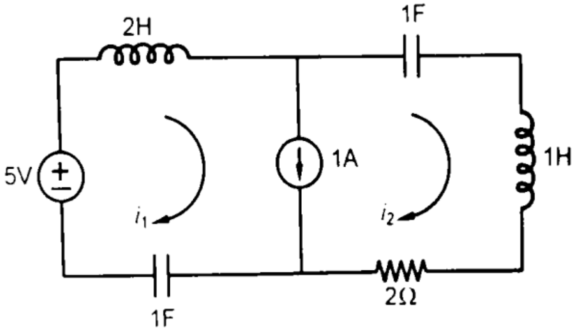
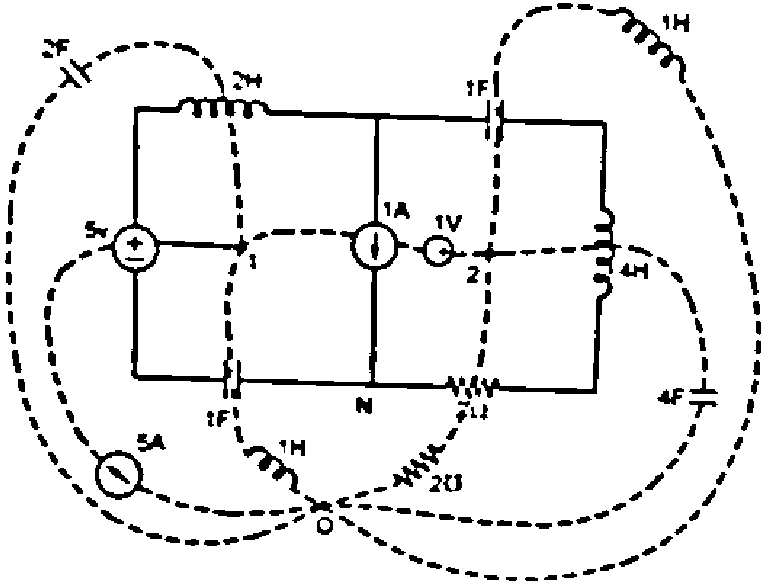
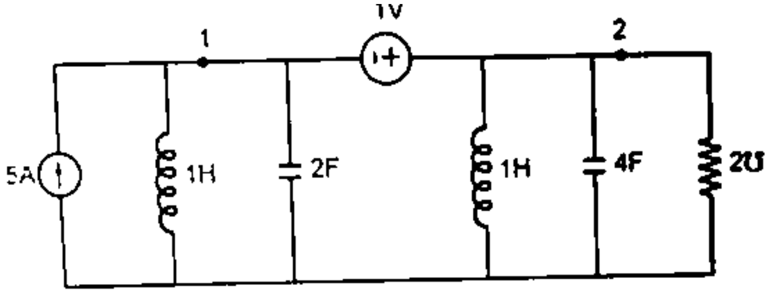
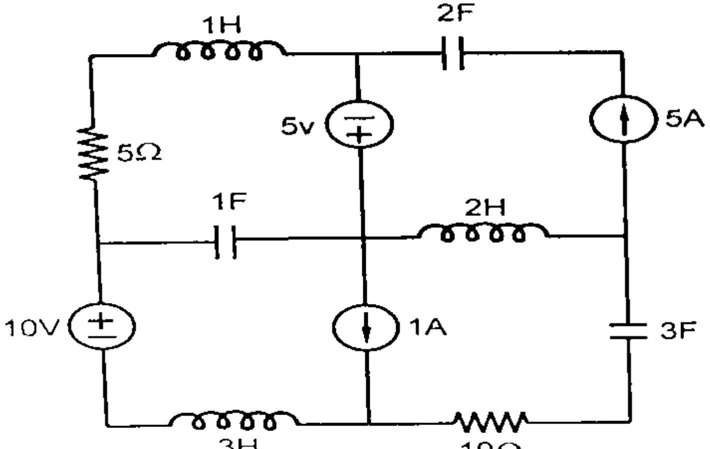
Duality concept

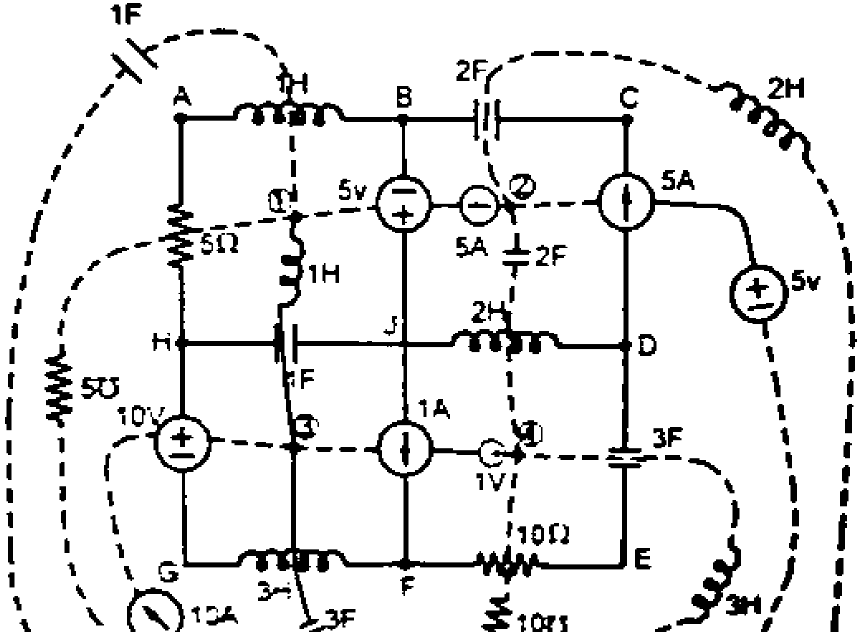
منبع ولتاژی که جریان ساعتگرد برای حلقه تولید می کند در مدار دوگان جهت منبع جریان به سمت گره متناظر خواهد بود
منبع جریانی که جهت آن برای حلقه ساعتگرد است در مدار دوگان پلاریته منفی منبع ولتاژ به سمت گره متناظر خواهد بود

