"In the Name of God"

Engineering probabilities and Statistics **CE 40181**
Sharif University of Technology

Homework 1
Wed, Oct 18 2017

Reminders:

- **Typing the solutions has extra marks.**
- **Collaboration is permitted, but you must write solutions *by yourself without* assistance.**
- **Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.**
- **Late submissions will be treated according to the course policy.**

Problems:

1. Describe the appropriate mathematical model of the sample space and the set of events for the problems below. It is not necessary to calculate the probability.
   (a) The probability that a 64 bit binary number has less than 10 bits of 1.
   (b) The probability that the difference between you and your friend's marks in this course becomes less than 2 marks (suppose that the mark can be any number from the continuous rang of 0 to 20).
   (c) The probability that after throwing two dices, their sum will be 10.
2. Suppose that we have a computer that can produce one random bit with probability $\frac{1}{2}$ (Computer gives 1 with probability 1/2 and 0 with probability 1/2). We need to produce other probabilities and we can use this computer as many times as we want. Answer the following questions and describe the reasons.
   (a) Can we produce the probability set $(\frac{1}{4}, \frac{3}{4})$ (Produce 1 with probability 1/4 and 0 with probability 3/4) using this computer a finite number of times?
   (b) Can we produce the probability set $(\frac{1}{3}, \frac{2}{3})$ using this computer a finite number of times?
   (c) Can we produce the probability set $(p, 1-p)$ for all rational numbers $0 < p < 1$ using this computer a finite number of times?
3. In a circular permutation of n different balls while $n > 4$, what is the probability that balls number 1 and 2 are adjacent but balls 1, 2 and 3 are not adjacent?

4. Shangool wants to spy on HabbeAngoor! For this purpose he needs to gain HabbeAngoor's laptop password. He knows that HabbeAngoor always chooses her passwords randomly from possible permutations of character's of her name. If Shangool writes a program that can test 10^3 passwords in one second and he has an hour of access to HabbeAngoor's laptop what is the probability that he can find the password (you should consider both lower case and upper case characters)?

5. A transmitter sends its first packet successfully with probability of 90% and fails with probability 10%. After each success, the probability of both failure and success do not change, but after each failure the probability of success decreases by 10%. What is the probability that this transmitter sends its third packet successfully?

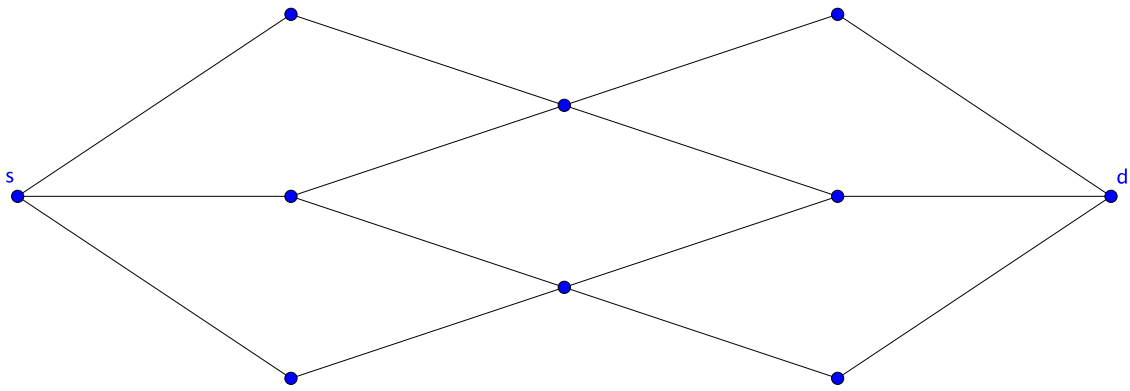
6. Suppose $A, B \subseteq \{1,2,3,4,5\}, C \subseteq \{1,2,3,4,6\}$ and A, B, C are chosen randomly among all possible subsets. What is the probability that $A \cap B \subseteq C \subseteq A \cup B$?

7. prove that:

   (a) $P(A|B \cap C) = \frac{P(A \cap B|C)}{P(B|C)}$

   (b) $P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_n)P(A_{n-1}|A_n) \cdots P(A_1|A_2 \cap A_3 \cap \cdots \cap A_n)$

8. We have a network in which each connection between two nodes is healthy with probability 50%. We have a source node shown by s and a destination node shown by d. s is connected to d if there is a healthy path with 5 nodes between s and d. What is the probability that s and d are connected?



9. A train and a bus arrive at a station at a random time between 8 and 9 AM. The Train waits for 10 minutes and the bus waits for x minutes. what should x be so that the probability that the train and the bus meet becomes 50%?

10. A transmitter sends 6 bits of data in each transmission. In order to check for effects of noise on the transmission, one bit of parity is sent with each transmission (so the transmitter sends 7 bits in each transmission). Suppose that each bit can change in transmission with the probability of 10%. What is the probability that the data becomes corrupted on its way and the receiver can't find out this corruption using the parity bit?

R problem

Consider a graph with 20 nodes $X_1, X_2, \ldots, X_{20}$. Write an R program to create all possible graphs with different number of edges starting from 1 to 190, i.e. $E = 1, 2, \ldots, 190$.

1. Report the number of graphs for each E value.
2. Report the proportion of connected graphs among all graphs with E edges, for all E values.
3. Now, assume that the graph is connected, hence each edge has its direction. Report the proportion of strongly connected graphs among all possible directed graphs. It is worthy to note that, a strongly connected graph is a graph in which there is a directed path between each two nodes.