# 1. D.C. Circuits Overview

**Direct Current (D.C.) circuits** involve the flow of electric charge in one constant direction. Unlike alternating current (AC) circuits, where current and voltage vary periodically, in D.C. circuits the values are typically constant (or in a steady state). The analysis of D.C. circuits forms the basis for understanding more complex behaviors in electronics and power systems. These circuits are governed by conservation laws (energy and charge) and rely on a set of core principles that we'll explore next.

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# 2. Voltage and Current Sources

## Independent Sources

- **Voltage Sources:**
  An independent voltage source is an element that maintains a fixed voltage across its terminals regardless of the current drawn from it. In ideal conditions, the voltage remains constant no matter what load is attached.
  *Example:* A battery providing 12 V regardless of the device's current draw (within practical limits).
- **Current Sources:**
  An independent current source delivers a constant current irrespective of the voltage across its terminals. The ideal current source adjusts its terminal voltage to maintain the specified current.
  *Example:* A current-regulated LED driver that ensures a fixed current flows through the LED array.

## Dependent (Controlled) Sources

Dependent sources are unique in that their output is a function of some other voltage or current in the circuit:

- **Voltage-Controlled Voltage Source (VCVS):**
  The output voltage is proportional to a voltage elsewhere in the circuit.
- **Current-Controlled Voltage Source (CCVS):**
  The output voltage is proportional to a current in another branch.
- **Voltage-Controlled Current Source (VCCS):**
  The output current is proportional to a voltage in the circuit.
- **Current-Controlled Current Source (CCCS):**
  The output current is proportional to another current.

These sources model real-world devices (like transistors and operational amplifiers) whose behavior is dictated by another parameter in the system. They are essential in linear circuit analysis and in designing amplifiers and other analog circuits.

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# 3. Units and Dimensions

Understanding the **units and dimensions** is crucial to ensure consistency and correctness in circuit analysis.

- **Voltage (V):** Measured in volts, it represents the potential difference between two points. One volt is defined as one joule per coulomb.
- **Current (I):** Measured in amperes (A), it represents the flow of electric charge. One ampere equals one coulomb per second.
- **Resistance (R):** Measured in ohms (Ω), it quantifies how much a component resists the flow of current. Ohm's law, which we'll discuss shortly, relates these three.
- **Dimensional Analysis:**
  This is a powerful tool used to check the validity of equations. For instance, in Ohm's law I=VRI = \frac{V}{R}I=RV, the dimensions on the right-hand side are:

  Volts (J/C)Ohms (V/A)=J/CV/A=A,\frac{\text{Volts (J/C)}}{\text{Ohms (V/A)}} = \frac{\text{J/C}}{\text{V/A}} = \text{A},Ohms (V/A)Volts (J/C)=V/AJ/C=A,

  confirming that the units match for current.

Understanding these dimensions is not only fundamental for analysis but also for designing and scaling circuits.

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# 4. Source Conversion

**Source conversion** is a method to simplify circuit analysis by transforming between equivalent representations of sources. Two common transformations are:

## Voltage Source with Series Resistance ↔ Current Source with Parallel Resistance

- **Voltage Source Representation:**
  Consider a voltage source VSV_SVS in series with a resistor RSR_SRS. When this combination is connected to a load, the voltage division between RSR_SRS and the load determines the current.
- **Current Source Representation:**
  The same behavior can be modeled with a current source ISI_SIS in parallel with a resistor RSR_SRS. The relationship between the two representations is given by:

  IS=VSRSI_S = \frac{V_S}{R_S}IS=RSVS

  and conversely,

  VS=IS×RS.V_S = I_S \times R_S.VS=IS×RS.

**Why Convert?**
Source conversion is useful because it allows you to simplify the circuit for easier analysis.

Depending on the circuit configuration (series or parallel combinations), one form might be more convenient than the other. This conversion is mathematically underpinned by Thevenin's and Norton's theorems, which show that any linear circuit can be represented by an equivalent voltage source with a series resistance or an equivalent current source with a parallel resistance.

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# 5. Ohm's Law

**Ohm's Law** is the cornerstone of circuit analysis for resistive components. It establishes a linear relationship between voltage (VVV), current (III), and resistance (RRR):

V=I×RV = I \times RV=I×R

### Key Points:

- **Linearity:**
  The law assumes that the resistor's behavior is linear, meaning that if you double the voltage, the current also doubles, assuming constant resistance.
- **Practical Implications:**
  In a simple circuit, knowing any two of these quantities allows you to compute the third. For example, if you know the voltage across a resistor and its resistance, you can determine the current flowing through it:

  I=VR.I = \frac{V}{R}.I=RV.