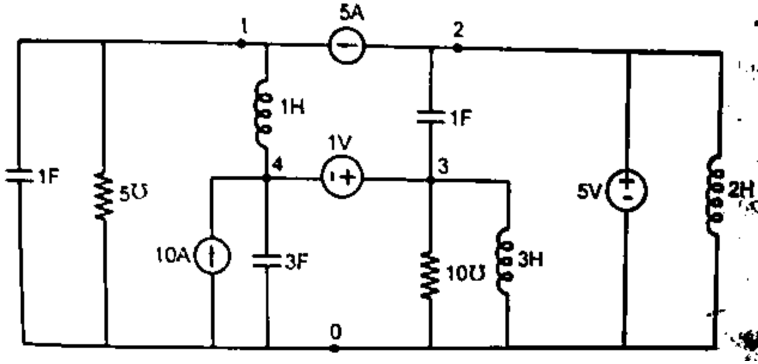


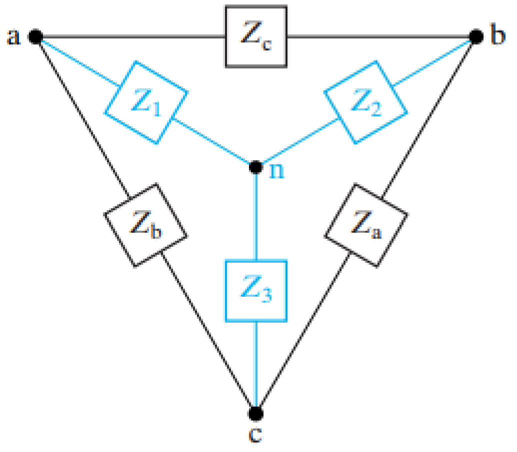










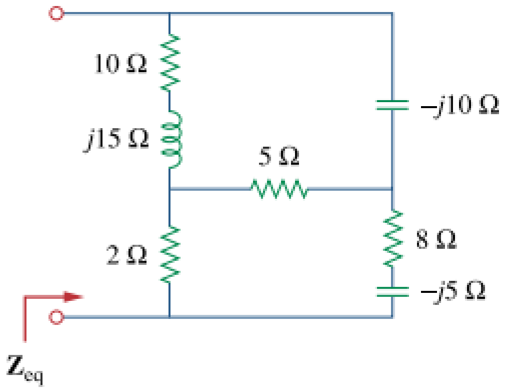


$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}, \qquad Z_1 = \frac{Z_bZ_c}{Z_a + Z_b + Z_c},$$

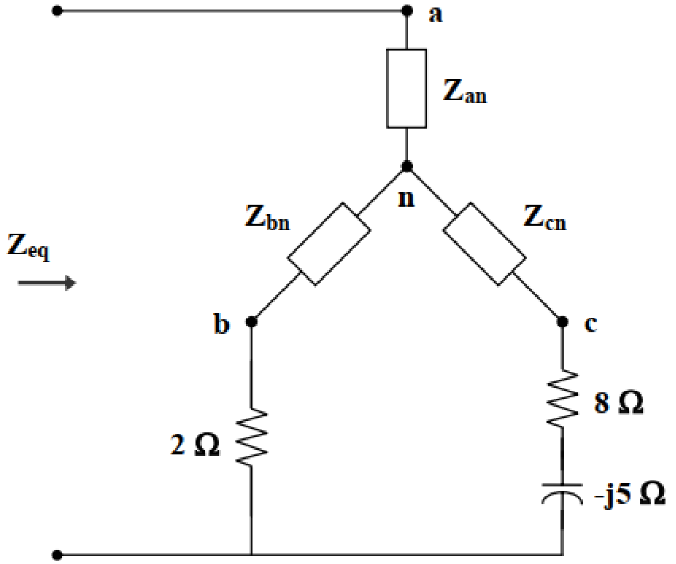
$$Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}, \qquad Z_2 = \frac{Z_cZ_a}{Z_a + Z_b + Z_c},$$

$$Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}. \qquad Z_3 = \frac{Z_aZ_b}{Z_a + Z_b + Z_c}.$$

Find the equivalent impedance



Make a delta-to-wye transformation as shown in the figure below.



$$\mathbf{Z}_{an} = \frac{(-j10)(10 + j15)}{5 - j10 + 10 + j15} = \frac{(10)(15 - j10)}{15 + j5} = 7 - j9$$

$$\mathbf{Z}_{bn} = \frac{(5)(10 + j15)}{15 + j5} = 4.5 + j3.5$$

$$\mathbf{Z}_{cn} = \frac{(5)(-j10)}{15 + j5} = -1 - j3$$

$$\mathbf{Z}_{eq} = \mathbf{Z}_{an} + (\mathbf{Z}_{bn} + 2) \,||\, (\mathbf{Z}_{cn} + 8 - j5)$$

$$\mathbf{Z}_{eq} = 7 - j9 + (6.5 + j3.5) \,||\, (7 - j8)$$

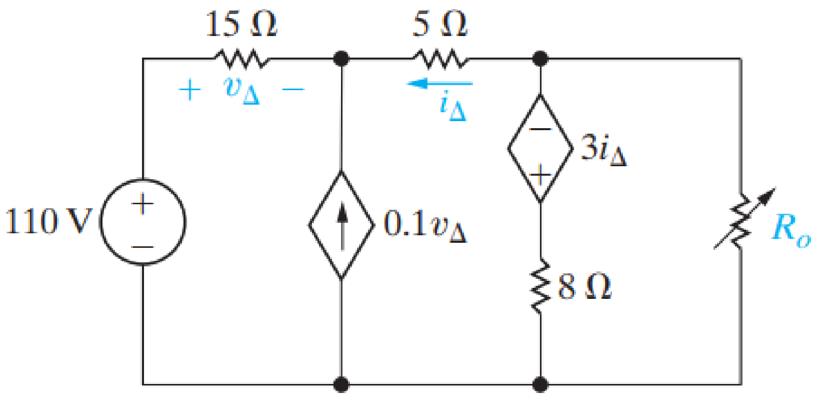
$$\mathbf{Z}_{eq} = 7 - j9 + \frac{(6.5 + j3.5)(7 - j8)}{13.5 - j4.5}$$

