Module - 2

Numerical Methods - II

Part - 1 Numerical Differentiation

① Derivatives of $y = f(x)$ based on Newton's forward Interpolation formula

② Derivatives of $y = f(x)$ based on Newton's Backward Interpolation formula.

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Part-1.

Numerical Differentiation

Method 1) Derivatives of $y = f(x)$ based on Newton's forward Interpolation formula:

We know that Newton's forward Interpolation formula is

$$f(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots\cdots + \cdots \quad (1)$$

where $p = \dfrac{x - x_0}{h}$ $\Rightarrow$ $\boxed{x = x_0 + ph}$

Here,
$x \to$ estimation value
$x_0 \to$ initial value
$h \to$ interval

put $x = x_0 + ph$ in eqn (1)

$$f(x_0+ph) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots\cdots$$

$$f(x_0+ph) = y_0 + p\Delta y_0 + \frac{(p^2-p)}{2!}\Delta^2 y_0 + \frac{(p^3-3p^2+2p)}{3!}\Delta^3 y_0 + \frac{p^4-6p^3+11p^2-6p}{4!}\Delta^4 y_0 + \cdots$$

Differentiate w.r.t. p both sides

$$h f'(x_0+ph) = \Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{3p^2-6p+2}{3!}\Delta^3 y_0 + \frac{1}{24}(4p^3-18p^2+22p-6)\Delta^4 y_0 + \cdots$$

Again, diff.

$$h^2 f''(x_0+ph) = \Delta^2 y_0 + \frac{(6p-6)}{6}\Delta^3 y_0 + \frac{(12p^2-36p+22)}{24}\Delta^4 y_0$$

Again differentiating

$$h^3 f'''(x_0+ph) = \Delta^3 y_0 + \frac{1}{12}(12p-18)\Delta^4 y_0 + \cdots$$

Hence,

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{3!}\Delta^3 y_0 + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6p-6)}{3!}\Delta^3 y_0 + \cdots + \frac{(12p^2-36p+22)}{24}\Delta^4 y_0\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{1}{12}(12p-18)\Delta^4 y_0 + \cdots\right]$$

Derivatives of $y = f(x)$ based on Newton's forward interpolation formula.

(i) $$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - \frac{1}{6}\Delta^6 y_0 + \cdots\right]$$

(ii) $$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots + \frac{137}{180}\Delta^6 y_0\right]$$

put p=0 (p=0) (p=0)

(iii) $$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \frac{7}{4}\Delta^5 y_0 - \cdots\right]$$

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Ques. Given that

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y :	7.989	8.403	8.781	9.129	9.451	9.750	10.031

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, at $x = 1.1$ and $x = 1.6$.

$$\because \quad \frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6p-6)}{3!}\Delta^3 y_0 + \dots\right]$$

put $p=0$

$$\left(\frac{d^2y}{dx^2}\right)_{1.5} = \frac{1}{(0.5)^2}\left[\Delta^2 y_0 + \Delta^3 y_0 + \dots\right]$$

$$= \frac{1}{(0.5)^2}\left[3.00 - 0.75\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{1.5} = \frac{1}{0.25} \times 2.25 = 9$$

Ques find the first and second derivative at $x = 1.5$

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.0	13.625	24.00	38.875	59.00

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.5	3.375				
		3.625			
2.0	7.0		3		
		6.625		0.75	
2.5	13.625		3.75		0
		10.375		0.75	
3.0	24.00		4.5		0
		14.875		0.75	
3.5	38.875		5.25		
		20.125			
4.0	59.00				

Here,

$x_0 \rightarrow 1.5$

$h \rightarrow 0.5$

$x \longrightarrow 1.5$

$\Rightarrow p = \dfrac{x - x_0}{h} = \dfrac{1.5 - 1.5}{0.5}$

$\boxed{p = 0}$ — ⓐ

from table $\Delta y_0 = 3.625$, $\Delta^2 y_0 = 3$, $\Delta^3 y_0 = 0.75$, $\Delta^4 y_0 = 0$

we know that Newton forward Interpolation formula of derivative

$$\left(\frac{dy}{dx}\right) = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{3p^2 - 6p + 2}{3!}\Delta^3 y_0 + \cdots\cdots\right] \quad \text{—— (1)}$$

putting $p = 0$ in eqn (1)

$$\left(\frac{dy}{dx}\right)_{1.5} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{dy}{dx}\right)_{1.5} = \frac{1}{0.5}\left[3.625 - \frac{1}{2}\times 3.0 + \frac{1}{3}\times 0.75 - \frac{1}{4}\times 0\right]$$

$$= \frac{2.735}{0.5} = 4.75 \quad \text{Ans}$$

Ques 2. find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of tabulated function at point $x = 1$.

x	1	3	5	7	9
$f(x)$	85.3	74.5	67.0	60.5	54.3

Soln

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	85.3				
		-10.8			
3	74.5		3.3		
		-7.5		-2.3	
5	67.0		1.0		1.6
		-6.5		-0.7	
7	60.5		0.3		
		-6.2			
9	54.3				

