طراحی الگوریتم ها

جلسه هجدهم: شاخه و حد

سجاد شیرعلی شهرضا

بهار 1402

یکشنبه، 24 اردیبهشت 1402

اطلاع رسانی

- امتحانک سوم: یکشنبه آینده، 31 اردیبهشت
- تمدید تمرین سوم تا صبح پنجشنبه، 28 اردیبهشت

روش شاخه و حد

Situation

- Sometimes it is not easy to design a DP or greedy solution
- Checking all possible answers is not feasible either

Situation -> Idea

- Sometimes it is not easy to design a DP or greedy solution
- Checking all possible answers is not feasible either
- Idea: stop exploring a path if we know it would not give us a better answer
  - Keep an estimate of how good the answer will be
  - Continue the path with the highest possible answer
  - Stop a path if it cannot provide a better solution

0/1 Knapsack

- n items
  - Each has a weight w_i and value v_i
- A knapsack (i.e., bag) with capacity W
- Goal: fill the bag such that
  - Total weight of items is less than or equal to W
    - $\Sigma w_i \leq W$
  - The value of selected items is maximised
    - Σv_i is maximised

Capacity: **16**

Item:	🥑	🧲	🍇	🐨
Weight:	2	5	10	5
Value:	40	30	50	10

Greedy Approach

- Sort items based on unit value
- Start with item with highest unit value
- Add item if it fits in the bag

Capacity: **16**

Item:	🥑	🧲	🍇	🐨
Weight:	**2**	**5**	**10**	**5**
Value:	**40**	**30**	**50**	**10**
Unit value:	**20**	**6**	**5**	**2**

Greedy Approach - Not Working!

- Sort items based on unit value
- Start with item with highest unit value
- Add item if it fits in the bag
- Final answer is not necessarily the best one
  - Will select avocado, magnet, and quala with value 80
  - Best answer: select avocado and grape with value 90

Capacity: **16**

Item:	🥑	🧲	🍇	🐨
Weight:	**2**	**5**	**10**	**5**
Value:	**40**	**30**	**50**	**10**
Unit value:	**20**	**6**	**5**	**2**

Branch and Bound Algorithm

- Start with an empty answer as partial answer
- Select the partial answer with highest bound
- Expand the partial answer by deciding whether to pick the next item
  - If selecting the item, update weight and value
- Calculate an upper bound for the answer
  - Fill the remaining capacity with highest remaining items in terms value/unit
- Continue while there is a partial answer that is not eliminated

Capacity: **16**

Item:	🥑	🧲	🍇	🐨
Weight:	**2**	**5**	**10**	**5**
Value:	**40**	**30**	**50**	**10**
Unit value:	**20**	**6**	**5**	**2**

0/1 Knapsack with Branch and Bound

- Start with an empty answer

```
weight = ?
value  = ?
bound  = ?
```

Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Start with an empty answer
- Calculate upper bound for the value
  - bound = 16 * 20 = 320

```
weight = 0
value  = 0
bound  = 320
```

Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 320

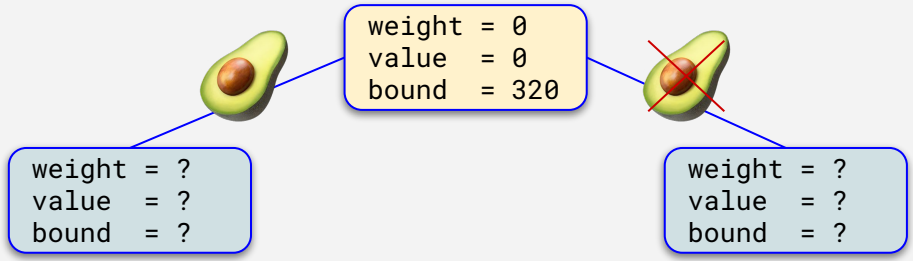
```
weight = 0
value  = 0
bound  = 320
```

Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 320

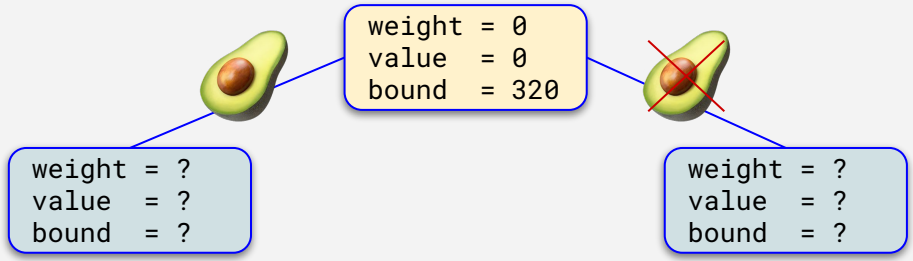



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Avocado

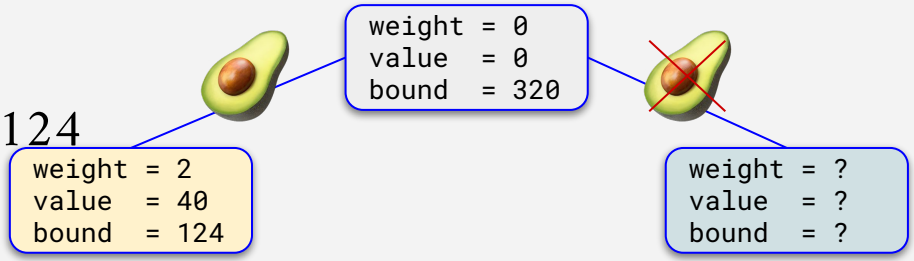



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Avocado
  - Found an answer with value 0 + 40 = 40
  - weight = 0 + 2 = 2
  - bound = 40 + (16-2) * 6 = 124

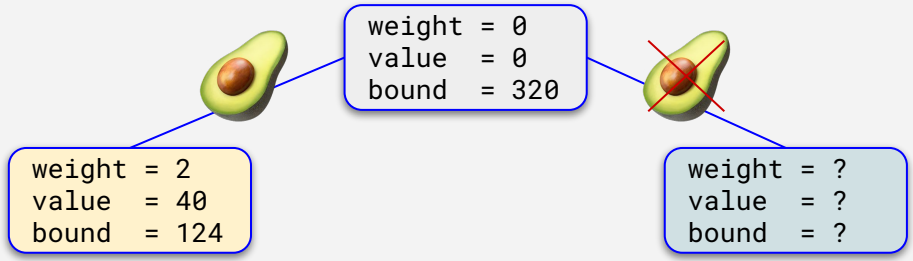



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Avocado

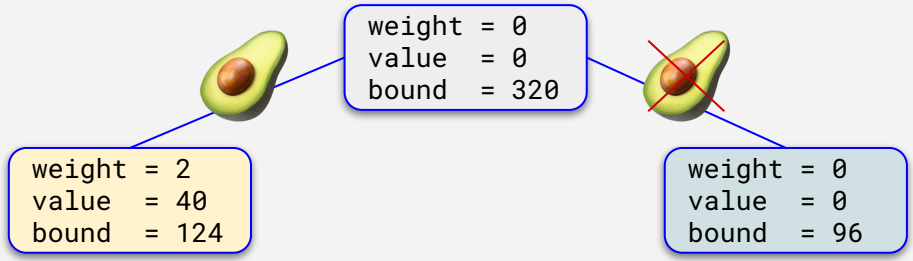



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Avocado
  - value = 0 + 0 = 0
  - wight = 0 + 0 = 0
  - bound = 16 * 6 = 96

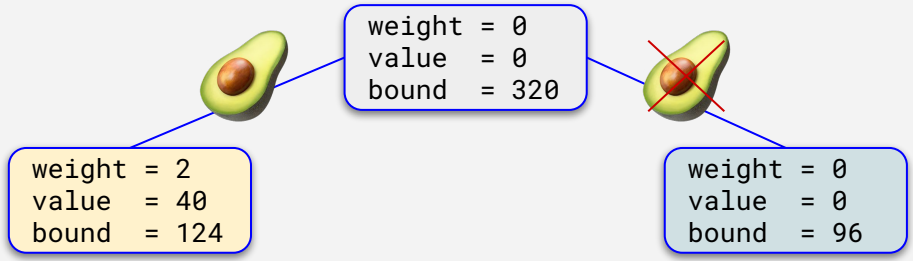


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 124

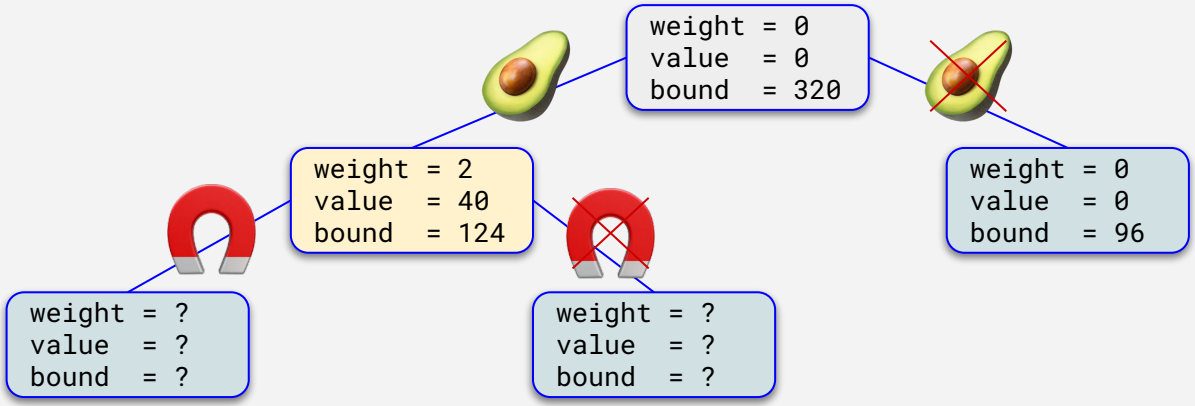



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 124

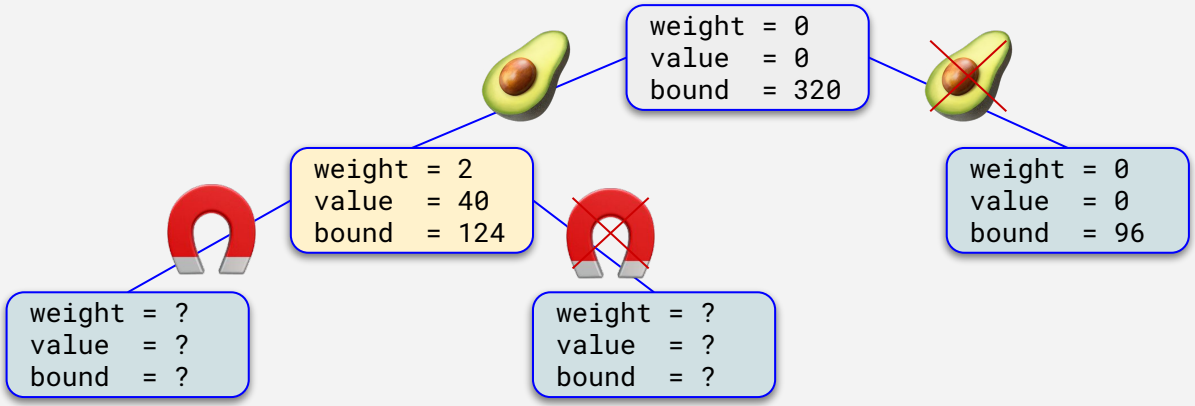



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Magnet

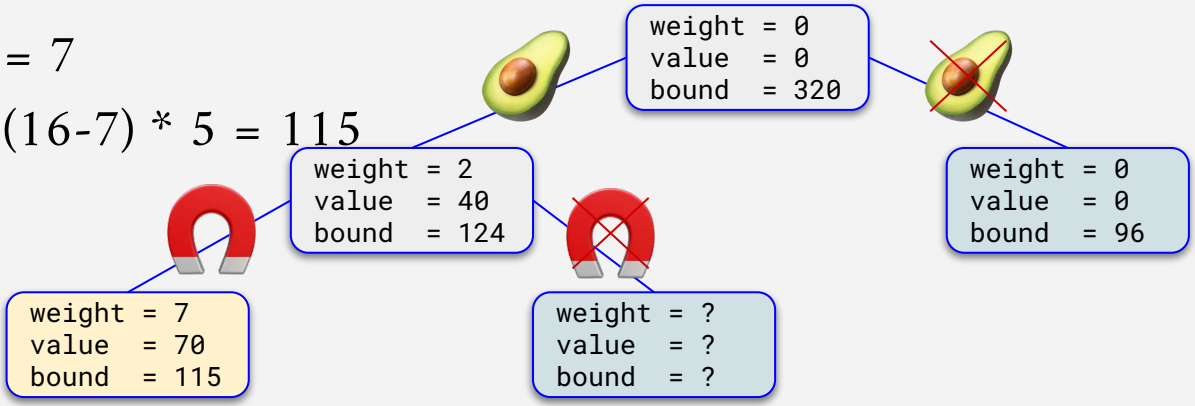



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Magnet
  - Found an answer with value 40 + 30 = 70
  - wight = 2 + 5 = 7
  - bound = 70 + (16-7) * 5 = 115

