Unit 5: Push down Automata

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DATE

* Introduction to Push Down Automata (PDA)

PDA is a machine that accepts the languages formed by type-2 grammar i.e. Context free grammar. It can be considered as ε-NFA with the addition of the data-structure called stack which acts as the memory for PDA. Stack is based on last in first out. PDA is more powerful than finite state machines.

Informally PDA is an abstract machine consisting of
- Input tape
- Finite state control
- A stack

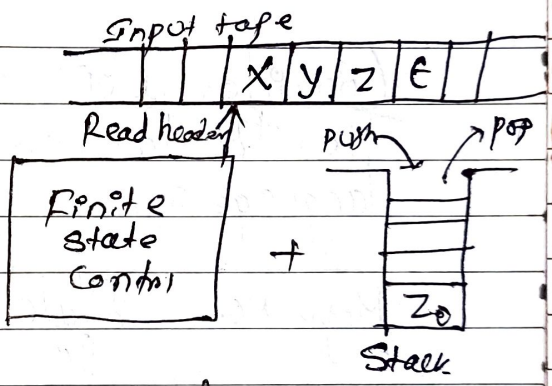

fig: Block diagram of PDA

Moves of PDA

Each moves of the machine is determined by:
- The current state (Q)
- Next input symbol (Σ)
- Symbol in the top of stack (Γ)

The moves consist of either changing state / staying on the same state & replacing the stack top by string of zero or more symbols.
Push and pop operation can be done in stack.

Formally, A PDA is defined by seven tuple $(Q, \Sigma, \delta, q_0, f, \Gamma, Z_0)$ where

Q - Finite set of states
q_0 = Start state
Σ - Finite set of input symbols
δ - Transition function: $Q \times \{\Sigma \cup \epsilon\} \times \Gamma \rightarrow Q \times \Gamma^*$
Γ - Stack alphabet (Push into stack)
Z_0 - Stack start symbol (Initial Stack Symbol)
f - Set of final states

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PAGE

DATE

Note: If $r^* = \epsilon$ then stack is popped.
If $r^* = r$ then stack is unchanged
If $r^* = yz$ then r is replaced by z and y is pushed into the stack.

* Representation of PDA.

PDA can be represented by using two techniques

1) Transition table

Consider a CFL defined by a grammar $a^n b^n$. Then the transition table used to represent this language is:

Move No.-	State	Input	Stack symbol	Move(s)
1	q_0	a	z_0	(q_0, az)
2	q_0	a	a	(q_0, aa)
3	q_0	b	a	(q_1, ϵ)
4	q_1	b	a	(q_1, ϵ)
5	q_1	ϵ	z_0	(q_f, z_0)

Here, q_0 is starting state of PDA, q_f is final state of PDA and z_0 is the initial stack symbol

DATE

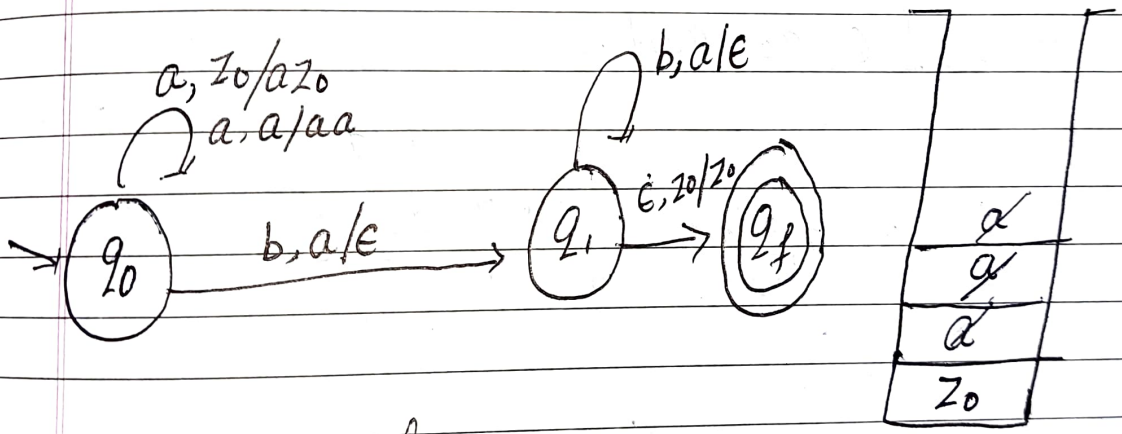
2) Transition diagram

We can use transition diagram to represent a PDA, where

- Any state is represented by a node.
- Any arrow labeled with "start" indicates the start state & doubly circle state are accepting/final state
- The arc corresponds to transition of PDA i.e. X/α means: $\delta(q, a, X) = (p, \alpha)$ ~~from arc for arc state~~ for arc from state p to q.

Let's take an example of $L = \{ a^n b^n / n \geq 1 \}$

⇒ Lets take: a a a b b b ϵ



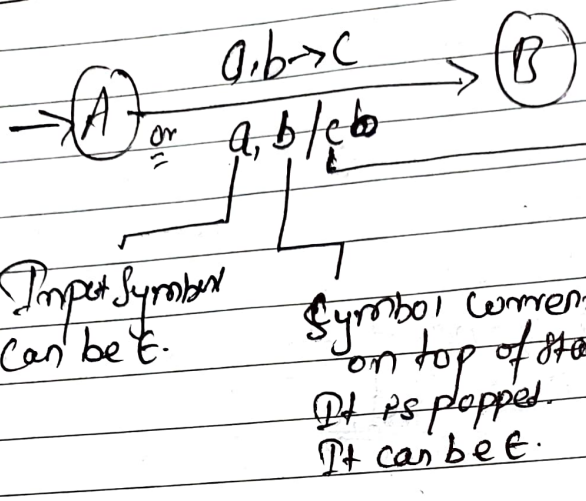
Stack Transition function:-

$\delta(q_0, a, Z_0) = (q_0, a Z_0)$
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_1, \epsilon)$
$\delta(q_1, b, a) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

PAGE

DATE

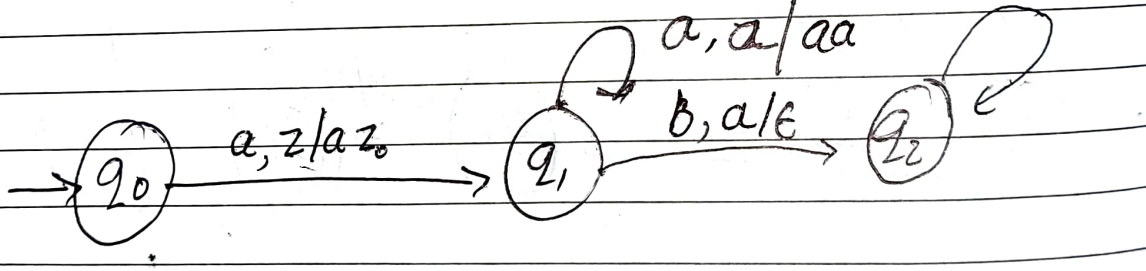
* Graphical Notation of PDA



Let's take another Example of PDA

$L = \{ a^n b^n \mid n \geq 0 \}$

$\therefore L = \{ \epsilon, ab, aabb, \ldots\ldots \}$



Transition function :

