Overview of terms in SFE

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Contents

1 Introduction

Notation:

Ω	volume (sub)domain
Γ	surface (sub)domain
t	time
y	any function
$\underline{y}$	any vector function
$\underline{n}$	unit outward normal
q, s	scalar test function
p, r	scalar unknown or parameter function
$\bar{p}$	scalar parameter function
$\underline{v}$	vector test function
$\underline{w}$, $\underline{u}$	vector unknown or parameter function
$\underline{b}$	vector parameter function
$\underline{\underline{e}}(\underline{u})$	Cauchy strain tensor ($\frac{1}{2}((\nabla u) + (\nabla u)^T)$)
$\underline{f}$	vector volume forces
ρ	density
ν	kinematic viscosity
c	any constant
δ_{ij}, $\underline{\underline{I}}$	Kronecker delta, identity matrix

The suffix "$_0$" denotes a quatity related to a previous time step.
General syntax of a term call:

```
<term_name>.<region>( <arg1>, <arg2>, ... )
```

In the following, `<virtual>` corresponds to a test function, `<state>` to a unknown function and `<parameter>` to a known function arguments.

2 Terms in termsMass

2.1 dw_mass

Class: MassTerm
Description: Inertial forces term (constant density).
Definition:

$$\int_\Omega \rho \underline{v} \cdot \frac{\underline{u} - \underline{u}_0}{\Delta t}$$

Arguments:

material.rho	...	ρ
ts.dt	...	Δt
parameter	...	$\underline{u}_0$

Syntax: `dw_mass.<region>( <ts>, <material>, <virtual>, <state>, <parameter> )`

2.2 dw_mass_scalar

Class: MassScalarTerm
Description: Scalar field mass matrix/rezidual.
Definition:

$$\int_\Omega qp$$

Syntax: `dw_mass_scalar.<region>( <virtual>, <state> )`

2.3 dw_mass_scalar_fine_coarse

Class: MassScalarFineCoarseTerm
Description: Scalar field mass matrix/rezidual for coarse to fine grid interpolation. Field p_H belong to the coarse grid, test field q_h to the fine grid.
Definition:

$$\int_\Omega q_h p_H$$

Syntax: `dw_mass_scalar_fine_coarse.<region>( <virtual>, <state>, <iemaps>, <pbase> )`

2.4 dw_mass_vector

Class: MassVectorTerm
Description: Vector field mass matrix/rezidual.
Definition:

$$\int_\Omega \rho \, \underline{v} \cdot \underline{u}$$

Syntax: `dw_mass_vector.<region>( <material>, <virtual>, <state> )`

3 Terms in termsBasic

3.1 d_surface_dot

Class: DotProductSurfaceTerm
Description: Surface $L^2(\Gamma)$ dot product for both scalar and vector fields.
Definition:

$$\int_\Gamma pr, \int_\Gamma \underline{u} \cdot \underline{w}$$

Syntax: `d_surface_dot.<region>( <parameter_1>, <parameter_2> )`

3.2 d_surface_integrate

Class: IntegrateSurfaceTerm
Definition:

$$\int_\Gamma y, \text{ for vectors: } \int_\Gamma \underline{y} \cdot \underline{n}$$

Syntax: `d_surface_integrate.<region>( <parameter> )`

3.3 d_volume

Class: VolumeTerm
Description: Volume of a domain. Uses approximation of the parameter variable.
Definition:

$$\int_\Omega 1$$

Syntax: `d_volume.<region>( <parameter> )`

3.4 d_volume_dot

Class: DotProductVolumeTerm
Description: Volume $L^2(\Omega)$ dot product for both scalar and vector fields.
Definition:

$$\int_\Omega p r , \int_\Omega \underline{u} \cdot \underline{w}$$

Syntax: d_volume_dot.<region>(<parameter_1>, <parameter_2>)

3.5 d_volume_integrate

Class: IntegrateVolumeTerm
Definition:

$$\int_\Omega y$$

Syntax: d_volume_integrate.<region>(<parameter>)

