# Numerical Method - 1

## Algebraic And Transcendental Equations

**Polynomial** → An expression of form
$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$
is called polynomial of degree 'n'.

**Example:** $f(x) = x^3 + 4x^2 + 5x + 2$
is polynomial of degree 3.

**Polynomial or Algebraic Eq^n's**
If $f(x) = 0$ then it is called
Polynomial eq^n.

**Transcendental Equation**
If algebraic equation contains
exponential, logarithmic or
trigonometric functions, then
it is called Transcendental Eq^n.

**Example:** (i) $x \log_{10} x = 1.2$
(ii) $xe^x = 2$
(iii) $3x = \cos x + 1$.

**Roots:-** The value of equation that
satisfies the equation is called
root of that eq^n.

* Eq^n of $n^{th}$ degree has only 'n' roots.
* Every eq^n of odd degree has
atleast one real root.



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# NOTE FOR CALCULATOR

1. Always reset your Calculator
SHIFT + MODE + 3 + =(2 times)

2. Set it to correct decimal places as asked:
(i) Press MODE button until it shows. Fix Sci Norm
1 2 3

Choose fix option by pressing 1
It will ask Fix 0~9?.
Choose decimal places.

3. While solving any question that involves trigonometric ratio ~~use~~ fix calculator on Radian mode by using:

Pressing MODE until show.
Deg Rad Gra
1 2 3

Choose Radian option using/pressing 2 button.

4. For $\log_{10}$ use simple log and for $\log_e$ use ln

5. If 5 decimal places is asked set on 6 decimal places always fix on one digit extra.



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## -ION OR BOLZANO METHOD

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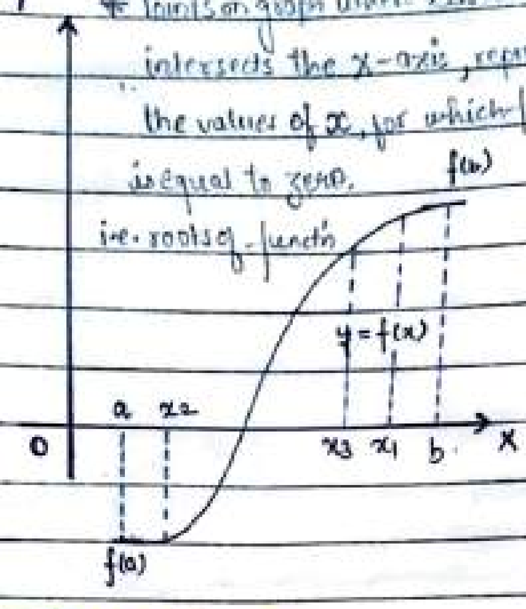
### Concept of Bisection Method

* Points on graph where curve intersects the $x$-axis, represents the values of $x$, for which function is equal to zero. i.e. roots of functn




### WORKING RULE

1. Let $f(x) = 0$, be the given eqn.

2. Find $a$ and $b$ such that, $\underset{(-ve)}{f(a) < 0}$ and $\underset{(+ve)}{f(b) > 0}$ {or $f(a).f(b) < 0$}

3. Find 1st approximate root using Bisection method.

$$x_1 = \frac{a+b}{2}$$

Now, calculate $f(x_1)$ and examine it's sign.

(3.1) If $f(x_1) < 0 \Rightarrow$ root lies b/w $x_1$ and $b$ and second approximate root is given by, $x_2 = \frac{x_1 + b}{2}$

(3.2) If $f(x_1) > 0 \Rightarrow$ root lies b/w $a$ and $x_1$, and second approximate root is given by, $x_2 = \frac{a + x_1}{2}$

Now calculate $f(x_2)$ and check sign and repeat (3.1) and (3.2) to find desired accuracy.

Ques. Find the root of the equation $x^3 - x - 4 = 0$ between 1 and 2 to ~~four~~ three places of decimal using Bisection Method.

Soln. Let $f(x) = x^3 - x - 4 = 0$

Here $a = 1$ and $b = 2$

$\therefore f(a) = f(1) = 1^3 - 1 - 4 = -4$, (−ve) and

$f(b) = f(2) = 2^3 - 2 - 4 = 2$. (+ve)

Clearly $f(a) \cdot f(b) < 0$. Therefore, roots of $f(x)$ lies b/w 1 and 2.

1st approximation $x_1 = \frac{a+b}{2}$

$\Rightarrow x_1 = \frac{1+2}{2} = 1.5$.

Now, since $f(1) = -4$ −ve

$f(2) = 2$ +ve

and $f(x_1) = f(1.5) = -2.1250$ −ve.

Therefore root lies in interval $(1.5, 2)$.

2nd approximation $x_2 = \frac{1.5+2}{2}$

$\Rightarrow x_2 = 1.7500$

and $f(1.7500) = -0.3906$

$f(x_2) = -0.3906$ (−ve)

Clearly $f(1.7500)$ is −ve and $f(2)$ is positive.

Therefore root lies in interval $(1.7500, 2)$.

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3rd approximation:

$$x_3 = \frac{1.7500 + 2}{2}$$

$x_3 = 1.8750$ and

~~and~~ $f(x_3) = f(1.8750) = 0.7168$
+ve

Since $f(1.7500)$ is negative and $f(1.8750)$ is positive.
Therefore, root lies in interval
$(1.7500, 1.8750)$.

4th approximation

$$x_4 = \frac{1.7500 + 1.8750}{2}$$

$x_4 = 1.8125$ and

$f(x_4) = f(1.8125) = 0.1418$ (+ve)

Therefore, root lies b/w interval
$(1.7500, 1.8125)$

5th approximation

$$x_5 = \frac{1.7500 + 1.8125}{2}$$

$x_5 = 1.7813$, and

$f(x_5) = f(1.7813) = -0.1296$ (-ve)

Therefore root lies b/w interval
$(1.7813, 1.8125)$

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## $6^{th}$ approximation:

$$x_6 = \frac{1.7813 + 1.8125}{2}$$

$x_6 = 1.7969$ and

$f(x_6) = f(1.7969) = 0.0050$ +ve

Therefore, root lies b/w interval $(1.7813, 1.7969)$.

## $7^{th}$ approximation:

$$x_7 = \frac{1.7813 + 1.7969}{2}$$

$x_7 = 1.7891$ and

$f(x_7) = f(1.7891) = -0.0624$ -ve

Therefore root lies b/w interval $(1.7891, 1.7969)$.

## $8^{th}$ approximation

$$x_8 = \frac{1.7891 + 1.7969}{2}$$

$x_8 = 1.7930$ and

$f(x_8) = f(1.7930) = -0.0288$ -ve

Therefore root lies between interval $(1.7930, 1.7969)$

## $9^{th}$ approximation

$$x_9 = \frac{1.7930 + 1.7969}{2}$$

$x_9 = 1.7950$ and

$f(x_9) = f(1.7950) = -0.0119$ -ve

Therefore root lies in interval $(1.7950, 1.7969)$



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**$10^{th}$ approximation**

$$x_{10} = \frac{1.7950 + 1.7969}{2}$$

$x_{10} = 1.7960$ and

$f(x_{10}) = f(1.7960) = -0.0032$ -ve

Therefore, root lies b/w interval

$(1.7960, 1.7969)$

**$11^{th}$ approximation**

$$x_{11} = \frac{1.7960 + 1.7969}{2}$$

$x_{11} = 1.7965$ and

$f(x_{11}) = f(1.7965) = 0.0011$ +ve

Therefore, root lies b/w interval.

$(1.7960, 1.7965)$

**$12^{th}$ approximation**

$$x_{12} = \frac{1.7960 + 1.7965}{2}$$

$x_{12} = 1.7963.$

Since, $x_{10} = x_{11} = x_{12} = 1.796$ upto three decimal places, hence the root of given equation is $1.796$.

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# Solution of Algebraic and Transcendental Equations

**Polynomials** → An expression of form

$$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$$

is called Polynomial of degree 'n'

Example : ① $f(x) = x^3 + 4x^2 + 5x + 2$ (polynomial of degree 3)

② $f(x) = x^2 + 5x + 6$ (Polynomial of degree 2)

**Equations** If $f(x) = 0$ then this is called Polynomial equation

Ex: $x^3 + 4x^2 + 5x + 2 = 0$
(is polynomial or algebraic equation of degree 3)

Transcendental Equation → If Algebraic equation contains logarithmic terms or Exponential terms or trigonometric terms then they are called Transcendental equations.

**Example** — ⓐ $3x = \cos x + 1$
ⓑ $xe^x = 2$
ⓒ $x \log_{10} x = 1.2$

**Roots** :- The value of eqn that satisfies the equation is called a root.

**Ex.** : $f(x) = x^2 - 5x + 6 = 0$
$= x^2 - 3x - 2x + 6 = 0$
$= x(x-3) - 2(x-3) = 0$
$\Rightarrow (x-3) = 0, (x-2) = 0$
$x = 3, \quad x = 2$

then $x = 3, 2$ are roots of eqn

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# Bisection Method

## Working Rule

1) Let $f(x) = 0$ —— (1)
be the given equation. find a and b such that $f(a) < 0$ and $f(b) > 0$ $\{$or $f(a).f(b) < 0\}$

(2) find $1^{st}$ approx. root using bisection method.

$$x_1 = \frac{a+b}{2}$$, calculate $f(x_1)$ and examine its sign.

(2.1) If $f(x_1) < 0 \Rightarrow$ root lies b/w $x_1$ and $b$.

and second approximation or approx. root is given by

$$x_2 = \frac{x_1 + b}{2}.$$

(2.2) If $f(x_1) > 0 \Rightarrow$ root lies b/w a and $x_1$

and second approx. root is given by.

$$x_2 = \frac{a + x_1}{2}.$$

Now calculate $f(x_2)$ and repeat step. (2.1) and (2.2) until required accuracy.

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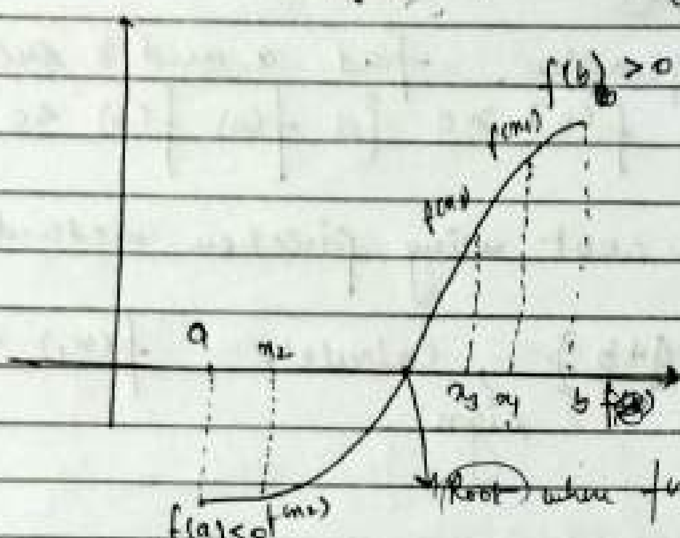
## Concept of Bisection method.

$f(x) = 0$ given eqn.

* Root means where the function intersects x-axis.