Here

$p = \frac{x - x_0}{h}$

$p = \frac{1-1}{2}$

$\boxed{p = 0}$

from table

$\Delta y_0 = -10.8$, $\Delta^2 y_0 = 3.3$, $\Delta^3 y_0 = -2.3$

and $\Delta^4 y_0 = 1.6$

at $p = 0$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0\right] \quad \text{--- (1)}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \dots\right] \quad \text{--- (2)}$$

from (1) $$\left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{2}\left[-10.8 - \frac{1}{2}\times(3.3) + \frac{1}{3}(-2.3) - \frac{1}{4}(1.6)\right]$$

$$\left(\frac{dy}{dx}\right)_{x=1} = -\frac{13.617}{2} = -6.808$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=1} = \frac{1}{4}\left[3.3 - (-2.3) + \frac{11}{12}(1.6)\right]$$

$$= \frac{1}{4}\left[3.3 + 2.3 + 1.466\right]$$

$$= \frac{1}{4}\left[7.066\right] =$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 1.767 \quad \text{Ans}$$

Method 2 Derivatives using Newton Gregory Interpolation formula.

By Newton Backward formula

$$f(x_n+ph) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$+ \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \ldots \quad \text{--- (1)}$$

where $p = \frac{x - x_n}{h}$ | Here, $x \rightarrow$ estimation value, $x_n \rightarrow$ last value, $h \rightarrow$ interval

Differentiating w.r.t. p both sides

$$h f'(x_n+ph) = \nabla y_n + \frac{(2p+1)}{2!}\nabla^2 y_n + \frac{(3p^2+6p+2)}{3!}\nabla^3 y_n$$

$$+ \frac{(4p^3+18p^2+22p+6)}{4!}\nabla^4 y_n + \ldots \quad \text{--- (2)}$$

Again diff

$$h^2 f''(x_n+ph) = \nabla^2 y_n + \frac{(6p+6)}{3!}\nabla^3 y_n + \frac{(12p^2+36p+22)}{4!}\nabla^4 y_n + \ldots \quad \text{--- (3)}$$

Again diff.

$$h^3 f'''(x_n+ph) = \nabla^3 y_n + \frac{(24p+36)}{4!}\nabla^4 y_n \quad \text{--- (4)}$$

Hence,

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{(2p+1)}{2}\nabla^2 y_n + \frac{(3p^2+6p+2)}{3!}\nabla^3 y_n + \frac{(4p^3+18p^2+22p+6)}{4!}\nabla^4 y_n + \cdots\right]$$

Also

$$\frac{d^2y}{dx^2} = \frac{1}{h}\left[\nabla^2 y_n + \frac{(6p+6)}{3!}\nabla^3 y_n + \frac{(12p^2+36p+22)}{4!}\nabla^4 y_n + \cdots\right]$$

and

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{(24p+36)}{4!}\nabla^4 y_n + \cdots\right]$$

(i) $$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots\right]$$

(ii) $$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right]$$

(iii) $$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \frac{7}{4}\nabla^5 y_n + \cdots\right]$$

Que find $f'(x)$ and $f''(x)$ at $x = 6$ given that

x	4.5	5.0	5.5	6.0	6.5	7.0	7.5
$f(x)$	9.69	12.90	16.71	21.18	26.37	32.34	39.15

Sol. to find $f'(x)$ and $f''(x)$ at $x = 6$, So taking $x_0 = 6$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
4.5	9.69				
		3.21			
5.0	12.90		0.60		
		3.81		0.06	
5.5	16.71		0.66		0
		4.47		0.06	
$x_0 =$ 6.0	y_0 21.18		0.72		0
		Δy_0 5.19		0.06	
6.5	26.37		$\Delta^2 y_0$ 0.78		0
		5.97		$\Delta^3 y_0$ 0.06	
7.0	32.34		0.84		
		6.81			
7.5	39.15				

again, as $x = x_0$
$p = 0$

Using Derivative of forward difference formula

$$\left(\frac{dy}{dx}\right)_{x=6} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + - - -\right]$$

or

$$= \frac{1}{0.5}\left[5.19 - \frac{1}{2}(0.78) + \frac{1}{3}(0.06)\right]$$

$$= 9.64$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=6} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \cdots\right]$$

or

$$f''(6) = \frac{1}{0.25}\left[0.78 - 0.06\right] = 2.88$$

Que Given that

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at

$x = 1.1$ and $x = 1.6$.