$$\mathbf{Z}_{eq} = 7 - j9 + 5.511 - j0.2$$

$$\mathbf{Z}_{eq} = 12.51 - j9.2 = \underline{\mathbf{15.53\angle\text{-}36.33°\ \Omega}}$$

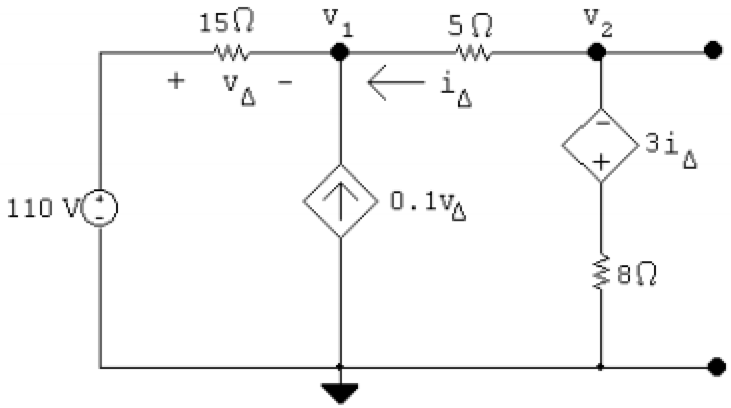
a) Find the value of R_o.

b) Find the maximum power.



بدست آوردن V_t

[a] Open circuit voltage



Node voltage equations:

$$\frac{v_1 - 110}{15} - 0.1v_\Delta + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2 + 3i_\Delta}{8} = 0$$

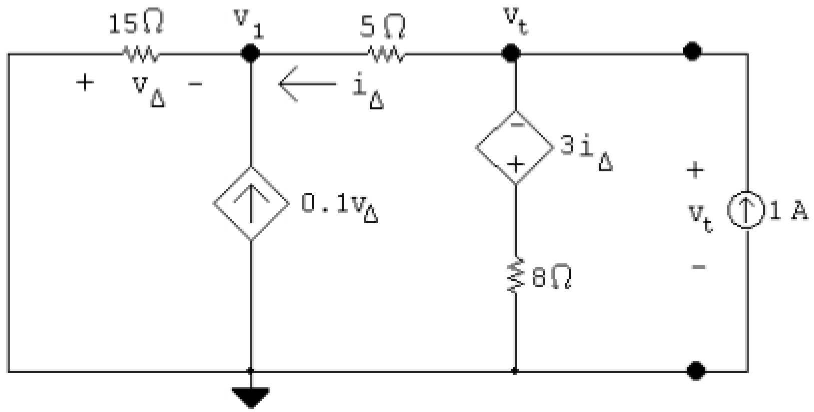
Constraint equations:

$$i_\Delta = \frac{v_2 - v_1}{5}; \qquad v_\Delta = 110 - v_1$$

Solving, $v_2 = 55$ V $= v_{\text{Th}}$

بدست آوردن R_{th}

Thévenin resistance using a test source:



$$\frac{v_1}{15} - 0.1v_\Delta + \frac{v_1 - v_t}{5} = 0$$

$$\frac{v_t - v_1}{5} + \frac{v_t + 3i_\Delta}{8} - 1 = 0$$

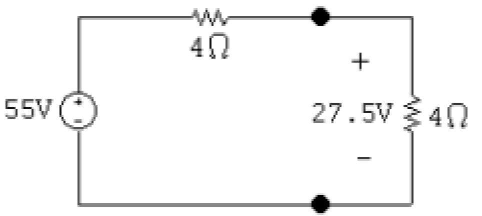
$$i_\Delta = \frac{v_t - v_1}{5}; \qquad v_\Delta = -v_1$$

Solving, $v_t = 4$ V.

$$R_{\text{Th}} = \frac{v_t}{1} = 4\,\Omega$$

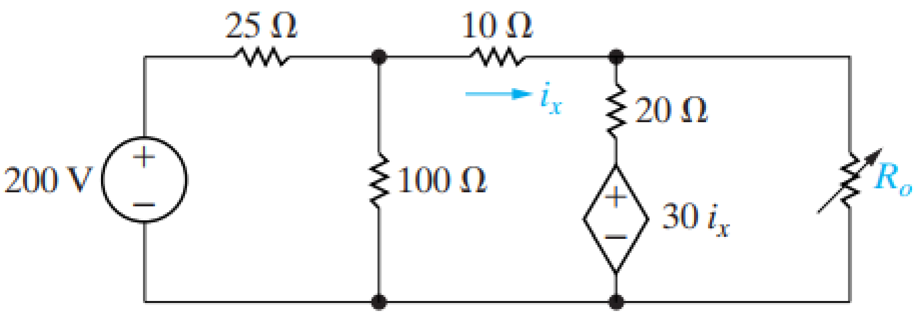
$$\therefore \quad R_o = R_{\text{Th}} = 4\,\Omega$$

[b]

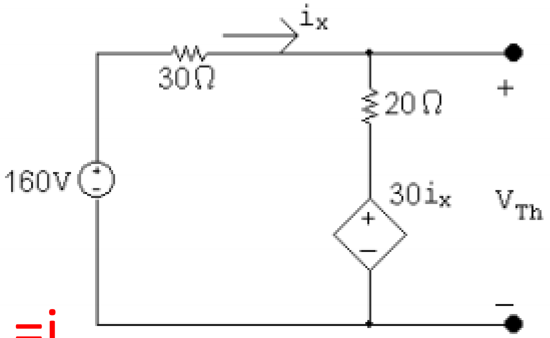


$$p_{\max} = \frac{(27.5)^2}{4} = 189.0625\,\text{W}$$

The variable resistor (R_o) in the circuit in Fig. P4.88 is adjusted until the power dissipated in the resistor is 250 W. Find the values of R_o that satisfy this condition.



