**"In the Name of God"**

**Engineering probabilities and Statistics** **CE 40181**
**Sharif University of Technology**

# Homework 6
# Wed, Jan 03 2018

**Reminders:**

- **Typing the solutions has extra marks.**
- **Collaboration is permitted, but you must write solutions *by yourself without* assistance.**
- **Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.**
- **Late submissions will be treated according to the course policy.**

**Problems:**

1. Your head TA wants to select a random sample of 10 students from your class -CE 40181. Which of the following strategies is the best:

   **a)**Select 10 students from the last row.
   **b)**At the morning, wait near the door and select 10 students who first enter the class.
   **c)**Select 10 of your friends from the class. If you do not have 10 friends in CE 40181, you should find 10 friends first!
   **d)**Assign a number to each student in your class and use a random number generator to pick 10 students.

2. A sample of 5 observations, $(X_1 = \frac{1}{3}, X_2 = \frac{3}{3}, X_3 = \frac{4}{10}, X_4 = \frac{7}{10}, X_5 = \frac{9}{10})$ is collected from a continuous distribution with density

   $$f(x) = \theta x^{\theta-1} \text{ for } 0 < x < 1.$$

   By the method of maximum likelihood and the method of moments, estimate $\theta$.

3. Suppose $N$ random Intel processors lifetime of which usually follows the Poisson distribution with mean $\lambda$, are being examined and the results are $Y_1, Y_2, ..., Y_N$. Find the Maximum likelihood estimator $\hat{\lambda}$ for $\lambda$.

4. Installation of a certain chipset takes a random period of time with a standard deviation of 5 minutes. A SUT hardware student installs this hardware on 64 different computers with the average time of 42 minutes. Compute a 95% confidence interval for the mean installation time.

5. For measuring typing speed of the CE student, 12 students are randomly chosen and tested. The speeds (word/min) of the students were:

$$63, 58, 60, 58, 70, 58, 61, 67, 59, 65, 60, 64.$$

Estimate the variance of the type speeds with a 95% confidence interval.
**Hint**: To make your estimation, you should make some assumption for the distribution of speeds.

6. $X = (X_1, X_2, ..., X_n)$ is random sample from the geometric distribution with unknown parameter $p \in (0, 1)$. Prove that ML estimator of $p$ is $U = \frac{1}{M}$.

7. Suppose a wireless network in which Transmitter Transmit the signal $X \sim N(0, \sigma_X^2)$ over the channel. Assume that the receiver received signal

$$Y = X + T$$

where $T \sim N(0, \sigma_T^2)$ is independent from $X$.
Find ML(Maximum Likelihood) and MAP(Maximum A Posteriori) estimates of $X$, when $Y = y$ is observed.

**R Problem**

So far, we learned how to simulate random process based on *probabilistic models*. Now we want to face the *data* generated under such probabilistic model. Our goal is to capture the *behavior of the model* by means of statistics. Statistics help us to extract information, and furthermore knowledge from the data.
In other words, when you *estimate* a parameter, in fact, you estimate a feature of the model. When *test* for differences between two groups based on the data collected, you are aiming to extract the underlying fact in your population.

## General Math Data

The supplementary file data#6.csv has been provided by prof. Moghaddasi which contains total scores of general math in fall 2004.

1. Use t.test in order to check if there is any significant differences between boys and girls.

2. Use t.test, pairwise, in order to check if there is any significant differences among professors.

3. Check if progress is possible, to do so, write a code in order to predict final score given midterm score (you can use maximum likelihood for parameter estimation). Compare your prediction and the reality. Report for differences.

4. Use ggplot to:

   - draw a boxplot among departments.
   - draw two histograms, one for girls and one for boys, both in one figure.
   - draw density plots, for each department, all in one figure.