$f'(1.1) = 3.9578$ $\quad f''(1.1) = -3.742$

$f'(1.6) = 2.7476$ $\quad f''(1.6) = -0.7144$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.0	7.989						
		0.414					
1.1	8.402		-0.036				
		0.378		0.006			
1.2	8.781		-0.030		-0.002		
		0.348		0.004		0.001	
1.3	9.129		-0.026		-0.001		0.002
		0.322		0.003		0.003	
1.4	9.451		-0.023		0.002		
		0.299		0.005			
1.5	9.750		-0.018				
		0.281					
1.6	10.031						

To find $f(1.1)$ by forward formula. $x_0 = 1.1$, $x = 1.1$

Here $y_0 = 7.989$, $h = 0.1$ and $p = 0$

we have,

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - \frac{1}{6}\Delta^6 y_0\right]$$

$$\left(\frac{dy}{dx}\right)_{x=1.1} = \frac{1}{0.1}\left[0.378 + \frac{1}{2}(0.03) + \frac{1}{3}(0.004) + \frac{1}{4}(0.001) + \frac{1}{5}(0.003)\right]$$

$$\left(\frac{dy}{dx}\right)_{x=1.1} = 3.9518$$

and $$\left(\frac{d^2y}{dx^2}\right)_{x=1.1} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.030 - 0.004 + \frac{11}{12}(-0.001) - \frac{5}{6}(0.003)\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=1.1} = -3.742.$$

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Numerical Integration

Let $I = \int_a^b f(x)\,dx$, where $y = f(x)$ be given for certain equidistant values of arguments say, $x_0, (x_0+h), (x_0+2h), (x_0+3h), \ldots, (x_0+nh)$.
Let the range $(b-a)$ be divided into n equal parts each of width 'h'. Then, $h = \frac{b-a}{n}$ since $b-a=nh$

1.) Trapezoidal Rule.

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+\cdots+y_{n-1})\right]$$

$$= \frac{h}{2}\left[(\text{sum of } 1^{st} \text{ and last term}) + 2(\text{sum of remaining terms})\right]$$

2.) Simpson's One-Third $(1/3)^{rd}$ Rule:

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+y_6+\cdots) + 4(y_1+y_3+y_5+\cdots)\right]$$

$$= \frac{h}{3}\left[(\text{sum of } 1^{st} \text{ and last}) + 2(\text{sum of even terms}) + 4(\text{sum of odd terms})\right]$$

3) Simpson's $3/8^{th}$ Rule.

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n) + 2(y_3+y_6+y_9+\cdots) + 3(y_1+y_2+y_4+y_5+\cdots)\right]$$

$$= \frac{3h}{8}\left[(\text{sum of } 1^{st} \text{ and last term}) + 2(\text{sum of multiple of three}) + 3(\text{sum of remaining terms})\right]$$

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Note :- ① There is no restriction ~~is~~ for number of intervals in Trapezoidal Rule

② In Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule, the number of subintervals must be ~~even~~ multiple of 2

③ In Simpson's $\left(\frac{3}{8}\right)^{th}$ rule, the number of subintervals must be multiple of 3.

④ To get more accuracy, divide given interval into maximum number of subintervals.

and in Weddles subinterval multiple of 6.

To solve all the three or four methods take $n = 6$ as its applicable in all cases. i.e. (i) Trapezoidal $n =$ multiple of any

(ii) Simpson's 1/3rd $n =$ multiple of 2

(iii) Simpson's 3/8th, $n =$ multiple of 3

(iv) Weddles — $n = 6$.

Que find the first and second derivatives of the function at point $x = 1.2$.

x	1	2	3	4	5
y	0	1	5	6	8

Sol

To find $f'(x)$ and $f''(x)$ at $x = 1.2$, which is near to $x_0 = 1$. Using Newton's forward interpolation formula.

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots \quad (1)$$

where

$$p = \frac{x - x_0}{h} \Rightarrow x = x_0 + ph \quad (\text{ie } x_0 + ph = 1.2)$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	0				
		1			
2	1		3		
		4		-6	
3	5		-3		10
		1		4	
4	6		1		
		2			
5	8				

from (1)

$$f(x_0+ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots$$

$$\Rightarrow f(x_0+ph) = y_0 + p\Delta y_0 + \frac{(p^2-p)}{2!}\Delta^2 y_0 + \frac{(p^3-3p^2+2p)}{3!}\Delta^3 y_0 + \frac{p^4-6p^3+11p^2-6p}{4!}\Delta^4 y_0 + \cdots$$

Diff. w.r.t. to p both sides.

$$\Rightarrow hf'(x_0+ph) = \Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{3!}\Delta^3 y_0 + \frac{(4p^3-18p^2+22p-6)}{4!}\Delta^4 y_0 + \cdots \quad (3)$$

Again diff.

$$h^2 f''(x_0+ph) = \Delta^2 y_0 + \frac{(6p-6)}{6}\Delta^3 y_0 + \frac{(12p^2-36p+22)}{24} \times \Delta^4 y_0 + \cdots \quad (4)$$

putting $h = 1$, $x_0 = 1$, $p = \frac{x-x_0}{h} = \frac{1.2-1}{1}$

$$\boxed{p = 0.2}$$

and $x = 1.2$ in ~~(3)~~ ~~(4)~~ (3) and (4) we get

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$$f'(1.2) = 1 + \frac{(0.4-1)\times 3}{2} + \frac{[3(0.2)^2 - 6\times(0.2) + 2]}{6}\times(-6) + \frac{[4(0.2)^3 - 12\times(0.2)^2 + 22\times(0.2) - 6]}{24} + \cdots$$

$$= 1 - (0.9) - (0.92) - (0.853)$$

$$= 1 - 2.673 = -1.673$$

$f'(1.2) = -1.673$ (−1.773) | Ally $f''(1.2) = 8.13$ (14.166)

