. With this simple tree, let's understand them one by one.

-

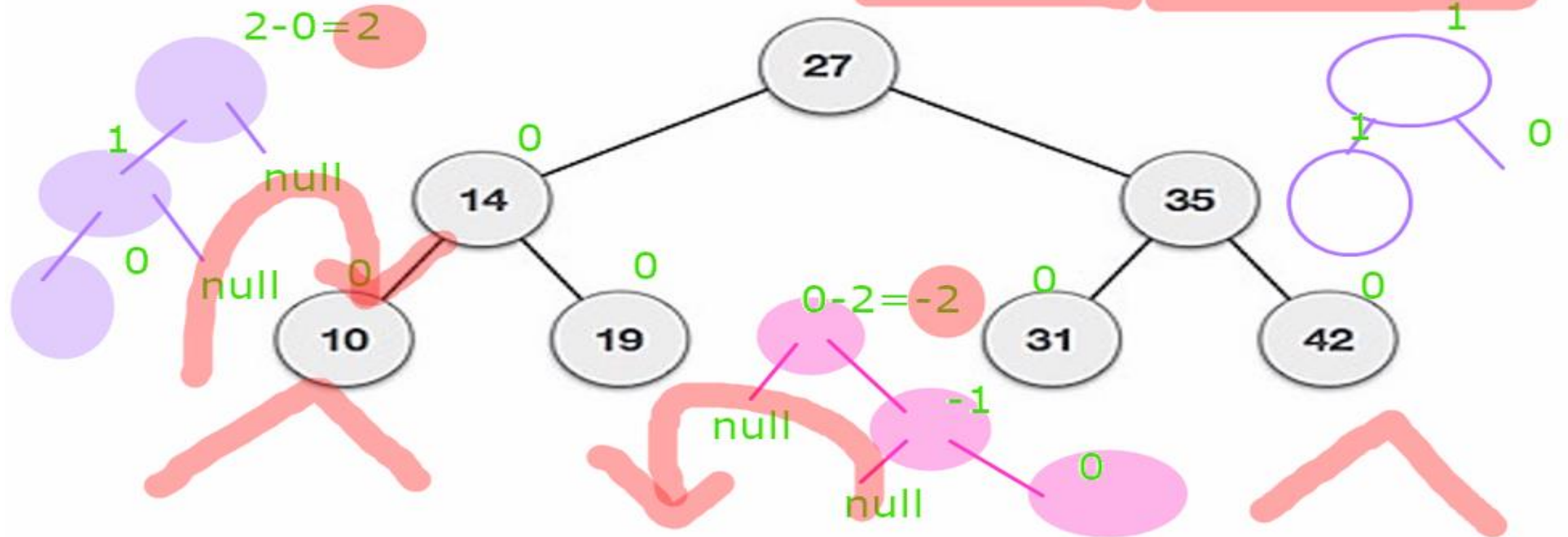
# Example

$0, 1 \leq 2$  Need for balance the tree

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Balance Factor :  $\text{height(LST)} - \text{height(RST)}$

Tree is not balance , then we apply rotations to balance that tree.



