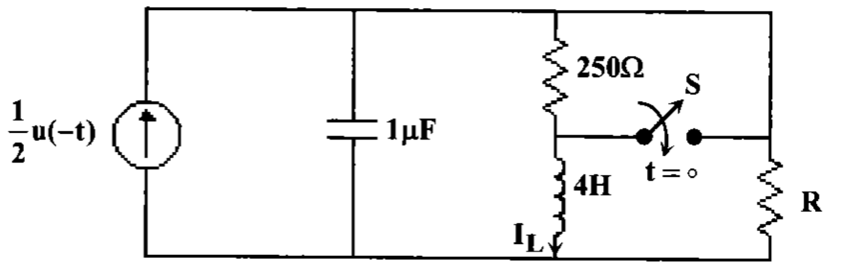
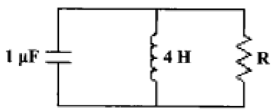
جواب تمرینات سری چهارم

۲-



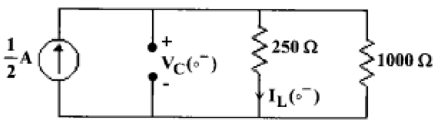
اگر مدار در $t > ۰$ در حالت میرایی بحرانی باشد، داریم:



$$\alpha = \omega_۰ \Rightarrow \frac{۱}{۲RC} = \frac{۱}{\sqrt{LC}}$$

$$\frac{۱}{۲ \times R \times ۱۰^{-۶}} = \frac{۱}{\sqrt{۴ \times ۱۰^{-۶}}} \Rightarrow R = ۱۰۰۰\,\Omega$$

حال مدار در $t = ۰^-$ تحلیل می‌شود.



$$I_L(۰^{\pm}) = \frac{۱}{۲} \times \frac{۱۰۰۰}{۱۰۰۰ + ۲۵۰} = ۰/۴A$$

$$V_C(۰^{\pm}) = \frac{۱}{۲}(۲۵۰ \| ۱۰۰۰) = ۱۰۰\,V$$

$$۲\alpha = \frac{۱}{RC} = \frac{۱}{۱۰۰۰ \times ۱۰^{-۶}} = ۱۰۰۰ \qquad \text{و} \qquad \omega_۰^۲ = \frac{۱}{LC} = \frac{۱}{۴ \times ۱۰^{-۶}} = ۲۵۰۰۰۰$$

$$S^۲ + ۲\alpha s + \omega_۰^۲ = ۰ \Rightarrow S^۲ + ۱۰۰۰S + ۲۵۰۰۰۰ = ۰ \Rightarrow S_۱, S_۲ = -۵۰۰ \Rightarrow I_L(t) = (A + Bt)e^{-۵۰۰t}u(t)$$

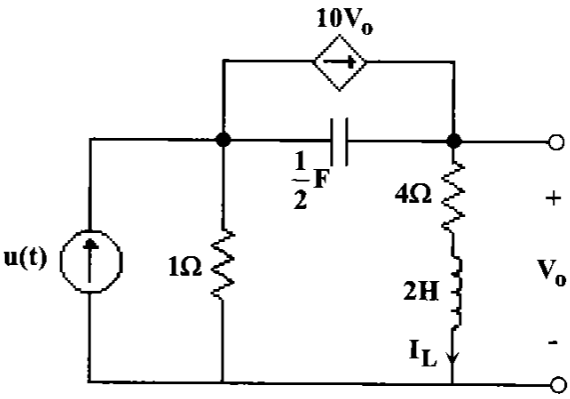
حال ضریب A و B را محاسبه می‌کنیم.