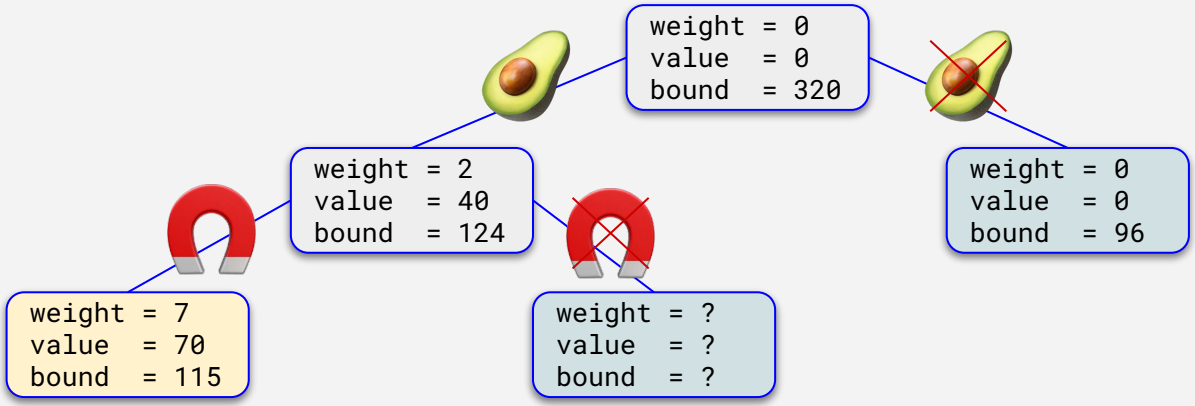



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Magnet

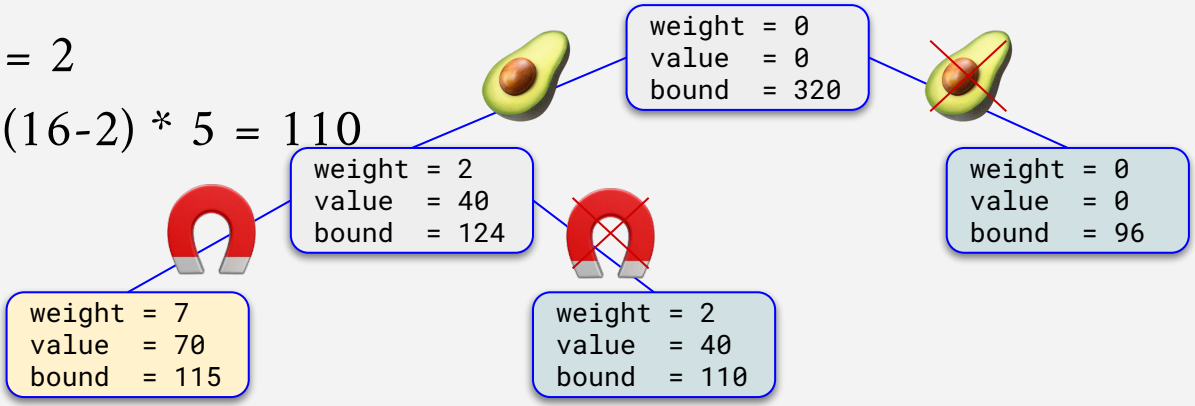


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Magnet
  - value = 40 + 0 = 40
  - wight = 2 + 0 = 2
  - bound = 40 + (16-2) * 5 = 110

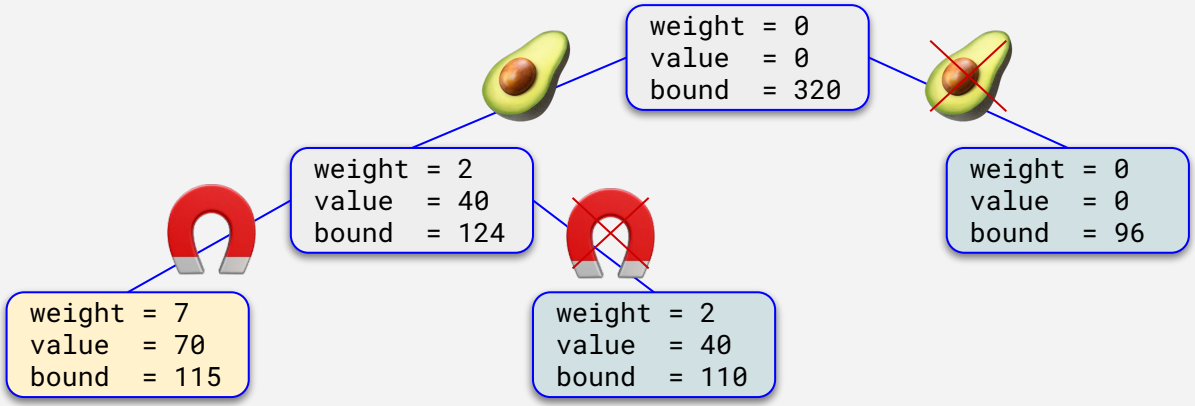



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 115

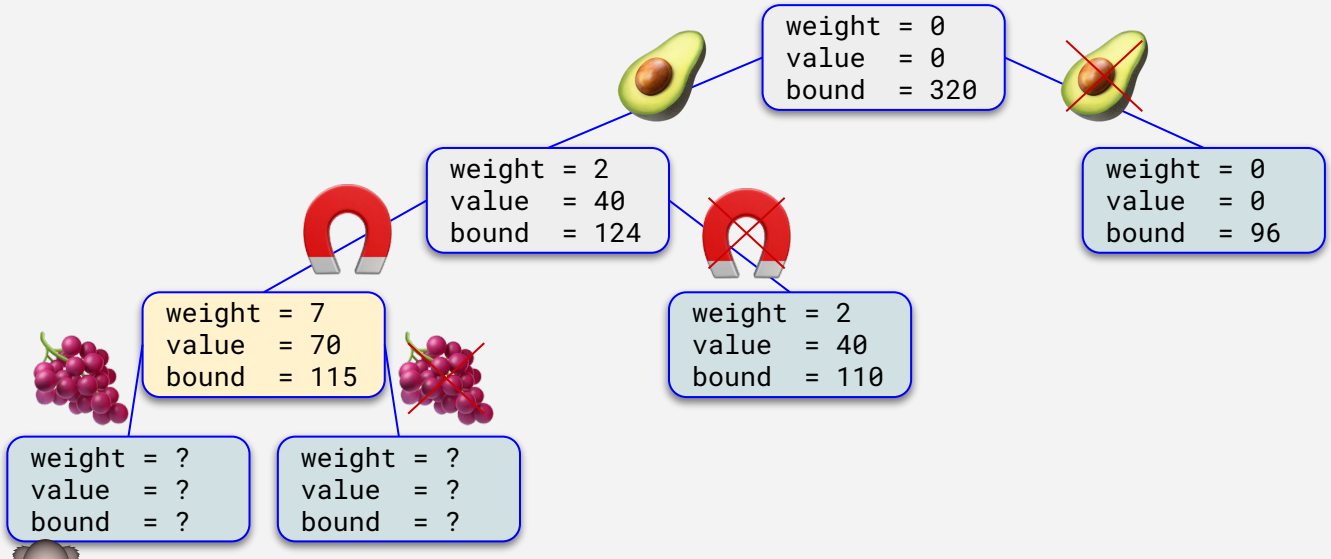



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 115

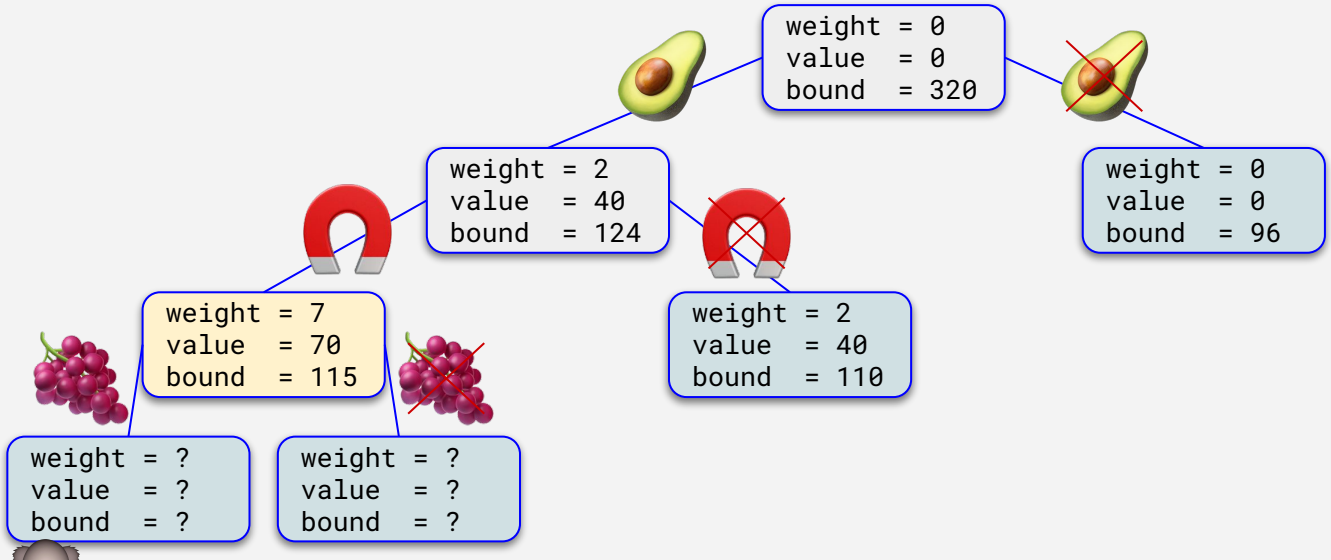


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Grape

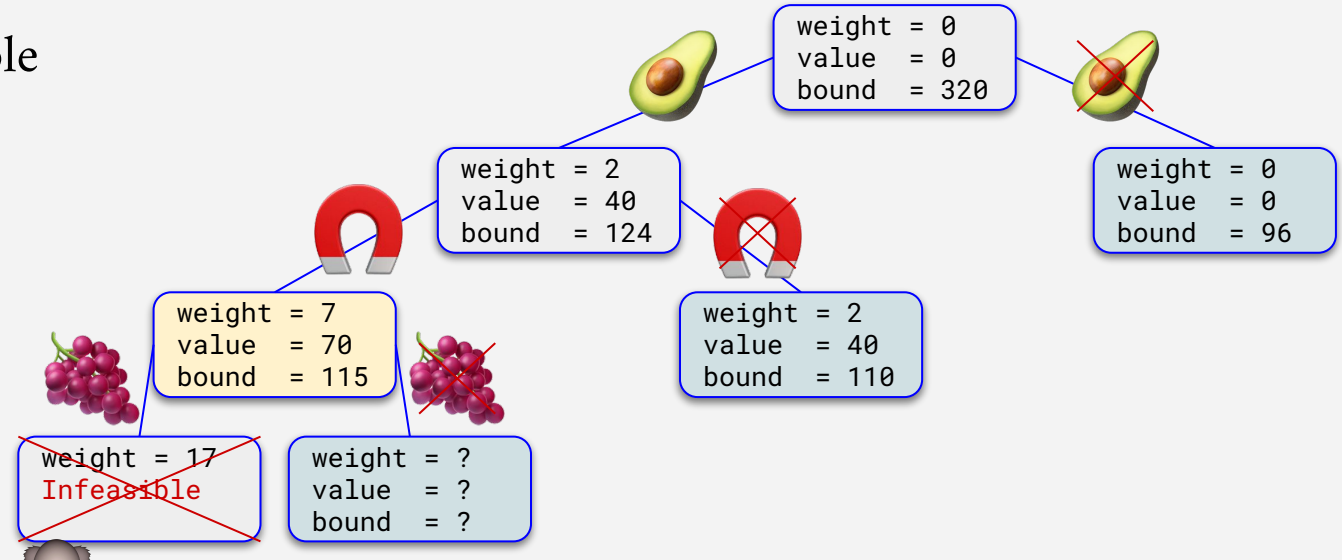


Capacity: **16**

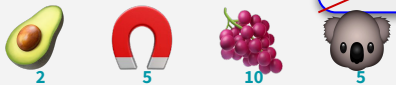
Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Grape
  - wight = 7 + 10 = 17 > 16
  - Infeasible

