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ABOUT COSMATT

Title and Authors

COSMATT™

Compro's Online Servo Motor Analysis and Training Tool

By: John Durrant, Graham Elvis, Roger Brookes

Compro Technologies

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Authors

John Durrant – Retired Servo Engineer

I qualified as an Electrical Engineer, with an electronic bias. I spent a few years in the Thyristor drive development laboratory of a major UK electrical equipment manufacturer, where I was responsible for the development of their small drives range. After that I was part of a start-up enterprise designing and manufacturing small electrical equipment including motor controls.

In the 1980s I joined Electro-Craft, one of the pioneers of brushless servo motor, providing application support for the European Sales activity. This involved evaluating proposed machine designs and recommending the correct servo components, including position controllers. It also involved on-site troubleshooting. For a while I was also responsible for the design of some of the controllers and a range of small drives. Electro-Craft was eventually absorbed into Rockwell Automation, and I took over responsibility for the application support of Allen-Bradley Motion Control Products in Europe.

During this period, I helped develop the System sizing tool used by Electro-Craft globally, and was later involved with the other authors in creating Motion Analyzer, the world class sizing and selection tool for Allen-Bradley.

Graham Elvis – Retired Servo Engineer

My formal qualifications are in Mechanical Engineering which included some electrical knowledge and control theory. I spent several years in a Machine Tool company R&D department which involved much electronic equipment and I frequently worked on vibration analysis and the design of structures. This was to become extremely relevant when it came to servo performance issues.

For the last 20 years of my career I specialized in servo systems for a global automation company. I had to specify servo components for industrial automation systems, perform startups and troubleshoot. In the early days I wrote my own servo sizing software using GW

Basic and was later involved with the other authors in creating a world class sizing and selection tool.

Roger Brookes – Electrical & Electronics Engineer

After an interest in motors and control from an early age, I obtained an Honours Degree in Engineering specializing in electronics, electric motors, and servos. After specialising as servo design engineer at International Computers developing dynamic peripherals, I took up a European technical sales and support position with Electro-Craft, the leading high performance motor company. Electro-Craft went on to develop full servo-systems, so I prepared tools for mathematical analysis of dynamic applications to simplify motor selection and optimise mechanics. These tools firstly ran programmable calculators, later pocket PCs as these became available, and finally as full suites for evolving PCs and laptops. Electro-Craft integrated into Rockwell where I managed the European Motion Control Division, and participated in the motion analysis software development.

Following Rockwell I became an independent consultant for motion control, working with Motion Specialist integrators and OEMs, integrating large numbers of servo-motors in demanding applications. The Integration was enabled utilizing the advanced motion analysis software that I continued be extensively involved in developing.

Contents

Chapter 1: Fundamentals
The necessary knowledge of the topics required for understanding the course is presented in this chapter. There is a chance that the user may already be familiar with some of these topics. So, a Pre-test section is provided which will help the user in knowing his / her current understanding of the fundamentals.

Motion Profiles:
This chapter deals with the formation and analysis of a “Motion Profile” associated with single or multiple axes. It also discusses the approaches to optimise the profile shape and build longer profiles.

How to Analyse a Rotary Load:
Motion profiles include calculation of inertia, load torque, RMS torque and so on. This chapter will help the user in calculating the associated parameters and plotting the torque-speed requirements on a chart.

Servo Motor Selection:
This chapter will guide the user in comparing the application requirements to motor ratings. The normal requirement is to find the lowest cost solution that will satisfy the specified performance and accuracy requirements.

Transmission Ratio Optimisation:
Motors are often connected to a load mass or load inertia through a transmission system. This

may be a gearbox, toothed belt/pulley system or some other transmission mechanism. The effect of transmission ratio as well as the optimisation required will be explained in this chapter.

Linear Loads:
A linear load in this context is a load that moves in a straight line, usually guided by rails or similar constraints. It will be driven either via a mechanism that translates from rotary to linear motion such as a screw-type or a tangential-type drive, or directly by a linear motor. Author will examine the screw-type and tangential-type mechanisms in this chapter.

Application Examples:
In this chapter, author will through some typical examples which frequently appear in industrial applications. It will discuss about multiple axes involved in real-world examples of servo motors.

Course Assessment:
After studying the industrial application of servo motors, this chapter will provide numerous questions that deal with tweaking and adjusting of parameters to optimize the cost, reliability and performance of the motors.

Foreword

COSMATT is a training course on the sizing of electric servo motors and the associated analysis of servo systems. The name COSMATT is an acronym for Compro's Online Servo Motor Analysis and Training Tool.

COSMATT is meant for engineers who want to learn the basics of sizing servo motors. It also provides a useful online reference for motion control professionals. The course material covers a range of topics that are essential for understanding how to properly size electric servo motors. The course starts with a chapter on Fundamentals to refresh the key engineering concepts used in servo sizing, including - units, mass & inertia, force & torque, electrical quantities, motion equations, RMS and other basic concepts. It then takes the user through a series of topics that are related to the key aspects of servo sizing. These include Motion Profiles, Analysing Rotary and Linear Loads, Servo Motor Selection and Transmission Ratio Optimisation.

The current version of COSMATT is a basic level course. The authors and Compro also have plans to create an advanced version of the course, which will address many of the more complex topics associated with servo sizing.

The training approach used in COSMATT includes a mix of theory and practice problems, with assessment problems at the end of each chapter (topic). A complete chapter at the end of the course is devoted to application examples, where all the principles taught in earlier chapters are applied to a typical 10 axis production line. The user is first shown how to solve the 10 axes and then asked to test his or her understanding by independently solving a variation of the same 10 axes.

The course utilizes the latest software and digital technology, to create a highly interactive state-of-the-art online learning experience. It contains several tools which engage the user and help enhance learning and problem solving, including - a motion profile editor, a torque speed curve tool and an inertia calculator. These online tools create an engaging experience and should help improve the learning experience. COSMATT also makes extensive use of a spreadsheet based tool (comproDLS Leonardo) to provide a real life experience in working with spreadsheet based tables and assessment problems.

COSMATT includes an elementary online sizing tool to promote self-exploration and experimentation using the concepts learned from the course. While this tool is not intended for full feature sizing it can be used to illustrate the concepts presented in this course. It is recommended that the user first complete the training course before exploring the sizing tool.

COSMATT has been authored by John Durrant, Graham Elvis and Roger Brookes. All have many years of experience in the field of motion control and servo sizing, and have worked with Compro for over 15 years. It has been my privilege to have been associated with the authors for almost 20 years. I first met the authors when they were working at the Rockwell center of excellence in Crewe, UK. We worked together for many years on Motion Analyzer, Rockwell's well known servo sizing tool. COSMATT attempts to pass on the many years of knowledge gained by the authors in their professional careers to future generation of engineers involved with motion control.

COSMATT has been developed by Compro Technologies, a company with extensive experience in developing motion control and sizing software solutions. Compro's past clients in the field of sizing and selection software include Rockwell Automation, Danaher Motion (Pacific Scientific, Kollmorgen, Giddings and Lewis Controls, Warner Electric), Parker Hannifin, Emerson Control Techniques and Thompson Industries (Philips Airpax). Compro also has extensive experience in developing online learning solutions and has recently developed a new learning suite (comproDLS) to help publishers create highly effective online digital products.

COSMATT has been created using the comproDLS learning suite and offers numerous customization options for manufacturers who wish to adopt it for their training needs. The customizations include branding, manufacturer specific product (motor) data, customized content, class management tools and authoring tools. Compro also invites inquiries from manufacturers who may wish to use comproDLS product suite to develop high quality digital solutions related to other training needs. Please contact Compro for additional information www.comprotechnologies.com)

User feedback and comments are important and most welcome. They will help guide the authors to evolve the course make it more relevant.

Happy sizing.

Dr. Gurdial Singh

gsingh@comprotechnologies.com

Introduction

A servo system is usually defined as a system whereby the motion and position of a mechanism are controlled using a feedback device, by which the system can detect errors and act to correct them. The corrective action can be performed using electric motors, or pneumatic or hydraulic cylinders.

This is a course of instruction about how electric servo systems work and in the optimal choice of servo motors and drives (amplifiers) to achieve a specified performance at minimum cost. Nevertheless, much of the content applies to all servo systems irrespective of the type. It is designed to help both those with little knowledge of the subject to become conversant and those with existing knowledge to appreciate the finer points of the subject. The content is modularized and covers everything from fundamentals to sophisticated applications. This allows users to skip subjects they are already comfortable with. As will become apparent, it is tedious and time consuming to manually calculate all possible permutations of a proposed system so the course content is accompanied by the COSMATT sizing calculation support tool, which combines the methods and widgets from the course to provide a rapid sizing environment.

Learning Objectives

- Gain adequate knowledge of the fundamentals of dynamic motion and mechanical and electrical properties
- Building Basic motion profiles in response to move requirements
- Understanding differences between the treatment of linear and rotary motion
- Understanding effects of changing the shape of the profile
- Understanding how to evaluate a rotary load, incorporating inertia and including friction losses
- Applying a motion profile to calculate the torque and speed requirements
- Understanding how peak and continuous operating points are displayed in tabular and graphical forms
- Compare the torque and speed required by the rotary load with the capabilities of various motors
- Understand impact of adding coupling, motor inertia and losses
- Plot the torque-speed curve of the motor with the operating points of the load
- Be able to select an appropriate drive for the motor
- Understanding the impact of transmission ratio on the choice of motor
- Get to know the commonly available types of mechanical transmission systems
- Learn how to deal with the added inertia and losses
- Analyzing simple linear applications, including linear motion profiles and load configurations
- To get an understanding of screw-type, and tangential (belt, chain or rack and pinion) mechanical systems and the impact on the transmission ratio

Chapter 1: Fundamentals

1.1: Introduction

The COSMATT Course is a digital e-learning course that explains how servo systems work and how to select and optimize electric servo system components.

Before we start with the course it is necessary to have knowledge of high school level mathematics and basic engineering. Fundamentals chapter will help in providing a necessary knowledge of the topics that are needed for this course.

Pretest

There is a chance that you may already be familiar with some of these topics. Therefore, we have designed a Pretest section to understand how much you already know. The pretest is intended to judge your prerequisite knowledge for Servo System Training course. Therefore, it is recommended to complete the pretest.

Based on your performance in the test, some topics of Fundamentals will be automatically marked as complete and you may skip reading those topics.

Following topics will be covered in this chapter:

- Units
- Radians
- Electrical Quantities
- Dynamic Equations
- Mass
- Inertia
- Force
- RMS
- Torque
- Power & Energy
- How a Servo Axis Works

Learning Objectives

- Gain adequate knowledge of the fundamentals of dynamic motion and mechanical and electrical properties.
  - To understand how SI units make engineering calculations more straightforward
  - To understand this fundamental unit of angle
  - To appreciate basic electrical theory necessary for the understanding of drive systems
  - To understand how dynamic quantities are necessary for the understanding of drive systems
  - To understand the concept of mass and how it differs from weight
  - To understand the concept of inertia in rotating objects

- To understand the concept of force
- To understand how RMS applies to current and torque in a drive system
- To understand how torque is derived and is applied to drive systems
- To understand the concept of Power and Energy in electrical theory
- To understand the working of Servo Axis

1.2: Pre-test

Question 1 - How many Watts in 3.3 kW?

a. I don't know.
b. 330
c. 3,300
d. 3,300,000

Question 2 - What is the SI unit of speed?

e. I don't know.
f. mm/sec
g. m/sec
h. km/hr
i. Furlongs/Fortnight

Question 3 - What is the SI Unit of Inertia?

j. I don't know.
k. gm-cm^2
l. kg-cm^2
m. kg-m^2

Question 4 - I want to move a turntable through three revolutions – how many radians is that?

n. π
o. 2*π
p. 6*π
q. 12*π

Question 5 - I am using a conveyer belt to move a load. The diameter of the drive pulley is 100mm. How many radians will I need to turn the pulley through in order to move my load through 1 meter.

r. 6* π
s. 10
t. 20
u. 10*π

Question 6 - Calculate the value of Current (A), if the Voltage is 12.5V and Resistance is 6.25Ω?

a. 1A
b. 0.5A
c. 2 A
d. None of the above

Question 7 - Calculate Power(W), if Voltage is 10V and Current is 2.5A?

a. 250W
b. 25W
c. 2.5W
d. None of the above

Question 8 - Calculate the Acceleration of a train when it is moving steadily from 4.0 m/s to 20 m/s in 100 s is

a. 1 m/s^2
b. 2 m/s^2
c. 0.16 m/s^2
d. 3 m/s^2

Question 9 - If an object weighs one Ton on the earth, what is its mass on the Moon, roughly?

v. 1 Ton
w. 6 Tons
x. 1/6th of a Ton
y. Zero
z. I don't know.

Question 10 If an object weighs one Ton on the earth, what does it weigh in the International Space-station?

aa. 1 Ton
bb. 6 Tons
cc. 1/6th of a Ton
dd. Zero

Question 11 If an object weighs one Ton on the earth, what is its mass in the International Space-station?

ee. 1 Ton
ff. 6 Tons
gg. 1/6th of a Ton
hh. Zero

Question 12 A rotating disk has an inertia of 1 Kg-m^2. What would the inertia be in the International Space-station?

ii. 1 Kg-m^2

jj. 6 Kg-m^2
kk. 1/6th of a Kg-m^2
ll. Zero

Question 13 If a shopping trolley weighing 50kg is pushed with a force of 100N, its acceleration will be

a. 5000 m/s^2
b. 50 m/s^2
c. 150 m/s^2
d. 2 m/s^2

Question 14 What does the RMS value of a cyclically varying current mean?

a. It is the equivalent to the steady AC current
b. It is the equivalent to the steady DC current
c. It is just another term for cyclically varying current
d. None of the above

Question 15 A motor needs to provide 10 Nm of torque to a conveyer to move it. The drive roller is 200mm Diameter. What force is required at the circumference of the drive roller to move the conveyer?

a. 1 N
b. 10 N
c. 20 N
d. 50 N
e. 100 N

Question 16 In order to do work, energy is

a. transferred or converted
b. used up
c. lost
d. lost or transferred

Question 17 Power is defined as

a. P = E/t, where E is energy and t is time
b. P = t/E, where t is time and E is energy
c. P = E/d, where E is energy and d is distance
d. P = d/E, where d is distance and E is energy

Question 18 If a bulb uses energy of 100 J and remains on for 25 s, power consumed by the bulb will be

a. 125 W
b. 4 W
c. 2500 W
d. 75 W

1.3: Units

Units of Measure

Units of measure have evolved over centuries so that there are now multiple units for most quantities such as length, having dozens of variations.

Usage is broadly split between the USA ("British" or "Imperial" units) and Europe ("Metric" or "SI" units). Other countries tend to follow one or other of these according to their historic associations. Some, such as the UK, use a mix of the two. This has caused generations of engineering students to puzzle over whether to multiply or divide by G (or not) and struggle to remember conversion constants.

"SI" or "System International d'Unités", announced by the "General Conference on Weights and Measures" in 1960, is the only coherent system of units. That is, a set of seven "base" units are defined and all others are derived from them. The practical result of this is that no conversion constants (including G) are required. This greatly simplifies calculations and reduces the chance of error. Note that "Metric" is a term that is used rather loosely to cover "European" as opposed to "American" units but the name "SI" is very specific.

Examples

Mechanical – Imperial units

For linear

$$Power(HP) = Force\ (lbs) \times Velocity\ (ft/s) \times 550$$

For rotational

$$Power(HP) = Torque\ (lbf.ft) \times Speed\ (rpm)/5252$$

Mechanical – SI units

For linear

$$Power(W) = Force\ (N) \times Velocity\ (m/s)$$

For rotational

$$Power(W) = Torque\ (Nm) \times Speed\ (radians/s)$$

See Chapter 1.4 - Radians for further details.

Electrical

$$Power\ (W)\ =\ Voltage\ (V) \times Current\ (A)$$

Note that by using "SI" units electrical and mechanical (rotational or linear) power results are identical and there is no need for conversion constants.

In the SI system fundamental units such as meter may be given a prefix to scale up or down, positive powers indicate a scale up and negative powers indicate a scale down.

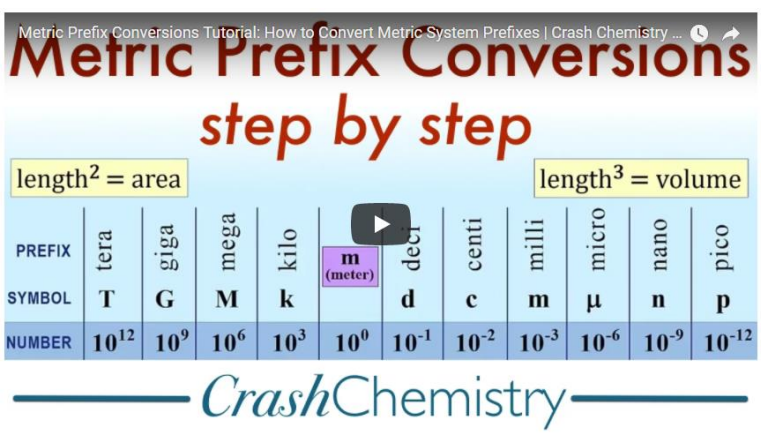
Prefix	Symbol	Notation
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	K	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
peco	p	10^{-12}

Table 1

Such prefixes extend all the way from $Y\ (Yota\ldots)\ \times 10^{24}$ to $y\ (Yocto\ldots)\ /\ 10^{-24}$ in steps of 10^3 surely covering any possible requirement . 10^{24} is roughly the number of stars in the universe!

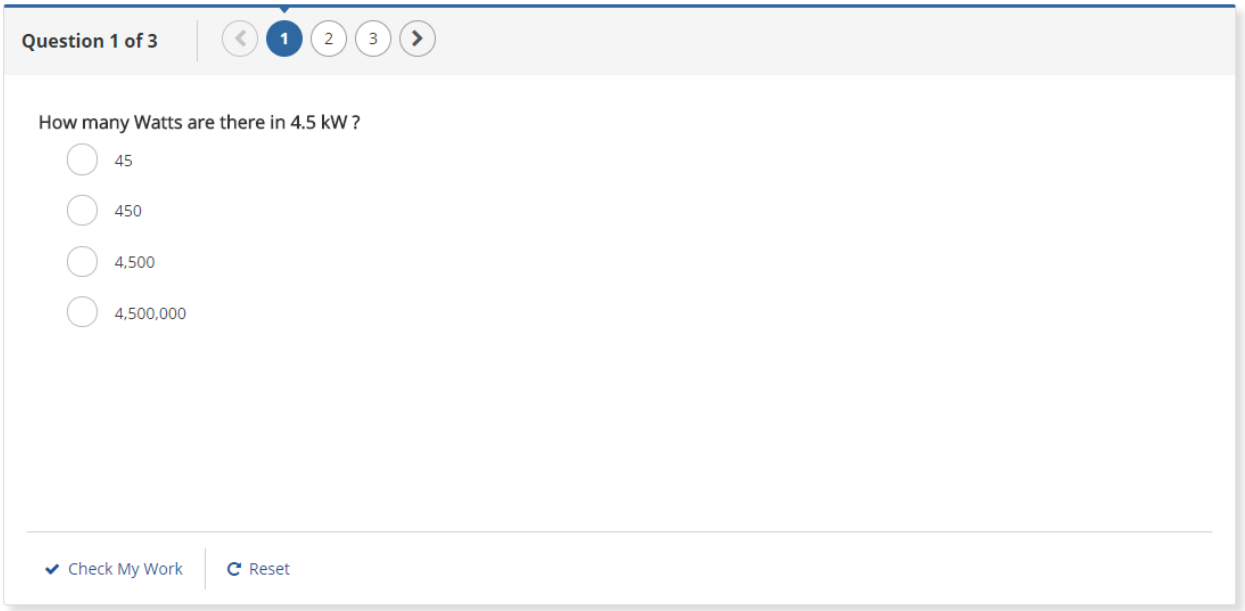
Finally, it is common in reports of engineering matters to see formulas where the units are not specified rigorously – this is extremely bad practice and should be avoided at all times.

Want to learn more, check out the video below:



Web search "Units of Measurement", "SI units" for further information.

Test Your Understanding



1.4: Radians

Unit of Angle

The Radian was defined by James Thomson as the angle subtended by an arc with the same length as the radius of a circle.

It is an angle of 57.296°

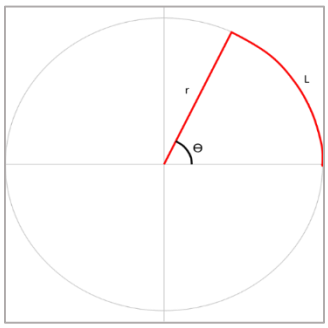


Figure 1

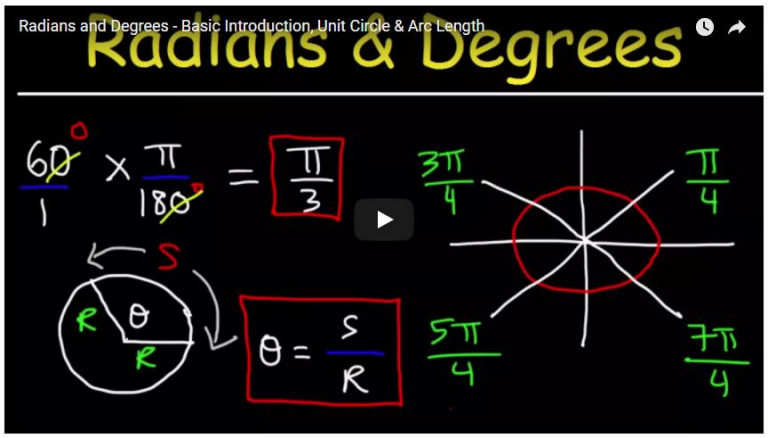
The length of the arc L equals that of the radius r for an angle Θ of 1 radian.

There are $2 \times \pi$ radians in 360°.

It is important to understand what a radian is for motion control as the fundamental unit for rotary acceleration is the Radian per second. One RPM is $(2 \times \pi)/60$ radians per second, which is roughly one-tenth.

Thus, for example, let's say you need to accelerate an inertia of 0.01 kg-m^2 to 1000 rpm in one second, roughly what magnitude torque do you need? The acceleration rate is 1000 revolutions per minute per second – clearly a very awkward unit. You can convert to $1000 / 60 = 16.67$ revolutions per second squared but to get to the fundamental unit for torque calculation you need to multiply by $2 \times \pi$ to get radians per second, giving about 100 radians per second. Multiply directly by the inertia to get a torque requirement of about 1Nm.

Want to learn more, check out the video below:



Web search "radians" for further information.

Test Your Understanding

Question 1 of 2 1 2

I want to move a turntable through 6 revolutions – how many radians is that?

- pi
- 2pi
- 6pi
- 12pi

Check My Work | Reset

1.5: Electrical Quantities

Flow of Electricity

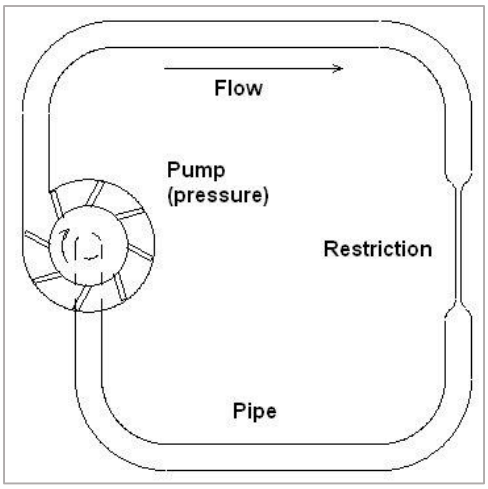


Figure 1

The flow of electricity can be likened to the flow of liquid in a closed loop, the liquid flow rate increases with increasing pressure (produced by the pump) and decreases with increasing restriction.

$$Flow = \frac{Pressure}{Restriction}$$

Note that the pipe circuit must be a complete ring for this to work. Many hydraulic systems work in this way.

Electrical voltage is equivalent to liquid pressure and current is equivalent to flow rate. Resistance is equivalent to restriction.

$$Current = \frac{Voltage}{Resistance}$$

Again, the loop must be completed (by copper wire in this case) for current to flow.

The convention is that current “flow” is from positive to negative though, in reality, electrons flow from negative to positive.

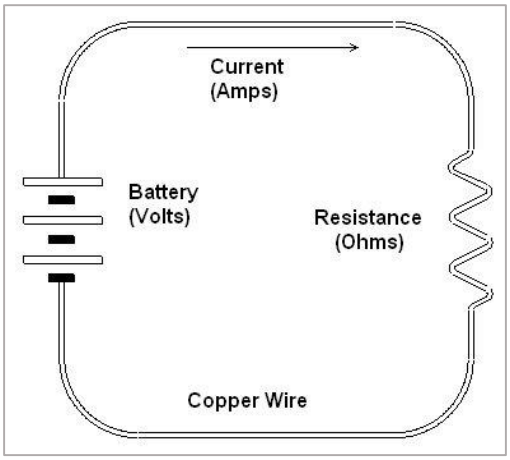


Figure 2

The other essential concept is that of power which is defined as "the rate of doing work".

For hydraulic systems: $Power = Pressure \times Flow\ Rate$

For electrical systems: $Power\ (Watts) = Voltage \times Current$

Alternating Current

The current produced by a battery flows in one direction only – known as "Direct Current" (DC). The earliest public power supplies were DC but were soon superseded by "Alternating Current" (AC) where the voltage and current swing positive and negative in a sinusoidal manner (50Hz or 60Hz depending on Country). A major driver for this change was the fact that AC voltage is easily stepped up or down by transformers and high voltage suffers fewer losses on long transmission lines.

DC voltage can now be changed easily by solid-state electronic devices. The Drives that control the speed and position of motors are such devices – the incoming AC power is rectified (converted to DC) then chopped up into a varying AC voltage and frequency.

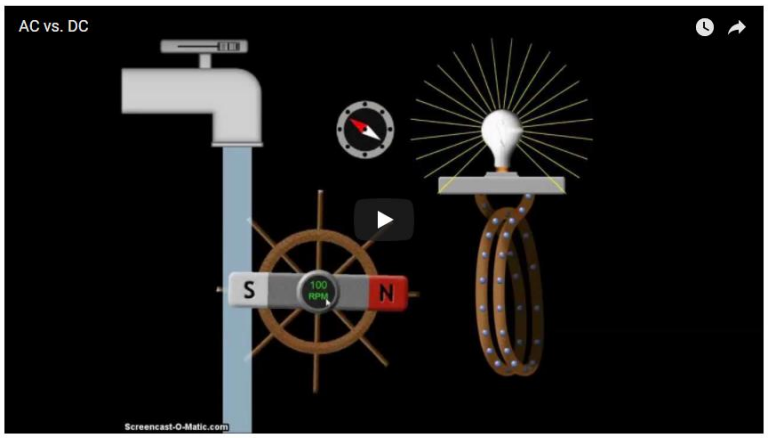
Units

By convention, units are indicated by a suffix:

Ω is the Greek letter omega

A lower case "m" preceding the suffix means "milli" (Latin for 1/1000th) e.g. 100mA = 0.1A, see Chapter Chapter 1.3 - Units .

Want to learn more, check out the video below:



Web search "basic electricity" for further information.

Test Your Understanding

Question 1 of 2 1 2

Calculate Power(W), if Voltage is 12.5V and Current is 4A?

- 250W
- 25W
- 50W
- None of the above

Check My Work | Reset

1.6: Motion Equations

The basic equations of motion are generally taught at school. They are

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

Where: -

Initial Velocity = u

Final Velocity = v

Distance Moved = s

Duration = t

Acceleration = a

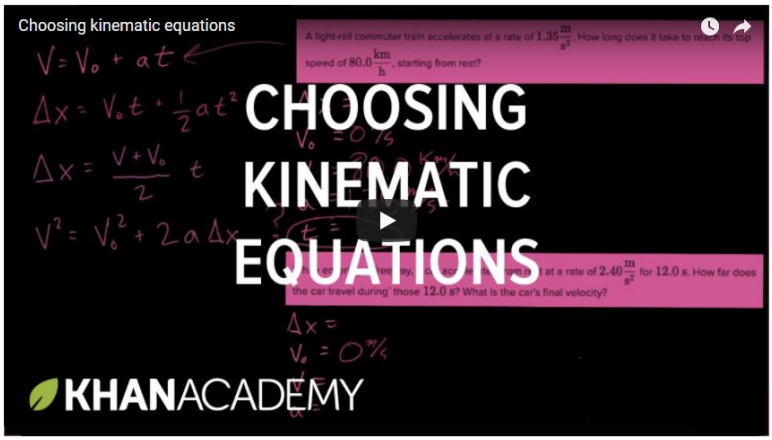
These equations apply to a solid mass moving in a straight line with constant (or zero) acceleration. They can be used to describe a single element of a motion sequence, which for a machine will often be a simple index move comprising an acceleration element, a constant speed element (which we describe as a cruise element), and a deceleration element bringing the speed back to zero. Often there is a subsequent element with no motion, which we describe as a dwell.

It is important to realize that when stringing Motion Elements together in this way, the initial velocity of an element is the same as the final velocity of the previous element – i.e. velocity is continuous. If it were not so, the acceleration at the element boundary (where one Motion Element gives way to the next) would be infinite, which is not physically possible. Equally, elapsed time and position are continuous, however, acceleration is not – it can change instantaneously at the element boundary.

In creating a Motion Profile, it is normal to assume this continuity so that the initial velocity and position of a Motion Element is always known. The exception is the first element in the series where they should be specified, although sometimes it is assumed that the initial values are zero. Thus, when specifying a Motion Element, the user would typically give two pieces of data, usually the duration of the element and the final velocity, or the duration of the element and the distance moved.

Generally, a minimum of three of the variables must be known to enable the other two to be evaluated.

Want to learn more, check out the video below:



Test Your Understanding

Question 1 of 1

Calculate the Acceleration of a train when it is moving steadily from 20.0 m/s to 90.0m/s in 70s is

- 1 m/s^2
- 2 m/s^2
- 0.16 m/s^2
- 3 m/s^2

Check My Work | Reset

1.7: Fundamentals– Mass

Weight and Mass

It is vitally important to distinguish clearly between weight and mass. It is unfortunate that these terms tend to be used interchangeably in common usage, not helped by the use of the term “pounds” to mean either mass or weight depending on context.

Newton’s first law of motion states that: - “Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it”. In other words, to cause an object to accelerate (increase its velocity), a force must be applied to it.

It is a common experience that a boat displacing several tons can be set into motion (accelerated), albeit slowly, by a person simply pushing hard on it. Likewise, a dinghy can be accelerated at the same rate but with much less force. The resistance or reluctance of the

vessels to accelerate is proportional to their MASS. If the same two vessels could be floated on the moon the force required to accelerate them at the same rate would be the same.

Weight, on the other hand, is proportional to mass times gravity. The weight of either boat would be one-sixth if weighed on the moon compared to here on Earth.

In the Imperial system, pounds mass is designated by “lb” whilst pounds force (or weight) is “lbf”. The weight of a 1.0lb mass on Earth is 1.0lbf. The weight of 1.0lb on the moon is 0.16lbf.

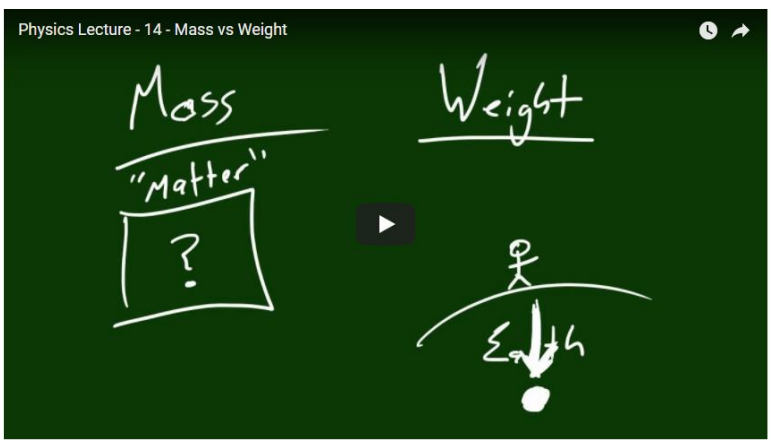
$$Force(lbf) = Mass\ (lb) \times Acceleration\ \left(\frac{ft}{s^2}\right) / 32.2$$

Great care must be taken not to confuse the two in calculations.

In the System International (SI), mass is designated by “kg” (kilogram) where force is “N” Newton’s. The weight of a 1.0kg mass on Earth is about 9.8N. The weight of 1.0kg on the moon is about 1.63N.

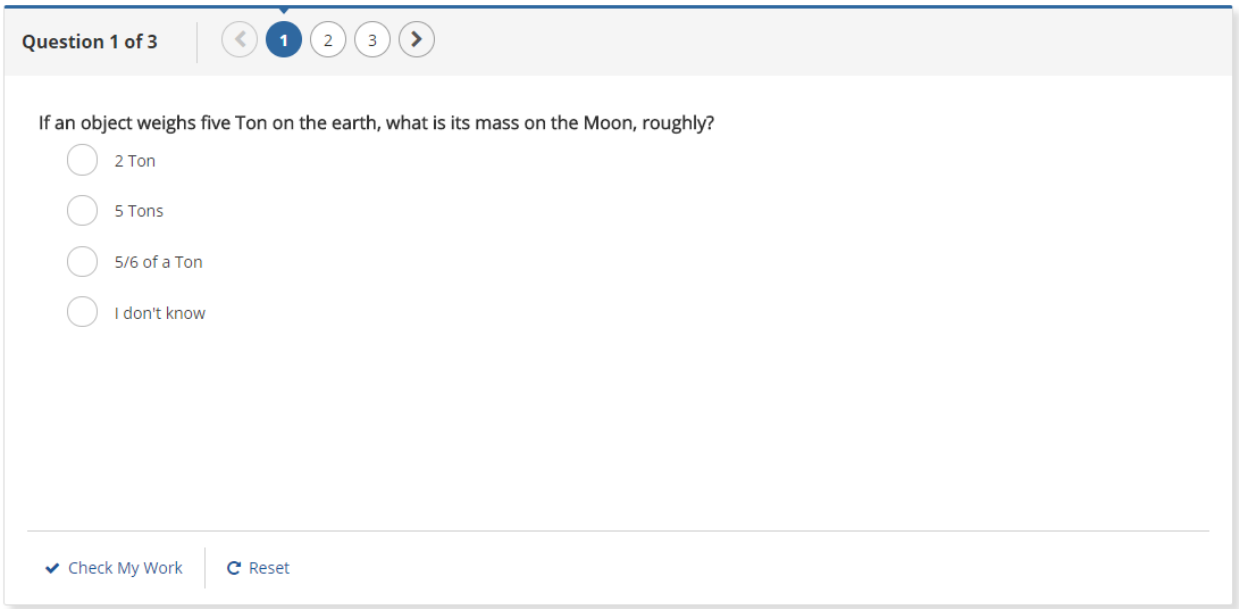
$$Force(N) = Mass\ (kg) \times Acceleration\ (\frac{m}{s^2})$$

Want to learn more, check out the video below:



Web search “mass” for further information.

Test Your Understanding



1.8: Moment of Inertia

Definition of Inertia

Newton's first law of motion states that: - "Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it." This applies to rotary motion as well as linear.

In servo terminology, Inertia (strictly "polar moment of inertia") is the rotary equivalent of mass. The equivalent of force is torque.

Every rotating object has inertia and requires torque to accelerate it.

$$Torque(lbf \times ft) = Inertia\ (lb \times ft^2) \times Acceleration\ \left(\frac{rpm}{sec}\right) \times \frac{2\pi}{60} \times \frac{1}{G(32\ \times \frac{ft}{sec^2})}$$

In S.I. Units

$$Torque(N \times m) = Inertia\ (kg \times m^2) \times Acceleration \left(\frac{rad}{sec^2}\right)$$

In a servo axis it is generally very important to reduce inertia to the minimum possible, in order to reduce the power required to move it, and hence the size and cost of the drive.

Calculating Inertia

Inertia is defined as

$$J = m \times r^2$$

Where J = Inertia, m = mass, r = Radius of Gyration; the radius at which the mass is effectively concentrated. A length of tube with an infinitely thin wall would have a Radius of Gyration equal to its physical diameter.

Example: Inertia of a Solid Cylinder

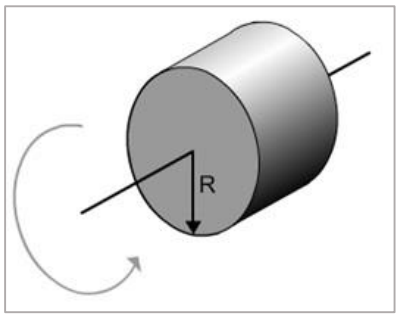


Figure 1: r = Radius of cylinder

$$Inertia\ of\ a\ solid\ cylinder = \frac{1}{2} \times mr^2 \ldots\ldots where\ m = mass, r = radius$$

There are formulae available for numerous shapes.

Moment of Inertia varies according to the axis of rotation under consideration.

Click here to see Moment of Inertia of some standard shapes.

Offset Loads

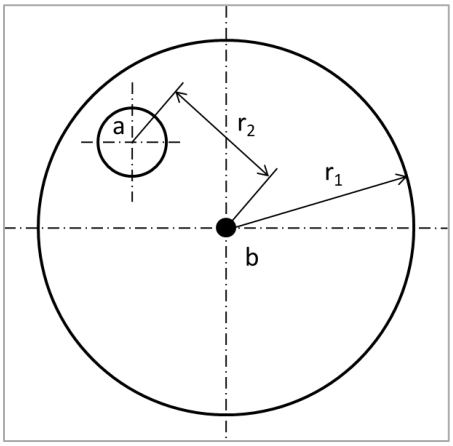


Figure 2: a = centre of rotation, b = centre of gravity

The cylinder has its Centre of Gravity(CG) at its geometrical centre but the centre of rotation is offset by r_2.

Now, there are two components of inertia:

1. Rotation of the cylinder about its own CG.
2. Rotation of its mass about the centre of rotation.

Entity	Formula
Inertia of the cylinder about its CG	$J_1 = 0.5 \times m \times r_1^2$
Inertia of its mass	$J_2 = m \times r_2^2$
Total Inertia	$J_1 + J_2$

Table 1

This assumes that the cylinder is rigidly connected to the rotating shaft. An exception would be a bicycle pedal. In this case its CG orbits around the axle so J_2 applies but it does not rotate about its own CG so J_1 does not.

Linear / Rotary equivalence

If a rotary motor drives, via some mechanism, a mass of M kilograms in a straight line so that a change in shaft angle of θradians results in a movement of L metres then it's effective radiusR_{eff} will be given by:

$$R_{eff} = \frac{L}{\theta}$$

Then the effective inertia J_{eff} will be given by:

$$J_{eff} = M \times {R_{eff}}^2$$

A simple example is a screw with lead L driving a load of mass M.

One rotation ($2 \times \pi$radians) results in a movement L so the effective radius is:-

$$R_{eff} = \frac{L}{2 \times \pi}$$

$$J_{eff} = M \times {R_{eff}}^2$$

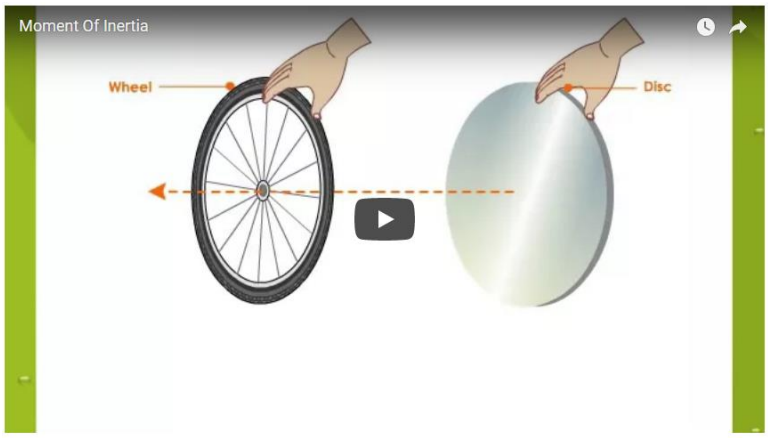
This concept is taken much further in Load Types – Variable Inertia Loads.

Non - Standard Shapes

1. It is sometimes possible to break a component down into standard shapes such as cylinders, cones or slabs. In this case the offset calculation above would be needed for each element.
2. Some sizing software allows complex shapes to be created with some limitations.
3. The ultimate method is to use 3D CAD (3 Dimensional Computer Aided Design) software which will calculate masses, CG positions and inertia about any axis.

The COSMATT sizing software which accompanies this course has an inertia calculation tool. Click **here** to open it.

Want to learn more, check out the video below:



Web search "inertia" and "inertia of common shapes" for further information.

Test Your Understanding

Question 1 of 1 1

A rotating disk has an inertia of 4 Kg-m^2. What would the inertia be in the International Space-station?

- 4 Kg-m^2
- 6 Kg-m^2
- 1/6 Kg-m^2
- Zero

Check My Work | Reset

1.9: Force

Magnitude of push or pull

"a force is any interaction that, when unopposed, will change the motion of an object" (Nave, C. R. (2014). "Force". Hyper physics. Dept. of Physics and Astronomy, Georgia State University).

Force is a push or pull which, when applied to an unrestrained object, causes it to change its velocity (to accelerate). Typical units are Newton or pounds force.

Force is a vector quantity so has direction, amplitude and sign.

Different types of Force: -

- Applied Force
- Gravitational Force
- Normal Force
- Frictional Force
- Air Resistance Force
- Tension Force
- Spring Force

Drawing free body diagram of a body helps us visualize the net force acting on a body and its direction. They are used to show the relative magnitude and direction of all forces acting upon an object in a given situation.

Want to learn more, check out the video below:

Typical Force units are: -

SI – Newton (N)

Metric – Kilograms Force (kgm)

British (Imperial) – Pounds Force (lbf)

Test Your Understanding

Question 1 of 1 1

If a shopping trolley weighing 25kg is pushed with a force of 150 N, its acceleration will be

- 5000 m/s^2
- 6 m/s^2
- 150 m/s^2
- 2 m/s^2

Check My Work | Reset

1.10: RMS

RMS

"RMS" or "Root Mean Squared" is used as a way to give a single value to a cyclically varying quantity, the most common example being the AC supply voltage or current of an electrical supply.

Applied to electrical current, RMS is the equivalent steady DC current of a cyclically varying current - that which would produce the same heating effect in a resistive load.

However, it can equally be applied to non-sinusoidal currents, such as the current required by a servo axis throughout the motion profile. This is important as the prime selection criterion for the motor in a servo axis is usually how hot it gets.

In an electric motor, current is directly proportional to torque so we calculate RMS torque to compare with the maximum continuous torque rating of the motor (since torque relates directly to current this defines the heating effect on the motor windings).

Method

Following is a sample of simple RMS calculations,

Segment	Time [t]	Motor Torque [T_m]	$T^2_m * t$
Accelerate	0.1	2	0.4
Cruise	0.1	0.1	0.001
Decelerate	0.1	-2	0.4
Wait	0.2	0	0
Totals	0.5		0.801

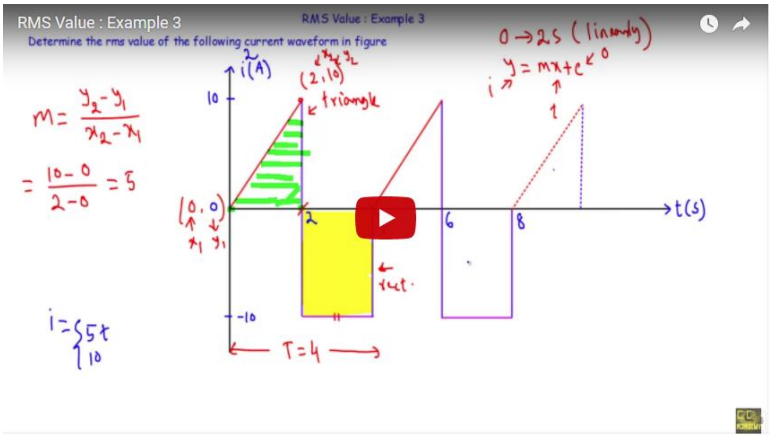
Table 1

1. Enter the time for each segment of the profile graph into column 1.
2. Enter the torque for each segment (assumed to be constant within the segment) into column 2.
3. Column 3 is the square of column 2 multiplied by column 1.
4. Columns 1 and 3 are summed (blue). The mathematical representation of a sum is the symbol ∑.
5. From the column totals we can compute the mean of the acceleration squared figure – divide the sum of the squares (0.801) by the overall time (0.5 s). Taking the square root of the result gives the RMS acceleration which is 1.265.

$$RMS = \sqrt{(\sum(Acc^2 * time))/(\sum time)}$$

$$RMS = \sqrt{\frac{0.801}{0.5}} = 1.265$$

Want to learn more, check out the video below:



Web search "root mean square" for further information.

Test Your Understanding

Question 1 of 1

What does the RMS value of a cyclically varying current mean?

- It is the equivalent to the steady AC current
- It is the equivalent to the steady DC current
- It is just another term for cyclically varying current
- None of the above

Check My Work | Reset

1.11: Torque

Servomotors are mainly defined by their torque rating rather than their power, so it is important to understand what torque is and what it does.

