طراحي الگوريتم ها

جلسه هجدهم: شاخه و حد

سجاد شیرعلی شهرضا بهار 1402 یکشنبه، 24 اردیبهشت 1402

اطلاع رساني

- امتحانک سوم: یکشنبه آینده، 31 اردیبهشت
 تمدید تمرین سوم تا صبح پنجشنبه، 28 اردیبهشت

روش شاخه و حد

Situation

- Sometimes it is not easy to design a DP or greedy solution
- Checking all possible answers is not feasible either

Situation -> Idea

- Sometimes it is not easy to design a DP or greedy solution
- Checking all possible answers is not feasible either
- Idea: stop exploring a path if we know it would not give us a better answer
 - Keep an estimate of how good the answer will be
 - Continue the path with the highest possible answer
 - Stop a path if it cannot provide a better solution

0/1 Knapsack

- n items
 - Each has a weight w_i and value v_i
- A knapsack (i.e., bag) with capacity *W*
- Goal: fill the bag such that
 - \circ Total weight of items is less than or equal to W
 - $\Sigma w_i \leq W$
 - The value of selected items is maximised
 - Σv_i is maximised



Capacity: 16



Value:









0

Greedy Approach

- Sort items based on unit value
- Start with item with highest unit value
- Add item if it fits in the bag



Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

Greedy Approach - Not Working!

- Sort items based on unit value
- Start with item with highest unit value
- Add item if it fits in the bag
- Final answer is not necessarily the best one
 - Will select avocado, magnet, and quala with value 80
 - Best answer: select avocado and grape with value 90



Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Jnit value:	20	6	5	2

Branch and Bound Algorithm

- Start with an empty answer as partial answer
- Select the partial answer with highest bound
- Expand the partial answer by deciding whether to pick the next item
 - o If selecting the item, update weight and value
- Calculate an upper bound for the answer
 - Fill the remaining capacity with highest remaining items in terms value/unit
- Continue while there is a partial answer that is not eliminated



Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
nit value:	20	6	5	2

• Start with an empty answer

weight = ?
value = ?
bound = ?





- Start with an empty answer
- Calculate upper bound for the value

$$\circ$$
 bound = 16 * 20 = 320

weight = 0 value = 0 bound = 320





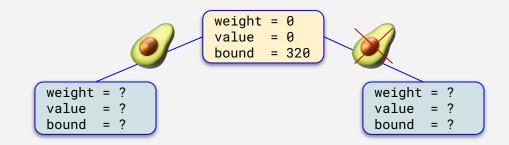
• Expand node with bound 320

weight = 0 value = 0 bound = 320





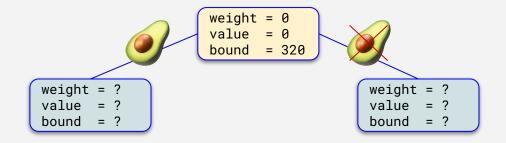
• Expand node with bound 320





Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Jnit value:	20	6	5	2

• Pick Avocado





Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Jnit value:	20	6	5	2

- Pick Avocado
 - \circ Found an answer with value 0 + 40 = 40
 - \circ weight = 0 + 2 = 2
 - 9 bound = 40 + (16-2) * 6 = 124

weight = 2 value = 40 bound = 124 weight = 0 value = 0 bound = 320

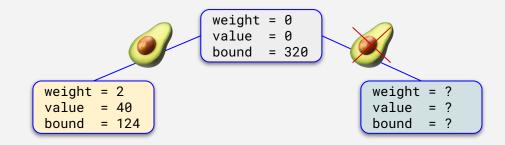


weight = ?
value = ?
bound = ?