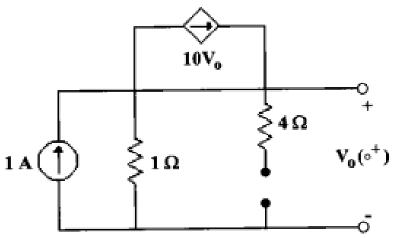
$$I_L(۰^+) = ۰/۴ = (A + B \times ۰)e^{-۵۰۰ \times ۰}u(۰) \Rightarrow A = ۰/۴$$

$$\frac{dI_L(۰^+)}{dt} = \frac{V_L(۰^+)}{L} = \frac{V_C(۰^+)}{L} = \frac{۱۰۰}{۴} = ۲۵\frac{A}{sec} \Rightarrow \frac{dI(t)}{dt} = -۵۰۰e^{-۵۰۰t}(A + Bt) + Be^{-۵۰۰t}u(t)$$

$$\frac{dI(۰^+)}{dt} = \frac{dI_L(۰^+)}{dt} = ۲۵ = -۵۰۰[۰/۴ + B \times ۰] + B \Rightarrow B = ۲۲۵ \Rightarrow \mathbf{I(t) = (۰/۴ + ۲۲۵t)e^{-۵۰۰t}u(t)}$$

۳-





ابتدا مدار را در $t=0^+$ تحلیل می‌شود.　　　$V_o(0^+)=1\times1=1V$

حال مدار را دوباره تحلیل می‌کنیم.

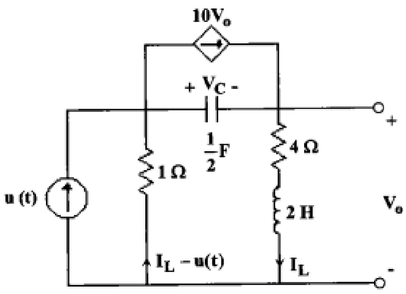
$$V_o(t)=2\frac{dI_L}{dt}+4I_L$$

$$V_o(0^+)=1=2\frac{dI_L(0^+)}{dt}+0 \Rightarrow \frac{dI_L(0^+)}{dt}=\frac{1}{2}\left(\frac{A}{sec}\right)$$

با نوشتن KVL در حلقه وسطی داریم:

$$4I_L+2\frac{dI_L}{dt}+1\times(I_L-u(t))+2\int_0^t(I_L-10V_o)dt=0$$

با مشتق‌گیری از معادله بالا داریم:

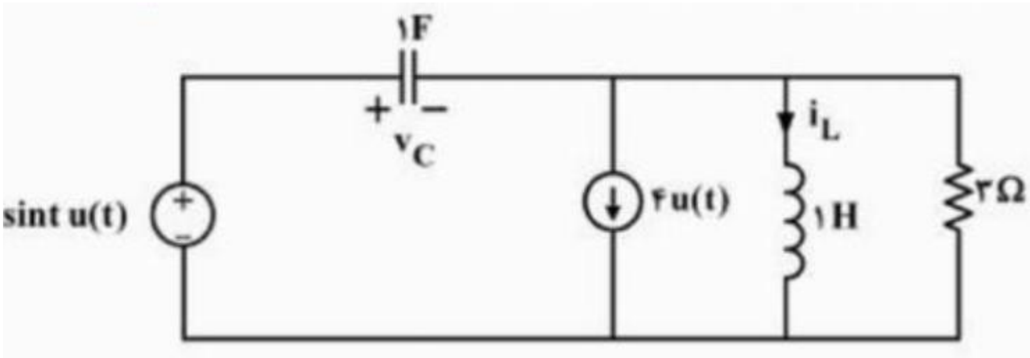


$$2\frac{d^2I_L}{dt^2}+\frac{dI_L}{dt}-\delta(t)+2I_L-20V_o+\frac{4dI_L}{dt}=0$$

$$2\frac{d^2I_L(0^+)}{dt^2}+\frac{dI_L(0^+)}{dt}-\delta(0^+)+2I_L(0^+)-20V_o(0^+)+4\frac{dI_L(0^+)}{dt}=0$$

$$2\frac{d^2I_L(0^+)}{dt^2}+\frac{1}{2}-0+2\times0-20\times1+4\times\frac{1}{2}=0 \Rightarrow \frac{d^2I_L(0^+)}{dt^2}=8/75\left(\frac{A^2}{sec^2}\right)$$