Definition of Torque

Torque is the rotary equivalent of Force and is defined as $Force \times Radius$:

$$Torque(Nm) = Force\ (N) \times Radius\ (m)$$

$$Torque(lbf.ft) = Force\ (lbf) \times Radius(ft)$$

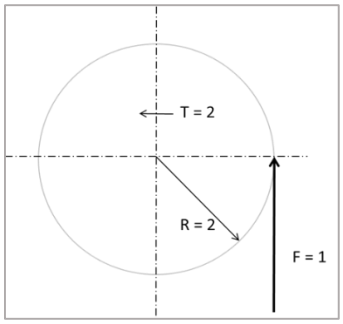


Figure 1

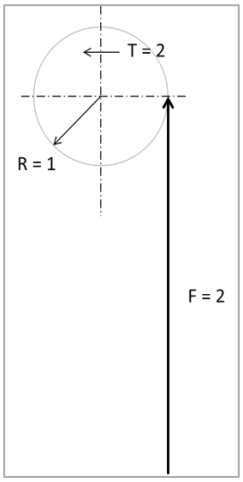


Figure 2

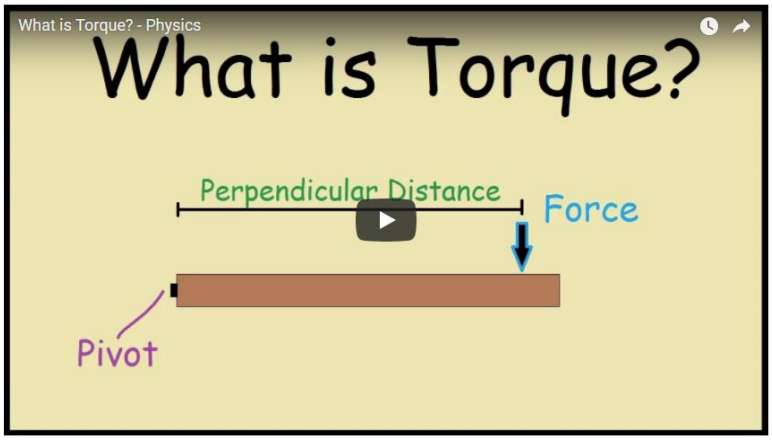
In both cases, the resultant torque is the same (2 torque units). In Figure 2, Force is double but Radius is half of the values in Figure 1.

Torque units

These always include both Length and Force:

- Nm (Newton metres)
- lbf.ins
- lbf.ft

Want to learn more, check out the video below:



Web search "torque" for further information.

Test Your Understanding

Question 1 of 1

A motor needs to provide 20 Nm of torque to a conveyer to move it. The drive roller is 100mm Diameter. What force is required at the circumference of the drive roller to move the conveyer?

- It is the equivalent to the steady AC current
- It is the equivalent to the steady DC current
- It is just another term for cyclically varying current
- None of the above

Check My Work | Reset

1.12: Power & Energy

Definition of Energy

Energy (or work done) is defined as:

Linear:

$$Force \times Distance\ Moved\ (regardless\ of\ speed)$$

In SI units

$$Energy\ (Joules) = Force\ (N) \times Distance\ Moved\ (m)$$

In Imperial units

$$Energy\ (lbf.ft)\ =\ Force\ (lbf)\ \times\ Distance\ Moved\ (ft)$$

Rotary:

$$Torque\ \times\ Angle\ Moved\ (regardless\ of\ speed)$$

In SI units

$$Energy\ (Joules)\ =\ Torque\ (Nm)\ \times\ Angle\ Moved\ (radians)$$

In Imperial units

$$Energy\ (ft.lbs)\ =\ Torque\ (lb.ft)\ \times\ Angle\ Moved\ (deg)\ /\ 57.3$$

Note that 1 radian = 57.3^{o}.

If one were to apply a force to an object that did not move as a result, no work would be done (zero energy).

Definition of Power

Power is defined as the rate at which work is done:

Linear:

$$\frac{(Force \times Distance)}{time}$$

$$Power(HP) = Force\ (lbf)\ \times Velocity\ \left(\frac{ft}{sec}\right) \times 550$$

$$Power(W) = Force\ (N) \times Velocity\ (m/sec)$$

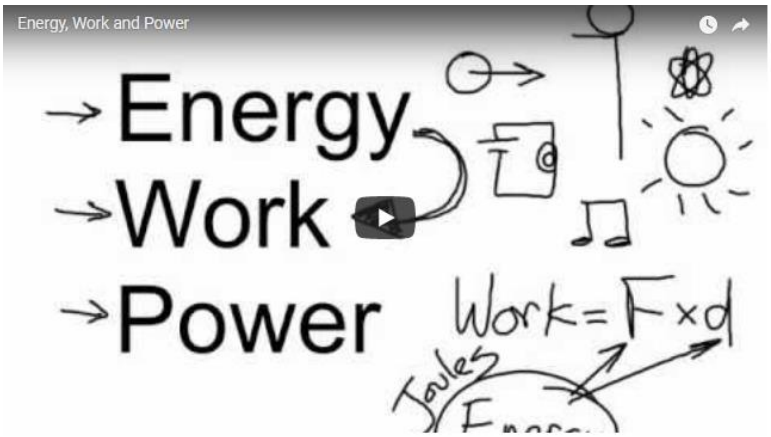
Rotary:

$$\frac{(Torque \times Velocity)}{time}$$

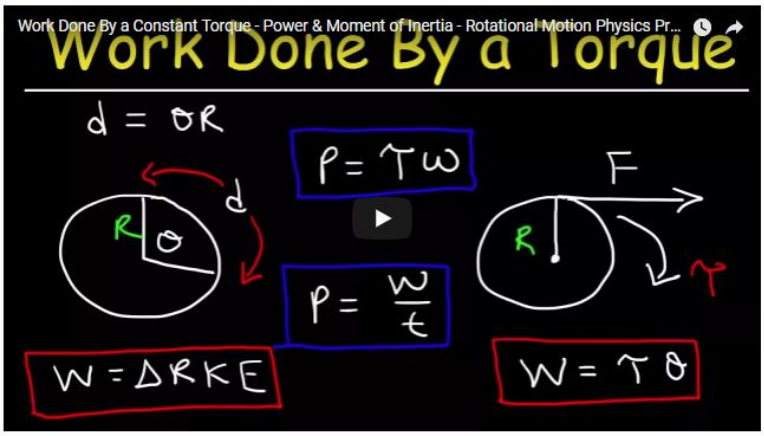
$$Power(W) = Torque\ (Nm) \times Speed\ (rad/sec)$$

$$Power(HP) = Torque\ (lb.ft) \times Speed\ (rpm)\ /\ 5252$$

Want to learn more, check out the video below:



Power in Circular Motion:



Web search "power physics" for further information.

Test Your Understanding

Question 1 of 3 1 2 3

In order to do work, energy is

- transferred or converted
- used up
- lost
- lost or transferred

Check My Work | Reset

1.13: How Servo Axis Works

Purpose of Servo Axis

The primary purpose of a Servo Axis is to accurately control the position of a load by following a position command generated by the Motion Control System.

The principle of operation is that the Servo Drive compares the actual (feedback) position of its motor with the command position and varies its speed and direction such as to minimize the error.

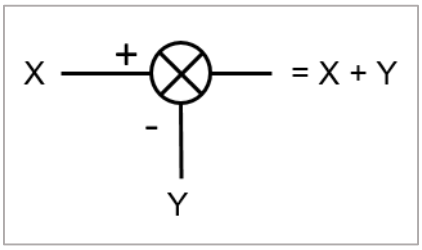


Figure 1

In loop diagrams the comparison of feedback to command is performed by a "Summing Junction" shown in Figure 1. The (+/−) signs indicate that the input Y is subtracted from input X to create the output.

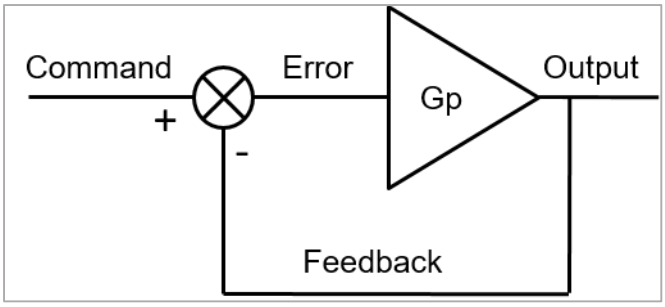


Figure 2

Figure 2, shows the basic servo loop. The output of the Summing Junction is the error. This is amplified by the Gain Gp to create the output. This, in turn, is fed back to the Summing Junction.

The gain Gp is known as "Proportional" gain because the output is always a fixed multiple of the input – output is proportional to input. The higher the gain, the smaller will be the error but, if the gain is set too high, the output will be unstable, tending to oscillate.

In Servo Axis control three such loops are nested together:

1. Position loop:
    a. Input Position command and feedback
    b. Output velocity command.

2. Velocity loop:
    a. Input Velocity command and feedback
    b. Output Current command.
3. Current loop:
    a. Input Current command and feedback
    b. Output Voltage to the motor.

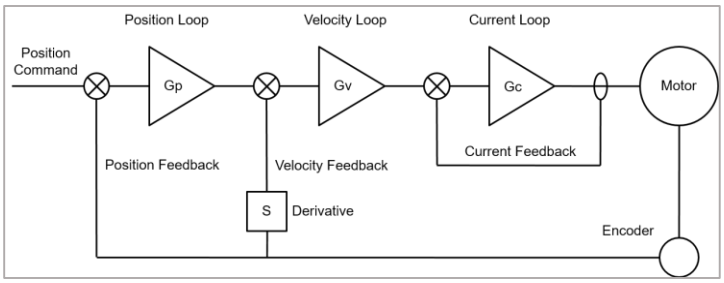


Figure 3

A major reason for nesting loops is that each loop adds damping to the previous one – this greatly reduces the tendency to instability allowing higher values of gain to be used.

Position feedback is provided, typically, by a motor mounted Encoder. This produces a large number of pulses for each motor revolution.

Velocity feedback is provided by deriving the pulse train frequency ("S" box).

Current feedback is by a transducer directly measuring current in the motor windings.

Since, for most servo motors, the torque is proportional to current the current loop may also be considered to be a Torque Loop.

"Tuning" is the term used to describe the adjustment of the various tuning (controller) parameters to optimize performance. The subject is covered in detail in the advanced course. For the basic course, it is important to understand that mechanical design can have a major effect on the ability to tune the system successfully, and is often the limiting factor in dynamic performance and accuracy.

Chapter 2 – Motion Profiles

2.1: Introduction

What is a Motion Profile?

The purpose of all servo systems is to move some kind of load and the way in which the load is moved is known as the move profile, or more often the **motion profile**. A **motion profile** can be as simple as a movement from point A to point B on a single axis, or it may be a complex move in which multiple axes need to move precisely in coordination. An example velocity profile is shown in Figure 1. The total distance traveled is found by calculating the area under the graph of velocity against time. The slope of the velocity curve represents the acceleration or deceleration at that particular instant. There are several types of motion profiles used with servo control systems. The most often used are **Constant Velocity**, **Trapezoidal**, and **S-Curve** motion profiles.

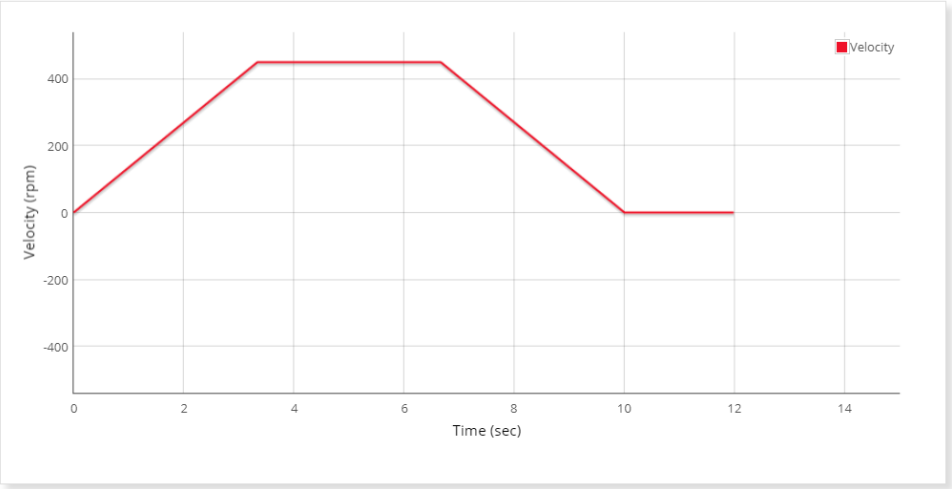


Figure 1

Following topics will be covered in this chapter:

- Basic Profiles
- Building a custom profile
- Introducing the Profile Widget
- Optimising the Profile Shape
- Building Longer Profiles
- Absolute Moves

Learning Objectives:

- Building basic motion profiles in response to move requirements
- Differences between the treatment of linear and rotary motion
- The effects of changing the shape of the profile

2.2: Basic Profiles

There are two main reasons why a motion profile is needed:

1. To calculate the speed and torque requirements for the selection of the motor.
2. To write the motion program in a motion controller.

Here we are only interested in the first of these. The profile enables us to establish the maximum speed and maximum acceleration requirements, both of which are major criteria in choosing a motor. You will learn more about that in Chapter 3 - How to Analyse a Rotary Loads and Chapter 4 - Servo Motor Selection.

Index Profiles

In servo systems, an "Axis" is a mechanism driven by a single motor whether rotary or linear. In most cases the way an axis is required to move is specified by the machine designer, but this may be a written description and may not specify every part of the motion cycle exactly. To evaluate the motion properly, a precise motion profile will be required.

The motion profile is simply a way of defining how an Axis moves with respect to time. Usually it is shown graphically.

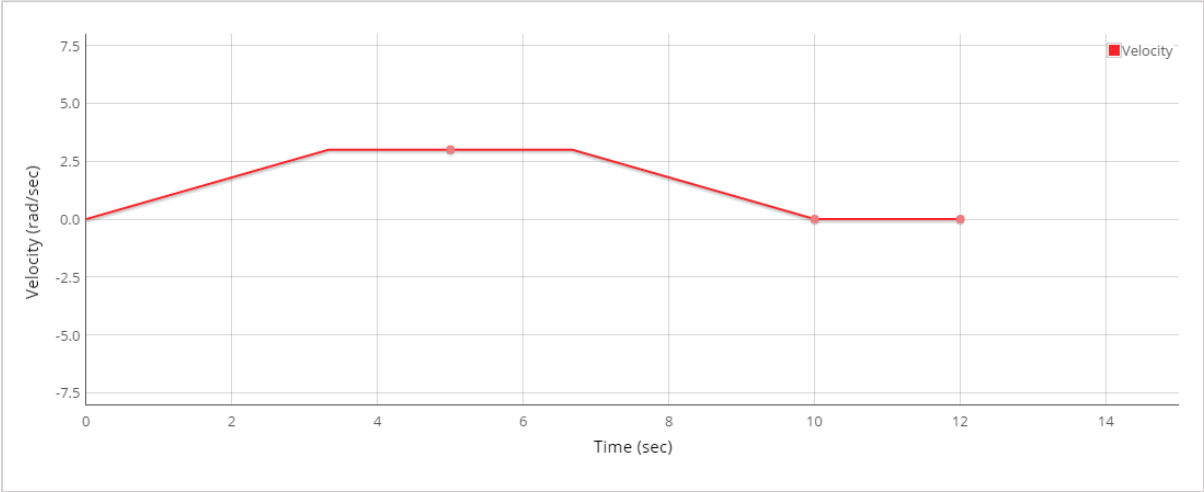


Figure 1

Figure 1, above, displays a simple index motion profile defined in terms of velocity, the most common representation, in this case accelerating, cruising at constant speed, decelerating back to a standstill then pausing or dwelling. For sizing purposes, the cycle is usually assumed to repeat indefinitely. Note that velocity could be linear or rotary; the diagrams are the same. Note also that the velocity profile is continuous, i.e. there can be no step changes, (which would involve infinite acceleration). This also means that the velocity at the end of the profile must be the same as at the start.

It is possible to define a profile in terms of position or acceleration against time but most people find that velocity is the most intuitive.

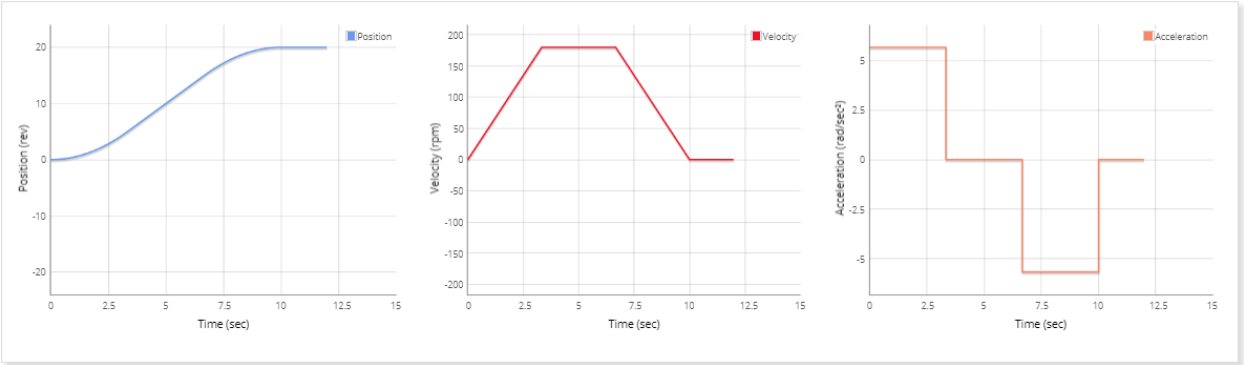


Figure 2

The Blue Graph is the equivalent position profile, velocity is the rate of change (or differential / derivative) of position. See how in the second segment the position is changing at a constant rate (constant velocity). This is not as easy to visualize as the velocity profile for most people but it is just as valid mathematically.

The Orange Graph is the same simple profile shown in terms of acceleration. Acceleration is the rate of change (or differential / derivative) of velocity. The constant positive value of the first segment corresponds to the constantly increasing velocity. In the second segment acceleration is zero since velocity is constant. In the third segment the velocity is decreasing at a constant rate so the acceleration is a constant negative value.

Although, it is possible to represent a profile in terms of acceleration, it would not normally be used alone. However, it is important to be able to visualize the acceleration profile especially when advanced profiles are used with non-linear accelerations. These are dealt with in the advanced course

> It is vital to become so familiar with these representations that one may mentally switch between them.

Linear and Rotary motion

Note, while the diagrams are the same for linear and rotary motion, the units are different which is extremely important. A linear profile may be specified in velocity terms using meter per second (m/sec) as the units, in which case the acceleration will be in meter per second squared (m/sec^2). This allows direct calculation of the required forces in Newton.

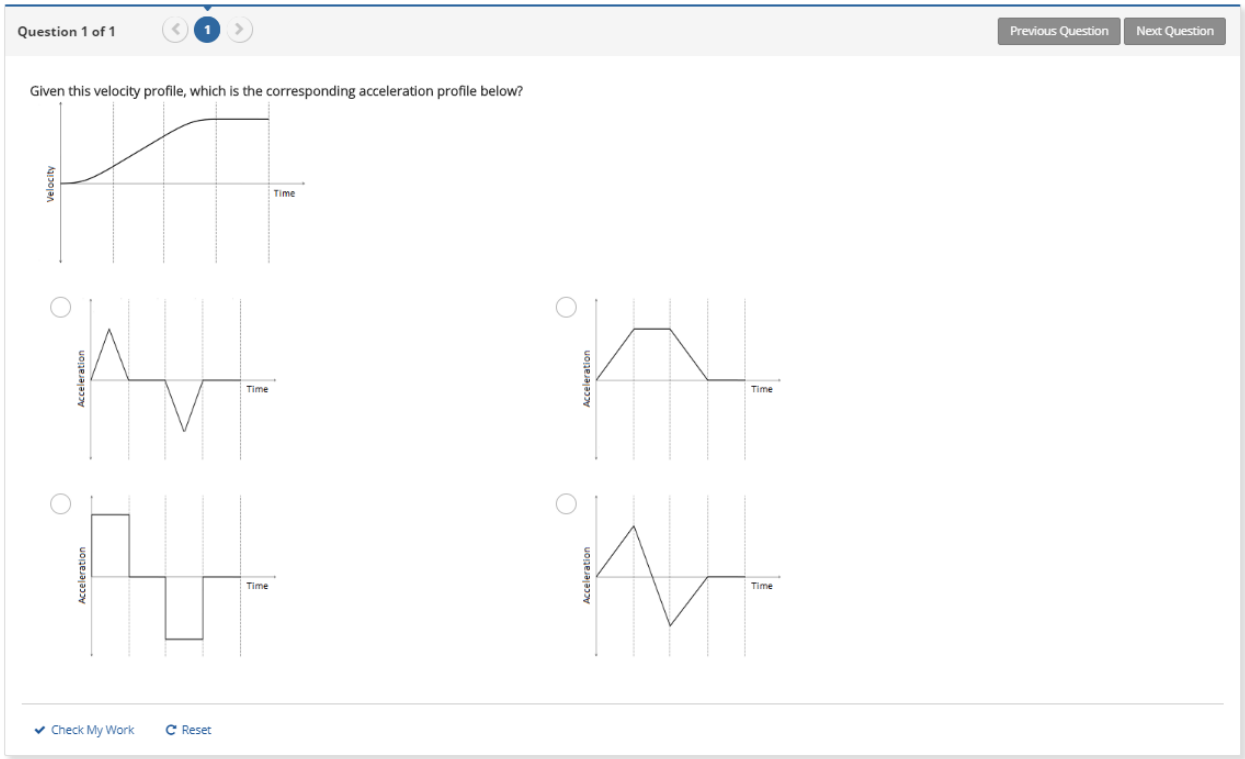
In the rotary case, the distance is usually specified in the familiar units of degrees or revolutions, rather than the fundamental unit, the radian. If you are programming a controller, you will generally not have the option of programming in radians. You will usually be using revolutions or degrees.

Thus speed will often be specified in rpm giving acceleration units of rpm per second, or in degrees per second, giving acceleration in degrees per second squared. Both these acceleration units are awkward to use when calculating the required torques (see Chapter 3 - How to Analyse a Rotary Load). Using the fundamental unit (the radian) solves this problem, allowing you to directly calculate torque in Newton-meter (Nm). However, this introduces rounding errors in most cases, due to the need to use π in the calculations. A good compromise is to stick with revolutions or degrees for distance and speed and only convert to radians per second squared for the acceleration, as these units are necessary for direct calculation of torque – you will learn more about that in chapter Rotary Loads.

Advanced profiles

In this section we have only dealt with motion where the acceleration is constant or zero, where there is Infinite jerk (i.e. instant change of acceleration). Jerk is the usual term for the rate of change of acceleration and this is a controlled variable in motion profiles such as a cubic spline, common in robotics. Also motion profiles can be built using standard cam equations – all this is dealt with in the advanced course.

Test your understanding



2.3: Building a Custom Profile

How to build a profile

Let's start with a simple rotary example. We could build a profile to move a wheel, let's say 3.25 revolutions in 0.25 seconds followed by a dwell of 0.05 seconds. Symmetrical profiles are

normally used for indexes, but the profile doesn't have to be symmetrical, so for this example we will make the acceleration time 90ms, the cruise time 75ms and the deceleration time 85ms. A faster deceleration is sometimes used when there is some friction to help when slowing the machine.

The first step is to calculate the maximum speed. Since the acc/dec (acceleration/deceleration) segments have a triangular velocity profile, the average speed during each of these segments must be half the maximum speed. We can use this fact to calculate what the maximum speed must be.

Let's call the maximum speed V_m. The total move distance (TMD) is thus given by

$$\mathrm{TMD} = (\mathrm{t}_1 * \ \mathrm{V_m}/2\ ) + (\mathrm{t}_2 * \ \mathrm{V_m}) + (\mathrm{t}_3 * \ \mathrm{V_m}/2\ )$$

Where t_1, t_2 and t_3 are the durations of segments 1, 2 and 3 respectively.

Substituting what we know for this example (time units have been changed from ms to sec): -

$$\mathrm{TMD} = (0.09\ * \mathrm{V_m}/2\ ) + (0.075 * \mathrm{V_m}) + (0.085\ * \ \mathrm{V_m}/2\ )$$

$$\mathrm{TMD} = \mathrm{V_m} * [\left(\frac{0.09 + 0.085}{2}\right) + 0.075]$$

$$\mathrm{TMD} = \mathrm{V_m} * [(0.175/2) + 0.075]$$

$$\mathrm{TMD} = 0.1625 * \ \mathrm{V_m}$$

And we know that TMD is 3.25 revolutions, so $\mathrm{V_m} = \frac{3.25}{0.1625} = 20\ \ \mathrm{rev/sec}$, or 1200 rpm.

The profile table

We can now create a table of what we know:

Segment Type	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	Distance Travelled [s]
	(rev/s)	(rev/s)	(s)	(rev/s²)	(rev)
Accel	0	20	0.09	?	?
Cruise	20	20	0.075	?	?
Decel	20	0	0.085	?	?
Dwell	0	0	0.05	?	?

Table 1

This table could be created using spreadsheet software like Excel, Google Sheets or LibreOffice. If you use a spreadsheet application, save the results as we will expand on them as we go through the course.

It is fairly easy to fill in the unknowns using the basic motion equations. Taking the first segment and using the formula $\mathrm{v} \ = \ \mathrm{u} \ + \ \mathrm{a} * \mathrm{t}$, rewritten as $\mathrm{a} = (\mathrm{v} - \mathrm{u})/\mathrm{t}$

Acceleration $a = \frac{(20-0)}{0.09} = 222.22\ rev/s^2$

The distance travelled can be calculated using $s = 1/2 * (u + v) * t$

Distance $s = \left(\frac{0+20}{2}\right) * 0.09 = 0.9\ rev$

You can fill in the other unknown numbers using the same equations. The final table will be as below.

Segment Type	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	Distance Travelled [s]
	(rev/s)	(rev/s)	(s)	(rev/s²)	(rev)
Accel	0	20	0.09	222	0.9
Cruise	20	20	0.075	0	1.5
Decel	20	0	0.085	-235	0.85
Dwell	0	0	0.05	0	0

Table 2

Graphically, the Velocity vs. Time profile looks like this:

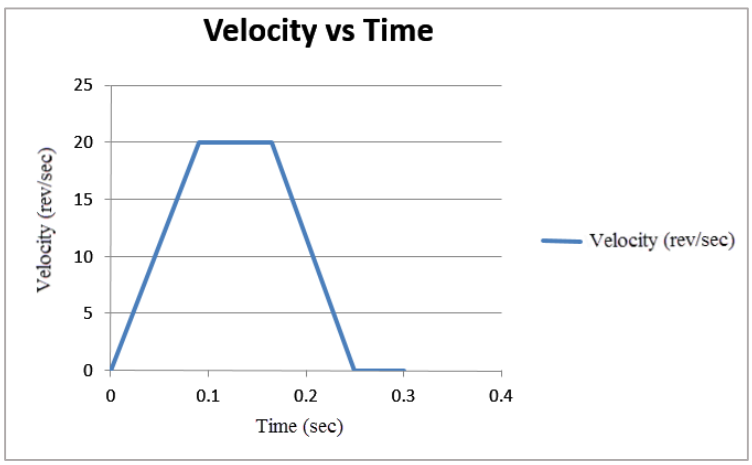


Figure 1

RMS acceleration

When we move on to calculating the torque or force, which we will cover in Chapter 3 - How to Analyse a Rotary Loads, it is useful to have a figure for the RMS acceleration, which has a direct effect on the motor heat loss. Here's the calculation for the profile we are using in this example. For each segment we need the segment duration and the square of the acceleration (first two columns), which we multiply together to give the figures in the third column.

Segment Type	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	$a^2 * t$
	(rev/s)	(rev/s)	(s)	(rev/s^2)	
Accel	0	20	0.09	222	4440
Cruise	20	20	0.075	0	0
Decel	20	0	0.085	-235	4710
Dwell	0	0	0.05	0	0
Total			0.3	-13.1	9150

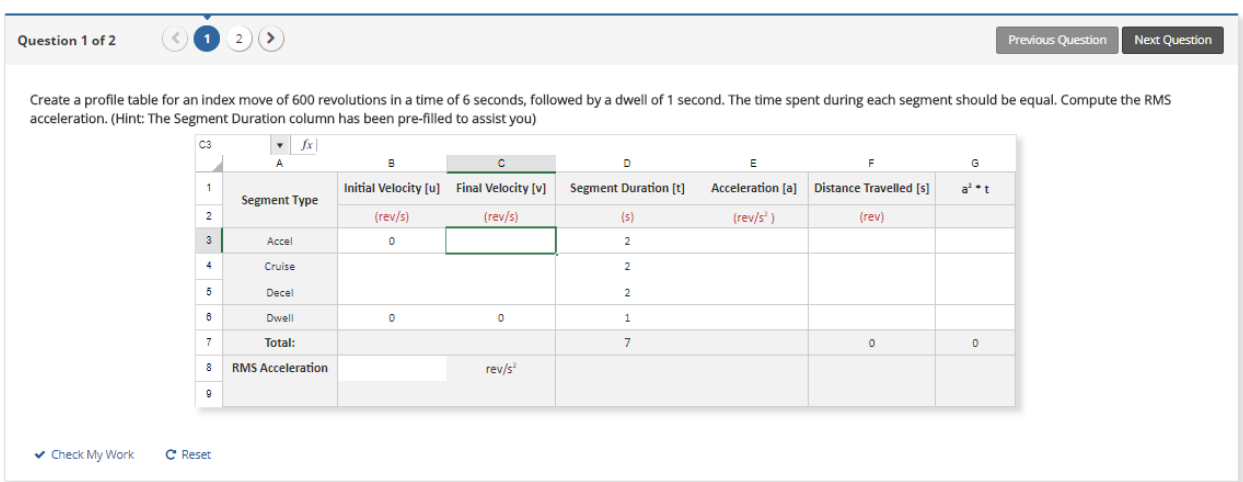
Table 3

From the column totals we can compute the mean of the acceleration squared figure – divide the sum of the squares (9150.33) by the overall time (0.3 s). Taking the square root of the result gives the RMS acceleration which is 175 rev/s squared.

$$RMS = \sqrt{(\sum(Acc^2 * time))/(\sum time)}$$

$$RMS = \sqrt{\frac{9150.33}{0.3}} = 175\ (\mathrm{rev}/s^2)$$

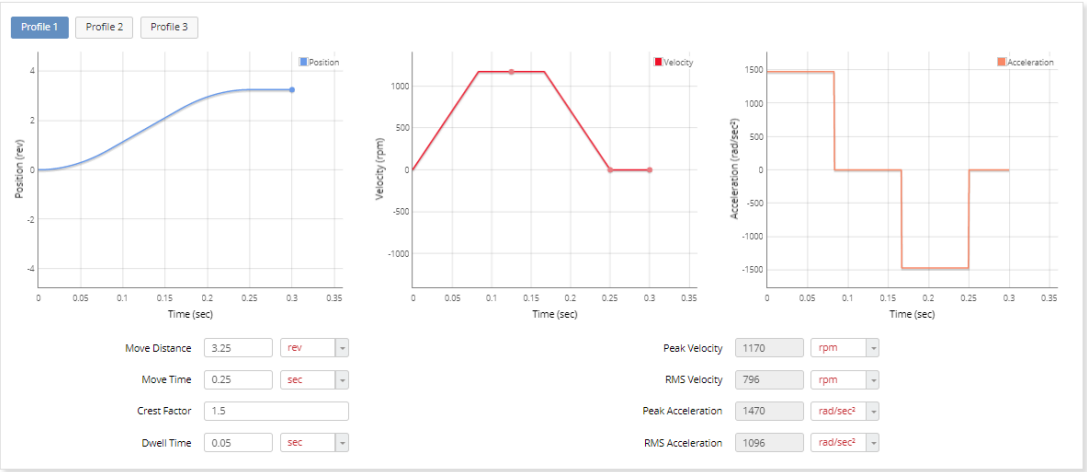
Test your understanding



2.4: Introducing the Profile Widget

Calculating a simple move profile isn't too hard as we saw in the last section (Building a Custom Profile), but it is useful to be able to use a software tool which will do much of the calculation

for you, while giving you an instantaneous graphical representation of your entries. Here is such a tool, the Motion Profile Widget.



The graph above shows three plots related to a single motion profile, all plotted against time. The first is the position, the second velocity and the third acceleration. Two other profiles are available, selected via the tabs at the top of the graphs.

Profile 1 is a typical example of a symmetrical motion index with a dwell time after the move. You can change the index as you please in two ways:

1. On the graphs there are prominent dots. These are active handles which can be moved using the mouse – try it now. As you move the handle on one graph you will see the others changing to match the new data. You will also see the data in the text boxes below changing.
2. There are text boxes below the graphs on the left to allow direct entry of data. Enter some numbers to see how the graphs change immediately. The data on the right shows some important resulting parameters.

   The Widget has a parameter labeled “Crest Factor”. This is the ratio of the peak to average velocity (analogous to the Crest Factor of an electrical waveform = Peak/rms). It is useful to allow the user to see the effect of varying this factor for a given time and distance.

You can also change the units using the list boxes to the side of the numerical data.

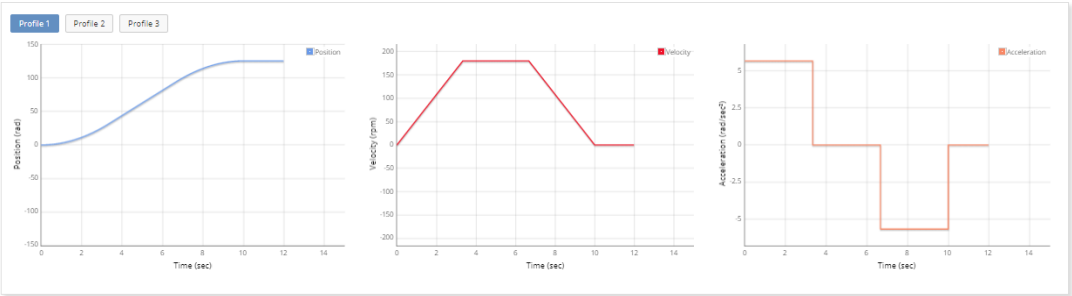
Profile 2 has a triangular motion profile, while Profile 3 has a constant velocity throughout the move. Note the change in Crest factor. We’ll discuss these different shapes in the next section.

Usefully, the widget calculates some results which will be needed for selecting the motor. These can be seen at the bottom right. Possibly the most useful is the RMS acceleration which, as mentioned in the last section, relates directly to the motor heat loss. The reason for this is explained in more detail in the advanced course.

Test your understanding

2.5: Optimising the Profile Shape

We have seen how we can create a simple motion profile and how the shape can be changed, but what are the practical implications of changing the shape? Here's the widget again, with the three pre-selected profiles.



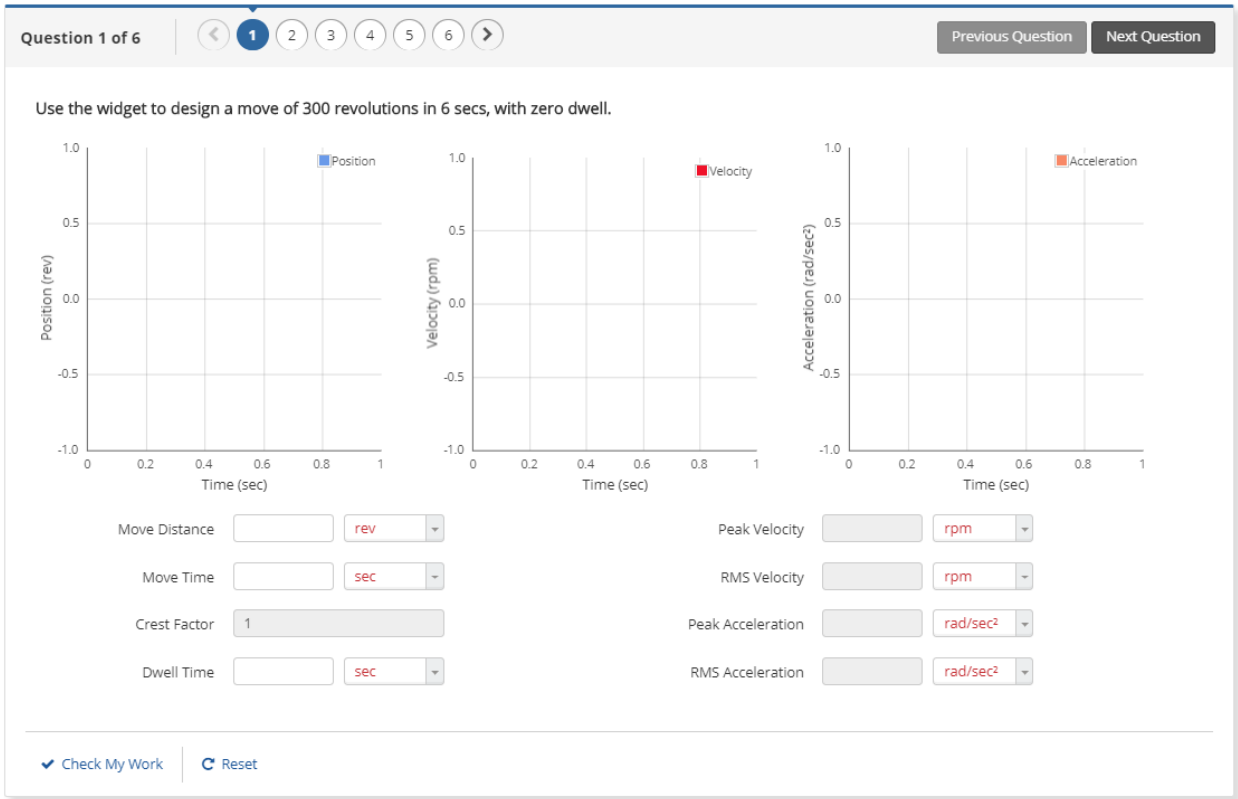
- Profile 1, is a "1/3, 1/3, 1/3" profile where the Crest Factor is 1.5 ($peak\ velocity = V_{avg} * 1.5$) where V_{avg} is the average velocity of the moving part of the profile (not including the dwell). This produces the lowest value of RMS torque and therefore the lowest possible heating of the motor windings.
- Profile 2, is a triangular move with Crest Factor 2.0 ($peak\ velocity = V_{avg} * 2$) that gives the lowest value of acceleration.

- Profile 3, shows an instantaneous acceleration/deceleration not physically possible for an index move, but using very high rates of acceleration, it can be approximated. This gives the lowest possible value of velocity equal to the average velocity of the move (V_{avg}) - Crest Factor = 1.0 . Note that the complete cycle average is lower due to the dwell segment.

Web search trapezoidal move for further information.

All these profiles are symmetrical i.e. the acceleration and deceleration times are the same. The widget does not allow for skewed (asymmetric) profiles such as our example in Chapter 2.3 - Building a Custom Profile but there can be some advantage in skewing the profile. For example, if the machine you wish to move has a high level of friction you will need less torque to decelerate so you may as well decelerate a little faster and save some time. Subtleties such as this are covered in more detail in the advanced course.

Test your understanding



2.6: Building a Longer Profile - Absolute moves

You can extend the technique which we used in Chapter 2.3 - Building a Custom Profile to build a longer profile with more segments. Here's an application example:

An indexing table is required to move from its start point at zero degrees to a position at 120 degrees in 120 milliseconds. It must remain stationary for 100 (ms) while a manufacturing operation takes place. Then move again to 270 deg, this time in 150 ms. After a further dwell of 200 ms while the workpiece is ejected, it must return to its start point and again dwell for 330 ms while a new workpiece is loaded, before repeating the cycle. The production rate required is 40 products per minute i.e. a cycle time of 1.5 seconds.

The profile shape for these indexes is not specified and it is not stated which direction is used for the return-to-zero move. In a rotary application it could be forwards or backwards but often there is a restraint due to the mechanical configuration or cabling which prevents repetitive moves in one direction. These are questions you would ask of the machine designer of course.

For this exercise we will use the 1/3, 1/3, 1/3 shape which gives the lowest RMS acceleration and the return to zero will be backwards.

Before we start a table, there are some preliminary calculations to be made for each index:

a) The first index moves 120 degrees in 120 ms which is an average speed of 1000 deg/s. We know from the last section that for the chosen profile shape the cruise velocity is 1.5 times the average i.e. 1500 deg/s. For index 2 the final position is 270 degrees, so the move distance is 150 degrees. We can use the same logic to show the cruise speed is again 1500 deg/s.
b) The return move will be minus 270 degrees. Adding all the specified times gives us a total of 900 ms, so by subtracting from the total cycle time of 1500 ms, we know the return move time must be 600 ms. Using the same logic again gives a cruise speed of -675 deg/s.

Thus we can start a table with the information we have:

Segment Type	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	Distance Travelled [s]
	(deg/s)	(deg/s)	(s)	(deg/s^2)	(deg)
Accel	0	1500	0.04	?	?
Cruise	1500	1500	0.04	?	?
Decel	1500	0	0.04	?	?
Dwell	0	0	0.1	0	0
Accel	0	1500	0.05	?	?
Cruise	1500	1500	0.05	?	?
Decel	1500	0	0.05	?	?
Dwell	0	0	0.2	0	0
Accel	0	-675	0.2	?	?
Cruise	-675	-675	0.2	?	?
Decel	-675	0	0.2	?	?
Dwell	0	0	0.33	0	0

Table 1

You can complete the information using the motion equations, to give:

Segment Type	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	Distance Travelled [s]
	(deg/s)	(deg/s)	(s)	(deg/s²)	(deg)
Accel	0	1500	0.04	37500	30
Cruise	1500	1500	0.04	0	60
Decel	1500	0	0.04	-37500	30
Dwell	0	0	0.1	0	0
Accel	0	1500	0.05	30000	37.5
Cruise	1500	1500	0.05	0	75
Decel	1500	0	0.05	-30000	37.5
Dwell	0	0	0.2	0	0
Accel	0	-675	0.2	-3380	-67.5
Cruise	-675	-675	0.2	0	-135
Decel	-675	0	0.2	3380	-67.5
Dwell	0	0	0.33	0	0

Table 2

The velocity profile graph looks like this:

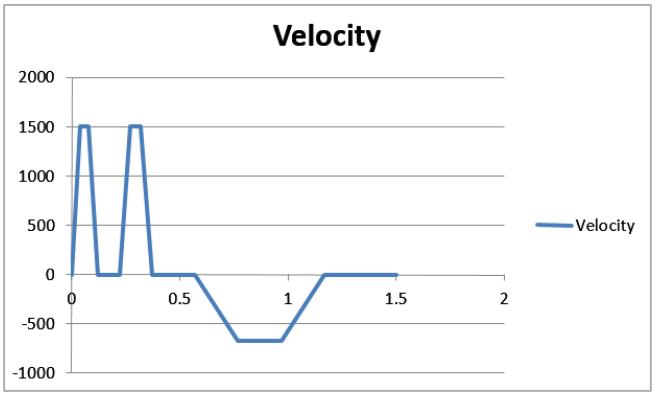


Figure 1

Absolute values

Time and distance are shown in the table above using incremental values – the duration of the segment rather than the absolute elapsed time, and the distance travelled within the segment rather than the target position. To make it easier to check these figures, it may be useful to show the corresponding absolute figures, and often the machine designer will specify the move in absolute terms. In the table below, two columns have been added to show these values – scroll to the right to see them.

Segment Type	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	Distance Travelled [s]	Absolute or Elapsed Time
	(deg/s)	(deg/s)	(s)	(deg/s^2)	(deg)	(s)
Accel	0	1500	0.04	37500	30	0.04
Cruise	1500	1500	0.04	0	60	0.08
Decel	1500	0	0.04	-37500	30	0.12
Dwell	0	0	0.1	0	0	0.22
Accel	0	1500	0.05	30000	37.5	0.27
Cruise	1500	1500	0.05	0	75	0.32
Decel	1500	0	0.05	-30000	37.5	0.37
Dwell	0	0	0.2	0	0	0.57
Accel	0	-675	0.2	-3380	-67.5	0.77
Cruise	-675	-675	0.2	0	-135	0.97
Decel	-675	0	0.2	3380	-67.5	1.17
Dewell	0	0	0.33	0	0	1.5

Table 3

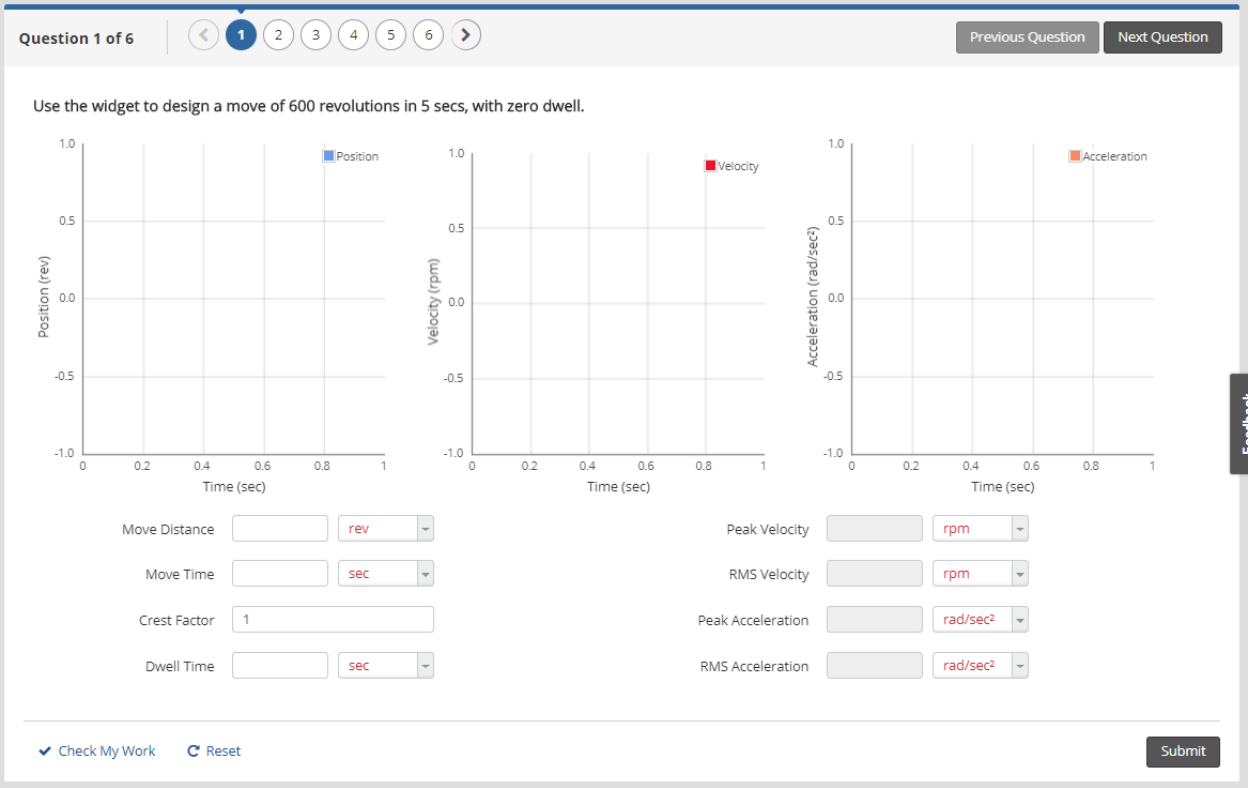
It is easy to see now that the total cycle time is 1.5 seconds, as specified, and that the second index correctly moves to a target position of 270 degrees after the second index and back to zero at the end of the cycle.