Ques: Find 1st and 2nd derivative of the function at $x = 1.1$.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.0	0				
		0.128			
1.2	0.128		0.288		
		0.416		0.048	
1.4	2.544		0.336		0
		0.752		0.048	
1.6	1.296		0.384		0
		0.136		0.048	
1.8	2.432		0.432		
		1.568			
2.0	4.000				

$f'(1.1) = 0.63$

$f''(1.1) = 6.60$

Proceeding same as above here $p = \frac{x - x_0}{h}$

$p = \frac{1.1 - 1.0}{0.2} = 0.5$, $h = 0.2$

Weddle's Rule for $n = 6$.

$$\int_{x_0}^{x_6} y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

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Que. find $\int_0^1 \frac{1}{1+x^2} dx$ by (i) Trapezoidal Rule
(ii) Simpson's $1/3^{rd}$ Rule
(iii) Simpson's $3/8^{th}$ Rule

Soln Here, $f(x) = \frac{1}{1+x^2}$, taking $n=6$.

$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$.

Data table

x	$y = f(x)$
$x_0 = 0$	$y_0 = 1$
$x_1 = 1/6$	$y_1 = 0.97297$
$x_2 = 2/6$	$y_2 = 0.9$
$x_3 = 3/6$	$y_3 = 0.8$
$x_4 = 4/6$	$y_4 = 0.69231$
$x_5 = 5/6$	$y_5 = 0.59016$
$x_6 = 6/6 = 1$	$y_6 = 0.5$

(i) Trapezoidal Rule $(n=6)$

$$\int_0^1 \frac{1}{1+x^2} dx = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{12}\left[(1+0.5) + 2\{0.97297 + 0.9 + 0.8 + 0.69231 + 0.59016\}\right]$$

$$= \frac{1}{12}\left[1.5 + 2\{3.95544\}\right] = \frac{1}{12}\left[1.5 + 7.91088\right]$$

$$= \frac{1}{12}\left[9.41088\right] = 0.78424$$

Qus. Use Simpson's 1/3rd Rule to find $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates.

Soln $f(x) = e^{-x^2}$, $n = 6$, $h = \frac{0.6-0}{6} = 0.1$

x	y
$x_0 = 0$	$y_0 = 1$
$x_1 = 0.1$	$y_1 = 0.99$
$x_2 = 0.2$	$y_2 = 0.96$
$x_3 = 0.3$	$y_3 = 0.9139$
$x_4 = 0.4$	$y_4 = 0.85214$
$x_5 = 0.5$	$y_5 = 0.7788$
$x_6 = 0.6$	$y_6 = 0.69767$

By Simpson's 1/3rd Rule

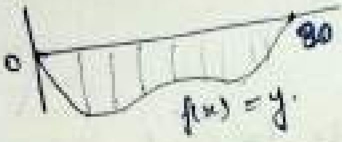
$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)\right]$$

$$\int_0^{0.6} e^{-x^2}dx = \frac{0.1}{3}\left[(1+0.69767) + 2(0.96+0.85214) + 4(0.99+0.9139+0.7788)\right]$$

$$= \frac{0.1}{3}\left[1.69767 + 2(1.81214) + 4(2.68270)\right]$$

$$= \frac{0.1}{3}\left[1.69767 + 3.62428 + 10.73080\right]$$

$$= \frac{0.1}{3}\left[16.05275\right] = \frac{1.60528}{3} = 0.53509$$



Area $= \int_0^{80} y \cdot dx$

Que. A river is 80 feet wide. The depth y of the river at a distance 'x' from one bank to is given by

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

find approx. area of cross section of river by (i) Simpson's 1/3rd (ii) Trapezoidal Rule

Soln Here n = 8 $\quad h = \frac{b-a}{n} = \frac{80}{8} = 10$

Simpson's 1/3rd.
Area of cross section of river.

$$\int_0^{80} y \cdot dx = \frac{10}{3}\left[(y_0+y_8) + 2(y_2+y_4+y_6) + 4(y_1+y_3+y_5+y_7)\right]$$

$$= \frac{10}{3}\left[(0+3) + 2(7+12+14) + 4(4+9+15+8)\right]$$

$$= \frac{10}{3} \times \left[3+66+144\right] = \frac{2130}{3} = 710$$

By Trapezoidal Rule. $\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_8) + 2(y_1+\cdots+y_7)\right]$

$$\int_0^{80} y \cdot dx = \frac{10}{2}\left[(0+3) + 2(4+7+9+12+15+14+8)\right]$$

$$= \frac{10}{2} \times \left[3+138\right]$$

$$= \frac{1410}{2} = 705$$

* Calculate in Radian mode

Que. Evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by dividing the interval into 6 parts, using Simpson's $1/3^{rd}$ Rule & Weddle's Rule