- **Limitations:**
  Not all circuit elements obey Ohm's Law. Non-linear components (like diodes, transistors in certain regions, etc.) have a more complex relationship between voltage and current.

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# 6. Kirchhoff's Laws

Kirchhoff's laws are fundamental in analyzing the conservation principles within electrical circuits.

### Kirchhoff's Current Law (KCL)

- **Statement:**
  The algebraic sum of currents entering a node (junction) is zero. In other words, the total current entering a node must equal the total current leaving it.
  Mathematically: ∑Iin=∑Iout\sum I_{\text{in}} = \sum I_{\text{out}}∑Iin=∑Iout
- **Physical Basis:**
  This law is a direct consequence of the conservation of charge. Charge cannot accumulate at a node in a steady-state circuit.

### Kirchhoff's Voltage Law (KVL)

- **Statement:**
  The algebraic sum of the potential differences (voltage drops and rises) around any closed loop in a circuit must be zero.
  Mathematically: ∑V=0\sum V = 0∑V=0
- **Physical Basis:**
  This is based on the conservation of energy. As a charge moves around a closed loop, the energy it gains and loses must cancel out.

### Application in Circuit Analysis:

- **Node Voltage Analysis (Using KCL):**
  By applying KCL at various nodes, you can write a system of equations that, when solved, gives you the unknown node voltages.
- **Mesh Analysis (Using KVL):**
  Writing KVL for each independent loop (mesh) allows you to solve for loop currents, which in turn provide insights into the branch currents and voltages.

Both laws are interrelated and can be used in conjunction to analyze complex circuits. They are indispensable when dealing with networks containing multiple sources (both independent and dependent), resistances, and other passive elements.

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# Bringing It All Together

1. **Understanding Sources:**
   Grasping the difference between voltage and current sources, and recognizing when a source is independent or dependent, is essential for modeling real-world devices.
2. **Ensuring Consistency with Units:**
   Always verifying equations with proper units and dimensions prevents errors and reinforces the physical significance of each parameter.
3. **Leveraging Source Conversion:**
   This technique simplifies circuit analysis by allowing you to switch between equivalent circuit representations. It connects theoretical principles (like Thevenin's and Norton's theorems) with practical circuit design.
4. **Applying Fundamental Laws:**
   Ohm's Law provides the direct relationship between current, voltage, and resistance. Kirchhoff's laws—current for nodes and voltage for loops—offer the conservation-based framework that underpins circuit analysis.
5. **Deep Analytical Insight:**
   Each concept builds upon the other. For example, by using Ohm's Law within the framework of Kirchhoff's laws, you can tackle complex circuits step by step, ensuring every conservation principle holds true. Source conversion then lets you reinterpret the circuit in a way that might simplify these analyses further.

# 1. Superposition Theorem

The **Superposition Theorem** states that in a **linear circuit** with multiple independent sources (voltage or current sources), the **response (voltage or current) at any element** in the circuit is the **algebraic sum** of the responses **due to each independent source acting alone**, while all other independent sources are replaced by their internal impedances.

## Mathematical Statement

If a circuit has **n** independent sources, the response **R_total** at any element is given by:

Rtotal=R1+R2+R3+...+RnR_{total} = R_1 + R_2 + R_3 + ... + R_nRtotal=R1+R2+R3+...+Rn

where:

- R1R_1R1 is the response due to the first source alone (all other sources turned off).
- R2R_2R2 is the response due to the second source alone (all other sources turned off).
- R3R_3R3 is the response due to the third source alone (all other sources turned off).
- and so on...

## Steps to Apply Superposition Theorem

1. **Consider One Source at a Time**
   - Keep one independent source active.
   - Replace all other independent voltage sources with **short circuits** (since an ideal voltage source has zero internal resistance).
   - Replace all other independent current sources with **open circuits** (since an ideal current source has infinite internal resistance).
2. **Solve the Circuit for the Chosen Source**
   - Find voltages and currents using **Ohm's Law**, **Kirchhoff's Voltage Law (KVL)**, and **Kirchhoff's Current Law (KCL)**.
3. **Repeat for Each Source**
   - Repeat the process for each independent source in the circuit.
4. **Sum the Individual Responses**
   - Algebraically sum up the individual responses to get the total voltage or current.

