TUTORIAL→2

(1)

i	j
1	2
3	3
6	4
10	5
	⋮

$$\frac{k(k+1)}{2} = n$$

$k^2 = n \Rightarrow k = \sqrt{n}$

$TC \rightarrow O(\sqrt{n})$

(2) $T(0)=0$

$T(1)=0$

$T(n) = T(n-1) + T(n-2) + 1$

Let $T(n-1) \simeq T(n-2)$

$T(n) = 2T(n-1)+1$

using backward substitution

$T(n) = 2\cdot2(T(n-2)+1)+1$

$= 4(T(n-2)) + 3$

$T(n-2) = 2T(n-3)+1$

$= 2(2(2(T(n-3)+1)+1)+1)$

$= 8T(n-3)+7$

$T(n) = 2^k T(n-k) + 2^k - 1$

$T(0)=0$

$n-k=0$

$n=k$

$T(n) = 2^n T(n-n) + 2^n - 1$

$= 2^n + 2^n$

$TC \rightarrow O(2^n)$

③ n^2

```
void func(int n)
{
  for(int i=1; i<=n; i++)
  {
    for (j=1; j<=n; j++)
    {
        //some task for O(1)
    }
  }
}
```

n^3

```
void func(int n)
{
  four(int i=1 to n)
  {
    for (int j= 1 to n)
    {
      for(int k=1 to n)
      {
         //some O(1) task
      }
    }
  }
}
```

log(log n)

```
=> void func(int n)
   {
     for (int i=n; i>1; i=pow(i,k))
     {  //some O(1) task
     }
   }
```

(4) $T(n) = T(n/4) + T(n/2) + cn^2$

assume $T(n/2) >= T(n/4)$

$T(n) = 2T(n/2) + cn^2$

$c = \log_b a$

$= \log_2 2 = 1$

$\therefore n^c < f(n)$

$t.c. = O(n^2)$

(5) i j

i	j
1	n times
2	n/2 times
3	n/3 times
|	|
n	n/n times

$\boxed{TC = O(n\log n)}$

(6) $i = 2, 2^k, (2^k)^k, (2^{k^2})^k = 2^{k^3} \ldots 2^{k \log k(\log n)}$

$2^{k \log k(\log n)} = n$

$2^{\log 1} = 1$

$\Rightarrow TC = O(\log(\log(n)))$

[Page → 6]

(7) $T(n) = T(99n/100) + T(n/100) + O(n)$

~~1st level~~

Taking one branch 99% and other 1%.

$T(n) = T(99n/100) + T(n/100) + O(n)$

1st level $= n$

2nd level $= \frac{99n}{100} + \frac{n}{100} = n$

So IIIrd remains same for any kind of partition.

$\therefore$ If we take longer branch $= O(n\log_{100/99} n)$

for shorter branch $= \Omega(n\log_{10} n)$

Either way base complexity of $O(n\log n)$ remains.

(8) (a) $100 < \sqrt{n} < \log(\log n) < \log n < n < n\log n < \log(n!) <$
$n^2 < n! < 2^n < 4^n < 2^{2n}$

(b) $1 < \log(\log n) < \sqrt{\log n} < \log n < \log 2n < n < n\log n = \log(1) <$
$\log(1) < 2n < 4n < 2(2^n) < n! < n^2$

(c) $96 < \log_2(n) < \log(2!) < n\log_2 n < n\log_6 n < 5n < n! < 8n^2 < 7n^3 <$
$O(2n)$