It is very important to be aware of the difference between incremental and absolute values and to know which is being used when constructing the profile.

The Profile Widget which you have seen earlier in this chapter is limited to indexes and is not as useful for a more complex profile like this example. However, the COSMATT sizing tool has a fully functional profile generator which can handle an unlimited number of segments. Also this chapter has only covered segments involving constant or zero acceleration. The COSMATT tool can deal with more complicated segments like T-S-Curves and cubic splines, which you can learn about on the advanced course.

In the next chapter, we will look at what is required to make a rotary load move in accordance with a Motion Profile.

2.7: Assessment



Chapter 3 – How to Analyse a Rotary Load

3.1: Introduction

In Chapter 2.1 - Move Profiles we learnt how to define the motion profile that the axis will be required to achieve. The next step towards specifying a motor is to calculate the torque requirements.

A motor is a means of generating mechanical power using electricity. For many types of industrial motor, we are given a continuous power rating which the motor can supply indefinitely, and a peak rating available for extremely short periods. While these power ratings of the motor are useful they don't give us enough information for servo motors, where we need to be much more detailed in terms of speed and torque capabilities. You will learn more about that in Chapter 4.1 - Servo Motor Selection, but for now we will look at calculating the load torque.

The peak torque requirement determines what the chosen motor must be capable of delivering for a very short period, the continuous (RMS) torque requirement determines what torque the motor must be able to produce continuously without overheating and of course, the motor must be capable of at least the maximum speed required by the motion cycle.

We already know the speed requirement from the profile calculations, so now we need to compute the torque requirements, for which we need to know about the load.

What exactly do we mean by the term "Load"?

The payload is the target component which is to be moved by the axis; in the case of a rotary payload, it is generally in the form of a rotating mechanism which will have inertia. The motor which drives the axis will need to overcome some kind of resistance in order to rotate this payload. This is seen as a load torque. It will include any additional torque which is required to move any mechanical components which connect the motor to the payload such as shafts and couplings and perhaps gearboxes, collectively called the transmission. Put together, these components all constitute the "load" as seen by the motor.

The most obvious resistance is friction and the motor must develop sufficient torque to overcome any friction or it cannot move. However, torque is also needed in order to change the speed of rotating inertias (the payload and transmission components). There may also be externally applied torque to overcome, such as would occur in an elevator motor where gravity is ultimately applying an additional torque. All these torques add together to create the total load torque which the motor must overcome in order to move the mechanism.

Thus, the load torque consists of:

- Acceleration or deceleration torque
- Externally applied torque (e.g. due to gravity or spring force)
- Friction torque

In many applications, the motion requirement is very dynamic i.e. there are frequent sharp changes of speed in the motion profile, so the torque required to accelerate and decelerate the inertia of the mechanisms is often dominant. The inertia of the motor itself also contributes to this part of the load torque and is dealt with in Chapter 4.1 - Servo Motor Selection.

Following topics will be covered in this chapter:

- Inertia Calculations
- Inertia Calculator
- Computing the Load Torque
- Calculating the RMS Value
- Representing the Instantaneous Torque & Speed Requirements on a Chart

Learning Objectives:

- To analyse a rotary load.
- Calculating inertia and including friction losses and additional externally applied loads.
- Apply a motion profile to calculate the torque and speed requirements.
- To understand how peak and continuous operating points are displayed in tabular and graphical forms.

3.2: Inertia Calculations

A Rotary Load is any driven load that rotates about an axis; normally connected either directly to a motor by a coupling or via a transmission system such as belts, chains, gearboxes etc.

A direct coupled, rotary Load Axis has the basic components as shown in Figure 1.

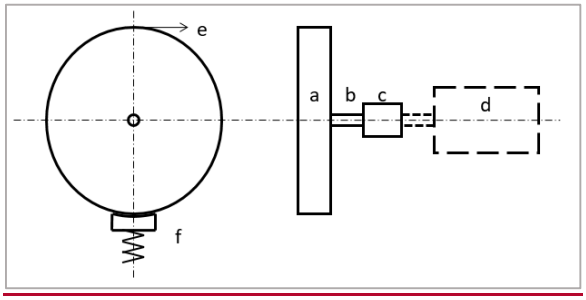


Figure 1

The above figure shows an axial view of a disk (left) and a side view (right). In this figure, the various elements are represented by :-

a represents Load Inertia,

b represents Drive Shaft,

c represents Coupling,

d represents Motor,

e represents External Torque,

f represents Friction Torque

Inertia

As we mentioned in introduction to this chapter, the most important parameter in fast accelerating servo systems is the inertia of the load as "seen" by the motor. In Figure 1 above, the inertial load is the sum of the inertias of components a, b and c.

Normally the mechanical designer will be able to provide this data but, if not, it will be necessary to calculate it. Most rotating loads may be broken down into simple shapes, cylinders, cubes etc. Taking a simple example of a solid disk:-

First calculate the volume of the disk:-

$$Volume\ of\ a\ solid\ cylinder = l\,\pi\,r^2 \quad where\ l = length, \qquad r = radius$$

Calculate its mass: -

$$Mass\ = Volume * Density$$

Then calculate its inertia: -

$$Inertia\ of\ a\ solid\ cylinder = \frac{1}{2}mr^2 \quad where\ m = mass, \qquad r = radius$$

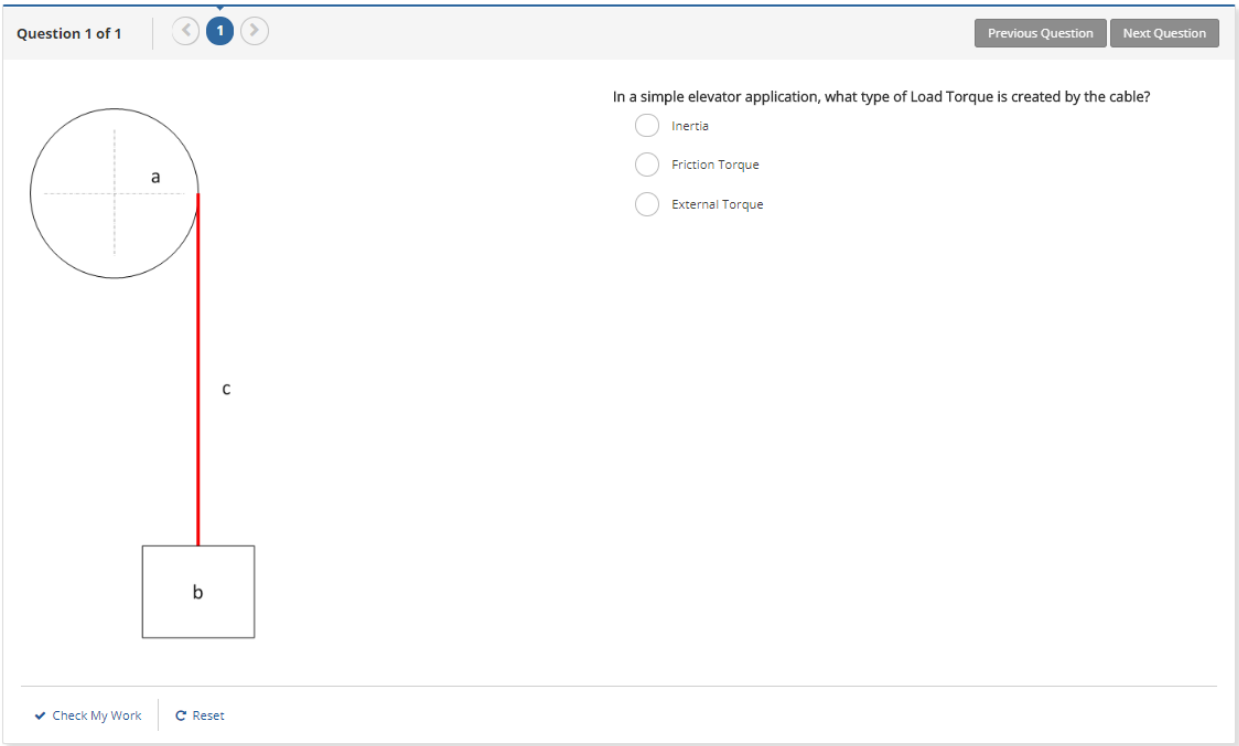
If the object can be reduced to a series of primary shapes, calculate the inertia of each as shown above then add them together. Chapter 1.8 – Moment of Inertia provides further information on more complex shapes.

As we mentioned earlier, the shaft (b), and coupling (c) will have inertia too and these must be included in the total load inertia.

There is another way in which a mechanical component can add to the inertia, and that is if it is seen as a point mass at the circumference of the disk, for example, a cable or belt at the circumference of a belt/pulley transmission system or hoist. In this case, the formula for this component of inertia is:-

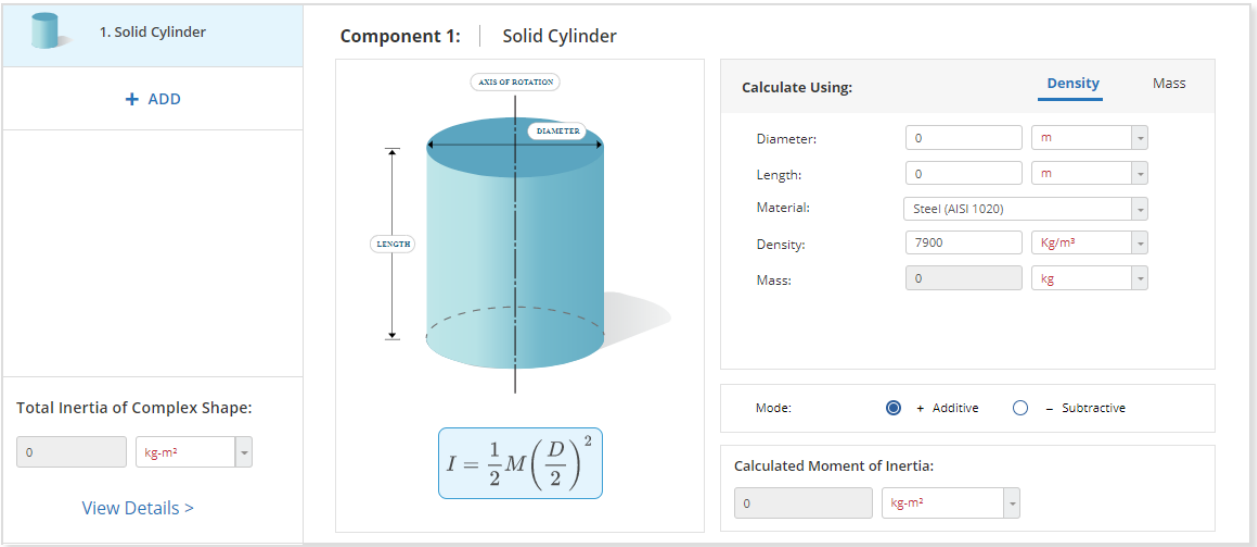
$$Inertia\ of\ a\ point\ mass\ on\ the\ circumference = mr^2 \quad where\ m = mass,\ \ r = radius$$

Test your understanding



3.3: Inertia Calculator

Computing the inertia of a complex shape can be tedious, so here is an inertia calculator which makes the process a lot easier.



Inertia Calculator

This calculator allows you to enter a simple disk or cylinder, and then build it up into a more complex shape such as a pulley wheel with a hub, web and rim as shown below.

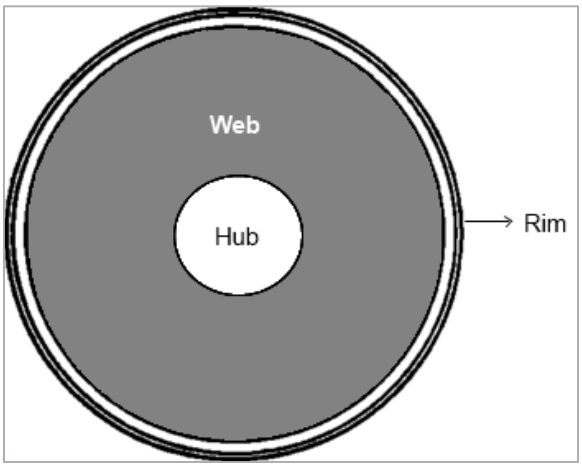


Figure 1

Let's try a solid cylinder first. You can choose to use either the mass, if you know it, or the material density; choose the mass option first – select "Mass" at the top right. Enter a diameter of 25 mm and a mass of 0.078 kg. You will immediately see that it has an inertia of 6.09 x 10^{-6} kg-m^2.

This is a quick way to get an inertia, but typically you will be faced with a more complex shape like a pulley where you don't know the mass of the individual parts, but you do know the material they are made of. So now select "Density" and enter the diameter and length – say 25 mm diameter and 20 mm length. Let's say this pulley is made of steel – click on the drop-down Material field and select Steel. The density of steel appears immediately in the next field down, and the mass immediately below that – in this example 77.6 gm.

Note that at any time you can change the units on the right. As we saw before, the moment of inertia appears in the field at the bottom of the calculator. Try a few different numbers to see what happens.

Now add a web. This would be in the form of a hollow cylinder, so click on "ADD" on the left, a "Add New Shape" pop-up appears, from which select hollow cylinder and then click Add. For the inside diameter enter 25 mm, since that is the diameter of the hub. Then, say 100 mm for the outside diameter and 10 mm for the length, and again choose steel. You will see the inertia of the web alone at the bottom right, and the total inertia of the two items added at the bottom left, with details of each shape in the left-hand pane.

Try adding a rim of length 25 mm, and outer diameter of 120 mm – what result do you get?

You may have noticed that there is an option to add a shape which can be tagged as "Subtractive". For a pulley wheel, this could be the central shaft aperture. We entered a solid hub in our shape above, but we could now subtract the inertia of the shaft aperture, as that would be part of the motor shaft, for example. Try subtracting an aperture to allow for a 10 mm shaft. What difference does it make?

As you build up the shape, you can click at the bottom left where it says "Total Inertia of Complex Shape" and a summary of all your shapes will appear.

3.4: Computing the Load Torque

In Chapter 2.1 - Move Profiles, we learned how to construct a motion profile table. You will recall that one of the columns was acceleration. We will be using acceleration to calculate torque, because torque is equal to acceleration multiplied by inertia as mentioned in Chapter 1.8 - Moment of Inertia. We must pay attention to the units, and calculation is easier if we convert the acceleration to radians/sec^2 first.

Here's the motion profile table again; an extra column for acceleration in radians/sec^2 has been added on the right.

Since we know that ($1\ rev\ =\ 2 * \pi\ rad$), we have the following table :-

Segment Type	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	Distance travelled [s]	Acceleration [a]
	(rev/s)	(rev/s)	(s)	(rev/s^2)	(rev)	(rad/s^2)
Accel	0	20	0.09	222	0.9	1400
Cruise	20	20	0.075	0	1.5	0
Decel	20	0	0.085	-235	0.85	-1480
Dwell	0	0	0.05	0	0	0

Table 1

We can now extend the table to calculate a torque requirement for each segment of motion by adding two columns. One column is the load inertia, generally this will be the same in every segment. However there are some situations where it changes, for example, where a workpiece is ejected from the machine part way through the motion cycle. The second new column is the accelerating torque, the result of multiplying acceleration and inertia. In the example table below, an inertia of 0.006 kg-m^2 is used for the load inertia.

Segment Type	Motion Profile					Load Calculations		
	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	Distance travelled [s]	Acceleration [a]	Load Inertia [J_L]	Acceleratio
	(rev/s)	(rev/s)	(s)	(rev/s2)	(rev)	(rad/s2)	(kg-m2)	(
Accel	0	20	0.09	222	0.9	1400	0.006	8
Cruise	20	20	0.075	0	1.5	0	0.006	
Decel	20	0	0.085	-235	0.85	-1480	0.006	-
Dwell	0	0	0.05	0	0	0	0.006	

Table 2

Getting some idea of the continuous torque

The table above gives us an indication of the peak torque needed from the motor in order to accelerate the load. This is the highest absolute value of torque in the acceleration torque column. Note that in this example, the peak torque of 8.87 Nm is in the third segment, because the rate of deceleration is greater than that of acceleration.

We can also get an early idea of the continuous torque requirement at this stage. In Chapter 2.3 - Building a Custom Profile, we found that the RMS acceleration of this profile is 175 rev/s^2; if we convert this to rad/s^2 and multiply by the inertia in kg-m^2, it immediately gives us a figure for RMS torque, one of the key parameters in choosing a motor.

In this case, $175 * 2 * \pi$ gives us a figure of about 1097 rad/s^2; multiply this with the inertia of 0.006 kg-m^2 and we get a figure of about 6.58 Nm.

This shortcut must only be taken as an indication however because, as we saw in Section 1, there are usually two other factors which add to the Load Torque. These are :-

Friction (f)

All rotating systems have friction, seen as a torque which resists motion; the torque is usually taken to be constant and it always opposes the direction of motion. While stationary, there is no friction torque.

In a spreadsheet, the easiest way of ensuring that the sign of the friction torque is correct is to multiply it by the sign of the average velocity. In excel the formulae would be SIGN((u+v)/2), however the division by 2 is superfluous as it doesn't change the sign. So we usually use SIGN(u+v). If the average velocity is zero, the result of the function will be zero.

Externally applied Torque (e)

This represents any torque required to do work. It will typically remain acting in the same direction whatever the direction of rotation, and will be present when the system is stationary. A good example is the torque due to a gravitational load in an elevator application.

If these are present, we can add them to the table. For our example, we'll make the friction torque 2 Nm, and the externally applied torque 0.5 Nm, which will give us a table like this:-

Segment Type	Motion Profile							
	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	Distance travelled [s]	Acceleration [a]	Load Inertia [J_L]	Accelerati
	(rev/s)	(rev/s)	(s)	(rev/s^2)	(rev)	(rad/s^2)	(kg-m^2)	
Accel	0	20	0.09	222	0.9	1400	0.006	
Cruise	20	20	0.075	0	1.5	0	0.006	
Decel	20	0	0.085	-235	0.85	-1480	0.006	
Dwell	0	0	0.05	0	0	0	0.006	

Table 3

Note how the friction torque is zero during the dwell (the fourth segment), and that before we added the friction, the greatest torque requirement was in the third segment; now it is in the first. The added loads have made a considerable difference to the peak torque figures.

We now know all the instantaneous speed and torque requirements which the motor must be able to achieve through the motion cycle. We can extract some important data from the table; remember that the key factors for choosing a motor are the peak and the continuous torque requirements and the maximum speed.

Looking down the table we can see that the greatest instantaneous requirement is for a torque of 10.878 Nm, and the maximum speed is 20 rev/s (1,200 rpm). We'll work out the continuous (RMS) torque in the next section.

Test your understanding

Question 1 of 1 (1) Previous Question Next Question

Compute the peak operating points for a more complicated motion cycle as seen in Chapter 2 - 'Building Longer Profiles Absolute Moves'. Given the load inertia for the application of 0.5 kg-m², a friction load of 2.3 Nm, and an externally applied torque of 0.4 Nm. Fill in the blank cells (you can use cell formulae and the copy/fill commands of MS Excel here). Note that you can scroll the table to see the columns to the right.

I5 fx

	A	B	C	D	E	F	G	H	I
1	Segment Number	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Acceleration [a]	Distance travelled [s]	Acceleration [a]	Load Inertia [J_L]	Acceleration Torque [T_a]
2		(rev/s)	(rev/s)	(s)	(rev/s²)	(rev)	(rad/s²)	(kg-m²)	(Nm)
3	1	0	1500	0.04	37500	30		0.05	
4	2	1500	1500	0.04	0	60		0.05	
5	3	1500	0	0.04	-37500	30		0.05	
6	4	0	0	0.1	0	0		0.05	
7	5	0	1500	0.05	30000	37.5		0.05	
8	6	1500	1500	0.05	0	75		0.05	
9	7	1500	0	0.05	-30000	37.5		0.05	
10	8	0	0	0.2	0	0		0.05	
11	9	0	-675	0.2	-3380	-67.5		0.05	
12	10	-675	-675	0.2	0	-135		0.05	
13	11	-675	0	0.2	3380	-67.5		0.05	
14	12	0	0	0.33	0	0		0.05	

Check My Work Reset

3.5: Calculating the RMS Value

We have seen already in the last section, Chapter 3.4 – Computing the Load Torque, how to evaluate the peak requirements of the motor. Now we need to re-compute the RMS torque for the cycle. We already know it is at least 6.58 Nm due to the shortcut we took in the last section, but now we have added some other torque loading, so we need to recompute the result.

We can't use the RMS acceleration shortcut anymore because the added torques are not dependent on the acceleration. Instead we must work out the RMS torque in detail. The calculation method is the one we used to compute the RMS acceleration in Chapter 2.3- Building a Custom Profile (see Chapter 1.10 - RMS)

We need to consider the segment duration and the torque during the segment. For clarity, it is useful to copy these to a new table :-

Segment Duration [t]	Total Segment Torque [Ts]
(s)	(Nm)
0.09	10.9
0.075	2.5
0.085	-6.37
0.05	0.5

Table 4

We add one more column to show the square of the torque multiplied by the time, thus:-

Segment Duration [t]	Total Segment Torque [T_s]	T_s^2 * Time
(s)	(Nm)	
0.09	10.9	10.6
0.075	2.5	0.469
0.085	-6.37	3.45
0.05	0.5	0.0125
0.3	**Totals**	**14.6**

Table 5

We also add cells to contain the totals of the first and third columns. From this, we can compute the mean of the torque squared figure – divide the sum of the squares (14.58) by the overall time (0.3 s). Taking the square root of the result gives the RMS torque as shown below. The continuous rating of the motor must be greater than this.

$$RMS = \sqrt{(\sum(Tq^2 * \ time))/(\sum time)}$$

$$RMS = \sqrt{\frac{14.58}{0.3}} = 6.971\ Nm$$

It is interesting to note that this is not too much greater than the approximate figure of 6.58 Nm due solely to the inertial load that we obtained when we used the shortcut in the last section. This illustrates how inertial loads tend to dominate the continuous torque

requirements in indexing moves like this, even when there is additional loading from friction and external torques.

This is because while the friction loads are adding to the overall torque during acceleration, they subtract during the deceleration and are thus partly cancelled out in a start-stop move. This is why the RMS acceleration shortcut can be a useful indicator, allowing us to make an approximate guess at the likely size of the motor we will need.

Now that we have established the instantaneous torque/speed and the continuous load requirements, we are ready to choose a motor - Chapter 4 - Servo Motor Selection explains the next step.

Test your understanding

Question 1 of 1 (1) Previous Question Next Question

Again using the multiple move application from the assessment question in section 4, compute the RMS torque requirement.

A1:A2 fx Segment

	A	B	C	D
1	Segment	Segment Duration [t]	Total Segment Torque [Ts]	T_s^2 * Time
2		(s)	(Nm)	
3	1	0.04	35.4	
4	2	0.04	2.7	
5	3	0.04	-30	
6	4	0.1	0.4	
7	5	0.05	28.9	
8	6	0.05	2.7	
9	7	0.05	-23.5	
10	8	0.2	0.4	
11	9	0.2	-4.85	
12	10	0.2	-1.9	
13	11	0.2	1.05	
14	12	0.33	0.4	
15				
16	Totals	1.5		0
17		RMS Torque		

Check My Work Reset

3.6: Representing the Instantaneous Torque & Speed Requirements on a Chart

To evaluate the requirements graphically, it helps to produce charts of the important instantaneous operating points. From the example application in **Chapter 3.4 - Computing the Load Torque**, we can extract data to produce another table. We take the initial and final point in each segment and enter the speed and the total torque, thus :-

Segment	Elapsed Time	Speed	Torque [T_s]
	(s)	(rpm)	(Nm)
1	0	0	10.9
1	0.09	1200	10.9
2	0.09	1200	2.5
2	0.165	1200	2.5
3	0.165	1200	-6.37
3	0.25	0	-6.37
4	0.25	0	0.5
4	0.3	0	0.5

Table 6

Why two points for each segment? This is because, for example, at the end of the first segment the speed is 1200 rpm and the torque is 10.8718 Nm, but immediately after that at the start of the second segment, the speed is the same but the torque has instantly reduced to 2.5 Nm, so it is a different operating point, even though it is at the same time point in the cycle.

A time-based plot looks like this (the speed has been changed to RPM/100 for scaling purposes) :-

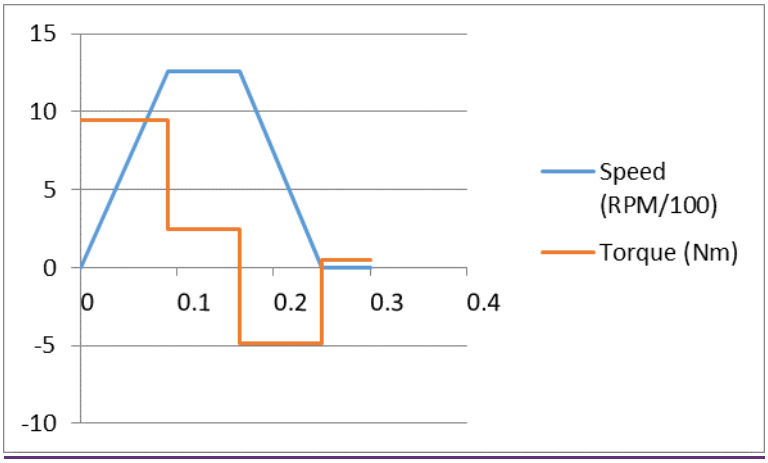
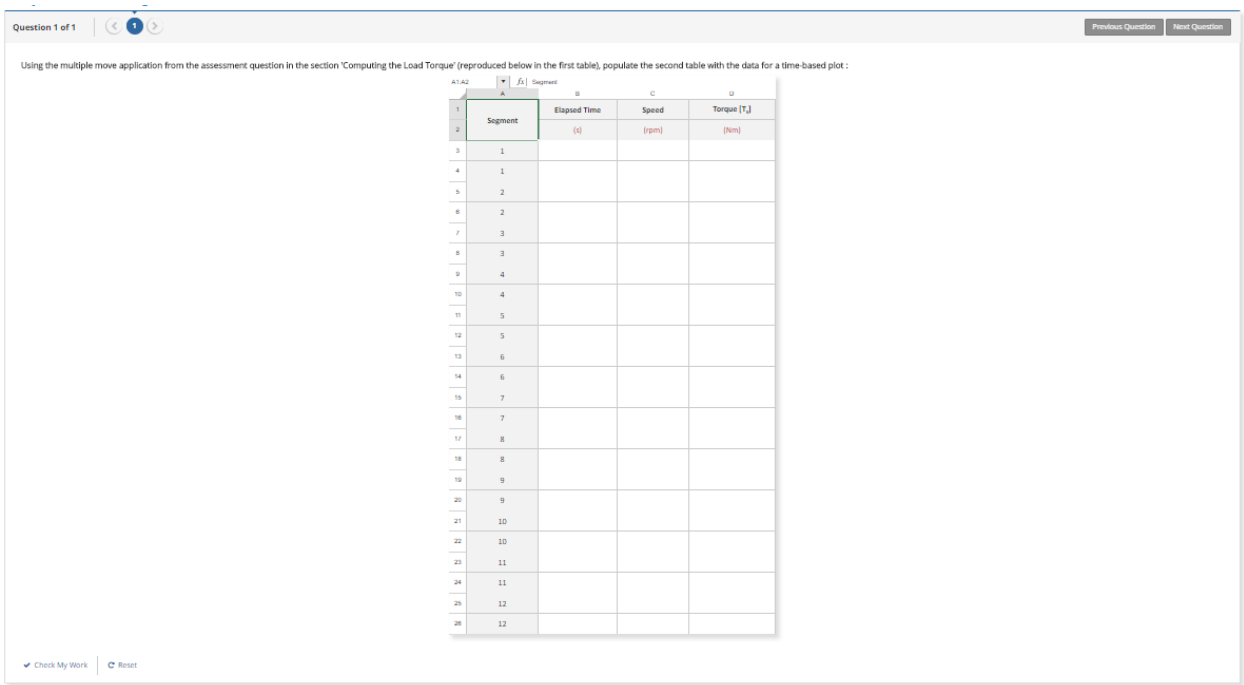


Figure 1

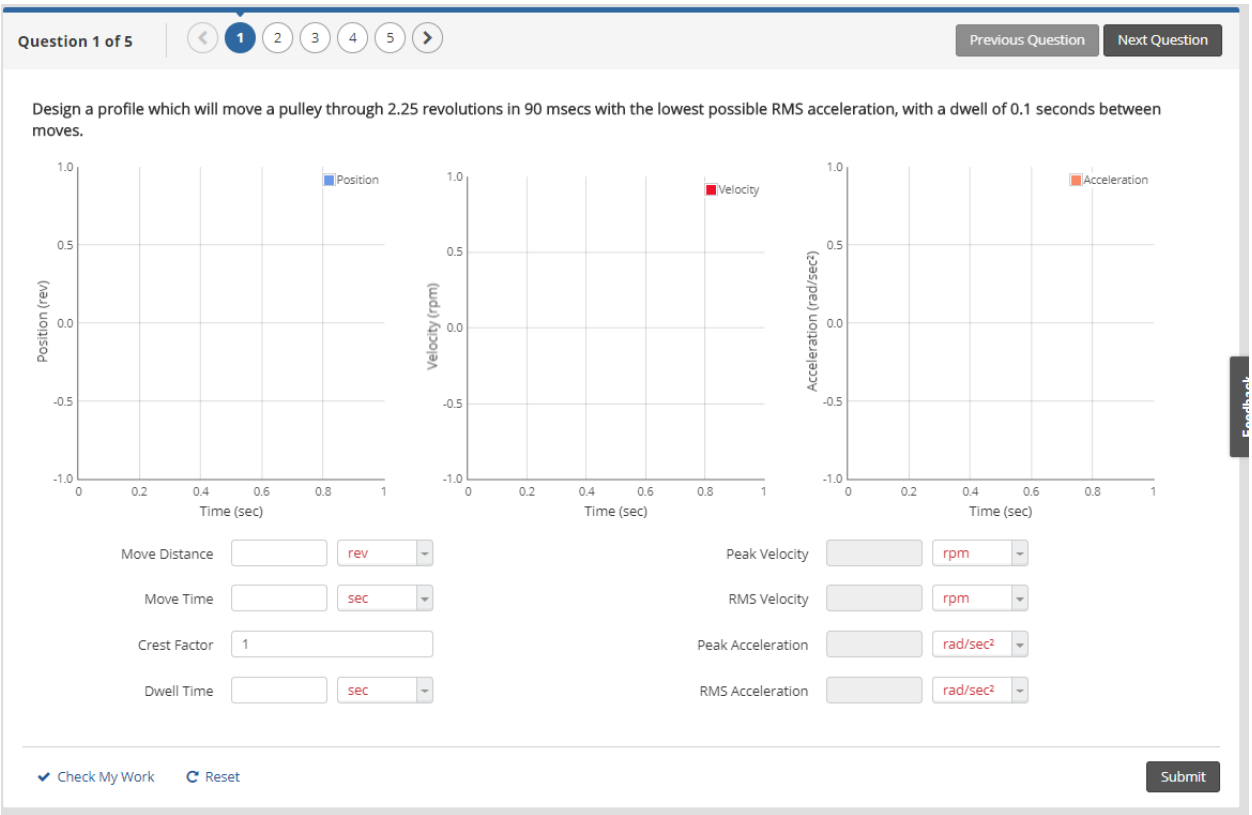
It is easy to see that the motor must be able to produce at least 10 Nm at 1200 rpm.

Manufacturers often provide motor performance data using a Torque vs. Speed graph, so it is useful to understand how the application data fit in. In the next chapter we will learn more about this.

Test your understanding



3.7: Assessment



Chapter 4- Servo Motor Selection

4.1: Introduction

Most industrial servo systems use a brushless servo motor which is driven by an electronic power supply known as an amplifier or drive. These motors can convert electrical power to mechanical power (known as motoring) but they can also act as generators and convert mechanical power back into electrical power (known as regenerating), in a fully controlled stepless manner. There are other types of a motor which can be used for servo applications, these are dealt with in the advanced course.

The normal requirement is to find the lowest cost solution that will satisfy the specified performance and accuracy requirements. Lowest cost generally equates to the smallest motor and drive that will deliver with appropriate margins. We will deal with the drive in Chapter 4.6 - Drive Selection. In this chapter, we are concerned with selecting the motor.

First, the application torque and speed requirements must be established, as we saw in Chapter 3 - How to Analyse a Rotary Load.

For each segment of the required profile:-

1. The acceleration torque must be calculated and added to any friction and external torque, as we saw in Chapter 3 - How to Analyse a Rotary Load. The worst case (highest) torque is used for sizing. This is the absolute value of torque - i.e. irrespective of the sign.
2. The maximum required speed is established, again irrespective of the sign.
3. The RMS (continuous) torque is calculated for the whole profile.

Following topics will be covered in this chapter:

- Motor ratings
- Comparing the application requirements to the motor ratings
- Adding the motor inertia
- Introducing the T- S widget
- Drive Selection

Learning Objectives:

By the end of this chapter, the student will be able to:

- Compare the torque and speed required by the rotary load with the capabilities of various motors.
- Understand impact of adding coupling, motor inertia and losses.
- Plot the torque-speed curve of the motor with the operating points of the load.

- Be able to select an appropriate drive for the motor.

4.2: Motor Ratnigs

As we said in the introduction, the most important parameters for sizing are continuous (RMS) torque, peak torque and maximum speed. Many motor manufacturers publish Torque/Speed curves for their motors in the form shown in Figure 1.

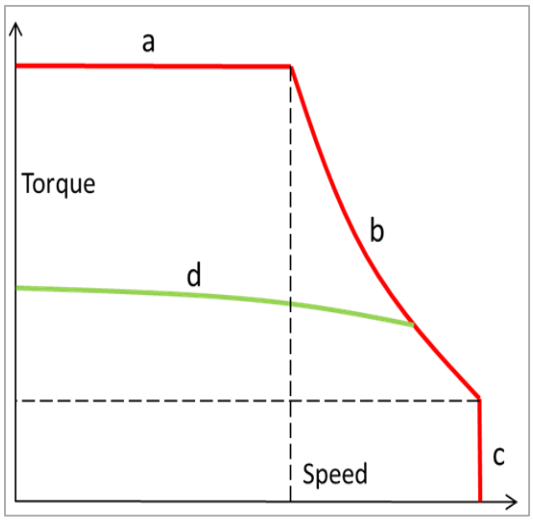


Figure 2

Figure 1, shows a typical Torque/Speed curve for a brushless servo motor. The red lines **a**, **b** and **c** define the limits of operation; the motor is unable to produce more torque or more speed than these lines indicate. The green curve **d** is a thermal limit. The motor may operate outside this limit for a short period, but it will overheat if the limit is exceeded for too long.

The limiting factors for each of the lines are:-

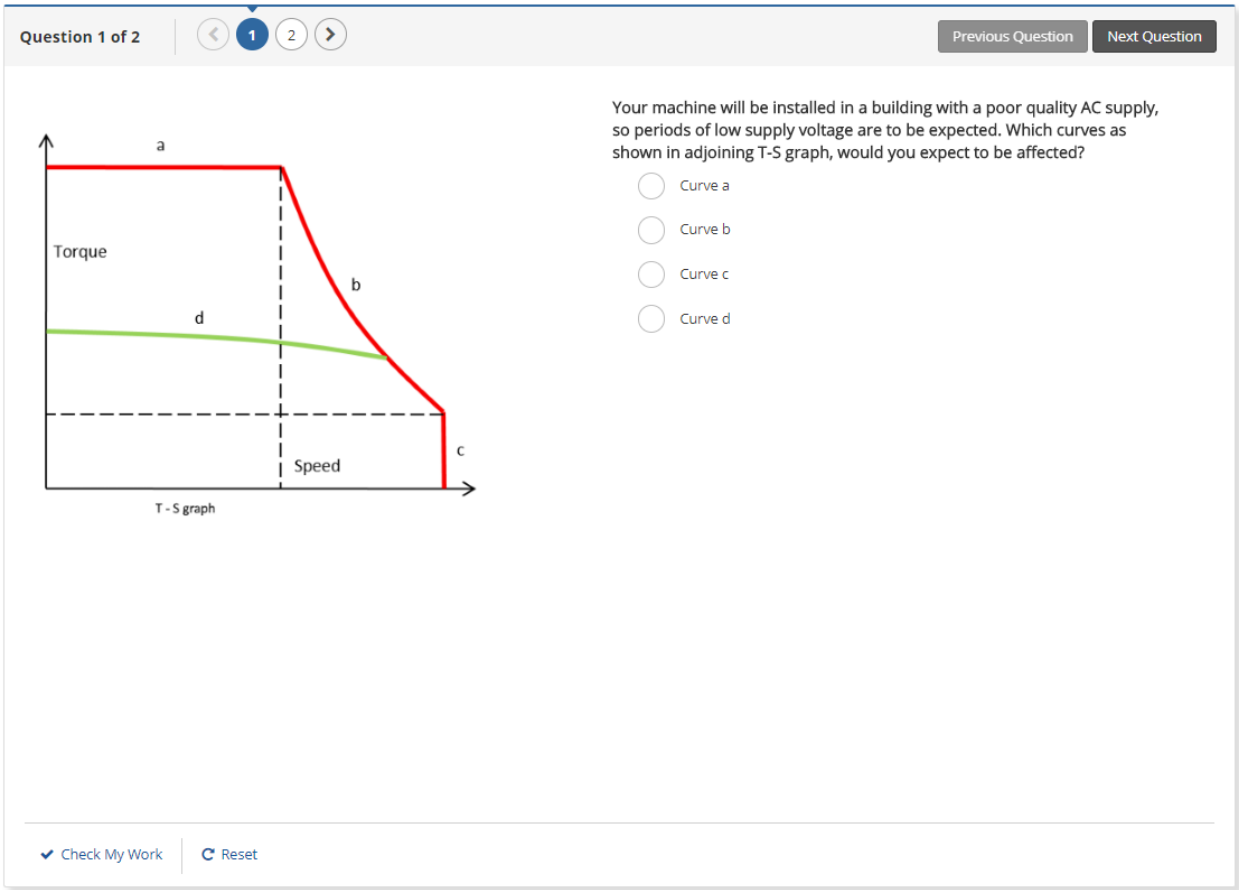
- The red curve **a** is limited by peak current capacity.
- The red curve **b** is limited by supply voltage.
- The red curve **c** is limited by mechanical parameters.
- The green curve **d** is limited by the maximum winding temperature.

Increasing the supply voltage will move section **b** to the right. Conversely, if supply voltage falls, it moves left and could impinge on the operating points. It is therefore necessary to choose the motor size using the lowest likely voltage limit.

The motor temperature is mainly determined by resistive losses in the stator winding and is therefore dependent on the RMS current, but there is also a speed-related heat loss which is generally small. That is why curve **d** gradually slopes downwards as the speed increases.

Some manufacturers do not give T-S curves, just numerical data. If you are trying to match your application to a motor with only numerical data, beware of the voltage limiting effect. We will return to that later.

Test your understanding



4.3: Comparing the Application Requirements to the Motor Ratings

As stated in Chapter 4.2 - Motor Ratings, three parameters are crucial:

1. Peak torque, which must remain below the red curve in T-S graph. If the manufacturer only gives a figure for peak torque, it is important to bear the voltage limitation in mind (curve **b** in Figure 1 of Chapter 4.2 - Motor Ratings).
2. Peak speed must remain to the left of the red curves in T-S graph.
3. Continuous (RMS) torque must remain below the green curve **d** in T-S graph. Because of the fact that the heat loss is in part related to the square of the speed, this should be checked at the RMS speed. However, RMS speed is tedious to calculate, so to be safe, the RMS torque should be checked against the motor rating at the maximum application speed.

In each case, it is wise to include some margin or headroom. Suggested values are:

1. Peak torque < 80%
2. Peak speed < 90%
3. RMS torque < 70%

This margin depends on the reliability of the data available, as not every customer is on top of this aspect.

In Chapter 3 - How to Analyse a Rotary Load, we learned how to find the operating points and the RMS torque rating which the motor must be able to achieve in order to satisfy the application requirements. For simple sizing, we can take the worst of the instantaneous points. The two most important are the highest required torque, irrespective of sign, and the highest required speed, also irrespective of sign, i.e. the highest absolute values of each.

Note that the highest torque and the highest speed could occur at different operating points. If so, we will put them together to obtain a single point representing the highest limit of the required performance.

In more complex profiles, there may be another operating point which does not have the highest torque or the highest speed, but which nevertheless can be close to the voltage-limited part of the curve. This would normally be the point where the application requires the highest power. In most applications, the highest power point is also the point of highest torque and speed, but this isn't always the case. This aspect is dealt with more thoroughly in the advanced course.

For this basic course, we will just take the highest speed and torque figures. We can take the table we developed in Chapter 3.6 - Representing the Instantaneous Torque and Speed Requirements on a Chart, Table 6), reproduced here: -

Segment	Elapsed Time	Velocity	Torque
	(s)	(rpm)	(Nm)
1	0	0	10.9
1	0.09	1200	10.9
2	0.09	1200	2.5
2	0.165	1200	2.5
3	0.165	1200	-6.37
3	0.25	0	-6.37
4	0.25	0	0.5
4	0.3	0	0.5

Table 1

It is easy to see that the highest torque point is at the end of segment 1 and in this example, it is also the point where the speed is highest. Thus the limit of required performance is **10.88 Nm at 1200 rpm.**

For the RMS torque point, the correct comparison point with the continuous curve (curve **d** in Figure 1 of Chapter 4.2 - Motor Ratings) is at the RMS speed, but that is tedious to calculate and it is normally adequate to use the maximum application speed. The RMS torque for the above table, calculated in Chapter 3.5 - Calculating the RMS Value was 6.971 Nm; thus, the point representing the continuous requirement is **6.971 Nm at 1200 rpm**.

We have created a table of reasonably realistic data for a fictional range of motors: -

Motor Number	Motor Model	Voltage	Maximum Speed	Peak Stall Torque	Continuous Stall Torque at 40 °C	Inertia
		(V)	(rpm)	(Nm)	(Nm)	(kg-cm^2)
1	CPB-1-01	440	4500	2.7	1	0.1
2	CPB-1-02	440	5000	4.4	1.7	0.15
3	CPB-1-03	440	4500	7.5	2.3	0.25
4	CPB-2-01	440	6000	3.8	2	0.65
5	CPB-2-02	440	6000	7.8	3.4	0.8
6	CPB-2-03	440	5000	11	4.2	1.2
7	CPB-3-01	440	4000	21	9	4
8	CPB-3-02	440	3500	29	10	6
9	CPB-4-01	440	3000	50	20	15
10	CPB-4-02	440	3000	68	25	22
11	CPB-4-03	440	3000	75	32	29

Table 2

We'll choose motor No 7 (CPB-3-01) which is the smallest motor which can meet the RMS torqu e requirement.

A Torque-speed (T-S) curve is available for this motor. The peak and continuous operating points can be superimposed on the T-S curve to give a chart like this: -

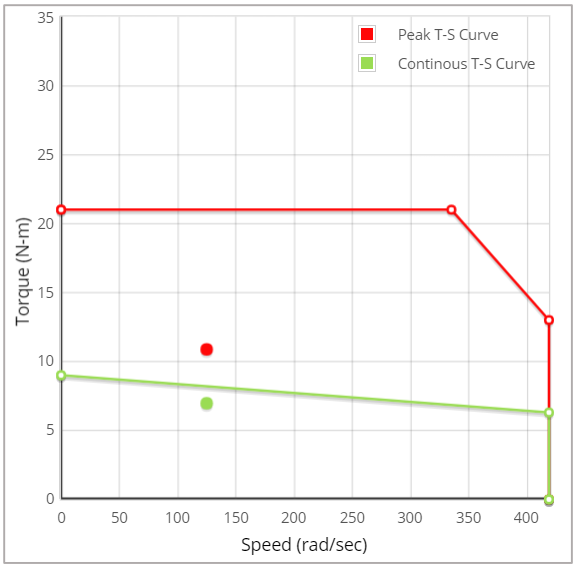


Figure 2

The RMS torque must be below curve **d**, and the peak torque and peak speed must both be within the total area. In this case both the operating points are well within the area covered by the motor, so the motor looks like it can easily do the job. However, the added inertia of the motor often has a significant influence. After the initial selection, the inertia of the candidate motor must be added in and the calculations repeated to check it still satisfies the requirements. We'll do that in the next section.

There are also other more minor considerations: -

Motor Losses

Motors also have losses which add to the torque requirement. These are usually small so we'll ignore them for the basic course, they will be covered in the advanced course.

Ambient Temperature

The Continuous or RMS torque output of a motor will be quoted at a specific ambient temperature, typically 40°C. Above this a de-rating factor supplied by the manufacturer must be applied.

Altitude

The Continuous or RMS torque output of a motor will be quoted at a maximum altitude, typically 1000 ft to 3000 ft above mean sea level. Above this, a de-rating factor supplied by the manufacturer must be applied.

Inertia matching

The stability of the system when under control is affected by the Inertia match, i.e. the ratio of the load inertia to the motor inertia. The load inertia, in this case, is the total inertia of the payload and any other mechanical components such as couplings, shafts etc., i.e. everything

but the motor. When the load inertia is more dominant the control system may become more difficult to tune, and dynamic performance may be reduced. Analysis of how to deal with inertia Matching is covered in detail in the advanced course, but we will look at some simple pointers as we go on through this basic course.