$\delta(q_0, a, z_0) = (q_1, az)$

$\delta(q_1, a, a) = (q_1, aa)$

$\delta(q_1, b, a) = (q_2, \epsilon)$

$\delta(q_2, b, a) = (q_2, \epsilon)$

$\delta(q_0, \epsilon, z_0) = (q_f, z_0)$

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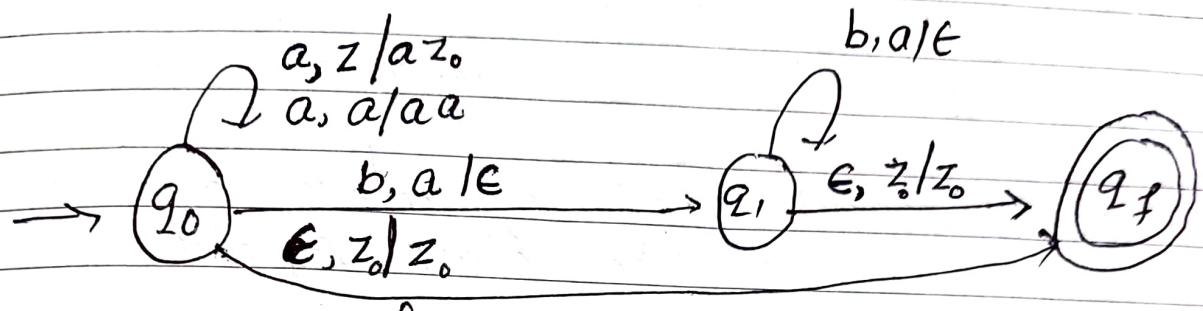
PAGE

Let's take another example of PDA

$L = \{a^n b^n \mid n \geq 0\}$



Transition function is defined a

$\delta(q_0, a, z_0) = (q_0, az_0)$
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_1, \epsilon)$
$\delta(q_1, b, a) = (q_1, \epsilon)$
$\delta(q_0, \epsilon, z_0) = (q_f, z_0)$
$\delta(q_0, \epsilon, z_0) = (q_f, z_0)$

Example:

Note: If given stack alphabet $\Gamma = \{B, G\}$ use this:-

$\delta(q_0, a, z_0) = (q_0, Bz)$
$\delta(q_0, a, B) = (q_0, BB)$
$\delta(q_0, b, B) = (q_1, \epsilon)$
$\delta(q_1, b, B) = (q_1, \epsilon)$
$\delta(q_0, \epsilon, z_0) = (q_f, z)$

Let's derive a string (Instantaneous description)

$(q_0, aabb, z_0) \vdash (q_0, abb, az_0)$
$\vdash (q_0, bb, aaz_0) \vdash (q_1, b, az_0)$
$\vdash (q_1, \epsilon, z_0) \vdash (q_f, z_0)$

which is accepted

DATE

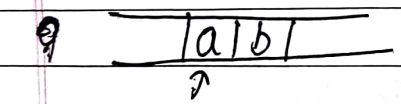
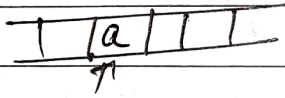
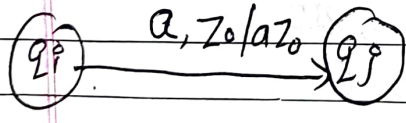
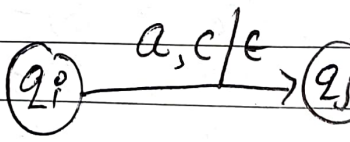
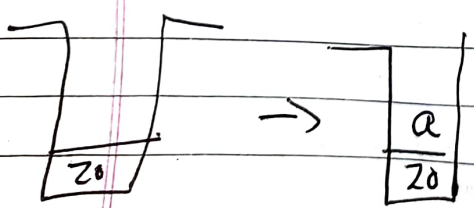
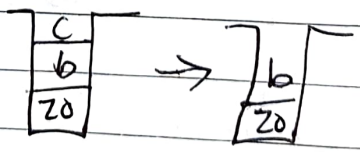
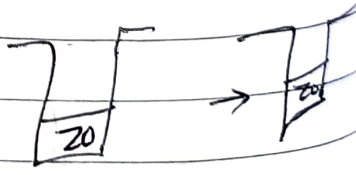
* Operations on PDA.

In PDA the stack head always scans the top symbol of the stack. It performs two basic operations & one extra operation.

1) Push operation :- Push operations add a new symbol from the stack symbol 'Γ' at the top of the stack.

2) Pop operation :- Pop operations remove the top symbol from the stack.

3) Skip operation :- It neither removes nor adds symbol from in stack.

Push	Pop	Skip
		
		
		
$\delta(q_0, a, Z_0) = \delta(q_1, aZ_0)$	$\delta(q_1, a, c) = \delta(q_2, \epsilon)$	$\delta(q_i, a, Z_0) = \delta($

DATE

* Instantaneous Description for PDA. (ID)

Any configuration of a PDA can be described by a triplet (q, w, r) where.

q - is the state (current)

w - is the remaining input

r - is the stack contents (current)

Such a description by triple is called ID of PDA. It is helpful for describing changes in the state, input and stack of the PDA.

ID is an informal notation of how a PDA "computes" a input string and make a decision that string is accepted or rejected.

Formally, Let $P = \{Q, \Sigma, \delta, q_0, F, \Gamma, Z_0\}$ be a PDA. Then, we define a relation $\vdash$ "yields" as:

$(q, aw, z\alpha) \vdash (p, w, \beta\alpha)$ if $\delta(q, a, z) = (p, \beta)$ and a may be $\in$)

Eg: For the PDA described earlier for $L = \{a^n b^n \mid n \geq 1\}$ lets check accepting sequence of ID's for string aabb.

NOTE: don't write δ here.

$(q_0, aabb, Z_0) \vdash (q_0, abb, aZ_0)$

$\vdash (q_0, bb, aaZ_0)$

$\vdash (q_1, b, aZ_0)$

$\vdash (q_1, \in, Z_0)$

$\vdash (q_f, \in, Z_0)$ Accepted.

PAGE

DATE

* Language of PDA

It refers to the set of strings that is accepted by the PDA. The acceptance of any string by a PDA can be defined in two ways.

1) Acceptance by final state.

According to this, language of PDA is set of all string that causes PDA to enter in its accepting state after consuming all the alphabets of the string.

Given a PDA P, the language accepted by final state, L(P) is:

$\{ w \mid (q, w, z_0) \vdash^* (q', \epsilon, \gamma) \}$ where $q' \in F$ and $\gamma \in \Gamma^*$.

2) Acceptance by empty stack

According to this language of PDA is a set of all string that causes PDA to enter in its accepting state and emptying its stack after consuming all the alphabets of the string.