**Note**

① In trigonometric function set Calci on Radian mode

* The points at ~~which~~ where its graph ~~intercep~~ intersects or touches. the x-axis, representing the values of x for which function equals zero

Since $x_{10}$ and $x_{11}$ are same upto two decimal places the root is 0.73

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$f(x) = x - \cos x = 0$

$\underset{A}{f(0)} = -1$ $\qquad$ $\underset{B}{f(1)} = 0.4597$

Since, $f(0) < 0$ and $f(1) > 0$, the root lies b/w. 0 and 1.

$$C = \frac{A+B}{2} \quad ; \quad C - \cos C.$$

use Alpha and Calc last.

| No. | A (−ve) | B (+ve) | $x_0 = a+b/2$ | Sign of $f(x)$. |
|---|---|---|---|---|
| 1 | 0 | 1 | 0.5 | −0. 3775. −ve |
| 2 | 0.5 | 1 | 0.75 | 0.0183 (+ve) |
| 3 | 0.5 | 0.75 | ~~0.75~~ 0.6250 | −0.1860 (−ve) |
| 4 | 0.625 | 0.75 | ~~0.75~~ 0.6875 | −0.0853 (−ve) |
| 5 | 0.6875 | 0.75 | ~~0.75~~ 0.7188 | −0.0339 (−ve) |
| 6 | 0.7188 | 0.75 | ~~0.75~~ 0.7344 | −0.0078 (−ve) |
| 7 | 0.7344 | 0.75 | 0.7422 | 0.0052 (+ve) |
| 8 | 0.7344 | 0.7422 | 0.7383 | −0.0015 (−ve) |
| 9 | 0.7383 | 0.7422 | 0.7403 | 0.0020 (+ve) |
| 10 | 0.7383 | 0.7403 | 0.7393 | 0.0004 (+ve) |
| 11 | 0.7383 | 0.7393 | 0.7388 | −0.0005 (−ve) |
| 12 | 0.7388 | 0.7393. | 0.7391 | −0.0001 |
| 13. | 0.7391 | 0.7393 | 0.7392 | 0.0002 |

Here. $x_{11} = x_{12} = 0.739$ upto 3 decimal places. Hence, root of given eqn is 0.739.

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Que 2 find the real root of the equation $x\log_{10} x = 1.2$ by Bisection Method correct to three decimal place.

Soln Let $f(x) = x\log_{10} x - 1.2 = 0$

taking $f(1) = -1.2000$

$f(2) = -0.5979$ −ve

$f(3) = 0.2314$ +ve

Hence root lies b/w interval (2, 3).

| Approximation No. | A | B | Approx. $x = \frac{a+b}{2}$ | $f(x)$ | Sign of $f(x)$ |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 2.5000 | −0.2051 | −ve. |
| 2 | 2.5000 | 3 | 2.7500 | 0.0082 | +ve. |
| 3 | 2.5000 | 2.7500 | 2.6250 | −0.0998 | −ve |
| 4 | 2.6250 | 2.7500 | 2.6875 | −0.0461 | −ve |
| 5 | 2.6875 | 2.7500 | 2.7188 | −0.0191 | −ve. |
| 6 | 2.7188 | 2.7500 | 2.7344 | −0.0054 | −ve |
| 7 | 2.7344 | 2.7500 | 2.7422 | 0.0014 | +ve |
| 8 | 2.7344 | 2.7422 | 2.7383 | −0.0020 | −ve |
| 9 | 2.7383 | 2.7422 | 2.7403 | −0.0003 | −ve |
| 10 | 2.7403 | 2.7422 | 2.7413 | 0.0005 | +ve |
| 11 | 2.7403 | 2.7413 | 2.7408 | 0.0001 | +ve |
| 12 | 2.7403 | 2.7408 | 2.7406 | −0.0001 | −ve |
| 13 | 2.7406 | 2.7408 | 2.7407 | 0.0000 | — |

Here, ∴ $x_{11} = x_{12} = x_{13} \approx 2.740$ upto 3 decimal places. Hence root of given function is 2.740

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①

Que. find the root of eqn $x^3+x-1$ which is near to 1. upto 3 decimal places.

Soln

Here, $f(x) = x^3+x-1$

Now,

$f(0) = -1$ -ve

$f(1) = 1$ +ve

$\because$ $f(0).f(1) < 0$, therefore root lies in interval (0,1)

| No. | a | b | $x=\frac{a+b}{2}$ | f(x) | sign |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0.5000 | -4.3750 | -ve |
| 2 | 0.5000 | 1 | 0.7500 | -4.3281 | -ve |
| 3 | 0.7500 | 1 | 0.8750 | -4.2051 | -ve |
| 4 | 0.8750 | 1 | 0.9375 | -4.1135 | -ve |
| 5 | 0.9375 | 1 | 0.9688 | -4.0596 | -ve |
| 6 | 0.9688 | 1 | 0.9844 | -4.0305 | -ve |
| 7 | 0.9844 | 1 | 0.9922 | -4.0154 | -ve |
| 8 | 0.9922 | 1 | 0.9961 | -4.0078 | -ve |
| 9 | 0.9961 | 1 | 0.9981 | -4.0039 | -ve |
| 10 | 0.9981 | 1 | 0.9991 | -4.0019 | -ve |
| 11 | 0.9991 | 1 | 0.9996 | -4.0009 | -ve |
| 12 | 0.9996 | 1 | 0.9998 | -4.0004 | -ve |

Here, $x_{10} = x_{11} = x_{12} = 0.999$ upto 3 decimal places.

Therefore, root ~~g~~ of given equation is 0.999.

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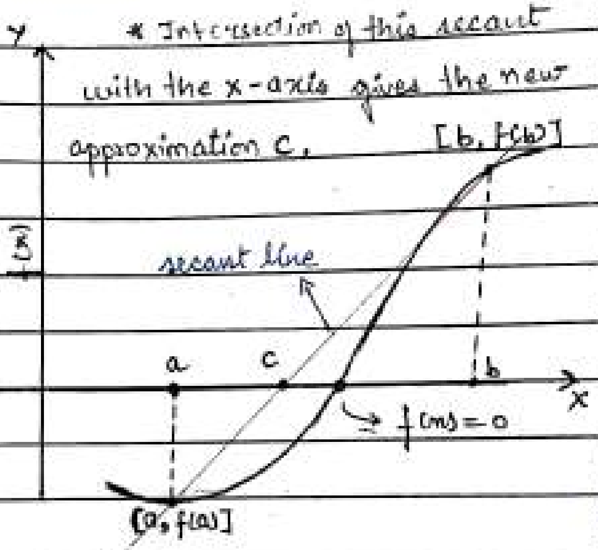
# Regula Falsi Method or False Position Method

Regula falsi method is similar to the bisection method, but it improves convergence by using a weighted average based function values.




**Working Rule:** Let $f(x) = 0$ —①

1. Select two initial points $(a, b)$ such that $f(a)$ and $f(b)$ have opposite signs [i.e. function crosses the x-axis between them].
   i.e. $f(a) \cdot f(b) < 0$

2. Compute new approximation $(c)$ using the formula:

$$c = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

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3. Check if $f(c) \approx 0$ (or meets) the desired accuracy.

4. update a or b based on the sign of $f(c)$ and continue iterating.

5. Repeat until convergence

For Calculator

Que. $x \log x - 1.2$.

$A : B : C = A \log A - 1.2 :$
$D = B \log B - 1.2 :$

$$X = \frac{AD - BC}{D - C} :$$

$E = X \log X - 1.2$.

Here $A = a$
$B = b$
$C = f(a)$
$D = f(b)$
$x_n = X$
and $E = X \log X - 1.2$.

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Ques: find a real root of the eq^n $x^3 - 9x + 1 = 0$ by Regula falsi.

Sol: Here, $f(x) = x^3 - 9x + 1 = 0$

$f(1) = -7$, $f(2) = -9$

$f(3) = 1$

Since $f(2) \cdot f(3) < 0$ therefore root lies in interval $(2, 3)$.

| No. | a | b | f(a) | f(b) | $x_n = \frac{a f(b) - b f(a)}{f(b) - f(a)}$ | $f(x_n)$ |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | -9 | 1 | $x_1 = 2.9$ | -0.7110 |
| 2 | 2.9 | 3 | -0.7110 | 1 | $x_2 = 2.9416$ | -0.0215 |
| 3 | 2.9416 | 3 | -0.0207 | 1 | $x_3 = 2.9428$ | -0.0006 |
| 4 | 2.9428 | 3 | -0.0003 | 1 | $x_4 = 2.9428$ | 0.0000 |

Here = $x_3 = x_4 = 2.942$, upto 3 decimal places.

So, root of given eq^n is 2.942

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Ques find the real root of the equation $x\log_{10}x - 1.2 = 0$ by Regula falsi method upto 4 decimal places.

Soln Tabular form:-

| No. | a | b | f(a) | f(b) | $x_n = \frac{af(b)-bf(a)}{f(b)-f(a)}$ | $f(x_n)$ |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | -0.59794 | 0.23136 | 2.72101 | -0.01709 |
| 2 | 2.72101 | 3 | -0.01709 | 0.23136 | 2.74021 | -0.00038 |
| 3 | 2.74021 | 3 | -0.00038 | 0.23136 | 2.74064 | -0.00001 |
| 4 | 2.74064 | 3 | -0.00001 | 0.23136 | 2.74065 | 0.0000 |

Complete Solution:

Here $f(x) = x\log_{10}x - 1.2 = 0$

$f(2) = 2\log_{10}2 - 1.2 = -0.59794$ (-ve)

$f(3) = 3\log_{10}3 - 1.2 = 0.23136$ (+ve)

atleast a root lies in (2,3).

taking $a = 2$ and $b = 3$.

$f(2) = -0.59794$ , $f(3) = 0.23136$.

By Regula falsi method,

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$\Rightarrow x_1 = \frac{2(0.23136) - 3(-0.59794)}{0.23136 - (-0.59794)}$$

$\Rightarrow x_1 = 2.72102$.

Also,

$f(x_1) = f(2.72102) = -0.01709 < 0$

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Since $f(2.72102)$ and $f(3)$ are of opposite sign.
So the root lies between $(2.72102, 3)$

$$x_2 = \frac{2.72102\,(0.23136) - 3\,(-0.01709)}{(0.23136) - (-0.01709)}$$

$$x_2 = 2.74021.$$

Thus. $f(x_2) = f(2.74021) = -0.00038 < 0$

Since $f(2.74021)$ and $f(3)$ are of opposite sign
So the root lies between $(2.74021, 3)$

$$x_3 = \frac{2.74021\,(0.23136) - 3(-0.00038)}{0.23136 - (-0.00038)}$$

$$x_3 = 2.74064$$

Again $f(x_3) = f(2.74064) = -0.00001 < 0$

Since $f(2.74064)$ and $f(3)$ are of opposite sign
so, the root lies between $(2.74064, 3)$.

$$x_4 = \frac{2.74064\,(0.23136) - 3\,(-0.00001)}{0.23136 - (-0.00001)}$$

$$x_4 = 2.74065$$

Here $x_3 = x_4 = 2.7406$ upto four decimal places.

Hence $2.7406$ is the required root of given eqn.

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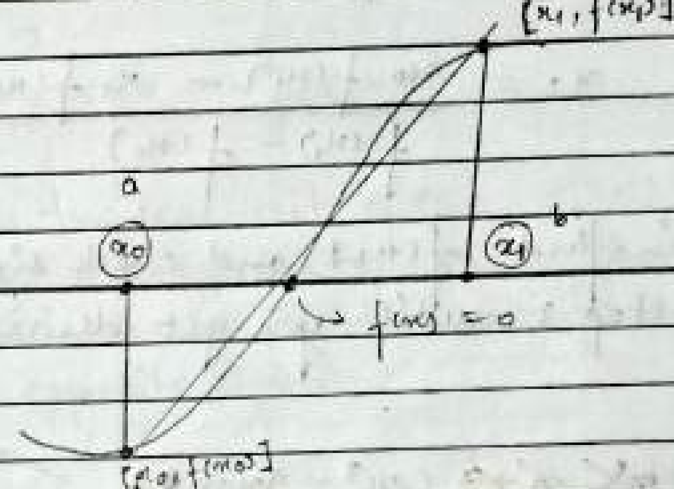
# Regula falsi Method. or Method of false Position.




formula used:

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

<u>Working Rule</u> :- ① Given $f(x) = 0$ —— ①

② find $x_0$ and $x_1$ such that $f(x_0) < 0$ and $f(x_1) > 0$ i.e $f(x_0) \cdot f(x_1) < 0 \Rightarrow$ roots of ① lies in interval $(x_0, x_1)$

③ find first approximate root by Regula falsi method

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}.$$

or for $(a, b)$

$$x_n = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

find $f(x_2)$ and check its sign

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(2.1) If $f(x_2) < 0$ then replace $x_0 = x_2$
(2.2) If $f(x_2) > 0$ then replace $x_1 = x_2$

And find second approx root using.