Test your understanding

Question 1 of 2 (1) (2)

In Chapter 3, section 3.5, you completed an assessment question on a 12-segment profile. The data table is reproduced here. Specify the maximum torque and maximum speed.

A1:A2 | fx | Segment Number

	A	B	C	D	E
1	Segment Number	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Total Segment Torque
2		(rpm)	(rpm)	(sec)	(Nm)
3	1	0	1500	0.04	35.4
4	2	1500	1500	0.04	2.7
5	3	1500	0	0.04	-30
6	4	0	0	0.1	0.4
7	5	0	1500	0.05	28.9
8	6	1500	1500	0.05	2.7
9	7	1500	0	0.05	-23.5
10	8	0	0	0.2	0.4
11	9	0	-675	0.2	-4.85
12	10	-675	-675	0.2	-1.9
13	11	-675	0	0.2	1.05
14	12	0	0	0.33	0.4
15					
16	Peak Torque		Nm		
17	Maximum Velocity		rpm		
18					

Check My Work | Reset

4.4: Adding the Motor Inertia

In Chapter 3 - How to Analyse a Rotary Load, we saw how to work through an example application to construct a table of application operating points. This is the table from that example: -

Initial Velocity [u]	Final Velocity [v]	Acceleration [a]	Segment Duration [t]	Distance Travelled [s]	Acceleration [a]	Load Inertia
(rev/sec)	(rev/sec)	(rev/sec^2)	(sec)	(rev)	(rad/sec^2)	(kg-m^2)
0	20	222	0.09	0.9	1400	0.006
20	20	0	0.075	1.5	0	0.006
20	0	-235	0.085	0.85	-1480	0.006
0	0	0	0.05	0	0	0.006

Table 3

The RMS torque as calculated in Chapter 3 - How to Analyse a Rotary Load is 6.971 Nm. We need a motor which can at least achieve this requirement, such as motor No 7 which we tried out in the last section. However, once we have selected the motor, we need to assess the effect of the added inertia of the motor and the coupling used to connect it to the load. For this example, the motor has a continuous rating of 9 Nm, a peak rating of 21 Nm, and is capable of 4,000 rpm. The first check is on the speed capability; in this case, it's fine since we only need to reach 1200 rpm. Next check is the torque capability.

From the data table in the last section, we saw that Motor No 7 has a rotor inertia of 4 kg-cm^2, or 0.0004 kg-m^2. We also need to include the inertia of the necessary coupling, but for now, we'll assume it is negligible. This added inertia will require additional acceleration torque.

A quick way to assess the effect is to take the RMS and peak accelerations and multiply with the inertia of the proposed motor, then check the results against the motor ratings. It should be noted however that this short-cut can produce a result which is higher than the actual requirement, particularly in cases where the friction or externally applied torque is enough to offset the deceleration torque. The method can be used to give a reasonable indication, but you should perform a full calculation of the RMS torque before finalizing the choice of motor.

Since, in common with many applications, the example application does not have high friction or external torque, we will use the RMS acceleration short-cut to home in on a motor selection.

Using this method to check the RMS torque requirement, the calculation is: -

$$Total\ RMS\ Torque = \ RMS\ load\ torque + (RMS\ acceleration * \ added\ inertia)$$

The RMS acceleration is calculated automatically by the profile widget, so if you are using that, you will already have that number available. Otherwise, you can quickly calculate it, as we saw in Chapter 2 - Move Profiles. For the example profile, the RMS acceleration is 1097 radians/sec^2.

So, Total RMS torque is: -

$$Total\ RMS\ Torque = 6.971 + (1097 * 0.0004) = \ 7.41\ Nm$$

Motor No 7 has a rating of about 8 Nm at 1200 rpm based on the T-S curve we saw in the last section, so it will satisfy the RMS torque requirement albeit with a smaller margin than we

would normally like. Normally we would recommend that the RMS torque should only be 70% of the motor's rating, to allow for higher than expected ambient temperatures (on hot days, for example) or increased friction loss as the machine gets older. In this case, the torque requirement is 93% of the motor rating. If we applied the 70% margin we would need this motor to have a rating of about 10.75 Nm, so we would have to consider a bigger motor, but that would have an increased inertia and require a recalculation – it will also be more expensive. We'll return to that later – for now we'll proceed with this motor.

Now we must check the peak torque requirement. We need to check the segments individually because the highest acceleration (here in segment 3) does not always occur in the same segment as the highest **Total load torque** (here in segment 1). We can add three columns to the table (for display purposes we can hide some of the columns we don't currently need).

Segment Duration [t]	Acceleration [a]	Load Inertia	Total Load Torque	Motor and Coupling Inertia	Motor Acceleration Torque	Total Motor To
(sec)	(rad/sec^2)	(kg-m^2)	(Nm)	(kg-m^2)	(Nm)	(Nm)
0.09	1400	0.006	10.9	0.0004	0.559	11.4
0.075	0	0.006	2.5	0.0004	0	2.5
0.085	-1480	0.006	-6.37	0.0004	-0.591	-6.96
0.05	0	0.006	0.5	0.0004	0	0.5

Table 4

Of course, the additional inertia will be the same throughout, but adding it to the table in its own column makes it easier to calculate using a spreadsheet application. We can see the added acceleration torque, and that the peak torque required has risen from 10.878 Nm to 11.436 Nm.

Here is the T-S curve with the recalculated operating points: -

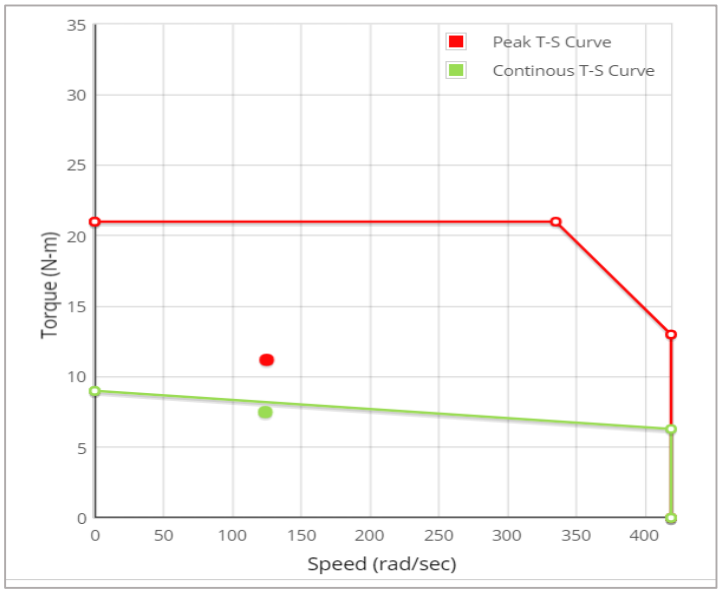


Figure 4

Our selected motor has a peak capability of 21 Nm, so our selection is OK as far as peak torque is concerned, as the requirement is only 55% of the motor's capability.

If you only have numerical data for the motor, the information is insufficient to check the ability of the motor as there could be a problem with the voltage limit curve (Curve b above). You need to obtain a T-S curve from the manufacturer, taking care to make sure the voltage at which it is specified applies to your situation.

We should also note the Inertia Ratio; in this example, the motor inertia is 0.0004 kg-m^2, but the load inertia is 0.006 kg-m^2. This would make the Inertia Ratio equal to .006/0.0004 = 15:1.

This would be considered by many to be too high, and a larger motor would be sought, once again requiring a recalculation.

The high inertia ratio of this example is apparently acceptable in sizing terms, but it could have a detrimental effect on the dynamic performance, depending on the mechanical design and the control system configuration – this is covered in detail in the advanced course.

Test your understanding

Question 1 of 1 1

Using the Motion Profile from Chapter 2 section 5 with the added loads used in the chapter 3 assessments, coupled to a motor through a shaft and coupling with an inertia which adds up to 0.0012 kg-m^2. Find the RMS and peak torques.

A1:A2 | fx | Segment Number

	A	B	C	D	E	F	G
1	Segment Number	Segment duration [t]	Acceleration [a]	Load Inertia	Total Load Torque	Motor Inertia	Motor Acceleration Torque
2		(sec)	(rad/sec^2)	(kg-m^2)	(Nm)	(kg-m^2)	(Nm)
3	1	0.04	654	0.05	35.4	0.002	
4	2	0.04	0	0.05	2.7	0.002	
5	3	0.04	-654	0.05	-30	0.002	
6	4	0.1	0	0.05	0.4	0.002	
7	5	0.05	524	0.05	28.9	0.002	
8	6	0.05	0	0.05	2.7	0.002	
9	7	0.05	-524	0.05	-23.5	0.002	
10	8	0.2	0	0.05	0.4	0.002	
11	9	0.2	-58.9	0.05	-4.85	0.002	
12	10	0.2	0	0.05	-1.9	0.002	
13	11	0.2	58.9	0.05	1.05	0.002	
14	12	0.33	0	0.05	0.4	0.002	
15	Total	1.5					0
16							
17	RMS Torque		Nm				
18	Peak Torque		Nm				
19							

4.5: Introducing the T-S Widget

In the previous section, we chose motor no. 7 for our sample application; based on the data table it looked as if it would be a good choice, since it has an RMS stall torque rating of 9Nm and we only needed 6.97 Nm, however after adding the motor inertia to our load, recalculating the RMS torque and comparing the result to the motor T-S curve we found that the margin of motor performance over the application requirement was perhaps insufficient. To go back to the table, choose another motor, look up the T-S curve and check the results, perhaps many times, is tedious, so here is a widget which does much of the work for you.

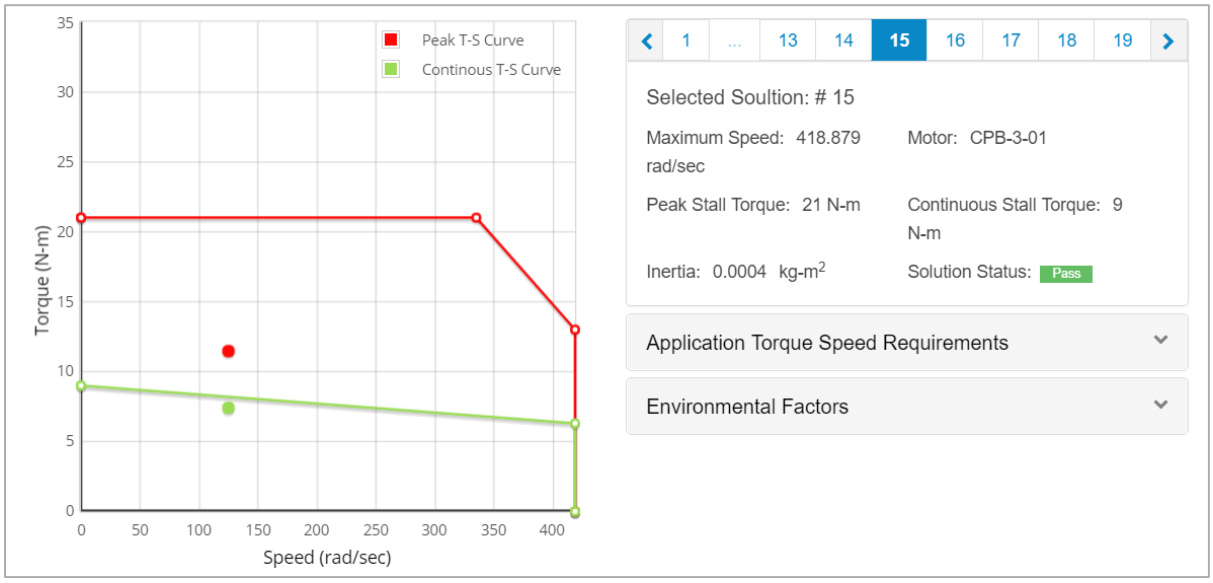


Figure 5

The data for motor no. 7 is showing here, but you can change the selection using the top box. You can work up and down the available motors using the left and right arrows, or go directly to a motor by clicking on the number indicating its position in the library.

To use the tool, first click on the button “Application Torque Speed Requirements”. Five data entry boxes will appear, allowing you to set the peak torque, continuous torques, peak speed, peak acceleration and RMS acceleration. The load data for our application is loaded already.

To the right are five boxes which show the resulting conditions at the motor; the torques are slightly higher as the effect of the motor inertia is added – you can see that they match the values we calculated in the last section. The calculated speeds and accelerations are the same as the application values, but you will see why these calculated values are there in the next chapter.

Enter some different torque and speed values, and you will see the red and green dots moving accordingly. Watch the Solution Status as you change the selection of motor using the top box. If they go above the curve with the matching colour, the solution status changes from “Pass” to “Fail”. Re-enter the original data, which we derived in Chapter 3 - How to Analyse a Rotary Load – these were: -

Peak Torque	10.9	Nm
RMS Torque	6.97	Nm
Peak Speed	1200	rad/sec
Peak Acceleration	1478	rad/sec^2
RMS Acceleration	1097	rad/sec^2

Table 5

You can also change the motor selection and see the T-S curve change. You will also see the motor side torques change due to the different inertias, so you don't need to do those tedious calculations. If you start with a small motor such as motor no. 5, then click the right-hand arrow to increase the size in steps, you will see the "fail" change to a "pass" when you get to the first motor which will satisfy the application, motor no. 7. You can continue up the range and see the margin for the RMS torque improving as you do so.

The tool also allows you to check the main environmental factors. Click on the "Environmental factors" button and you will see sliders for the ambient temperature and altitude for the motor. Move the sliders to see the effects.

Robustness

You will probably have noticed that the obtain a good margin for the RMS torque, we need to go up the motor range to motor no. 9, which will be a much bigger and more expensive motor. There is a dilemma here, a temptation to accept a tighter margin to save cost, at the risk that the application may fail at some point in the future, in other words it would not be as **robust** a solution.

A robust solution is one which can tolerate changes of factors such as ambient temperature, lubrication, wear and tear of the mechanical components, etc., as well as imprecise data. Robustness is an important practical requirement, and we will return to the concept as we refine the solution.

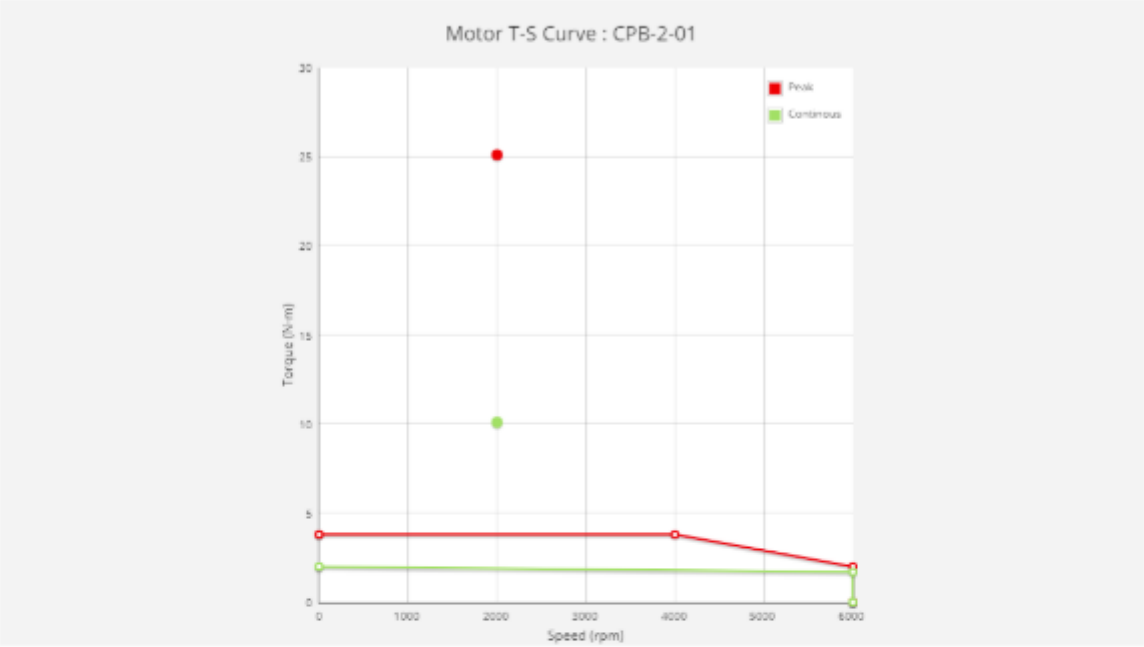
Transmission Ratio

Rather than keep increasing the size of the motor, we can improve our solution by using a transmission system. The **Transmission Ratio** is very important factor in the sizing and performance of a servo system. Up to now we have only considered the case where the motor is directly coupled to the load and while our selection is perhaps ok, we may be able to use a smaller and hence cheaper motor if we add a transmission system to gear the motor to run at a higher speed. The Transmission ratio has profound effects on most of the parameters, particularly the inertia ratio, and is dealt with in detail in the next Chapter 5 - Transmission Ratio Optimisation.

Test your understanding

Question 1 of 2 1 2

An application has a peak speed of 2000 rpm, with a peak torque of 25 Nm and an RMS torque of 10 Nm. The peak acceleration is 1478 rad/s², and the rms acceleration is 1097 rad/s². If the ambient temperature is 20 degrees C, altitude is 0 m then choose from the standard library the smallest motor which can satisfy the application.



1 2 3 4 5 6 7 ... 11

Selected Motor: #4 (CPB-2-01) Fail

Maximum Speed: 6000 rpm

Inertia: 6.50x10⁻⁵ kg-m²

Peak Stall Torque: 3.8 N-m

Continuous Stall Torque: 2 N-m

Application Torque Speed Requirements

	Load Side		Motor Side
Peak Torque:	25	N-m	25.1 N-m
RMS Torque:	10	N-m	10.1 N-m
Peak Speed:	2000	rpm	2000 rpm
Peak Acceleration:	1478	rad/sec²	1478 rad/sec²
RMS Acceleration:	1097	rad/sec²	1097 rad/sec²

Environmental Factors

Check My Work | Reset

4.6: Drive Selection

You should select a motor and drive which are supplied by the manufacturer as a matched pair, as there are some subtleties involved in matching a motor and a drive which can cause problems. The subject of drive-to-motor matching is covered in the advanced course.

Here are some key parameters which you should check to make sure the motor/drive system you have selected will satisfy the application requirements: -

Voltage

The drive's rated voltage should be compatible with the specified power supply and the number of phases (single or three-phase). Sometimes drives are able to work off a range of supply voltages, the main effect is on the speed range. Make sure the Torque-Speed data you are using to select the motor is available with the supply voltage which you will be using.

Peak Current

The peak torque rating for the motor/drive combination should exceed that calculated to be required by the application by an appropriate margin. Sometimes the peak torque is limited by the drive rather than the motor, so make sure the rating you check is for the combination.

Continuous (RMS) Current

The continuous torque rating for the motor/drive combination should exceed that calculated to be required by the application by an appropriate margin. Again, make sure you check that the rating is for the combination rather than just for the motor.

Ambient Temperature

The Continuous or RMS current rating of a drive will be quoted at a specific ambient temperature, typically 25°C. Above this, a de-rating factor supplied by the manufacturer must be applied. Note that this will most probably not be the same temperature experienced by the motor, which will have its own specified ambient temperature.

Altitude

The Continuous or RMS current rating of a drive will be quoted at a maximum altitude, typically 330m (1000ft) above mean sea level. Above this a de-rating factor supplied by the manufacturer must be applied.

Drive Architectures

The simplest form of drive is a single-axis unit which simply connects to the AC supply and regulates power to the motor as required. However, there are other architectures, the most common of which is a multi-axis architecture where several drives are connected to a DC bus which is supplied from the AC power source using a single converter. If you have several axes on your machine, a multi-axis drive system may be more economical, but you need to weigh up the costs carefully. Drive architectures are dealt with in detail in the advanced course.

Braking Energy

In order to accelerate a load, the motor must supply it with mechanical energy, the drive in turn supplying electrical energy to the motor. When the load is decelerated, energy may flow back through the motor to the drive and the drive system needs to be able to deal with this energy.

In our example application, segment 3 requires a negative torque of -5.48 Nm while still rotating in a forward direction. Multiplying the torque by the speed at the start of the segment, we get a negative (therefore regenerative) power of 689 watts. This power reduces as the speed reduces, so since the velocity profile is triangular, the average power being regenerated during the segment is half the peak, i.e. 345 watts. This power is only present during this segment, i.e. for 85 milliseconds out of the complete cycle of 300 millisecs. Thus the maximum **continuous** power which the shunt will need to handle is $345 \times 85 / 300 = 195\ watts$. In practice, much of the regenerated energy will be absorbed by storage capacitors in the drive, while even more will be absorbed by losses in the motor windings and the drive's transistors.

However, it is quite complex to evaluate these factors, so you need to ensure that your drive has a shunt which can handle this amount of power, to be safe.

Dump Resistors

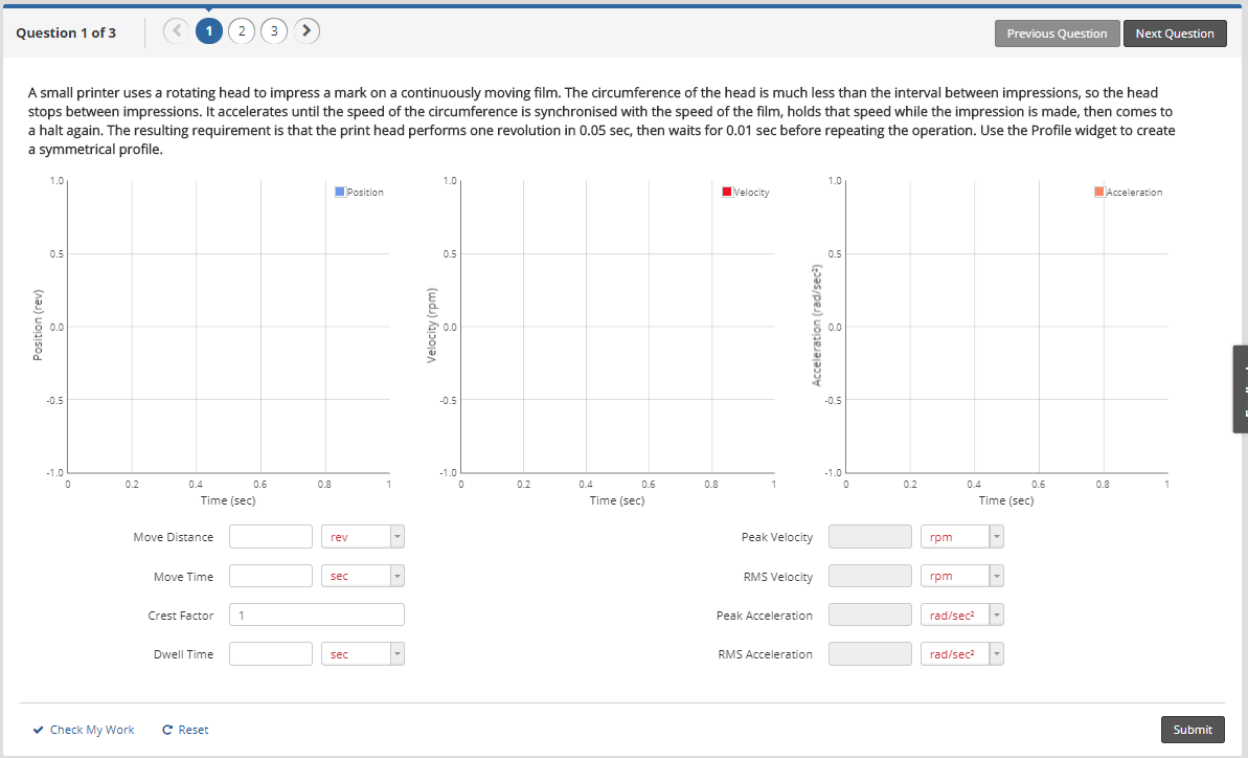
Drives may be fitted with an internal "dump" function; this is a transistor that turns on at the maximum bus voltage connecting a power resistor across the bus that is sized to handle the surplus energy. Some drives do not have this capability at all, where other drives may require a separate module to be connected. Multi-axis drives usually have an advantage here as the DC bus which serves several axes is usually connected to a single shunt.

Regeneration to the supply

A more expensive option is for the drive system to contain a second transistor bridge that switches the DC bus power back to AC power which is then "regenerated" back into the mains supply. The advantage of this solution is that there is no limit on the amount of energy, and the efficiency of operation is much better. This capability is usually only available on high power drives.

The subject of regeneration is covered in depth in the advanced course.

4.7: Assessment



Chapter 5 – Transmission Ratio Optimisation

5.1: Introduction

Up to now we have only studied a directly coupled motor, however motors are often connected to the load mass or load inertia through a transmission system. This may be a gearbox, toothed belt/pulley system or some other transmission mechanism. Descriptions of the most common types for rotary loads are covered in Chapter 5.4 - Adding a Transmission Mechanism. In the case of linear loads the mechanism which converts the rotary motion of the motor into the linear motion required to move the load is in itself a transmission system, and this is covered in Chapter 6 - Linear Loads.

Changing the transmission ratio between motor and load is the single most powerful tool at the disposal of the servo engineer for optimizing motor size and system performance, and often the cost of adding extra gearing components is more than offset by the reduction of the cost of the motor/drive system. It is important to check the potential for cost saving in all applications.

The following generic principles apply whichever physical transmission is chosen.

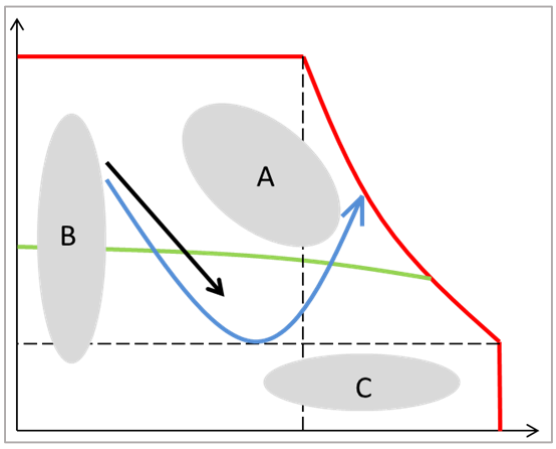


Figure 1

When you have found the key operating points for your application, and you begin the process of selecting the motor, you may find that with the best selection available, the operating points are grouped in part of the T-S chart which is not ideal. In Figure 1:

- The optimum peak T/S point is in the area marked **A** in Figure 1. This makes good use of the motor's capabilities.
- If the point should fall in the area marked **B**, the speed capability of the motor is wasted.
- If the point should fall in the area marked **C**, the torque capability of the motor is wasted.

The peak torque operating point can be moved around by inserting or changing a ratio between motor and load. Intuitively, increasing the transmission ratio should move the operating point

in the direction of the black arrow. In practice, after initially heading in that direction it may actually starts to head back upwards as shown by the blue arrow, as the inertia of the motor becomes more significant than the reflected inertia of the load. There is often a clear optimum ratio. This also holds true for other parameters such as RMS torque.

If the result is a position between **B** and **C,** it is clear that a smaller (and lower cost) motor would probably be sufficient. To find out you will need to perform a recalculation of the application, including a ratio – this will probably need repeating several times to find the ratio which works best. This is covered in the next section.

Note that once you have found what you consider to be an optimum ratio, you will need to choose a physical transmission mechanism. This mechanism will add its own inertia and losses, so increasing the number of recalculations required. We will look at this aspect in Chapter 5.4 - Adding a Transmission Mechanism .

Thus the whole exercise may need to be repeated many times until an optimum solution is found.

It becomes clear that a software package such as **COSMATT** that automates most of this process speeds up selection dramatically and covers more permutations than a manual calculation could ever do.

Following topics will be covered in this chapter:

- Adding a Ratio
- Using the T-S Widget to Help with Optimisation
- Adding a Transmission Mechanism
- The Effect of Inertia Ratio

Learning Objectives:

- Understanding the impact of transmission ratio on the choice of motor
- Get to know the commonly available types of mechanical transmission systems
- Learn how to deal with the added inertia and losses

5.2: Adding a Ratio

If it looks like adding a transmission will be beneficial, we should first try to establish the likely effect of adding a ratio in our calculations to increase the motor speed. Once we have decided that adding some gearing is going to be beneficial, we will need to examine the effect of the additional loads and losses that a real mechanism will add. We'll look at that in the Chapter 5.4 - Adding a Transmission Mechanism, but first we'll just examine the effect of gearing.

In Chapter 3.4 - Computing the Load Torque, we developed an application example, giving us the following table:

Initial Velocity [u]	Final Velocity [v]	Acceleration [a]	Segment Duration [t]	Distance Travelled [s]	Acceleration [a]	Load Inertia	Acceleration Torque	Friction T
(rev/sec)	(rev/sec)	(rev/sec²)	(sec)	(rev)	(rad/sec²)	(kg-m²)	(Nm)	(Nm
0	20	222	0.09	0.9	1400	0.006	8.38	2
20	20	0	0.075	1.5	0	0.006	0	2
20	0	-235	0.085	0.85	-1480	0.006	-8.87	2
0	0	0	0.05	0	0	0.006	0	0

Table 1

This gave us an RMS torque of 6.971 Nm which we then checked in Chapter 4 - Servo Motor Selection as being available from our chosen motor at the maximum application speed of 1200 rpm. The peak requirement was 10.878 Nm, again at 1200 rpm. When we chose a motor (no. 7) the torque increased due to the motor inertia. The RMS torque, at the increased value of 7.409 Nm, was at nearly 94% of the motor rating, and we would prefer if there was a greater margin. The T-S curve looked like this:

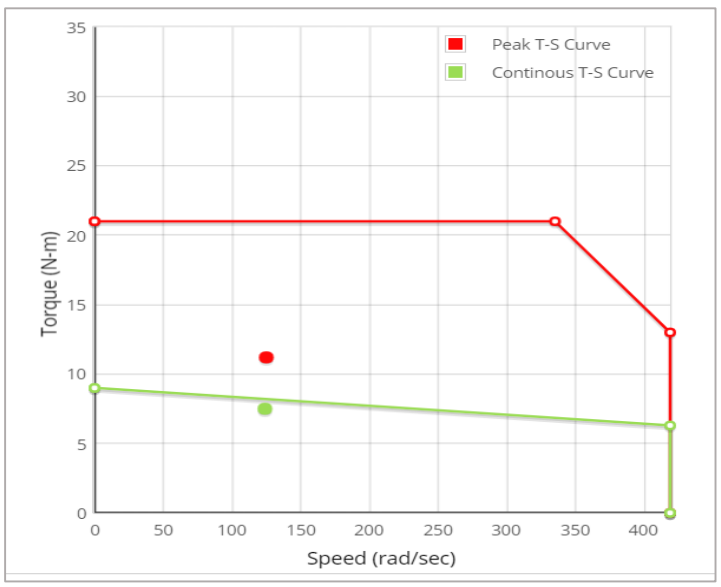


Figure 1

Using the T-S widget we looked at bigger motors, but the potential increase in cost created a dilemma – should we cut our margins and accept a risk that the application could fail, or accept the higher cost? There is another option, which is to add a transmission system to improve the utilisation of the motor.

Based on what you learned in the last section, it is clear that we are not employing the full speed capability of this motor, so let's examine the effect of a gear ratio on the above application, assuming a perfect lossless transmission at first. The distance moved, speed and acceleration are multiplied by the ratio, while the torque is divided by the ratio. The inertia of the load as reflected to the motor is reduced by the square of the ratio.

Thus we will have to think about two sets of parameters – those on the load side and those on the motor side. We refer to the load-side parameters as being reflected to the motor side by the ratio.

Motor Number	Motor Model	Voltage	Maximum Speed	Peak Stall Torque	Continuous Stall Torque at 40 °C	Inertia
		(V)	(rpm)	(Nm)	(Nm)	(kg-cm²)
1	CPB-1-01	440	4500	2.7	1	0.1
2	CPB-1-02	440	5000	4.4	1.7	0.15
3	CPB-1-03	440	4500	7.5	2.3	0.25
4	CPB-2-01	440	6000	3.8	2	0.65
5	CPB-2-02	440	6000	7.8	3.4	0.8
6	CPB-2-03	440	5000	11	4.2	1.2
7	CPB-3-01	440	4000	21	9	4
8	CPB-3-02	440	3500	29	10	6
9	CPB-4-01	440	3000	50	20	15
10	CPB-4-02	440	3000	68	25	22
11	CPB-4-03	440	3000	75	32	29

Table 2

In this example, a ratio of 3:1 will give a required motor speed of $1200 \times 3 = 3600\ rpm$, still within the range of this motor, and will reduce the RMS torque requirement of the load to $\frac{6.971}{3} = 2.32\ Nm$ and the peak requirement to $\frac{10.877}{3} = 3.63\ Nm$ before the motor is added.

Here we must remember that the motor inertia will require extra torque during acceleration and deceleration. To get a quick idea of the added torque, we can multiply the RMS and peak accelerations on the motor side by the motor inertia, and add this to the RMS and peak torques of the load on the motor side to check against the motor capability. So,

$$Added\ RMS\ Torque\ = RMS\ Acceleration \times Ratio \times Motor\ Inertia$$

and

$$Total\ RMS\ Torque\ = Added\ RMS\ Torque + Load\ RMS\ Torque \div Ratio$$

Thus the additional torque added by the motor inertia is $0.0004 \times 3 \times 1097 = 1.32\ Nm$, bringing the total RMS torque to 3.64 Nm, which is under 50% utilisation of the motor rating. We also need to check the peak torque and as we saw in Chapter 4 - Servo Motor Selection we need to be careful, as peak torque and peak acceleration are not in the same segment.

For this we need a new table:

Load Side				Motor side (No 7)				
Acceleration [a]	Load Inertia [J_L]	Total Torque [T_t]	Ratio	Acceleration [a]	Reflected Load Torque	Reflected Load Inertia	Motor Inertia	Motor Ac
(rad/sec)	(kg-m^2)	(Nm)		(rad/sec^2)	(Nm)	(kg-m^2)	(kg-m^2)	
1400	0.006	10.9	3	4190	3.63	0.000667	0.0004	
0	0.006	2.5	3	0	0.833	0.000667	0.0004	
-1480	0.006	-6.37	3	-4440	-2.12	0.000667	0.0004	
0	0.006	0.5	3	0	0.167	0.000667	0.0004	

Table 3

We have the important original data on the left of the table, under the heading "Load Side". In the middle is the gear ratio – as with the load inertia it won't change through the motion cycle, but it's easier for creating a spreadsheet to put it in a column. On the right side everything is geared to the motor side (scroll across to see it if necessary), and you can see that the peak torque is still in the first segment, even though the added torque due to motor acceleration is greater in the third segment. The peak torque at 5.3 Nm is utilising only 17% of the motor peak rating.

Of course, there will be an additional cost associated with adding the gearing mechanism, but sometimes this comes for free, as the design of the machine may require some kind of transmission mechanism because of its design. It is always worth checking! Also there are additional loads and losses with a real transmission – we'll look at that in a later section.

However it appears at this stage that there is an opportunity here to choose a smaller motor which is capable of this new speed and torque requirement, and see how it fits. The reduction in cost will often be more than the cost of adding a transmission. A smaller motor will also have a reduced rotor inertia, which helps even more.

In the **COSMATT** motor library, the next motor down from motor no. 7 is motor no. 6, which has an inertia of 0.00012 kg-m^2. Using the above formulae, the RMS torque becomes $(6.971\ /\ 3) + (1097 \times 3 \times .00012) = 2.715\ Nm$, which is 65% utilisation since the motor has a rating of $4.2\ Nm$. The peak torque table is now like this:-

Load Side				Motor side (No 6)				
Acceleration [a]	Load Inertia [J_L]	Total Torque [T_t]	Ratio	Acceleration [a]	Reflected Load Torque	Reflected Load Inertia	Motor Inertia	Motor Ac
(rad/sec)	(kg-m^2)	(Nm)		(rad/sec^2)	(Nm)	(kg-m^2)	(kg-m^2)	
1400	0.006	10.9	3	4190	3.63	0.000667	0.00012	
0	0.006	2.5	3	0	0.833	0.000667	0.00012	
-1480	0.006	-6.37	3	-4440	-2.12	0.000667	0.00012	
0	0.006	0.5	3	0	0.167	0.000667	0.00012	

Table 4

The peak requirement is $4.13\ Nm$ and the motor is capable of $11\ Nm$, so the peak utilisation is still low at only 40%. We should look at the T-S curve to check for voltage limitation.

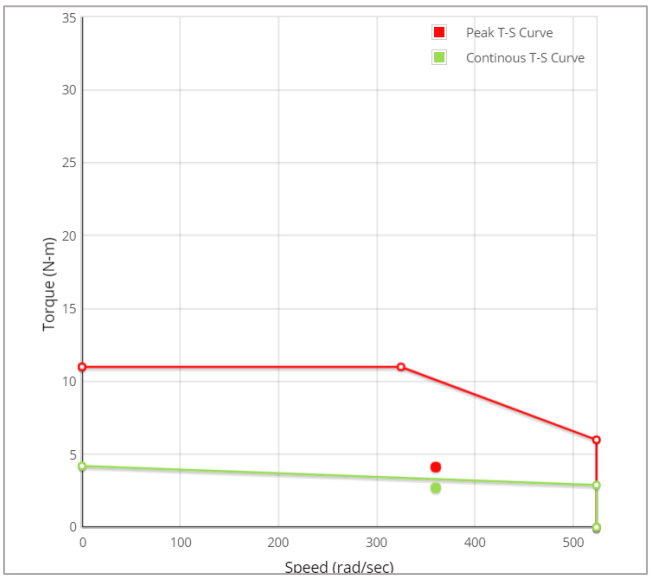


Figure 2

Or maybe we can go even smaller and use motor no. 5. Again the inertia is reduced, now to $0.00008\ kg - m^2$, so the RMS torque utilisation is $(6.971\ /\ 3) + (1097 \times 3 \times .00008) = 2.58\ Nm$, about 75% of the motor rating – maybe a step too far if we wish to keep the utilisation below 70%, particularly since we need to remember there will be added torque due to the additional transmission mechanics.

However motor no. 5 has a higher speed range. Maybe we can use it if we push the ratio up to 4:1. The new maximum speed of $4{,}800\ rpm$ is utilising 80% of the speed rating of the motor and we are usually happy at up to 90% utilisation for speed. That gives us $(6.971\ /\ 4) + (1097 \times 4 \times .00008) = 2.1\ Nm$, and RMS torque utilisation is down to 62%.

The peak table changes to:

Load Side				Motor side (No 5)				
Acceleration [a]	Load Inertia [J_L]	Total Torque [T_L]	Ratio	Acceleration [a]	Reflected Load Torque	Reflected Load Inertia	Motor Inertia	Motor Ac
(rad/sec)	(kg-m^2)	(Nm)		(rad/sec^2)	(Nm)	(kg-m^2)	(kg-m^2)	
1400	0.006	10.9	3	4190	3.63	0.000667	8.00x10^{-5}	
0	0.006	2.5	3	0	0.833	0.000667	8.00x10^{-5}	
-1480	0.006	-6.37	3	-4440	-2.12	0.000667	8.00x10^{-5}	
0	0.006	0.5	3	0	0.167	0.000667	8.00x10^{-5}	

Table 5

Giving a new peak torque requirement of only $3.17\ Nm$, an utilisation of 41%. Again, we need to check the T-S curve:

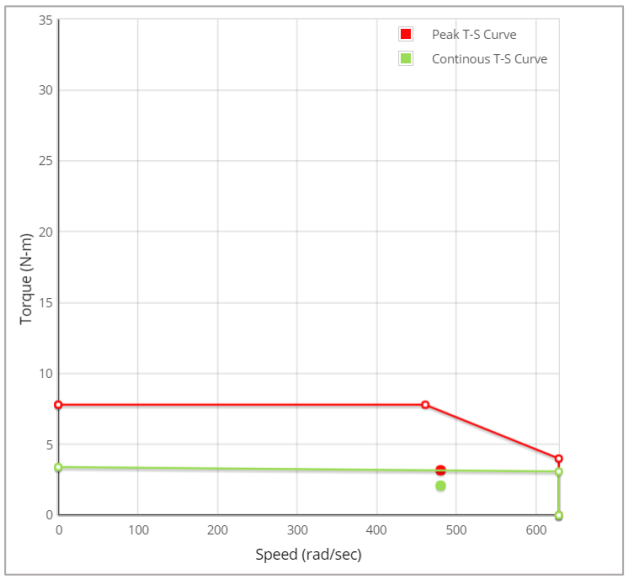


Figure 3

This shows that we are still well clear of the voltage limit line.

We should also note that since the inertia is reduced by the square of the ratio, the reflected load inertia will now be $0.00038\ kg - m^2$ which the motor inertia is under 5 times. In the original direct drive case with motor no. 7 it was 15:1, so it seems we can reduce the dominance of the load considerably by using a transmission, even while we are using a smaller and cheaper motor.

It is obviously a little tedious to need to recalculate until you find the best prospective solution, so in the next section we will be looking at how an added enhancement to the T-S Widget – a ratio slider - helps to make this a lot easier.

Test your understanding

Question 1 of 1 1

Use Motor No. 5 from the table and transmission ratio 5:1 to compute the RMS torque requirement.

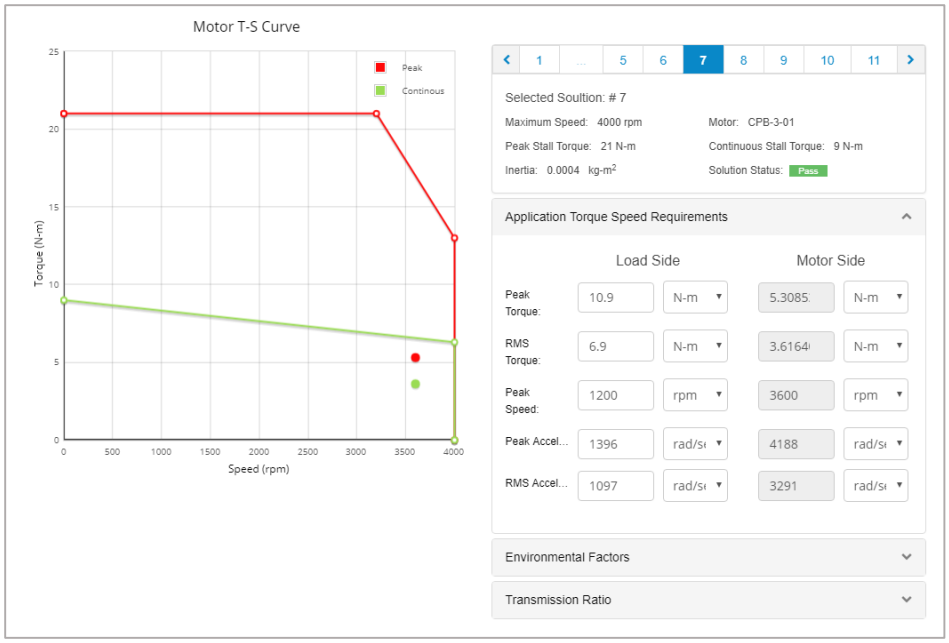
Q2 fx Total Motor Torque

	A	B	C	D	E	F	G
1	Load Side						
2	Initial Velocity [u]	Final Velocity [v]	Segment Duration [t]	Distance Travelled [s]	Acceleration [a]	Load Inertia [J_L]	Total Torque [T_t]
3	(rev/s)	(rev/s)	(s)	(rev)	(rad/s^2)	(kg-m^2)	(Nm)
4	0	20	0.09	0.9	1400	0.006	10.9
5	20	20	0.075	1.5	0	0.006	2.5
6	20	0	0.085	0.85	-1480	0.006	-6.37
7	0	0	0.05	0	0	0.006	0.5
8	Total		0.3				
9							
10	RMS Torque	0	Nm				
11							

Check My Work Reset

5.3: Using a T-S Widget to Help with Optimisation

In the last section you saw how adding a transmission with a ratio of 3:1 allowed us to use a smaller motor. We also saw that we needed to check quite a few possible motors and transmission ratios in order to find the best solution. Wouldn't it be easier to use the T-S widget to quickly check the effects of changing the ratio? Here's the T-S widget with a ratio adjustment capability added.

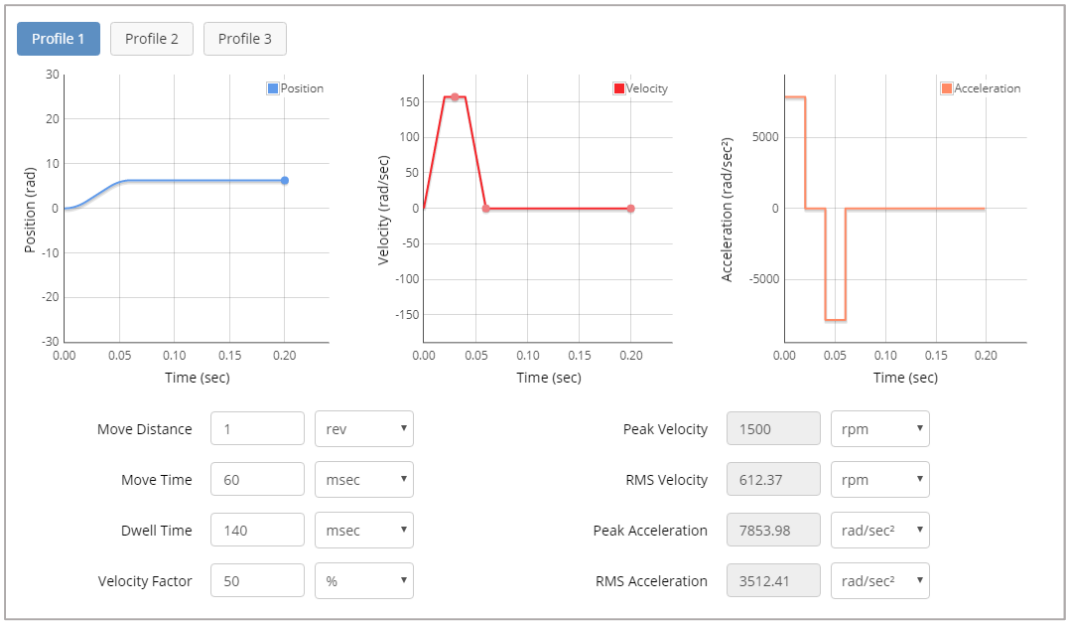


The original values for the original application have been entered already, with motor no. 7 selected.