Simpson's $1/3^{rd}$ Rule

Soln Here $h = \frac{b-a}{n} = \frac{\pi/2}{6} = \frac{\pi}{12}$.

$$\int_{x_0}^{x_6} y\,dx = \frac{h}{3}\left[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)\right] \quad \text{——} (1)$$

$x = \theta \qquad y = \sqrt{\cos\theta}$

$x_0 = 0 \qquad y_0 = 1$

$x_1 = \pi/12 \qquad y_1 = 0.98281$

$x_2 = \pi/6 \qquad y_2 = 0.93060$

$x_3 = \pi/4 \qquad y_3 = 0.84089$

$x_4 = \pi/3 \qquad y_4 = 0.70710$

$x_5 = 5\pi/12 \qquad y_5 = 0.50874$

$x_6 = \pi/2 \qquad y_6 = 0$

Hence, from (1)

$$\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta = \frac{1}{3}\cdot\frac{\pi}{12}\Big[(1+0) + 2(0.93060 + 0.70710) + 4(0.98281 + 0.84089 + 0.50874)\Big]$$

$$= \frac{\pi}{36}\left[1 + 3.2754 + 0.932976\right] = \frac{\pi}{36} \times 13.60516$$

$= 1.187274$ Ans

Ques. Find an approximate value of log5 by calculating to 4 decimal places, by Simpson's 1/3 Rule,

$$\int_0^5 \frac{dx}{4x+5},$$ dividing range into 10 equal parts.

Soln Here range of integration (0,5) is divided into 10 equal parts.

$a=0,\ b=5,\ n=10$.

$h = \frac{5-0}{10} = 0.5$ and $f(x) = 4x+5$

Then,

x	$y = 1/4x+5$
$x_0 = 0$	$y_0 = 0.2$
$x_1 = 0.5$	$y_1 = 0.14286$
$x_2 = 1$	$y_2 = 0.11111$
$x_3 = 1.5$	$y_3 = 0.090901$
$x_4 = 2$	$y_4 = 0.07692$
$x_5 = 2.5$	$y_5 = 0.06667$
$x_6 = 3$	$y_6 = 0.05882$
$x_7 = 3.5$	$y_7 = 0.05263$
$x_8 = 4$	$y_8 = 0.04762$
$x_9 = 4.5$	$y_9 = 0.04348$
$x_{10} = 5$	$y_{10} = 0.04000$

By Simpson's 1/3rd Rule for $n=10$

$$\int_{x_0}^{x_{10}} y\,dx = \frac{h}{3}\left[(y_0+y_{10}) + 2(y_2+y_4+\dots+y_8) + 4(y_1+y_3+\dots+y_9)\right]$$

$$= \frac{0.5}{3}\left[(0.2+0.04000) + 2(0.11111+0.07692+0.05882+0.04762) + 4(0.14286+0.090901+0.06667+0.05263+0.04348)\right]$$

$$= \frac{0.5}{3}\left[(0.24) + 2(0.29446) + 4(0.396541)\right]$$

$$= \frac{0.5}{3}\left[0.24 + 0.58892 + 1.586164\right]$$

$$= 0.16666\left[0.24 + 0.58892 + 1.586164\right] = 0.16666\left[2.415084\right] = 0.4025$$

$$\text{or} \int_0^5 \frac{dx}{4x+5} = \frac{0.5}{3}\left[2.41515\right] = 0.40253 \quad \text{--- (1)}$$

Now value of Actual integral is

$$\int_0^5 \frac{dx}{4x+5} = \frac{1}{4}\left[\log(4x+5)\right]_0^5$$

let $u = 4x+5$

$$\frac{du}{dx} = 4$$

$$\Rightarrow dx = \frac{du}{4}$$

$$\int \frac{1}{u} \cdot \frac{du}{4} = \frac{1}{4}\int \frac{1}{u} \cdot du$$

$$= \frac{1}{4}\log u + c.$$

$$= \frac{1}{4}\log(4x+5)$$

$$= \frac{1}{4}\left[\log 25 - \log 5\right]$$

$$= \frac{1}{4}\left[\log 5^2 - \log 5\right]$$

$$= \frac{1}{4}\left[2\log 5 - \log 5\right]$$

$$= \frac{1}{4}\log 5$$

$$\Rightarrow 0.40253 = \frac{1}{4}\log 5 \quad \text{from (1)}$$

$$\log 5 = 1.61012$$

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Ques find the value of $\int_1^2 \frac{dx}{x}$ by Simpson's Rule

Hence obtain approx. value of $\log_e 2$

Soln Here $f(x) = \frac{1}{x}$ let $n = 6$.