# AVL Rotations

•To make itself balanced, an AVL tree may perform four kinds of rotations –

1. Left rotation

2. Right rotation

3. Left-Right rotation

4. Right-Left rotation

Left  
Right

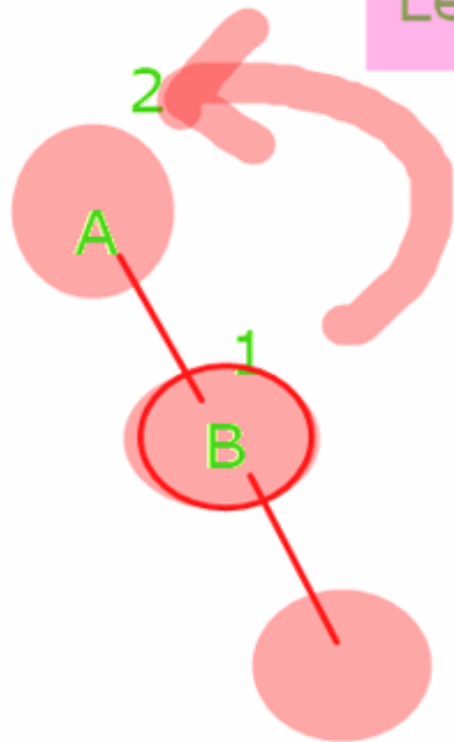
=> Single Rotation

LR Rotation  
RL Rotation

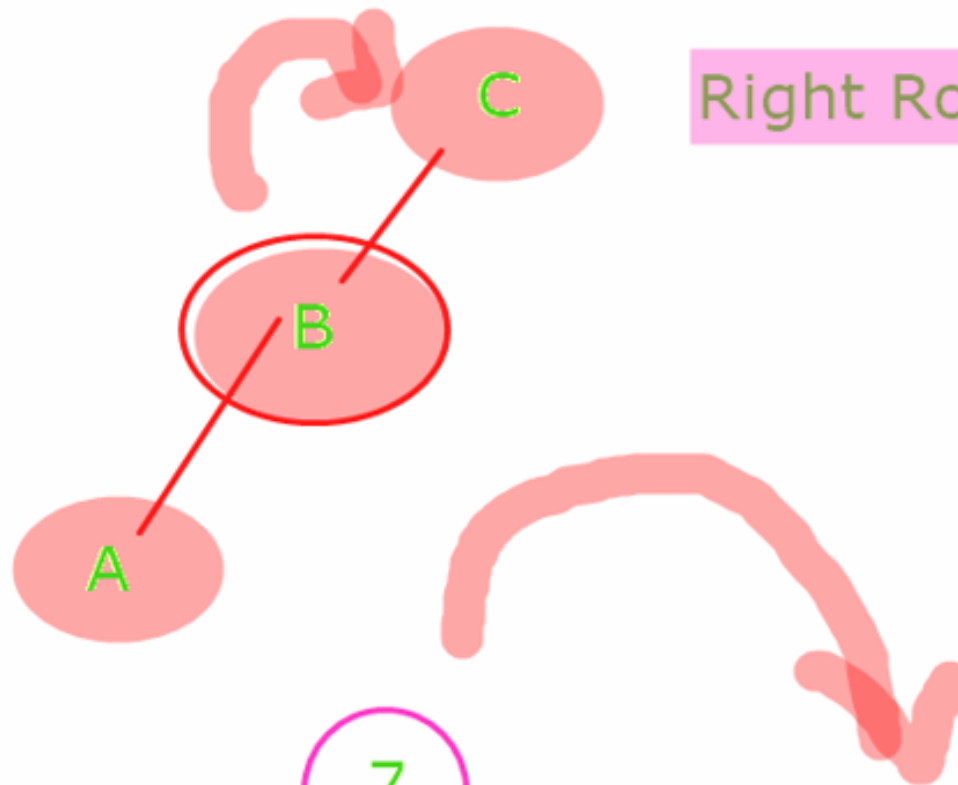
=> Double Rotation

•First two rotations are single rotations and next two rotations are double rotations. Two have an unbalanced tree we at least need a tree of height 2. With this simple tree, let's understand them one by one.