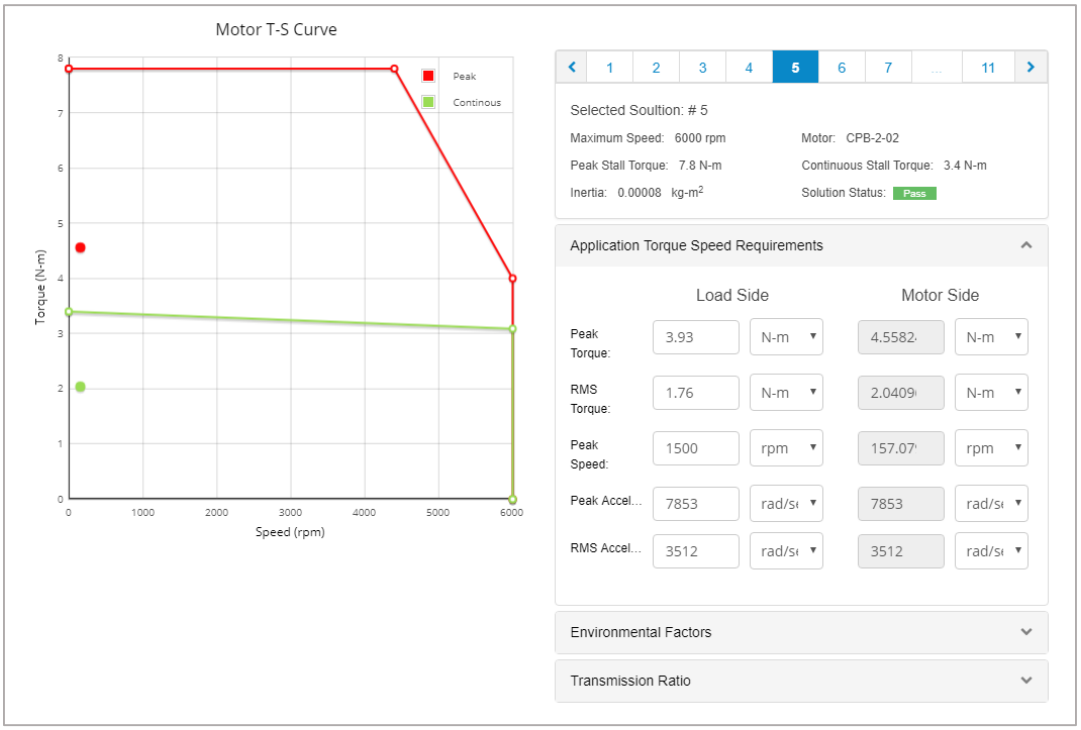
Click on the “Transmission Ratio” panel and a box will appear which allows you to change the ratio. Move it up to 3:1 - you will see that the result matches with what you saw in the last section. Choose motor no. 6 (click the left arrow on the motor selector), and again you will see a similar result to what you saw in the last section for that motor. Try different motors with different ratios.

Click on the “Application Torque/Speed Requirements” panel, if it isn’t already open. While you are changing the ratio you will see that the values for the motor-side torques, the peak speed and the accelerations are changing to match the ratio changes. The accelerations are both being applied to the inertia of the chosen motor, so the additional torque required to accelerate the motor is being automatically added. Note that the peak acceleration figure is from segment 1, not segment 3. Although the acceleration in segment 3 is greater, we already know that segment 1 is the peak torque segment.

Now here’s an example of a quick selection, running right through the process. We start by using the Motion Profile widget and enter a trapezoidal index moving one revolution in $60\ ms$ followed by a dwell of $140\ ms$.



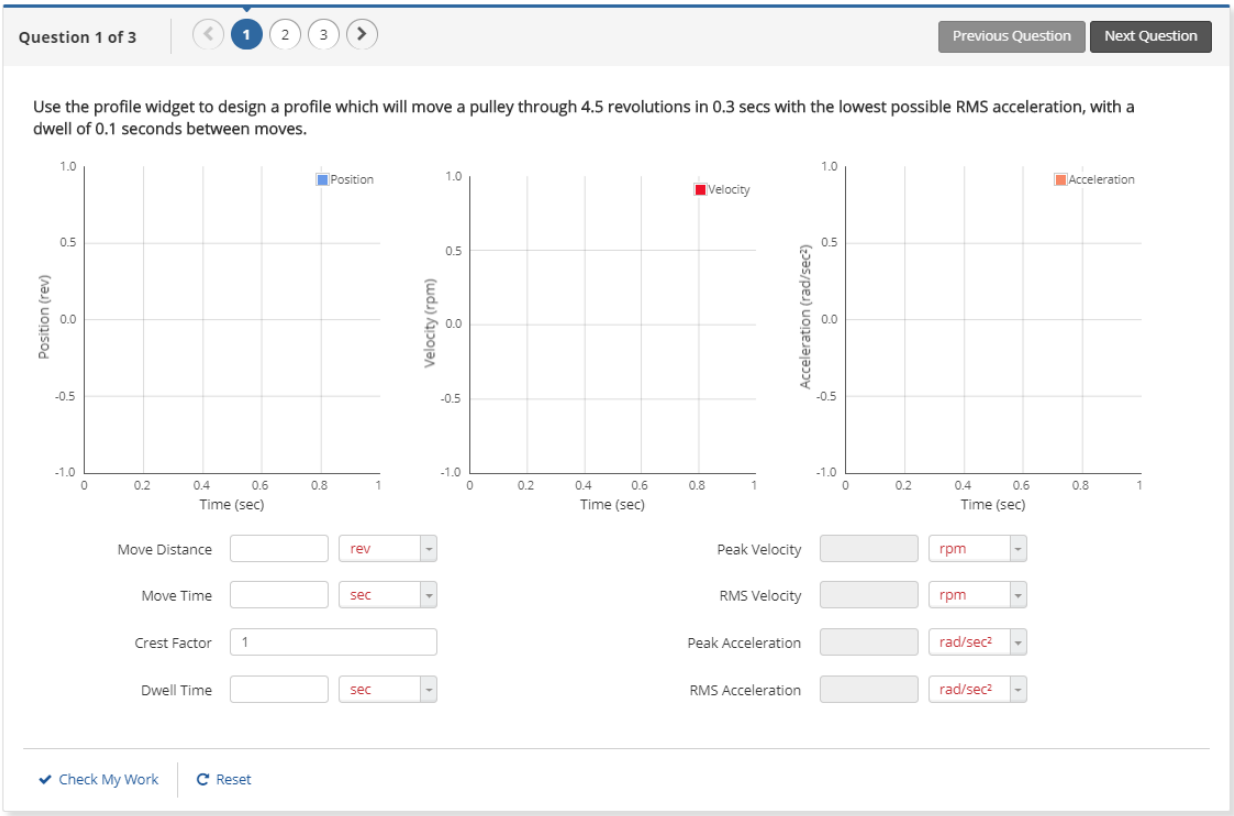
The load has an inertia of 5 $kg - cm^2$ with negligible friction and no externally applied load, which means that with a peak acceleration of 7854 rad/s^2 the peak torque requirement is 3.93 Nm, and with an RMS acceleration of 3512 rad/s^2 the RMS torque is 1.76 Nm. Now let's put those numbers into the T-S widget.



The motor no. 5 looks like a good candidate. Let's check what changing the ratio does. You should find that no ratio improves the situation enough to use a smaller motor, and in fact the higher ratios make it worse.

We must always remember though, that the widget doesn't know anything about the additional inertia and losses imposed by transmission mechanism, so these results, while indicative of a solution, are not always accurate enough to make a final selection. The next section will show you how to account for the mechanical parameters of the transmission mechanism.

Test your understanding



5.4: Adding a Transmission Mechanism

Up to now we have not considered the additional loads and losses imposed by adding a transmission mechanism, let alone the additional cost.

Types of Transmission

For rotary loads, only two types of transmission mechanism are commonly used, pulleys or a gearbox.

Generally, the simplest, lowest cost mechanism is a toothed-belt or chain and two pulleys, but this is limited to very low ratios, typically less than 4:1. The ratio is set by the ratio of teeth on the pulleys, which is the same as their pitch circle diameters (PCD). Do not attempt to measure diameters to determine the ratio; use the number of teeth or the manufacturer's specified PCD. Whether you can use this arrangement will of course depend on the design of the machine, as

the rotational axis of the motor will be displaced sideways from that of the load as shown in Figure 1 below.

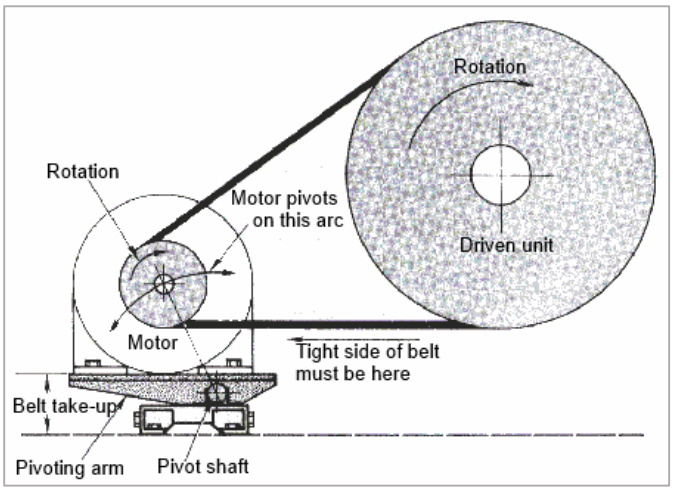


Figure 1

Otherwise the usual choice is to use a planetary gearbox which will offer a wide range of ratios from 3:1 to several hundred to 1, but will be costlier. While a worm gearbox is a common choice for machines, it is not often used in servo applications because of the much higher friction losses. Figure 2 shows a typical example of servomotor/gearbox combination.

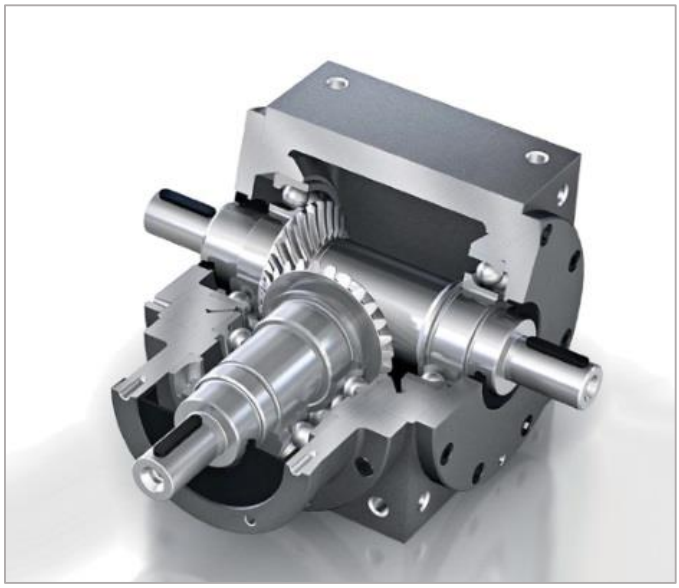


Figure 2

Accuracy is also an important consideration. You are trying to control the position of the payload, but the encoder which is giving you feedback will be in the motor – so the input to output positional accuracy of the mechanism you choose will be important. Planetary gearboxes are the superior solution if high accuracy is required as spur gearboxes usually have much higher backlash.

For some applications, it may make sense to use two transmission mechanisms in series, where a high gear ratio is required. This can add some additional complication to calculations and the use of a sizing tool such as COSMATT will make things a lot easier.

Examining the Effect of the Transmission Losses

Let's look at the effect of the transmission mechanism on loads and losses. We will use the example from Chapter 5.2 - Adding a Ratio after we had recalculated it using the 3:1 ratio and motor no. 6, where the RMS torque utilisation was a safe 65%. We will examine how the use of a pulley system of gearing would affect the solution.

We had a table looking like this

Load Side				Motor side (No 6)				
Acceleration [a]	Load Inertia [J_L]	Total Torque [T_t]	Ratio	Acceleration [a]	Reflected Load Torque	Reflected Load Inertia	Motor Inertia	Motor Ac
(rad/sec)	(kg-m^2)	(Nm)		(rad/sec^2)	(Nm)	(kg-m^2)	(kg-m^2)	
1400	0.006	10.9	3	4190	3.63	0.000667	0.00012	
0	0.006	2.5	3	0	0.833	0.000667	0.00012	
-1480	0.006	-6.37	3	-4440	-2.12	0.000667	0.00012	
0	0.006	0.5	3	0	0.167	0.000667	0.00012	

Table 1

The RMS result was $2.715\ Nm$.

For the transmission, we could specify a toothed belt and pulley system with a pulley on the motor side of say $30\ mm$ diameter. A belt of $10\ mm$ width would be OK, so the pulley would be $10\ mm$ thick. The inertia of such a pulley in aluminium will be about $0.021\ kg - m^2$. As already mentioned, the ratio would be determined by the number of teeth, not the pulley diameters, but for this exercise we will assume the diameter of the second pulley can be estimated with reasonable accuracy by multiplying the smaller pulley diameter by the ratio. Thus the second (load side) pulley will have a diameter of about $90\ mm$ and again, a thickness of $10\ mm$. This would give an inertia of about $1.74\ kg - m^2$.

Another factor is the mass of the belt, which of course depends on the length. You would want it to be short to maintain stiffness of the transmission system (more on that in the next section), and light to keep the load as low as possible, but to illustrate the calculations we will use a figure of $0.1\ kg$ for the belt mass. This appears to the system as a circumferential mass on the motor-side pulley, so it will present an inertia on the motor side of $0.225\ kg - m^2$.

We can now change the table above. This can be done in two ways – you can either add the load-side pulley inertia to the load side and the belt and motor-side pulley inertias to the motor side, or you can refer the entire pulley inertia to the motor side. In practice it is best to do the latter.

The load-side pulley inertia is $1.74\ kg - m^2$ divide that by the square of the ratio to refer it to the motor side, and you get $\frac{1.74}{3^2} = 0.193\ kg - m^2$ making the total referred inertia of the transmission $(0.021 + 0.193 + 0.225)$, or about $0.44\ kg - m^2$.

We have added two new two columns on the right hands side, for transmission inertia and transmission acceleration torque:

Load Side							
Segment Duration [T]	Acceleration [a]	Load Inertia [J_L]	Total Torque [T_t]	Ratio	Acceleration [a]	Reflected Load Torque	Reflected Load Inertia
(s)	(rad/sec)	(kg-m^2)	(Nm)		(rad/sec^2)	(Nm)	(kg-m^2)
0.09	1400	0.006	10.9	3	4190	3.63	0.000667
0.075	0	0.006	2.5	3	0	0.833	0.000667
0.085	-1480	0.006	-6.37	3	-4440	-2.12	0.000667
0.05	0	0.006	0.5	3	0	0.167	0.000667

Table 2

You can see that the effect of adding the transmission increases the peak torque by $0.18\ Nm$. If we also recalculate the RMS torque we find that it has increased from $2.71\ Nm$ to $2.84\ Nm$. We need to recheck that the motor is still capable of doing the job, but the change is very small and we already had an adequate safety margin, so we can be confident that the belt transmission system will not cause a problem in this example.

We mentioned in Chapter 5.2 - Adding a Ratio that there will also be additional losses, which will be in the usual form of friction torque. You can usually ignore them in the calculations for a belt transmission as they will be small, but your chosen solution should have a little extra capacity to allow for them.

Adding a Gearbox

In some ways calculating the added effect of a gearbox is easier, as the inertia of a gearbox intended for use with servomotors will usually be specified as being with respect to the motor side, so you can just add the figure in the “Transmission Inertia” column directly. However, gearboxes will have greater frictional losses than a pulley system, as they usually have shaft seals to prevent leakage of the lubricant, and the churning of the lubricant during motion will also cause losses.

The problem is that the manufacturers do not specify these losses, so you will need to make an estimate. What they do usually specify is the efficiency but this can be misleading. Most of the losses in a gearbox are speed-related, not torque related, so they exist even if there is no load – just rotating the unloaded gearbox can require a significant amount of torque (particularly on a cold morning), and of course the efficiency of an unloaded gearbox is zero, because there is no output.

You can get some idea of the frictional losses in a gearbox by taking the efficiency figure and applying it to the rated torque for the box, so for example if a gearbox has a rated output torque of $25\ Nm$ and it is 90% efficient, you can infer that the loss torque **reflected to the load-side** is $2.5\ Nm$. If the gearbox ratio is 10:1, this would mean the motor-side loss torque would be about $0.25\ Nm$. You should use this number in your calculations although you need to remember that it is only an approximation.

Thus in our example, if you choose a gearbox it should be rated for a continuous output torque of something near the peak torque of the application to keep the mechanical wear and tear as low as possible. Let's assume a 3:1 ratio gearbox with a stated efficiency of 95%. Remember this efficiency only applies at full torque and speed. The highest load-side torque is $9.8\ Nm$, so let's say we chose a gearbox with a rating of$10\ Nm$. We would then estimate the loss torque as

$$RatedTorque \times \frac{(1 - efficiency)}{ratio}$$

i.e.

$$10 \times \frac{1 - 0.95}{3} = 0.167\ Nm.$$

We would add this as if it were a constant friction torque (i.e. zero when at rest, and opposing the direction of motion).

Such a gearbox would have a very small inertia – for this example let's just use the same figure as we got for the pulley system. Thus our table gets one more column for the loss torque.

Load Side					Motor side (No 6)						
Segment Duration [T]	Acceleration [a]	Load Inertia [J_L]	Total Torque [T_L]	Ratio	Acceleration [a]	Reflected Load Torque	Reflected Load Inertia	Motor Inertia	Motor Acceleration Torque	Transmission Inertia	Transmission Acceleration Torqu
(s)	(rad/sec)	(kg-m^2)	(Nm)		(rad/sec^2)	(Nm)	(kg-m^2)	(kg-m^2)	(Nm)	(kg-m^2)	(Nm)
0.09	1400	0.006	10.9	3	4190	3.63	0.000667	0.00012	0.503	0.44	0.184
0.075	0	0.006	2.5	3	0	0.833	0.000667	0.00012	0	0.44	0
0.085	-1480	0.006	-6.37	3	-4440	-2.12	0.000667	0.00012	-0.532	0.44	-0.195
0.05	0	0.006	0.5	3	0	0.167	0.000667	0.00012	0	0.44	0

Table 3

This has now increased our peak from $4.13\ Nm$ to $4.48\ Nm$, and our RMS from $2.71\ Nm$ to $2.96\ Nm$, again, only a small increase well within the capability of the motor.

The table has become fairly large and while it isn't complex for this simple example, it does get to be quite a chore producing tables for more complex motions. This is where a tool such as COSMATT becomes invaluable.

So now we have covered everything you need to do for a basic rotary system. In the next chapter we'll look at linear systems.

5.5: The Effect of Inertia Ratio

The inertia ratio is the ratio at any point in the mechanical chain between the load-side inertia and the motor-side inertia. Usually unless the point at which it is measured is specified, it can be taken as the ratio of the load-side inertia to the motor's rotor inertia at the motor shaft. It is important because of the effect it has on the stability of the position control system. Some manufacturers arbitrarily suggest a maximum inertia ratio of 3:1 or perhaps 5:1. In practice much higher ratios may be used with satisfactory results if you know what you are doing.

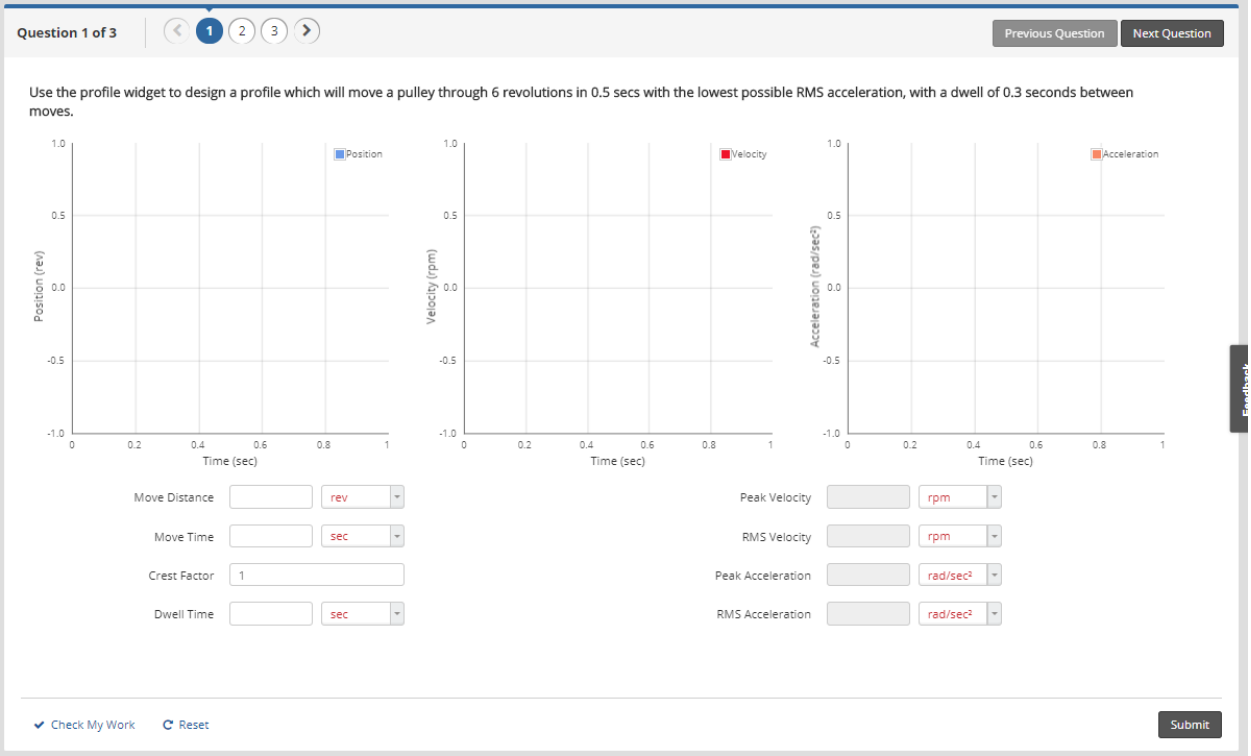
The stability of the system will be heavily dependent not on the inertia ratio alone**,** but also the level of backlash and compliance between the load and the motor. This is dealt with in detail in the advanced course.

Inertia matching is not well understood by most, but it is greatly affected by the transmission ratio, since as we have already seen, the effect of gearing up the motor speed is to reduce the load inertia by the **square** of the ratio used.

For the basic course it is sufficient to be aware that you should try to reduce the reflected inertia at the motor shaft as much as you can, by reducing the inertia of the rotating components or by gearing the system to reduce the inertia of the machine as reflected at the motor. Perhaps even more important is to ensure that the mechanical system is as torsionally rigid as possible.

When choosing a coupling, make sure it is specified as having high torsional rigidity. If the manufacturer does not specify the torsional rigidity it is probably best to use a different make. When choosing a gearbox, use one which is specified for use with the motor you are using, and if your inertia ratio is high, use a low-backlash version. If you use a pulley system, a toothed belt is better than a chain, but choose a servo duty belt with minimum stretch and use aluminium pulleys if possible. The more the load inertia dominates, the more you need to pay attention to these points.

5.6: Assessment



Chapter 6 – Linear Loads

6.1: Introduction

A linear load in this context is a load that moves in a straight line, usually guided by rails or similar constraints. It will be driven either via a mechanism that translates from rotary to linear motion such as a screw-type or a tangential-type drive, or directly by a linear motor. We will examine the screw-type and tangential-type mechanisms later in this chapter; linear motors are dealt with in the advanced course.

In this context the term "Linear" refers to the geometry of the device, not to be confused with mathematical linearity, which refers to a proportional relationship. For example, a Crank mechanism is a mechanism that converts rotary motion to linear motion, however, the mathematical relationship between the rotation of the motor and the movement of the linear part of the mechanism is not a mathematically linear relationship. You can find out more about crank mechanisms in the advanced course.

Following topics will be covered in this chapter:

- Conversion to Rotary
- Losses and Gravity Loading
- Screw-type Mechanisms
- Tangential Mechanisms
- Summary of Key Points

Learning Objectives:

- Analyse linear applications
- Linear motion profiles & linear load configurations
- The transmission ratio
- Screw-type mechanisms
- Tangential mechanisms (Belt, Chain or Rack and Pinion)

6.2: Conversion to Rotary

The linear load parameters may be converted to equivalent rotary parameters by a simple concept:-

Whatever the geometry of the mechanism the load will move a constant distance for every rotation of the motor (Note: This does not apply to crank or lever-type mechanisms – these non-linear mechanisms are covered in the advanced course). Thus an "equivalent pulley" may be thought of with a radius (R_{eq}) that produces the same linear movement (L) per motor revolution. This conceptual pulley adds no mass or friction of its own.

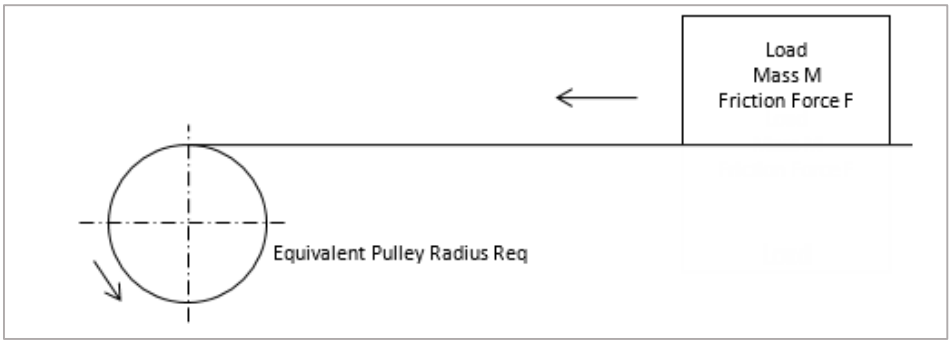


Figure 1 - The load moves a Length L for each motor revolution

$$Circumference\ L = 2\pi R_{eq}$$

$$Equivalent\ Radius\ R_{eq} = L/2\pi$$

The Equivalent Radius Point is the point within the mechanism at which linear motion is converted to rotary motion, and linear forces are converted to torque. Once the Equivalent Radius is calculated, these parameters are easily converted:-

$$Inertia = M\ \times\ {R_{eq}}^2$$

$$Torque = Force\ \times\ R_{eq}$$

$$Rotary\ Velocity = Linear\ Velocity\ /R_{eq}$$

$$Rotary\ acceleration = Linear\ acceleration\ /\ R_{eq}$$

$$Rotary\ distance\ moved = Linear\ distance\ moved\ /\ R_{eq}$$

Note that the equivalent radius has units of linear distance/rotary distance, such as metres per radian.

This is a general result which is applicable to any linear to rotary mechanism. It becomes particularly important in non-linear mechanisms such as cranks or four-bar linkages, where the equivalent radius is variable as the mechanism moves, and can be described by a mathematical function which depends on the position of the mechanism. This is dealt with in the advanced course.

For more commonly used mechanisms the equivalent radius is fairly easy to calculate, but you need to be careful. It is best to consider the linear distance moved for one turn, then divide by 2π. For example, in a toothed belt or chain system, a pulley may have 100 $teeth$. In this case, the linear distance moved for 1 $revolution$ is 100 times the tooth pitch, not the circumference of the pulley.

The mechanism will itself present additional inertia and friction to the motor, according to its design. These values must be added, together with any other elements connected to the motor such as shafts and couplings.

Test your understanding

Question 1 of 3 1 2 3

A toothed belt has a tooth pitch of 10mm. It is driven by a pulley with 80 teeth. What is the equivalent radius?

- 3.1415 mm/rad
- 254.6479 mm/rad
- 127.3239 mm/rad

Check My Work | Reset

6.3: Losses and Gravity Loading

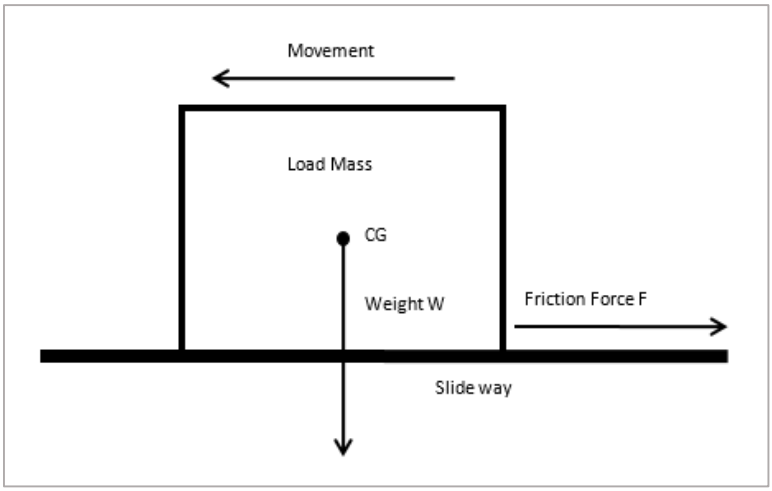


Figure 1 - A horizontal linear load

Mass

Often the most important parameter in fast-moving servo systems is the mass of the load, which appears to the motor as an inertia. When a linear load is driven by a rotary motor the mass is converted to an equivalent inertia by the conversion mechanism. We commonly refer to this as "reflected" inertia.

Friction

In most linear applications, the friction force seen by the motor tends to be dominated by the friction caused by the weight of the load on its slideway.

$$Friction\ force\ F = W\mu, where\ W = weight, \mu = coefficient\ of\ friction$$

External Force

Any additional force which is acting on the mass, such as gravity or spring forces. This is separate from the force required to accelerate the mass and the friction force.

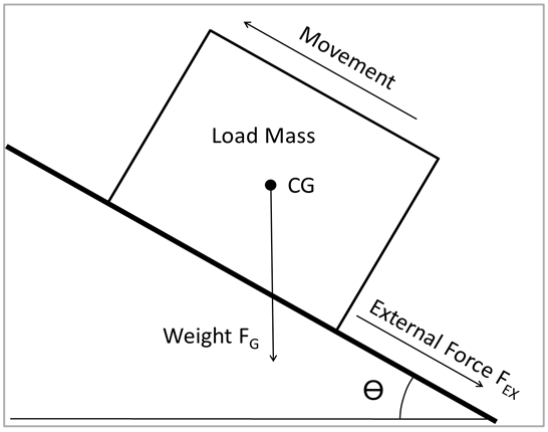


Figure 2 - An inclined load where gravity creates an external force.

$$Gravity\ force\ F_G = W\ sin(\theta)$$

$$where\ W = weight, \theta = angle\ of\ inclination$$

$$Friction\ force\ F = \mu W cos(\theta)$$

$$where\ W = weight, \mu = coefficient\ of\ friction, \theta = angle\ of\ inclination$$

The friction and gravity forces must be added to the forces required to accelerate or decelerate the load mass when it is moved, in exactly the same way as rotary losses are all added together. However, in linear applications, there will always be additional torque requirements imposed by the mechanism itself.

The mechanism will, in addition to the payload, impose additional linear masses and forces which must also be “reflected” to the rotary side of the mechanism, where they will add to the torque. There will also be rotating inertia and rotary friction losses to add. All these must be summed correctly to calculate the final torque requirement. Some of the additional torque imposed by the mechanism is dependent on the motion, of course.

We will study how to calculate the torque for the relatively simple types of motion profile we have encountered in the earlier chapters, but many profiles involve variable acceleration, making the calculations more complex. For these situations, a sizing application such as the COSMATT tool can save a considerable amount of time and effort, and also reduce the likelihood of errors considerably.

In the following sections, we will examine the two most common mechanism types to find out what sort of additional loads and losses we are likely to encounter.

Test your understanding

Question 1 of 2 1 2

The weight of a loaded table in a linear mechanism is 10 kg. It slides horizontally on slides with a coefficient of friction of 0.1. What force is necessary to overcome the friction when moving slowly?

- 98 N
- 9.8 N
- 0.98 N

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6.4: Screw-type Mechanism

Lead screws or ball screws are commonly used in industrial applications. A rotating threaded shaft is used to move a nut along a linear path. Usually, these mechanisms cannot reach high speeds for reasons which are dealt with more fully in the Advanced course. The nut is usually attached to a table which is guided by rails, using linear bearings or lubricated slides.

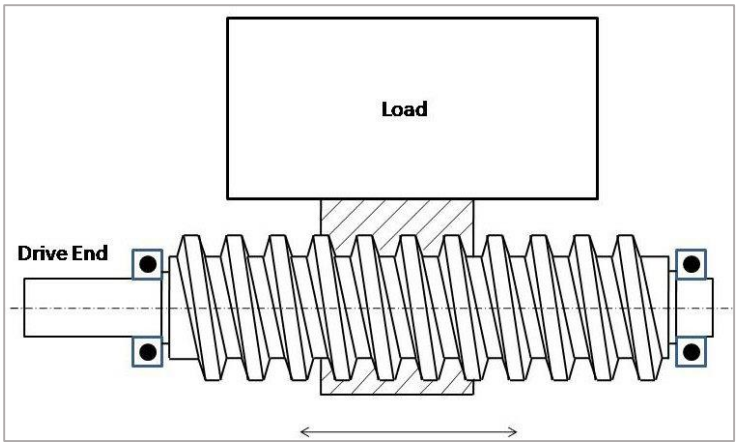

Figure 3 - Plain (Acme) Threaded Screw and Nut

The plain screw is rarely used for servo axes. It has backlash and a high level of friction.

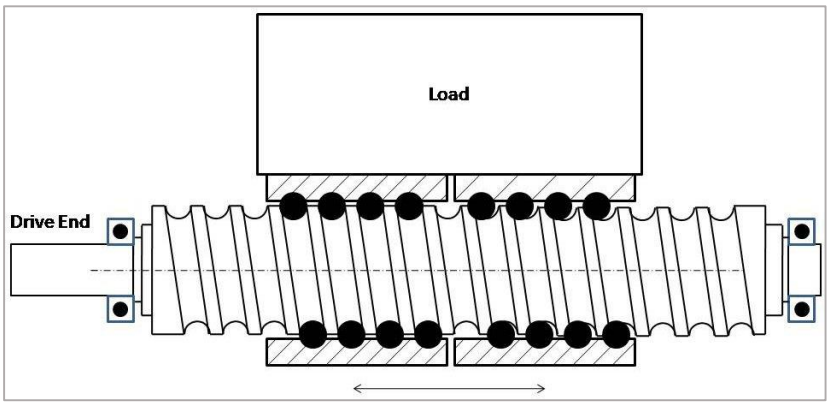

Figure 4 - Ball screw with two nuts

The ball screw has its internal and external “threads” separated by steel balls to virtually eliminate friction. As each ball exits the raceway a tube scoops it up and returns it to the opposite end to re-circulate.

To eliminate backlash, two nuts are fitted and adjusted so that they are “pre-loaded” against each other. This pre-load is set by measuring the friction torque as specified by the ball screw manufacturer. Higher pre-load produces greater stiffness but at the cost of higher friction (and heat) and reduced life.

Whirling Speed

Every rotating shaft has a critical speed at which it “whirls”, that is, it bends outwards with a real danger of suffering permanent damage. This critical speed decreases with increasing length and reducing diameter. It also depends on how the shaft is supported (position of bearings) - worst case is when one end is unsupported (cantilevered).

Ball screw manufacturers publish the limiting conditions for each of their screws.

Specifying Screws

Screws are specified by "**Lead**" and **Diameter**: -

"Lead" (the key parameter) is the linear movement per revolution while "Pitch" is the axial distance between threads.

Usually, Lead and Pitch are identical but occasionally a screw will have two or more "Starts". A two start screw has two separate helical grooves or threads, reminiscent of DNA.

Additional loads and losses

In addition to the payload, linear moving masses will include the table and the nut. These masses are added to the payload when calculating forces. Also, the screw itself has inertia, which adds to the motor side inertial loading.

There will be linear friction between the table and the slide, as explained in Chapter 6.2 – Conversion to Rotary, but there will also be a rotary friction loss due to friction between the nut and the screw thread in a lead screw, which is seen as a torque loss on the motor side. In a ball screw the friction is low, but if preload is applied as it usually is, that will be seen as a friction torque, as mentioned above. The manufacturer of the ball screw should specify this torque in their data as "preload" or "drag" torque. They will sometimes specify efficiency of the conversion from rotary to linear motion.

It can sometimes be quite difficult to find the information you need on commercial lead screws or ball screws – the information is often not readily available in catalogues and you will need to contact the manufacturers to get the details you need, or make an intelligent estimation if you are a skilled mechanical engineer.

The Linear System

We can create a table of the data which affects movement, as follows: -

Screw example data		
Data	**Value**	**Unit**
Payload	20	kg
Table mass	1	kg
Slideway friction coefficient	0.01	per unit
Inclination	45	degrees
Screw lead	0.012	m
Rotating inertia	0.005	kg-m^2
Preload/ rotary friction/drag torque	0.02	Nm

Table 1

We apply an associated motion profile, such as this (note the linear units since the profile is specified at the table/payload). The table moves forward, dwells, then returns to the start position followed by a further dwell.

Segment No.	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]
	(m/s)	(m/s)	(s)	(m/s^2)	(m)
1	0	0.2	0.09	2.22	0.009
2	0.2	0.2	0.075	0	0.015
3	0.2	0	0.085	-2.35	0.0085
4	0	0	0.05	0	0
5	0	-0.2	0.09	-2.22	-0.009
6	-0.2	-0.2	0.075	0	-0.015
7	-0.2	0	0.085	2.35	-0.0085
8	0	0	0.05	0	0

Table 2

Next we add columns to represent the linear part of the mechanical system: we usually assume that positive velocity is up the slope and forces that oppose the motion are positive.

Mass

The payload could vary from segment to segment – for example, if the payload is a work piece which is ejected from the machine at some point in the motion cycle. It is simply added to the table mass where appropriate.

External Force

In this case gravity is the only external force.

$$External\ Force = \ Gravity\ Force = Mass \times \ g \times sin(inclination)$$

$$e.g. External\ Force \ = \ 21 \times 9.81 \times 0.7071 \ = \ 145.67\ N$$

Friction Force

In this case we need to account for the inclination of the slide.

$$Friction\ Force = Mass \times Coeff\ of\ Friction \times g \times cos(inclination) \times Sign\ of\ Direction$$

$$e.g. Friction\ Force \ = \ 21 \times 0.01 \times 9.81 \times 0.7071 \times 1 \ = \ 1.457\ N$$

Sign of Direction

Friction always opposes the direction of motion so, in spreadsheets, we need to apply a "Sign of Direction" ± 1 to the friction force that mirrors the sign of the velocity. We also assume that the friction force is zero at zero speed, so, "Sign of Direction" becomes zero. To create a "Sign of Direction" column in MS Excel use this function "$= IF(E3 > 0, 1, IF(E3 < 0, \ -1, 0))$" where $E3$ is the cell containing the distance (s) in row 3. Search for excel nested if statements for further information.

Segment No	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External Force	Friction Force	T
	(m/s)	(m/s)	(s)	(m/s²)	(m)		(kg)	(N)	(N)	
1	0	0.2	0.09	2.22	0.009	1	21	146	1.46	
2	0.2	0.2	0.075	0	0.015	1	21	146	1.46	
3	0.2	0	0.085	-2.35	0.0085	1	21	146	1.46	
4	0	0	0.05	0	0	0	21	146	0	
5	0	-0.2	0.09	-2.22	-0.009	-1	21	146	-1.46	
6	-0.2	-0.2	0.075	0	-0.015	-1	21	146	-1.46	
7	-0.2	0	0.085	2.35	-0.0085	-1	21	146	-1.46	
8	0	0	0.05	0	0	0	21	146	0	

Table 3

Checking the Ball Screw thrust rating

The Total Force column is needed to check the thrust rating.

$$Total\ Force = Acceleration\ Force + External\ Force + Friction\ Force$$

$$Total\ Force = (Mass \times Acceleration) + External\ Force + Friction\ Force$$

$$e.g.\ Total\ Force = 21 \times 2.22 + 145.67 + 1.457 = 193.8\ N$$

The peak force of $193.8N$ occurs in segment 1. This must lie within the thrust capacity of the Ball Screw. Thrust capacity rises with screw diameter.

The Total Force column also shows that:-

- The torque at constant speed in segment 2 is higher than that at constant speed in segment 6 due to the friction acting in opposite directions.
- This is a case where gravity dominates the system: a smaller motor may be possible if the load were counter-balanced – this is covered in the advanced course.

The Equivalent Rotary System

Now we must transform table 3 data into an equivalent rotary system, whilst adding in the rotary components of the ball screw:-

Equivalent Radius (R_{eq})

$$R_{eq} = \text{Lead}/(2\pi\ )$$

$$R_{eq} = 0.012/(2\pi) = 0.00191\ m/radian$$

It is very important to know the units of the Equivalent Radius, to make sure you know what units to use on the rotary side of the calculations.

Profile

$$u\ (rotary) = u\ (linear)\ /\ R_{eq}$$

$$v\ (rotary) = v\ (linear)\ /\ R_{eq}$$

$$e.g.\ v\ (rotary) = 0.2\ /\ 0.00191 = 104.7\ rad/s$$

The time column does not change, so we can compute the other two columns by either using the standard motion equations or by using the ratio again, which will be easier.

$$a\ (rotary) = a\ (linear)/\ R_{eq}$$

$$s\ (rotary) = s\ (linear)/\ R_{eq}$$

Efficiency

The efficiency of most Ball Screws is very high and so may be ignored. As the lead reduces in comparison with diameter the efficiency tends to fall. If the manufacturer's data indicates low efficiency (say, below 70%) then it needs to be taken into account. Plain (Acme) screws are always inefficient. The treatment of efficiency is not straight forward and will be covered in the advanced course.

Load Inertia

Comprises the effective inertia of the mass plus the rotating inertia, which is primarily the inertia of the screw. Efficiency affects the acceleration force so we increase the effective inertia of the mass to achieve the correct acceleration torque in subsequent calculations.

$$Total\ \ Inertia\ =\ Mass \times {R_{eq}}^2 +\ Rotating\ Inertia$$

$$e.g.\ Total\ \ Inertia\ =\ 21 \times 0.00191^2\ + 0.005 = 0.00508\ kgm^2$$

Load External Torque

$$External\ Torque\ =\ External\ Force \times R_{eq}$$

$$e.g.\ External\ Torque\ =\ 145.67 \times 0.00191 = 0.278\ Nm$$

Load Friction Torque

Comprises friction due to the linear slide plus rotary friction due to the ball screw pre-load.

$$Friction\ Torque\ =\ Friction\ Force \times R_{eq} + PreLoad\ Torque \times Sign\ of\ Direction$$

$$e.g. Friction\ Torque\ =\ 1.457\ \times\ 0.00191\ +\ 0.02\ \times 1\ =\ 0.023\ Nm$$

Segment No	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia	Accel. Torque	Load external torque	Load
	(rad/s)	(rad/s)	(s)	(rad/s²)	(rad)	(kg-m²)	(Nm)	(Nm)	
1	0	105	0.09	1160	4.71	0.00508	5.91	0.278	
2	105	105	0.075	0	7.85	0.00508	0	0.278	
3	105	0	0.085	-1230	4.45	0.00508	-6.26	0.278	
4	0	0	0.05	0	0	0.00508	0	0.278	
5	0	-105	0.09	-1160	-4.71	0.00508	-5.91	0.278	
6	-105	-105	0.075	0	-7.85	0.00508	0	0.278	
7	-105	0	0.085	1230	-4.45	0.00508	6.26	0.278	
8	0	0	0.05	0	0	0.00508	0	0.278	

Table 4

Checking the Ball Screw speed rating

Maximum speed is seen to be $104.72\ rad/s$.

$$Speed\ (rpm) = Speed\ (rad/s) \times 60/2\pi$$

$$Speed\ =\ 104.72\ /\ 2\pi\ =\ 1000\ rpm$$

This must be safely inside the Whirling Speed rating. For a given length, whirling speed increases with screw diameter and varies according to how the screw is supported.

Completing the sizing exercise

- Table 4 is **identical in layout** to Table 3 in Chapter 3.4 - Computing the Load Torque. The following processes for adding a motor (and, possibly, a gearbox) and optimising the transmission ratio are now exactly the same as for a rotary load. Even complex loads may be transposed and simplified in this way making the whole sizing exercise easier.
- Note that changing the effective radius has the same effect as changing the transmission ratio. Sometimes a change of screw lead will avoid having to add a gearbox or allow a smaller motor. Keep in mind the "Whirling Speed" limit when reducing the lead since this will increase the rotational speed.
- The inertia column is the total inertia seen by the motor, which will vary if the load mass changes (only slightly in this case). This variation can have subtle effects on the tuning of the control system, as explained in the advanced course.

Test your understanding

Question 1 of 2 1 2

A flying shear is driven using a ballscrew. The maximum permissible speed of the ballscrew is 1000 rpm. The shear performs a forward move of 500 mm at a speed of 0.2 m/s. The maximum available acceleration/deceleration is 1 m/s2. This is followed by a return move at maximum speed of 0.5 m/s in the shortest possible time. There are no dwells. The data for the axis is given in the table below. Compute the linear table of the axis.

A3:B3 | fx | Maximum Payload

	A	B	C	D	E	F	G	H	I
1	Punch-press, Y-axis								
2	Data		Value	Units					
3	Maximum Payload		250	kg					
4	Table mass		100	kg					
5	Slideway friction coefficient		0.005	per unit					
6	Inclination		0	degrees					
7	Screw lead		0.02	m					
8	Rotating inertia		0.004	kg-m²					
9	Preload/Rotary friction/Drag torque		0.5	Nm					
10									
11	Segment No	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External F
12		(m/s)	(m/s)	(s)	(m/s²)	(m)		(kg)	(N)
13	1	0	0.2	0.2					
14	2	0.2	0.2	2.3					
15	3	0.2	0	0.2					
16	4	0	-0.4	0.4					
17	5	-0.4	-0.4	0.85					
18	6	-0.4	0	0.4					
19									
20	Equivalent Radius		m/rad						
21	Peak Force		N						

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6.5: Tangential Mechanisms

There are a number of different rotary-to-linear mechanisms which can be grouped into the Tangential category, the common factor being that the conversion between rotary and linear motion occurs at the circumference of a pulley or a wheel. The calculations needed to deal with all these mechanisms are very similar.