We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to

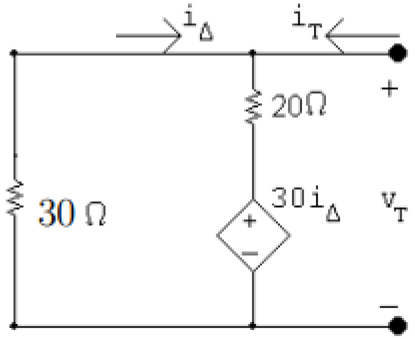


$i_x = i_\Delta$

$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2\,\text{A}$$

$$V_{\text{Th}} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100\,\text{V}$$

Using the test-source method to find the Thévenin resistance gives

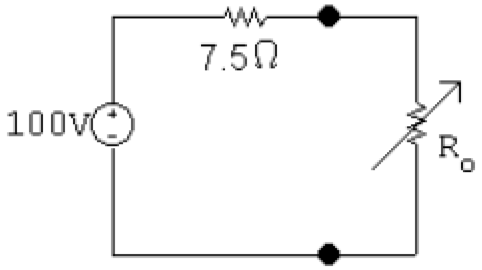


$$i_{\text{T}} = \frac{v_{\text{T}}}{30} + \frac{v_{\text{T}} - 30(-v_{\text{T}}/30)}{20}$$

$$\frac{i_{\text{T}}}{v_{\text{T}}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{\text{Th}} = \frac{v_{\text{T}}}{i_{\text{T}}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25} R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

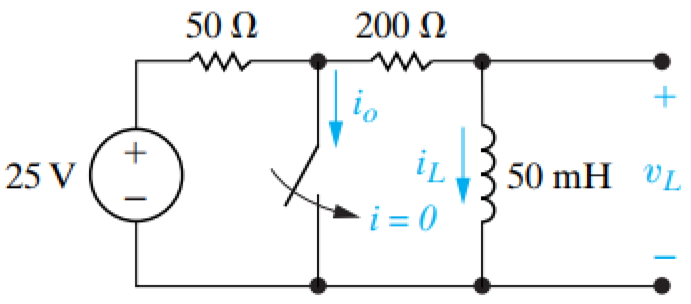
$$R_o^2 - 25R_o + 56.25 = 0$$

$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

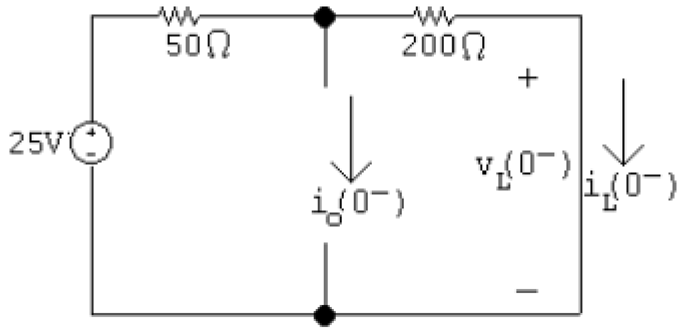
$$R_o = 22.5\,\Omega$$

$$R_o = 2.5\,\Omega$$

a) Find $i_o(0^-)$, $i_L(0^-)$, and $v_L(0^-)$.
b) Find $i_o(0^+)$, $i_L(0^+)$, and $v_L(0^+)$.
c) Find $i_o(\infty)$, $i_L(\infty)$, and $v_L(\infty)$.



[a] For $t = 0^-$ the circuit is:

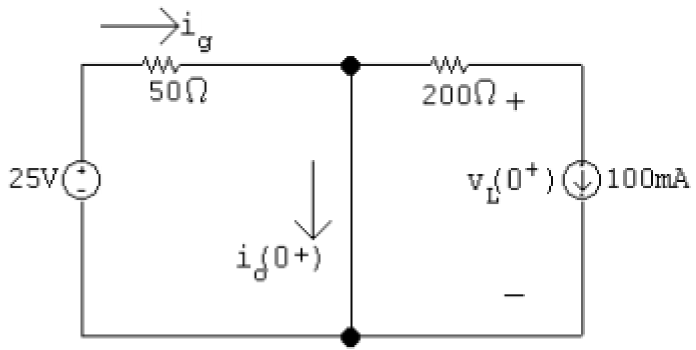


$i_o(0^-) = 0$ since the switch is open

$$i_L(0^-) = \frac{25}{250} = 0.1 = 100\,\text{mA}$$

$v_L(0^-) = 0$ since the inductor behaves like a short circuit

[b] For $t = 0^+$ the circuit is:



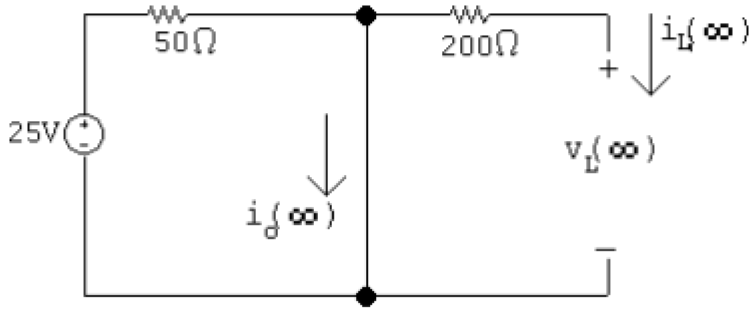
$$i_L(0^+) = i_L(0^-) = 100\,\text{mA}$$

$$i_g = \frac{25}{50} = 0.5 = 500\,\text{mA}$$

$$i_o(0^+) = i_g - i_L(0^+) = 500 - 100 = 400\,\text{mA}$$

$$200 i_L(0^+) + v_L(0^+) = 0 \qquad \therefore \qquad v_L(0^+) = -200 i_L(0^+) = -20\,\text{V}$$

[c] As $t \to \infty$ the circuit is:

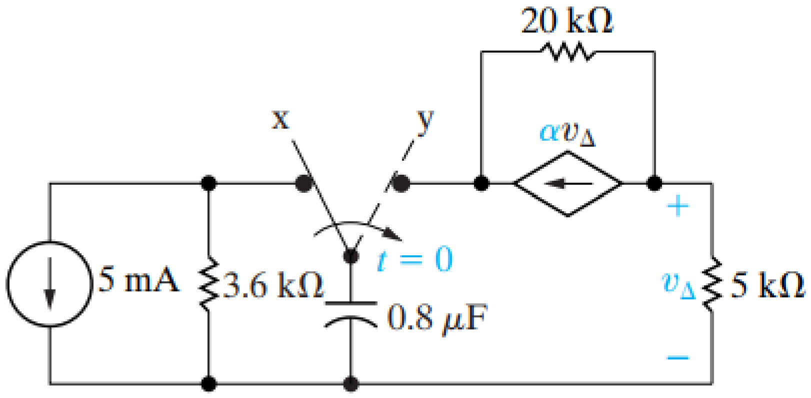


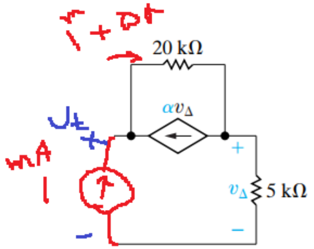
$$i_L(\infty) = 0; \qquad v_L(\infty) = 0$$

$$i_o(\infty) = \frac{25}{50} = 500\,\text{mA}$$

a) Find α so that the time constant for $t > 0$ is 40 ms.

b) For the α found in (a), find v_Δ.





$V_\Delta = ۵^v$

KVL: $V_t = ۲۰^k(۱^m + ۵\alpha) + ۵$

$V_t = ۲۵ + ۱۰۰^k\alpha$

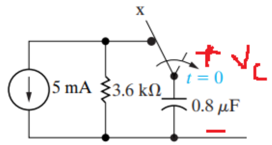
$R_{th} = \frac{V_t}{۱^m} = ۲۵^k + ۱۰^۸\alpha$

$\tau = R_{th}C = ۸۰۰^{nF}(۲۵^k + ۱۰^۸\alpha) = ۴۰^m$

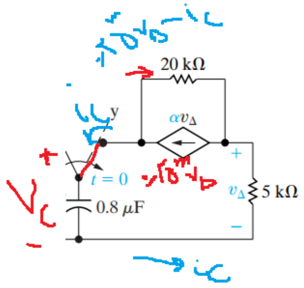
$۲۰^m + ۸۰\alpha = ۴۰^m$

$\alpha = \frac{۲۰^m}{۸۰} = ۰/۲۵^m$

$t<0$



$V_c(0^-) = -۳,۶^k \times ۵^m = -۱۸$



$t>0$

$-V_c + ۲۰^k(۰,۱۵ v_\Delta - i_c) - ۵^k i_c = 0$

$i_c = C\frac{dv_c}{dt} = ۰,۸^\mu \frac{dV_c}{dt}$ $\quad V_\Delta = -۵^k i_c$

$+V_c + ۵۰\cdot(۰,۸^\mu)\frac{dV_c}{dt} = 0$

$۰,۰۴\frac{dV_c}{dt} + V_c =$

$\frac{dV_c}{dt} + ۲۵ V_c = 0$

$V_c(t) = Ae^{-۲۵t} + B$

$V_c(0) = -۱۸ = A + B$

$V_c(\infty) = 0 = B$

$V_c(t) = -۱۸ e^{-۲۵t}$

$V_\Delta = -۵^k i_c = -۵^k\left(۰,۸^\mu \frac{dV_c}{dt}\right)$

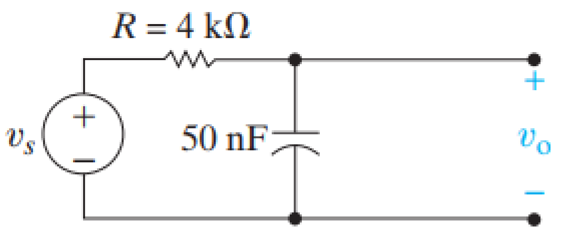
$V_\Delta = -۴^m\left(۴۵۰ e^{-۲۵t}\right)$

$V_\Delta = -۱,۸\, e^{-۲۵t}$

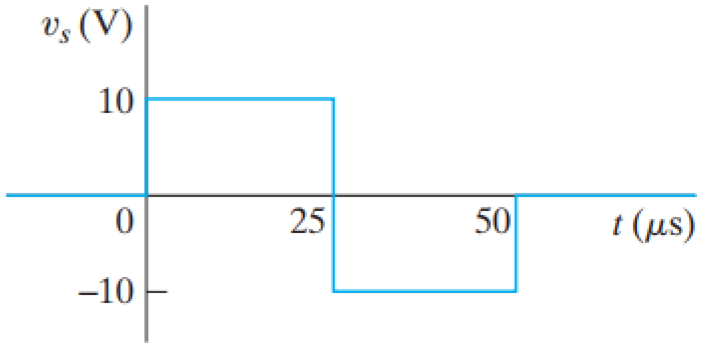
a) Derive the expressions for $v_o(t)$ that apply in the intervals $t < 0$; $0 \le t \le 25\ \mu s$; $25\ \mu s \le t \le 50\ \mu s$; and $50\ \mu s \le t < \infty$.

b) Sketch v_o and v_s on the same coordinate axes.

c) Repeat (a) and (b) with R reduced to 800 Ω.