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Again find $f(x_2)$ and check sign and repeat step 3, until you get desired accuracy.

① $\cancel{x^3 - x + 9}$ $x^3 - 9x + 1$ $\longrightarrow 2.9428$ $\quad 0.\cancel{3604}$

$x^3 - 2x - 5 \longrightarrow 2.09.$

$x \log_{10} x - 1.2 = 0 \longrightarrow 2.7406$

$x e^x = 2. \longrightarrow 0.8526.$

$x e^x = \cos x \longrightarrow 0.5174.$

$3x + \sin x - e^x \longrightarrow 0.36042$

$\cos x = 3x - 1. \longrightarrow 0.607$

$f(a)\cdot f(b) < 0$

$-ve$ $+ve$

Ques) Find the real root of the equation $x^3 - 9x + 1 = 0$ by the method of false position.

Soln let $f(x) = x^3 - 9x + 1 = 0$

$f(1) = -7$ $f(0) = 1$

$f(2) = -9$ $f(3) = 1$

* To find the real root of eqn $f(x) = 0$, we consider a sufficiently small interval $(a, b)$ where $a < b$ such that $f(a)$ and $f(b)$ will have opposite signs.

| $f(x)$ | $a$ ($-ve$) | $b$ ($+ve$) | $f(a)$ | $f(b)$ | $x = \frac{a f(b) - b f(a)}{f(b) - f(a)}$ |
|---|---|---|---|---|---|
| $-0.7110$ | 2 | 3 | $-9$ | 1 | $x_1 = 2.9$ |
| $-0.0215$ | 2.9 | 3 | $-0.7110$ | 1.000 | $x_2 = 2.9416$ |
| $-0.0006$ | 2.9416 | 3 | $-0.0207$ | 1 | $x_3 = 2.9428$ |
| 0.0000 | 2.9428 | 3 | $-0.0003$ | 1 | $x_4 = 2.9428$ |

Here $x_3 = x_4 = 2.9428$

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| | a | b | f(a) | f(b) | $x_{n}$ = | $f(x_n)$ |
|---|---|---|---|---|---|---|
| 0 | 2 | 3 | -0.59794 | 0.23136 | 2.72101 | -0.01709 |
| 1 | 2.72101 | 3 | -0.01709 | 0.23136 | 2.74021 | -0.00038 |
| 2 | 2.74021 | 3 | -0.00038 | 0.23136 | 2.74064 | -0.00001 |
| 3 | 2.74064 | 3 | -0.00001 | 0.23136 | 2.74065 | 0.00000 |

root = 2.74065

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Ques. find the real root of the equation $x \log_{10} x - 1.2 = 0$ by Regula-falsi method upto 4 decimal places.

Soln. $f(x) = x \log x - 1.2$

$f(1) = 1 \log 1 - 1.2 = -1.20000$

$f(2) = 2 \log 2 - 1.2 = -0.59794$ (-ve)

$f(3) = 3 \log 3 - 1.2 = 0.23136$. (+ve)

Here $f(2) \cdot f(3) < 0$. Hence atleast a root lies b/w 2 and 3.

taking $a = 2$ and $b = 3$.

$f(2) = -0.59794$, $f(3) = 0.23136$.

By method of false Position

1st approx, $x_1 = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$

$x_1 = \dfrac{2 f(3) - 3 f(2)}{f(3) - f(2)} = \dfrac{2(0.23136) - 3(-0.59794)}{0.23136 - (-0.59794)}$

$x_1 = 2.72101$

Now. $f(x_1) = 2.72102 \log 2.72102 - 1.2$

$= -0.01709 < 0$

Here $A : B : C = \overset{f(a)}{A \log A - 1.2} : D = \overset{f(b)}{B \log B - 1.2}$ :

$X = \dfrac{AD - BC}{D - C}$ : $X \log_{10} X - 1.2$.

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Que (9) find the root of the eq^n ~~$x^3 - 5x - 7$~~

$x^3 - 5x - 7 = 0$ which lies b/w 2 and 3 by method of false Position.

Sol^n

$f(x) = x^3 - 5x - 7$

$f(2) = -9$, $f(3) = 5$

$\because f(2) \cdot f(3) < 0$, then atleast a root lies b/w. $(2, 3)$.

$= \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

| | a | b | f(a) | f(b) | $x_n$ | $f(x_n)$ |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | -9 | 5 | 2.6429 | -1.7547 |
| 2 | 2.6429 | 3 | -1.7541 | 5 | 2.7356 | -0.2054 |
| 3 | 2.7356 | 3 | -0.2061 | 5 | 2.7461 | -0.0225 |
| 4 | 2.7461 | 3 | -0.0220 | 5 | 2.7472 | -0.0029 |
| 5 | 2.7472 | 3 | -0.0026 | 5 | 2.7473 | -0.0003 |
| 6 | 2.7473 | 3 | -0.0008 | 5 | 2.7473 | -0.0001 |

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Q(ii) Regula-falsi upto 3 decimal places

$f(x) = x e^{x} - \cos x.$

$f(0) = -1.$ , $f(1) = 2.1780$

$\because f(0) \cdot f(1) < 0.$ root lies b/w $(0, 1)$.

| No. | a | b | f(a) | f(b) | $x_n$ | $f(x_n)$ |
|---|---|---|---|---|---|---|
| | 0 | 1 | -1 | 2.1780 | 0.3147 | -0.5199 |
| | 0.3147 | 1 | -0.5198 | 2.1780 | 0.4467 | -0.2035 |
| | 0.4467 | 1 | -0.2036 | 2.1780 | 0.4940 | -0.0708 |
| | 0.4940 | 1 | -0.0708 | 2.1780 | 0.5099 | -0.0236 |
| | 0.5099 | 1 | -0.0237 | 2.1780 | 0.5152 | -0.0078 |
| | 0.5152 | 1 | -0.0078 | 2.1780 | 0.5169 | -0.0025 |
| | 0.5169 | 1 | -0.0026 | 2.1780 | 0.5175 | -0.0009 |
| | 0.5175 | 1 | -0.0008 | 2.1780 | 0.5177 | -0.0003. |

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# NEWTON'S RAPHSON'S METHOD

Newton Raphsons method is an iterative numerical technique used to find the roots of a real-valued function. It follows the general formula.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

where, $n = 0, 1, 2, 3$ ------.

## WORKING RULE :-

1. Let $f(x)$ be the given function.
2. Calculate, $f(a) < 0$ and $f(b) > 0$, then root lies between a and b.

3. Now select an initial approximation,

$$x_0 = \frac{a+b}{2}$$

4. Calculate $f(x_n)$ and $f'(x_n)$.

5. Then apply Newton Raphson's formula.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

where $n = 0, 1, 2, 3$ ------.

6.) Repeat the steps to get desired root.

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$A : B = A^4 - A - 10 : C = 4A^3 - 1 :$

$D = A - (B \div C)$.

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Ques

Ques. Find a real root of equation $x^4 - x - 10 = 0$ upto 3 decimal place using Newton Raphson Method.

Soln.

$$f(x) = x^4 - x - 10 = 0 \quad , \quad f'(x) = 4x^3 - 1$$

Now, $f(0) = -10 < 0$

$$\left.\begin{aligned} f(1) &= -10 < 0 \\ f(2) &= 4 > 0 \end{aligned}\right\}$$

Hence root lies between 1 and 2.

Taking initial approximation,

$$x_0 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$$

Now, by Newton Raphson's Method, General formula.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1st approximation

at $n=0$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 1.5$
$f(x_0) = -6.4375$
$f'(x_0) = 12.5000$
$x_1 = 2.0150$

$$x_1 = 1.5 - \left[\frac{(1.5)^4 - 1.5 - 10}{4(1.5)^3 - 1}\right]$$

$$x_1 = 1.5 - \left[\frac{-6.4375}{12.5000}\right]$$

$$x_1 = 2.0150$$

2nd approximation at $n = 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.0150 - \frac{4.4704}{31.7254}$$

$$x_2 = 1.8741$$

$x_1 = 2.0150$
$f(x_1) = 4.704$
$f'(x_1) = 31.7254$
$x_2 = 1.8741$

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3rd approximation: at $n = 2$.

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_2 = 1.8741$

$f(x_2) = 0.4618$

$f'(x_2) = 25.3292$

$x_3 = 1.8559$

$$x_3 = 1.8741 - \frac{0.4618}{25.3292}$$

$$x_3 = 1.8559$$

4th approximation: at $n = 3$.

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$x_3 = 1.8559$

$f(x_3) = 0.0077$

$f'(x_3) = 24.5696$

$x_4 = 1.8556$

$$x_4 = 1.8559 - \frac{0.0077}{24.5696}$$

$$x_4 = 1.8556.$$

Since. $x_3$ and $x_4$ = 1.855 upto 3 decimal place.

Hence, required root is 1.855.

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Que. 2. find a root by Newton Raphson method of equation $x^3 - 3x + 1 = 0$ correct to 3 decimal places

Soln $f(x) = x^3 - 3x + 1$ — ①

$f'(x) = 3x^2 - 3$

Now, $f(0) = 1$ and $f(1) = -1$.

So, root lies b/w $(0, 1)$.

Initial Approximation, $x_0 = \frac{0+1}{2} = 0.5$

1st approximation: at $n = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0.5 - \frac{(-0.3750)}{(-2.2500)}$$

$$x_1 = 0.3333$$

2nd approx. at $n = 2$.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.3333 - \frac{0.0371}{(-2.6667)}$$

$$x_2 = 0.3472.$$

3rd approx at $n = 3$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.3472 - \frac{0.0003}{(-2.6384)}$$

$$x_3 = 0.3473.$$

Here, $x_2 = x_3 = 0.347$ upto 3 decimal places.

Hence required root is $0.347$.

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Ques. Apply Newton-Raphson Method to solve $3x - \cos x - 1 = 0$ correct to three decimal places

Soln. Here $f(x) = 3x - \cos x - 1$, $f'(x) = 3 + \sin x$

$$f(0) = -2 < 0$$
$$f(1) = 1 > 0$$

So, root lies between $(0, 1)$.

Initial approximation

$$x_0 = \frac{0+1}{2} = 0.5$$

1st approx at $n=0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \cancel{0.5 - \frac{(-0.5)}{3.4}}$$

$$x_1 = 0.5 - \frac{(-0.378)}{3.479}$$

$$x_1 = 0.609.$$

2nd approx at $n=1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.609 - \frac{0.007}{3.572}$$

$$x_2 = 0.607$$

3rd approx at $n=2$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.607 - \frac{0.000}{3.570}$$

$$x_3 = 0.607$$

Here, $x_2 = x_3 = 0.607$ upto 3 decimal places.

Hence required root is $0.607$.

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Ques. Newton Raphson's

$f(x) = x\sin x + \cos x$ , nearby $x = \pi$

$f(x) = x\sin x + \cos x$

$f'(x) = x\cos x + \sin x + \sin x = x\cos x$

$x_0 = 3.1416$ value of $\pi$.

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3.1416 -$

$x_1 = 2.8233$.

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.7986$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7984$.