Rack and pinion

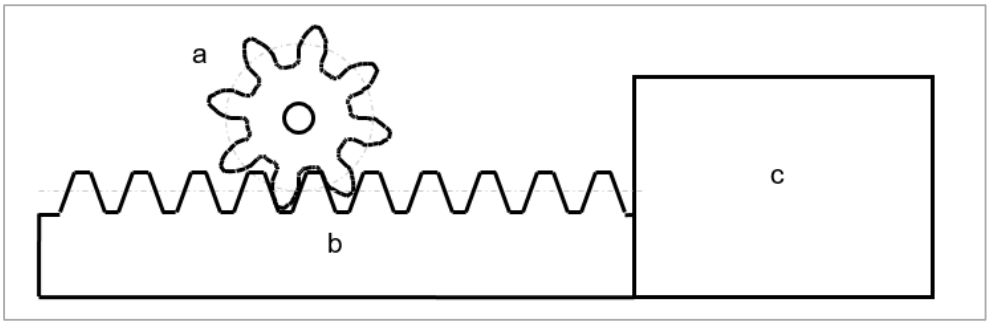


Figure 5 - Rack & Pinion

There will inevitably be some backlash between the rack and pinion which may limit the dynamic performance; this aspect is dealt with in the Advanced Course.

It is possible to have two motors each driving a pinion engaged with the rack. The two are arranged to provide opposing torques to take up any backlash; this is somewhat analogous to a class AB amplifier, or to the preload in a Ball screw.

Equivalent Radius

$$R_{eq} = Pitch\ Circle\ Diameter/(2\pi)$$

or

$$R_{eq} = Teeth \times Module/(2\pi)$$

Belt drive

This includes conveyor systems, and belt (or chain) driven actuators. A belt driven actuator is essentially a conveyor with a table and slide mechanism attached to the belt; this obviously limits the travel as with lead screw or ball screw actuator. Belt-driven actuators are not as mechanically stiff as a screw-type, but can achieve much higher speeds.

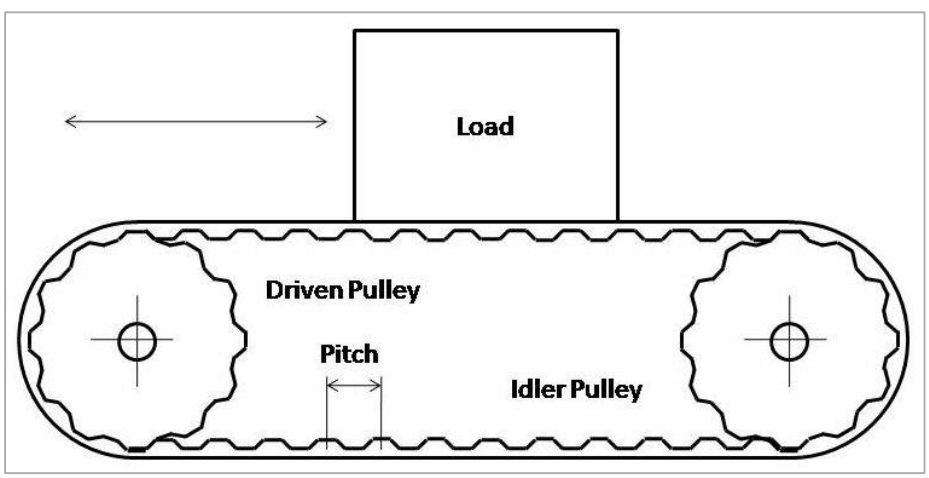


Figure 6 – Toothed Belt

Note that smooth or V belts are not used for servo axes because the position feedback (monitoring) is normally taken from the motor – any slippage of the belt would not be detected, causing position errors. The belt must be tensioned to provide sufficient stiffness.

Equivalent Radius

$$R_{eq} = Pitch \times Number\ of\ Teeth/(2\pi)$$

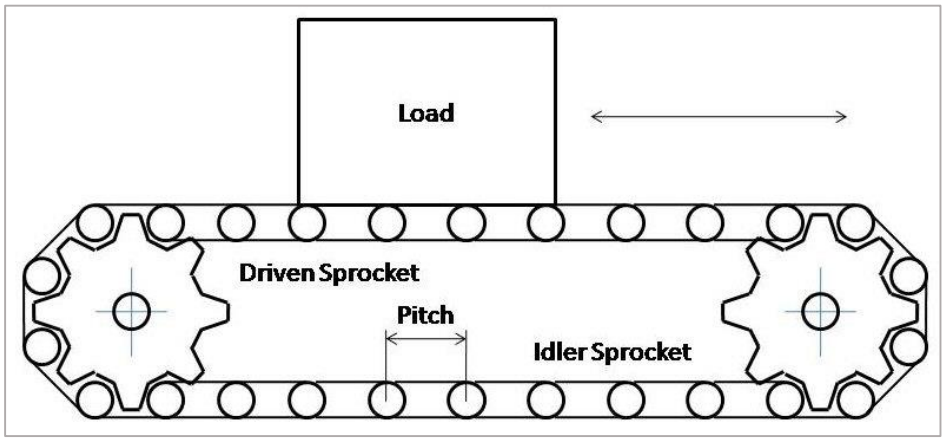


Figure 7 – Chain & Sprocket

Not common in servo systems but sometimes used in lower speed higher stiffness applications. A tensioner is required to eliminate backlash.

Note that there is some unevenness in the drive as each roller engages with the teeth of the sprocket so is not appropriate when smooth movement is critical.

Equivalent Radius

$$R_{eq} = Pitch \times Number\ of\ Teeth/(2\pi)$$

Additional loads and losses

For rack and pinion actuators, as with the screw-type actuators, it is necessary to add all the linearly moving masses together to obtain a total mass and to evaluate all the sources of sliding friction to obtain a total friction loading. Externally applied forces such as gravity will also require evaluation as in Chapter 6.3 - Losses and Gravity Loading. Then once these have been converted to an equivalent inertia, the inertia of the pinion must be added. Likewise, once the friction force is converted to a torque, and rotary bearing losses must be added.

In the same way, for a belt or chain-type actuators where there is a table and slides, the table will add to the payload to give a total linear moving mass. However, there is a subtlety where the mass of the belt or chain is concerned. They also add to the moving mass, but not to the moving **weight**, as they effectively are balanced on opposite sides on the complete mechanism. This effect must be handled correctly when the linear moving path is inclined or vertical.

Again, once moving mass is converted to inertia, the combined inertia of the pulley system must be added; here there is another subtlety if the pulleys are not all the same size. This is dealt with in the advanced course – for this course, we will assume that there are only two pulleys and that they are identical, as is often the case.

Again, once the linear friction forces are converted to rotary, any bearing losses at the pulley bearings must be added.

The Linear System

Again, we can create a table of data which affects movement. This is a suitable table for a belt-type mechanism; it would be different for a rack mechanism: -

Belt example data		
Data	**Value**	**Units**
Table mass	20	kg
Belt mass	1	kg
Slideway friction coefficient	0.01	per unit
Inclination	45	degrees
Number of Teeth on drive pulley	20	teeth
Tooth pitch	0.005	m
Total Rotating inertia	0.005	kg-m^2
Rotary friction loss	0.1	Nm

Table 5

We apply an associated motion profile, such as this (note the linear units since the profile is specified at the table/payload). The table moves forward, dwells, then returns to the start position followed by a further dwell.

Segment No.	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]
	(m/s)	(m/s)	(s)	(m/s^2)	(m)
1	0	0.2	0.09	2.22	0.009
2	0.2	0.2	0.075	0	0.015
3	0.2	0	0.085	-2.35	0.0085
4	0	0	0.05	0	0
5	0	-0.2	0.09	-2.22	-0.009
6	-0.2	-0.2	0.075	0	-0.015
7	-0.2	0	0.085	2.35	-0.0085
8	0	0	0.05	0	0

Table 6

Now we can complete the table of linear loading. We need to be careful in the case of the belt-type mechanisms about what constitutes the Total Linear Mass – it's a matter of remembering that Mass and Weight are different quantities. When calculating the friction due to gravity, the weight of the belt will generally not contribute to the friction loss since it isn't part of the table. Also, it balances around the complete mechanism so it also doesn't contribute to the gravitational loading in vertical movement. However, it **is** part of the Total Linear Moving Mass which needs to be accelerated during motion. We usually assume that positive velocity is up the slope and forces that oppose the motion are positive.

Mass

The payload could vary from segment to segment – for example, if the payload is a work piece which is ejected from the machine at some point in the motion cycle. It is simply added to the table mass where appropriate.

In this case there is no "payload".

$$Mass = Table\ Mass + Belt\ Mass$$

$$Mass = 20 + 1 = 21\ kg$$

External Force

In this case gravity is the only external force.

$$External\ Force = Gravity\ Force = Mass \times g \times sin(inclination)$$

$$e.g.\ External\ Force = 21 \times 9.81 \times 0.7071 = 145.67\ N$$

Friction

In this case we need to account for the inclination of the slide.

$$Friction\ Force = Mass \times Coeff\ of\ Friction \times g \times cos(inclination) \times Sign\ of\ Direction$$

$$e.g.\ Friction\ Force\ \ = \ 21 \times 0.01 \times 9.81 \times 0.7071 \times 1\ \ = \ 1.457\ N$$

Friction always opposes the direction of motion so, in spreadsheets, we need to apply a "Sign of Direction" ± 1 to the friction force that mirrors the sign of the velocity. We also assume that the friction force is zero at zero speed so "Sign of Direction" becomes zero. To create a "Sign of Direction" column in MS Excel use this function "$= IF(E3 > 0,1, IF(E3 < 0, -1,0))$" where $E3$ is the cell containing the distance (s) in row 3. Search for excel nested if statements for further information.

Segment No	Initial Velocity [u] (m/s)	Final Velocity [v] (m/s)	Time [t] (s)	Acceleration [a] (m/s²)	Distance [s] (m)	Sign of direction	Mass (kg)	External Force (N)	Friction Force (N)
1	0	0.2	0.09	2.22	0.009	1	21	146	1.46
2	0.2	0.2	0.075	0	0.015	1	21	146	1.46
3	0.2	0	0.085	-2.35	0.0085	1	21	146	1.46
4	0	0	0.05	0	0	0	21	146	0
5	0	-0.2	0.09	-2.22	-0.009	-1	21	146	-1.46
6	-0.2	-0.2	0.075	0	-0.015	-1	21	146	-1.46
7	-0.2	0	0.085	2.35	-0.0085	-1	21	146	-1.46
8	0	0	0.05	0	0	0	21	146	0

Table 7

The optional Total Force column is added when, say, the belt tension needs to be checked. The peak would represent the absolute minimum belt tension required to prevent sagging. In practice, much higher tensions are usually required to achieve sufficient stiffness, the importance of which will be covered in the advanced course.

$$Total\ Force = Acceleration\ Force + External\ Force + Friction\ Force$$

$$Total\ Force = (Mass \times Acceleration) + External\ Force + Friction\ Force$$

$$Total\ Force = (21 \times 2.22) + 145.66 + 1.457 = 193.8\ N$$

This column shows that:-

- The peak force of 193.8 occurs in segment 1.
- The torque at constant speed in segment 2 is higher than that at constant speed in segment 6 due to the friction acting in opposite directions.
- This is a case where gravity dominates the system: a smaller motor may be possible if the load were counter-balanced – this is covered in the advanced course.

The Equivalent Rotary System

Now we must transform table 7 data into the equivalent rotary system, whilst adding in the rotary components of the belt drive:-

Equivalent Radius (R_{eq})

The "pitch" of the rack/belt/chain must be established first. The machine manufacturer will normally be able to supply this information. If the parts are manufactured in Europe they will be metric so the pitch will normally be a whole number of millimetres. American components will, typically, have a whole number of teeth per inch.

Then the equivalent radius is given by

$$R_{eq} = Pitch \times Number\ of\ Teeth/(2\pi)$$

$$R_{eq} = 0.005 \times 20/(2\pi) = 0.0159\ m$$

Sometimes the Pitch Circle Diameter (PCD) is specified, then

$$R_{eq} = PCD/2$$

Do not attempt to measure the PCD as it is not possible without specialist equipment.

It is very important to know the units of the Equivalent Radius, to make sure you know what units to use on the rotary side of the calculations.

Profile

$$u\ (rotary) = u\ (linear)\ /\ R_{eq}$$

$$v\ (rotary) = v\ (linear)\ /\ R_{eq}$$

$$e.g.\ v\ (rotary) = 0.2\ /\ 0.0159 = 12.58\ rad/s$$

The time column does not change, so we can compute the other two columns by either using the standard motion equations or by using the ratio again, which will be easier.

$$a\ (rotary) = a\ (linear)/\ R_{eq}$$

$$s\ (rotary) = s\ (linear)/\ R_{eq}$$

Load Inertia

Comprises the effective inertia of the Table & Belt masses plus the pulley inertia.

$$Total\ \ Inertia = (Table\ Mass + Belt\ Mass) \times {R_{eq}}^2 + Pulley\ Inertias$$

$$e.g.\ Total\ \ Inertia = 21 \times 0.0159^2 + 0.005 = 0.0103\ kgm^2$$

Load External Torque

$$External\ Torque\ =\ External\ Force \times R_{eq}$$

$$e.g. External\ Torque\ =\ 145.67 \times 0.0159\ =\ 2.32\ Nm$$

Load Friction Torque

Comprises friction due to the linear slide plus rotary friction loss.

$$Friction\ Torque\ =\ Friction\ Force \times R_{eq} + Friction\ Torque \times Sign\ of\ Direction$$

$$e.g.\ Friction\ Torque\ =\ 1.457 \times 0.0159\ +\ 0.1 = 0.123\ Nm$$

Segment No	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia	Accel. Torque	Load external torque	Loa
	(m/s)	(m/s)	(s)	(m/s^2)	(m)	($kg\text{-}m^2$)	(Nm)	(Nm)	
1	0	12.6	0.09	140	0.566	0.0103	1.44	2.32	
2	12.6	12.6	0.075	0	0.943	0.0103	0	2.32	
3	12.6	0	0.085	-148	0.534	0.0103	-1.52	2.32	
4	0	0	0.05	0	0	0.0103	0	2.32	
5	0	-12.6	0.09	-140	-0.566	0.0103	-1.44	2.32	
6	-12.6	-12.6	0.075	0	-0.943	0.0103	0	2.32	
7	-12.6	0	0.085	148	-0.534	0.0103	1.52	2.32	
8	0	0	0.05	0	0	0.0103	0	2.32	

Table 8

Completing the sizing exercise

- Table 8 is **identical in layout** to Table 3 in Chapter 3.4 - Computing the Load Torque. The following processes for adding a motor (and, possibly, a gearbox) and optimising the transmission ratio are now exactly the same as for a rotary load. Even complex loads may be transposed and simplified in this way making the whole sizing exercise easier.
- Note that changing the effective radius has the same effect as changing the transmission ratio. Sometimes a change of pinion/pulley/sprocket diameter will avoid having to add a gearbox or allow a smaller motor.
- The inertia column is the total inertia seen by the motor, which will vary if the load mass changes (none in this case). This variation can have subtle effects on the tuning of the control system, as explained in the advanced course.

Test your understanding

Question 1 of 2 (1) (2)

A vertical lift is operated using a belt. The lift raises a product vertically through 150mm in 0.6 seconds. The product is then removed, taking 0.4 seconds, and the lift immediately returns to its lower level in 0.3 seconds before waiting another 0.4 seconds for the next product to be loaded. The operation repeats continuously. The product weighs 25 kg. Given below is the data for the belt. Compute the Linear Table of the axis.

A1:D1 | fx Lift

	A	B	C	D	E	F	G	H	I
1	Lift								
2	Data		Value	Units					
3	Table mass		5	kg					
4	Belt mass		3	kg					
5	Slideway friction coefficient		0.05	per unit					
6	Inclination		90	degrees					
7	Number of teeth on drive pulley		20	teeths					
8	Tooth pitch		0.01	m					
9	Total Rotating Inertia		0.01	kg-m²					
10	Rotary Friction Loss		0.5	Nm					
11									
12	Segment No	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External Forc
13		(m/s)	(m/s)	(s)	(m/s²)	(m)		(kg)	(N)
14	1	0	0.375	0.2					
15	2	0.375	0.375	0.2					
16	3	0.375	0	0.2					
17	4	0	0	0.4					
18	5	0	-0.75	0.1					
19	6	-0.75	-0.75	0.1					
20	7	-0.75	0	0.1					
21	8	0	0	0.4					
22									
23	Equivalent Radius		m/rad						
24	Peak Force		N						

Check My Work | Reset

6.6: Summary of key Points

What we have seen is that any mechanism which converts between rotary and linear motion can be analysed in two parts, the linear part and the rotary part. We can then convert the

motion and force information from linear to rotary by using a simple concept, the equivalent radius, which applies at the exact point where the conversion takes place.

We have studied two common types of mechanism for linear to rotary conversion, but as we mentioned more types of mechanism exist, such as crank mechanisms and four-bar linkages. While these are non-linear (i.e. the equivalent radius varies as the mechanism moves), the same principle applies. This is all examined in more detail in the Advanced Course, and the COSMATT tool can be used to speed analysis.

It should also be noted that in the two examples above, we used the same motion profile and load. However, because of the nature of the mechanisms, the screw example requires a realistic motor speed of about $1000\ rpm$, while the belt example only peaks at about $40\ rpm$ and would probably need a gearbox to match to a motor. Nevertheless, the torque requirement of the belt mechanism is not a great deal more than the screw mechanism, despite the much lower speed, so the power utilisation and hence the efficiency is much greater. After gearing to obtain a suitable motor speed, a much smaller motor could be used.

Don't forget that inertia matching between the load-side and the motor-side can also be important as we saw in Chapter 4.5 – Introducing the T-S Widget. The quality of the transmission mechanism is an important factor – a system with low or zero backlash and high stiffness is necessary to prevent instability in the control system, particularly if the load side inertia is dominant. In the examples, the inertia of the belt mechanism as reflected to the motor is much greater unless reduced by gearing on the rotary side. The accuracy of the mechanical system may also be an issue as the feedback to the control system will usually be taken from the encoder in the motor. In machines where very high accuracy is required a linear encoder may be added to overcome transmission errors, but these are costly.

In the case of the non-linear mechanisms, the ratio of load-side to motor-side inertia will depend on the position of the mechanism since the Effective Radius is variable. This causes a number of subtle but significant problems, which are dealt with in the Advanced Course.

Chapter 7 – Application Examples

7.1: Introduction

By now you know how to evaluate a simple application requirement and choose a servo motor which is capable of achieving the mechanical requirements of the application. In this final chapter we will work through some typical examples which frequently appear in industrial applications. Before that we will recap some of the factors you need to take into account when designing your system solution.

Servo systems are typically optimised for one or more of three common factors.

- **COST**: Servo systems represent significant cost, so machine designers seek to minimise this cost so as to be competitive suppliers.
- **RELIABILITY & ROBUSTNESS**: Sometimes Large numbers of servos are used on a single manufacturing line so reliability and robustness are essential as downtime is very expensive. Robustness refers to the ability to cope with changes in power supply, temperature, machinery wear, and sometimes product changes.
- **PERFORMANCE**: Productivity is improved by decreasing cycle time at bottleneck stations on a production line.

Let's review the major choices in designing an application solution:

1. **Direct Drive**:
   Direct drive is the simplest and most reliable but inertia ratio / stability may be an issue. However, compliance and backlash issues are minimised. Of course, the cost of transmission components can be avoided, but the motor may be larger and costlier than it could be if gearing were used.

2. **Geared Systems**:
   Gearing often helps utilize the motor's full speed range and has several advantages:
   1. May reduce cost by using smaller motors and drives despite the cost of gearing.
   2. May allow higher performance and shorter cycle times.

3. May offer an improved inertia ratio and hence stability and robustness. Note that compliance (softness) or backlash (play) in the gearing will tend to degrade stability.
4. Applications are often not possible ungeared.
5. Below are types of Gearing:
    a. Gearbox: Reliable and long life: expensive: have high / very high losses that need more torque and power: worn gearboxes can cause problems: ratios up to $1000:1$.
    b. Toothed belt: Low cost: shorter life: ting tension may be tricky: has compliance: ratios < 10 per stage.
    c. Chain and sprocket: medium cost: longer life: lubrication required: needs tensioner to avoid backlash: ratios < 7.

3. **Typical Applications**:
   Many applications can be divided into one or more of these groups but can cover a wide range of sizes depending on the product and speed.
   a. Pick and place
   b. Rotary turret
   c. Transfer mechanism
   d. Stacker
   e. Packaging
   f. Machining
   g. Web handling

Each servo drive system is referred to as an “axis”. Bearing these factors in mind, we will now look at a typical complex production line.

7.2: Example Application

Example: Typical Complex Production Line

As part of a production line a constant speed belt conveyer feeds components to the Infeed Belt. The component is detected by a photo cell and the belt decelerates to an accurate position. A “pick and place” mechanism moves the component to Station 1 of a rotary indexing table. At Station 2 a stud is friction welded to the component. At Station 3 a plastic cap is placed over the stud to protect the thread. At Station 4 a second “pick and place” mechanism passes the component to the outfeed belt which, in turn, passes the component on to another constant speed conveyor belt.

The “pick and place” mechanisms each consist of two axes- Lift and Traverse.

In this welding process a stud is rotated at high speed then forced into contact with the component until friction creates enough heat to melt the metal surfaces.

There are seven different servo applications involved in the process (we are assuming that the pairs of Lift, Traverse and Belt axes are identical for sizing purposes). This has been simplified for training purposes.

1. Belt conveyer (stop to deliver / remove components)
2. Lift (components from belt and place on table and vice versa)
3. Traverse (moves the lift mechanism between the belt and the table)
4. Indexing Rotary table (90° steps so as to stop at load, weld, trim and unload stations)
5. Friction Welder rotation (high friction torque heats the component parts till they fuse together)
6. Welder Feed (applies high pressure for the welding)
7. Trimmer (feeds a tool in to clean any flashing from the welding process)
8. Lift (components from belt and place on table and vice versa)
9. Traverse (moves the lift mechanism between the belt and the table)
10. Belt conveyer (stop to deliver / remove components)

Below is diagram for Index Table and infeed/outfeed belts (shown at 1 second in the diagram in Figures 3 & 4): -

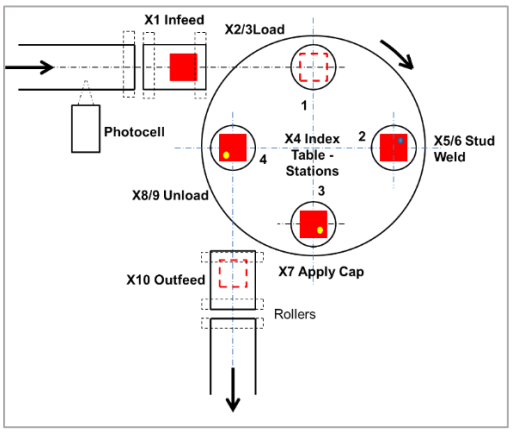


Figure 1

Side view of infeed belt and lift/traverse (pick and place) axes:

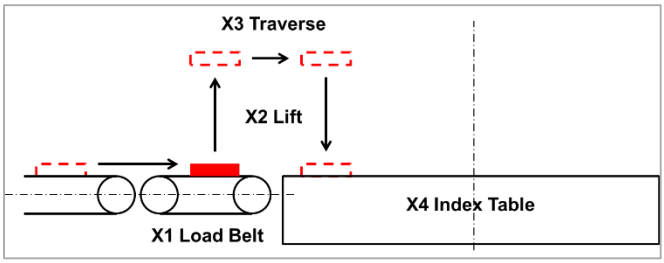


Figure 2

The required production rate is 15 parts per minute, i.e. the cycle time for each axis will be four seconds. Often an OEM will provide a Timing Diagram to aid in understanding the interdependencies between axes.

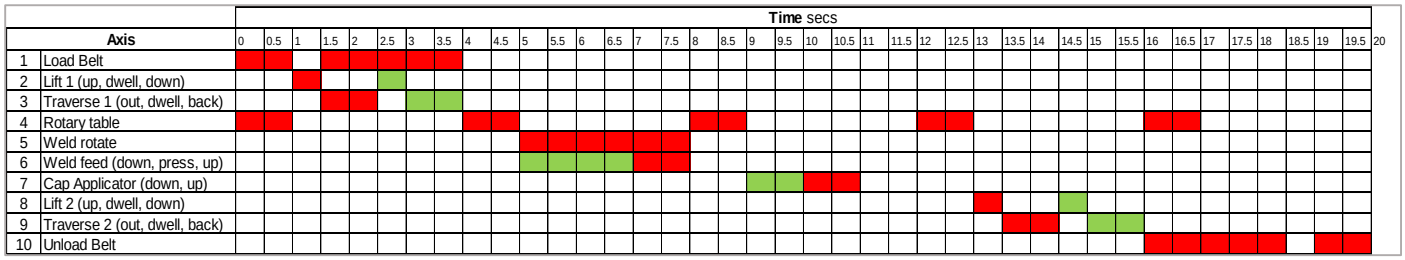


Figure 3 - Timing Diagram for a single component

Below is the timing diagram compressed into one cycle.

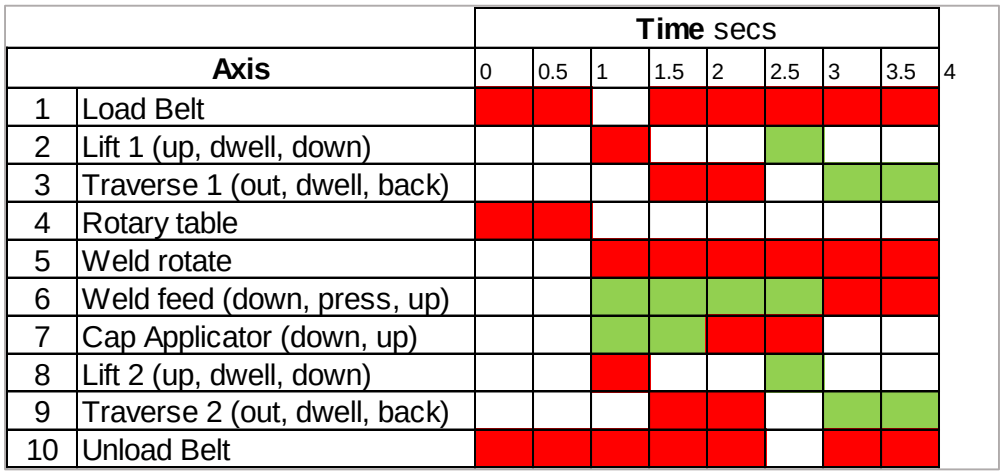


Figure 4 - Timing Diagram (compressed into one cycle)

The diagram shows, for example, that The Load belt must stop before Lift 1 can start upward, Traverse 1 must wait for Lift 1 to stop before it can move and so on.

Note that a real application would be a little more complex – for instance, after the belt stops a small period of time will be needed to actuate the gripper device (typically pneumatically operated), similarly after the Lift 1 axis stops at the bottom the gripper needs to be released.

These applications will be studied in the next few sections.

7.3: Axis 1: Infeed Belt

Infeed - a belt, sliding on a flat surface, accepts a component from a non-stop belt (supplied by others) and stops to allow the pick and place by lifter to remove it.

A subtle feature of this application is that the 45 kg load is passed to this belt from another running at the same speed. If it were dropped onto the belt from a standstill its mass would need to be accelerated by the belt which would cause a "pulse" of torque. This will be covered in the advanced course.

Axis Specification

- Delivery Belt speed: 0.3 m/s
- Dwell time: 0.5 sec (for pick up or drop)
- Mass of component: 45 kg
- Coefficient of friction: 0.1
- Mass of belt: 5 kg
- Inertia of rollers and shafts: 0.00156 kg-m^2
- Diameter of drive roller: 50 mm
- Rotary losses: 0.2 Nm

First, let's figure out the motion cycle requirement: -

- The peak speed must match the delivery belt at 0.3 m/s
- The belt should accelerate and decelerate in 0.5 s
- Dwell time 0.5 s
- Total move time is thus 3.5 s (2.5 s cruise)
- Move distance is not important

Note that the component is transferred to this belt smoothly followed very soon by deceleration to zero so the component mass is only significant during deceleration (acceleration for the outfeed belt).

This data can be gathered into tables to describe the linear requirements: -

Mechanical Parameters		
Name	**Value**	**Unit**
Component Mass	45	kg
Belt Mass	5	kg
Friction Coefficient	0.1	per unit
Inclination	0	degrees
Roller Diameter	0.05	m
Total Rotating inertia	0.00156	kg-m^2
Rotary Friction Loss	0.2	Nm

Table 2

Profile Parameters		
Name	**Value**	**Unit**
Acceleration/Deceleration time	0.5	s
Cruise Time	2.5	s
Dwell Time	0.5	s
Peak Velocity	0.3	m/s

Table 3

Now we can create a Linear Motion profile table here from the Mechanical and Profile tables: -

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External Force	Fri
	(m/s)	(m/s)	(s)	(m/s^2)	(m)		(kg)	(N)	
1	0	0.3	0.5	0.6	0.075	1	5	0	
2	0.3	0.3	2.5	0	0.75	1	5	0	
3	0.3	0	0.5	-0.6	0.075	1	50	0	
4	0	0	0.5	0	0	0	5	0	
Total			4		0.9				

Table 4

From above table following points can be noticed: -

1. The component is only on the belt during segment 3, deceleration (plus a small period of cruise that can be ignored).
2. Strictly, only half the belt mass should be included in the weight for the friction calculation but is small enough to be ignored.
3. The optional Total Force column allows the belt rating to be checked.
4. The times and distances are summed to check they reach the correct total – helps to eliminate errors.

The equivalent radius is the radius of the primary roller, i.e. the drive roller being driven by the motor, which has a radius of 0.025 m.

$$\text{Equivalent Radius} = 0.025 \text{ m}$$

We can then create the rotary profile data table using the equivalent radius: -

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque [T_{ext}]	Frictio
	(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)	(kg-m^2)	(Nm)	
1	0	12	0.5	24	3	0.00469	0	
2	12	12	2.5	0	30	0.00469	0	
3	12	0	0.5	-24	3	0.0328	0	
4	0	0	0.5	0	0	0.00469	0	
Total			4		36			

Table 5

T-S Widget Motor Selection

At this point we could generate the required data for the T-S widget (the optional columns above will be require

T-S Widget Data		
Segment	Total Torque^2 * Time [$T_{tot}^2.\Delta t$]	Acceleration^2 * Time [$a^2.\Delta t$]
1	0.0946	288
2	0.26	0
3	0.204	288
4	0	0
Total	0.559	576

Table 6

We gather the requisite data together: -

Torque-Speed Requirements	Value	Unit
Peak Torque	0.639	Nm
RMS Torque	0.374	Nm
Peak Speed	115	rpm
Peak Acceleration	24	rad/s^2
RMS Acceleration	12	rad/s^2

Table 7

Feed these values into the T-S widget.

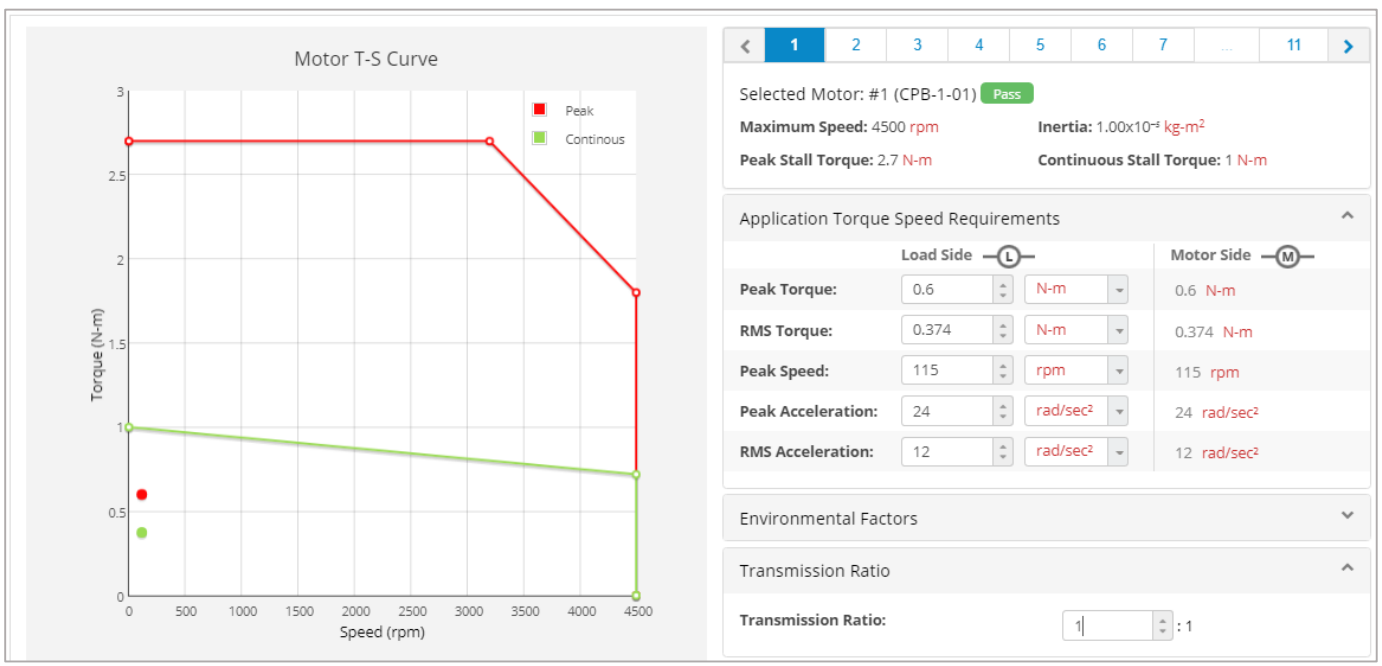


Figure 5

Selecting motor # 1, the very low speed indicates that some gearing could be useful to reduce the inertia ratio, which is very high (the highest load inertia is 0.0328 kg-m^2, and the motor is only 0.00001 kg-m^2). To keep cost down, we'll try to use a belt transmission, using the highest ratio we can, say 5:1.

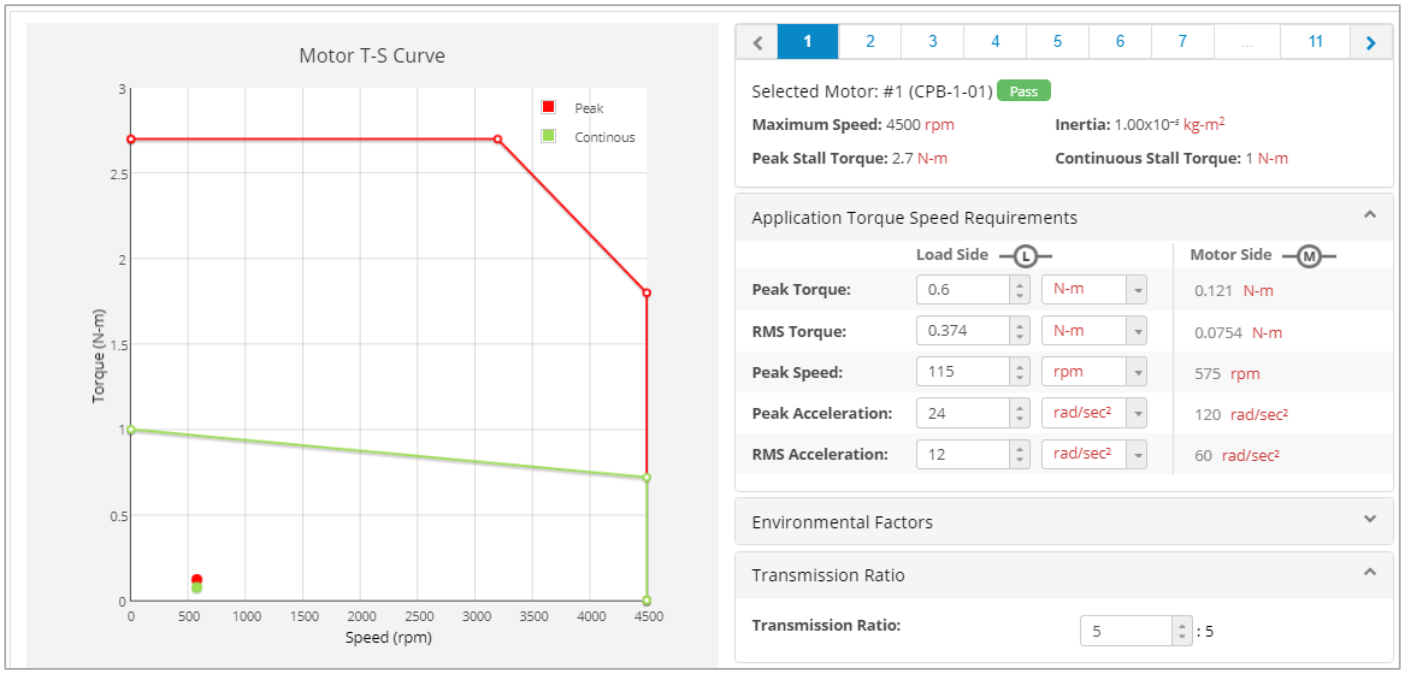


Figure 6

Because the application is well within the capability of the motor we won't go on to evaluate the additional penalty of the inertia and losses of the belt transmission for this axis, but in a practical situation that would be necessary as part of the final design. We'll do that for one of the later axes.

Manual Motor Selection

Now we can refer the data to a sample motor taking into account the motor's own inertia.

Motor Data (CPB-1-01)		
Type	Value	Unit
Maximum Speed	4500	rpm
Peak Torque	2.7	Nm
RMS Torque	1	Nm
Inertia	1.00×10^{-5}	kg-m^2

	Rotary Motion - Motor Side							
Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [TL]	Load Inertia [JL]	Total Inertia [Jtot]	Acc
	(rpm)	(rpm)	(s)	(rad/s^2)	(Nm)	(kg-m^2)	(kg-m^2)	
1	0	115	0.5	24	0.323	0.00468	0.0047	
2	115	115	2.5	0	0.323	0.00468	0.0047	
3	115	0	0.5	-24	1.43	0.0328	0.0328	
4	0	0	0.5	0	0	0.00468	0.0047	
Total			4					
RMS Torque	0.374							
J Ratio Minimum	468	: 1						
J Ratio Maximum	3280	: 1						
J Range	7	: 1						

Table 8

Note that both the maximum value of load/motor inertia ratio and the variation through the cycle will limit the attainable stiffness and dynamic response which could be a problem. This subject is dealt with in detail in the Advanced Course.

We now have the data required to verify the motor from manufacturer's data: -

1. RMS torque – 0.374 Nm (Motor RMS torque – 1.0)
2. Peak torque – 0.639 Nm (Motor Peak torque – 2.7)
3. Peak speed – 115 rpm (Motor Peak speed – 4500)

Clearly the motor is capable of driving the load through the specified profile.

But the load/motor inertia ratio, at over 3000:1, is extremely high and could be difficult to tune. The low peak motor speed of 115 rpm is also low, not making good use of the motor characteristics. Both these parameters could be improved by adding a transmission ratio. Try the effect of changing the transmission ratio in the table above.

This subject is covered in detail in the advanced course.

Note – The T-S Widget will, typically, over-estimate the RMS torque a little due to the shortcut of using RMS acceleration. It should not be relied upon for serious sizing but is a useful tool to quickly show the effect of changing the gearing ratio and homing in on the optimum motor. The manual method should always be used for a final check. COSMATT is a rigorous sizing tool under development that will automate all the calculations described in this course.

We will try adding a gearing ratio of 5:1 as we did in the T-S widget: -

Transmission Data		
Ratio*	5	: 1
Transmission Inertia	0	kg-m²

* Try the Effect of Changing the Transmission Ratio

Motor Data (CPB-1-01)		
Type	Value	Unit
Maximum Speed	4500	rpm
Peak Torque	2.7	Nm
RMS Torque	1	Nm
Inertia	1.00×10^{-5}	kg-m²

					Rotary Motion - Motor Side		
Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]	Total Inertia [J_{to}
	(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)	(kg-m²)
1	0	573	0.5	120	0.0645	0.000187	0.000197
2	573	573	2.5	0	0.0645	0.000187	0.000197
3	573	0	0.5	-120	0.285	0.00131	0.00132
4	0	0	0.5	0	0	0.000187	0.000197
Total			4				
RMS Torque	0.0747						
J Ratio Minimum	18.7	: 1					
J Ratio Maximum	131	: 1					
J Range	7	: 1					

Table 9

Now we are able to check the motor rating: -

1. RMS torque – 0.0754 Nm (Motor RMS torque – 1.0)
2. Peak torque – 0.127 Nm (Motor Peak torque – 2.7)
3. Peak speed – 573 rpm (Motor Peak speed – 4500)

Even less arduous for the motor – a smaller motor would be considered if one were available.

Note how much lower the load/motor inertia ratio is – this would make tuning easier but it is still high. Try increasing the gearing ratio to see what could be achieved. However, in practical terms this would mean using a gearbox rather than a belt transmission, with associated cost implications.

An alternative way to reduce the inertia ratio would be to use a larger motor (or a medium inertia motor from another range). Any part of the transmission that is rigidly coupled to the motor will effectively raise the motor inertia with the same benefits. Note also that the inertia ratio changes by a factor of about 7:1 through the cycle which can be a problem when tuning. Again, these points are covered in the Advanced Course.

The utilisation is so low that a lower cost alternative, such as a stepper drive, might be considered. The high inertia ratio has the potential to make tuning difficult, so an open-loop stepper (which does not require tuning) could be a better solution.

7.4: Axis 2: Component Lift

The component is lifted from the belt and then replaced at the same height on the rotary table by a rack and pinion mechanism.

Axis Specification

- Lift Height: 250 mm
- Move time: 0.5 sec each way. Since the cycle time of the machine is 4 seconds that means a dwell of 1.5 seconds after each move
- Pinion PCD: 20 mm
- Pinion inertia: 0.00000781 kg-m^2
- Rack and moving mass: 2 kg
- Component mass: 45 kg
- Rotary friction: 0.1 Nm

Mechanical Parameters		
Name	**Value**	**Unit**
Component Mass	45	kg
Slide Mass	2	kg
Friction Coefficient	0.1	per unit
Inclination	90	degrees
Pinion PCD	0.02	m
Pinion Inertia	7.81×10^{-6}	kg-m^2
Rotary Friction Loss	0.1	Nm

Table 10

Profile Parameters		
Name	**Value**	**Unit**
Move Distance	0.25	m
Move Time (up)	0.5	s
Average Velocity	0.5	m/s
Peak Velocity	0.75	m/s
Dwell Time	1	s
Move Time (down)	0.5	s
Average Velocity	-0.5	m/s
Peak Velocity	-0.75	m/s
Dwell Time	2	s

Table 11

As in the first axis, the initial step is to create the profile. We can again use the profile widget to get some initial values, but this is an up/down move so we will need to extend the profile when populating the linear data table.

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External Force	Fric
	(m/s)	(m/s)	(s)	(m/s²)	(m)		(kg)	(N)	
1	0	0.75	0.167	4.5	0.0625	1	47	461	
2	0.75	0.75	0.167	0	0.125	1	47	461	
3	0.75	0	0.167	-4.5	0.0625	1	47	461	
4	0	0	1	0	0	0	47	461	
5	0	-0.75	0.167	-4.5	-0.0625	-1	47	461	
6	-0.75	-0.75	0.167	0	-0.125	-1	47	461	
7	-0.75	0	0.167	4.5	-0.0625	-1	47	461	
8	0	0	2	0	0	0	2	19.6	

Table 12

The equivalent radius is half the PCD of the pinion, 10 mm or 0.01 m.

$$\text{Equivalent Radius} = 0.01 \text{ m}$$

This gives us the rotary data table thus: -

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque [T_{ext}]	Friction
	(rad/s)	(rad/s)	(s)	(rad/s²)	(rad)	(kg-m²)	(Nm)	(
1	0	75	0.167	450	6.25	0.00471	4.61	
2	75	75	0.167	0	12.5	0.00471	4.61	
3	75	0	0.167	-450	6.25	0.00471	4.61	
4	0	0	0.167	0	0	0.00471	4.61	
5	0	-75	0.167	-450	-6.25	0.00471	4.61	
6	-75	-75	0.167	0	-12.5	0.00471	4.61	
7	-75	0	0.167	450	-6.25	0.00471	4.61	
8	0	0	2	0	0	0.000208	0.196	

Table 13

T-S Widget Motor Selection

At this point we could generate the required data for the T-S widget (the optional columns above will be required): -

T-S Widget Data		
Segment	**Total Torque^2 * Time [$T_{tot}^2.\Delta t$]**	**Acceleration^2 * Time [$a^2.\Delta t$]**
1	7.77	33800
2	3.7	0
3	1.12	33800
4	21.3	0
5	0.954	33800
6	3.39	0
7	7.32	33800
8	0.077	0
Total	45.6	135000

Table 14

Torque-Speed Requirements	Value	Unit
Peak Torque	6.83	Nm
RMS Torque	3.38	Nm
Peak Speed	716	rpm
Peak Acceleration	450	rad/s^2
RMS Acceleration	184	rad/s^2

Table 15

Using direct drive, the smallest possible motor (with no margin at all) is #5, so we would probably select #6 or even #7 to give a robust solution.