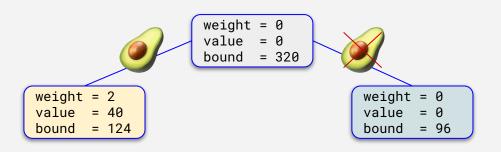
• Do not pick Avocado





Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Jnit value:	20	6	5	2

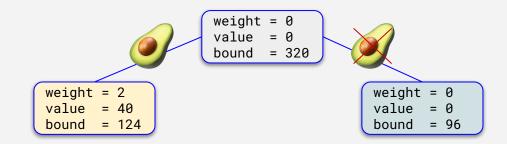
- Do not pick Avocado
 - \circ value = 0 + 0 = 0
 - $\circ \quad \text{wight} = 0 + 0 = 0$
 - \circ bound = 16 * 6 = 96







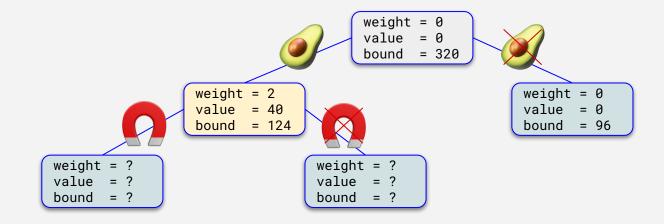
• Expand node with bound 124





Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Jnit value:	20	6	5	2

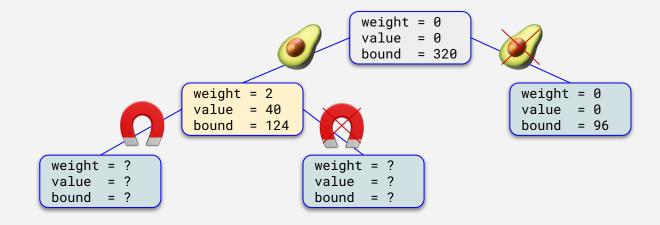
• Expand node with bound 124





Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Jnit value:	20	6	5	2

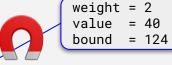
Pick Magnet





Item: Weight:	2		10	1
Value:	40	30	50	10
Unit value:	20	6	5	2

- Pick Magnet
 - Found an answer with value 40 + 30 = 70
 - \circ wight = 2 + 5 = 7
 - bound = 70 + (16-7) * 5 = 115



weight = 7 value = 70 bound = 115

weight = ? value = ? bound = ?