Taking $h = \frac{2-1}{6} = \frac{1}{6}$.

x	$y = 1/x$
$x_0 = 1$	$y_0 = 1$
$x_1 = 7/6$	$y_1 = 0.85714$
$x_2 = 8/6$	$y_2 = 0.75000$
$x_3 = 9/6$	$y_3 = 0.66667$
$x_4 = 10/6$	$y_4 = 1.66667$
$x_5 = 11/6$	$y_5 = 1.83333$
$x_6 = 2$	$y_6 = 0.5$

$$\int_{x_0}^{x_6} y \, dx = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$\int_1^2 \frac{dx}{x} = \frac{1}{18}\left[(1.5) + 4(3.35714) + 2(2.41667)\right]$$

$$= \frac{1}{18}\left[1.5 + 13.42856 + 4.83334\right]$$

$= 0.08556[19.76190]$

$= 1.08797 \longrightarrow 0.693169.$ $\longleftarrow$ (1)

<u>Now</u>

$$\int_1^2 \frac{dx}{x} = \left[\log_e x\right]_1^2 = \log_e 2 - \log_e 1$$

$$= \log_e 2 \quad \left[\because \log 1 = 0\right] \longleftarrow (2)$$

from (1) & (2)

$$\log_e 2 = 0.693169..$$

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Ques Compute the value of π ~~using~~ from the formula $\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule with 10 subintervals.

$$\pi = 3.13992$$

Ques

Ques. A body is in form of solid revolution. The diameter D in cms of the sections at distance x cm from one end are given below. Estimate volume of solid.

x	0	2.5	5.0	7.5	10.0	12.5	15.0
D	5	5.5	6.0	6.75	6.25	5.5	4.0
	D_0	D_1	D_2	D_3	D_4	D_5	D_6

Soln. Here $h = 2.5$, $n = 6$ (i.e. 7 ordinates are given)

Therefore required volume of solid is given by

$$V = \int_a^b \pi (\text{radius})^2 dx$$

$$V = \int_0^{15.0} \pi \left(\frac{D}{2}\right)^2 dx \qquad \left[\because r = \frac{D}{2}\right]$$

$$V = \frac{\pi}{4} \int_0^{15.0} D^2 dx.$$

By Simpson's 1/3 rule.

$$V = \frac{\pi}{4} \cdot \frac{h}{3} \left[(D_0^2 + D_6^2) + 4(D_1^2 + D_3^2 + D_5^2) + 2(D_2^2 + D_4^2) \right] \quad (1)$$

Here. $D_0 = 5$, $D_1 = 5.5$, $D_2 = 6.0$, $D_3 = 6.75$, $D_4 = 6.25$, $D_5 = 5.5$, $D_6 = 4.0$.

from (1)

$$V = \frac{\pi}{4} \times \frac{(2.5)}{3} \left[(25 + 16) + 4(30.25 + 45.5625 + 30.25) + 2(36.00 + 39.0625) \right]$$

$$V = \frac{7.8540}{12} \times \left[41 + 424.25 + 150.125 \right] = 402.76 \text{ cu. cms.}$$

Hence, $V = 402.76$ cu. cms.

Qun. The following data give the velocity v of a particle at time t :-

t(sec)	0	2	4	6	8	10	12
v (m/sec)	4.	6	16	34	60	94	136

find the distance moved by particle in 12 seconds and also accelaration at $t = 2$ seconds.

Soln Let D = distance , v = velocity , t = time.

$\therefore \quad \frac{dD}{dt} = v \quad \Rightarrow \quad D = \int_0^{12} v\,dt$

Here 7 ordinates are given i.e $\boxed{n = 6}$ and $h = \frac{12-0}{6}$

$\Rightarrow \boxed{h = 2}$

from table, $v_0 = 4$, $v_1 = 6$, $v_2 = 16$, $v_3 = 34$,

$v_4 = 60$, $v_5 = 94$, $v_6 = 136$.

$\therefore \quad S = \int_0^{12} v\,dt = \frac{h}{3}\left[(v_0 + v_6) + 2(v_2 + v_4) + 4(v_1 + v_3 + v_5) \right]$

$= \frac{2}{3}\left[(4 + 136) + 2(16 + 60) + 4(6 + 34 + 94) \right]$

$= \frac{1}{3} \times [1656] = 552$ metres

$\text{Acc.} = \frac{v}{t} \Rightarrow$ at $t = 2$ sec. vel. = 6m/sec

$\therefore \quad \text{Acc} = \frac{6}{2} = 3 \text{ m/sec}^2$

Solution Of Simultaneous Linear Algebraic Equation.

Method: Gauss Elimination Method

1) Reduce Given matrix in form $AX = B$
2) Form Augmented matrix $A:B$.
3) Apply row operation (only) and reduce $A:B$ into upper triangular matrix or Echelon form.
4.) Solve the equations thus formed and get value of unknowns.

Ques. Solve by Gauss Elimination method.

$$\left.\begin{aligned} 2x - y + 3z &= 9 \\ x + y + z &= 6 \\ x - y + z &= 2 \end{aligned}\right\} \longrightarrow (1)$$

Soln.