3.6 dw_volume_integrate

Class: IntegrateVolumeOperatorTerm
Definition:

$$\int_\Omega q$$

Syntax: dw_volume_integrate.<region>(<virtual>)

4 Terms in termsLaplace

4.1 dw_laplace

Class: LaplaceTerm
Description: Laplace term (constant parameter).
Definition:

$$c \int_\Omega \nabla s : \nabla r$$

Syntax: dw_laplace.<region>(<material>, <virtual>, <state>)

5 Terms in termsNavierStokes

5.1 d_div

Class: DivIntegratedTerm
Description: Integrated divergence term (weak form).
Definition:

$$\int_\Omega \bar{p} \, \nabla \cdot \underline{w}$$

Syntax: d_div.<region>(<parameter_1>, <parameter_2>)

5.2 dq_grad

Class: GradQTerm
Description: Gradient term (weak form) in quadrature points.
Definition:

$$(\nabla p)|_{qp}$$

Syntax: `dq_grad.<region>( <state> )`

5.3 dq_lin_convect

Class: LinearConvectQTerm
Description: Linearized convective term evaluated in quadrature points.
Definition:

$$((\underline{b} \cdot \nabla)\underline{u})|_{qp}$$

Syntax: `dq_lin_convect.<region>( <parameter>, <state> )`

5.4 dw_convect

Class: ConvectTerm
Description: Nonlinear convective term.
Definition:

$$\int_{\Omega} ((\underline{u} \cdot \nabla)\underline{u}) \cdot \underline{v}$$

Syntax: `dw_convect.<region>( <virtual>, <state> )`

5.5 dw_div

Class: DivTerm
Description: Divergence term (weak form).
Definition:

$$\int_{\Omega} q \ \nabla \cdot \underline{u}$$

Syntax: `dw_div.<region>( <virtual>, <state> )`

5.6 dw_div_grad

Class: DivGradTerm
Description: Diffusion term.
Definition:

$$\int_{\Omega} \nu \ \nabla \underline{v} : \nabla \underline{u}$$

Syntax: `dw_div_grad.<region>( <material>, <virtual>, <state> )`

5.7 dw_div_r

Class: DivRTerm
Description: Divergence term (weak form) with a known field (to use on a right-hand side).
Definition:

$$\int_{\Omega} q \ \nabla \cdot \underline{w}$$

Syntax: `dw_div_r.<region>( <virtual>, <parameter> )`

5.8 dw_grad

Class: GradTerm
Description: Gradient term (weak form).
Definition:

$$\int_\Omega p \; \nabla \cdot \underline{v}$$

Syntax: `dw_grad.<region>( <virtual>, <state> )`

5.9 dw_gradDt

Class: GradDtTerm
Description: Gradient term (weak form) with time-discretized $\dot{p}$.
Definition:

$$\int_\Omega \frac{p - p_0}{\Delta t} \nabla \cdot \underline{v}$$

Arguments:

ts.dt	...	Δt
parameter	...	p_0

Syntax: `dw_gradDt.<region>( <ts>, <virtual>, <state>, <parameter> )`

5.10 dw_lin_convect

Class: LinearConvectTerm
Description: Linearized convective term.
Definition:

$$\int_\Omega ((\underline{b} \cdot \nabla) \underline{u}) \cdot \underline{v}$$

Syntax: `dw_lin_convect.<region>( <virtual>, <parameter>, <state> )`

5.11 dw_st_grad_div

Class: GradDivStabilizationTerm
Description: Grad-div stabilization term (γ is a global stabilization parameter).
Definition:

$$\gamma \int_\Omega (\nabla \cdot \underline{u}) \cdot (\nabla \cdot \underline{v})$$