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## Example 1: Application of Superposition Theorem

### Circuit Description

Consider a simple **series-parallel resistive circuit** with two independent voltage sources:

- V1=10VV_1 = 10VV1=10V connected in series with a resistor R1=5ΩR_1 = 5\OmegaR1=5Ω and a parallel network.
- V2=20VV_2 = 20VV2=20V connected in series with R2=10ΩR_2 = 10\OmegaR2=10Ω, which is part of the parallel network with R3=15ΩR_3 = 15\OmegaR3=15Ω.

### Solution Using Superposition

1. **Step 1: Consider V1V_1V1 Alone**
   - Turn off V2V_2V2 by replacing it with a short circuit.
   - Solve for voltages and currents using KVL/KCL.
2. **Step 2: Consider V2V_2V2 Alone**
   - Turn off V1V_1V1 by replacing it with a short circuit.
   - Solve for voltages and currents.
3. **Step 3: Add Individual Responses**
   - Add the currents and voltages from both cases.

**Key Observations**

- Superposition is useful when analyzing circuits with **multiple sources**, especially when sources contribute to different parts of the circuit.
- It **cannot be used for power calculations** directly since power is proportional to the square of voltage or current.

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# 2. Thevenin’s Theorem

The **Thevenin’s Theorem** states that any **linear circuit with resistances and independent sources** can be replaced by an **equivalent circuit** consisting of:

- A **single voltage source** (VthV_{th}Vth, called Thevenin voltage).
- A **single series resistance** (RthR_{th}Rth, called Thevenin resistance).
- Connected to the original circuit’s **load resistance**.

## Steps to Apply Thevenin’s Theorem

1. **Remove the Load Resistance**
   - If analyzing a circuit for a specific load, temporarily remove it.
2. **Find Thevenin Voltage (VthV_{th}Vth)**
   - Find the **open-circuit voltage** across the terminals where the load was connected.
   - Use **KVL/KCL, voltage division, or node/mesh analysis**.
3. **Find Thevenin Resistance (RthR_{th}Rth)**
   - **Turn off all independent sources**:
     - Replace **voltage sources with short circuits**.
     - Replace **current sources with open circuits**.
   - Find the equivalent resistance seen at the open terminals.
4. **Reattach the Load Resistance**
   - The new **simplified circuit** is a simple **series circuit** consisting of:
     - VthV_{th}Vth
     - RthR_{th}Rth
     - RLR_LRL (load resistance)
   - Solve for the required voltage or current using **Ohm’s Law**.

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### Example 2: Application of Thevenin's Theorem

#### Circuit Description

Consider a circuit where:

- V=24VV = 24VV=24V
- R1=4ΩR_1 = 4\OmegaR1=4Ω, R2=6ΩR_2 = 6\OmegaR2=6Ω, R3=12ΩR_3 = 12\OmegaR3=12Ω (connected in some series-parallel combination).
- We need to find the voltage across a **load resistor** RL=8ΩR_L = 8\OmegaRL=8Ω.

#### Solution Using Thevenin's Theorem

1. **Step 1: Remove RLR_LRL**
   - Temporarily disconnect RLR_LRL from the circuit.
2. **Step 2: Find VthV_{th}Vth**
   - Find the **open-circuit voltage** across the terminals where RLR_LRL was connected.
   - Use **voltage division** or **node voltage analysis**.
3. **Step 3: Find RthR_{th}Rth**
   - Turn off the independent source (replace VVV with a short circuit).
   - Calculate the equivalent resistance seen from the terminals.
4. **Step 4: Solve for VLV_LVL**
   - Use the Thevenin equivalent circuit to compute:

     VL=Vth×RLRth+RLV_L = V_{th} \times \frac{R_L}{R_{th} + R_L}VL =Vth×Rth+RLRL

#### Key Observations

- **Thevenin's Theorem simplifies complex circuits** into one voltage source and one resistor.
- **It is useful in circuits with varying loads**, as you can replace a large portion of the circuit with its Thevenin equivalent.
- Thevenin resistance is useful in impedance matching applications.

# 1. Power & Energy in Electrical Circuits

## a. Fundamental Definitions

- **Power (P):**
  The instantaneous power in a circuit element is defined as the product of the instantaneous voltage v(t)v(t)v(t) across the element and the current i(t)i(t)i(t) through it:

  p(t)=v(t)·i(t)p(t) = v(t) \cdot i(t)p(t)=v(t)·i(t)

  For DC circuits or resistive loads, this often simplifies to

P=IVP = IVP=IV

where III and VVV are constant values.

