Master of Science (M.Sc.) Semester—I (C.B.C.S.) (Computer Science) Examination

DISCRETE MATHEMATICAL STRUCTURE

Paper—1

Paper—I

Time : Three Hours] [Maximum Marks : 80

N.B. :— ALL questions are compulsory and carry equal marks.

EITHER

1. (A) Define LCM and GCD.

Prove that if a and b are two positive integers, then $GCD(a, b) \cdot LCM(a, b) = a \cdot b$. 8

(B) Define elementary product and elementary sum.

Obtain disjunctive normal forms of :

(a) $P \wedge (P \rightarrow Q)$

(b) $\neg (P \vee Q) \rightleftarrows (P \wedge Q)$ 8

OR

(C) Prove the statement is true by using mathematical induction :

$$1 + 2^1 + 2^2 + ... + 2^n = 2^{n+1} - 1.$$ 8

(D) State the rules of generalization and specification.

Show that :

$$(x)\ (P(x) \rightarrow Q(x)) \wedge (x)\ (Q(x) \rightarrow R(x)) \Rightarrow (x)\ (P(x) \rightarrow R(x)).$$ 8

EITHER

2. (A) State and prove the pigeonhole principle. Compute :

(i) ${}_{7}C_{4}$

(ii) ${}_{16}C_{5}$ 8

(B) Let A = {1, 2, 3, 4, 5} and R = {(1, 1), (1, 2), (2, 3), (3, 5), (3, 4), (4, 5)}.

Draw digraph of R and compute R^2 and R^∞. 8

OR

(C) Define :

(i) Transposition

(ii) Even and odd permutation.

Determine the permutation :

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$$

is even or odd. 8

(D) Let A = {1, 2, 3, 4} and B = {a, b, c}. Let R = {(1, a), (1, b), (2, b), (2, c), (3, b), (4, a)} and S = {(1, b), (2, c), (3, b), (4, b)}. Compute : (a) $\overline{R}$, (b) $R \cap S$, (c) $R \cup S$ (d) R^{-1}. 8

EITHER

3. (A) Define Hamiltonian path and circuit. Prove—Let the number of edges of G be m. Then G has a Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$, where n is no. of vertices. 8

(B) Let A = {1, 2, 3, 4, 12, 24}. Consider the partial order of divisibility on A, that is, if a and b ∈ A, a ≤ b if and only if a | b. Draw the Hasse diagram of the poset (A, ≤). 8

OR

(C) If $(L_1, \leq)$ and $(L_2, \leq)$ are lattices, then (L, ≤) is a lattice. Where $L = L_1 \times L_2$ and the partial order ≤ of L is the product partial order. 8

(D) Define tree and spanning tree. Prove that—A tree with 'n' vertices has n – 1 edges. 8

EITHER

4. (A) Consider the binary operation ∗ defined on the set A = {a, b, c, d} by the table :

∗	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Compute :

(i) c ∗ d and d ∗ c

(ii) b ∗ d and d ∗ b

(iii) a ∗ (b ∗ c) and (a ∗ b) ∗ c

(iv) Is commutative or associative ? 8

(B) What is state transition table ? Draw the digraph of the machine whose state transition table is shown below :

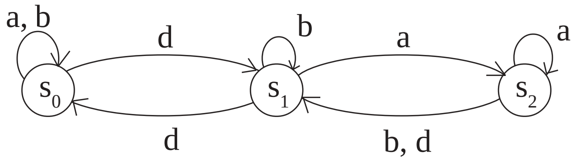
	0	1	2
s_0	s_1	s_0	s_2
s_1	s_0	s_0	s_1
s_2	s_2	s_0	s_2

8

OR

(C) Let G be the set of all non-zero real numbers and let $a * b = \frac{ab}{2}$. Show that (G, ∗) is an Abelian group. 8

(D) Let $S = \{s_0, s_1, s_2\}$ and I = {a, b, d}. Consider the finite-state machine M = (S, I, F) defined by the digraph as shown in figure below :



Compute the functions f_{bad}, f_{add} and f_{badadd} and verify that :

$$f_{add} \circ f_{bad} = f_{badadd}.$$ 8

5. (A) Prove that :

$$\overline{A \cup B} = \overline{A} \cap \overline{B}.$$ 4

(B) How many different seven person committees can be formed each containing three women from an available set of 20 women and four men from an available set of 30 men ? 4

(C) Define :

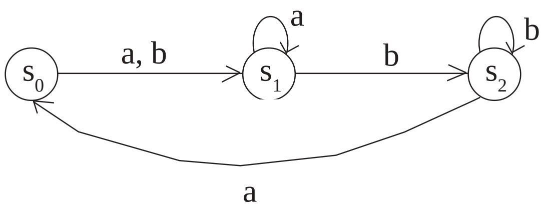
(i) Graph

(ii) Subgraph

(iii) Discrete graph

(iv) Complete graph. 4

(D) Construct the state transition table of the finite state machine of the digraph.



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