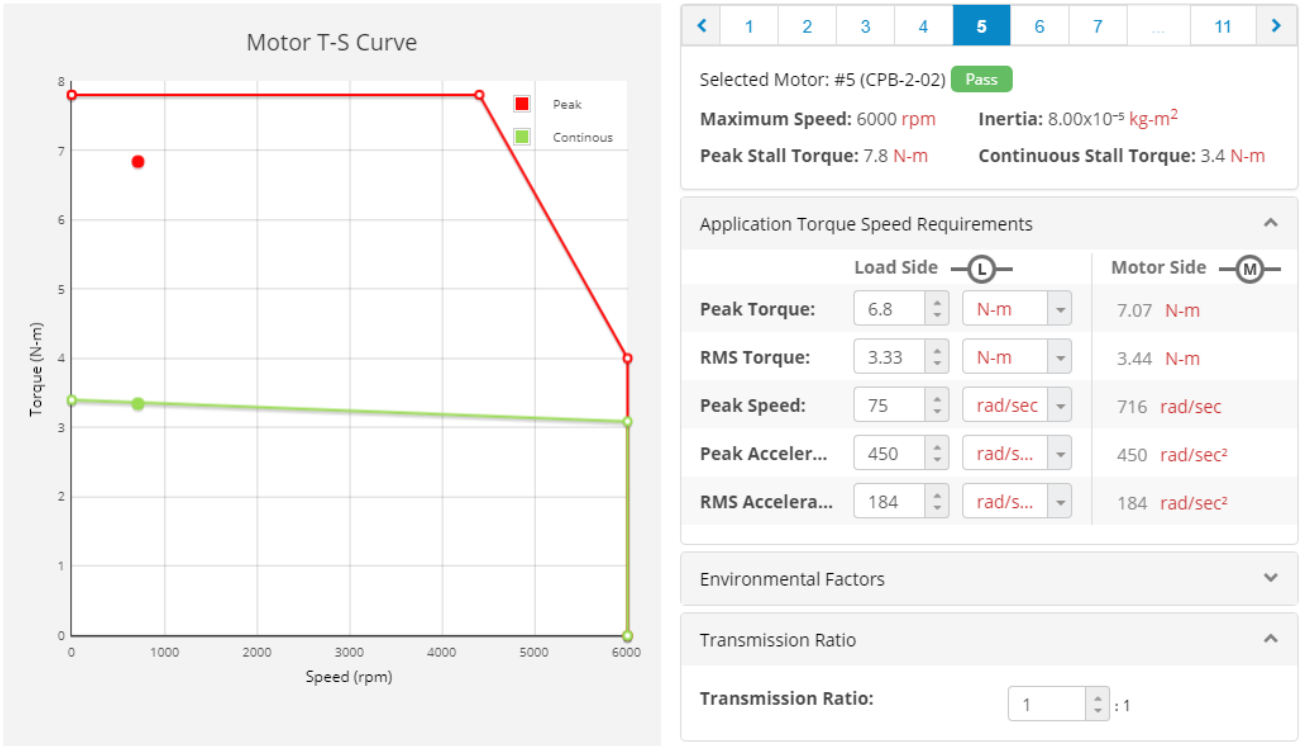


Figure 7

Again, we would immediately consider gearing, and it is possible to use a ratio of 6:1 to allow the use of motor no. 1. However, 6:1 ratio would probably necessitate using a gearbox which could be expensive. It would probably be more economic to use a lower ratio with motor no. 2, say 4:1, using a pulley-type transmission.

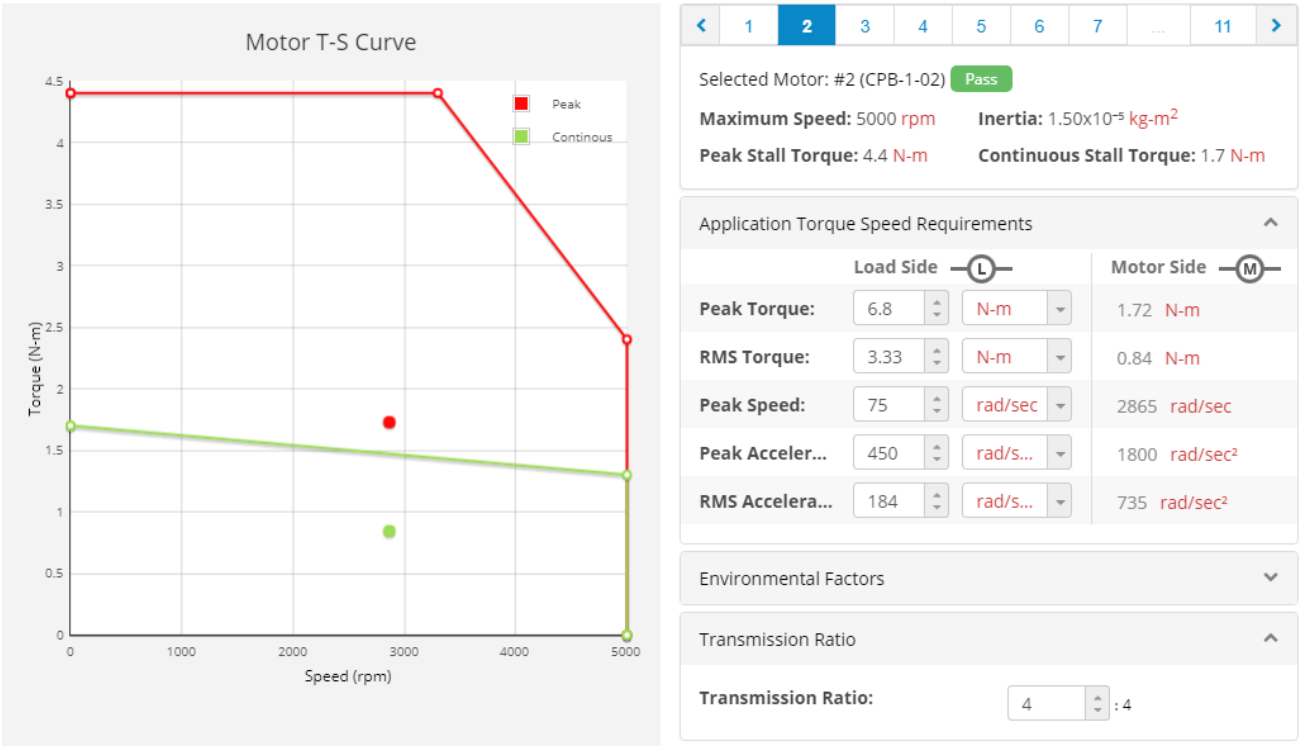


Figure 8

This time we will evaluate the pulley system for the additional loading due to its inertia. Note that the RMS torque after gearing and adding the motor inertia is 0.84 Nm.
The inertia of the pulley system is 0.00032 kg-m^2 so since after gearing, the RMS acceleration is 735 r/s^2 and the additional RMS torque is (735 * 0.00032) = 0.24 Nm, bringing the total RMS torque up to 1.08 Nm. This will not bring the motor loading too close to the RMS limit line (green in the T-S chart above), so the solution is acceptable.
However, there is a problem with this method of computing the additional torque, which you were warned about in Chapter 4.4 – Adding the Motor Inertia. Note that in this application, because of the gravitational load, the load torque is never negative even during deceleration and reverse motion. That means the short cut using the RMS acceleration will produce an invalid result – it will be higher than the real requirement.
Thus, having made a preliminary selection using the T-S widget, we need to perform a thorough RMS torque calculation: -

Manual Motor Selection

Transmission Data		
Ratio*	1	: 1
Transmission Inertia	0.00032	kg-m²

* Try the Effect of Changing the Transmission Ratio

Motor Data (CPB-1-01)		
Type	Value	Unit
Maximum Speed	4500	rpm
Peak Torque	2.7	Nm
RMS Torque	1	Nm
Inertia	1.00×10^{-5}	kg-m²

					Rotary Motion - Motor Side		
Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]	Total Inertia [J_m
	(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)	(kg-m²)
1	0	716	0.167	450	4.71	0.00471	0.00504
2	716	115	0.167	0	4.71	0.00471	0.00504
3	716	0	0.167	-450	4.71	0.00471	0.00504
4	0	0	1	0	4.61	0.00471	0.00504
5	0	-716	0.167	-450	4.51	0.00471	0.00504
6	-716	-716	0.167	0	4.51	0.00471	0.00504
7	-716	0	0.167	450	4.51	0.00471	0.00504
8	0	0	2	0	0.196	0.000208	0.000538
Total			4				
RMS Torque	3.39						
J Ratio Minimum	52.8	: 1					
J Ratio Maximum	503	: 1					
J Range	9.53	: 1					

Table 16

So, the actual RMS torque requirement is 14% less than the short cut using RMS acceleration indicated. This can be significant, so you need to watch out for applications such as this, where friction torque or externally applied torque is high. In addition, the 4:1 gearing has reduced the load/motor inertia ratio to more sensible levels.

7.5: Axis 3: Traverse Mechanism

The Lift mechanism travels along a beam from the belt to the required position over the rotary table, ready to place the component. This mechanism is a toothed belt.

Axis Specification

- Move distance: 500 mm
- Move Time: in - 1 s and out - 1 s
- Dwell time: in - 0.5 s and out - 1.5 s
- Moving Mass: 5 kg
- Belt Mass: 200 gm
- Pulley diameters (2): 50 mm
- Pulley mass (each): 100 gm
- Friction coefficient: 0.01
- Rotary friction loss: negligible

Mechanical Parameters		
Name	**Value**	**Unit**
Component Mass	45	kg
Table and Belt Mass	5.2	kg
Friction Coefficient	0.01	per unit
Inclination	0	degrees
Pulley Diameter	0.05	m
Pulley Mass x 2	0.2	kg
Rotary Friction Loss	0	Nm
Pulley Inertia	0.000125	kg-m^2

Table 17

Profile Parameters		
Name	**Value**	**Unit**
Move Distance	0.5	m
Move Time	1	s
Average Velocity	0.5	m/s
Peak Velocity	0.75	m/s
Dwell Time - Out	0.5	s
Dwell Time - Return	1.5	s

Table 18

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External Force	Fric
	(m/s)	(m/s)	(s)	(m/s^2)	(m)		(kg)	(N)	
1	0	0.75	0.333	2.25	0.125	1	50.2	0	
2	0.75	0.75	0.333	0	0.25	1	50.2	0	
3	0.75	0	0.333	-2.25	0.125	1	50.2	0	
4	0	0	0.333	0	0	0	5.2	0	
5	0	-0.75	0.333	-2.25	-0.125	-1	5.2	0	
6	-0.75	-0.75	0.333	0	-0.25	-1	5.2	0	
7	-0.75	0	0.333	2.25	-0.125	-1	5.2	0	
8	0	0	1.5	0	0	0	5.2	0	

Table 19

$$\text{Equivalent Radius} = 0.0125 \text{ m}$$

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque [T_{ext}]	Friction
	(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)	(kg-m^2)	(Nm)	(
1	0	60	0.333	180	10	0.00797	0	0.
2	60	60	0.333	0	20	0.00797	0	0.
3	60	0	0.333	-180	10	0.00797	0	0.
4	0	0	0.5	0	0	0.000937	0	
5	0	-60	0.333	-180	-10	0.000937	0	0.
6	-60	-60	0.333	0	-20	0.000937	0	0.
7	-60	0	0.333	180	-10	0.000937	0	0.
8	0	0	1.5	0	0	0.000937	0	

Table 20

T-S Widget Motor Selection

At this point we could generate the required data for the T-S widget (the optional columns above will be required): -

T-S Widget Data		
Segment	**Total Torque^2 * Time [$T_{tot}^2.\Delta t$]**	**Acceleration^2 * Time [$a^2.\Delta t$]**
1	0.746	10800
2	0.00126	0
3	0.628	10800
4	0	0
5	0.00879	10800
6	1.36×10^{-5}	0
7	0.0102	10800
8	0	0
Total	1.39	43200

Table 21

Torque-Speed Requirements	Value	Unit
Peak Torque	1.5	Nm
RMS Torque	0.59	Nm
Peak Speed	573	rpm
Peak Acceleration	180	rad/s²
RMS Acceleration	104	rad/s²

Table 22

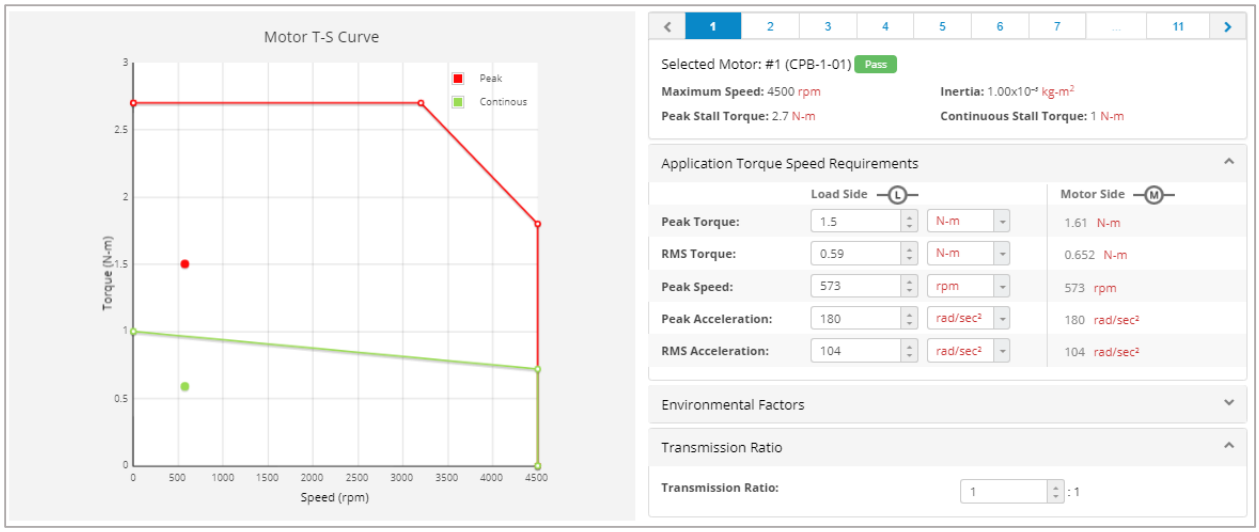


Figure 9

Motor #1 will suffice with direct drive. Get the load inertia from the table and compare it to the motor inertia shown in the widget. You will see that the inertia ratio in the loaded part of the motion cycle is very large.

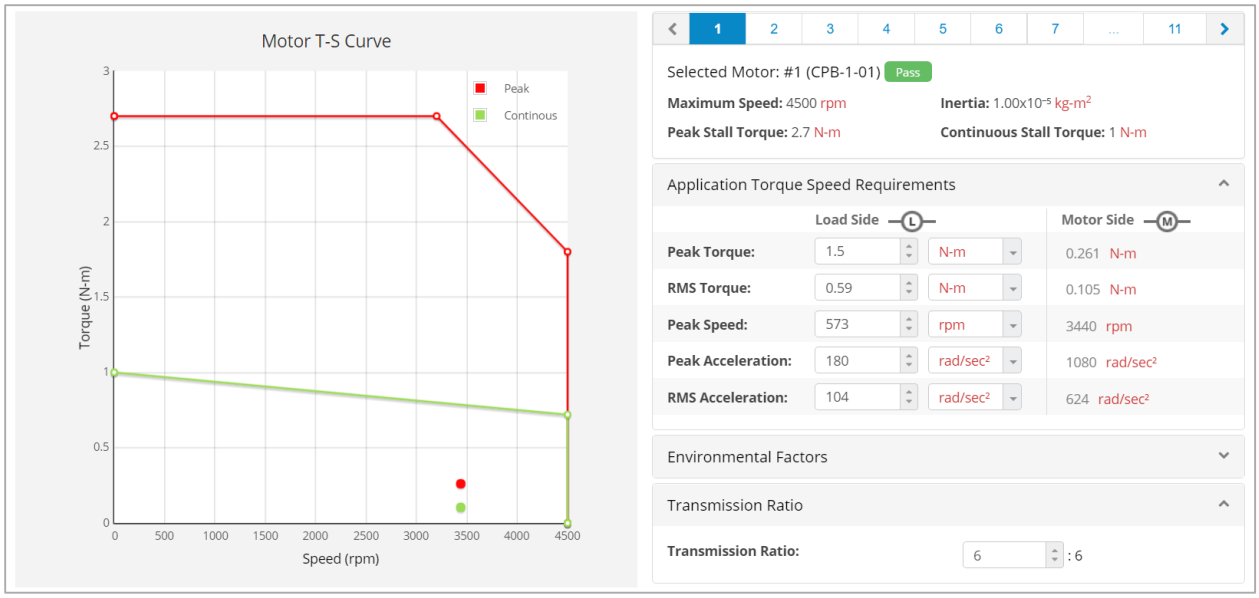


Figure 10

By adjusting the transmission ratio, we find that Motor #1 will also work at a ratio of 6:1, reducing the maximum inertia ratio by a factor of 36 – the square of the ratio.

Manual Motor Selection Check

Using the Widget result let's go with the 6:1 gearing ratio: -

Transmission Data		
Ratio*	6	: 1
Transmission Inertia	0	kg-m²

* Try the Effect of Changing the Transmission Ratio

Motor Data (CPB-1-01)		
Type	Value	Unit
Maximum Speed	4500	rpm
Peak Torque	2.7	Nm
RMS Torque	1	Nm
Inertia	1.00×10^{-5}	kg-m²

					Rotary Motion - Motor Side		
Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]	Total Inertia [J_{to}
	(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)	(kg-m²)
1	0	3440	0.333	1080	0.0103	0.000221	0.000231
2	3440	3440	0.333	0	0.0103	0.000221	0.000231
3	3440	0	0.333	-1080	0.0103	0.000221	0.000231
4	0	0	0.5	0	0	2.60×10^{-5}	3.60×10^{-5}
5	0	-3440	0.333	-1080	0.00106	2.60×10^{-5}	3.60×10^{-5}
6	-3440	-3440	0.333	0	0.00106	2.60×10^{-5}	3.60×10^{-5}
7	-3440	0	0.333	1080	0.00106	2.60×10^{-5}	3.60×10^{-5}
8	0	0	1.5	0	0	2.60×10^{-5}	3.60×10^{-5}
Total			4				
RMS Torque	0.103						
J Ratio Minimum	2.6	: 1					
J Ratio Maximum	22.1	: 1					
J Range	8.5	: 1					

Table 23

The inertia ratio is now reasonable making tuning easier. Try re-entering a gearing ratio of 1:1 to see the difference.

7.6: Axis 4: Rotary Index Table

The table indexes a quarter revolution in four steps to stop at load, two locations for component processing and unload stations.

Axis Specification

- Component mass: 3 x 45 kg (see the machine diagram - Figure 1 in Chapter 7.2 - Example Application. One station will always be empty)
- Component mounting radius: 0.5 m
- Mass of Table: 200 kg (Assume it is a uniform disk)
- Diameter of Table: 1.4 m
- Move time: 1.0 s
- Dwell time: 3.0 s
- Move distance: 1/4 revolution

Mechanical Parameters		
Name	**Value**	**Unit**
Component Mass	135	kg
Component Mounting Radius	0.5	m
Table Diameter	1.4	m
Table Mass	200	kg
Rotary Friction Loss	5	Nm
Table Inertia	49	kg-m^2

Table 24

Profile Parameters		
Name	**Value**	**Unit**
Move Distance	1.57	radians
Total Move Time	1	s
Average Velocity	1.57	rad/s
Peak Velocity	2.36	rad/s
Dwell Time	3	s

Table 25

When calculating the total moment of inertia, we are taking the component mass as being at a single point. In fact, there would be a very small additional inertia due to the rotation of the component itself (Refer: Chapter 1.8 - Inertia), which we can treat as negligible.

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Load Inertia [J_L]	Exterr
	(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)		(kg-m^2)	
1	0	2.36	0.333	7.07	0.393	1	82.8	
2	2.36	2.36	0.333	0	0.785	1	82.8	
3	2.36	0	0.333	-7.07	0.393	1	82.8	
4	0	0	3	0	0	0	82.8	

Table 26

T-S Widget Motor Selection

At this point we could generate the required data for the T-S widget (the optional columns above will be required):-

T-S Widget Data		
Segment	**Total Torque^2 * Time [$T_{tot}^2.\Delta t$]**	**Acceleration^2 * Time [$a^2.\Delta t$]**
1	116000	16.7
2	8.33	0
3	112000	16.7
4	0	0
Total	228000	33.3

Table 27

Torque-Speed Requirements	Value	Unit
Peak Torque	590	Nm
RMS Torque	239	Nm
Peak Speed	22.5	rpm
Peak Acceleration	7.07	rad/s^2
RMS Acceleration	2.89	rad/s^2

Table 28

Looking at the extreme low speed and the very high torques it is clear that a large gearing ratio will be needed. 100:1 is the maximum available in the T-S widget so we will try that first:-

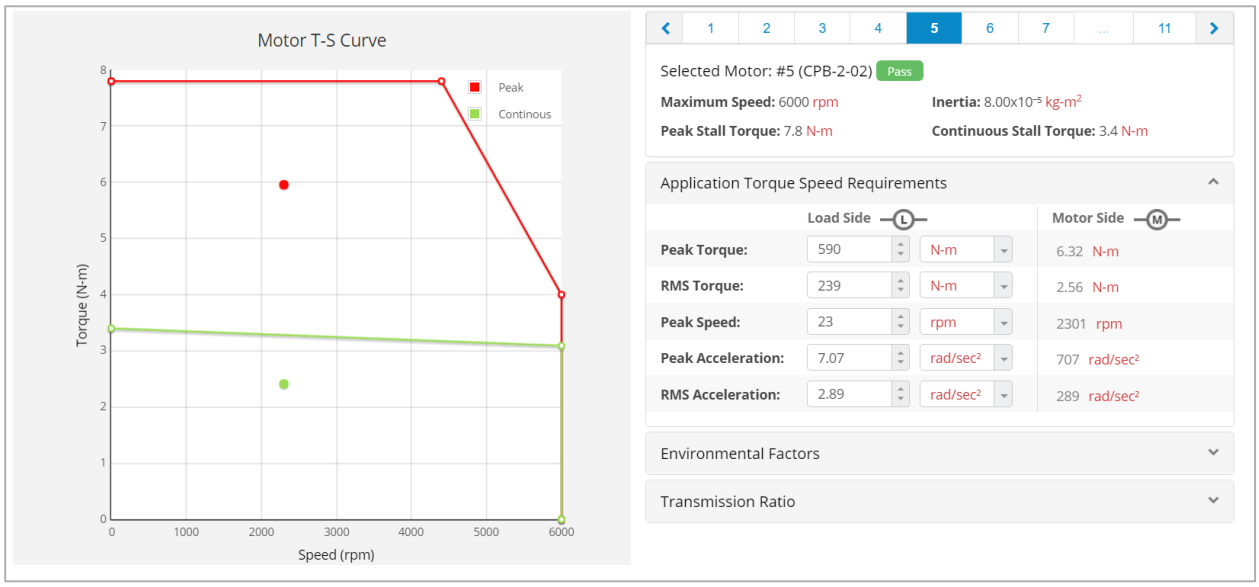


Figure 11

Motor #5 looks good so let's check manually: -

Manual Motor Selection Check

Transmission Data		
Ratio*	100	: 1
Transmission Inertia	0	kg-m^2

* Try the Effect of Changing the Transmission Ratio

Motor Data (CPB-2-02)		
Type	**Value**	**Unit**
Maximum Speed	6000	rpm
Peak Torque	7.8	Nm
RMS Torque	3.4	Nm
Inertia	8.00x10^{-5}	kg-m^2

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load
	(rpm)	(rpm)	(s)	(rad/s^2)	
1	0	2250	0.333	707	
2	2250	2250	0.333	0	
3	2250	0	0.333	-707	
4	0	0	3	0	
Total			4		
RMS Torque	2.41				
J Ratio Minimum	103	: 1			
J Ratio Maximum	103	: 1			
J Range	1	: 1			

Table 29

This confirms the widget result. The inertia ratio is very high so it may require a higher inertia motor – use the widget to find a motor giving an inertia ratio of less than 10:1. Such a motor would clearly be under-utilised, but large motor generally have a lower speed range so trying to use an even higher gear ratio to improve the inertia match further would potentially take the maximum speed above the range of any motors available to you.

7.7: Axis 5: Welder Rotate

An auto loaded stud is pressed into the component with a high friction torque so that the metal melts and the parts are fused together. This is the friction servo.

Application Specification

- Rotation speed: 2000 rpm
- Torque: 25 Nm
- Rotation time: 3.0 sec
- Dwell time: 1.0 sec

Mechanical Parameters		
Name	Value	Unit
Spindle Inertia	0.04	kg-m²
Rotary Friction Loss	25	Nm

Table 30

Profile Parameters		
Name	Value	Unit
Acceleration/Deceleration Time	0.5	sec
Cruise Time	2	sec
Peak Velocity	2000	rpm
Dwell Time	1	sec

Table 31

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Total Inertia [J_{tot}]	External
	(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)		(kg-m^2)	(
1	0	209	0.5	419	52.4	1	0.04	
2	209	209	2	0	419	1	0.04	
3	209	0	0.5	-419	52.4	1	0.04	
4	0	0	1	0	0	0	0.04	

Table 32

T-S Widget Motor Selection

At this point we could generate the required data for the T-S widget (the optional columns above will be required): -

T-S widget data		
Segment	Total Torque^2 * Time [$T_{tot}^2.\Delta t$]	Acceleration^2 * Time [$a^2.\Delta t$]
1	872	87700
2	1250	0
3	34	87700
4	625	0
5	2780	175000

Table 33

Torque-Speed Requirements	Value	Unit
Peak Torque	41.8	Nm
RMS Torque	26.4	Nm
Peak Speed	2000	rpm
Peak Acceleration	419	rad/s^2
RMS Acceleration	209	rad/s^2

Table 34

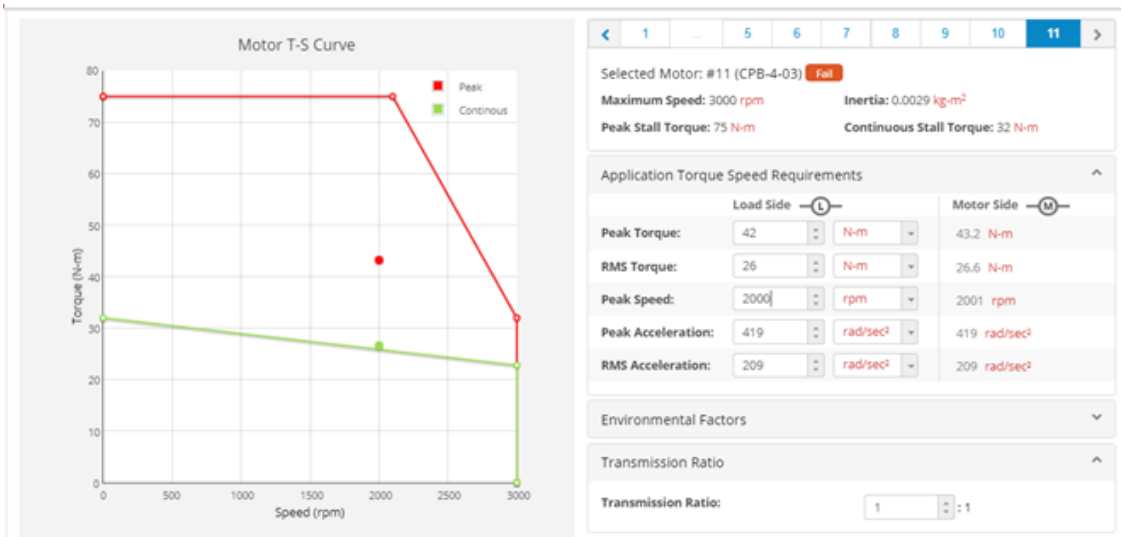


Figure 12

The biggest available motor is marginal on RMS torque. We have the following options: -

1. Find a bigger motor outside the current range.
2. Add a modest reduction ratio of, say, 1.2:1 (the widget only supports integer numbers, and 2:1 will be too high)
3. Add forced air cooling (if the option is available)

We could do a simple manual calculation of the five parameters using a ratio of 1.2, thus: -

Torque-Speed Requirements	Transmission Ratio = 1		Transmission Ratio = 1.2	
	Value	Unit	Value	
Peak Torque	41.8	Nm	34.8	
RMS Torque	26.4	Nm	22	
Peak Speed	2000	rpm	2400	
Peak Acceleration	419	rad/s²	503	r
RMS Acceleration	209	rad/s²	251	r

Table 35

Entering these numbers into the widget gives: -

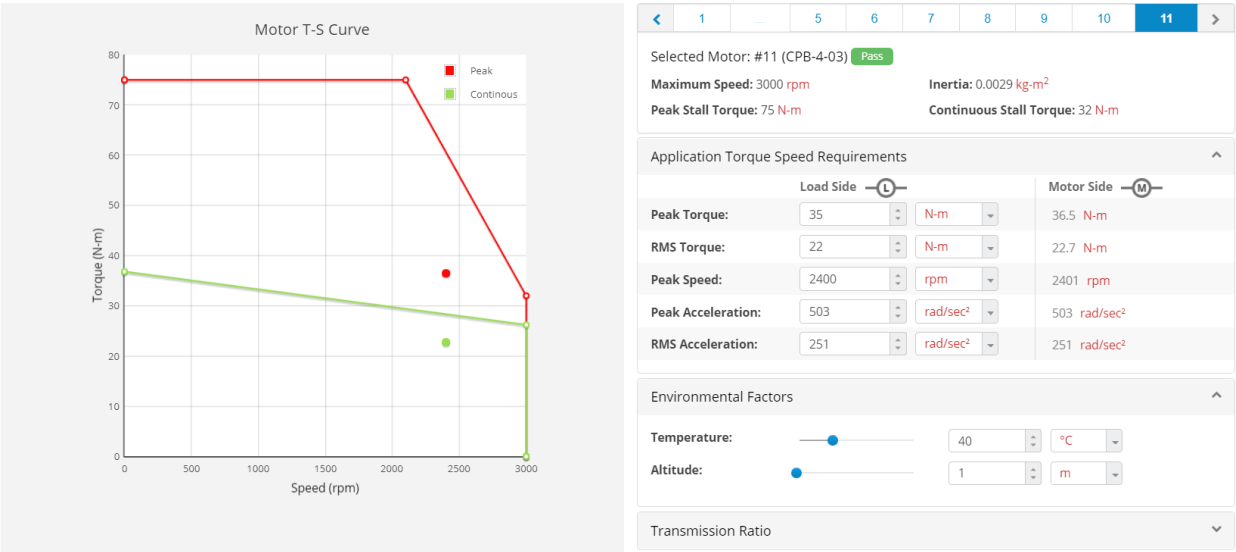


Figure 13

This confirms that motor #11 looks like a good fit with a gearing ratio of 1.2:1. Now we can finalise the result manually.

Manual Motor Selection Check

Transmission Data		
Ratio*	1.2	: 1
Transmission Inertia	0	kg-m²

* Try the Effect of Changing the Transmission Ratio

Motor Data (CPB-4-03)		
Type	Value	Unit
Maximum Speed	3000	rpm
Peak Torque	75	Nm
RMS Torque	32	Nm
Inertia	0.0029	kg-m²

					Rotary Motion - Motor Side		
Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]	Total Inertia [J_{to}
	(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)	(kg-m²)
1	0	2400	0.5	503	20.8	0.0278	0.0307
2	2400	2400	2	0	20.8	0.0278	0.0307
3	2400	0	0.5	-503	20.8	0.0278	0.0307
4	0	0	1	0	20.8	0.0278	0.0307
Total			4				
RMS Torque	22.2						
J Ratio Minimum	9.58	: 1					
J Ratio Maximum	9.58	: 1					
J Range	1	: 1					

Table 36

This shows good correlation with the T-S widget.

Changing the ratio to 1.2:1 lowers the RMS torque to 22.2 Nm at 2400 rpm which is inside the RMS torque envelope. If you think the margin looks a little tight (since the rating at this speed is about 28 Nm - a utilisation of about 78%), remember that T-S curve is for a 40-degree ambient at the motor. If the ambient was reduced to 25 degrees, how would that affect the margin?

7.8: Axis 6: Welder Feed (Belt screw)

This mechanism presses the auto-loaded stud so that friction welding occurs.

Application Specification

- Moving mass: 20 kg
- Distance: 20 mm
- Move Time: 0.1 s down: Dwell 2.8 s: 0.5s up: Dwell 1.0 s
- Weld Force: 1000 N
- Ball screw lead: 10 mm
- Ball screw Inertia: 7.8E-6 kg-m2
- Rotary friction may be ignored

Mechanical Parameters		
Name	**Value**	**Unit**
Sliding Mass	20	kg
Friction Coefficient	0	per unit
Inclination	90	degrees
Ballscrew Lead	0.01	m
Ballscrew Inertia	7.81×10^{-6}	kg-m^2
Rotary Friction Loss	0	Nm
Weld Force	10000	N

Table 37

Profile Parameters		
Name	**Value**	**Unit**
Fast-feed Distance	-0.02	m
Fast-feed time	0.1	s
Average Velocity	-0.2	m/s
Peak Velocity	-0.3	m/s
Dwell (pressure) Time	2.8	s
Return Time	0.1	s
Dwell Time	1	s

Table 38

Note that the positive move is taken to be downward hence the negative feed distance.

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External Force	Friction Force	Total Force
	(m/s)	(m/s)	(s)	(m/s^2)	(m)		(kg)	(N)	(N)	(N)
1	0	-0.3	0.0333	-9	-0.005	-1	20	-196	0	-376
2	-0.3	-0.3	0.0333	0	-0.01	-1	20	-196	0	-196
3	-0.3	0	0.0333	9	-0.005	-1	20	-196	0	-16.2
4	0	0	2.8	0	0	0	20	9800	0	9800
5	0	0.3	0.0333	9	0.005	1	20	-196	0	-16.2
6	0.3	0.3	0.0333	0	0.01	1	20	-196	0	-196
7	0.3	0	0.0333	-9	0.005	1	20	-196	0	-376
8	0	0	1	0	0	0	20	-196	0	-196

Table 39

The external (gravity) force is negative whilst the feed force is positive.

$$\text{Equivalent Radius} = 0.001592 \text{ m}$$

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque [T_{ext}]	Friction
	(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)	(kg-m^2)	(Nm)	(
1	0	-188	0.0333	-5650	-3.14	5.85x10^{-5}	-0.312	
2	-188	-188	0.0333	0	-6.28	5.85x10^{-5}	-0.312	
3	-188	0	0.0333	5650	-3.14	5.85x10^{-5}	-0.312	
4	0	0	2.8	0	0	5.85x10^{-5}	15.6	
5	0	188	0.0333	5650	3.14	5.85x10^{-5}	-0.312	
6	188	188	0.0333	0	6.28	5.85x10^{-5}	-0.312	
7	188	0	0.0333	-5650	3.14	5.85x10^{-5}	-0.312	
8	0	0	1	0	0	5.85x10^{-5}	-0.312	

Table 40

T-S Widget Motor Selection

At this point we could generate the required data for the T-S widget (the optional columns above will be required): -

T-S Widget Data		
Segment	Total Torque^2 * Time [$T_{tot}^2.\Delta t$]	Acceleration^2 * Time [$a^2_{2,\Delta}$]
1	0.0138	1.07x10^6
2	0.00325	0
3	1.13x10^{-5}	1.07x10^6
4	682	0
5	1.13x10^{-5}	1.07x10^6
6	0.00325	0
7	0.0138	1.07x10^6
8	0.0975	0
Totals	682	4.26x10^6

Table 41

Torque-Speed Requirements	Value	Unit
Peak Torque	15.6	Nm
RMS Torque	13.1	Nm
Peak Speed	1800	rpm
Peak Acceleration	5650	rad/s²
RMS Acceleration	1030	rad/s²

Table 42

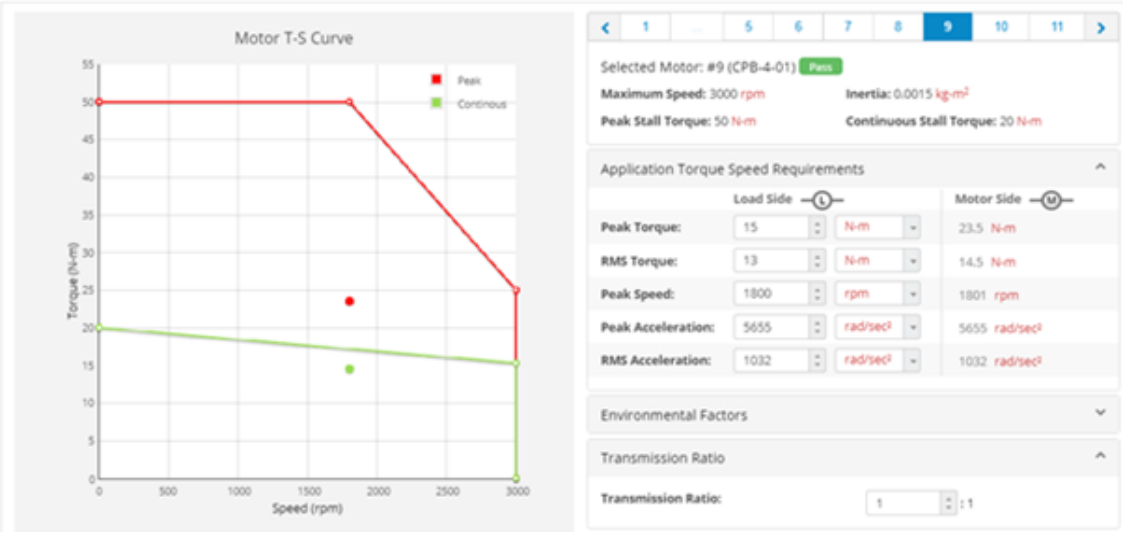


Figure 14

The #9 motor looks a good fit with direct drive.

Manual Motor Selection Check

Transmission Data		
Ratio*	1	: 1
Transmission Inertia	0	kg-m²

* Try the Effect of Changing the Transmission Ratio

Motor Data (CPB-4-01)		
Type	Value	Unit
Maximum Speed	3000	rpm
Peak Torque	50	Nm
RMS Torque	20	Nm
Inertia	0.0015	kg-m²

						Rotary Motion - Motor Side	
Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]	Total Inertia [J_{to}
	(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)	(kg-m²)
1	0	-1800	0.0333	-5650	-0.312	5.85×10^{-5}	0.00156
2	-1800	-1800	0.0333	0	-0.312	5.85×10^{-5}	0.00156
3	-1800	0	0.0333	5650	-0.312	5.85×10^{-5}	0.00156
4	0	0	2.8	0	15.6	5.85×10^{-5}	0.00156
5	0	1800	0.0333	5650	-0.312	5.85×10^{-5}	0.00156
6	1800	1800	0.0333	0	-0.312	5.85×10^{-5}	0.00156
7	1800	0	0.0333	-5650	-0.312	5.85×10^{-5}	0.00156
8	0	0	1	0	-0.312	5.85×10^{-5}	0.00156
Total			4				
RMS Torque	13.2						
J Ratio Minimum	0.039	: 1					
J Ratio Maximum	0.039	: 1					
J Range	1	: 1					

Table 43

The RMS torque of 13 Nm is well within the motor rating, and inertia match is no problem on this axis.

Note that the widget result was higher at 14.5 Nm – this is another of the cases where using the RMS acceleration short cut is inaccurate, because in several segments the load torque and the

acceleration torque have opposite signs. The RMS acceleration method only works properly if the torques are all additive, otherwise it is pessimistic, meaning you may choose a larger motor than you need.

7.9: Axis 7: Cap Applicator (Rack & Pinion)

A plastic cap is applied to the stud to protect the thread. It moves down to press the cap over the stud then returns using a ball screw for smooth motion.

Application Specification

- Distance: 100 mm
- Time: 1.0 s each way (no dwell)
- Dwell: 2 s
- Vertical Slide mass: 10 kg
- Friction Coefficient: 0.1
- Rotary Friction: 0.1 Nm
- Pinion PCD: 20 mm
- Pinion Inertia: 7.8E-6 kg-m²

Mechanical Parameters		
Name	**Value**	**Unit**
Slide mass	10	kg
Friction coefficient	0.1	per unit
Inclination	90	degrees
Pinion PCD	0.02	m
Pinion inertia	7.81×10^{-6}	kg-m²
Rotary friction loss	0.1	Nm

Table 44

Profile Parameters		
Name	**Value**	**Unit**
Move	0.1	m
Move time (down)	1	s
Average Velocity	0.1	m/s
Peak Velocity	0.15	m/s
Dwell	0	s
Move time (up)	1	s
Average Velocity	-0.1	m/s
Peak Velocity	-0.15	m/s
Dwell	2	s

Table 45

Segment	Initial Velocity [u] (m/s)	Final Velocity [v] (m/s)	Time [t] (s)	Acceleration [a] (m/s^2)	Distance [s] (m)	Sign of direction	Mass (kg)	External Force (N)	Friction Force (N)	Total Force (N)
1	0	0.15	0.333	0.45	0.025	1	10	-98.1	0	-93.6
2	0.15	0.15	0.333	0	0.05	1	10	-98.1	0	-98.1
3	0.15	0	0.333	-0.45	0.025	1	10	-98.1	0	-103
4	0	0	1	0	0	0	10	-98.1	0	-98.1
5	0	-0.15	0.333	-0.45	-0.025	-1	10	-98.1	0	-103
6	-0.15	-0.15	0.333	0	-0.05	-1	10	-98.1	0	-98.1
7	-0.15	0	0.333	0.45	-0.025	-1	10	-98.1	0	-93.6
8	0	0	2	0	0	0	10	-98.1	0	-98.1

Table 46

$$\text{Equivalent Radius} = 0.01\ \text{m}$$

Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque [T_{ext}]	Friction Torque [T_f]
	(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)	(kg-m^2)	(Nm)	(Nm)
1	0	15	0.333	45	2.5	0.00101	-0.981	0.1
2	15	15	0.333	0	5	0.00101	-0.981	0.1
3	15	0	0.333	-45	2.5	0.00101	-0.981	0.1
4	0	0	1	0	0	0.00101	-0.981	0
5	0	-15	0.333	-45	-2.5	0.00101	-0.981	-0.1
6	-15	-15	0.333	0	-5	0.00101	-0.981	-0.1
7	-15	0	0.333	45	-2.5	0.00101	-0.981	-0.1
8	0	0	2	0	0	0.00101	-0.981	0

Table 47

T-S widget Motor Selection

At this point we could generate the required data for the T-S widget (the optional columns above will be required)

T-S Widget Data		
Segment	Total Torque^2 * Time [$T_{tot}^2.\Delta t$]	Acceleration^2 * Time [$a^2.\Delta t$]
1	0.233	675
2	0.259	0
3	0.286	675
4	0.962	0
5	0.423	675
6	0.39	0
7	0.358	675
8	1.92	0
Totals	4.83	2700

Table 48

Torque-Speed Requirements	Value	Unit
Peak Torque	1.13	Nm
RMS Torque	0.983	Nm
Peak Speed	143	rpm
Peak Acceleration	45	rad/s^2
RMS Acceleration	23.2	rad/s^2

Table 49

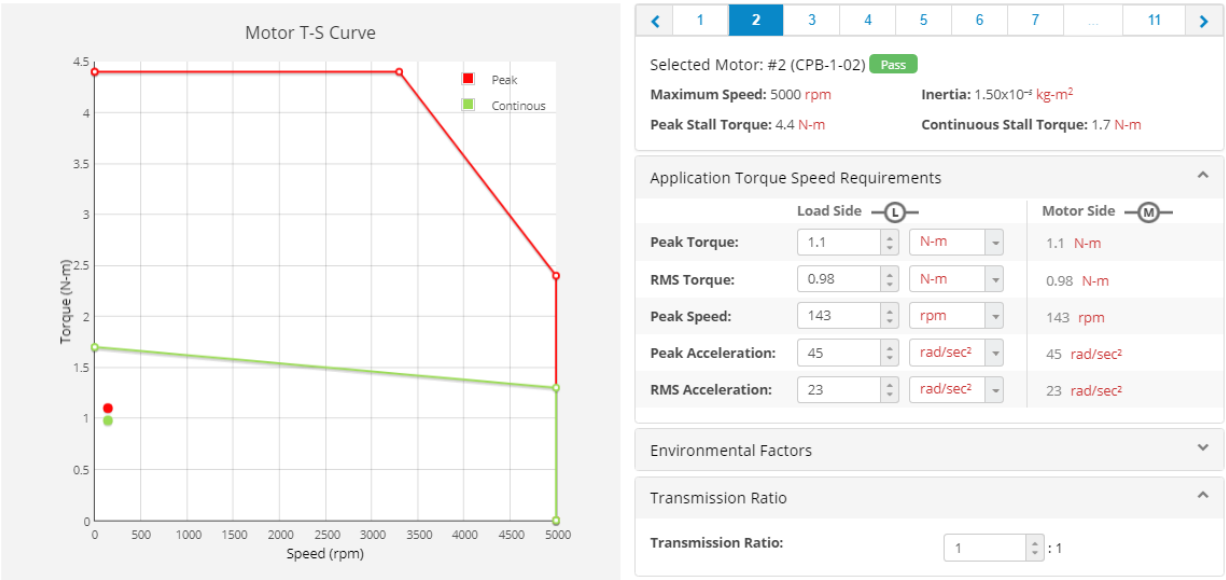


Figure 15

Motor #2 will easily perform the duty although the speed is very low.

Manual Motor Selection Check

Transmission Data		
Ratio*	1	:1
Transmission Inertia	0	kg-m²

* Try the Effect of Changing the Transmission Ratio

Motor Data (CPB-1-02)		
Type	Value	Unit
Maximum Speed	5000	rpm
Peak Torque	4.4	Nm
RMS Torque	1.7	Nm
Inertia	1.50×10^{-5}	kg-m²

	Rotary Motion - Motor Side							
Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]	Total Inertia [J_{tot}]	Accele
	(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)	(kg-m²)	
1	0	143	0.333	45	-0.881	0.00101	0.00102	
2	143	143	0.333	0	-0.881	0.00101	0.00102	
3	143	0	0.333	-45	-0.881	0.00101	0.00102	
4	0	0	1	0	-0.981	0.00101	0.00102	
5	0	-143	0.333	-45	-1.08	0.00101	0.00102	
6	-143	-143	0.333	0	-1.08	0.00101	0.00102	
7	-143	0	0.333	45	-1.08	0.00101	0.00102	
8	0	0	2	0	-0.981	0.00101	0.00102	
Total			5					
RMS Torque	0.983							
J Ratio Minimum	67.2	:1						
J Ratio Maximum	67.2	:1						
J Range	1	:1						

Table 50

The low speed suggests that the motor is under-utilised. The manual check also indicated a high inertia ratio. The addition of an appropriate gearing ratio would enable the smallest #1 motor to be used (it was very close) and bring the inertia ratio down to a sensible level.