Left Rotation

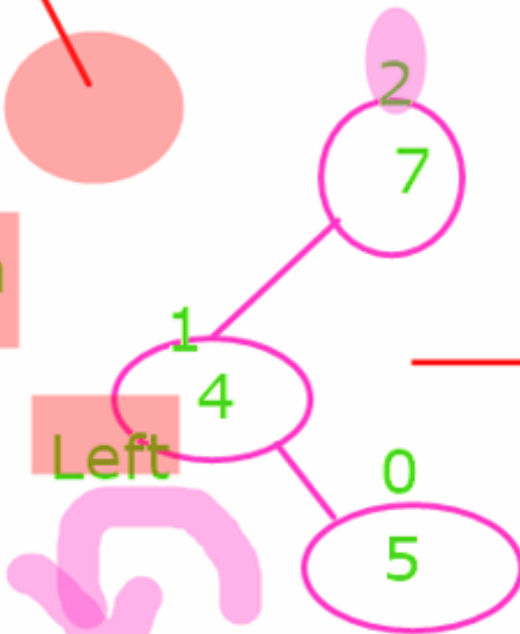


Right Rotation

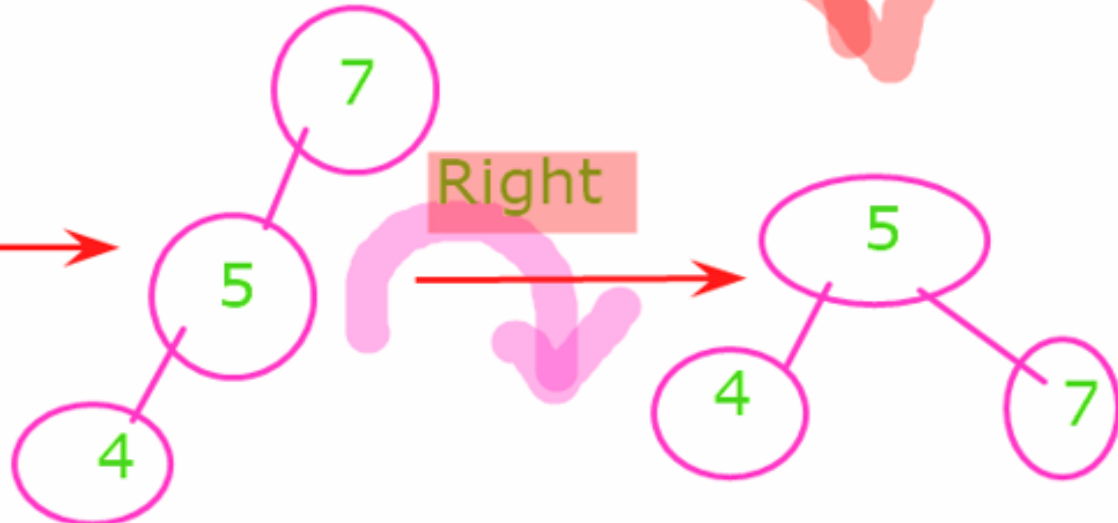


LR Rotation

Left

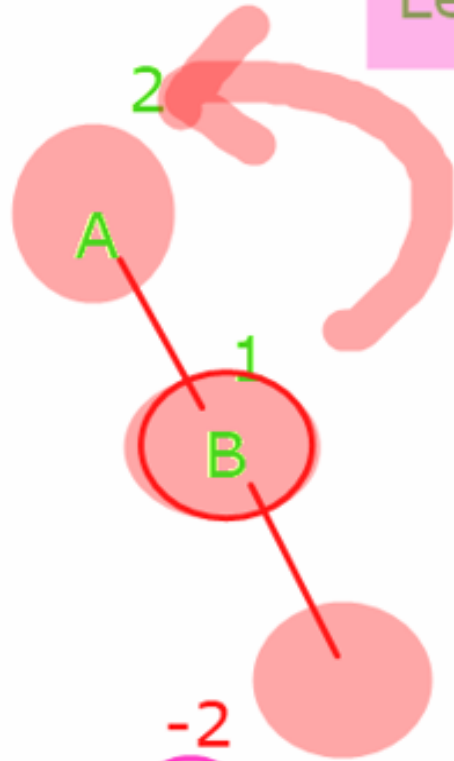


Right

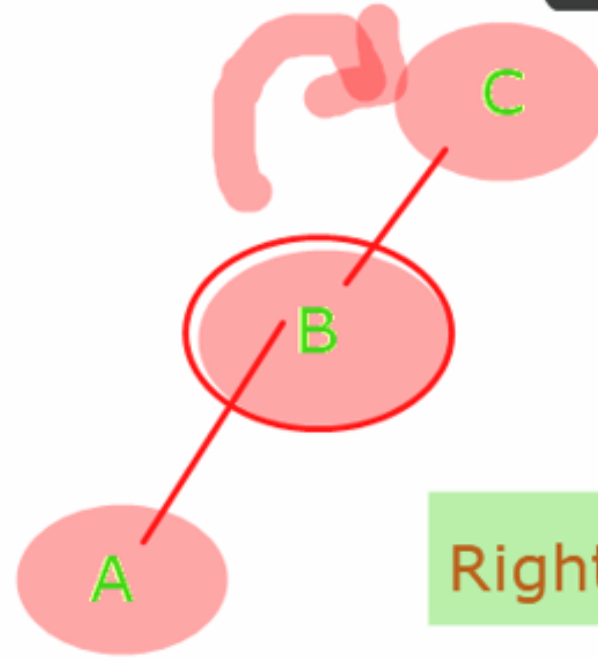




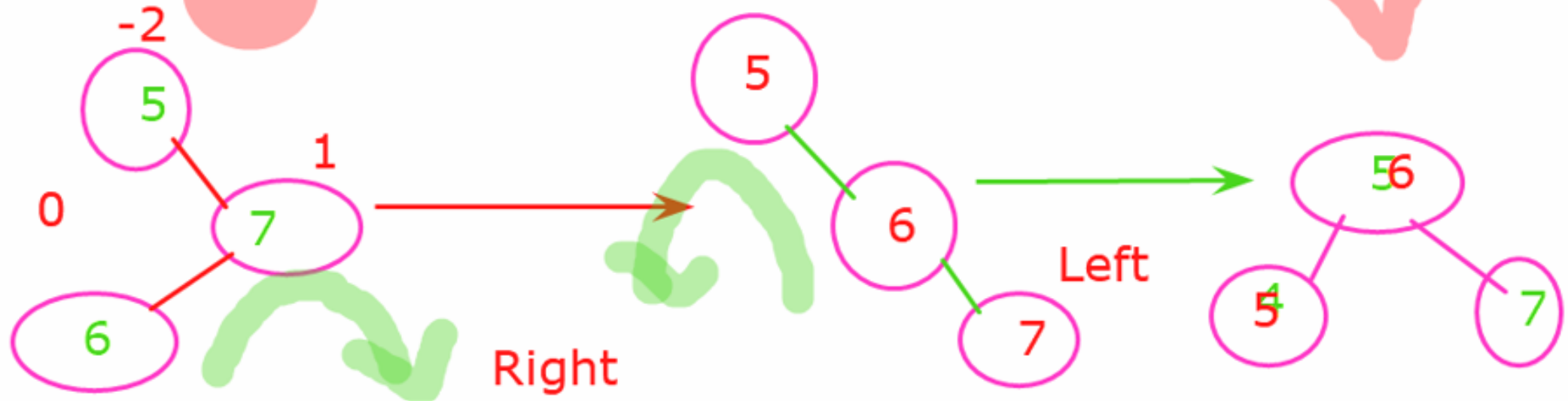
Left Rotation



Right Rotation



Right Left Rotation



# Heap

# Module I

# Kiran Waghmare



# Definition in Data Structure

- **Heap:**

- A special form of complete binary tree that key value of each node is no smaller (larger) than the key value of its children (if any).

- **Max-Heap:**

- root node has the largest key. A max tree is a tree in which the key value in each node is no smaller than the key values in its children. A max heap is a complete binary tree that is also a max tree.

- **Min-Heap:**

- root node has the smallest key. A min tree is a tree in which the key value in each node is no larger than the key values in its children. A min heap is a complete binary tree that is also a min tree.

- **Complete Binary Tree:**

- A complete binary tree is a binary tree in which every level, *except possibly the last*, is **completely filled, and all nodes are as far left as possible**

# Heap

- **Definition in Data Structure**

- **Heap:** A special form of **complete binary tree** that key value of each node is no smaller (larger) than the key value of its children (if any).

- **Max-Heap: root node has the largest key.**

- A **max tree** is a tree in which the key value in each node is **no smaller than** the key values in its children. A **max heap** is a **complete binary tree** that is also a max tree.