Given a PDA P, the language accepted by empty stack, L(P) is

$\{ w \mid (q, w, z_0) \vdash^* (q', \epsilon, \epsilon) \}$ where $q \in$

DATE

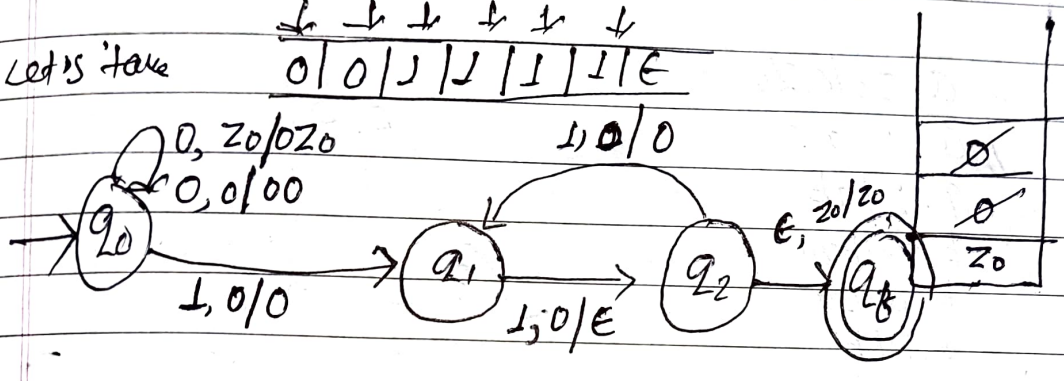
Some more examples of PDA

Q. $L = \{0^n 1^{2n}, n>0\}$

=) $L = \{011, 0011\,11, 000111111, \ldots\ldots\}$

Let's take

↓	↓	↓	↓	↓	↓
0	0	1	1	1	1	ϵ



$\therefore Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, Z_0\}$

$q_0 = \{q_0\}$

$F = \{q_f\}$

δ 's:

$\delta(q_0, 0, Z_0) = (q_0, 0Z_0)$

$\delta(q_0, 0, 0) = (q_0, 00)$

$\delta(q_0, 1, 0) = (q_1, 0)$

$\delta(q_1, 1, 0) = (q_2, \epsilon)$

$\delta(q_2, 1, 0) = (q_1, 0)$

$\delta(q_2, \epsilon, Z_0) = (q_f, Z_0)$

PAGE

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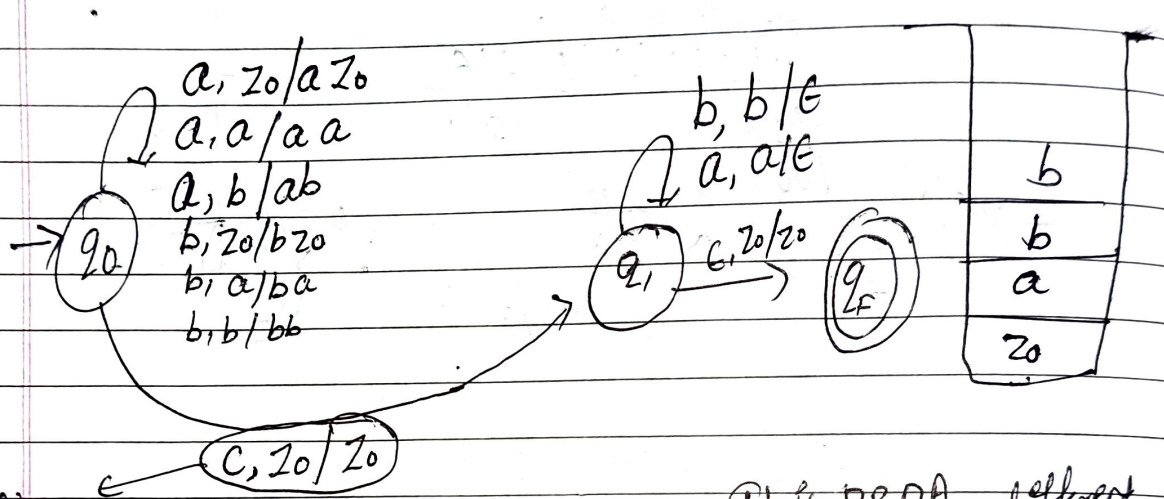
Q. $L = \{ wcw^R \mid w \in \{a,b\}^* \}$ (Palindrome)

$\Rightarrow$ $L = \{ \epsilon, ac, abcba, abbcbba, \text{-----} \}$

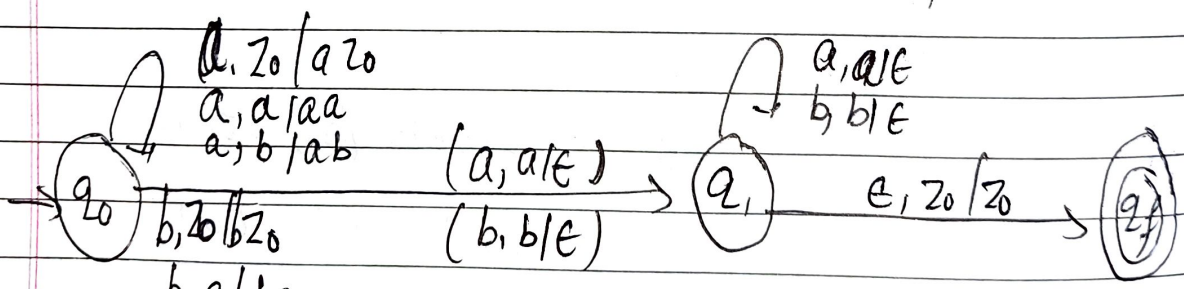
Let's take. | a | b | b | c | b | b | a | ε |



Note: If it was $\{a,b\}^+$ in the question, then no need to include this

Note: It is DPDA since there is different transition at each state

Q. $L = \{ wwR \mid w \in \{a,b\}^+ \}$ (even palindrome)



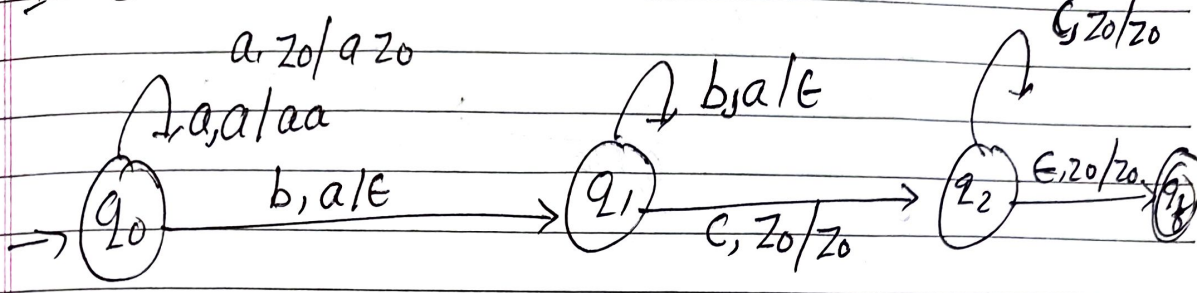
Note: It is NPDA as there are same transition (a, a/aa, a, a/ε)

Note: for $(a,b)^*$ connect q0 to qf directly with ε, Z0/Z0 or, you can also define ε, Z0/Z0 between q0 & q1