(a)



(b)

[a] $t < 0; \qquad v_o = 0$

$0 \le t \le 25\,\mu s$:

$$\tau = (4000)(50 \times 10^{-9}) = 0.2\,\text{ms}; \qquad 1/\tau = 5000$$

$$v_o = 10 - 10e^{-5000t}\,\text{V}, \qquad 0 \le t \le 25\,\mu s$$

$$v_o(25\,\mu s) = 10(1 - e^{-0.125}) = 1.175\,\text{V}$$

$25\,\mu s \le t \le 50\,\mu s$:

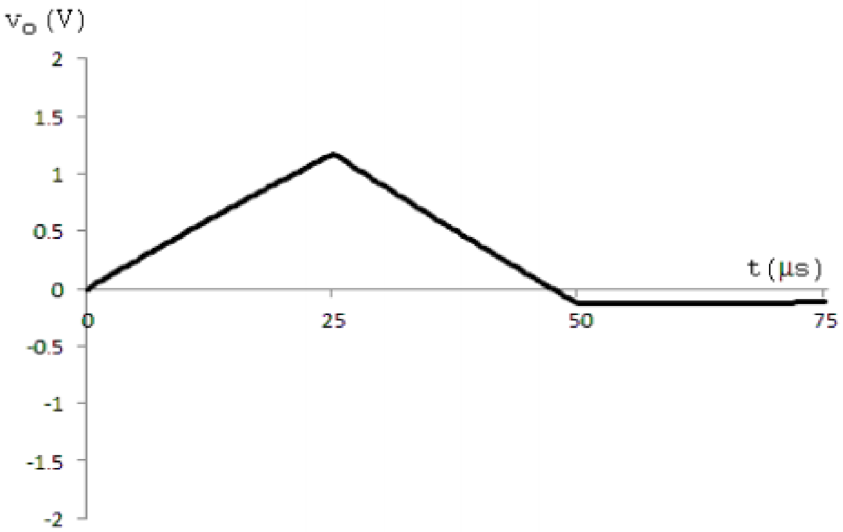
$$v_o = -10 + 11.175e^{-5000(t - 25\times 10^{-6})}\,\text{V}, \quad 25\,\mu s \le t \le 50\,\mu s$$

$$v_o(50\,\mu s) = -10 + 11.175e^{-0.125} = -0.138\,\text{V}$$

$t \ge 50\,\mu s$:

$$v_o = -0.138e^{-5000(t - 50\times 10^{-6})}\,\text{V}, \quad t \ge 50\,\mu s$$

[b]



[c] $t \leq 0: \qquad v_o = 0$

$0 \leq t \leq 25\,\mu$s:

$$\tau = (800)(50 \times 10^{-9}) = 40\,\mu\text{s} \quad 1/\tau = 25{,}000$$

$$v_o = 10 - 10e^{-25{,}000t}\,\text{V}, \qquad 0 \leq t \leq 25\,\mu\text{s}$$

$$v_o(25\,\mu\text{s}) = 10 - 10e^{-0.625} = 4.65\,\text{V}$$

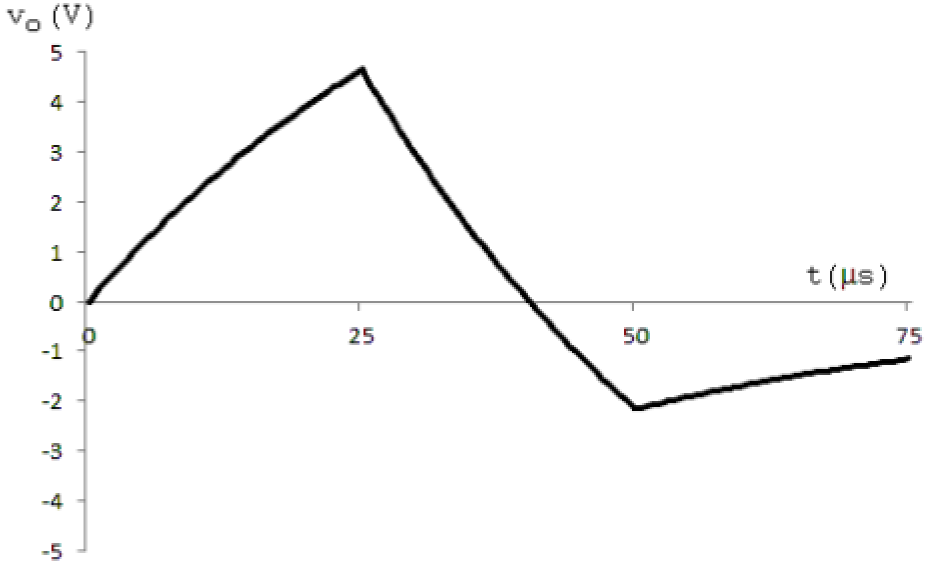
$25\,\mu\text{s} \leq t \leq 50\,\mu$s:

$$v_o = -10 + 14.65e^{-25{,}000(t-25\times10^{-6})}\,\text{V}, \qquad 25\,\mu\text{s} \leq t \leq 50\,\mu\text{s}$$