- **Energy (W):**
Energy is the time integral of power. For a period TTT, the energy delivered or absorbed by an element is

W=∫0Tp(t) dtW = \int_0^T p(t) \, dtW=∫0Tp(t)dt

In DC circuits with constant power, this becomes

W=P×TW = P \times TW=P×T

## b. Types of Power in AC Circuits

- **Instantaneous Power:**
p(t)p(t)p(t) as defined above, which varies with time in AC circuits.
- **Average (Real) Power:**
The average power over one full cycle of an AC waveform is given by

Pavg=VrmsIrmscos⁡ϕP_{avg} = V_{rms} I_{rms} \cos \phiPavg=VrmsIrmscosϕ

where VrmsV_{rms}Vrms and IrmsI_{rms}Irms are the root-mean-square values and ϕ\phiϕ is the phase angle between the voltage and current. The factor cos⁡ϕ\cos \phicosϕ is known as the power factor.

- **Reactive Power:**
In circuits with inductors and capacitors, energy is periodically stored and then returned to the circuit. The reactive power is given by

Q=VrmsIrmssin⁡ϕQ = V_{rms} I_{rms} \sin \phiQ=VrmsIrmssinϕ

and represents the energy oscillating between the source and reactive components.

- **Apparent Power:**
This is the product of the rms voltage and current without considering the phase angle:

S=VrmsIrmsS = V_{rms} I_{rms}S=VrmsIrms

Apparent power is a complex quantity, with real and reactive components forming a power triangle.

## c. Energy Storage Elements

- **Capacitors:**
Store energy in the electric field. The energy stored in a capacitor is WC=12CV2W_C = \frac{1}{2} C V^2WC=21CV2

- **Inductors:**
  Store energy in the magnetic field. The energy stored in an inductor is $W_L = \frac{1}{2} L I^2$

These concepts are foundational when analyzing transient behavior in circuits and understanding how energy is exchanged or dissipated over time.

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# 2. Mesh Analysis

## a. Overview

Mesh analysis is a systematic method based on Kirchhoff's Voltage Law (KVL), which states that the algebraic sum of all voltages around any closed loop (mesh) in a planar circuit is zero. This technique is especially effective in circuits that are planar (can be drawn on a plane without crossing wires).

## b. Step-by-Step Process

1. **Identify Meshes:**
   Define a mesh as a loop that does not contain any other loops within it. In a complex circuit, identify the smallest loops that cover the entire network.
2. **Assign Mesh Currents:**
   For each mesh, assign a current (typically in a clockwise direction) denoted by $I_1$, $I_2$, etc.
3. **Apply KVL to Each Mesh:**
   Write an equation for each mesh summing the voltage drops (products of resistance and mesh currents) and sources to zero.
   For example, for a mesh with resistors and a voltage source, you might have:

   $$V_s - I_1 R_1 - (I_1 - I_2)R_2 = 0$$

   where $(I_1 - I_2)$ appears if a resistor is shared between adjacent meshes.

4. **Solve the Simultaneous Equations:**
   The resulting set of linear equations can be solved using substitution, matrix methods, or other algebraic techniques to determine the mesh currents.

## c. Advantages and Considerations

- **Advantages:**
  Mesh analysis simplifies the handling of multiple loops and is systematic, reducing the risk of missing any voltage relationships.
- **Limitations:**
  It is best suited for planar circuits. For non-planar circuits, alternative methods (like nodal analysis) may be more effective.

# 3. Nodal Analysis

## a. Overview

Nodal analysis relies on Kirchhoff’s Current Law (KCL), which states that the sum of currents entering a node must equal the sum of currents leaving that node. This technique focuses on potential differences (node voltages) rather than currents in loops.

## b. Step-by-Step Process

1. **Identify Nodes:**
   Determine all the nodes in the circuit. Select one node as the reference node (ground) with a voltage of 0 V.
2. **Assign Node Voltages:**
   Label the remaining nodes with voltage variables, e.g., $V_1$, $V_2$, etc.
3. **Apply KCL to Each Node:**
   Write the current balance equation for each node (except the reference). For example, at node $V_1$, if it connects to resistors $R_1$, $R_2$ and a voltage source is present, you might have:

   $$\frac{V_1 - V_{ref}}{R_1} + \frac{V_1 - V_2}{R_2} + \cdots = 0$$

4. **Solve the Equations:**
   Solve the set of simultaneous equations to find the unknown node voltages. Once node voltages are known, branch currents can be determined using Ohm’s Law.