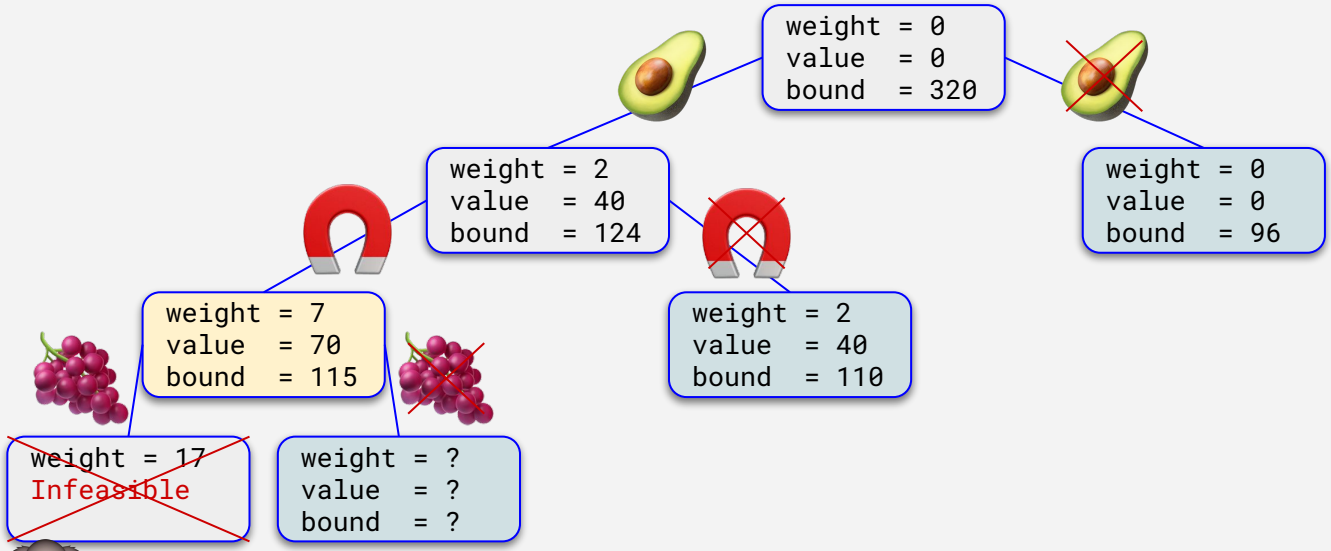



Capacity: **16**



0/1 Knapsack with Branch and Bound

- Do not pick Grape

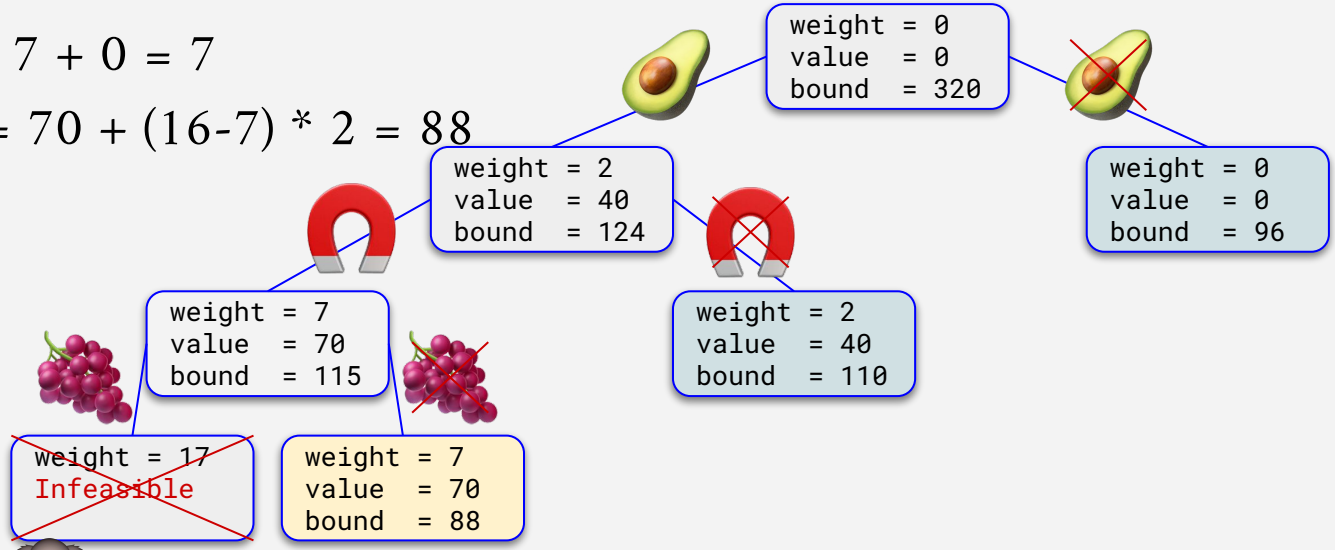


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Grape
  - value = 70 + 0 = 70
  - wight = 7 + 0 = 7
  - bound = 70 + (16-7) * 2 = 88

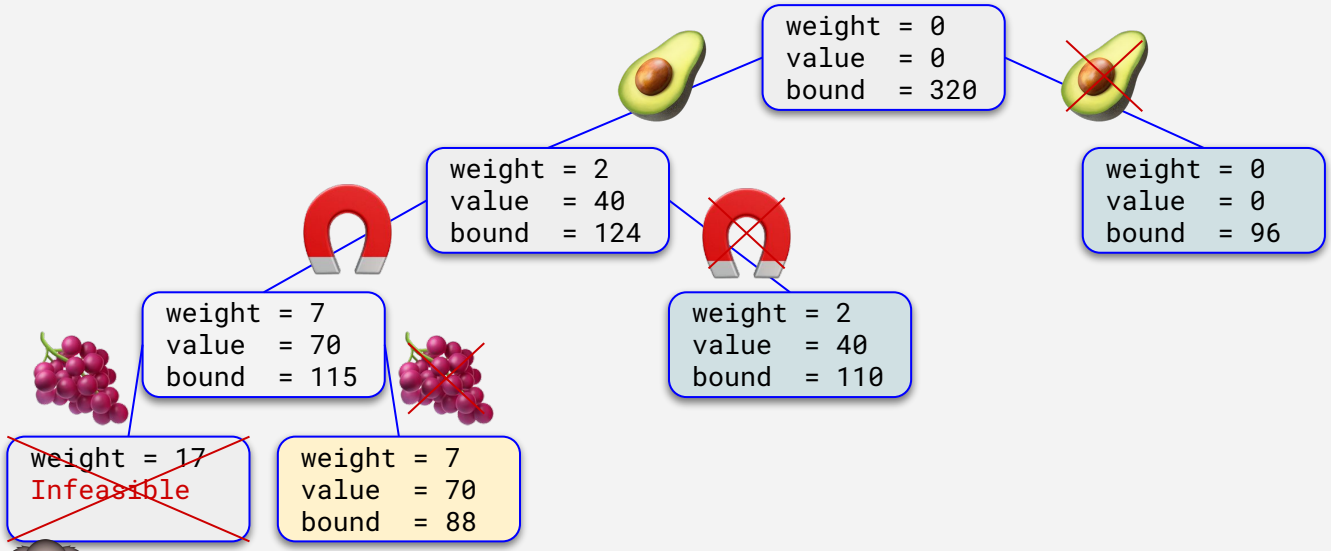


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 110

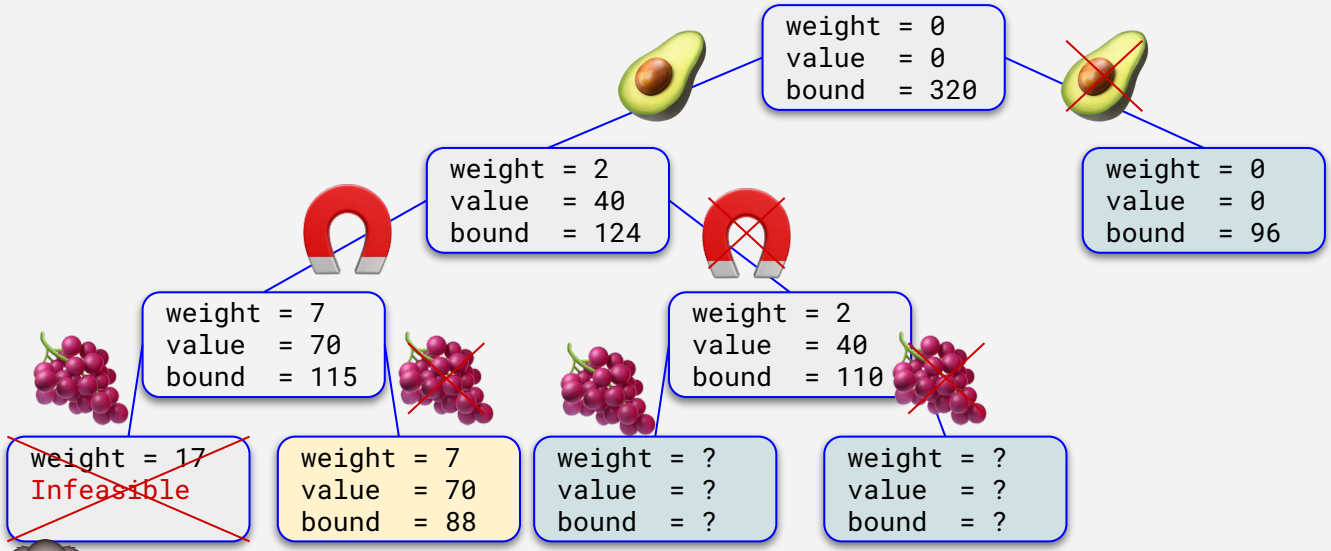


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 110

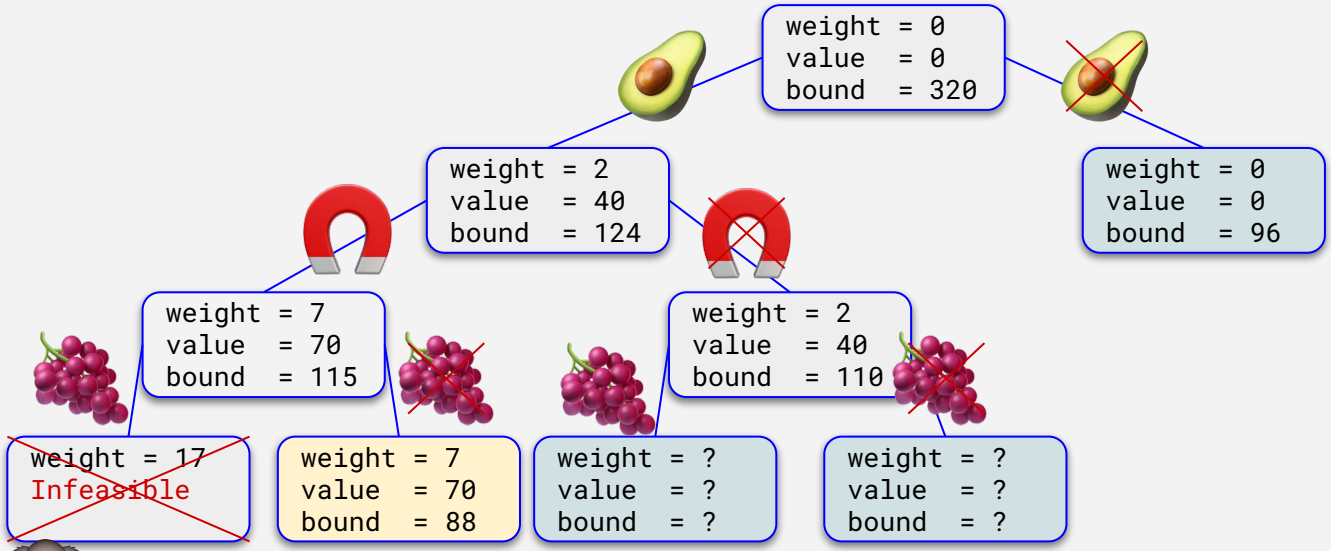


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Grape

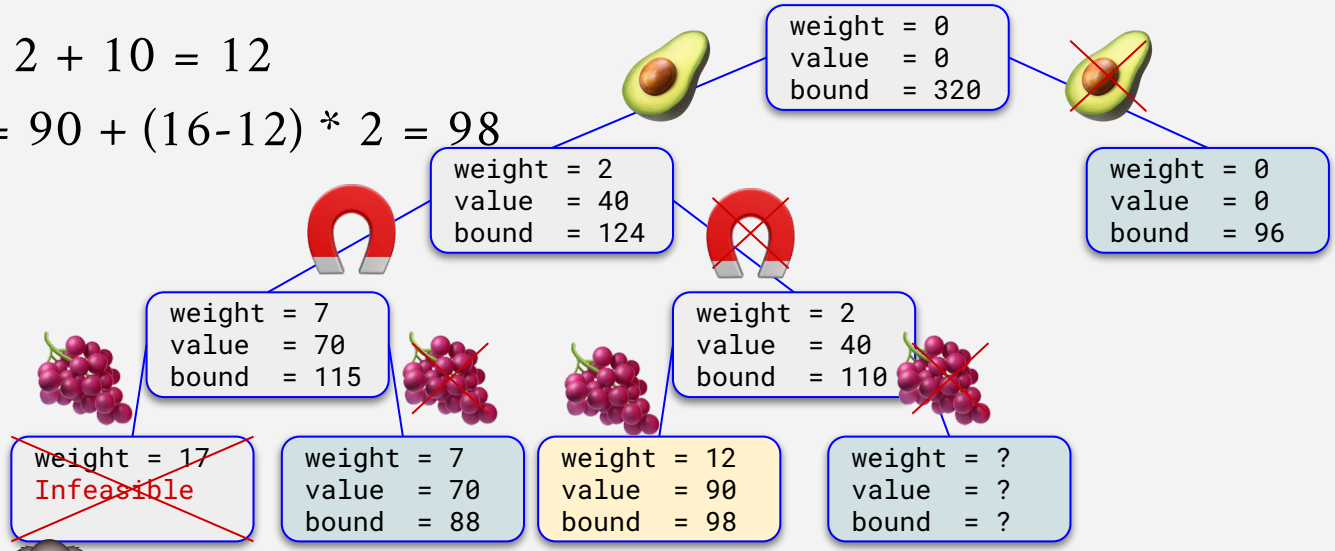


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Grape
  - Found an answer with value 40 + 50 = 90
  - wight = 2 + 10 = 12
  - bound = 90 + (16-12) * 2 = 98