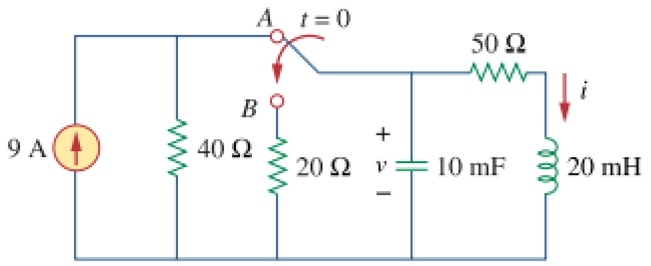
$$v_o(50\,\mu\text{s}) = -10 + 14.65e^{-0.625} = -2.16\,\text{V}$$

$t \geq 50\,\mu$s:

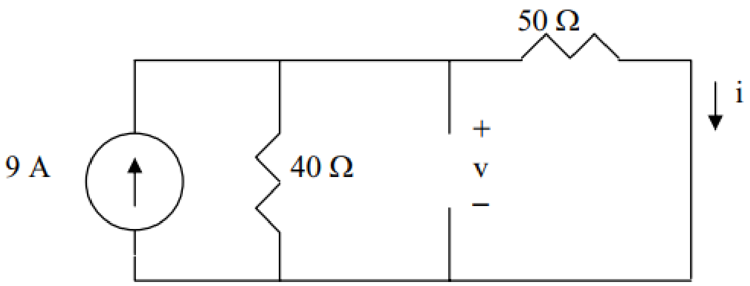
$$v_o = -2.16e^{-25{,}000(t-50\times10^{-6})}\,\text{V}, \quad t \geq 50\,\mu\text{s}$$



The switch moves from position A to B at $t=0$. Determine: (a) $i(0^+)$ and $v(0^+)$, (b) $di(0^+)/dt$, (c) $i(\infty)$ and $v(\infty)$.



(a) When the switch is at A, the circuit has reached steady state. Under this condition, the circuit is as shown below.



$$i(0^-) = \frac{40}{50+40}(9) = 4\text{A}, \qquad v(0^-) = 50i = 50\text{x}4 = 200\ \text{V}$$

$v(0^+) = v(0^-) = \underline{200\ \text{V}}$

$i(0^+) = i(0^-) = \underline{4\ \text{A}}$

(b) $$v_L = L\frac{di}{dt} \longrightarrow \frac{di(0)}{dt} = \frac{v_L(0^+)}{L}$$

At $t = 0^+$, the right hand loop becomes,

$-200 + 50\text{x}4 + v_L(0^+) = 0$ or $v_L(0^+) = 0$ and $(di(0^+)/dt) = \underline{\mathbf{0}}$.

$$i_c = C\frac{dv}{dt} \longrightarrow \frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

At $t = 0^+$, and looking at the current flowing out of the node at the top of the circuit,

$((200–0)/20) + i_C + 4 = 0$ or $i_C = –14$ A.

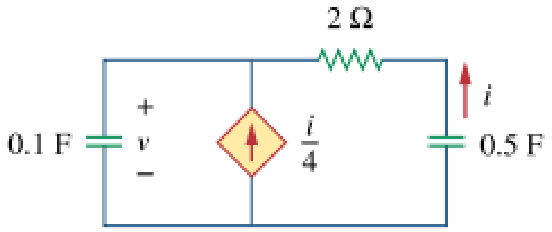
Therefore,

$dv(0^+)/dt = –14/0.01 = \underline{\mathbf{–1.4\ kV/s}}$.

(c) When the switch is in position B, the circuit reaches steady state. Since it is source-free, i and v decay to zero with time.

$\underline{i(\infty) = 0, v(\infty) = 0}$

find $v(t)$ for $t > 0$. Assume that $v(0^+) = 4$ V and $i(0^+) = 2$ A.



At the top node, writing a KCL equation produces,

$$i/4 + i = C_1 dv/dt, \ C_1 = 0.1$$

$$5i/4 = C_1 dv/dt = 0.1 dv/dt$$

$$i = 0.08 dv/dt \qquad (1)$$

But, $$v = -(2i + (1/C_2)\int idt), \ C_2 = 0.5$$

or $$-dv/dt = 2di/dt + 2i \qquad (2)$$

Substituting (1) into (2) gives,

$$-dv/dt = 0.16 d^2v/dt^2 + 0.16 dv/dt$$

$$0.16 d^2v/dt^2 + 0.16 dv/dt + dv/dt = 0, \text{ or } d^2v/dt^2 + 7.25 dv/dt = 0$$

Which leads to $s^2 + 7.25s = 0 = s(s + 7.25)$ or $s_{1,2} = 0, -7.25$

$$v(t) = A + Be^{-7.25t} \qquad (3)$$

$$v(0) = 4 = A + B \qquad (4)$$

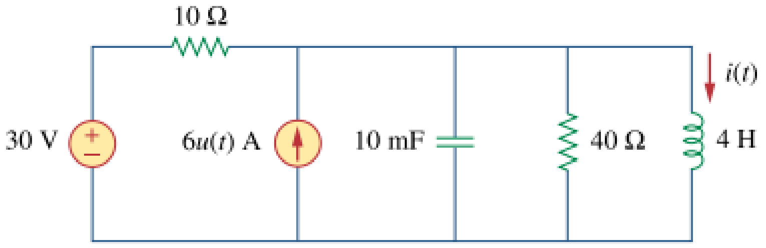
From (1), $i(0) = 2 = 0.08 dv(0+)/dt$ or $dv(0+)/dt = 25$

But, $dv/dt = -7.25Be^{-7.25t}$, which leads to,

$dv(0)/dt = -7.25B = 25$ or $B = -3.448$ and $A = 4 - B = 4 + 3.448 = 7.448$

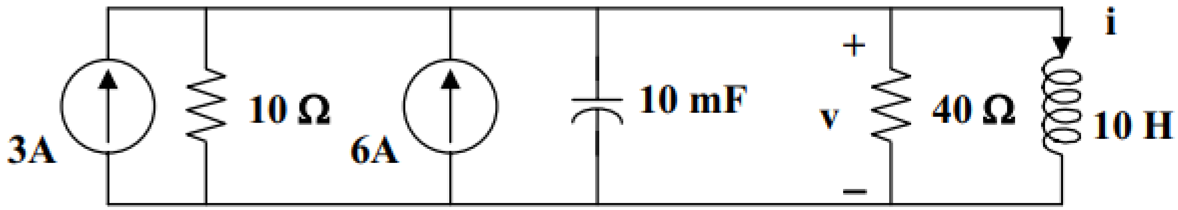
Thus, $v(t) = \underline{\mathbf{\{7.448 - 3.448e^{-7.25t}\}\ V}}$

find $i(t)$ for $t > 0$.