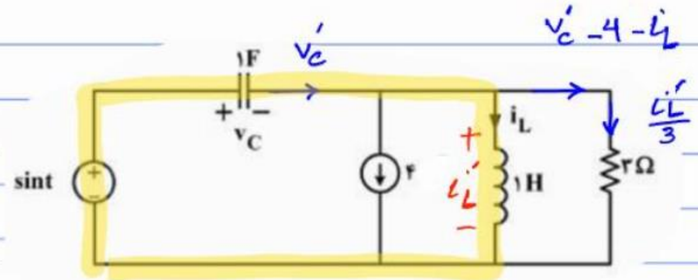
-۴



$$\begin{cases} i_L(0^+) = i_L(0^-) = 3A \\ v_c(0^-) = v_c(0^+) = 2V \end{cases}$$

مدار خاص نیست ←

$t > 0$



$$v_c' - 4 - i_L = \frac{i_L'}{3} \quad ①$$

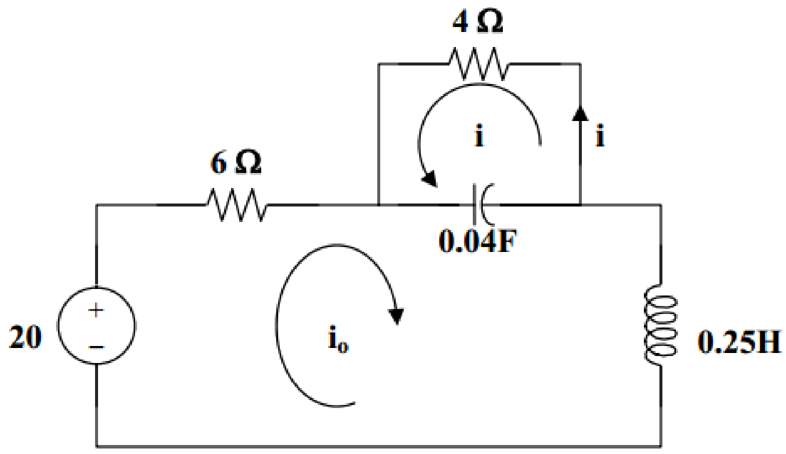
$$-\sin t + v_c + i_L' = 0 \quad ② \xrightarrow{\text{مشتق}} i_L'' = \cos t - v_c' \quad ③$$

$$t = 0^+ \text{ in } ۲ \rightarrow i_L'(0^+) = 0 - v_c(0^+) = -2 \xrightarrow[t=0^+]{\text{in } 1} v_c'(0^+) = 4 + 3 + \left(\frac{-2}{3}\right) = \frac{19}{3} \xrightarrow{\text{in } 3}$$

$$i_L''(0^+) = 1 - \frac{19}{3} = \frac{-16}{3} \; \frac{A}{s^2}$$

For $t < 0$, $i(0) = 0$ and $v(0) = 0$.

For $t > 0$, the circuit is as shown below.



Applying KVL to the larger loop,

$$-20 + 6i_o + 0.25di_o/dt + 25\int(i_o + i)dt = 0 \qquad (1)$$

For the smaller loop, $4i + 25\int(i + i_o)dt = 0$ or $\int(i + i_o)dt = -0.16i$ (2)

Taking the derivative, $4di/dt + 25(i + i_o) = 0$ or $i_o = -0.16di/dt - i$ (3)

and $di_o/dt = -0.16d^2i/dt^2 - di/dt$ (4)

From (1), (2), (3), and (4), $-20 - 0.96di/dt - 6i - 0.04d^2i/dt^2 - 0.25di/dt - 4i = 0$

Which becomes, $d^2i/dt^2 + 30.25di/dt + 250i = -500$

This leads to, $s^2 + 30.25s + 250 = 0$

$$\text{or } s_{1,2} = \frac{-30.25 \pm \sqrt{(30.25)^2 - 1000}}{2} = -15.125 \pm j4.608$$

Thus, $i(t) = I_s + e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t))$A.