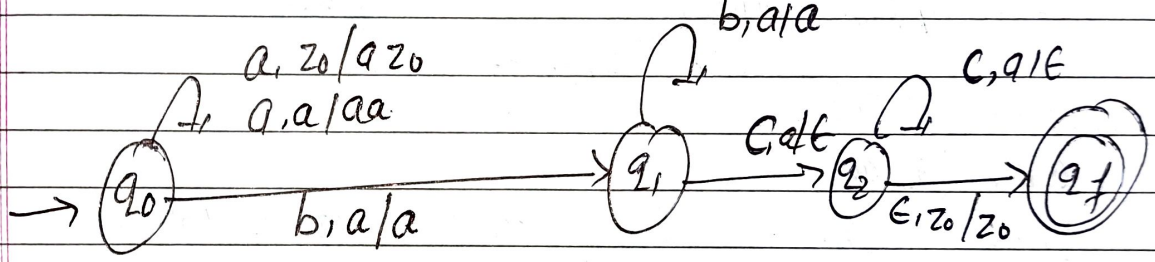
DATE

Q $L = a^n b^n c^m \mid n, m \geq 1$

⇒ It can be written as $a^n b^n c c^*$

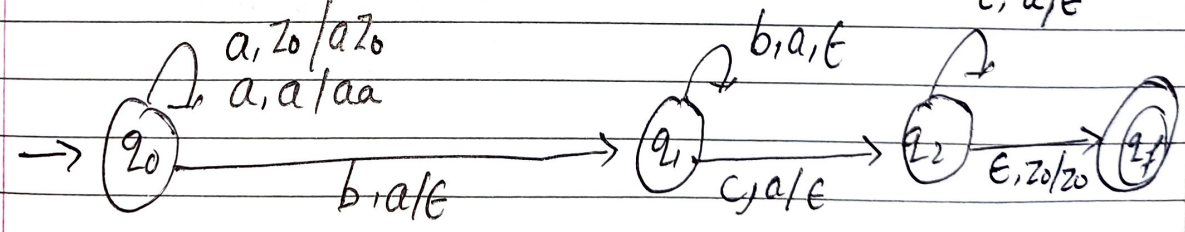


Q $L = a^n b^m c^n \mid m, n \geq 1$



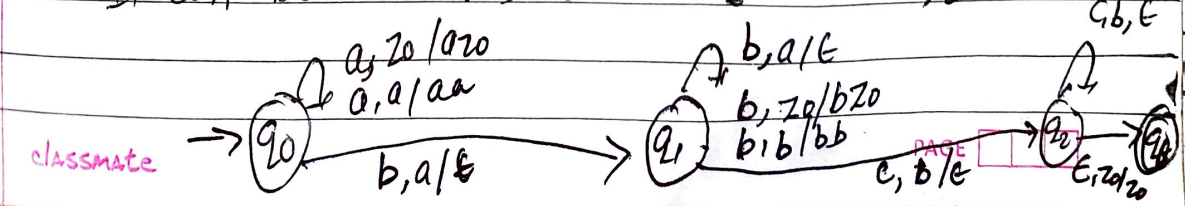
Q $L = a^{m+n} b^m c^n \mid m, n \geq 1$

$L = \{aabc, aaabbc, \ldots\}$



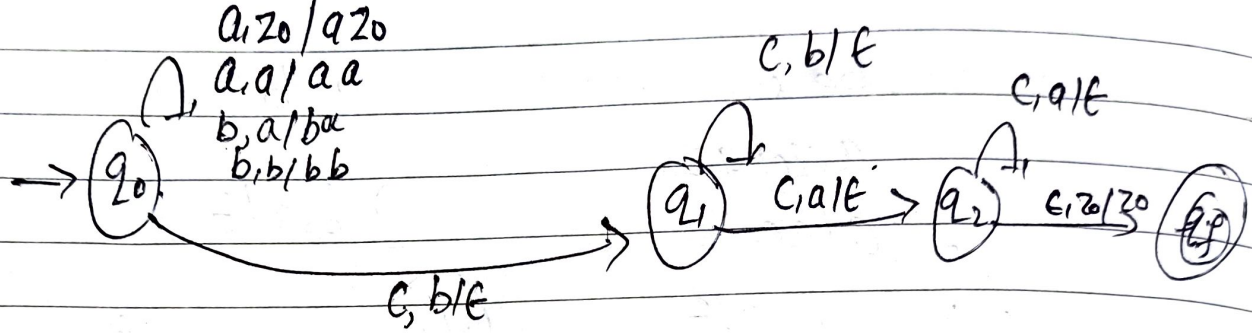
Q $L = a^n b^{m+n} c^m \mid m, n \geq 1$

It can be written as $a^n b^m b^n c^m$

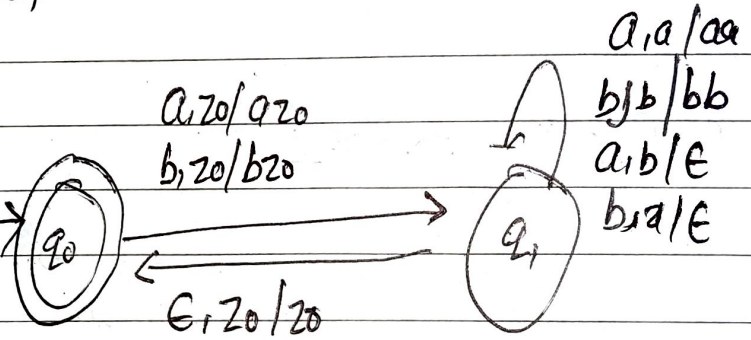


DATE

Q. $a^n b^m c^{m+n} \mid m,n \geq 1$



Q. Equal no. of a's and b's



$\delta(q_0, a, z_0) = (q_1, q z_0)$
$\delta(q_0, b, z_0) = (q_1, b z_0)$
$\delta(q_1, \epsilon, z_0) = (q_0, z_0)$
$\delta(q_1, a, a) = (q_1, aa)$
$\delta(q_1, b, b) = (q_1, bb)$
$\delta(q_1, a, b) = (q_1, \epsilon)$
$\delta(q_1, b, a) = (q_1, \epsilon)$
$\delta(q_1, b, z_0) = (q_0, b z_0)$
$\delta(q_1, a, z_0) = (q_0, a z_0)$

Lets check sequence of ID's for abba

$(q_0, abba, z_0) \vdash (q_1, bba, a z_0)$
$\vdash (q_1, ba, z_0)$
$\vdash (q_0, a, b z_0)$
$\vdash$

DATE

* Deterministic Push Down Automata (DPDA)

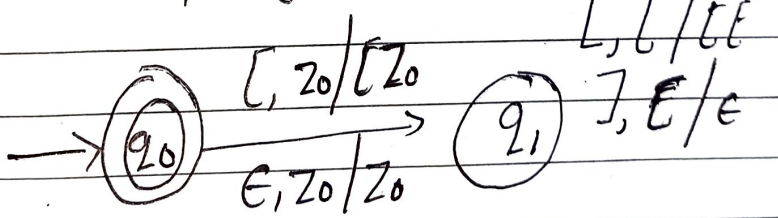
A PDA is said to be deterministic if for every input alphabet 'a' (a may be ϵ) and top of stack symbol, PDA has either unique transition (ie. moves only to one state) or its transition is not defined.

Formally, A PDA $P=\{Q, \Sigma, \delta, q_0, F, \Gamma, Z_0\}$ is deterministic if following two conditions are satisfied:

i) For any $q \in Q$, $X \in \Gamma$
if $\delta(q, a, X) \neq \phi$ for any $a \in \Sigma$ then
$\delta(q, \epsilon, X) = \phi$

ii) For any $q \in Q$, $a \in \Sigma \cup \{\epsilon\}$ and $X \in \Gamma$,
$\delta(q, a, X)$ has at most one element.

Ex: Construct DPDA that accepts balanced parenthesis. ie. equal no. opening & closing brackets. eg. [[ ]], [ ][ ]



PAGE

DATE

* Non-Deterministic PDA (NPDA)

A NPDA is basically a nfa with a stack added to it. It can have no unique transition

Formally, npda is a 7-tuple. $M = (Q, \Sigma, \delta, q_0, F, \Gamma, Z_0)$
where (explain all / same as PDA)

Transition function: $D: Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \rightarrow$ finite subset of $Q \times \Gamma^*$

DATE

Construction of PDA by Final state.