## c. Advantages and Considerations

- **Advantages:**
  Nodal analysis is particularly powerful when a circuit has many parallel branches. It reduces the number of equations compared to writing loop equations for every possible mesh.
- **Limitations:**
  When dealing with voltage sources that are not connected to the reference node, you may need to create supernodes—a technique that requires additional careful handling.

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# 4. Star-Delta (Wye-Delta) Transformation

## a. Purpose and Context

Complex resistor networks sometimes cannot be simplified by series-parallel reduction alone. Star-Delta (or Y-Δ) transformation is a method to convert a network of three resistors connected in a triangle (Δ) to an equivalent network connected in a star (Y) configuration—or vice versa. This conversion allows for easier analysis and simplification.

## b. The Transformations

### Delta-to-Star (Δ-to-Y) Transformation

Given a delta network with resistors RabR_{ab}Rab, RbcR_{bc}Rbc, and RacR_{ac}Rac between nodes aaa, bbb, and ccc, the equivalent star resistors RaR_aRa, RbR_bRb, and RcR_cRc (each connecting one node to a central point) are calculated by:

Ra=Rab·RacRab+Rbc+RacR_a = \frac{R_{ab} \cdot R_{ac}}{R_{ab} + R_{bc} + R_{ac}}Ra=Rab+Rbc+RacRab·Rac Rb=Rab·RbcRab+Rbc+RacR_b = \frac{R_{ab} \cdot R_{bc}}{R_{ab} + R_{bc} + R_{ac}}Rb=Rab+Rbc+RacRab·Rbc
Rc=Rac·RbcRab+Rbc+RacR_c = \frac{R_{ac} \cdot R_{bc}}{R_{ab} + R_{bc} + R_{ac}}Rc=Rab+Rbc+RacRac·Rbc

### Star-to-Delta (Y-to-Δ) Transformation

Conversely, if you start with a star network having resistors RaR_aRa, RbR_bRb, and RcR_cRc, the equivalent delta resistors are given by:

Rab=Ra+Rb+RaRbRc(or alternatively using symmetry)R_{ab} = \frac{R_a + R_b + \frac{R_a R_b}{R_c}}{\text{(or alternatively using symmetry)}}Rab
=(or alternatively using symmetry)Ra+Rb+RcRaRb

A more common form, ensuring symmetry, is:

Rab=RaRb+RbRc+RaRcRcR_{ab} = \frac{R_a R_b + R_b R_c + R_a R_c}{R_c}Rab=Rc
RaRb+RbRc+RaRc

with analogous formulas for RbcR_{bc}Rbc and RacR_{ac}Rac.

*(Note: There are various algebraically equivalent forms for the Y-to-Δ transformation. The key idea is that the impedance between any two nodes in the star configuration is matched to that in the delta configuration.)*

## c. Derivation Insight

- **Principle:**
  The transformation is derived by equating the resistance between any two nodes in both configurations. For example, in a star network, the resistance between nodes aaa and bbb is Ra+RbR_a + R_bRa+Rb. In the delta configuration, the equivalent is the parallel combination of the delta resistors. By setting these equal for every pair of nodes, the above formulas can be systematically derived.
- **Use Cases:**
  This transformation is extremely useful in three-phase circuits and bridge networks, where symmetry and simplification are key for analysis.

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# 5. Integration in Circuit Analysis

## a. Combining Methods

In practice, many circuit problems require a combination of these techniques. For example:

- **Mesh or nodal analysis** might be used initially to set up the network equations.
- **Star-Delta transformations** can simplify parts of the circuit, reducing a seemingly intractable network into one where series and parallel reductions are possible.

## b. Practical Implications

- **Design Optimization:**
  Understanding power, energy, and efficient methods of analysis enables engineers to design circuits that are both energy-efficient and cost-effective.
- **Transient Analysis:**
  In circuits involving energy storage (capacitors and inductors), knowing how energy is stored and released helps in analyzing transient responses, stability, and performance.
- **Control Systems:**
  Accurate nodal and mesh analysis underpin the development of control algorithms in power electronics, ensuring that systems perform reliably under dynamic conditions.