- **Min-Heap: root node has the smallest key.**

- A **min tree** is a tree in which the key value in each node is **no larger than** the key values in its children. A **min heap** is a **complete binary tree** that is also a min tree.

# Heap

- **Definition in Data Structure**

- **Heap:** A special form of **complete binary tree** that key value of each node is no smaller (larger) than the key value of its children (if any).

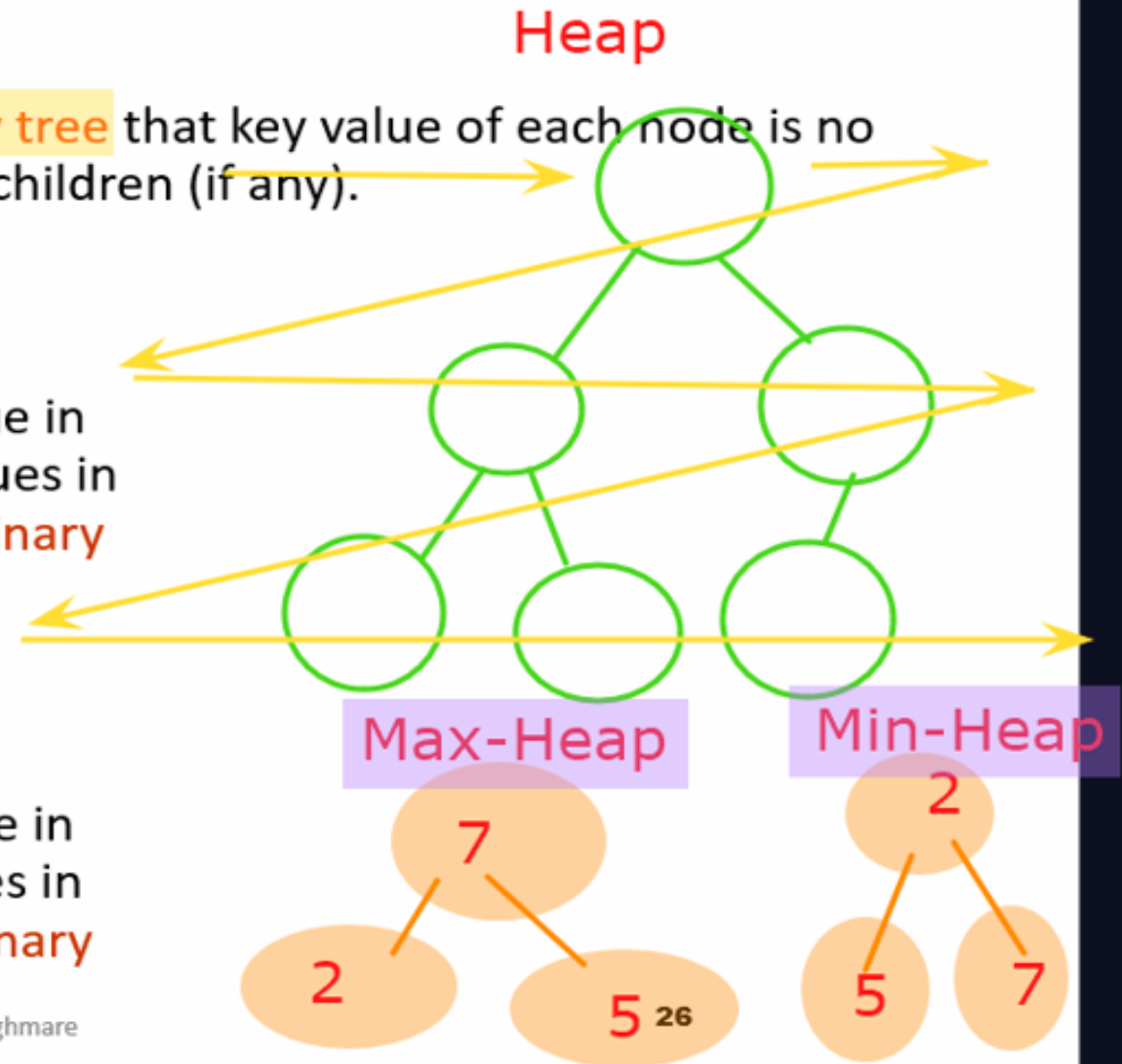
- **Max-Heap:** root node has the largest key.