A PDA accepts a string when after reading the entire string in the final state. From the starting state, we can make moves that endup in a final state with any stack values. The stack values are irrelevant as long as we endup in a final state.

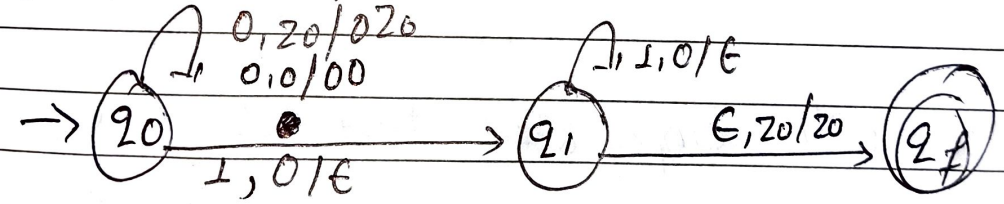
Let $P = (Q, \Sigma, \delta, q_0, F, \Gamma, Z_0)$ be a PDA. The language accepted by P by final state is:

$$L(P) = \{w \mid (q, w, Z_0) \vdash^* (q', \epsilon, r)\}$$

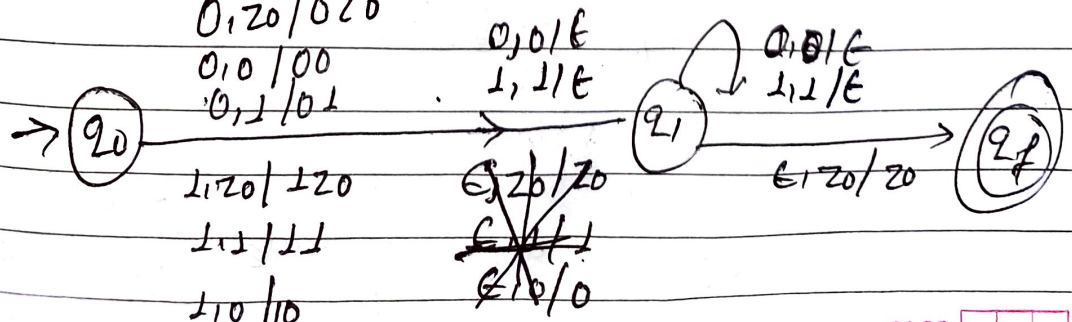
where, $q \in F$ for some state q' in F and any input stack string $r \in \Gamma^*$.

P Consumes w from the input & enters an accepting state.

Ex: PDA with final state with equal no. of 0's followed by equal no. of 1's. ie. $0^n 1^n \mid n \geq 1$



Ex: PDA with final state that accept the paindrom ie. $\{ww^R \mid w \in (0,1)^*\}$



DATE

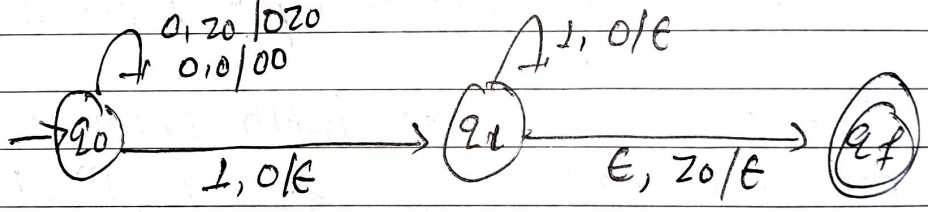
* Construction of PDA by Empty Stack

A PDA accepts a string when after reading the entire string, when PDA has emptied its stack. For a PDA $P: (Q, \Sigma, \delta, q_0, F, \Gamma, Z_0)$. The language accepted by empty stack is:

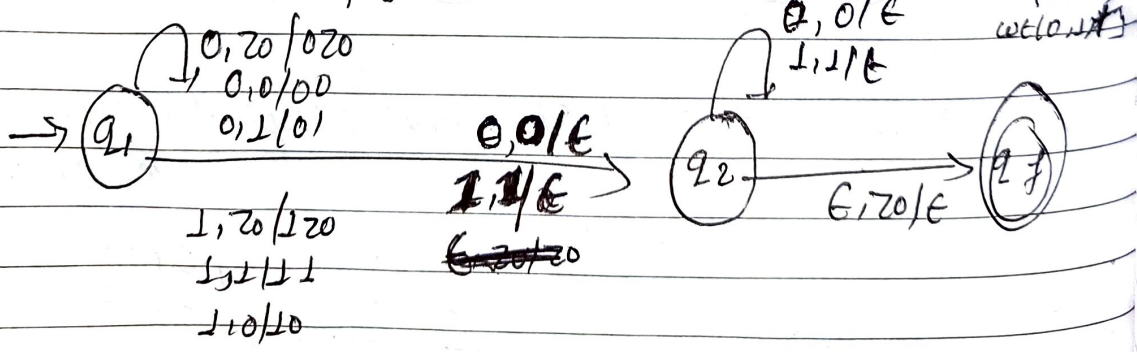
$$L(P): \{w \mid (q, w, Z_0) \vdash^* (q', \epsilon, \epsilon)\} \text{ where } q' \in Q$$

P can consume and at the same time empty the stack

Ex: PDA with empty stack with equal no. of zeros (0's) followed by equal no. of 1's.



Ex: PDA with empty stack that accept the palindrome (of even length)

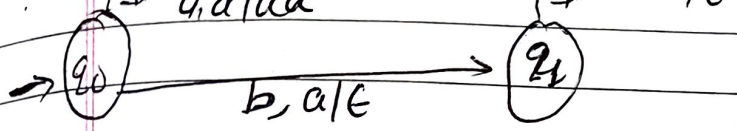


Note: You can also make q_f single circle since it doesn't need final state as the stack is empty.

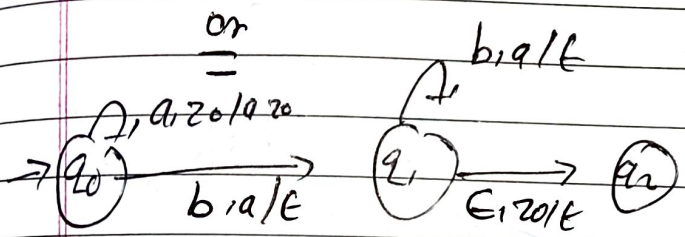
DATE

Ex: Design a PDA for $L = \{ a^n b^n / n \geq 1 \}$ both by empty stack & final state.