Given System of eqn (1) can be written in form $AX=B$ where

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now Augmented matrix

$$C = [A:B] = \left[\begin{array}{ccc:c} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{array}\right]$$

$R_3 \longleftrightarrow R_1$

$$= \left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 9 \end{array}\right]$$

$$= \begin{bmatrix} 1 & -1 & 1 & : & 2 \\ 1 & 1 & 1 & : & 6 \\ 2 & -1 & 3 & : & 9 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$$= \begin{bmatrix} 1 & -1 & 1 & : & 2 \\ 0 & 2 & 0 & : & 4 \\ 0 & 1 & 1 & : & 5 \end{bmatrix}$$

$R_2 \to R_2/2$
$$= \begin{bmatrix} 1 & -1 & 1 & : & 2 \\ 0 & 1 & 0 & : & 2 \\ 0 & 1 & 1 & : & 5 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 1 & -1 & 1 & : & 2 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}$$

which is in upper triangular form,

writting again in form $Ax = B$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$\left\{ \begin{array}{ll} x - y + z = 2 & \text{--- (1)} \\ \boxed{y = 2} & \text{--- (2)} \\ \boxed{z = 3} & \text{--- (3)} \end{array} \right.$$

from (2) and (3)

$$x - 2 + 3 = 2$$
$$x + 1 = 2 \quad \Rightarrow \boxed{x = 1}$$

Que. Solve using Gauss Elimination method.

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

Soln writing above system of eqn in matrix form $AX = B$.

where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

Augmented Matrix $[A:B] =$

$$= \left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

$R_3 \longleftrightarrow R_1$

$$= \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \end{array}\right]$$

$R_2 \longrightarrow R_2 - 3R_1 \; ; \quad R_3 \longrightarrow R_3 - 2R_1$

$$= \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & -7 & -17 & -22 \end{array}\right]$$

$R_2 \longrightarrow (-1) \times R_2 \quad R_3 \longleftrightarrow (-1) R_3$

$$= \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 0 & 10 & 24 & 30 \\ 0 & 7 & 17 & 22 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & 10 & 24 & 30 \\ 0 & 7 & 17 & 22 \end{array}\right]$$

$$R_2 \longrightarrow \frac{R_2}{10}$$

$$\approx \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & 1 & 12/5 & 3 \\ 0 & 7 & 17 & 22 \end{array}\right]$$

$$R_3 \longrightarrow R_3 - 7R_2$$

$$\approx \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & 1 & 12/5 & 3 \\ 0 & 0 & 1/5 & 1 \end{array}\right]$$

Again writing in form $AX = B$

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & 1 & 12/5 \\ 0 & 0 & 1/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 3 \\ 1 \end{bmatrix}$$

$$x + 4y + 9z = 16 \quad \text{--- (1)}$$

$$y + \frac{12}{5} z = 3 \quad \text{--- (2)}$$

$$\frac{z}{5} = 1 \Rightarrow \boxed{z = 5} \quad \text{--- (4)}$$

from (3) and (4)

$$y + \frac{12}{5} \times 5 = 3 \Rightarrow \boxed{y = -9} \quad \text{--- (5)}$$

from (5) and (4) and (1)

$$x + 4(-9) + 9(5) = 16$$

$$x - 36 + 45 = 16$$

$$\boxed{x = 7}$$

Ques3 Solve by Gauss' Elimination Method

$$\left.\begin{array}{l} 10x + y + 2z = 13 \\ 3x + 10y + z = 14 \\ 2x + 3y + 10z = 15 \end{array}\right\} \quad (1)$$

Soln writing above system of eqn as $AX = B$, where,

$$[A : B] = \left[\begin{array}{ccc|c} 10 & 1 & 2 & 13 \\ 3 & 10 & 1 & 14 \\ 2 & 3 & 10 & 15 \end{array}\right]$$

$R_2 \to R_2 - \left(\frac{3}{10}\right)R_1$; $\cancel{R_3 \to R_3 - \frac{1}{40}R_1}$

$R_3 \to R_3 - \left(\frac{1}{5}\right)R_1$, we get

$$[A : B] \sim \left[\begin{array}{ccc|c} 10 & 1 & 2 & 13 \\ 0 & 97/10 & 2/5 & 101/10 \\ 0 & 14/5 & 48/5 & 62/5 \end{array}\right]$$

$R_3 \to \frac{97}{2}R_3 - 14R_2$

$$\cancel{[A : B] \sim \left[\begin{array}{ccc|c} 10 & 1 & 2 & 13 \\ 0 & 97/10 & 2/5 & 101/10 \\ 0 & 0 & 460 & 460 \end{array}\right]}$$

$$[A : B] \sim \left[\begin{array}{ccc|c} 10 & 1 & 2 & 13 \\ 0 & 97/10 & 2/5 & 101/10 \\ 0 & 0 & 460 & 460 \end{array}\right]$$

$$[A:B] \sim \left[\begin{array}{ccc:c} 10 & 1 & 2 & 13 \\ 0 & 97/10 & 2/5 & 101/10 \\ 0 & 0 & 460 & 460 \end{array}\right]$$

which is in upper triangular form

Now.

$$10x + y + 2z = 13 \quad \text{—(1)}$$

$$\frac{97}{10}y + \frac{2}{5}z = \frac{101}{10} \quad \text{—(2)}$$

$$460z = 460 \quad \text{—(3)}$$

$$z = 1, \quad y = 1, \quad x = 1$$

METHOD - 2

GAUSS JORDAN METHOD

WORKING RULE

1. Reduce given system of eqn in matrix form $AX = B$.
2. form Augmented Matrix $C \sim [A:B]$
3. Reduce $[A:B]$ into ~~in~~ unit matrix using only row operation. ~~i.e.~~
4.) Solve system of eqn thus formed to get required solution.