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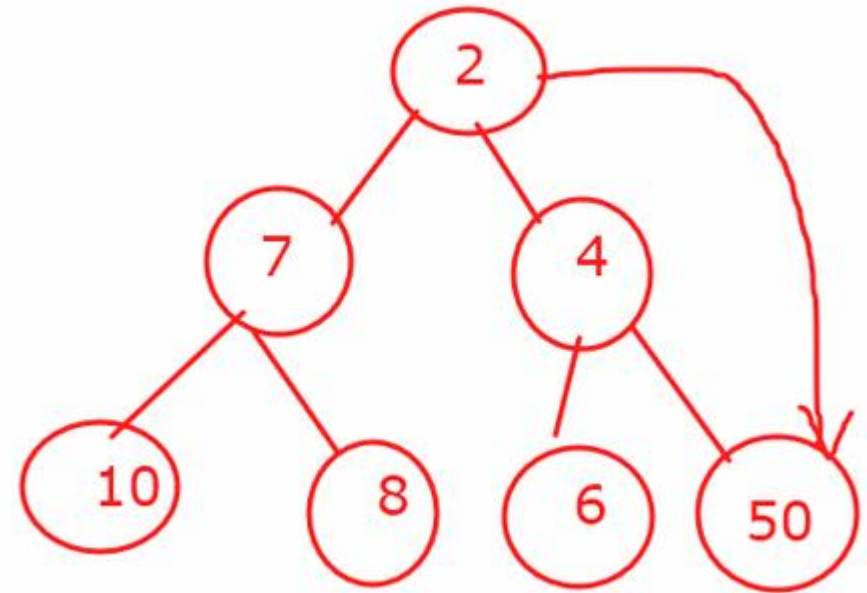
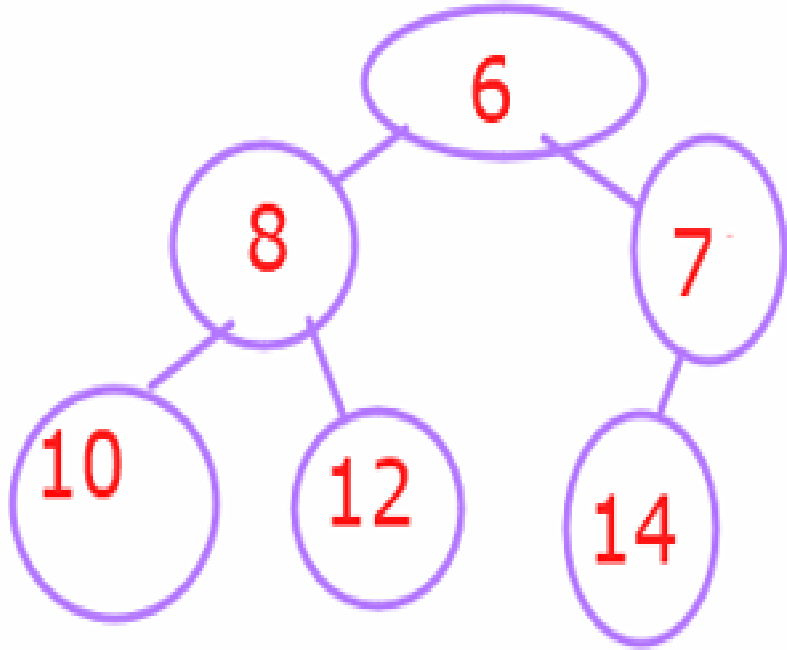
- **Min-Heap:** root node has the smallest key.