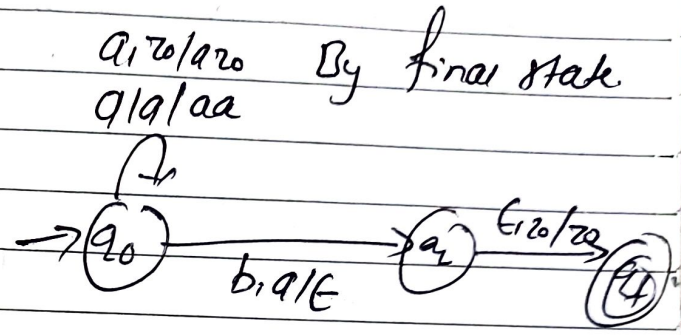
By empty stack



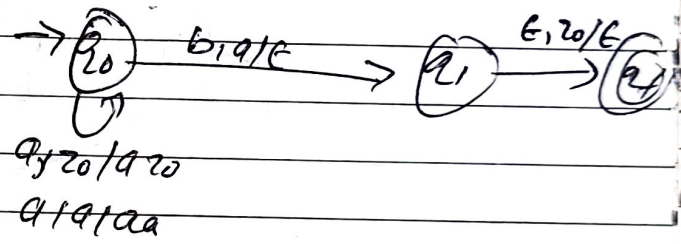
or



By final state

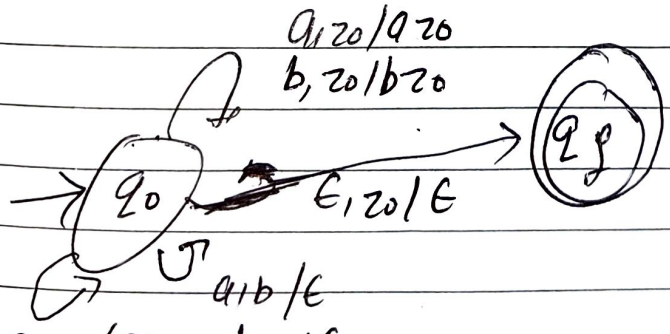


or



Ex: Design a PDA for $L = \{ w / n_a(w) = n_b(w) \}$

⇒ $L = \{ \epsilon, ab, ba, abba, abab, ababaabb, \ldots\ldots \}$



Lets check

$(q_0, abba, z_0)$

$\vdash (q_0, bba, az_0)$

$\vdash (q_0, ba, z_0)$

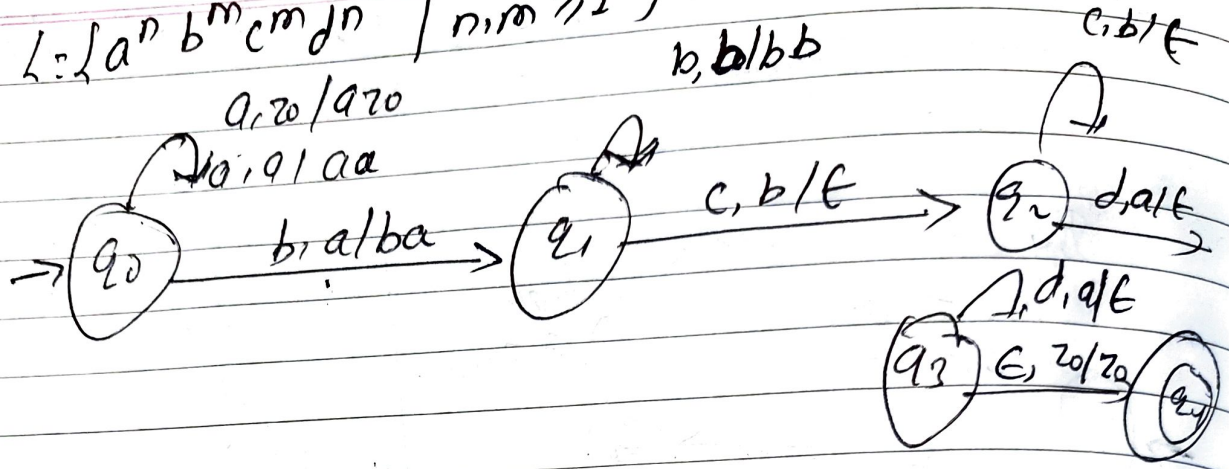
$\vdash (q_0, a, bz_0)$

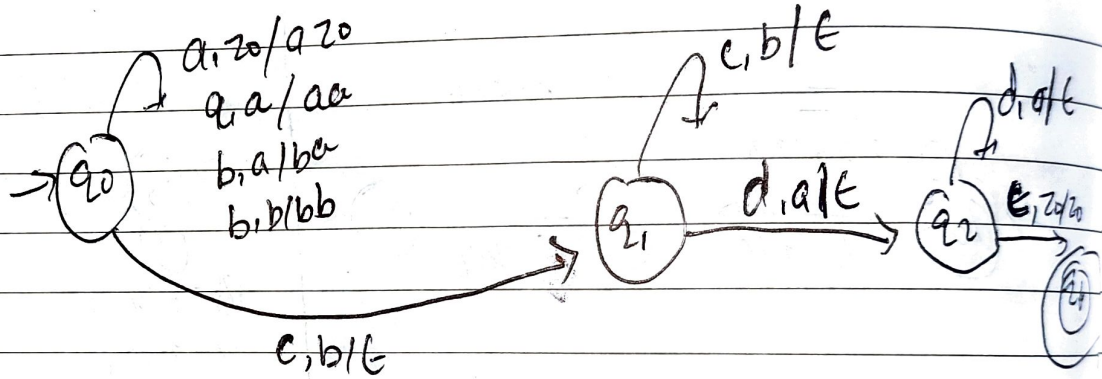
$\vdash (q_f, \epsilon, z_0)$

accepts

DATE

Ex: $L = \{a^n b^m c^m d^n \mid n, m \geq 1\}$





DATE

* Conversion of CFG to PDA.

Let, $G = (V, T, R, S)$ be a CFG, then we can construct PDA, accepts by empty stack. PDA,

$P = (\{q_0\}, T, VUT, \delta, q_0, S)$

where, transition function δ.

1. For each variable A

$$\delta(q_0, \epsilon, A) = \{(q_0, \beta) \mid A \to \beta \text{ is in } R\}$$

2. For each terminal a,

$$\delta(q_0, a, a) = (q_0, \epsilon)$$

Note: Given $P = (\{q_0\}, T, VUT, \delta, q_0, S)$

Compare with $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0)$

There is no final state F, as it is Empty.

$\therefore$ $Q = \{q_0\}$

$\Sigma = T$

$\Gamma = VUT$

$\delta = \delta$

$q_0 = q_0$

$z_0 = S$

where, $\{q_0\}$ = Set of state

q_0 = Start ~~Symbol~~ State

T = terminal symbol

VUT = variable (terminal & non-terminal)

PAGE

DATE

Ex. Construct a PDA for the following CFG & test whether "abbabb" is in N(P)

$G = (\{S, A\}, \{a, b\}, R, S)$

R: $S \to AA / a$

$A \to SA / b$

→ PDA, $P = (\{q_0\}, T, NUT, \delta, q, S)$

$= (\{q_0\}, \{a, b\}, \{S, A, a, b\}, \delta, q, S)$

Now, transition function δ is given by

1) For non-terminal: $\delta(q_0, \epsilon, S) = \{(q_0, AA), (q_0, a)\}$

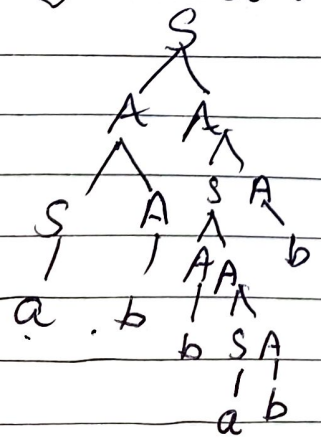
$\delta(q_0, \epsilon, A) = \{(q_0, SA), (q_0, b)\}$

2) For terminal

$\delta(q_0, a, a) = (q_0, \epsilon)$

$\delta(q_0, b, b) = (q_0, \epsilon)$

Now, let's derive abbabb using both CFG & PDA



So, abbabb is accepted.