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.7984$

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Q. Que find negative root of equation $x^3 - 21x + 3500 = 0$ correct to three decimal places using Newton Raphson's method

Soln

$f(x) = x^3 - 21x + 3500$

$f'(x) = 3x^2 - 21$.

-9 → +ve
-4 → +ve

$f(-15) = 440 > 0$ and $f(-16) = -260 < 0$

$\because f(15) \cdot f(16) < 0$.

$\Rightarrow$ root lies b/w -15 and -16.

Initial root,

$$x_0 = \frac{-15 + (-16)}{2} = -15.5$$

1st approx. $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} =$

$$x_1 = -15.5 - \frac{101.62500}{699.75000}$$

$$= -15.64523.$$

2nd approx.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = -15.64523 - \frac{(-0.98352)}{713.31967}$$

$$x_2 = -15.64385$$

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3rd approx

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = -15.64385 - \frac{0.00077}{713.19013}$$

$$= -15.64385$$

$x_2 = x_3 = -15.6438$ upto 4 decimal place.

Hence $-15.6438$ is required root.

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Que- find Determine Newton's Raphson iterative formula to find $k^{th}$ root of N.

$$x = \sqrt[k]{N} \qquad f(x) = x^k - N = 0$$

$$f'(x) = kx^{k-1}$$

Newton's formula -

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n^k - N)}{kx_n^{k-1}}$$

$$= \frac{x_n k x_n^{k-1} - x_n^k + N}{kx_n^{k-1}}$$

$$= \frac{kx_n^k - x_n^k + N}{kx_n^{k-1}}$$

$$= \frac{(k-1)x_n^k + N}{kx_n^{k-1}}$$

$$= \frac{1}{k}\left[\frac{(k-1)x_n^k}{x_n^{k-1}} + \frac{N}{x_n^{k-1}}\right]$$

$$x_{n+1} = \frac{1}{k} (k-1)x^n + \frac{N}{x_n^{k-1}}$$

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# Difference Operators

Let $y = f(x)$ be a given function and $h$ is any fixed value. Let $x = x_0, x_1, x_2, \dots, x_n$ such that,

$$x_1 = x_0 + h$$
$$x_2 = x_1 + h = x_0 + 2h$$
$$x_3 = x_2 + h = x_0 + 3h$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$x_n = x_{n-1} + h = x_0 + nh$$

Also,
$$y_0 = f(x_0)$$
$$y_1 = f(x_1) = f(x_0 + h)$$
$$y_2 = f(x_2) = f(x_0 + 2h)$$
$$\vdots \qquad \vdots$$
$$y_n = f(x_n) = f(x_0 + nh)$$

Note: Here $h \rightarrow$ length of interval and all $x_i$'s are equidistant.

1. Shifting Operator

Symbol – E and is defined as,

$$\boxed{E f(x) = f(x+h)}$$

$$E^2 f(x) = E[E f(x)] = E f(x+h) = f(x+2h)$$

- $E^2 f(x) = f(x+2h)$
- $E^{-1} f(x) = f(x-h)$ , • $E^{-2} = f(x-2h)$
- $E^{1/2} f(x) = f\left(x + \frac{h}{2}\right)$

\# $Ey_0 = E f(x_0) = f(x_0+h) = f(x_1) = y_1$

$\Rightarrow Ey_0 = y_1$ and • $Ey_1 = y_2$ , • $E^{-1} y_1 = y_0$

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* Forward value − current value.

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2. forward Difference Operator

Symbol − $\Delta$ defined as.

1st forward diff

$$\boxed{\Delta f(x) = f(x+h) - f(x)}$$

$f(x-h)$

Now.

$f(x)$

diff

$f(x+h)$

$$\Delta^2 f(x) = \Delta[\Delta f(x)]$$
$$= \Delta[f(x+h) - f(x)]$$

$$= \Delta f(x+h) - \Delta f(x)$$
$$= f(x+2h) - f(x+h) - \{f(x+h) - f(x)\}$$

$$= f(x+2h) - 2f(x+h) + f(x)$$

$$\Rightarrow \Delta^2 f(x) = f(x+2h) - 2f(x+h) + f(x)$$

* $\Delta y_0 = \Delta f(x_0) = f(x_0+h) - f(x_0)$
$= f(x_1) - f(x_0)$
$= y_1 - y_0$

$$\Rightarrow \Delta y_0 = y_1 - y_0$$

* $\Delta y_1 = y_2 - y_1$

$\Delta y_2 = y_3 - y_2$

* $\Delta^2 y_0 = \Delta(\Delta y_0) = \Delta(y_1 - y_0) = \Delta y_1 - \Delta y_0$

$= y_2 - y_1 - (y_1 - y_0) = y_2 - 2y_1 + y_0$

$$\Rightarrow \boxed{\Delta^2 y_0 = y_2 - 2y_1 + y_0}$$

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current value – Backward value

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3) **Backward Difference Operator**

Symbol – $\nabla$ and is defined as

$$\nabla f(x) = f(x) - f(x-h)$$

$$\nabla y_1 = \nabla f(x_1) = f(x_1) - f(x_1-h)$$
$$= f(x_1) - f(x_0+h-h)$$
$$= f(x_1) - f(x_0).$$
$$= y_1 - y_0.$$

$$\Rightarrow \nabla y_1 = y_1 - y_0.$$

* $\nabla y_2 = y_2 - y_1$
* $\nabla^2 y_2 = \nabla(y_2 - y_1) = \nabla y_2 - \nabla y_1$

$$= y_2 - y_1 - \{y_1 - y_0\} = y_2 - 2y_1 + y_0.$$

4.) **Central Difference Operator**

Symbol – $\delta$, defined as

$$\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$$

5.) **Averaging Operator**

Symbol – $\mu$ and is defined as

$$\mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$$

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# Relation between Difference Operator

Prove that

1. $\Delta = E - 1$
2. $\nabla = 1 - E^{-1}$
3. $\delta = E^{1/2} - E^{-1/2}$
4. $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$
5. $\left[E^{1/2} + E^{-1/2}\right](1+\Delta)^{1/2} = 2+\Delta$
6. $E = e^{hD}$, where $D = \frac{d}{dx}$
7. $hD = \log(1+\Delta) = -\log(1-\nabla)$
8. $(1+\Delta)(1-\nabla) = 1$

Ques Prove that, $\Delta = E - 1$

Since $\Delta f(x) = f(x+h) - f(x)$

$= E f(x) - f(x)$ $\{\because E f(x) = f(x+h)$

$\Delta f(x) = (E-1) f(x)$

{by left cancellation}

$\Rightarrow \Delta = E - 1$

Ques. Prove that, $\nabla = 1 - E^{-1}$

Since:

$\nabla f(x) = f(x) - f(x-h)$

$= f(x) - E^{-1} f(x)$

$\nabla f(x) = (1 - E^{-1}) f(x)$

By left cancell.

$\Rightarrow \nabla = 1 - E^{-1}$.

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3) Prove that $\delta = E^{1/2} - E^{-1/2}$.

$\because \delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$ by defn

$\Rightarrow \delta f(x) = E^{1/2} f(x) - E^{-1/2} f(x)$.

$\delta \cancel{f(x)} = E^{1/2} - E^{-1/2} \cancel{f(x)}$

$\rightarrow \delta = E^{1/2} - E^{-1/2}$.

4. Prove that $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$

$\because \mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$

$= \frac{1}{2}\left[E^{1/2} f(x) + E^{-1/2} f(x)\right]$

$\mu \cancel{f(x)} = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right] \cancel{f(x)}$.

$\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$.

5. Prove that $\left[E^{1/2} + E^{-1/2}\right](1+\Delta)^{1/2} = 2+\Delta$

L.H.S.

$= \left[E^{1/2} + E^{-1/2}\right](1+\Delta)^{1/2}$

$= \left[E^{1/2} + E^{-1/2}\right] E^{1/2}$ $\{\because 1+\Delta = E\}$

$= E + 1$

$= 1 + \Delta + 1$

$= 2 + \Delta$

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$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

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Que Prove that $(1+\Delta)(1-\nabla) = 1$

LHS

$= (1+\Delta)(1-\nabla)$

$= E \cdot E^{-1}$

$= 1$.

Que Prove that $E = e^{hD}$, where $D = \frac{d}{dx}$

By defn of shifting operator

$E f(x) = f(x+h)$. By Taylor's Theor.

$\Rightarrow E f(x) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \cdots$

$\Rightarrow E f(x) = f(x) + hD f(x) + \frac{h^2}{2!} D^2 f(x) + \frac{h^3}{3!} D^3 f(x) + \cdots$

$\Rightarrow E f(x) = \left\{ 1 + hD + \frac{h^2 D^2}{2!} + \frac{h^3 D^3}{3!} + \cdots \right\} f(x)$

$\Rightarrow E f(x) = e^{hD} f(x)$

$\Rightarrow E = e^{hD}$.

Que Prove that $hD = \log(1+\Delta) = -\log(1-\nabla)$

WKT $E = e^{hD}$ — (1)

Taking log both side.

$\log E = hD$ — (2)

$\log(1+\Delta) = hD$ — (3)

from (2) $-\log E = -hD$

$\log E^{-1} = -hD$

$\Rightarrow \log(1-\nabla) = -hD$.

$\Rightarrow -\log(1-\nabla) = hD$ — (4)

from (3) & (4) $hD = \log(1+\Delta) = -\log(1-\nabla)$

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Ques. Prove that $\left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x} = e^x$

LHS

$$= \left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$$

$$= \left[\frac{(E-1)^2}{E}\right] e^x \cdot \frac{e^{x+h}}{(E-1)^2 e^x} \quad \left\{\begin{array}{l}\Delta = E-1, \\ Ef(x) = f(x+h)\end{array}\right.$$

$$= \left[\frac{E^2-2E+1}{E}\right] e^x \cdot \frac{e^{x+h}}{[E^2-2E+1]\, e^x}$$

$$= \left[E - 2 + E^{-1}\right] e^x \cdot \frac{e^{x+h}}{e^{x+2h} - 2e^{x+h} + e^x}$$

$$= \left(e^{x+h} - 2e^x + e^{x-h}\right) \cdot \frac{e^x \cdot e^h}{e^{x+2h} - 2e^{x+h} + e^x}$$

$$= \frac{\cancel{\left(e^{x+2h} - 2e^{x+h} + e^x\right)} \cdot e^x}{\cancel{\left(e^{x+2h} - 2e^{x+h} + e^x\right)}}$$

$$= e^x$$

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$\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

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Ques Evaluate,

$$\left(\frac{\Delta^2}{E}\right)\sin(x+h) + \frac{\Delta^2 \sin(x+h)}{E\sin(x+h)}$$