Try it!

7.10: Axis 8, 9 & 10: Outfeed Axes

The last three axes can be expected to be mechanically identical to the corresponding infeed axes, as they perform the same operations. However, they need to be checked as subtle differences could exist which may affect the sizing.

Axis 8: Component lift to outfield belt

The component is lifted from the rotary table and then replaced at the same height on the outfeed belt by a rack and pinion mechanism. This Axis is identical to Axis 2: Component Lift. You are requested to refer Chapter 7.4 - Axis 2: Component Lift for details of this axis.

Studying the details of both these axes shows that the motion is equally loaded through the cycle, so the same solution can apply to both axis 2 and axis 8.

Axis 9: Traverse mechanism to outfeed belt

The Lift mechanism travels along a beam from the rotary table to the outfeed belt, ready to place the component on the belt. This is a converse action to axis 3, but can use the same mechanism. This Axis is identical to Axis 3: Traverse Mechanism. You are requested to refer Chapter 7.5 - Axis 3: Traverse Mechanism for details of this axis.

Studying the details of these axes shows that there is difference, in that the mechanism is loaded during the reverse part of the move rather than the forward part. However the move is horizontal and symmetrical, so this does not affect the sizing.

Axis 10: Outfeed belt

Stops to allow the pick and place lifter to place a component on it then accelerates to match the onward, non-stop belt (supplied by others).

This Axis is mechanically identical to Axis 1: Infeed Belt. You are requested to refer Chapter 7.3 - Axis 1: Infeed Belt for details of this axis.

Studying the details of these axes shows that there is a subtle but significant difference between infeed and outfeed. In the case of the infeed belt (axis 1) the 45 kg component mass has to be decelerated by the infeed motor where friction is helping to slow it down (segment 3). The outfeed belt, by contrast, sees the component mass during acceleration (segment 1) where friction is adding to the force required. Go back to axis 1 and try moving the component mass from segment 3 to segment 1 in the Linear Load table to see the effect ripple through the calculations. Will the motor still handle it?

Chapter 8 – Course Assessment

8.1: Introduction

To finally assess your understanding, re-evaluate the example application with a modified specification. A new model of the imaginary production machine is envisaged, with the same basic design, but the requirement is to make a smaller product with a higher production rate. To recap, here are the axes which need evaluating: -

1. Belt conveyer (stop to deliver / remove components)
2. Lift (components from belt and place on table and vice versa)
3. Traverse (moves the lift mechanism between the belt and the table)
4. Indexing Rotary table (90° steps so as to stop at load, weld, trim and unload stations)
5. Friction Welder rotation (high friction torque heats the component parts till they fuse together)
6. Welder Feed (applies high pressure for the welding)
7. Trimmer (feeds a tool in to clean any flashing from the welding process)
8. Lift (components from belt and place on table and vice versa)
9. Traverse (moves the lift mechanism between the belt and the table)
10. Belt conveyer (stop to deliver / remove components)

Index Table and infeed/outfeed belts: -

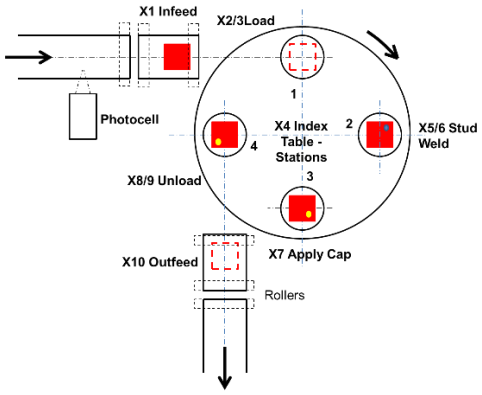


Figure 1

Side view of infeed belt and lift/traverse ("pick and place") axes: -

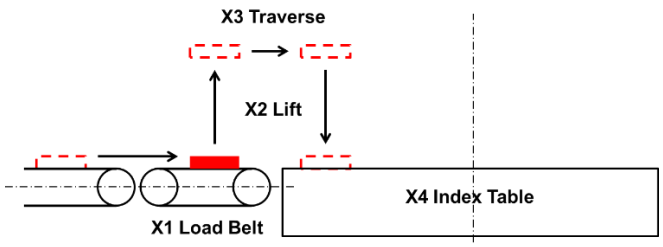


Figure 2

The new required production rate is 30 parts per minute, i.e. the cycle time for each axis will be two seconds, with a product mass of only 10 Kg. Some parts of the machine can be lightened and reduced in size, but others will remain the same to avoid the need to invest in redesigning, and to gain economies of scale.

Here is the new timing diagram: -

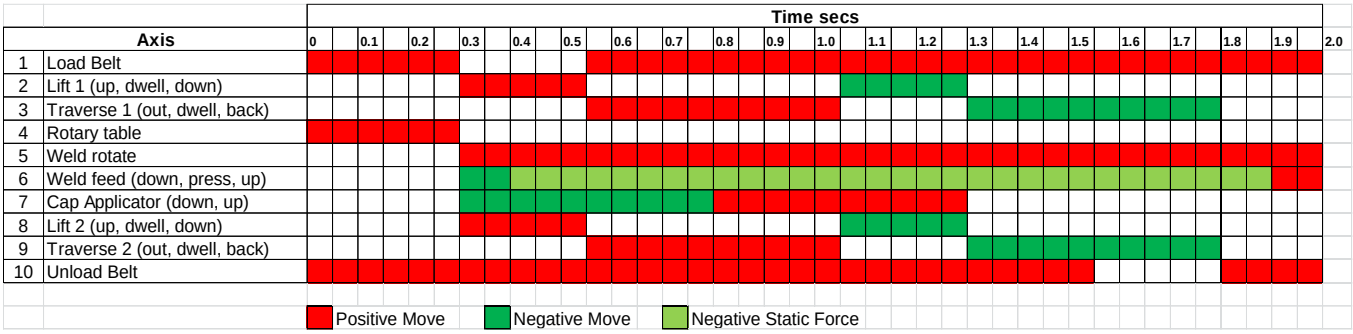


Figure 3

Your task in the next few sections is to specify the motors which will be required for the new design.

8.2: Axis 1: Infeed Belt Assessment

Selecting Suitable Motor for Axis 1

To Recap:

Infeed - a belt, sliding on a flat surface, accepts a component from a non-stop belt (supplied by others) and stops to allow the pick and place lifter to remove it.

A subtle feature of this application is that the 10 kg load is passed to this belt from another running at the same speed. If it were dropped onto the belt from a standstill its mass would need to be accelerated by the belt which would cause a 'pulse' of torque. This will be covered in the advanced course.

The specification of the mechanical parts of the infeed roller are retained from Chapter 7.3 - Axis 1: Infeed Belt to avoid a redesign

Axis Specifications

- Coefficient of friction: 0.1
- Mass of belt: 5 kg
- Inertia of rollers and shafts: 0.001156 kg-m^2
- Diameter of drive roller: 50 mm
- Rotary losses: 0.2 Nm

However, since the production rate is now 30 parts per minute, the infeed speed is increased to 600 mm/sec, and as mentioned above, the product mass is 10 kg instead of 45 kg.

First, figure out the motion cycle requirement:

The belt should now accelerate and decelerate in 0.25 s, and a dwell time of 0.25 s is required. Move distance is not important.

Gathering all the known entities and data points into Mechanical Parameters and Profile Parameters tables:

Mechanical Parameters		
Name	**Value**	**Unit**
Component Mass	10	kg
Belt Mass	5	kg
Friction Coefficient	0.1	per unit
Inclination	0	degrees
Roller Diameter	0.05	m
Total Rotating inertia	0.00156	kg-m^2
Rotary Friction Loss	0.2	Nm

Table 1

Profile Parameters		
Name	**Value**	**Unit**
Acceleration / Deacceleration time	0.25	s
Cruise Time	1.25	s
Dwell Time	0.25	s
Peak Velocity	0.25	m/s

Table 2

Create a Linear Profile table from above specification details using the below table. Click Submit button once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H	I	
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External Force	Fricti
2		(m/s)	(m/s)	(s)	(m/s²)	(m)		(kg)	(N)	
3	1									
4	2									
5	3									
6	4									
7	Total			0		0				
8										

Reset | Submit

Since the diameter of Drive Roller is 0.05 m, so the equivalent radius is 0.025 m.

Equivalent Radius	0.025	m

Compute the peak operating points for the axis and complete the equivalent Rotary Profile table below. Click Submit once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque [T_{ex}
2		(rad/s)	(rad/s)	(s)	(rad/s²)	rad	(kg-m²)	(Nm)
3								
4								
5								
6								
7	Total			0		0		
8								

Reset | Submit

T-S Widget Motor Selection

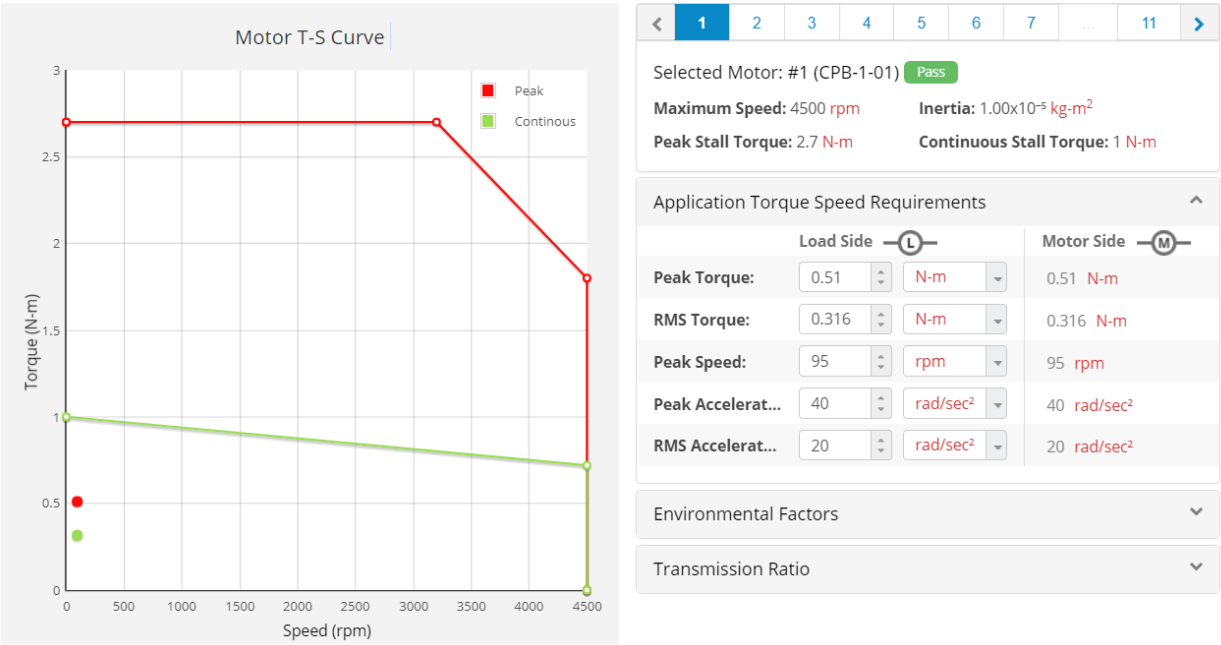
Generate the Torque - Speed Requirements for the axis in the table below. Click Submit button once you complete the table.

A1:D1 | fx | T-S Widget Data

	A	B	C	D
1	T-S Widget Data			
2	Segment	Total Torque^2 * Time [$T_{tot}^2.\Delta t$]	Acceleration^2 * Time [$a^2.\Delta t$]	
3				
4				
5				
6				
7	Total			
8				
9	Torque-Speed Requirements			
10	Peak Torque		Nm	
11	RMS Torque		Nm	
12	Peak Speed		rpm	
13	Peak Acceleration		rad/s^2	
14	RMS Acceleration		rad/s^2	
15				

Reset | Submit

Feed the values from above table into Torque -Speed widget given below. Adjust the transmission ratio to select the best suitable motor. Click Submit button once you complete the question.



Manual Motor Selection

Now refer the data to a sample motor (CPB-1-01) taking into account the motor's own inertia. Re-calculate the RMS torque without using the TS-Widget and enter a suitable transmission ratio so that the sample motor satisfies the Application's requirements.

A1:C1 | fx | Transmission Data

	A	B	C	D	E	F	G
1	Transmission Data						
2	Ratio		: 1				
3	Transmission Inertia	0	kg-m²				
4							
5	Motor Data (CPB-1-01)						
6	Type	Value	Unit				
7	Maximum Speed	4500	rpm				
8	Peak Torque	2.7	Nm				
9	RMS Torque	1	Nm				
10	Inertia	1.00×10^{-5}	kg-m²				
11							
12							Rotary Motion - Mot
13	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]
14		(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)
15	1						
16	2						
17	3						
18	4						
19	Total			0			
20							
21	RMS Torque		Nm				
22	J Ratio Minimum		: 1				
23	J Ratio Maximum		: 1				
24	J Range		: 1				
25							

Reset | Submit

8.3: Axis 2: Component Lift Assessment

Selecting Suitable Motor for Axis 2

The component is lifted from the belt and then replaced at the same height on the rotary table by a rack and pinion mechanism. Again, the mechanical specification is unchanged, but the profile is faster.

Axis Specifications

- Lift Height: 250 mm
- Move time: 0.25 sec each way
- Pinion PCD: 20 mm
- Pinion inertia: 0.00000781 kg-m^2
- Rack and moving mass: 2 kg
- Component mass: 10 kg
- Rotary friction: 0.1 Nm

Gathering all the known entities and data points into Mechanical Parameters and Profile Parameters tables:

Mechanical Parameters		
Name	**Value**	**Unit**
Component Mass	10	kg
Slide Mass	2	kg
Friction Coefficient	0.1	per unit
Inclination	90	degrees
Pinion PCD	0.02	m
Pinion Inertia	7.81×10^{-6}	kg-m^2
Rotary Friction Loss	0.1	Nm

Table 1

Profile Parameters		
Name	**Value**	**Unit**
Move Distance	0.25	m
Move Time (up)	0.25	s
Average Velocity	1	m/s
Peak Velocity	1.5	m/s
Dwell Time	0.5	s
Move Time (down)	0.25	s
Average Velocity	-1	m/s
Peak Velocity	-1.5	m/s
Dwell Time	1	s

Table 2

Create a Linear Profile table from above specification details using the below table. Click Submit button once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H	
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	Exter
2		(m/s)	(m/s)	(s)	(m/s^2)	(m)		(kg)	
3	1								
4	2								
5	3								
6	4								
7	5								
8	6								
9	7								
10	8								
11	Total			0		0			
12									

Equivalent Radius	0.01	m

Compute the peak operating points for the axis and complete the equivalent Rotary Profile table below. Click Submit once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque
2		(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)	(kg-m^2)	(Nm)
3	1							
4	2							
5	3							
6	4							
7	5							
8	6							
9	7							
10	8							
11	Total			0		0		
12								

Reset | Submit

T-S widget Motor Selection

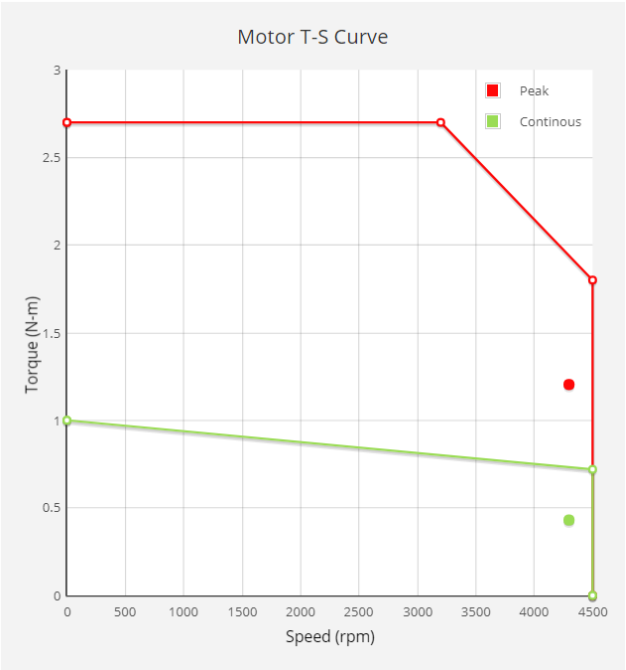
Generate the Torque - Speed Requirements for the axis in the table below. Click Submit button once you complete the table.

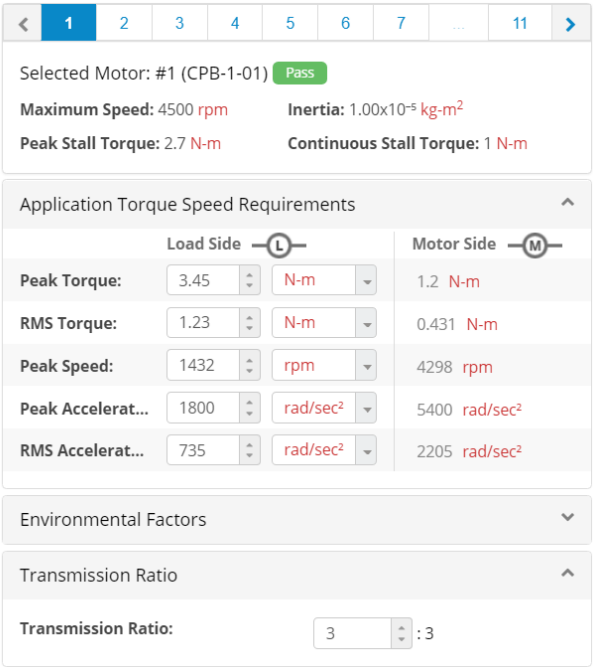
A1:D1 | fx | T-S Widget Data

	A	B	C	D
1	T-S Widget Data			
2	Segment	Total Torque² * Time [$T_{tot}^2.\Delta t$]	Acceleration² * Time [$a^2.\Delta t$]	
3				
4				
5				
6				
7				
8				
9				
10				
11	Total			
12				
13	Torque-Speed Requirements			
14	Peak Torque		Nm	
15	RMS Torque		Nm	
16	Peak Speed		rpm	
17	Peak Acceleration		rad/s²	
18	RMS Acceleration		rad/s²	
19				

Reset | Submit

Feed the values from above table into Torque -Speed widget given below. Adjust the transmission ratio to select the best suitable motor. Click Submit button once you complete the question.





Manual Motor Selection

Now refer the data to a sample motor (CPB-1-02) taking into account the motor's own inertia. Re-calculate the RMS torque without using the TS-Widget and enter a suitable transmission ratio so that the sample motor satisfies the Application's requirements.

A1:C1 | fx | Transmission Data

	A	B	C	D	E	F	G	H
1	Transmission Data							
2	Ratio		: 1					
3	Transmission Inertia	0.00032	kg-m²					
4								
5	Motor Data (CPB-1-02)							
6	Type	Value	Unit					
7	Maximum Speed	5000	rpm					
8	Peak Torque	4.4	Nm					
9	RMS Torque	1.7	Nm					
10	Inertia	1.30×10^{-5}	kg-m²					
11								
12							Rotary Motion - Motor Side	
13	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]	Total Inertia [J_{tc}
14		(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)	(kg-m²)
15	1							
16	2							
17	3							
18	4							
19	5							
20	6							
21	7							
22	8							
23	Total			0				
24								
25	RMS Torque		Nm					
26	J Ratio Minimum		: 1					
27	J Ratio Maximum		: 1					
28	J Range		: 1					
29								

Reset | Submit

8.4: Axis 3: Traverse Mechanism Assessment

Selecting Suitable Motor for Axis 3

The Lift mechanism travels along a beam from the belt to the required position over the rotary table, ready to place the component. This mechanism is a toothed belt as before and is unchanged to avoid redesign and gain economy of scale.

Axis Specifications

- Move distance: 500 mm.
- Move Time in: 0.5 sec and out: 0.5 sec
- Dwell time in: 0.25 sec and out: 0.75 sec
- Moving Mass: 5 kg
- Belt Mass: 200 gm
- Pulley diameters (2): 50 mm
- Pulley mass (each): 100 gm

Gathering all the known entities and data points into Mechanical Parameters and Profile Parameters tables:

Mechanical Parameters		
Name	**Value**	**Unit**
Component Mass	10	kg
Table & Belt Mass	5.2	kg
Friction Coefficient	0.01	per unit
Inclination	0	degrees
Pulley Diameter	0.05	m
Pulley Mass x2	0.2	kg
Rotary Friction Loss	0	Nm
Pulley Inertia	0.000125	kg-m^2

Table 1

Profile Parameters		
Name	**Value**	**Unit**
Move Distance	0.5	m
Move Time	0.5	s
Average Velocity	1	m/s
Peak Velocity	1.5	m/s
Dwell Time - out	0.25	s
Dwell Time - return	0.75	s

Table 2

Create a Linear Profile table from above specification details using the below table. Click Submit button once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H	
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	Exter
2		(m/s)	(m/s)	(s)	(m/s^2)	(m)		(kg)	
3	1								
4	2								
5	3								
6	4								
7	5								
8	6								
9	7								
10	8								
11	**Total**			0		0			
12									

Reset | Submit

Effective Radius	0.0125	m

Compute the peak operating points for the axis and complete the equivalent Rotary Profile table below. Click Submit once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque
2		(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)	(kg-m^2)	(Nm)
3	1							
4	2							
5	3							
6	4							
7	5							
8	6							
9	7							
10	8							
11	Total			0				
12								

Reset | Submit

T-S widget Motor Selection

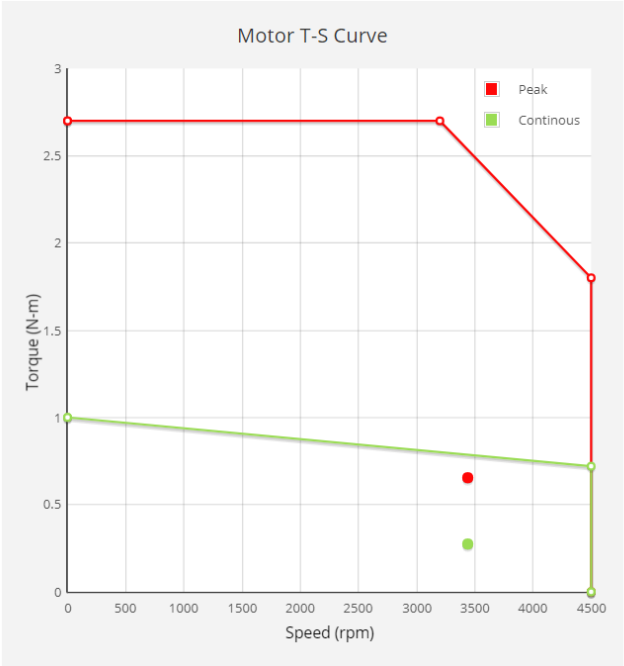
Generate the Torque - Speed Requirements for the axis in the table below. Click Submit button once you complete the table.

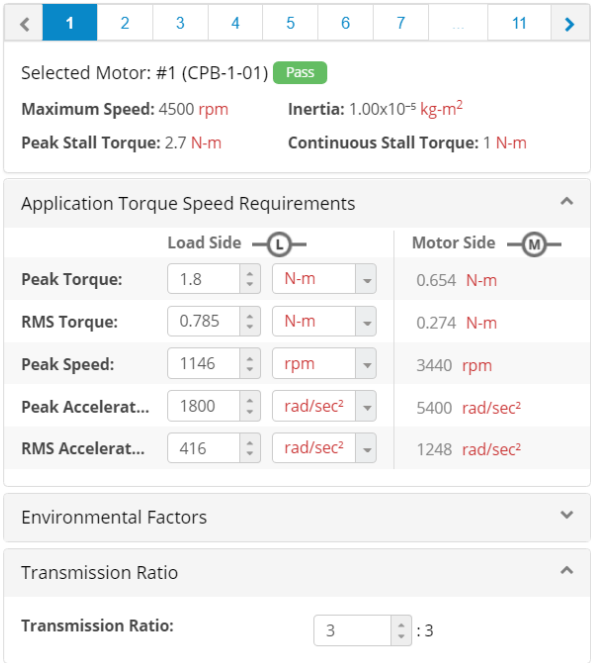
A1:D1 | fx | T-S Widget Data

	A	B	C	D
1	T-S Widget Data			
2	Segment	Total Torque² * Time [$T_{tot}^2.\Delta t$]	Acceleration² * Time [$a^2.\Delta t$]	
3				
4				
5				
6				
7				
8				
9				
10				
11	Total			
12				
13	Torque-Speed Requirements			
14	Peak Torque		Nm	
15	RMS Torque		Nm	
16	Peak Speed		rpm	
17	Peak Acceleration		rad/s²	
18	RMS Acceleration		rad/s²	
19				

Reset | Submit

Feed the values from above table into Torque -Speed widget given below. Adjust the transmission ratio to select the best suitable motor. Click Submit button once you complete the question.





Manual Motor Selection

Now refer the data to a sample motor (CPB-1-01) taking into account the motor's own inertia. Re-calculate the RMS torque without using the TS-Widget and enter a suitable transmission ratio so that the sample motor satisfies the Application's requirements.

A1:C1 | fx | Transmission Data

	A	B	C	D	E	F	G	H
1	Transmission Data							
2	Ratio		: 1					
3	Transmission Inertia	0.00032	kg-m²					
4								
5	Motor Data (CPB-1-02)							
6	Type	Value	Unit					
7	Maximum Speed	5000	rpm					
8	Peak Torque	4.4	Nm					
9	RMS Torque	1.7	Nm					
10	Inertia	1.30×10^{-4}	kg-m²					
11								
12							Rotary Motion - Motor Side	
13	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]	Total Inertia [$J_{...}$
14		(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)	(kg-m²)
15	1							
16	2							
17	3							
18	4							
19	5							
20	6							
21	7							
22	8							
23	Total			0				
24								
25	RMS Torque		Nm					
26	J Ratio Minimum		: 1					
27	J Ratio Maximum		: 1					
28	J Range		: 1					
29								

Reset Submit

8.5: Axis 4: Rotary Index Table Assessment

Selecting Suitable Motor for Axis 4

The table indexes a quarter revolution in four steps to stop at load, two locations for component processing and unload stations. For the smaller payload, a smaller table is possible, and a new welding process allows a shorter weld time, but the much shorter cycle time means we need to index the table as quickly as possible.

Axis Specifications

- Component mass: 3 x 10 kg
- Component mounting radius: 0.4 m
- Mass of Table: 60 kg (Assuming it is a uniform disk)
- Diameter of Table: 1 m
- Move time: 0.3 sec
- Dwell time: 1.7 sec
- Move distance: 1/4 revolution
- Table inertia: 7.5 kg-m^2

Gathering all the known entities and data points into Mechanical Parameters and Profile Parameters tables:

Mechanical Parameters		
Name	**Value**	**Unit**
Component mass	30	kg
Component mounting radius	0.4	m
Table Diameter	1	m
Table Mass	60	kg
Rotary friction loss	5	Nm
Table Inertia	7.5	kg-m^2

Table 1

Profile Parameters		
Name	**Value**	**Unit**
Move	1.57	rad
Total Move Time	0.3	s
Average Velocity	5.24	rad/s
Peak Velocity	7.85	rad/s
Dwell	1.7	s

Table 2

Compute the peak operating points for the axis and create a Rotary Profile table from above specification details using the below table. Click Submit button once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H	I
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Total Inertia [J_{tot}]	External To
2		(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)		(kg-m^2)	(N
3	1								
4	2								
5	3								
6	4								
7	Total			0		0			
8									

Reset | Submit

T-S Widget Motor Selection

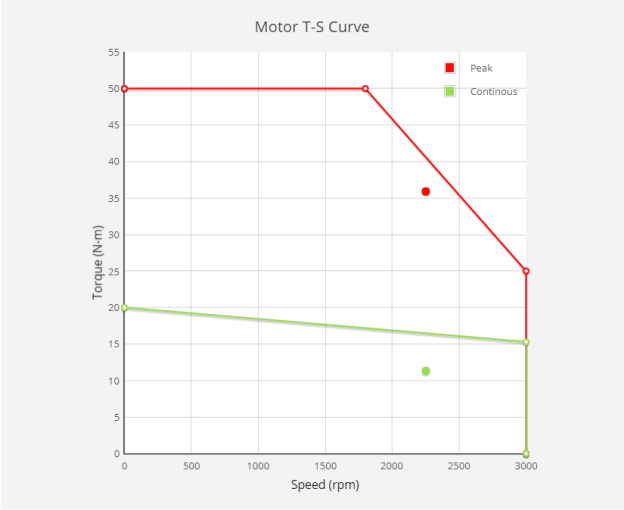
Generate the Torque - Speed Requirements for the axis in the table below. Click Submit button once you complete the table.

A1:D1 | fx | T-S Widget Data

	A	B	C	D
1	T-S Widget Data			
2	Segment	Total Torque^2 * Time [$T_{tot}^2.\Delta t$]	Acceleration^2 * Time [$a^2.\Delta t$]	
3				
4				
5				
6				
7	Total			
8				
9	Torque-Speed Requirements			
10	Peak Torque		Nm	
11	RMS Torque		Nm	
12	Peak Speed		rpm	
13	Peak Acceleration		rad/s^2	
14	RMS Acceleration		rad/s^2	
15				

Reset | Submit

Feed the values from above table into Torque -Speed widget given below. Adjust the transmission ratio to select the best suitable motor. Click Submit button once you complete the question.



1 ... 5 6 7 8 9 10 11

Selected Motor: #9 (CPB-4-01) Pass

Maximum Speed: 3000 rpm
Inertia: 0.0015 kg-m²
Peak Stall Torque: 50 N-m
Continuous Stall Torque: 20 N-m

Application Torque Speed Requirements

	Load Side		Motor Side
Peak Torque:	971	N-m	35.9 N-m
RMS Torque:	306	N-m	11.3 N-m
Peak Speed:	75	rpm	2251 rpm
Peak Acceleration:	78.5	rad/sec²	2356 rad/sec²
RMS Acceleration:	24.8	rad/sec²	745 rad/sec²

Environmental Factors

Transmission Ratio

Transmission Ratio: 30 : 30

Manual Motor Selection

Now refer the data to a sample motor (CPB-4-01) taking into account the motor's own inertia. Re-calculate the RMS torque without using the TS-Widget and enter a suitable transmission ratio so that the sample motor satisfies the Application's requirements.

A1:C1 | fx | Transmission Data

	A	B	C	D	E	F	G
1	Transmission Data						
2	Ratio		: 1				
3	Transmission Inertia	0	kg-m²				
4							
5	Motor Data (CPB-4-01)						
6	Type	Value	Unit				
7	Maximum Speed	3000	rpm				
8	Peak Torque	50	Nm				
9	RMS Torque	20	Nm				
10	Inertia	0.0015	kg-m²				
11							
12							Rotary Motion - Mot
13	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]	Load Inertia [J_L]
14		(rpm)	(rpm)	(s)	(rad/s²)	(Nm)	(kg-m²)
15	1						
16	2						
17	3						
18	4						
19	Total			0			
20							
21	RMS Torque		Nm				
22	J Ratio Minimum		: 1				
23	J Ratio Maximum		: 1				
24	J Range		: 1				
25							

Reset | Submit

8.6: Axis 5: Welder Rotate Assessment

Selecting Suitable Motor for Axis 5

An auto loaded stud is pressed into the component with a high friction torque so that the material melts and the parts are fused together. A new process and a smaller stud allows the weld time to be reduced to 1.7 seconds, and the spindle can be made smaller due to a reduced welding pressure requirement.

Axis Specifications

- Rotation speed: 2000 rpm
- Torque: 10 Nm
- Rotation time: 1.7 sec
- Dwell time: 0.3 sec

Gathering all the known entities and data points into Mechanical Parameters and Profile Parameters tables:

Mechanical Parameters		
Name	**Value**	**Unit**
Spindle Inertia	0.01	kg-m^2
Rotary friction loss	10	Nm

Table 1

Profile Parameters		
Name	**Value**	**Unit**
Acceleration/Deceleration Time	0.25	s
Cruise Time	1.2	s
Peak Velocity	2000	rpm
Dwell	0.3	s

Table 2

Compute the peak operating points for the axis and create a Rotary Profile table from above specification details using the below table. Click Submit button once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Total Inertia [J_{tot}]
2		(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)		(kg-m^2)
3	1							
4	2							
5	3							
6	4							
7	Total			0		0		
8								

Reset | Submit

T-S widget Motor Selection

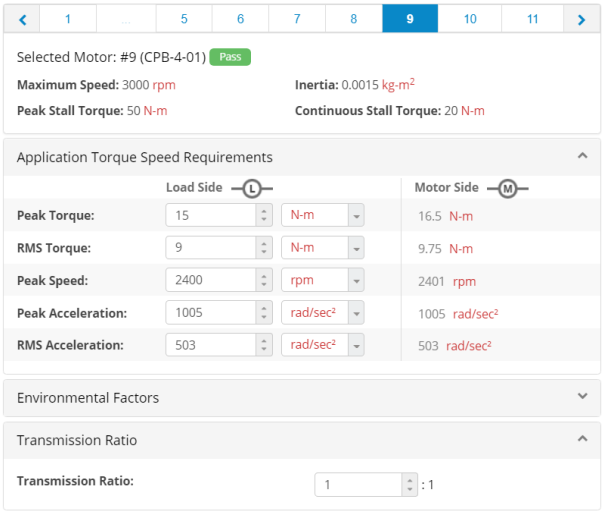
Generate the Torque - Speed Requirements for the axis widget with a suitable ratio (which will again be between 1 and 2 due to the speed) in the table below. Click Submit button once you complete the table.

A1:D1 | fx | T-S Widget Data

	A	B	C	D
1	T-S Widget Data			
2	Segment	Total Torque^2 * Time [$T_{tot}^2.\Delta t$]	Acceleration^2 * Time [$a^2.\Delta t$]	
3				
4				
5				
6				
7	Total			
8				
9	Torque-Speed Requirements			
10	Peak Torque		Nm	
11	RMS Torque		Nm	
12	Peak Speed		rpm	
13	Peak Acceleration		rad/s^2	
14	RMS Acceleration		rad/s^2	
15				

Reset | Submit

Feed the values from above table into Torque -Speed widget given below. Adjust the transmission ratio to select the best suitable motor. Click Submit button once you complete the question.



Manual Motor Selection

Now refer the data to a sample motor (CPB-4-01) taking into account the motor's own inertia. Re-calculate the RMS torque without using the TS-Widget and enter a suitable transmission ratio so that the sample motor satisfies the Application's requirements.

A1:C1 | fx | Transmission Data

	A	B	C	D	E	F
1	Transmission Data					
2	Ratio		: 1			
3	Transmission Inertia	0	kg-m²			
4						
5	Motor Data (CPB-4-01)					
6	Type	Value	Unit			
7	Maximum Speed	3000	rpm			
8	Peak Torque	50	Nm			
9	RMS Torque	20	Nm			
10	Inertia	0.0015	kg-m²			
11						
12						Rot:
13	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]
14		(rpm)	(rpm)	(s)	(rad/s²)	(Nm)
15	1					
16	2					
17	3					
18	4					
19	Total			0		
20						
21	RMS Torque		Nm			
22	J Ratio Minimum		: 1			
23	J Ratio Maximum		: 1			
24	J Range		: 1			
25						

Reset | Submit

8.7: Axis 6: Welder Feed (Ball Screw) Assessment

Selecting Suitable Motor for Axis 6

This mechanism presses the auto-loaded stud so that friction welding occurs. The smaller stud and the new welding process allows a reduced sliding mass and welding pressure, but the reduced time requires more rapid moves

Axis Specifications

- Moving mass: 10 kg
- Distance: 20 mm
- Move Time: Down - 0.1 sec, Dwell - 1.5 sec, Up - 0.1 sec, Dwell - 0.3 sec
- Weld Force: 6000 N
- Ball screw lead: 10 mm
- Ball screw Inertia: 5E-6 kg-m^2
- Rotary friction may be ignored

Gathering all the known entities and data points into Mechanical Parameters and Profile Parameters tables:

Mechanical Parameters		
Name	**Value**	**Unit**
Sliding Mass	10	kg
Friction Coefficient	0	per unit
Inclination	90	degrees
Ballscrew Lead	0.01	m
Ballscrew Inertia	5.00×10^{-6}	kg-m^2
Rotary Friction Loss	0	Nm
Weld Force	6000	N

Table 1

Profile Parameters		
Name	**Value**	**Unit**
Fast-feed Distance	-0.02	m
Fast-feed time	0.1	s
Average Velocity	-0.2	m/s
Peak Velocity	-0.3	m/s
Dwell (pressure) Time	1.5	s
Return Time	0.1	s
Dwell Time	0.3	s

Table 2

Create a Linear Profile table from above specification details using the below table. Click Submit button once you complete the table.

Remember that the positive move is taken to be downward hence the negative feed distance

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H	I
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External Force
2		(m/s)	(m/s)	(s)	(m/s^2)	(m)		(kg)	(N)
3	1								
4	2								
5	3								
6	4								
7	5								
8	6								
9	7								
10	8								
11	Total			0		0			
12									

Reset | Submit

The effective radius remains the same.

Effective Radius	0.00125	m

Compute the peak operating points for the axis and complete the equivalent Rotary Profile table below. Click Submit once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque
2		(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)	(kg-m^2)	(Nm)
3	1							
4	2							
5	3							
6	4							
7	5							
8	6							
9	7							
10	8							
11	Total			0				
12								

Reset | Submit

T-S Widget Motor Selection

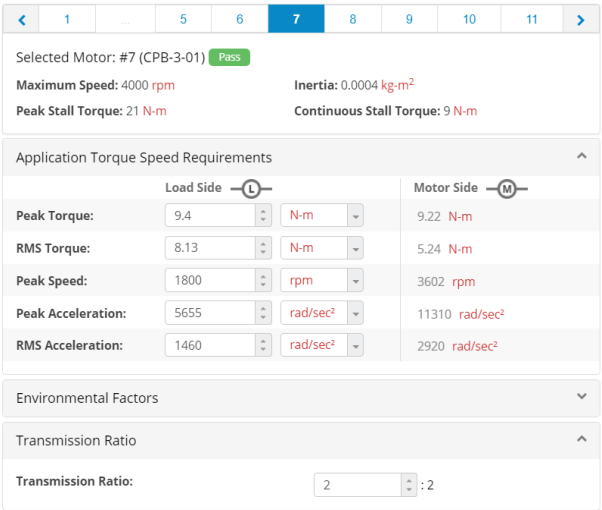
Generate the Torque - Speed Requirements for the axis - consider gearing this time in the table below. Click Submit button once you complete the table.

A1:D1 | fx | T-S Widget Data

	A	B	C	D
1	T-S Widget Data			
2	Segment	Total Torque^2 * Time [$T_{tot}^2.\Delta t$]	Acceleration^2 * Time [$a^2.\Delta t$]	
3				
4				
5				
6				
7				
8				
9				
10				
11	Total			
12				
13	Torque-Speed Requirements			
14	Peak Torque		Nm	
15	RMS Torque		Nm	
16	Peak Speed		rpm	
17	Peak Acceleration		rad/s^2	
18	RMS Acceleration		rad/s^2	
19				

Reset | Submit

Feed the values from above table into Torque -Speed widget given below. Adjust the transmission ratio to select the best suitable motor. Click Submit button once you complete the question.



Manual Motor Selection

Now refer the data to a sample motor (CPB-3-01) taking into account the motor's own inertia. Re-calculate the RMS torque without using the TS-Widget and enter a suitable transmission ratio so that the sample motor satisfies the Application's requirements.

A1:C1 | fx | Transmission Data

	A	B	C	D	E	F
1	Transmission Data					
2	Ratio		: 1			
3	Transmission Inertia	0	kg-m²			
4						
5	Motor Data (CPB-3-01)					
6	Type	Value	Unit			
7	Maximum Speed	4000	rpm			
8	Peak Torque	21	Nm			
9	RMS Torque	9	Nm			
10	Inertia	0.0004	kg-m²			
11						
12						Rotary Mo
13	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Load Torque [T_L]
14		(rpm)	(rpm)	(s)	(rad/s²)	(Nm)
15	1					
16	2					
17	3					
18	4					
19	5					
20	6					
21	7					
22	8					
23	Total			0		
24						
25	RMS Torque		Nm			
26	J Ratio Minimum		: 1			
27	J Ratio Maximum		: 1			
28	J Range		: 1			
29						

Reset | Submit

8.8: Axis 7: Cap Applicator (Rack & Pinion) Assessment

Selecting Suitable Motor for Axis 7

A plastic cap is applied to the stud to protect the thread. It moves down to press the cap over the stud then returns using a ball screw for smooth motion. The same mechanical setup is retained to save a redesign and gain economy of scale, but of course the move is quicker

Axis Specifications

- Distance: 100 mm
- Time: 0.5 sec each way (no dwell)
- Dwell: 1 sec

- Vertical Slide mass: 10 kg
- Friction Coefficient: 0.1
- Rotary Friction: 0.1 Nm
- Pinion PCD: 20 mm
- Pinion Inertia: 7.8E-6 kg-m²

Gathering all the known entities and data points into Mechanical Parameters and Profile Parameters tables:

Mechanical Parameters		
Name	**Value**	**Unit**
Slide mass	10	kg
Friction coefficient	0.1	per unit
Inclination	90	degrees
Pinion PCD	0.02	m
Pinion inertia	7.81×10^{-6}	kg-m²
Rotary friction loss	0.1	Nm

Table 1

Profile Parameters		
Name	**Value**	**Unit**
Move	0.1	m
Move time (down)	0.5	s
Average Velocity	0.2	m/s
Peak Velocity	0.3	m/s
Dwell	0	s
Move time (up)	0.5	s
Average Velocity	-0.2	m/s
Peak Velocity	-0.3	m/s
Dwell	1	s

Table 2

Create a Linear Profile table from above specification details using the below table. Click Submit button once you complete the table.

A1:A2 | fx | Segment

	A	B	C	D	E	F	G	H	I	J
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Sign of direction	Mass	External Force	Friction
2		(m/s)	(m/s)	(s)	(m/s²)	(m)		(kg)	(N)	(N
3	1									
4	2									
5	3									
6	4									
7	5									
8	6									
9	7									
10	8									
11	Total			0		0				
12										

Effective Radius	0.01	m

Compute the peak operating points for the axis and complete the equivalent Rotary Profile table below. Click Submit once you complete the table.

A1:A2 fx Segment

	A	B	C	D	E	F	G	H
1	Segment	Initial Velocity [u]	Final Velocity [v]	Time [t]	Acceleration [a]	Distance [s]	Load Inertia [J_L]	External Torque
2		(rad/s)	(rad/s)	(s)	(rad/s^2)	(rad)	(kg-m^2)	(Nm)
3	1							
4	2							
5	3							
6	4							
7	5							
8	6							
9	7							
10	8							
11	Total			0				
12								

T-S Widget Motor Selection

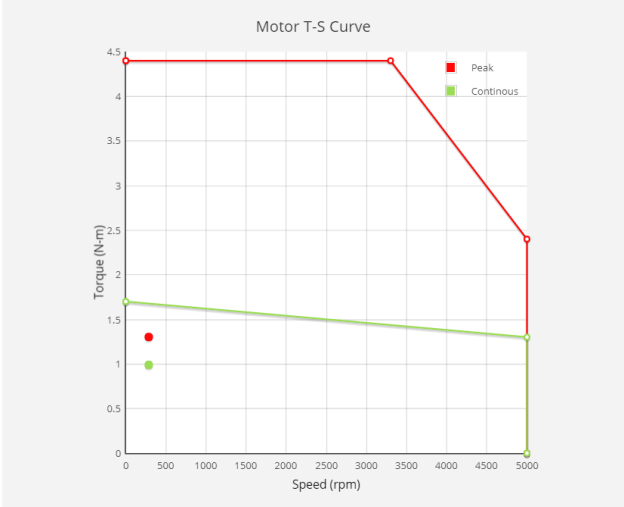
Generate the Torque - Speed Requirements for the axis in the table below. Click Submit button once you complete the table.

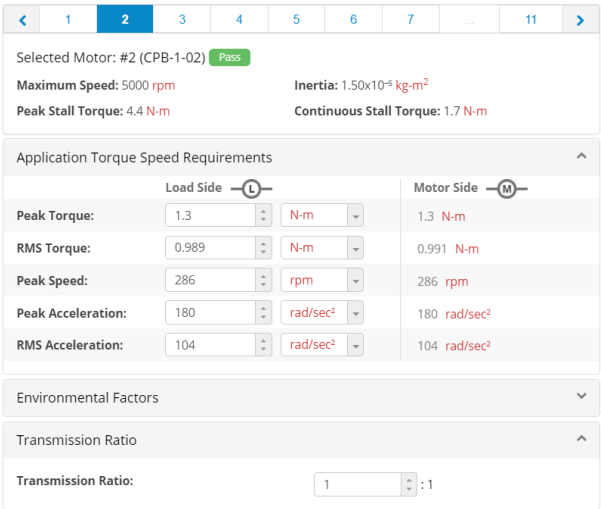
A1:D1 | fx | T-S Widget Data

	A	B	C	D
1	T-S Widget Data			
2	Segment	Total Torque^2 * Time [$T_{tot}^2.\Delta t$]	Acceleration^2 * Time [$a^2.\Delta t$]	
3				
4				
5				
6				
7				
8				
9				
10				
11	Total			
12				
13	Torque-Speed Requirements:			
14	Peak Torque		Nm	
15	RMS Torque		Nm	
16	Peak Speed		rpm	
17	Peak Acceleration		rad/s^2	
18	RMS Acceleration		rad/s^2	
19				

Reset | Submit

Feed the values from above table into Torque -Speed widget given below. Adjust the transmission ratio to select the best suitable motor. Click Submit button once you complete the question.





Manual Motor Selection

Now refer the data to a sample motor (CPB-1-01) taking into account the motor's own inertia. Re-calculate the RMS torque without using the TS-Widget and enter a suitable transmission ratio so that the sample motor satisfies the Application's requirements.