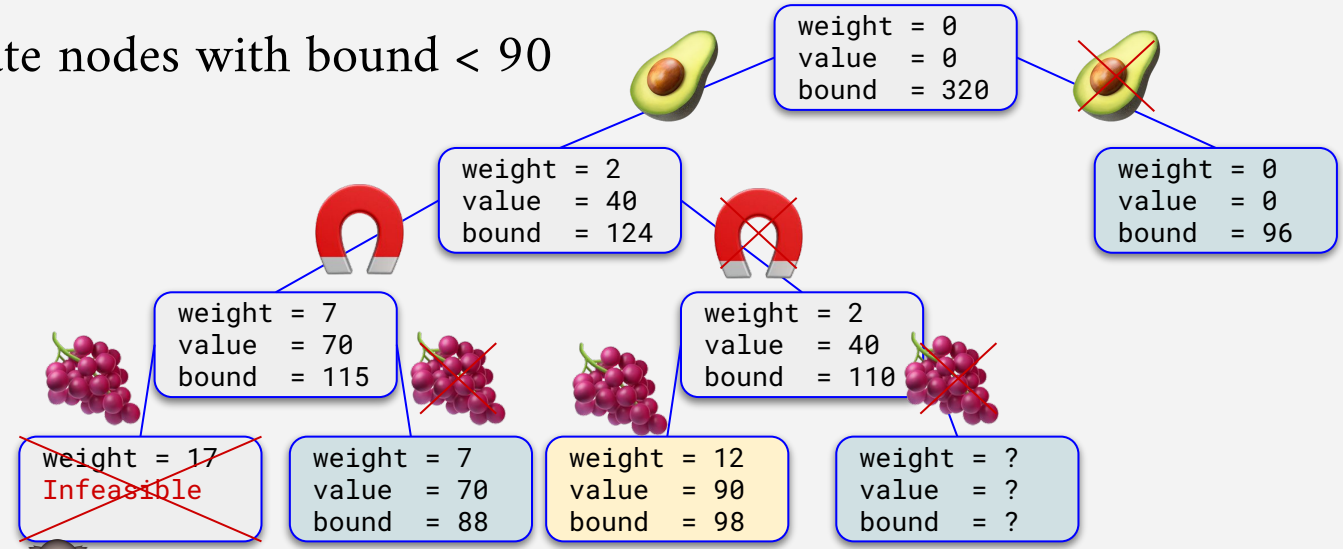


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Grape
  - Found an answer with value 90
  - Eliminate nodes with bound < 90

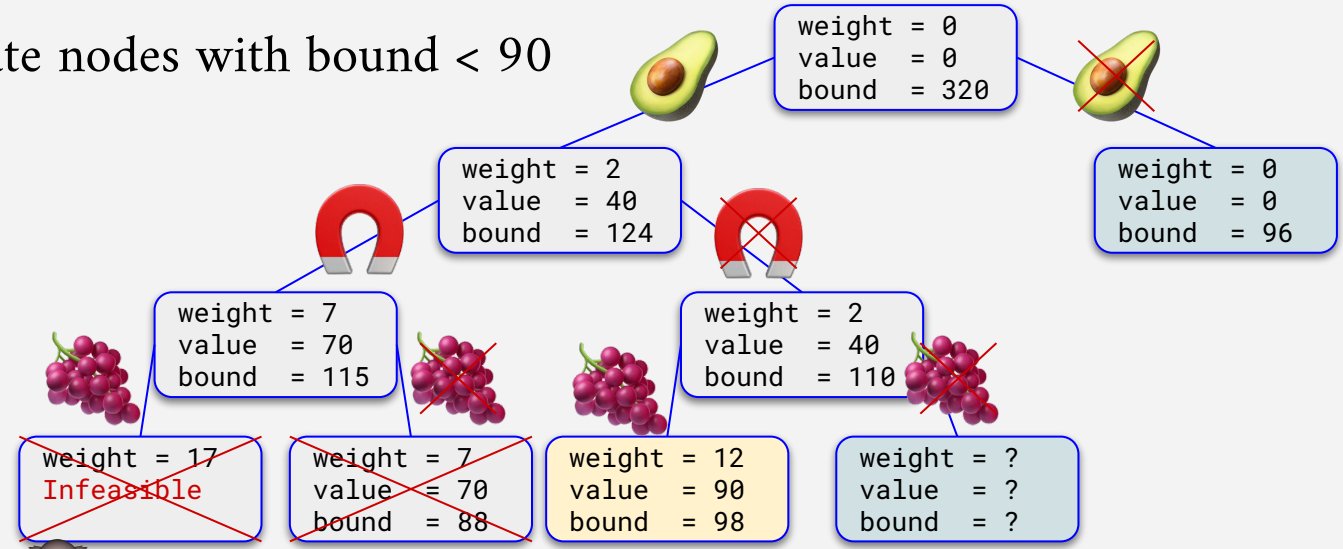


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Grape
  - Found an answer with value 90
  - Eliminate nodes with bound < 90

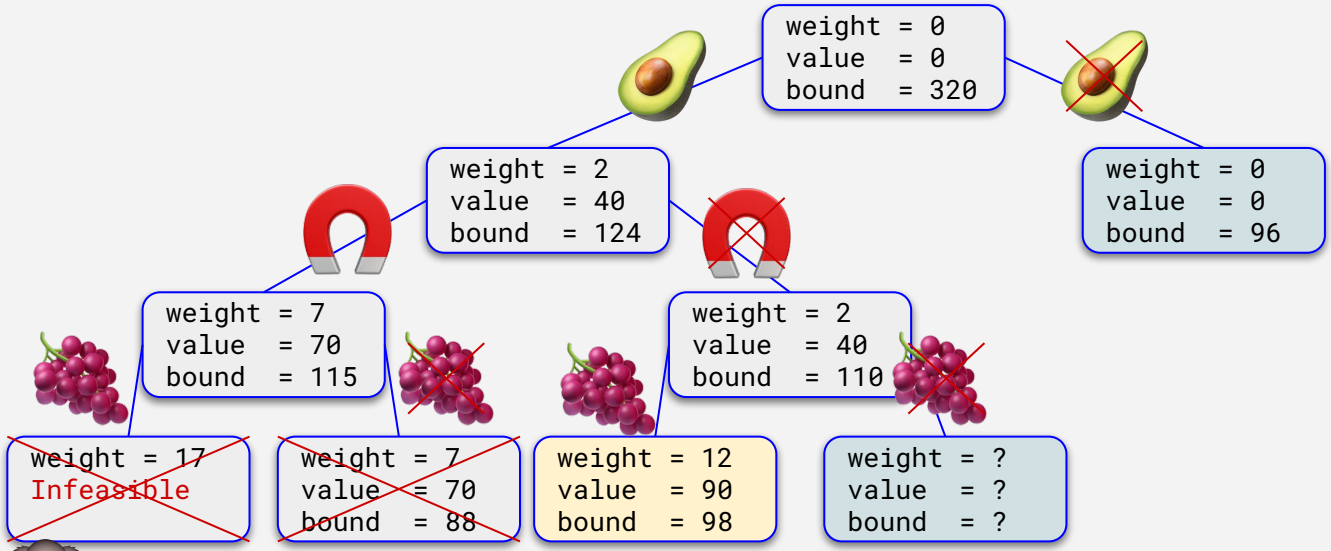


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Grape

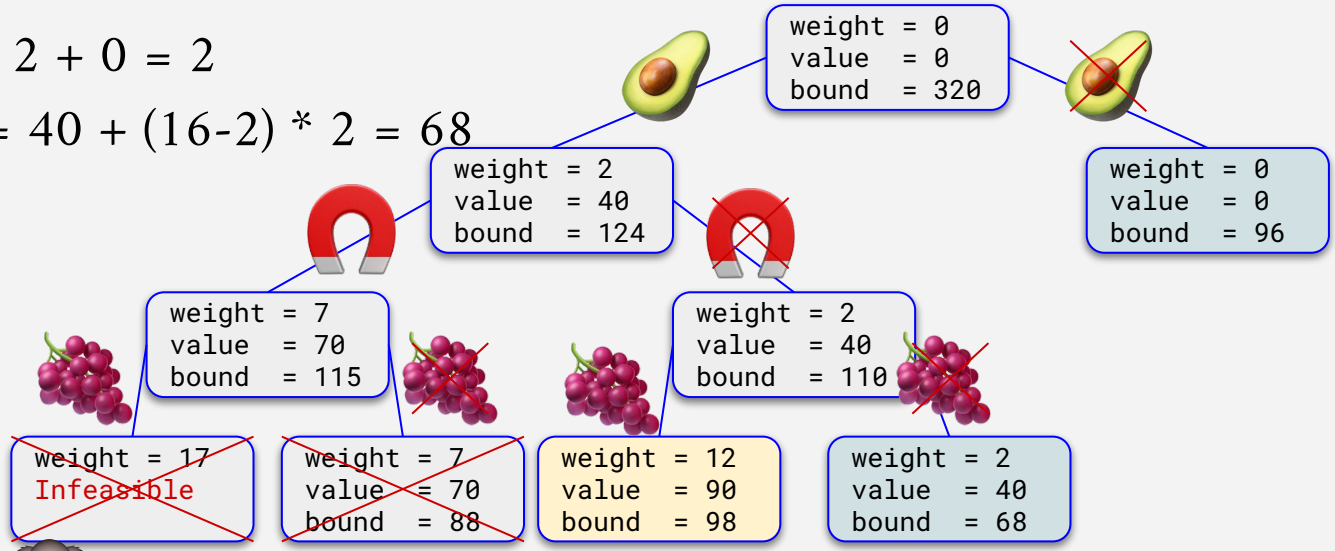


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Grape
  - value = 40 + 0 = 40
  - wight = 2 + 0 = 2
  - bound = 40 + (16-2) * 2 = 68

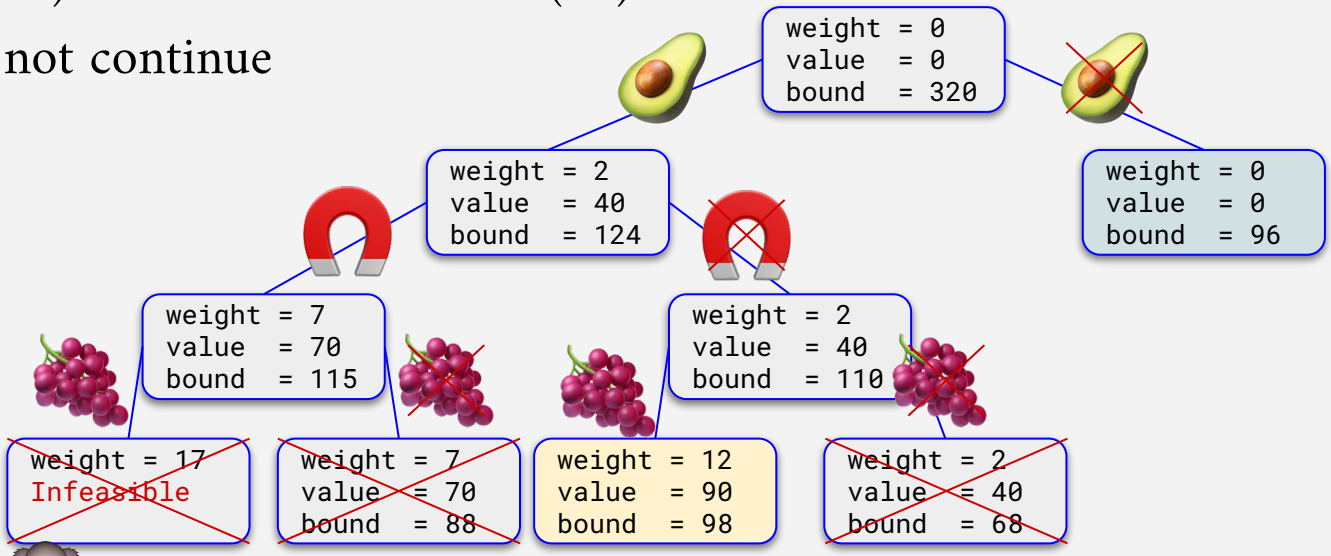


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Grape
  - Bound (68) < best known answer (90)
    - Do not continue