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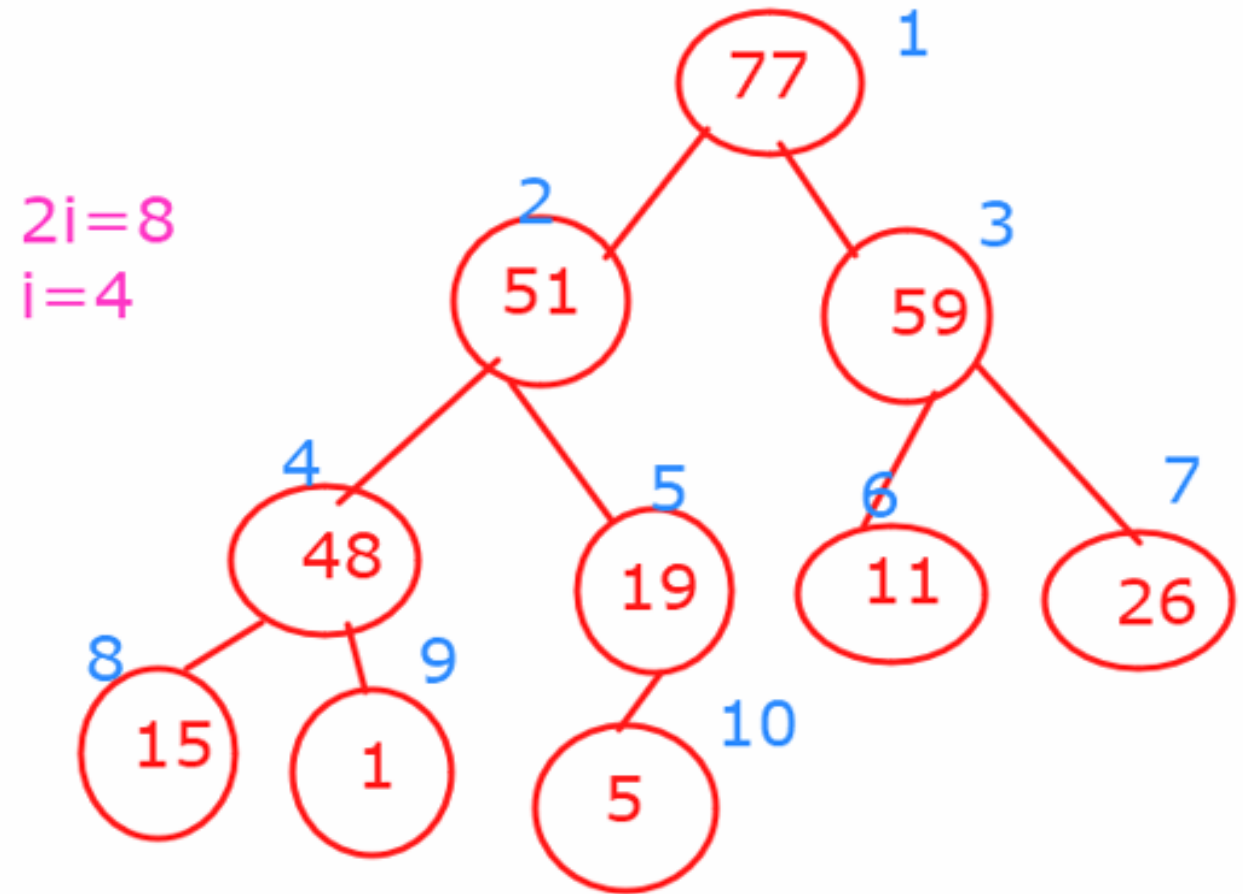
14, 12, 7, 10, 8, 6



# Heap Implementation:

Root LC RC

LC:  $2i$   
RC:  $2i+1$   
Parent:  $i/2$



Value:

77 51 59 48 19 11 26 15 1 5

Index:

1 2 3 4 5 6 7 8 9 10

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Heap:

2 3 7 1 8 5 6

8 3 7 1 2 5 6

7 3 6 1 2 5 8

6 3 5 1 2 7 8

5 3 2 1 6 7 8

3 1 2 5 6 7 8

2 1 3 5 6 7 8

1 2 3 5 6 7 8

# Thanks