Sol = $\left[\frac{(E-1)^2}{E}\right]\sin(x+h) + \frac{(E-1)^2\sin(x+h)}{\sin(x+2h)}$

$$= \left[\frac{E^2-2E+1}{E}\right]\sin(x+h) + \frac{\left[E^2-2E+1\right]\sin(x+h)}{\sin(x+2h)}$$

$$= \left[E-2+E^{-1}\right]\sin(x+h) + \frac{\sin(x+3h) - 2\sin(x+2h) + \sin(x+h)}{\sin(x+2h)}$$

$$= \sin(x+2h) - 2\sin(x+h) + \sin x + \frac{2\sin(x+2h)\cos h - 2\sin(x+2h)}{\sin(x+2h)}$$

$$= 2\sin(x+h)\cos h - 2\sin(x+h) + \frac{2\sin(x+2h)\left[\cos h - 1\right]}{\sin(x+2h)}$$

$$= 2\sin(x+h)\cos h - 2\sin(x+h) + 2(\cos h - 1).$$

$$= 2\sin(x+h)\left[\cos h - 1\right] + 2(\cos h - 1)$$

$$= 2(\cos h - 1)\left[\sin(x+h) + 1\right] \quad \text{: Ans}$$

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Que Evaluate

$$\Delta^2\left[\frac{5x+12}{x^2+5x+6}\right]$$

$$= \Delta^2\left[\frac{5x+12}{(x+2)(x+3)}\right]$$

Resolving into partial fraction

$$= \Delta^2\left[\frac{2}{x+2} + \frac{3}{x+3}\right]$$

$$= \Delta\left[\Delta\left(\frac{2}{x+2}\right) + \Delta\left(\frac{3}{x+3}\right)\right]$$

$$\{\because \Delta f(x) = f(x+h) - f(x)\}$$

$$= \Delta\left[2\left(\frac{1}{x+3} - \frac{1}{x+2}\right) + 3\left(\frac{1}{x+4} - \frac{1}{x+3}\right)\right]$$

$$= -2\Delta\left[\frac{1}{(x+2)(x+3)}\right] - 3\Delta\left[\frac{1}{(x+3)(x+4)}\right]$$

$$= -2\left[\frac{1}{(x+3)(x+4)} - \frac{1}{(x+2)(x+3)}\right] - 3\left[\frac{1}{(x+4)(x+5)} - \frac{1}{(x+3)(x+4)}\right]$$

$$= \frac{4}{(x+2)(x+3)(x+4)} + \frac{6}{(x+3)(x+4)(x+5)}$$

$$= \frac{2(5x+16)}{(x+2)(x+3)(x+4)(x+5)}$$

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# Factorial Notation or Polynomial

The factorial polynomial of degree 'n' is denoted by the symbol $x^{(n)}$ or $x^{[n]}$ (* read its as x to the power n factorial)

Thus,

$$x^{[n]} = x(x-h)(x-2h) \ldots (x-[n-1]h)$$

Here h is constant difference and n is positive integer, in particular h = 1, we have.

$$x^{[1]} = x$$
$$x^{[2]} = x(x-1)$$
$$x^{[3]} = x(x-1)(x-2) \text{ and so on.}$$

. . . . . .

* If n is negative integer then reciprocal factorial function of order n is denoted by $x^{(-n)}$ or $x^{[-n]}$

$$x^{(-n)} = \frac{1}{(x+h)(x+2h)(x+3h)\ldots(x+nh)}$$

when h = 1

$$x^{(-1)} = \frac{1}{(x+1)}$$

$$x^{(-2)} = \frac{1}{(x+1)(x+2)}$$

$$x^{(-3)} = \frac{1}{(x+1)(x+2)(x+3)}$$

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Remarks

1. $\Delta x^{(n)} = nhx^{[n-1]}$

example:

$$\Delta x^{(3)} = \frac{d}{dx} x^{(3)} = 3x^{(2)}.$$

Slly,

$$\Delta^2 x^{(n)} = n(n-1) h^2 x^{(n-2)}$$

2. $\Rightarrow$ $\boxed{\Delta^n x^{(n)} = n! \, h^n}$

3) $\Delta^k x^{(n)} = 0$ for $k > n$ and.

$$\Delta x^{(n)} = \frac{d}{dx} x^{(n)} = n x^{(n-1)}$$

4. $\frac{1}{\Delta} x^{(n)} = \int x^{(n)} dx = \frac{x^{(n+1)}}{n+1} + c$.

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Que. 1 Express $y = 2x^3 - 3x^2 + 3x - 10$ in factorial notation and show that $\Delta^3 y = 12$

Let $f(x) = 2x^3 - 3x^2 + 3x - 10$

Consider

$$f(x) = Ax^{(3)} + Bx^{(2)} + Cx^{(1)} + D \quad \text{--- (1)}$$

where A, B, C, D are constant, will find ~~this~~ these values using synthetic division method

| | | | | |
|---|---|---|---|---|
| 1 | 2 | -3 | 3 | -10 = D |
| | 0 | 2 | -1 | |
| 2 | 2 | -1 | 2 = C | |
| | 0 | 4 | | |
| 2 | 2 | 3 = B | | |
| | 0 | | | |
| | 2 = A | | | |

putting in eq (1)

required factorial notation

$$y = f(x) = 2x^{(3)} + 3x^{(2)} + 2x^{(1)} - 10 \quad \text{--- (2)}$$

$$= 2x(x-1)(x-2) + 3(x)(x-1) + 2x - 10.$$

from eq (2)

$$y = 2x^{(3)} + 3x^{(2)} + 2x^{(1)} - 10$$

$$\Delta y = 6x^{(2)} + 6x^{(1)} + 2$$

$$\Delta^2 y = 12x^{(1)} + 6$$

$$\Delta^3(y) = 12.$$

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Ques. find the function whose $1^{st}$ difference is $9x^2 + 11x + 5$

Soln Given first-difference i.e $\Delta f(x)$

$$\Delta f(x) = 9x^2 + 11x + 5$$

OR

$$\Delta f(x) = Ax^2 + Bx + C \quad \text{——} \; (1)$$

By synthetic Division method

| | | | |
|---|---|---|---|
| 1 | 9 | 11 | 5 = C |
| | 0 | 9 | |
| 2 | 9 | 20 = B | |
| | 0 | | |
| 3 | 9 = A | | |

putting values of A, B, C in (1).

$$\Delta f(x) = 9x^{(2)} + 20x^{(1)} + 5$$

$$\therefore f(x) = \frac{1}{\Delta}\left[9x^{(2)} + 20(x^{(1)} + 5\right] \qquad \left\{\because \frac{1}{\Delta} = \int dx\right.$$

$$f(x) = 9\frac{1}{\Delta}x^{(2)} + 20\frac{1}{\Delta}x^{(1)} + \frac{1}{\Delta}\cdot 5$$

$$f(x) = 9\int x^{(2)}dx + 20\int x^{(1)}dx + 5\int dx$$

$$f(x) = \overset{3}{\cancel{9}}\left(\frac{x^{(3)}}{\cancel{3}}\right) + \overset{10}{\cancel{20}}\left(\frac{x^{(2)}}{\cancel{2}}\right) + 5.[x] + C$$

$$f(x) = 3x^{(3)} + 10x^{(2)} + 5x + C$$

$$= 3x(x-1)(x-2) + 10x(x-1) + 5x + C$$

$$f(x) = 3x^2 + x^2 + x + C$$

where C is constant

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Ques Obtain the function whose first difference is $2x^3 + 3x^2 - 5x + 4$.

Soln first Difference :- $\Delta f(x)$ i.e.

$$\Delta f(x) = 2x^3 + 3x^2 - 5x + 4$$

let $\Delta f(x) = Ax^3 + Bx^2 + Cx + D$ —— ①

By synthetic division method.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | -5 | 4 = D |
| | 0 | 2 | 5 | |
| 2 | 2 | 5 | 0 = C | |
| | 0 | 4 | | |
| 3 | 2 | 9 = B | | |
| | 0 | | | |
| | 2 = A | | | |

from eqn ①

$$\Delta f(x) = 2x^{(3)} + 9x^{(2)} + 4$$

$$f(x) = \frac{1}{\Delta}\left[2x^{(3)} + 9x^{(2)} + 4\right]$$

$$= 2\frac{x^{(4)}}{\cancel{4}\,2} + \cancel{9}^{3}\frac{x^{(3)}}{\cancel{3}} + 4x + k$$

$$= \frac{x^{(4)}}{2} + 3x^{(3)} + 4x + k$$

(In factorial notation)

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Qus 4. Express $f(x) = x^3 - 2x^2 + x - 1$ in factorial notation and show that $\Delta^4 f(x) = 0$

Sol:

$$f(x) = x^3 - 2x^2 + x - 1 \quad \text{--- (1)}$$

let $f(x) = Ax^3 + Bx^2 + Cx + D \quad \text{--- (2)}$

By synthetic Division.

| | | | | |
|---|---|---|---|---|
| 1 | 1 | −2 | 1 | −1 = D |
| | 0 | 1 | −1 | |
| 2 | 1 | −1 | 0 = C | |
| | 0 | 2 | | |
| 3 | 1 | 1 = B | | |
| | 0 | | | |
| | 1 = A | | | |

putting in eqn (2)

$$f(x) = x^{(3)} + x^{(2)} - 1. \qquad \left\{ \because \Delta = \frac{d}{dx} \right\}$$

$$\Delta f(x) = \Delta \left[ x^{(3)} + x^{(2)} - 1 \right]$$

$$\Delta f(x) = 3x^{(2)} + 2x$$

$$\Delta^2 f(x) = 6x + 2$$

$$\Delta^3 f(x) = 6$$

$$\Delta^4 f(x) = 0 \qquad \text{H.P.}$$

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**Que.** Represent the function $f(x)$

$f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ and its successive difference in factorial notation.

**Soln.** $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$

Let

$$f(x) = Ax^{(4)} + Bx^{(3)} + Cx^{(2)} + Dx + E \quad \text{--- (1)}$$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | -12 | 24 | -30 | 9 = E |
| | 0 | 1 | -11 | 13 | |
| 2 | 1 | -11 | 13 | -17 = D | |
| | 0 | 2 | -18 | | |
| 3 | 1 | -9 | -5 = C | | |
| | 0 | 3 | | | |
| 4 | 1 | -6 = B | | | |
| | 0 | | | | |
| | 1 = A | | | | |

from eq (1) $f(x) = x^{(4)} - 6x^{(3)} - 5x^{(2)} - 17x + 9$ --- (2)

→ factorial notation.

Now Successive difference from (2)

$\Delta f(x) = 4x^{(3)} - 18x^{(2)} - 10x^{(1)} - 17$

$\Delta^2 f(x) = 12x^{(2)} - 36x^{(1)} - 10$

$\Delta^3 f(x) = 24x - 36$

$\Delta^4 f(x) = 24$

$\Delta^5 f(x) = 0$

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# Forward Difference Table

| Arguments $x$ | Entry $y = f(x)$ | 1st Difference $\Delta y$ | 2nd Difference $\Delta^2 y$ | 3rd Difference $\Delta^3 y$ | 4th Difference $\Delta^4 y$ |
|---|---|---|---|---|---|
| $x_0$ | $y_0$ | | | | |
| | | $\Delta y_0$ | | | |
| $x_1$ | $y_1$ | | $\Delta^2 y_0$ | | |
| | | $\Delta y_1$ | | $\Delta^3 y_0$ | |
| $x_2$ | $y_2$ | | $\Delta^2 y_1$ | | $\Delta^4 y_0$ |
| | | $\Delta y_2$ | | $\Delta^3 y_1$ | |
| $x_3$ | $y_3$ | | $\Delta^2 y_2$ | | |
| | | $\Delta y_3$ | | | |
| $x_4$ | $y_4$ | | | | |

Here $y_0 \longrightarrow$ is leading entry.

$\Delta y_0, \Delta^2 y_0, \Delta^3 y_0 \longrightarrow$ leading differences

* If given $n$ observations then $\Delta^n y = 0$

## Backward Difference table

$\nabla y_1 = y_1 - y_0$, $\quad \nabla^2 y_2 = \nabla y_2 - \nabla y_1$

| Arg $x$ | Entry $y$ | 1st Diff $\nabla y$ | 2nd Diff $\nabla^2 y$ | 3rd Diff $\nabla^3 y$ | 4th Diff $\nabla^4 y$ |
|---|---|---|---|---|---|
| $x_0$ | $y_0$ | | | | |
| | | $\nabla y_1$ | | | |
| $x_1$ | $y_1$ | | $\nabla^2 y_2$ | | |
| | | $\nabla y_2$ | | $\nabla^3 y_3$ | |
| $x_2$ | $y_2$ | | $\nabla^2 y_3$ | | $\nabla^4 y_4$ |
| | | $\nabla y_3$ | | $\nabla^3 y_4$ | |
| $x_3$ | $y_3$ | | $\nabla^2 y_4$ | | |
| | | $\nabla y_4$ | | | |
| $x_4$ | $y_4$ | | | | |

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Ques. Represent the function $f(x) \cancel{= x^4 - 12x^3 + 42x^2}$

$f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9$ and its successive difference in factorial notation in which interval of differencing is one.