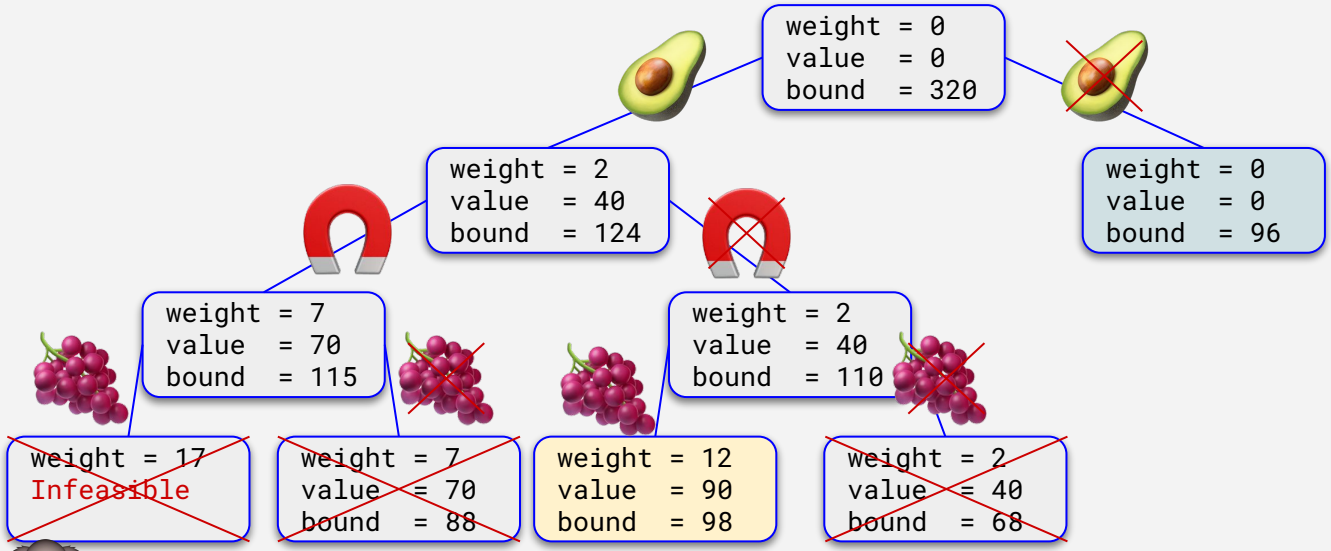


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand the node with bound 98

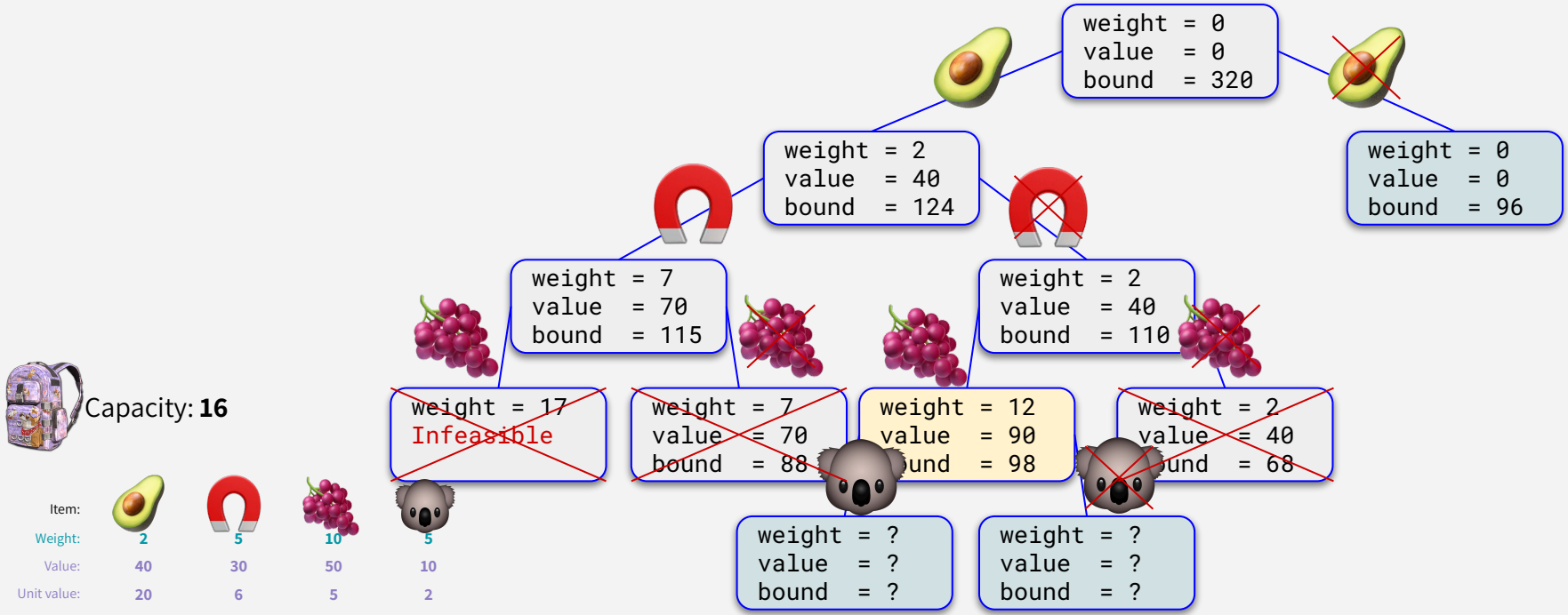


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

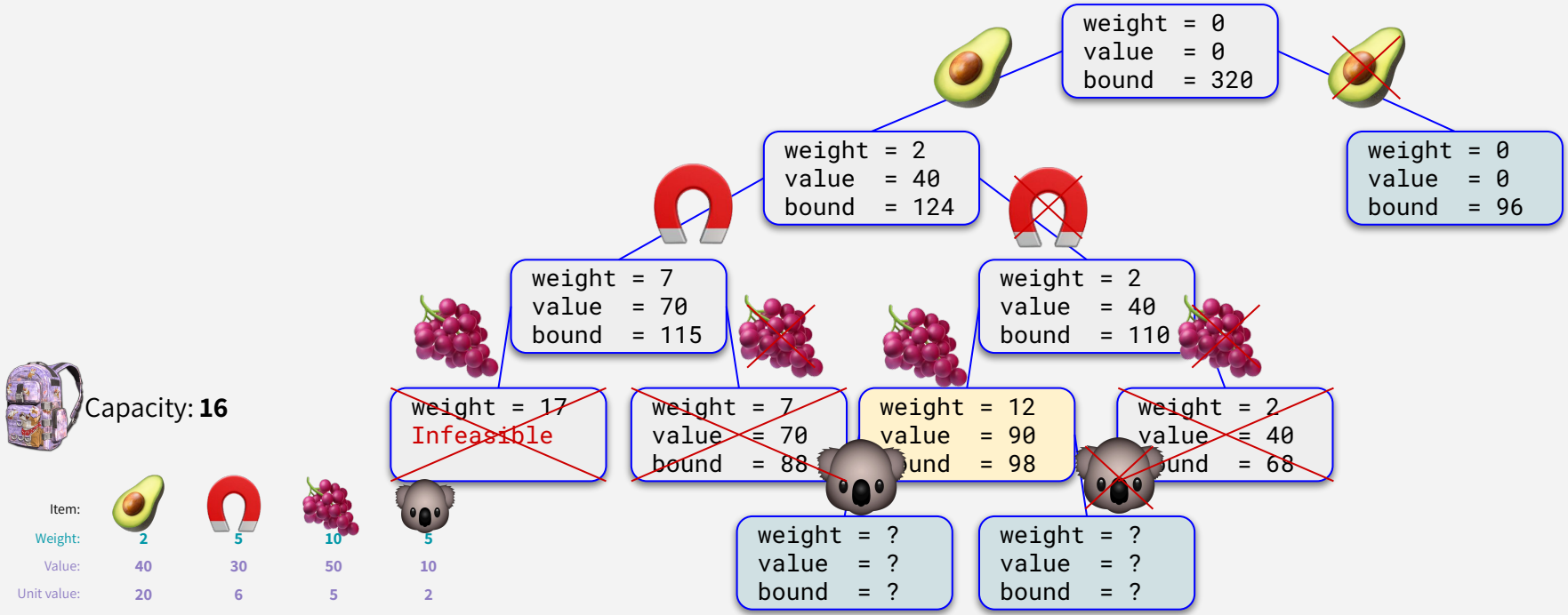
0/1 Knapsack with Branch and Bound

- Expand the node with bound 98



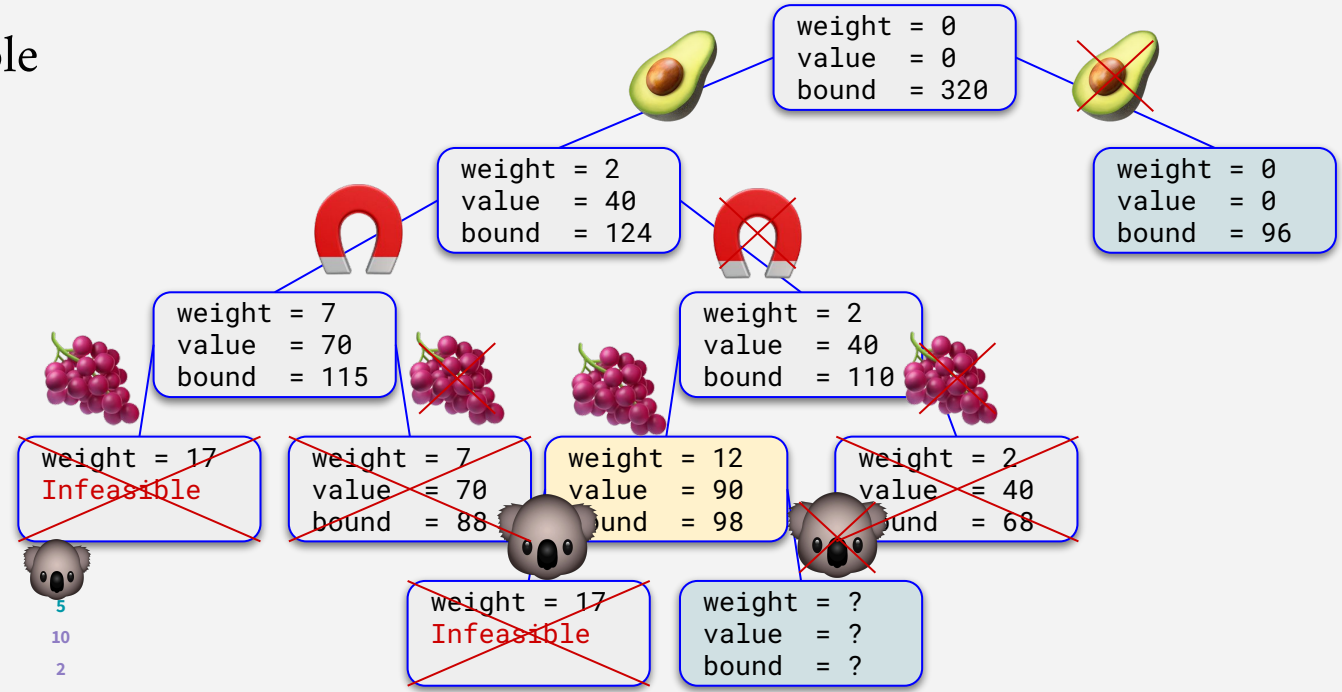
0/1 Knapsack with Branch and Bound

- Pick Quala



0/1 Knapsack with Branch and Bound