$(q_0, abbabb, S)$

$\vdash (q_0, abbabb, AA)$

~~$\vdash (q_0, bbabb, SAA)$~~

$\vdash (q_0, abbabb, SAA)$

$\vdash (q_0, abbabb, aAA)$

$\vdash (q_0, bbabb, AA)$

$\vdash (q_0, bbabb, bA)$

$\vdash (q_0, babb, A)$

PAGE

$\vdash (q_0, baabb, SA)$
$\vdash (q_0, babb, AAA)$
$\vdash (q_0, abb, bAA)$
$\vdash (q_0, abb, AA)$
$\vdash (q_0, abb, aAA)$
$\vdash (q_0, bb, AA)$
$\vdash (q_0, bb, bA)$
$\vdash (q_0, b, A)$
$\vdash (q_0, b, b)$
$\vdash (q_0, \epsilon, \epsilon)$

So, abbabb is accepted by PDA by N(P) or Empty stack PDA.

Q: Convert the following CFG to PDA

$S \rightarrow aS \mid aA$
$A \rightarrow bA \mid b$

PDA. $P = (\{q_0\}, \{a, b\}, \{S, A, a, b\}, \delta, q_0, S)$

Transition function δ is

$\delta(q_0, \epsilon, S) = (q_0, aS)$
$\delta(q_0, \epsilon, S) = (q_0, aA)$
$\delta(q_0, \epsilon, A) = (q_0, bA)$
$\delta(q_0, \epsilon, A) = (q_0, b)$
$\delta(q_0, a, a) = (q_0, \epsilon)$
$\delta(q_0, b, b) = (q_0, \epsilon)$

DATE

Let's Check String aabb

$(q_0, aabb, S)$
$\vdash (q_0, aabb, aS)$
$\vdash (q_0, abb, S)$
$\vdash (q_0, abb, aA)$
$\vdash (q_0, bb, A)$
$\vdash (q_0, bb, bA)$
$\vdash (q_0, b, A)$
$\vdash (q_0, b, b)$
$\vdash (q_0, \epsilon, \epsilon)$

So, String aabb is accepted by PDA by empty stack

Q. Convert the following CFG to PDA

$E \rightarrow T \mid E+T$
$T \rightarrow F \mid T*F$
$F \rightarrow a \mid (E)$

⇒ PDA $P = \{ \{q_0\}, \{q, *, +, (,)\}, \{q, *, +, (,), E, T, F\}, \delta, q_0, E \}$

where δ can be defined as

$\delta(q_0, \epsilon, E) = (q_0, T)$
$\delta(q_0, \epsilon, E) = (q_0, E+T)$
$\delta(q_0, \epsilon, T) = (q_0, F)$
$\delta(q_0, \epsilon, T) = (q_0, T*F)$
$\delta(q_0, \epsilon, F) = (q_0, a)$
$\delta(q_0, \epsilon, F) = (q_0, (E))$

$\delta(q_0, a, a) = (q_0, \epsilon)$

$\delta(q_0, *, *) = (q_0, \epsilon)$

$\delta(q_0, +, +) = (q_0, \epsilon)$

$\delta(q_0, (, () = (q_0, \epsilon)$

$\delta(q_0,),)) = (q_0, \epsilon)$

Lets Check String $a+(a*a)$

Eg: Convert the grammar defined by production to PDA:

$S \rightarrow 0SL \mid A$

$A \rightarrow 1S0 \mid S \mid \epsilon$

⇒ PDA equivalent to this grammar

$P = (\{q_0\}, \{0, 1\}, \{0, 1, S, A\}, \delta, q_0, S)$

where $Q = \{q_0\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, S, A\}$

$Z_0 = S$

$F = \phi$

And transition function δ is defined as

$\delta(q_0, \epsilon, S) = (q_0, 0SL)$

$\delta(q_0, \epsilon, S) = (q_0, A)$

$\delta(q_0, \epsilon, A) = (q_0, 1S0)$

$\delta(q_0, \epsilon, A) = (q_0, S)$

$\delta(q_0, \epsilon, A) = (q_0, \epsilon)$

$\delta(q_0, 0, 0) = (q_0, \epsilon)$

$\delta(q_0, 1, 1) = (q_0, \epsilon)$

DATE

Ex: Construct a PDA equivalent to following grammer defined by

$S \to aAA$

$A \to aS \mid bS \mid a$

Sol. $G = (V, T, P, S)$ be the grammar defined by

$S \to aAA$

$A \to aS \mid bS \mid a$

Now, we can define the PDA equivalent to the (CFG)

$M = (\{q_0\}, \{a, b\}, \{q, b, S, A\}, \delta, q_0, S)$

Where, δ is defined as:

$\delta(q_0, \epsilon, S) = (q_0, aAA)$

$\delta(q_0, \epsilon, A) = \{(q_0, aS), (q_0, bS), (q_0, a)\}$

$\delta(q_0, a, a) = (q_0, \epsilon)$

$\delta(q_0, b, b) = (q_0, \epsilon\}$

DATE

Let's check acceptance of aaabaaaaa

$(q_0, aaabaaaaa, S)$
$\vdash (q_0, aaabaaaaa, aAA)$
$\vdash (q_0, aabaaaaa, AA)$
$\vdash (q_0, aabaaaaa, aSA)$
$\vdash (q_0, abaaaaa, SA)$
$\vdash (q_0, abaaaaa, aAAA)$
$\vdash (q_0, baaaaa, AAA)$
$\vdash (q_0, baaaaa, bSAA)$
$\vdash (q_0, aaaaa, SAA)$
$\vdash (q_0, aaaaa, aAAAA)$
$\vdash (q_0, aaaa, AAAA)$
$\vdash (q_0, aaaa, aAAA)$
$\vdash (q_0, aaa, AAA)$
$\vdash (q_0, aaa, aAA)$
$\vdash (q_0, aa, AA)$
$\vdash (q_0, aa, aA)$
$\vdash (q_0, a, A)$
$\vdash (q_0, a, a)$
$\vdash (q_0, \epsilon, \epsilon)$

Hence, it is accepted by Empty Stack PDA

In CFG,

$S \rightarrow aAA$
$\rightarrow aaSA$
$\rightarrow aaaAAA$
$\rightarrow aaabSAA$
$\rightarrow aaabaAAAA$
$\rightarrow aaabaaAAA$
$\rightarrow aaabaaaAA$
$\rightarrow aaabaaaaA$
$\rightarrow aaabaaaaa$

PAGE

classmate

DATE

Note: If CFG is in GNF, you can do by this

$S \to 0BB$

$B \to 0S \mid 1S \mid 0$

⇒ Here, context free grammar G is

$G = (\{S, B\}, \{0, 1\}, P, S)$

Corresponding PDA is

$M = (\{q\}, \{0, 1\}, \{V, S, B\}, \delta, q, \phi)$

where, δ is,

$\delta(q, 0, S) \to (q, BB)$

$\delta(q, 0, B) \to (q, S)$

$\delta(q, 1, B) \to (q, S)$

$\delta(q, 0, B) \to (q, \epsilon)$

DATE

Conversion of PDA to CFG

Given a PDA $M = (Q, \Sigma, \delta, q_0, F, \Gamma, Z_0)$, we can obtain an equivalent CFG, $G = (V, T, P \text{ and } S)$ which generates the same language as accepted by the PDA M.