1st Method <u>Forward Method</u>

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ |
|---|---|---|---|---|---|
| 0 | $9 = f(0)$ | | | | |
| | | $01 = \Delta f(0)$ | | | |
| 1 | 10 | | $26 = \Delta^2 f(0)$ | | |
| | | 27 | | $-36 = \Delta^3 f(0)$ | |
| 2 | 37 | | $-10$ | | $24 = \Delta^4 f(0)$ |
| | | 17 | | $-12$ | |
| 3 | 54 | | $-22$ | | |
| | | $-5$ | | | |
| 4 | 49 | | | | |

In factorial Notation

$$f(x) = f(0) + \frac{\Delta f(0) x^{(1)}}{1!} + \frac{\Delta^2 f(0) x^{(2)}}{2!} + \frac{\Delta^3 f(0) x^{(3)}}{3!} + \frac{\Delta^4 f(0) x^{(4)}}{4!}$$

$$f(x) = 9 + x^{(1)} + 26\frac{x^{(2)}}{2} - \frac{36x^{(3)}}{6} + \frac{24x^{(4)}}{24}$$

factorial notation $\boxed{f(x) = 9 + x^{(1)} + 13x^{(2)} - 6x^{(3)} + x^{(4)}}$

Successive Differences.

$\Delta f(x) = 1 + 26x^{(1)} - 18x^{(2)} + 4x^{(3)}$

$\Delta^2 f(x) = 26 - 36x^{(1)} + 12x^{(2)}$

$\Delta^3 f(x) = -36 + 24x^{(1)}$

$\Delta^4 f(x) = 24$

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2nd Method    Direct Method.

Let $f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9$ — (1)

and its factorial notation is denoted by

$f(x) = A x^{(4)} + B x^{(3)} + C x^{(2)} + D x^{(1)} + E$ — (2)

from (1) and (2)

$\Rightarrow x^4 - 12x^3 + 42x^2 - 30x + 9$

$= Ax(x-1)(x-2)(x-3) + Bx(x-1)(x-2) + Cx(x-1) + Dx + E$ — (3)

put $x = 0$ in (3)

$\Rightarrow \boxed{E = 9}$

put $x = 1$ in (3)

$\Rightarrow 10 = D + E$

$\Rightarrow 10 = D + 9$

$\Rightarrow \boxed{D = 1}$

put $x = 2$ in (3)

$\Rightarrow 37 = 2C + 2D + E$

$\Rightarrow 37 = 2C + 2 + 9$

$\Rightarrow 37 = 2C + 11$

$\Rightarrow 2C = 37 - 11$

$\Rightarrow 2C = 26$

$\Rightarrow \boxed{C = 13}$

put $x = 3 \Rightarrow 54 = 6B + 6C + 3D + E$

$\Rightarrow 54 = 6B + 78 + 3 + 9$

$\Rightarrow 54 = 6B + 90$

$\Rightarrow 6B = 90 - 54$

$\Rightarrow 6B = -36$

$\Rightarrow \boxed{B = -6}$

put $x = 4 \Rightarrow 49 = 24A + 24B + 12C + 4D + E$

$49 = 24A - 144 + 156 + 4 + 9$

$49 = 24A + 25$

$24A = 49 - 25$

$24A = 24$

$\boxed{A = 1}$

Hence,

$f(x) = x^{(4)} - 6x^{(3)} + 13x^{(2)} + x^{(1)} + 9$

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## 3rd Method   Synthetic Division Method

Let $f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9$ — ①

and $f(x) = Ax^{(4)} + Bx^{(3)} + Cx^{(2)} + Dx^{(1)} + E$

be the factorial notation — ②

By synthetic Division.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | −12 | 42 | −30 | 9 = E |
| | 0 | 1 | −11 | 31 | |
| 2 | 1 | −11 | 31 | 1 = D | |
| | 0 | 2 | −18 | | |
| 3 | 1 | −9 | 13 = C | | |
| | 0 | 3 | | | |
| | 1 | −6 = B | | | |
| | 0 | | | | |
| | 1 = A | | | | |

put values of A, B, C, D, E in eq ②

$$f(x) = x^{(4)} - 6x^{(3)} + 13x^{(2)} + x^{(1)} + 9$$

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# INTERPOLATION

According to Theile.

"Interpolation is the art of reading between the lines of table."

It also means insertion or filling up intermediate terms of the series. Suppose we are given the following ~~values~~ values of $y = f(x)$ for a set of values of $x$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$: | $x_0$ | $x_1$ | $x_2$ | $x_3$ | - | - | $x_n$ |
| $y$: | $y_0$ | $y_1$ | $y_2$ | $y_3$ | - | - | $y_n$ |

Thus, the process of finding the values of $y$ corresponding to any value of $x$ between $x_0$ to $x_n$ is called "Interpolation".

Hence, Interpolation is technique of estimating the value of function for any intermediate value of the independent variable, while the process of computing the value of the function outside the given range is called "Extrapolation".

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Ques) Estimate the missing term in the following table.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 1 | 3 | 9 | — | 81 |

Soln By. Difference table.

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 0 | $y_0$ 1 | | | | |
| | | 2 | | | |
| 1 | $y_1$ 3 | | 4 | | |
| | | 6 | | $y_3 - 19$ | |
| 2 | $y_2$ 9 | | $y_3 - 15$ | | $124 - 4y_3$ |
| | | $y_3 - 9$ | | $105 - 3y_3$ | |
| 3 | $y_3$ | | $90 - 2y_3$ | | |
| | | $81 - y_3$ | | | |
| 4 | $y_4$ 81 | | | | |

Now. Here, $\Delta^4 y = 0$. {since 4 entries are given}

$\Rightarrow \quad 124 - 4y_3 = 0$

$\Rightarrow \quad 124 = 4y_3$

$\Rightarrow \quad y_3 = \frac{124}{4} = 31$

$\Rightarrow \boxed{y_3 = 31}$ Ans.

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Que 2. find the missing value in following data.

| $x$ | 2.0 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 |
|---|---|---|---|---|---|---|---|
| $y$ | 0.135 | $y_1$ | 0.111 | 0.100 | $y_4$ | 0.082 | 0.074 |

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ |
|---|---|---|---|---|---|---|
| 2.0 | 0.135 | $y_1 - 0.135$ | | | | |
| | | | $0.246 - 2y_1$ | | | |
| 2.1 | $y_1$ | $0.111 - y_1$ | | $-0.368 + 3y_1$ | | |
| | | | $-0.122 + y_1$ | | $y_4 + 0.401 - 4y_1$ | |
| 2.2 | 0.111 | $-0.011$ | | $y_4 + 0.033 - y_1$ | | $-0.163 - 5y_4 + 5y_1$ |
| | | | $y_4 - 0.089$ | | $0.238 - 4y_4 + y_1$ | |
| 2.3 | 0.100 | $y_4 - 0.100$ | | $0.271 - 3y_4$ | | |
| | | | $0.182 - 2y_4$ | | $-0.543 + 6y_4$ | $-0.781 + 10y_4 - y_1$ |
| 2.4 | $y_4$ | $0.082 - y_4$ | | $-0.272 + 3y_4$ | | |
| | | | $-0.09 + y_4$ | | | |
| 2.5 | 0.082 | $-0.008$ | | | | |
| 2.6 | 0.074 | | | | | |

No. of known values = 5

Therefore $\Delta^5 y = 0$

$$\left\{\begin{array}{ll} 5y_1 - 5y_4 = 0.163 & \text{—— (1)} \\ -y_1 + 10y_4 = 0.781 & \text{—— (2)} \end{array}\right.$$

$y_1 = 0.127$

$y_4 = 0.0904$

Qu Compute next three values.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | $-1$ | 1 | $-1$ | 1 | – | – | – |

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ |
|---|---|---|---|---|---|---|
| 0 | 1 | | | | | |
| | | $-2$ | | | | |
| 1 | $-1$ | | 4 | | | |
| | | 2 | | $-8$ | | |
| 2 | 1 | | $-4$ | | 16 | |
| | | $-2$ | | 8 | | $y_5-31$ |
| 3 | $-1$ | | 4 | | $y_5-15$ | |
| | | 2 | | $y_5-7$ | | $y_6-5y_5+26$ |
| 4 | 1 | | $y_5-3$ | | $y_6-4y_5+11$ | |
| | | $y_5-1$ | | $y_6-3y_5+4$ | | $y_7-5y_6+10y_5-16$ |
| 5 | $y_5$ | | $y_6-2y_5+1$ | | $y_7-4y_6+6y_5-5$ | |
| | | $y_6-y_5$ | | $y_7-3y_6+3y_5-1$ | | |
| 6 | $y_6$ | | $y_7-2y_6+y_5$ | | | |
| | | $y_7-y_6$ | | | | |
| 7 | $y_7$ | | | | | |

Since five values are given

$$\Delta^5 y = 0$$

$\Rightarrow$ $y_5 - 31 = 0$ —① $\Rightarrow$ $\boxed{y_5 = 31}$

$y_6 - 5y_5 + 26 = 0$ —②

$y_7 - 5y_6 + 10y_5 - 16 = 0$ —③

from ②

$y_6 - 5\times 31 + 26 = 0 \Rightarrow y_6 - 129 = 0 \Rightarrow \boxed{y_6 = 129}$

from ③

$y_7 - 5\times 129 + 10\times 31 - 16 = 0 \quad y_7 - 351 = 0 \Rightarrow \boxed{y_7 = 351}$

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Ques find the first term of the series whose second and subsequent terms are 8, 3, 0, −1, 0.

Sol

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ |
|---|---|---|---|---|---|---|
| 0 | $y_0$ | | | | | |
| | | $8-y_0$ | | | | |
| 1 | 8 | | $-13+y_0$ | | | |
| | | $-5$ | | $15-y_0$ | | |
| 2 | 3 | | 2 | | $-15+y_0$ | |
| | | $-3$ | | 0 | | $15-y_0$. |
| 3 | 0 | | 2 | | 0 | |
| | | $-1$ | | 0 | | |
| 4 | $-1$ | | 2 | | | |
| | | 1 | | | | |
| 5 | 0 | | | | | |

Since number of known data is 5

$\therefore \Delta^5 y = 0$

$\Rightarrow \quad 15 - y_0 = 0$

$\Rightarrow \boxed{y_0 = 15}$

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$\because \Delta = E - 1$

$y_0 = ?\ ,\ y_1 = 8\ ,\ y_2 = 3,\ \ y_3 = 0\ ,\ y_4 = -1\ \ y_5 = 0$

Soln Here no. of known data = 5 values.

Therefore $\Delta^5 y = 0$.

$\Rightarrow (E-1)^5 y = 0$

$\Rightarrow \left[E^5 - {}^5C_1 E^4 + {}^5C_2 E^3 - {}^5C_3 E^2 + {}^5C_4 E - {}^5C_5\right] y = 0$

$\Rightarrow E^5 - 5E^4 + 10E^3 - 10E^2 + 5E + 1]y = 0 \quad -①$

(i) putting $y = y_0$ in ①.