- Pick Quala
  - wight = 12 + 5 = 17 > 16
  - Infeasible

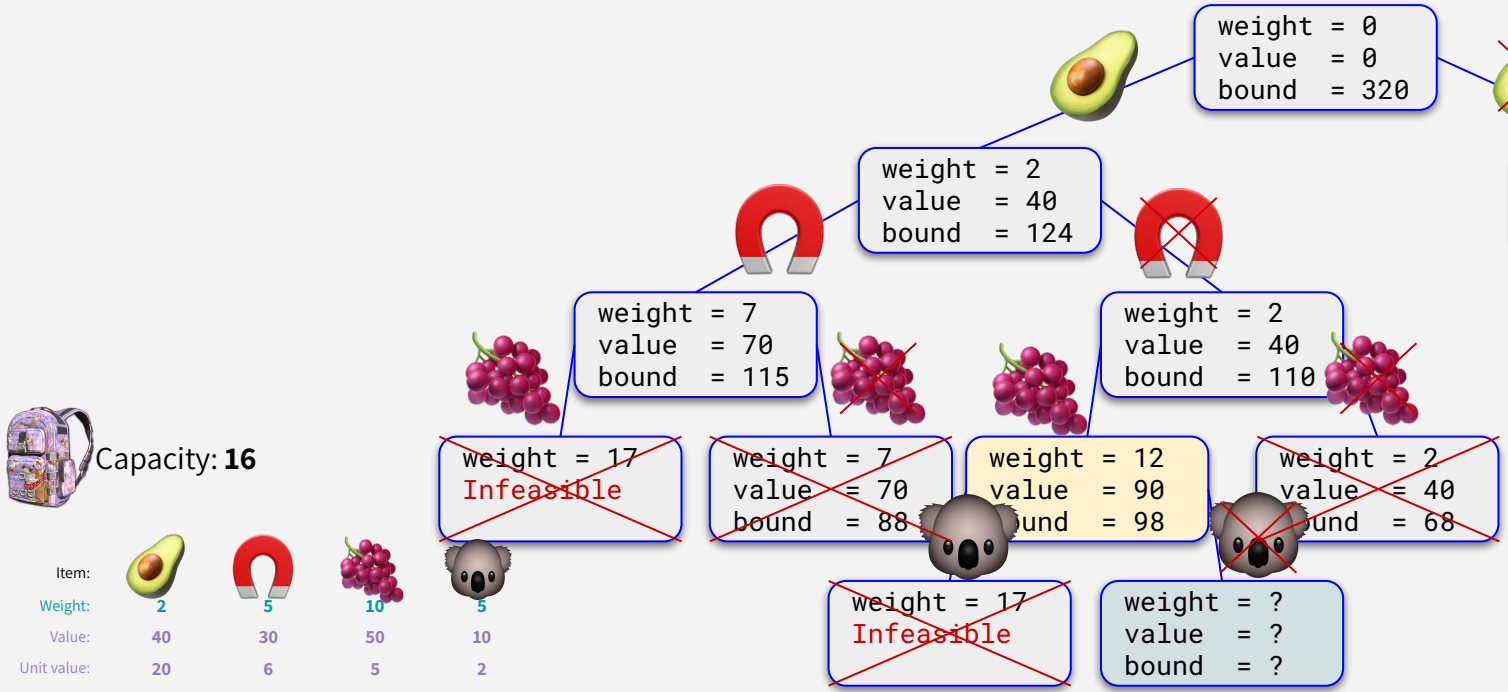


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

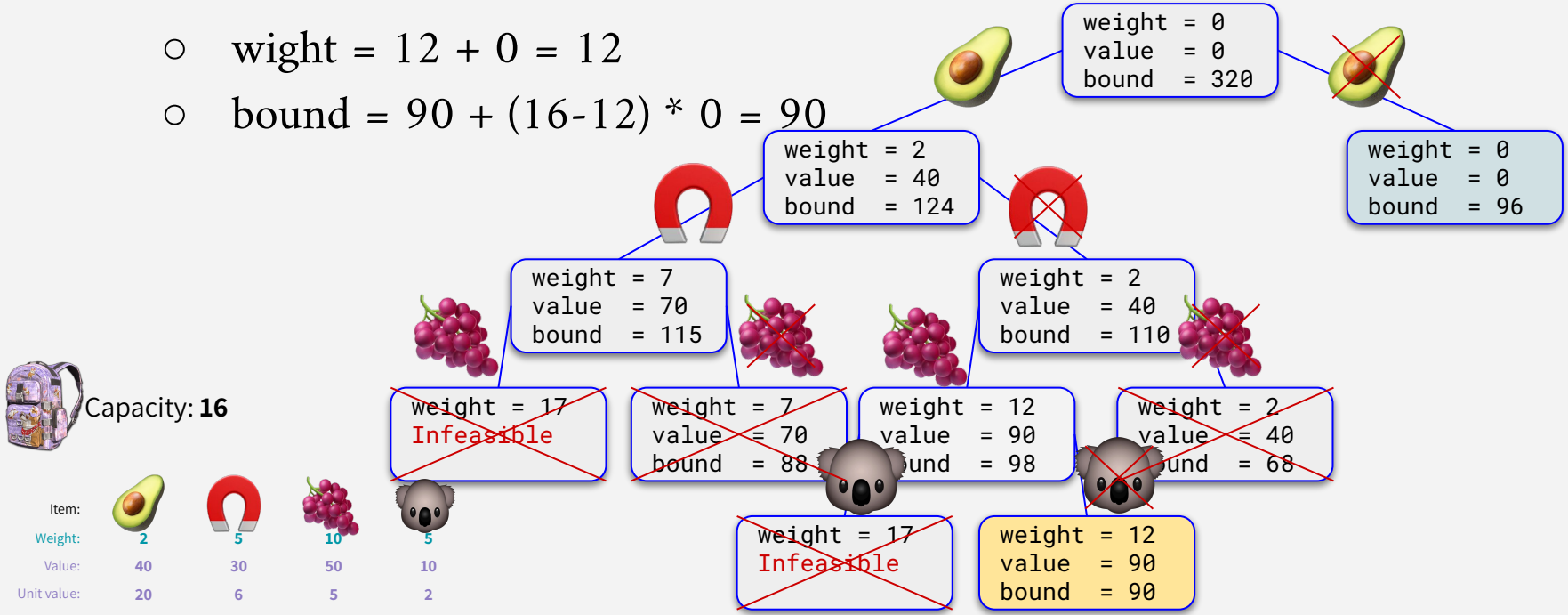
0/1 Knapsack with Branch and Bound

- Do not pick Quala



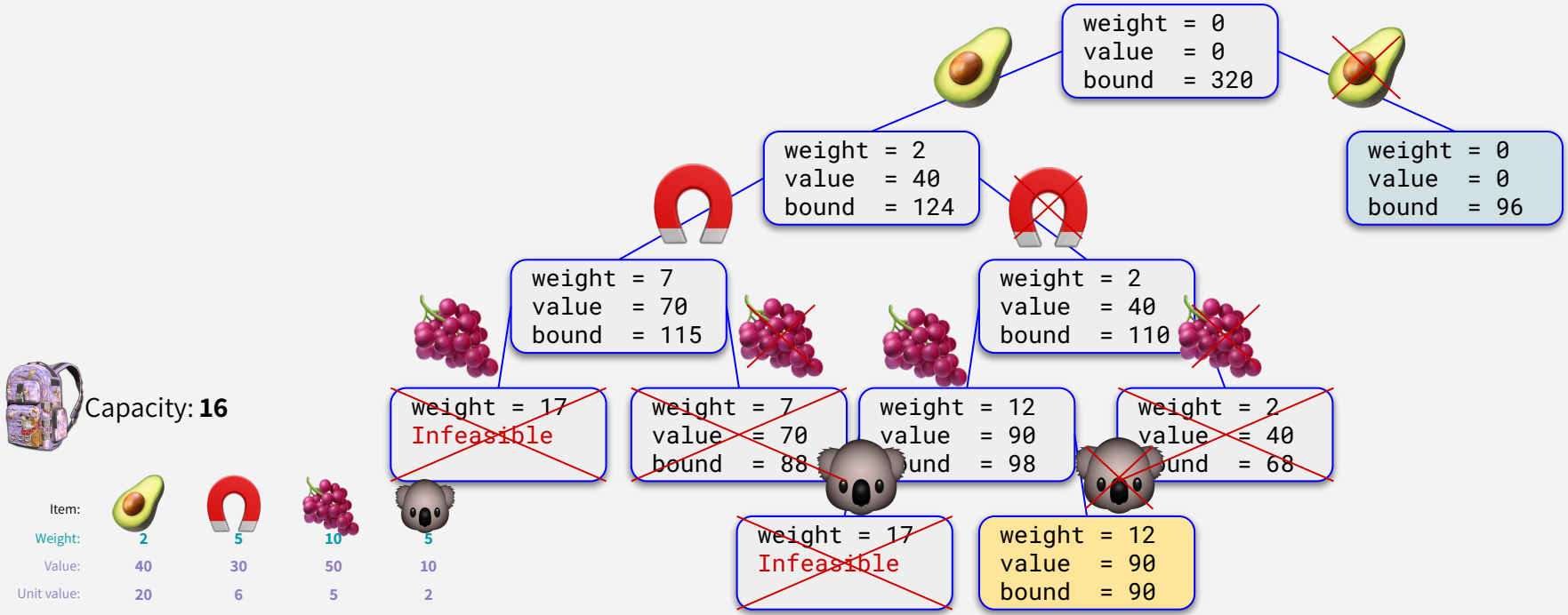
0/1 Knapsack with Branch and Bound

- Do not pick Quala
  - Found a final answer with value 90 + 0 = 90
  - wight = 12 + 0 = 12
  - bound = 90 + (16-12) * 0 = 90