→ The set of variables in the grammar consists of:

- The special symbol S, which is a start symbol.
- All the symbols of the form $[pXq]$; $p, q \in Q$ & $X \in \Gamma$
  i.e. $V = \{S\} \cup \{[pXq]\}$

→ The terminal in the grammar $T = \Sigma$

→ The production rule G is as follows:

- For all states $q \in Q$, $S \to [q_0 Z_0 q]$ is a production of G
- For any states $q, r \in Q$, $X \in \Gamma$ and $a \in \Sigma \cup \{\epsilon\}$

If $\delta(q, a, X) = (p, \epsilon)$ then $[pXq] \to a$

- If $\delta(q, a, X) = (r, Y_1 Y_2 \cdots Y_k)$; where $Y_1, Y_2, \cdots, Y_k \in \Gamma$ and $a \in \Sigma \cup \{\epsilon\}$ $k \geq 0$

Then for all lists of states $r_1, r_2, \cdots, r_k$, G has the production $[pXq] \to a [r Y_1 r_1][r_1 Y_2 r_2] \cdots [r_{k-1} Y_k r_k]$

DATE

Ex: Convert the following PDA to CFG.

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_1, a)$
$\delta(q_1, b, a) = (q_1, a)$
$\delta(q_1, a, a) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

:) Let $G = (V, T, P, S)$ be the equivalent CFG for the given pda where

$V = \{S\}$ $\quad$ S = Start state.

$T = \Sigma$

And P is defined by the following productions

(1) $S \rightarrow (q_0, Z_0, q_0)$
(2) $S \rightarrow (q_0, Z_0, q_1)$

For, $\delta(q_0, a, Z_0) = (q_0, aZ_0)$

(3) $[q_0, Z_0, q_0] \Rightarrow a\,[q_0, a, q_0]\,[q_0, Z_0, q_0]$
(4) $[q_0, Z_0, q_0] \Rightarrow a\,[q_0, a, q_1]\,[q_1, Z_0, q_0]$
(5) $[q_0, Z_0, q_1] \Rightarrow a\,[q_0, a, q_0]\,[q_0, Z_0, q_1]$
(6) $[q_0, Z_0, q_1] \Rightarrow a\,[q_0, a, q_1]\,[q_1, Z_0, q_1]$

For, $\delta(q_0, a, a) = (q_0, aa)$

(7) $[q_0, a, q_0] \Rightarrow [q_0, a, q_0]\,[q_0, a, q_0]$
(8) $[q_0, a, q_0] \Rightarrow [q_0, a, q_1]\,[q_1, a, q_0]$
(9) $[q_0, a, q_1] \Rightarrow [q_0, a, q_0]\,[q_0, a, q_1]$
(10) $[q_0, a, q_1] \Rightarrow [q_0, a, q_1]\,[q_1, a, q_1]$

classmate

PAGE

DATE

For, $\delta(q_0, b, a) = (q_1, a)$

(11) $[q_0, a, q_0] \Rightarrow b\,[q_1, a, q_0]$

(12) $[q_0, a, q_1] \Rightarrow b\,[q_1, a, q_1]$

For, $\delta(q_1, b, a) = (q_1, a)$

(13) $[q_1, a, q_0] \Rightarrow b\,[q_1, a, q_0]$

(14) $[q_1, a, q_1] \Rightarrow b\,[q_1, a, q_1]$

For, $\delta(q_1, a, a) = (q_1, \epsilon)$

(15) $[q_1, a, q_1] \Rightarrow a$

For, $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

(16) $[q_1, z_0, q_1] \Rightarrow \epsilon$

6. Convert the following PDA TO CFG

$\delta(q_0, 0, z_0) = (q_0, A z_0)$

$\delta(q_0, 1, A) = (q_0, AA)$

$\delta(q_0, 0, A) = (q_1, \epsilon)$

(1) $S \rightarrow (q_0, z_0, q_0)$

(2) $S \rightarrow (q_0, z_0, q_1)$

PAGE

DATE

For, $\delta(q_0, 0, Z) = (q_0, AZ)$

(3) $[q_0, Z_0, q_0] \rightarrow 0\,[q_0, A, q_0]\;[q_0, Z, q_0]$
(4) $[q_0, Z_0, q_0] \rightarrow 0\,[q_0, A, q_1]\;[q_1, Z, q_0]$
(5) $[q_0, Z_0, q_1] \rightarrow 0\,[q_0, A, q_0]\;[q_0, Z, q_1]$
(6) $[q_0, Z, q_1] \rightarrow 0\,[q_0, A, q_1]\;[q_1, Z, q_1]$

For $\delta(q_0, 1, A) = (q_0, AA)$

(7) $[q_0, A, q_0] \rightarrow 1\,[q_0, A, q_0]\;[q_0, A, q_0]$
(8) $[q_0, A, q_0] \rightarrow 1\,[q_0, A, q_1]\;[q_1, A, q_0]$
(9) $[q_0, A, q_1] \rightarrow 1\,[q_0, A, q_0]\;[q_0, A, q_1]$
(10) $[q_0, A, q_1] \rightarrow 1\,[q_0, A, q_1]\;[q_1, A, q_1]$

For $\delta(q_0, 0, A) = (q_1, \epsilon)$

(11) $[q_0, A, q_1] \rightarrow 0$

Note, you can also replace
$[q_0, Z_0, q_0]$ by A
$[q_0, Z_0, q_1]$ by B
$[q_1, Z_0, q_0]$ by C
$[q_1, Z_0, q_1]$ by D
$[q_0, A, q_0]$ by E
$[q_0, A, q_1]$ by F
$[q_1, A, q_0]$ by G
$[q_1, A, q_1]$ by H

And you get,
$S \rightarrow A \mid B$
$A \rightarrow 0EA \mid 0FC$
$B \rightarrow 0ED \mid 0FB$
$E \rightarrow 1EE \mid 1FG$
$F \rightarrow 1EF \mid FH \mid 0$

Where, A, B, C, D, E, F, G, H are non-terminals & 0, 1 are terminals

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Questions asked from this Chapter

DATE

Q. Construct a PDA that accept $L = \{a^n b^n \mid n >= 0\}$. Define a Push down Automata. (2078- 5 marks)

Q. Construct a PDA accepting strings over {0,1} with equal no. of 0s and 1s. Show by sequence of IDs that 0101 is accepted by this PDA. (2076-5 marks) (2075-6 marks) (2067- 6 marks)

Q How can you define the language accepted by a PDA? Explain how a PDA accepting language by empty stack is converted into an equivalent PDA accepting by final state & vice-versa. (2076-10 marks) (2076(old) - 6 marks) (2067 - 6 marks) (2068 - 6 marks)

Q. Construct a PDA that accepts the strings of language $L = \{WW^R \mid W \in (a,b)^*\}$ (2074- 6 marks)

Q. How can you convert a CFG into equivalent PDA? Explain with example. (2075-6 marks) (2072- 6 marks)

Q. What is PDA? How is it different from finite automata? (2073 - 6 marks) (2068- 6 marks)

Q. Give the formal definition of NPDA. How it differs with DPDA? Explain. (2067 - 6 marks) (2069 - 6 marks) (2070- 6 marks)

Q. Discuss the equivalent of PDA and CFG. Convert the grammar $S \to aAA$, $A \to as \mid bs \mid a$ to a PDA that accepts the same language by empty stack. (2073- 6 marks)

Q. Convert following grammar into equivalent PDA (2067 - 6 marks)

$S \to AAC$, $A \to aAb \mid \epsilon$, $C \to ac \mid b \mid ab$

PAGE

classmate