$\left[E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1\right] y_0 = 0$

$\Rightarrow y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$

$\Rightarrow 0 - 5(-1) + 10(0) - 10(3) + 5(8) - y_0 = 0$

$\Rightarrow 5 - 30 + 40 - y_0 = 0$

$\Rightarrow 15 - y_0 = 0$

$\boxed{y_0 = 15}$

$\left\{ (x+y)^n = {}^nC_0\, x^n y^0 + {}^nC_1\, x^{n-1} y^1 + {}^nC_2\, x^{n-2} y^2 + \cdots + {}^nC_n\, x^0 y^n \right\}$

${}^nC_0 = 1$

${}^nC_0 = 1$

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Que Given that $y_5 = 4$, $y_6 = 3$, $y_7 = 4$, ~~$y_8 = 10$~~

$y_8 = 10$ and $y_9 = 24$. find value of $\Delta^4 y_5$:
(i) By using difference table (ii) without using table

Soln (i) By difference Table

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 5 | 4 | | | | |
| | | -1 | | | |
| 6 | 3 | | 2 | | |
| | | 1 | | 3 | |
| 7 | 4 | | 5 | | $0 = \Delta^4 y_5$ |
| | | 6 | | 3 | |
| 8 | 10 | | 8 | | |
| | | 14 | | | |
| 9 | 24 | | | | |

Here. $\Delta^4 y_5 = 0$.

(ii) without table. $\Delta^4 y_5$ ∵ 5 entries given.

$(\because \Delta = E - 1)$ $\Delta^4 y_5 = (E-1)^4 y_5$

$\Rightarrow [E^4 - {}^4C_1 E^3 + {}^4C_2 E^2 - {}^4C_3 E + {}^4C_4] y_5$

$\because E^n y_x = y_{x+n}$ $\Rightarrow y_9 - 4y_8 + 6y_7 - 4y_6 + y_5$

$\Rightarrow 24 - 4(10) + 6(4) - 4(3) + 4$

$= 24 - 40 + 24 - 12 + 4$

$= 0.$ Ans.

$\Delta^4 y_5 = 0.$

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Ques Given.

| | | | | | |
|---|---|---|---|---|---|
| $x$ : | 1 | 2 | 3 | 4 | 5 |
| $y$ : | 2 | 5 | 10 | 17 | 26. |

find value of $\nabla^2 y_5$

| $x$ | $y$ | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ |
|---|---|---|---|---|
| 1 | $y_1 = 2$ | | | |
| | | 3 | | |
| 2 | $y_2 = 5$ | | 2 | |
| | | 5 | | 0 |
| 3 | $y_3 = 10$ | | 2 | |
| | | 7 | | 0 |
| 4 | $y_4 = 17$ | | 2 $\hookrightarrow = \nabla^2 y_5$ | |
| | | 9 | | |
| 5 | $y_5 = 26$ | | | |

from diff-table $\nabla^2 y_5 = 2$.

<u>Without Table</u>

$$\nabla^2 y_5 = (1-E^{-1})^2 y_5$$

$\left[ \because \nabla = (1-E^{-1}) \right.$

$\left. E^{-n} y_x = y_{x-n} \right.$

$$= (1 + E^{-2} - 2E^{-1}) y_5$$

$$= y_5 + y_3 - 2y_4.$$

$$= 26 + 10 - 2 \times 17$$

$$= 36 - 34$$

$$= 2$$

$$\boxed{\nabla^2 y_5 = 2}$$

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# INTERPOLATION WITH EQUAL INTERVALS :

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Interpolation using Newton's forward and Backward Difference formula.

1. **Newton's forward Interpolation formula.**

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \times \Delta^3 y_0 + \cdots$$

where $p = \dfrac{x - x_0}{h}$

Here, $x_0 \rightarrow$ first term
$h \rightarrow$ common difference
$x \rightarrow$ target point.

2) **Newton's Backward Interpolation formula**

$$y = f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

where $p = \dfrac{x - x_n}{h}$

Here.
$x \rightarrow$ target point
$x_n \rightarrow$ last term
$h \rightarrow$ common difference

* Though it is suggested that the forward formula should be used for interpolating $y$ at points near $x_0$ and backward formula should be used for interpolating near $x_n$, it is not necessary.

Either of the formula ~~should be~~ can be used for interpolation.

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**Que.** Using Newtons forward Interpolation formula find value of $f(1.6)$ if.

| $x$ | 1 | 1.4 | 1.8 | 2.2 |
|---|---|---|---|---|
| $y$ | 3.49 | 4.82 | 5.96 | 6.5 |

**Soln** Newton's forward Difference Table.

| $x$ | $y$ | $\Delta y_0$ | $\Delta^2 y_0$ | $\Delta^3 y_0$ |
|---|---|---|---|---|
| $x_0 = 1$ | $y_0$ 3.49 | | | |
| | | $\Delta y_0$ 1.33 | | |
| 1.4 | 4.82 | | $\Delta^2 y_0$ −0.19 | |
| | | 1.14 | | $\Delta^3 y_0$ −0.41 |
| 1.8 | 5.96 | | −0.60 | |
| | | 0.54 | | |
| 2.2 | 6.5 | | | |

Now, Newton's forward Interpolation formula.

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots \quad \text{--- (1)}$$

Now. $p = \frac{x - x_0}{h} = \frac{1.6 - 1}{0.4} = 1.5$

$\Rightarrow \boxed{p = 1.5}$

$$f(1.6) = 3.49 + \underset{([illegible])}{(1.5) \times (1.33)} + \underset{-0.07125}{\frac{(1.5)(0.5)}{2} \times (-0.19)}$$

$$+ \underset{0.025625}{\frac{(1.5)(0.5)(-0.5)}{6} \times (-0.41)} = 5.44$$

$= 5.57$

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Ques from the table, estimate the number of students who obtained marks between 40 and 45.

| Marks. | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|
| No. of students | 31 | 42 | 51 | 35 | 31 |

Sol^n To find the number of students who got marks between 40 and 45 i.e. difference b/w $f(40)$ and $f(45)$

Using forward difference Table.

| Marks less than $(x)$ | No. of stud. $y$ | $\Delta y_0$ | $\Delta^2 y_0$ | $\Delta^3 y_0$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 40 | $31 = y_0$ | | | | |
| | | $42 = \Delta y_0$ | | | |
| 50 | $31+42 = 73$ | | $9 = \Delta^2 y_0$ | | |
| | | 51 | | $-25 = \Delta^3 y_0$ | |
| 60 | $73+51 = 124$ | | $-16$ | | $37 = \Delta^4 y_0$ |
| | | 35 | | 12 | |
| 70 | $124+35 = 159$ | | $-4$ | | |
| | | 31 | | | |
| 80 | $159+31 = 190$ | | | | |

from table at $x = 40 \Rightarrow f(40) = 31$.

Now at $x = 45$.

Here, $p = \dfrac{x - x_0}{h} = \dfrac{45-40}{10} = 0.5$

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By Newton's forward formula.

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!} \cdot \Delta^4 y_0 \quad \text{--- (1)}$$

$$f(45) = 31 + (0.5)(42) + \frac{(0.5)(-0.5)}{2} \times 9$$

$$+ \frac{(0.5)(-0.5)(-1.5)}{6} \times (-25)$$

$$+ \frac{(0.5)(-0.5)(-1.5)(-2.5)}{24} \times 37$$

$$= 31 + 21 + (-1.13) + (-1.45)$$

$$f(45) = 49.42 \approx 49$$

Hence. No. of students getting marks b/w 40 and 45.

$$= f(45) - f(40) = 49 - 31 = 18 \quad \text{Ans}$$

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Ques: find the number of men getting wages between Rs. 10 and Rs. 15 from foll. data

| wage | 0-10 | 10-20 | 20-30 | 30-40 |
|---|---|---|---|---|
| freq. | 9 | 30 | 35 | 42 |

Soln: Difference Table

| wages less than ($x$) | frequency $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 10 | $9 = y_0$ | | | |
| | | $30 = \Delta y_0$ | | |
| 20 | $30+9 = 39$ | | $5 = \Delta^2 y_0$ | |
| | | 35 | | $2 = \Delta^3 y_0$ |
| 30 | $39+35 = 74$ | | 7 | |
| | | 42 | | |
| 40 | $74+42 = 116$ | | | |

Here, at $x = 10$, $f(10) = 9$.

To find $f(15)$ :- $p = \frac{x - x_0}{h} = \frac{15-10}{10} = 0.5$

Using Newton's forward formula,

$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$ :

$f(15)$

$f(x) = 9 + (0.5)(30) + \frac{(0.5)(-0.5)(5)}{2} + \frac{(0.5)(-0.5)(-1.5)(2)}{6}$

$f(15) = 9 + 15 + (-0.63) + (0.13) = 23.50 \approx 24$

Hence no. of men getting wages b/w ₹10 and ₹15

$= f(15) - f(10) = 24 - 9 = 15$

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Ques. Estimate f(42) from following data

| $x$ | 20 | 25 | 30 | 35 | 40 | 45 |
|---|---|---|---|---|---|---|
| $f(x)$ | 354 | 332 | 291 | 260 | 231 | 204 |

Sol. Difference table

| $x$ | $f(x)$ | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ | $\nabla^4 y$ | $\nabla^5 y$ |
|---|---|---|---|---|---|---|
| 20 | 354 | | | | | |
| | | -22 | | | | |
| 25 | 332 | | -19 | | | |
| | | -41 | | 29 | | |
| 30 | 291 | | 10 | | -37 | |
| | | -31 | | -8 | | 45 |
| 35 | 260 | | 2 | | 8 | |
| | | -29 | | 0 | | |
| 40 | 231 | | 2 | | | |
| | | -27 | | | | |
| 45 | 204 | | | | | |

To find $f(x)$ at $x = 42$ i.e. $f(42)$, so taking

$x_n = 45$ $\therefore$ $p = \frac{x - x_n}{h} = \frac{42-45}{5} = \frac{-3}{5}$

$\Rightarrow \boxed{p = -0.6}$

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Using Newton's Backward Difference formula

$$f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$+ \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!} \times \nabla^5 y_n .$$

$$f(42) = 204 + (-0.6)\times(-27) + \frac{(-0.6)(0.4)}{2!}\times(2) +$$

$$\frac{(-0.6)(0.4)(1.4)}{3!}\times(0) + \frac{(-0.6)(0.4)(1.4)(2.4)}{4!}\times(8)$$

$$+ \frac{(-0.6)(0.4)(1.4)(2.4)(3.4)}{5!}\times(45).$$

$$f(42) = 204 + 16.20 - 0.24 + 0 + 0.27 - 1.03$$

$$f(42) = 218.66 \approx 219$$

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Que from the following data find.
$f(7.5)$

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 |

Soln Difference-table.

| $x$ | $f(x)$ / $f=y$ | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ | $\nabla^4 y$ | $\nabla^5 y$ |
|---|---|---|---|---|---|---|
| 1 | 1 | | | | | |
| | | 7 | | | | |
| 2 | 8 | | 12 | | | |
| | | 19 | | 6 | | |
| 3 | 27 | | 18 | | 0 | |
| | | 37 | | 6 | | 0 |
| 4 | 64 | | 24 | | 0 | |
| | | 61 | | 6 | | 0 |
| 5 | 125 | | 30 | | 0 | |
| | | 91 | | 6 | | 0 |
| 6 | 216 | | 36 | | 0 | |
| | | 127 | | 6 | | |
| 7 | 343 | | 42 | | | |
| | | 169 | | | | |
| 8 | 512 | | | | | |

To find $f(x)$ at $x = 7.5$ i.e. $f(7.5) =$

Here $x = 7.5$ which is near to $x_n = 8$

$\therefore \quad p = \frac{x - x_n}{h} = \frac{7.5 - 8}{1} = -0.5$

$p = (-0.5)$

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Therefore, using Newton Backwards Interpolation formula

$$f(x) = y_n + p\nabla y_n + \frac{p(p+1)\cancel{(p+2)}}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$f(7.5) = 512 + (-0.5)(169) + \frac{(-0.5)(0.5)}{2} \times 42$$

$$+ \frac{(-0.5)(0.5)(1.5)}{3!} \times 6 + 0.$$

$$f(7.5) = 512 - 84.50 - 5.25 - 0.38$$

$$f(7.5) = 421.875 \approx 422.$$

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Ques. find the cubic polynomial which takes the following values:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | 1 | 2 | 1 | 10 |

Hence evaluate $f(4)$.