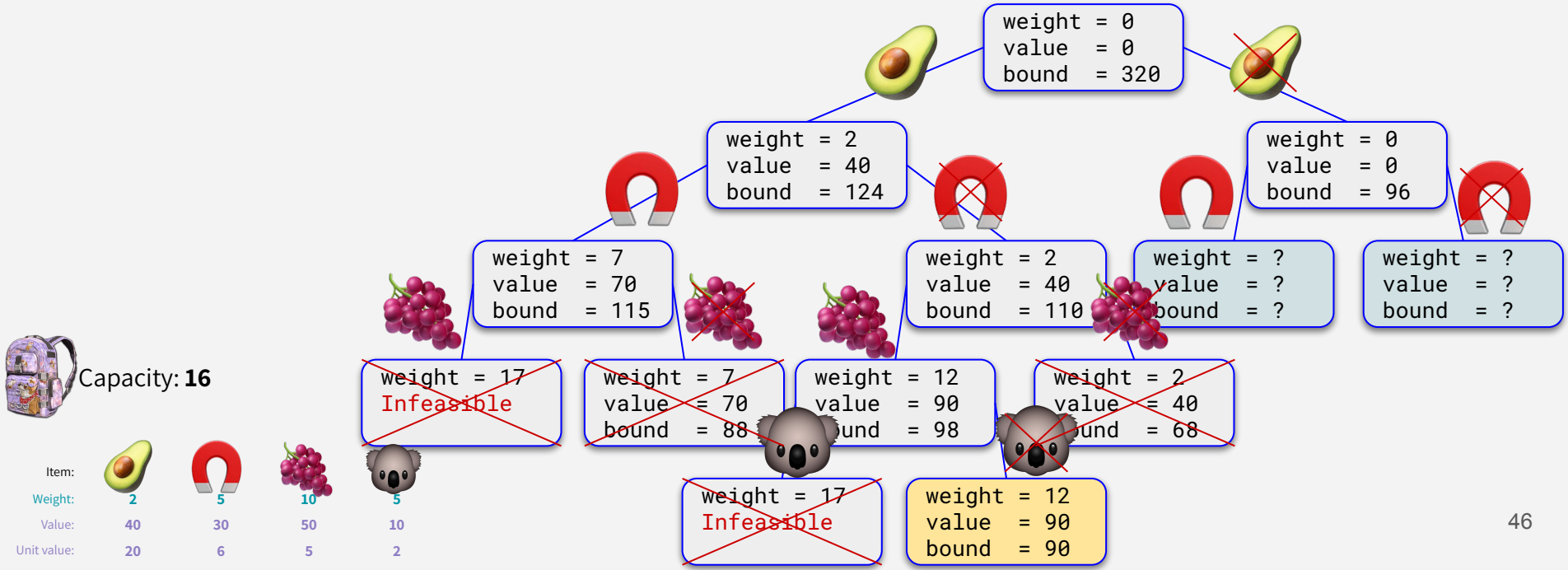
0/1 Knapsack with Branch and Bound

- Expand node with bound 96



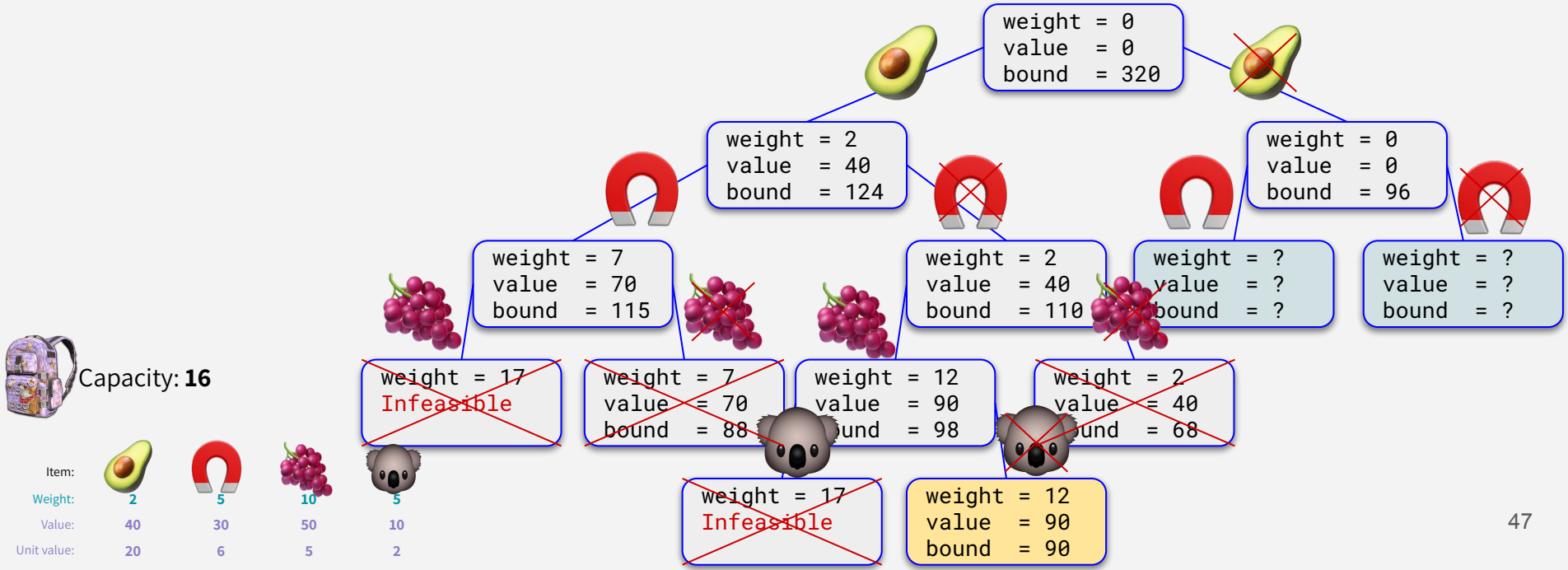
0/1 Knapsack with Branch and Bound

- Expand node with bound 96



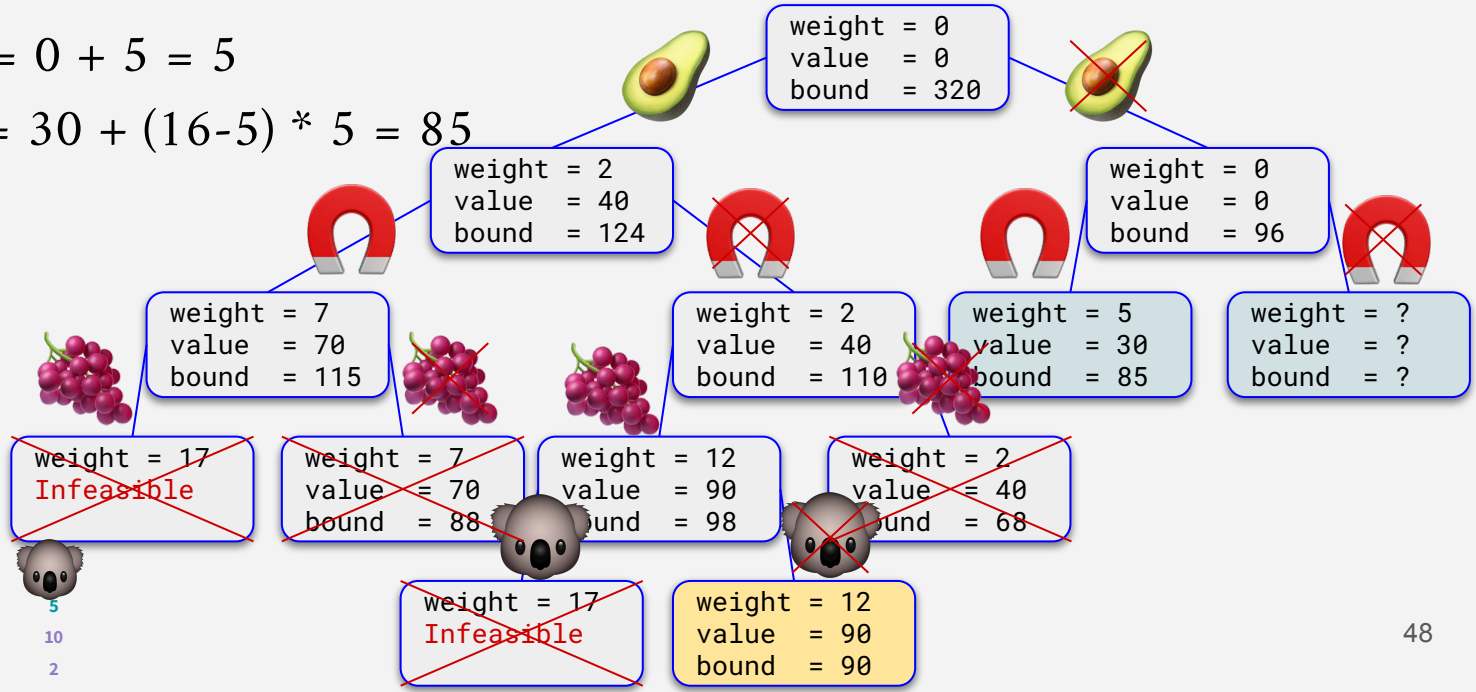
0/1 Knapsack with Branch and Bound

- Pick Magnet



0/1 Knapsack with Branch and Bound

- Pick Magnet
  - value = 0 + 30 = 30
  - weight = 0 + 5 = 5
  - bound = 30 + (16-5) * 5 = 85

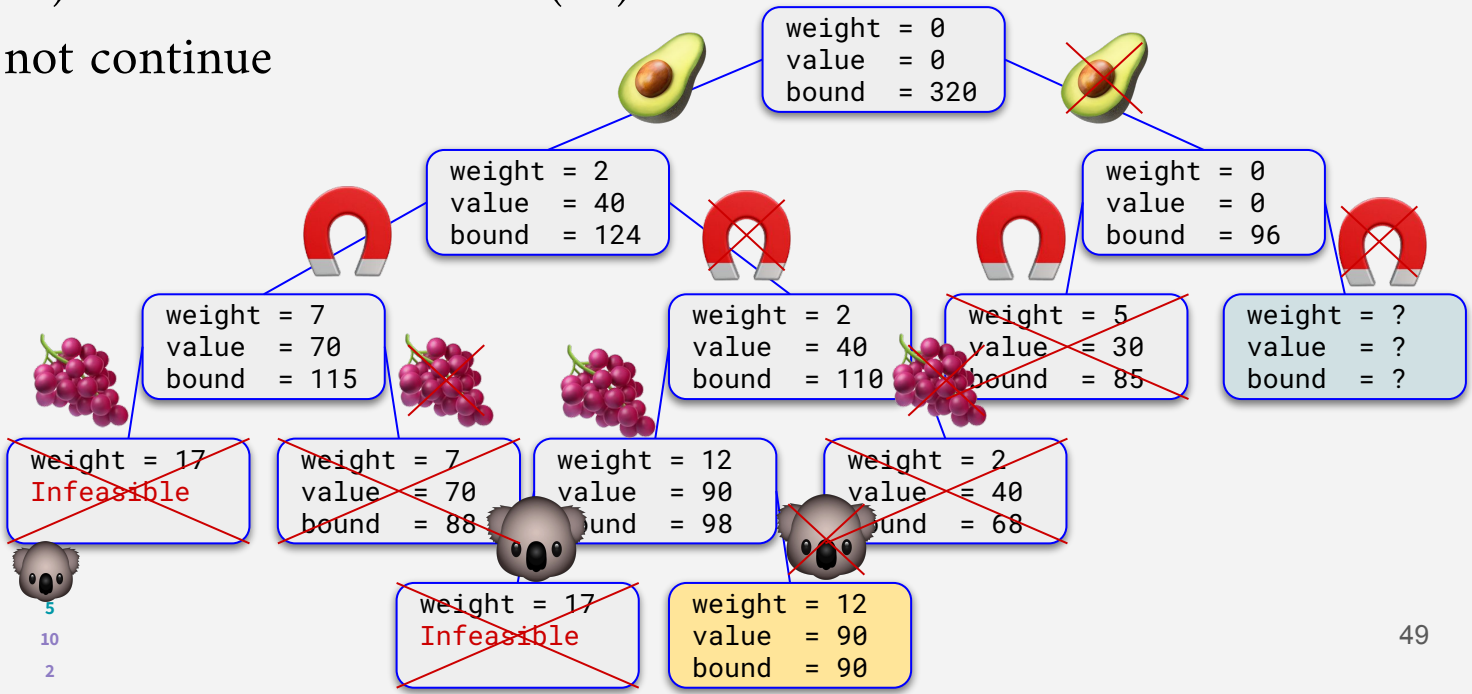


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Magnet
  - Bound (85) < best known answer (90)
    - Do not continue