Que. Apply Gauss Jordan method to find solution

$$\left.\begin{aligned} 10x + y + z &= 12 \\ 2x + 10y + z &= 13 \\ x + y + 5z &= 7 \end{aligned}\right\} \longrightarrow (1)$$

Soln. Given linear system of eqn (1) can be written in matrix form as $AX = B$;

~~where~~.

$$\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix}$$

Augmented matrix $[A:B]$ is

$$[A:B] = \begin{bmatrix} 10 & 1 & 1 & : & 12 \\ 2 & 10 & 1 & : & 13 \\ 1 & 1 & 5 & : & 7 \end{bmatrix}$$

$$[A:B] = \left[\begin{array}{ccc:c} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{array}\right]$$

op: $R_1 \longleftrightarrow R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{array}\right]$$

op: $R_2 \longrightarrow R_2 - 2R_1$; $R_3 \longrightarrow R_3 - 10R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{array}\right]$$

op: $R_2 \longrightarrow \left(\frac{1}{8}\right) R_2$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 5 & 7 \\ 0 & 1 & -9/8 & -1/8 \\ 0 & -9 & -49 & -58 \end{array}\right]$$

op: $R_3 \longrightarrow R_3 + 9R_2$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 5 & 7 \\ 0 & 1 & -9/8 & -1/8 \\ 0 & 0 & -473/8 & -473/8 \end{array}\right]$$

op: $\cancel{R_3 \longrightarrow \left(\frac{-473}{8}\right) R_3} \quad \cancel{R_3 \left(\frac{-8}{473}\right)}$

$R_3 \longrightarrow \left(\frac{-8}{473}\right) R_3$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 5 & 7 \\ 0 & 1 & -9/8 & -1/8 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

op:- $R_1 \to R_1 - R_2$

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 0 & 49/8 & 57/8 \\ 0 & 1 & -9/8 & -1/8 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

op: $R_2 \to R_2 + \frac{9}{8} R_3$

$R_1 \to R_1 - \frac{49}{8} R_3$

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

∴ from above

$x = 1, \quad y = 1, \quad z = 1.$

Ques. 2. Apply Gauss Jordan method to solve the eqns

$$10x + y + z = 9$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$

Soln. Augmented matrix,

$$[A:B] = \left[\begin{array}{ccc:c} 10 & 1 & 1 & 9 \\ 1 & 10 & 1 & 12 \\ 1 & 1 & 10 & 12 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$.

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 10 & 1 & 12 \\ 10 & 1 & 1 & 9 \\ 1 & 1 & 10 & 12 \end{array}\right]$$

$R_2 \longrightarrow R_2 - 10R_1 \quad ; \quad R_3 \longrightarrow R_3 - R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 10 & 1 & 12 \\ 0 & -99 & -9 & -111 \\ 0 & -9 & 9 & 0 \end{array}\right]$$

$R_2 \longrightarrow \left(\frac{-1}{9}\right)R_2 \quad , \quad R_3 \longrightarrow \left(\frac{+1}{9}\right)R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 10 & 1 & 12 \\ 0 & +11 & 1 & 12.333 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

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op: $R_1 \rightarrow 11R_1 - 10R_3$; $R_3 \longrightarrow 11R_3 + R_2$

$$[A:B] \sim \begin{bmatrix} 11 & 0 & 1 & : & 8.67 \\ 0 & 11 & 1 & : & 12.333 \\ 0 & 0 & 12 & : & 12.333 \end{bmatrix}$$

$R_3 \longrightarrow \frac{1}{12}(R_3)$

$$[A:B] \sim \begin{bmatrix} 11 & 0 & 1 & : & 8.67 \\ 0 & 11 & 1 & : & 12.333 \\ 0 & 0 & 1 & : & 1.02775 \end{bmatrix}$$

op: $R_1 \longrightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$$[A:B] \sim \begin{bmatrix} 11 & 0 & 0 & : & 7.64225 \\ 0 & 11 & 0 & : & 11.30525 \\ 0 & 0 & 1 & : & 1.02775 \end{bmatrix}$$

op:- $R_1 \longrightarrow \left(\frac{1}{11}\right)R_1$; $R_2 \rightarrow \left(\frac{1}{11}\right)R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 0 & 0 & : & 0.69475 \\ 0 & 1 & 0 & : & 1.02775 \\ 0 & 0 & 1 & : & 1.02775 \end{bmatrix}$$

$x = 0.69475$

$y = z = 1.02775$.