Soln

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 0 | 1 | | | |
| | | 1 | | |
| 1 | 2 | | -2 | |
| | | -1 | | 12 |
| 2 | 1 | | 10 | |
| | | 9 | | |
| 3 | 10 | | | |

Here $h = 1$ and $p = \frac{x - x_0}{h} = \frac{x-0}{1} = x$

By Newton's forward Diff. formula.

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$f(x) = 1 + x(1) + \frac{x(x-1)}{2!}\times(-2) + \frac{x(x-1)(x-2)}{3!}\times 12$$

$$= 1 + x - x(x-1) + 2x(x-1)(x-2)$$

$$= 1 + x + x^2 - x + 2x^3 - 6x^2 + 4x$$

$$f(x) = 2x^3 - 7x^2 + 6x + 1 \quad \text{—— (1)}$$

$f(4) = 41$

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Ques. The area of 'A' of a circle of diameter 'd' is given for following values.

| | | | | | |
|---|---|---|---|---|---|
| d : | 80 | 85 | 90 | 95 | 100 |
| A : | 5026 | 5674 | 6362 | 7088 | 7854 |

By using appropriate interpolation formulae, find approx. values for the areas of circles of diameter 82 and 91 resp. Ans. $f(82) = 5280$, $f(91) = 6504$.

Ques. The foll. table gives population of town. Using Newton's interpolation formula find increase population during 1946 to 1948. (1946 → 32.34) (1948 → 34.87)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Year | 1911 | 1921 | 1931 | 1941 | 1951 | 1961 | Ans. |
| Pop. | 12 | 15 | 20 | 27 | 39 | 52 | 2.530 |

Ques. A third degree polynomial passes through the points (0, −1), (1, 1), (2, 1) and (3, −2), find the polynomial.

Ans: $-\frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{8}{3}x - 1$

$\Rightarrow -0.17x^3 - 0.5x^2 + 2.67x - 1$ decimals.

Ques. The pressure p of wind corresponding to velocity v is given by following data. Estimate p when v = 15.

| | | | | |
|---|---|---|---|---|
| v : | 10 | 20 | 30 | 40 |
| p : | 1.1 | 2.0 | 4.4 | 7.9 |

Ans

$f(15) = 1.325$

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# INTERPOLATION WITH UNEQUAL INTERVALS

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① Lagranges Interpolation formula

② Newton's Divided Difference interval formula

Method - 1

LAGRANGES INTERPOLATION formula

· Let $f(x_0), f(x_1), \ldots, f(x_n)$ be the values of function $f(x)$ corresponding to the $(n+1)$ arguments $x_0, x_1, \ldots, x_n$ not necessarily equally spaced

Then, $y = f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} \times y_0$

$+ \frac{(x-x_0)(x-x_1)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} \times y_1$

$+ \ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})} \times y_n$

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Given the values

Ques.

| $x$: | 5 | 7 | 11 | 13 | 17 |
| --- | --- | --- | --- | --- | --- |
| $f(x)$: | 150 | 392 | 1452 | 2366 | 5202 |

Evaluate $f(9)$ using Lagrange's formula.

Soln. Lagrange's formula

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \times y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \times y_4$$

Here,

$x_0 = 5$, $x_1 = 7$, $x_2 = 11$, $x_3 = 13$, $x_4 = 17$

$y_0 = 150$, $y_1 = 392$, $y_2 = 1452$, $y_3 = 2366$, $y_4 = 5202$

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putting $x = 9$ and given values in (1) we get

$$f(9) = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)} \times 150$$

$$+ \frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)} \times 392$$

$$+ \frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)} \times 1452$$

$$+ \frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)} \times 2366$$

$$+ \frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)} \times 5202$$

$$f(9) = \frac{-128}{1152} \times 150 + \frac{256}{480} \times 392 + \frac{256}{288} \times 1452$$

$$+ \frac{(-128)}{384} \times 2366 + \frac{64}{2880} \times 5202$$

$$f(9) = \frac{-19200}{1152} + \frac{100352}{480} + \frac{371712}{288}$$

$$- \frac{302848}{384} + \frac{332928}{2880}$$

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$f(9) = -16.67 + 209 + 1291 - 789 + 116$

$f(9) = 810$

Ques Apply Lagrange's formula to find $f(15)$ if:-

| $x$ | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|
| $f(x)$ | 2420 | 1942 | 1497 | 1109 | 790 | 540 |

Ans. = $f(15) = 1295$ approx.

Ques. Given that

| $x$ | 4 | 5 | 7 | 10 | 11 | 13 |
|---|---|---|---|---|---|---|
| $f(x)$ | 48 | 100 | 294 | 900 | 1210 | 2028 |

Evaluate $f(8)$

Ans :- $f(8) = 448.$

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Method 2

## Newton's Divided Difference.

Divided Difference Table :-

| $x$ | $y=f(x)$ | $\Delta y$ | $\Delta^2 y$ |
|---|---|---|---|
| $x_0$ | $f(x_0)$ | | |
| | | $f(x_0,x_1)=\dfrac{f(x_1)-f(x_0)}{x_1-x_0}$ | |
| | | | $f(x_0,x_1,x_2)=\dfrac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$ |
| $x_1$ | $f(x_1)$ | | |
| | | $f(x_1,x_2)=\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$ | |
| | | | $f(x_1,x_2,x_3)=\dfrac{f(x_2,x_3)-f(x_1,x_2)}{x_3-x_1}$ |
| $x_2$ | $f(x_2)$ | | |
| | | $f(x_2,x_3)=\dfrac{f(x_3)-f(x_2)}{x_3-x_2}$ | |
| $x_3$ | $f(x_3)$ | | |

$\Delta^3 y$

$$f(x_0,x_1,x_2,x_3)=\frac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{(x_3-x_0)}$$

Now

$$f(x)=f(x_0)+(x-x_0)\Delta f(x_0)$$
$$+(x-x_0)(x-x_1)\Delta^2 f(x_0)+$$
$$+(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1})\Delta^n f(x_0)$$

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Que 4 Evaluate $f(9)$ using Newton's Divided Difference formula.

| $x$ | 5 | 7 | 11 | 13 | 17 |
|---|---|---|---|---|---|
| $f(x)$ | 150 | 392 | 1452 | 2366 | 5202 |

| $x$ | $y$ $f(x)$ | $\Delta y$ | $\Delta^2 y$ |
|---|---|---|---|
| 5 | 150 | $\Delta f(x_0)$ $\frac{392-150}{7-5} = 121$ | $\Delta^2 f(x_0)$ $\frac{265-121}{11-5} = 24$ |
| 7 | 392 | $\frac{1452-392}{11-7} = 265$ | $\frac{457-265}{13-7} = 32$ |
| 11 | 1452 | $\frac{2366-1452}{13-11} = 457$ | $\frac{709-457}{17-11} = 42$ |
| 13 | 2366 | $\frac{5202-2366}{17-13} = 709$ | |
| 17 | 5202 | | |

| $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|
| $\Delta^3 f(x_0)$ $\frac{32-24}{13-5} = 1$ | 0 |
| $\frac{42-32}{17-7} = 1$ | |

By divided difference formula.

$$\therefore f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0) \quad \text{--- (1)}$$

When $x = 9$

$$f(9) = 150 + (9-5)(121) + (9-5)(9-7)(24) + (9-5)(9-7)(9-11)(1) + 0$$

$$f(9) = 810$$

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Qun. Using Newton Divided Difference formula. find value of $f(8)$, $f(9)$, $f(15)$ using foll. data.

| $x$ | 4 | 5 | 7 | 10 | 11 | 13 |
|---|---|---|---|---|---|---|
| $f(x)$ | 48 | 100 | 294 | 900 | 1210 | 2028 |

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ |
|---|---|---|---|
| 4 | 48 | $\frac{100-48}{5-4} = 52$ | $= \frac{97-52}{7-4} = 15$ |
| 5 | 100 | $\frac{294-100}{7-5} = 97$ | $= \frac{202-97}{10-5} = 21$ |
| 7 | 294 | | |
| 10 | 900 | $\frac{900-294}{10-7} = 202$ | $= \frac{310-202}{11-7} = 27$ |
| 11 | 1210 | $\frac{1210-900}{11-10} = 310$ | |
| 13 | 2028 | | $= \frac{409-310}{13-10} = 33$ |
| | | $\frac{2028-1210}{13-11} = 409$ | |

| $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ |
|---|---|---|
| $\frac{21-15}{10-4} = 1$ | 0 | 0 |
| $\frac{27-21}{11-5} = 1$ | 0 | |
| $\frac{33-27}{13-\cancel{7}} = 1$ | | |

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| | x | y | Δy | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ |
|---|---|---|---|---|---|---|---|
| $x_0$ | 4 | 48 ($y_0$) | 52 ($\Delta y_0$) | | | | |
| | | | | 15 ($\Delta^2 y_0$) | | | |
| $x_1$ | 5 | 100 | 97 | | 1 ($\Delta^3 y_0$) | 0 | |
| | | | | 21 | | | 0 |
| $x_2$ | 7 | 294 | 202 | | 1 | 0 | |
| | | | | 27 | | | |
| $x_3$ | 10 | 900 | 310 | | 1 | | |
| | | | | 33 | | | |
| $x_4$ | 11 | 1210 | 409 | | | | |
| $x_5$ | 13 | 2028 | | | | | |

$$f(x) = f(x_0) + (x - x_0)\,\Delta f(x_0) + (x - x_0)(x - x_1)\,\Delta^2 f(x_0)$$
$$+ (x - x_0)(x - x_1)(x - x_2)\,\Delta^3 f(x_0)$$

at $x = 8$

$$f(8) = 48 + (8-4)(52) + (8-4)(8-5) \times (15)$$
$$+ (8-4)(8-5)(8-7) \cdot (1)$$

$$= 48 + 208 + 180 + 12$$
$$= 448$$

$$\boxed{f(8) = 448}$$

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at $x = 9$

$$f(9) = 48 + (9-4)\times(52) + (9-4)(9-5)\times(15) + (9-4)(9-5)(9-7)\times(1)$$

$$= 48 + 260 + 300 + 40$$

$$= 648$$

$$\boxed{f(9) = 648}$$

at $x = 15$

$$f(15) = 48 + (15-4)\,52 + (15-4)(15-5)\times(15) + (15-4)(15-5)(15-7)\times(1)$$

$$= 48 + 572 + 2400 + 880 = 3150$$

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Ques find $f'(10)$ from following data

| $x$ | 3 | 5 | 11 | 27 | 34 |
|---|---|---|---|---|---|
| $f(x)$ | -13 | 23 | 899 | 17315 | 35606 |

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ |
|---|---|---|---|---|---|
| 3 | -13 | | | | |
| | | 18 | | | |
| 5 | 23 | | 16 | | |
| | | 146 | | 1 | |
| 11 | 899 | | 40 | | 0 |
| | | 1026 | | 1 | |
| 27 | 17315 | | 69 | | |
| | | 2613 | | | |
| 34 | 35606 | | | | |

By Newtons Divided Diff formula

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0)$$
$$+ (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0). \longleftarrow (1)$$

putting values in (1)

$$f(x) = -13 + (x-3)18 + (x-3)(x-5)16 + (x-3)(x-5)(x-11)1$$

$$f(x) = x^3 - 3x^2 - 7x + 8.$$

On differentiating.

$$f'(x) = 3x^2 - 6x - 7 \qquad \text{put } x = 10$$
$$f'(10) = 3\times(10)^2 - 6(10) - 7$$
$$= 233$$

$$\boxed{f'(10) = 233}$$

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