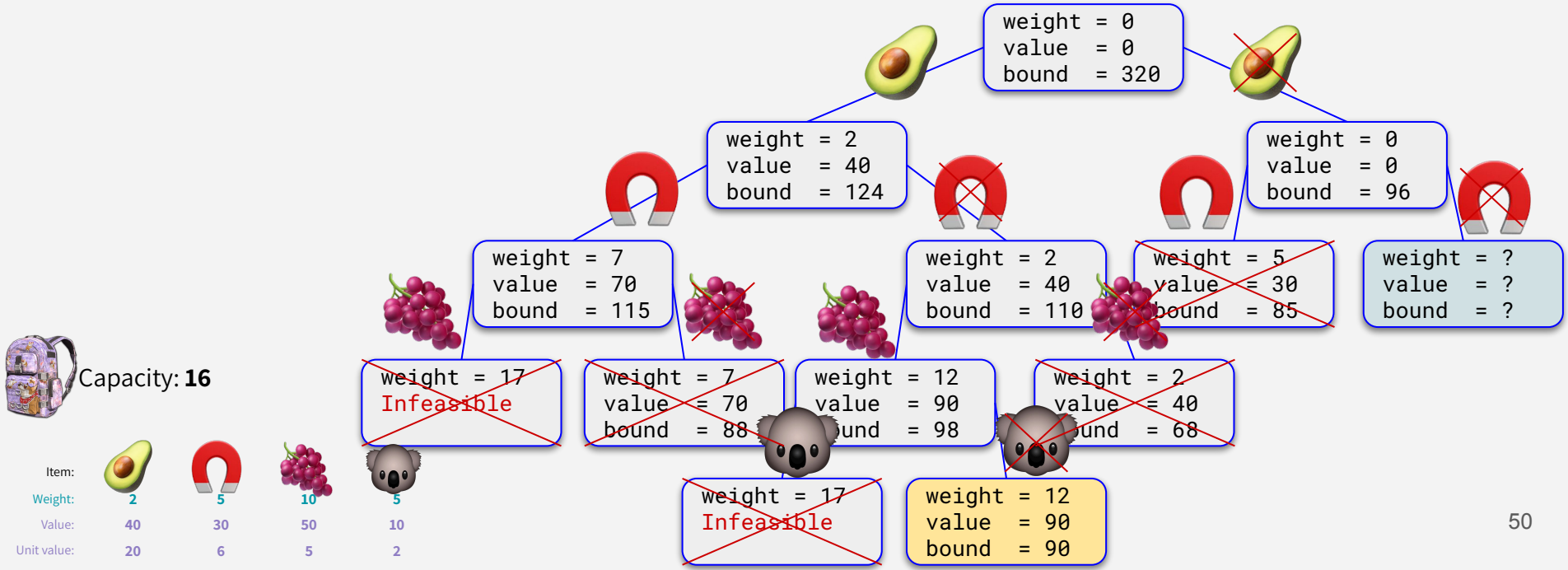



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

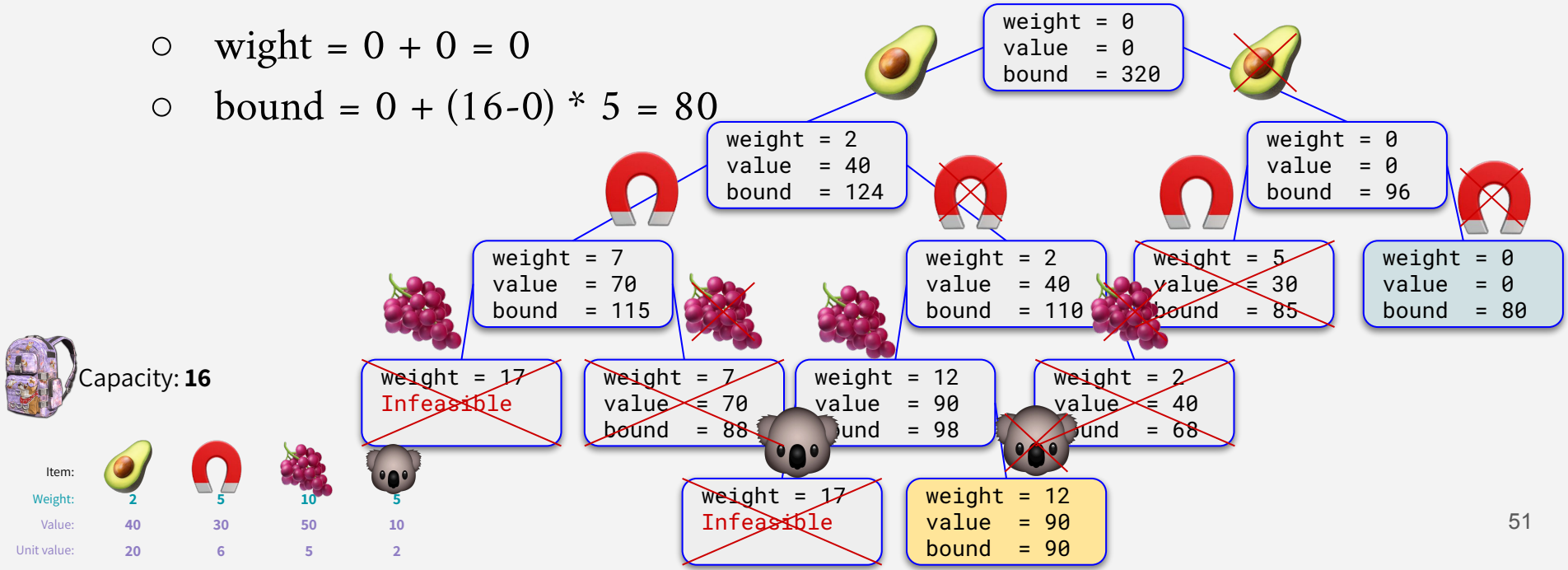
0/1 Knapsack with Branch and Bound

- Do not pick Magnet



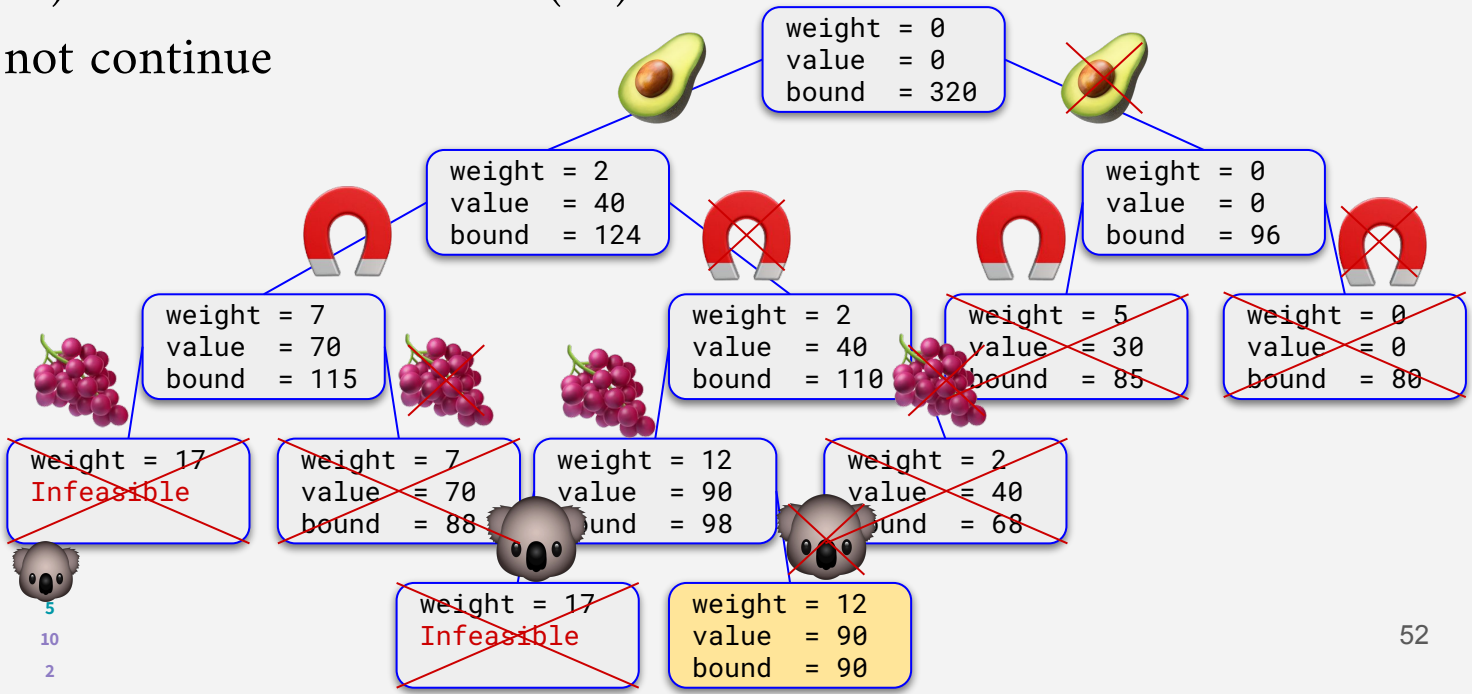
0/1 Knapsack with Branch and Bound

- Do not pick Magnet
  - value = 0 + 0 = 0
  - wight = 0 + 0 = 0
  - bound = 0 + (16-0) * 5 = 80

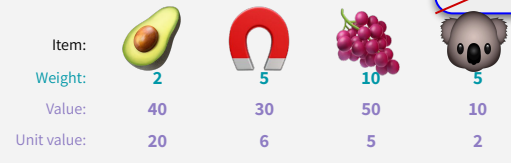


0/1 Knapsack with Branch and Bound

- Do not pick Magnet
  - Bound (80) < best known answer (90)
    - Do not continue

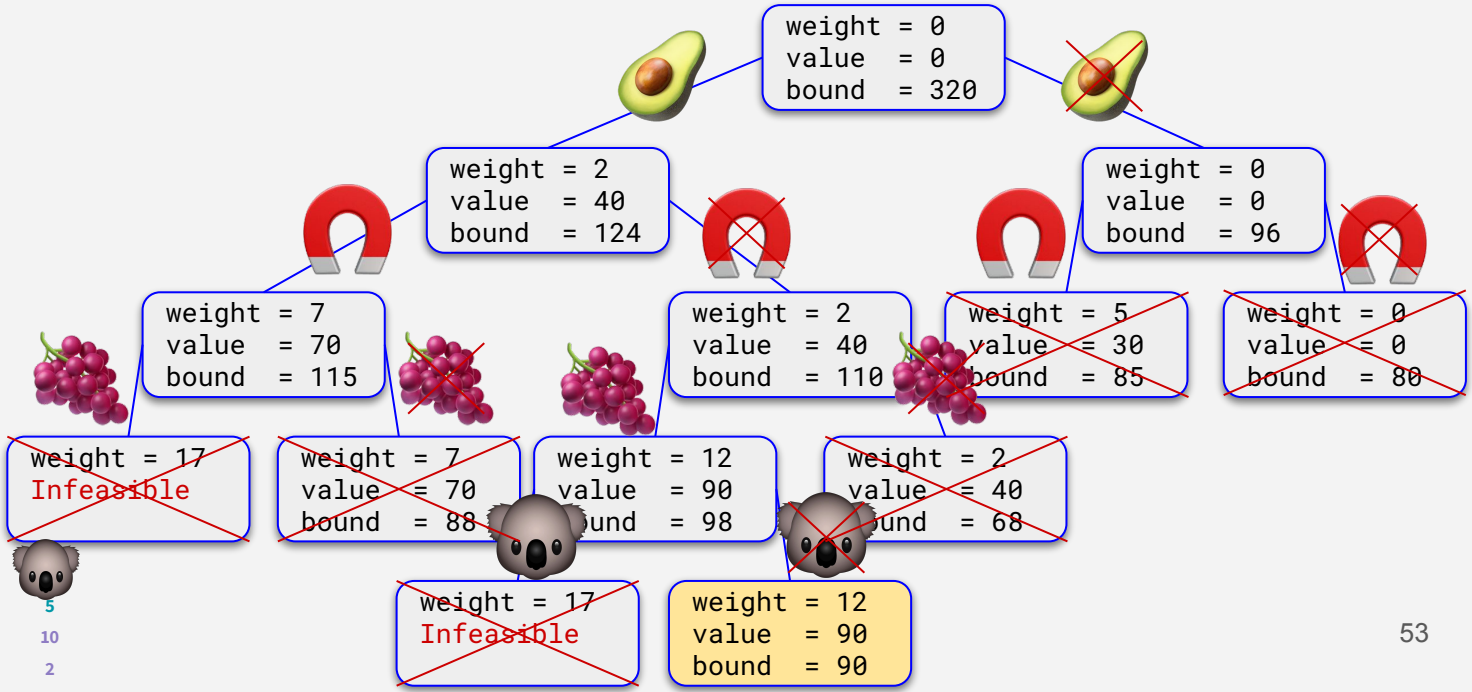


Capacity: **16**



0/1 Knapsack with Branch and Bound

- No more partial answer to expand
  - Current best answer is the best answer

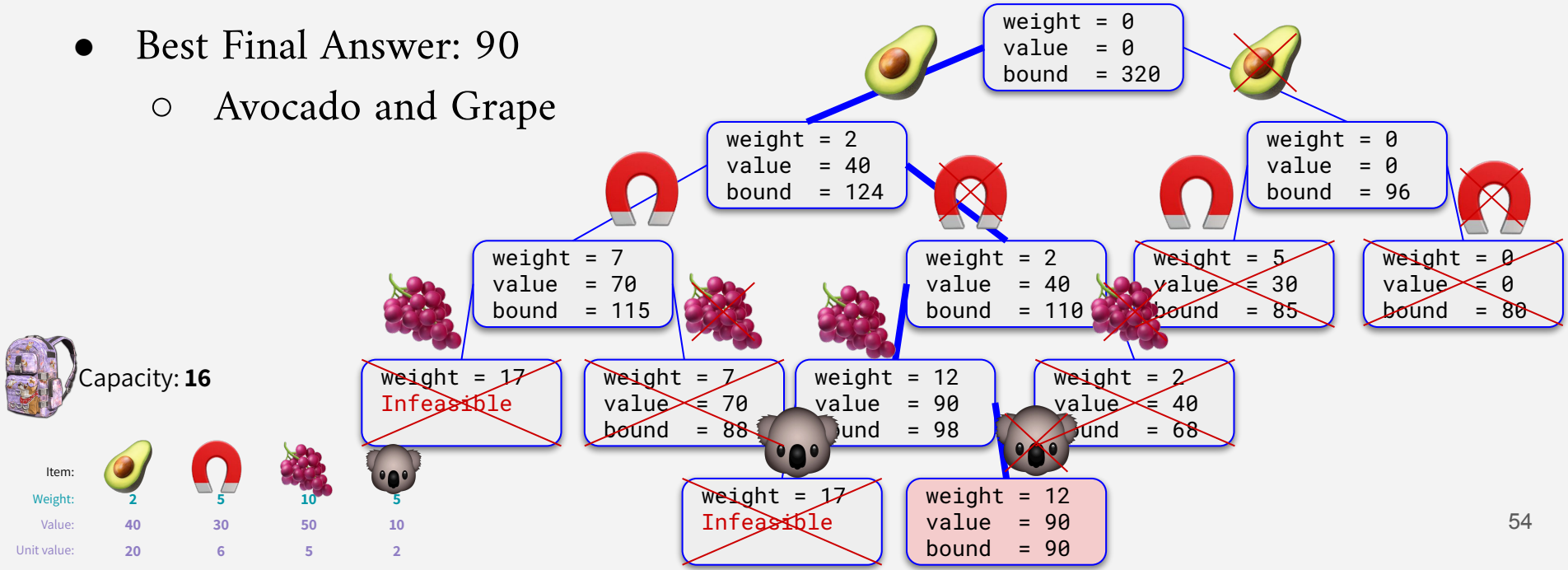


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- No more partial answer to expand
  - Current best answer is the best answer
- Best Final Answer: 90
  - Avocado and Grape



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