TUTORIAL → 4

(1) $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are constant and $f(n)$ is a +ve function.

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.58}$

which is $n^2 > n^{1.58}$

$\therefore$ case 3 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

(2) $T(n) = 4T(n/2) + n^2$

$a = 4 \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are const. and $f(n)$ is a positive function.

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$\therefore n^c = n^2$

$n^c = f(n)$

$\therefore$ case 2 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^2 \log n)}$

③ $T(n) = T(n/2) + 2^n$

$a=1 \quad b=2 \quad f(n)=2^n$

$\because$ a & b are constant and f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 1$

$= 0$

$\Rightarrow n^c = n^0 = 1$

$\because f(n) > n^c$

$\therefore$ case 3 is applied here

$\Rightarrow \boxed{T(n) = \Theta(2^n)}$

④ $T(n) = 2^n T(n/2) + n^n$

$a = 2^n \quad b=2 \quad f(n)=n^n$

$\because$ a is not constant, its value depends on n.

$\therefore$ Master's theorem is not applicable here.

⑤ $T(n) = 16T(n/4) + n$

$a=16 \quad b=4 \quad f(n)=n$

$\because$ a & b are constant and f(n) is a +ve function

$c = \log_b a$

$= \log_4 16 \Rightarrow \log_4 4^2 = 2\log_4 4 = 2$

$\Rightarrow n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applicable here

$\boxed{T(n) = \Theta(n^2)}$

(6) $T(n) = 2T(n/2) + n\log n$

$a=2 \quad b=2 \quad f(n) = n\log n$

∵ a, b are constant and f(n) is a +ve function

$c = \log_b a$

$= \log_2 2 = 1$

$n^c = n^1 = n$

$\because f(n) > n^c$

$\therefore f(n) > n^c$

case 3 is applied

$\Rightarrow \boxed{T(n) = \Theta(n\log n)}$

(7) $T(n) = 2T(n/2) + n/\log n$

$a=2 \; b=5 \; f(n) = n/\log n$

∵ a and b are constant & f(n) is a +ve function

$c = \log_b a$

$= \log_2 2 = 1$

$n^c = n^1 = n$

∵ non-polynomial difference b/w f(n) & n^c

∴ Master's theorem is not applicable.

(8) $T(n) = 2T(n/4) + n^{0.51}$

$a=2 \quad b=4 \quad f(n) = n^{0.51}$

$\because$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.50}$

$\because f(n) > n^c$

$\therefore$ case 3 is applicable

$\Rightarrow \boxed{T(n) = \Theta(n^{0.51})}$

(9) $T(n) = 0.5T(n/2) + 1/n$

$a=0.5 \quad b=2 \quad f(n) = 1/n$

$\because a < 1$

$\therefore$ Master's theorem is not applicable

(10) $T(n) = 16T(n/4) + n!$

$a=16 \quad b=4 \quad f(n) = n!$

$\because$ a & b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$n^c = n^2$

$\because f(n) > n^c$

$\therefore$ case 3 is applicable here

$\boxed{T(n) = \Theta(n!)}$

(11) $T(n) = 4T(n/2) + \log n$

$a=4 \quad b=2 \quad f(n) = \log n$

$\because$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_2 a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applied

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

(12) $\sqrt{n}\,T(n/2) + \log n$

$a = \sqrt{n} \quad b=2 \quad f(n) = \log n$

$\because$ a is not constant

$\therefore$ Master's theorem is not applicable

(13) $T(n) = 3T(n/2) + n$

$a=3 \quad b=2 \quad f(n) = n$

$\because$ a & b are constant & f(n) is a +ve f^n

$\therefore$ Master's Theorem is applicable

$c = \log_b a = \log_2 3 = 01.58$

$n^c = n^{1.58}$

$f(n) < n^c$

case 1 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^{1.58})}$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a=3$ $b=3$ $f(n) = \sqrt{n}$

$\because$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\because f(n) < n^c$

$\therefore$ case 1 is applicable

$\Rightarrow \boxed{T(n) = \Theta(n)}$

(15) $T(n) = 4T(n/2) + c.n$

$a=4$ $b=2$ $f(n) = c.n$

$\because$ a & b are constant and f(n) is a +ve function

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applicable here

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

(16) $T(n) = 3T(n/4) + n\log n$

$a=3$ $b=4$ $f(n) = n\log n$

$\because$ a & b are constant & f(n) is a +ve f^n

$\therefore$ Applicable

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

$\because f(n) > n^c$

case 3 is applicable here

$\Rightarrow \boxed{T(n) = \Theta(n \log n)}$

(17) $T(n) = 3T(n/3) + n/2$

$a = 3 \quad b = 3 \quad f(n) = n/2$

Applicable

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\because f(n) = n^c$

case 2 is applied here

$\boxed{T(n) = n \log n}$

(18) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6 \quad b = 3 \quad f(n) = n^2 \log n$

Applicable

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$f(n) > n^c$

case 3 is applied here

$\boxed{T(n) = \Theta(n^2 \log n)}$

Pr 7/9/19

(19) $T(n) = 4T(n/2) + n/\log n$

$a=4 \quad b=2 \quad f(n) = n/\log n$

$\because$ a and b are constant and $f(n)$ is a +ve function

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

case 1 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

(20) $T(n) = 64T(n/8) \quad n^2 \log n$

$\therefore$ a & b are constant but $f(n)$ is a -ve function

Master's theorem is not applicable here.

(21) $T(n) = 7T(n/3) + n^2$

$a=7 \quad b=3 \quad f(n) = n^2$

$\because$ a & b are constant &, $f(n)$ is +ve function

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

$f(n) > n^c$

$\therefore$ case 3 is applied here

$\boxed{T(n) = \Theta(n^2)}$

(22) $T(n) = \cancel{2}T(n/2) + n(2 - \cos n)$

$\because$ $f(n)$ is not regular function

$\therefore$ Master's theorem is not applicable here