For t = 0-, 4u(t) = 0, v(0) = 0, and i(0) = 30/10 = 3A.

For t > 0, we have a parallel RLC circuit.



$$I_s = 3 + 6 = 9A \text{ and } R = 10||40 = 8 \text{ ohms}$$

$$\alpha = 1/(2RC) = (1)/(2x8x0.01) = 25/4 = 6.25$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4x0.01} = 5$$

Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -2.5$$

Thus, $$i(t) = I_s + [Ae^{-10t}] + [Be^{-2.5t}], \quad I_s = 9$$

$$i(0) = 3 = 9 + A + B \text{ or } A + B = -6$$

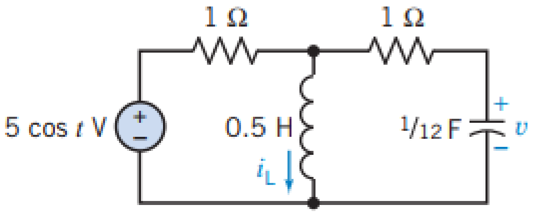
$$di/dt = [-10Ae^{-10t}] + [-2.5Be^{-2.5t}],$$

$$v(0) = 0 = Ldi(0)/dt \text{ or } di(0)/dt = 0 = -10A - 2.5B \text{ or } B = -4A$$

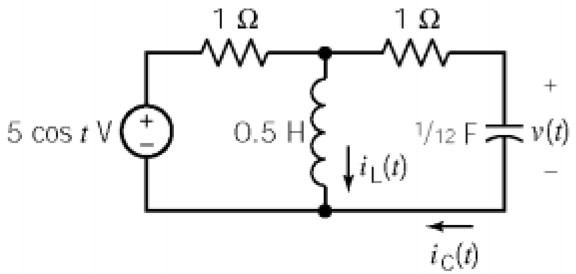
Thus, A = 2 and B = -8

Clearly, $$i(t) = \underline{\{ 9 + [2e^{-10t}] + [-8e^{-2.5t}] \} \text{ A}}$$

Find $v(t)$ for $t > 0$ for the circuit when $v(0) = 1$ V and $i_L(0) = 0$.



For $t > 0$



KCL at top node:

$$\left(0.5\frac{di_L(t)}{dt}-5\cos t\right)+i_L(t)+\frac{1}{12}\frac{dv(t)}{dt}=0 \qquad (1)$$

KVL for right mesh:

$$0.5\frac{di_L(t)}{dt}=\frac{1}{12}\frac{dv(t)}{dt}+v(t) \qquad (2)$$

Taking the derivative of these equations gives:

$$\frac{d}{dt}\text{ of }(1) \Rightarrow 0.5\frac{d^2i_L(t)}{dt^2}+\frac{di_L(t)}{dt}+\frac{1}{12}\frac{d^2v(t)}{dt^2}=-5\sin t \qquad (3)$$

$$\frac{d}{dt}\text{ of }(2) \Rightarrow 0.5\frac{d^2i_L(t)}{dt^2}=\frac{1}{12}\frac{d^2v(t)}{dt^2}+\frac{dv(t)}{dt} \qquad (4)$$

Solving for $\frac{d^2i_L(t)}{dt^2}$ in (4) and $\frac{di_L(t)}{dt}$ in (2) & plugging into (3) gives

$$\frac{d^2v(t)}{dt^2}+7\frac{dv(t)}{dt}+12v(t)=-30\sin t$$

The characteristic equation is: $s^2+7s+12=0$.

The natural frequencies are $s_{1,2}=-3,-4$.

The natural response is of the form $v_n(t) = A_1e^{-3t}+A_2e^{-4t}$. Try a forced response of the form $v_f(t)=B_1\cos t+B_2\sin t$. Substituting the forced response into the differential equation and equating like terms gives $B_1=\frac{21}{17}$ and $B_2=-\frac{33}{17}$.

$$v(t)=v_n(t)+v_f(t)=A_1e^{-3t}+A_2e^{-4t}+\frac{21}{17}\cos t-\frac{33}{17}\sin t$$

We will use the initial conditions to evaluate A_1 and A_2. We are given $i_L(0)=0$ and $v(0)=1$ V. Apply KVL to the outside loop to get

$$1\left[i_C(t)+i_L(t)\right]+1\left(i_C(t)\right)+v(t)-5\cos t=0$$

At t = 0+

$$i_C(0)=\frac{5\cos(0)+i_L(0)-v(0)}{2}=\frac{5+0-1}{2}=2 \text{ A}$$

$$\frac{dv(0)}{dt}=\frac{i_C(0)}{1/12}=\frac{2}{1/12}=24 \text{ V/s}$$

$$\left.\begin{aligned} v(0^+)&=1=A_1+A_2+\frac{21}{17}\\ \frac{dv(0^+)}{dt}&=24=-3A_1-4A_2-\frac{33}{17}\end{aligned}\right\} \Rightarrow \begin{aligned} A_1&=25\\ A_2&=-\frac{429}{17}\end{aligned}$$

Finally,

$$\therefore\ v(t)=25e^{-3t}-\frac{429e^{-4t}-21\cos t+33\sin t}{17} \text{ V}$$