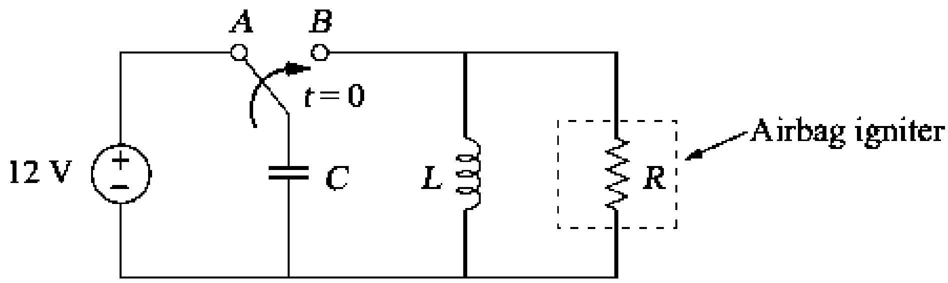
At t = 0, $i_o(0) = 0$ and $i(0) = 0 = I_s + A_1$ or $A_1 = -I_s$. As t approaches infinity, $i_o(\infty) = 20/10 = 2\text{A} = -i(\infty)$ or $i(\infty) = -2\text{A} = I_s$ and $A_1 = 2$.

In addition, from (3), we get $di(0)/dt = -6.25i_o(0) - 6.25i(0) = 0$.

$di/dt = 0 - 15.125\, e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t)) + e^{-15.125t}(-A_1 4.608\sin(4.608t) + A_2 4.608\cos(4.608t))$. At t=0, $di(0)/dt = 0 = -15.125A_1 + 4.608A_2 = -30.25 + 4.608A_2$ or $A_2 = 30.25/4.608 = 6.565$.

This leads to,

$$i(t) = \underline{\mathbf{(-2 + e^{-15.125t}(2\cos(4.608t) + 6.565\sin(4.608t))\ A}}$$



The voltage across the igniter is $v_R = v_C$ since the circuit is a parallel RLC type.

$$v_C(0) = 12, \text{ and } i_L(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2x3x1/30) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{60x10^{-3}x1/30} = 22.36$$

$\alpha < \omega_o$ produces an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm j21.794$$

$$v_C(t) = e^{-5t}(A\cos 21.794t + B\sin 21.794t) \quad (1)$$

$$v_C(0) = 12 = A$$

$$dv_C/dt = -5[(A\cos 21.794t + B\sin 21.794t)e^{-5t}]$$

$$+ 21.794[(-A\sin 21.794t + B\cos 21.794t)e^{-5t}] \quad (2)$$

$$dv_C(0)/dt = -5A + 21.794B$$

But, $dv_C(0)/dt = -[v_C(0) + Ri_L(0)]/(RC) = -(12 + 0)/(1/10) = -120$

Hence, $-120 = -5A + 21.794B$, leads to $B \; (5x12 - 120)/21.794 = -2.753$

At the peak value, $dv_C(t_o)/dt = 0$, i.e.,

$$0 = A + B\tan 21.794t_o + (A21.794/5)\tan 21.794t_o - 21.794B/5$$

$$(B + A21.794/5)\tan 21.794t_o = (21.794B/5) - A$$

$$\tan 21.794t_o = [(21.794B/5) - A]/(B + A21.794/5) = -24/49.55 = -0.484$$

$$\text{Therefore,} \quad 21.7945t_o = |-0.451|$$

$$t_o = |-0.451|/21.794 = \underline{\mathbf{20.68\ ms}}$$