Syntax: `dw_st_grad_div.<region>( <material>, <virtual>, <state> )`

5.12 dw_st_pspg_c

Class: PSPGCStabilizationTerm
Description: PSPG stabilization term, convective part (τ is a local stabilization parameter).
Definition:

$$\sum_{K \in \mathcal{T}_h} \int_{T_K} \tau_K \; ((\underline{b} \cdot \nabla) \underline{u}) \cdot \nabla q$$

Syntax: `dw_st_pspg_c.<region>( <material>, <virtual>, <parameter>, <state> )`

5.13 dw_st_pspg_p

Class: PSPGPStabilizationTerm
Description: PSPG stabilization term, pressure part (τ is a local stabilization parameter), cf. Laplace term.
Definition:

$$\sum_{K \in \mathcal{T}_h} \int_{T_K} \tau_K \ \nabla p \cdot \nabla q$$

Syntax: `dw_st_pspg_p.<region>( <material>, <virtual>, <state> )`

5.14 dw_st_supg_c

Class: SUPGCStabilizationTerm
Description: SUPG stabilization term, convective part (δ is a local stabilization parameter).
Definition:

$$\sum_{K \in \mathcal{T}_h} \int_{T_K} \delta_K \ ((\underline{b} \cdot \nabla)\underline{u}) \cdot ((\underline{b} \cdot \nabla)\underline{v})$$

Syntax: `dw_st_supg_c.<region>( <material>, <virtual>, <parameter>, <state> )`

5.15 dw_st_supg_p

Class: SUPGPStabilizationTerm
Description: SUPG stabilization term, pressure part (δ is a local stabilization parameter).
Definition:

$$\sum_{K \in \mathcal{T}_h} \int_{T_K} \delta_K \ \nabla p \cdot ((\underline{b} \cdot \nabla)\underline{v})$$

Syntax: `dw_st_supg_p.<region>( <material>, <virtual>, <parameter>, <state> )`

6 Terms in termsPoint

6.1 dw_point_lspring

Class: LinearPointSpringTerm
Description: Linear springs constraining movement of FE nodes in a reagion; use as a relaxed Dirichlet boundary conditions.
Definition:

$$\underline{f}^i = -k\underline{u}^i \quad \forall \text{ FE node } i \text{ in region}$$

Syntax: `dw_point_lspring.<region>( <material>, <virtual>, <state> )`

7 Terms in termsVolume

7.1 dw_volume_lvf

Class: LinearVolumeForceTerm
Description: Linear volume forces (weak form).
Definition:

$$\int_{\Omega} \underline{v} \cdot \underline{f}$$

Syntax: `dw_volume_lvf.<region>( <material>, <virtual> )`

8 Terms in termsSurface

8.1 dw_surface_ltr

Class: LinearTractionTerm
Description: Linear traction forces (weak form).
Definition:

$$\int_{\Gamma} \underline{v} \cdot \underline{\underline{\sigma}} \cdot \underline{n}, \text{ where, depending on dimension of 'material' argument,}$$
$$\underline{\underline{\sigma}} \cdot \underline{n} \text{ is } \bar{p} \underline{\underline{I}} \cdot \underline{n} \text{ for given scalar pressure, } \underline{f} \text{ for traction vector, and itself for a stress tensor}$$

Syntax: `dw_surface_ltr.<region>( <material>, <virtual> )`

9 Terms in termsLinElasticity

9.1 de_sdcc_strain

Class: SDCCStrainTerm
Description: Cauchy strain tensor averaged in elements.
Definition:

$$\text{vector of } \forall K \in \mathcal{T}_h : \int_{T_K} \underline{\underline{e}}(\underline{w})$$

Syntax: `de_sdcc_strain.<region>( <parameter> )`

9.2 dw_sdcc

Class: SDCCTerm
Description: Homogeneous isotropic linear elasticity term.
Definition:

$$\int_{\Omega} D_{ijkl}\ e_{ij}(\underline{v}) e_{kl}(\underline{u}) \text{ with } D_{ijkl} = \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \lambda\ \delta_{ij}\delta_{kl}$$

Syntax: `dw_sdcc.<region>( <material>, <virtual>, <state> )`