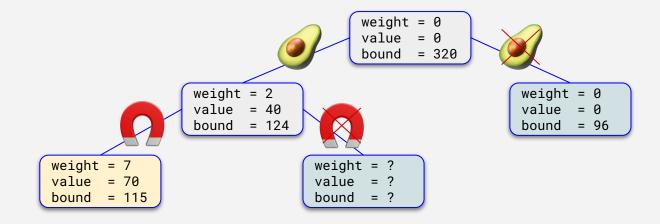


weight = 0 value = 0 bound = 96

Capacity: 16



• Do not pick Magnet





Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Jnit value:	20	6	5	2

weight = 7

bound = 115

value

• Do not pick Magnet

$$\circ$$
 value = 40 + 0 = 40

$$\circ$$
 wight = 2 + 0 = 2

o bound = 40 + (16-2) * 5 = 110

weight = 2 value = 40 bound = 124

weight = 2 value = 40 bound = 110

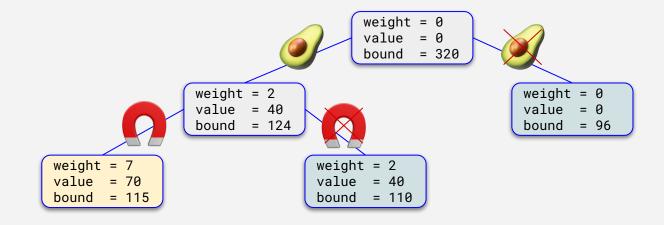
weight = 0
value = 0
bound = 320

weight = 0 value = 0 bound = 96

Capacity: **16**



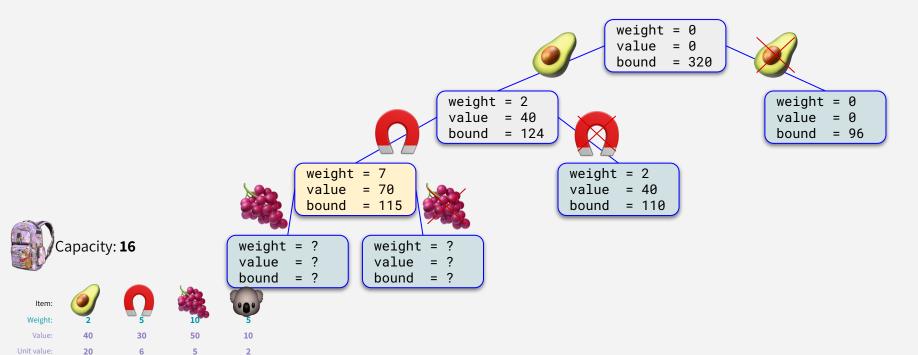
• Expand node with bound 115



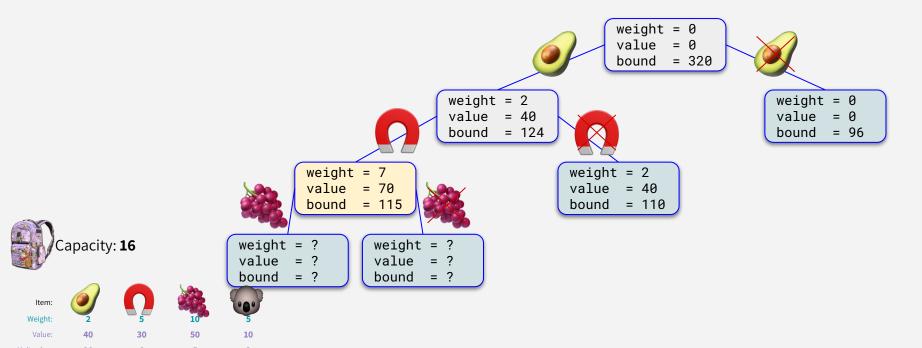


Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Jnit value:	20	6	5	2

• Expand node with bound 115



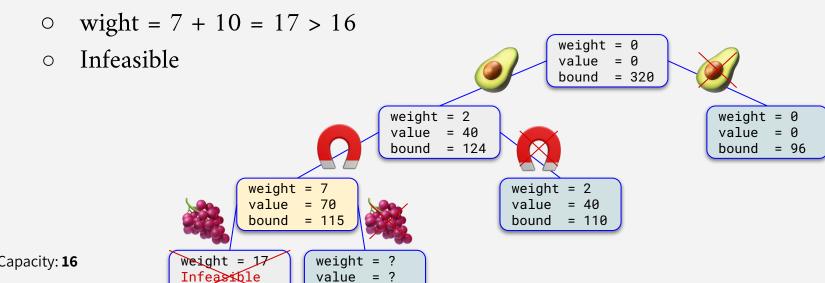
Pick Grape



26

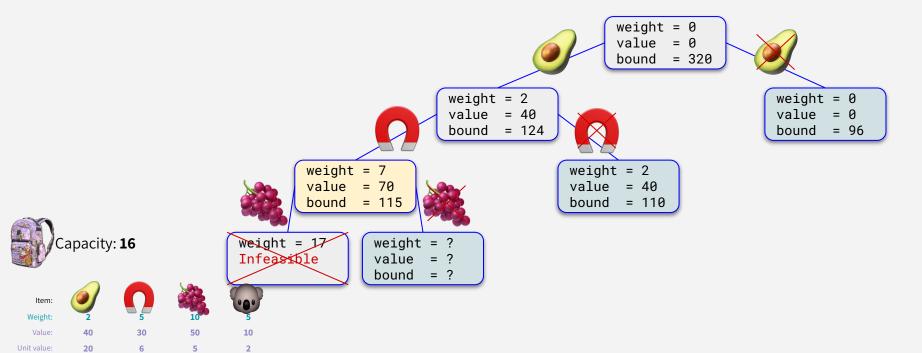
Pick Grape

Weight:

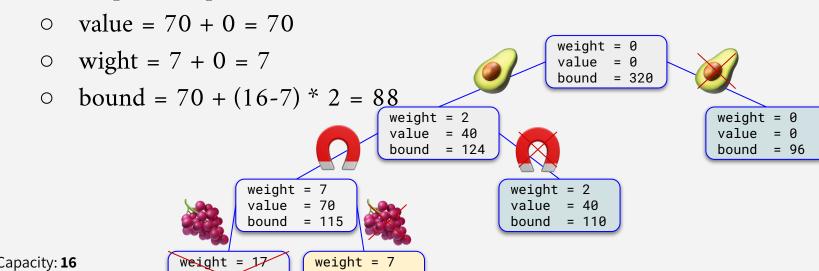


bound

• Do not pick Grape



Do not pick Grape



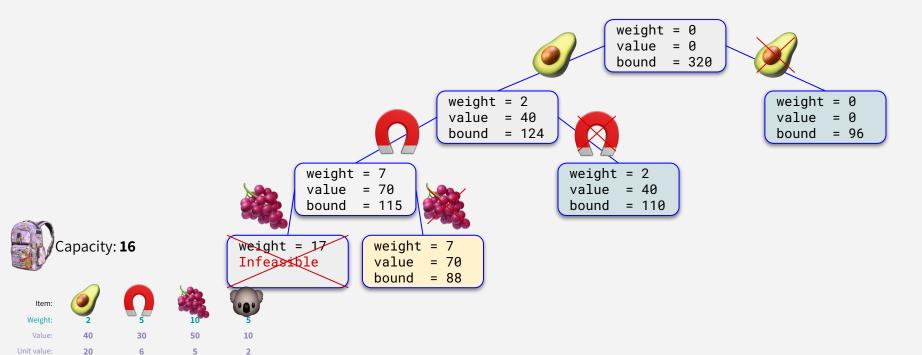
value

bound

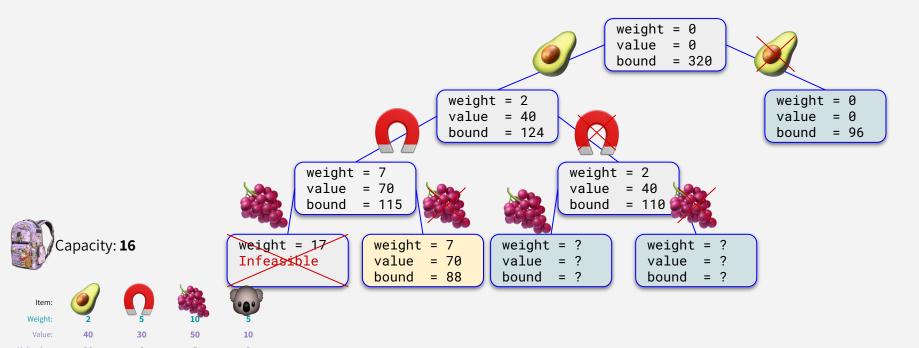
= 70

= 88

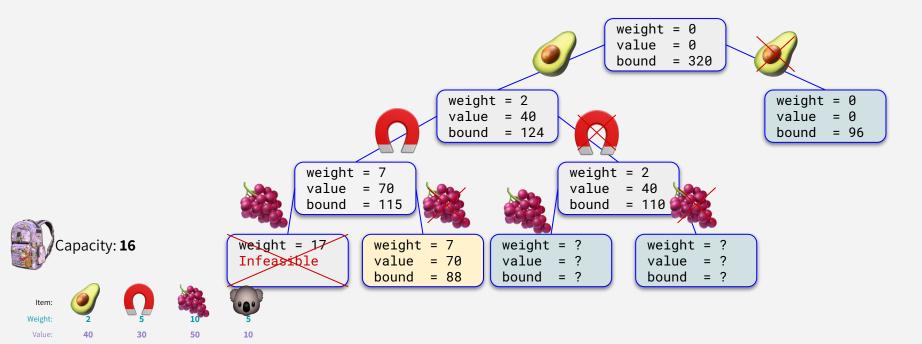
• Expand node with bound 110



• Expand node with bound 110

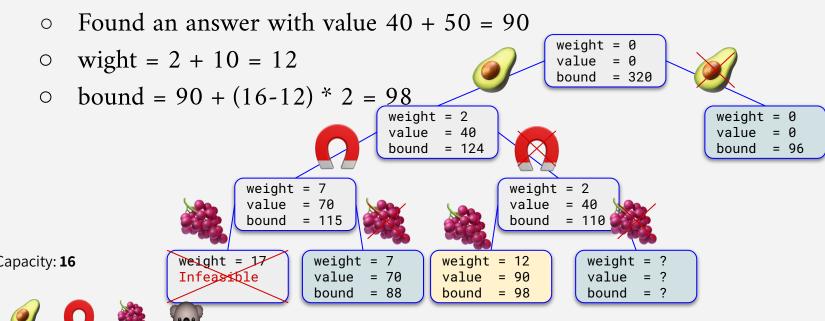


Pick Grape



Pick Grape

Weight:



Pick Grape

Weight:

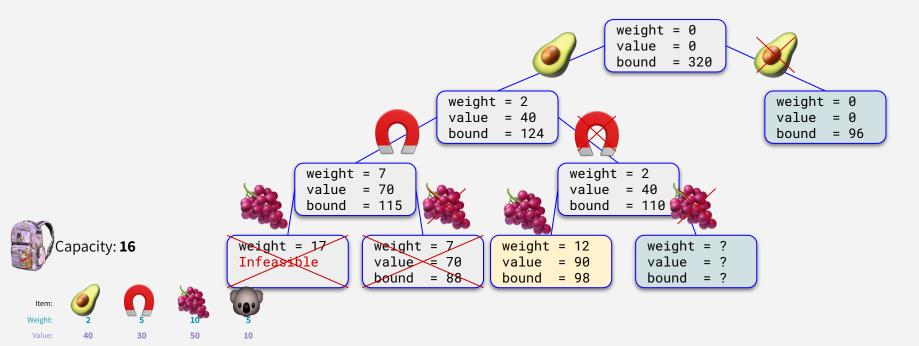
- Found an answer with value 90
- weight = 0Eliminate nodes with bound < 90 value = 0bound = 320weight = 2weight = 0value = 40value bound = 124bound = 96 weight = 7weight = 2= 70 value value = 40= 115 bound bound = 110Capacity: 16 weight = 17weight = 7weight = ?weight = 12 Infeasible = 90 value value = 70 value bound = 88 bound bound

• Pick Grape

Weight:

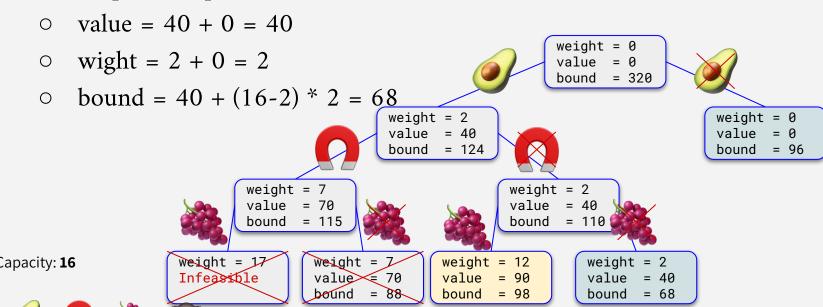
- Found an answer with value 90
- weight = 0Eliminate nodes with bound < 90 value = 0bound = 320 weight = 2weight = 0value = 40value bound = 124bound = 96 weight = 7weight = 2= 70 value value = 40= 115 bound bound = 110Capacity: 16 weight = 17weight = 7weight = ?weight = 12 Infeasible = 90 value value value bound bound

• Do not pick Grape



• Do not pick Grape

Weight:

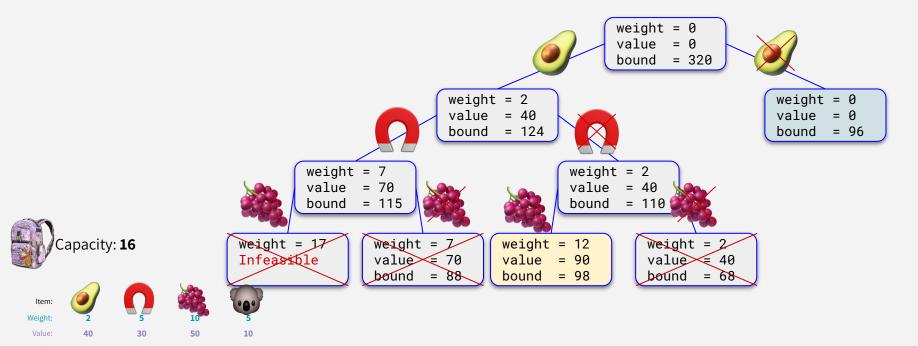


• Do not pick Grape

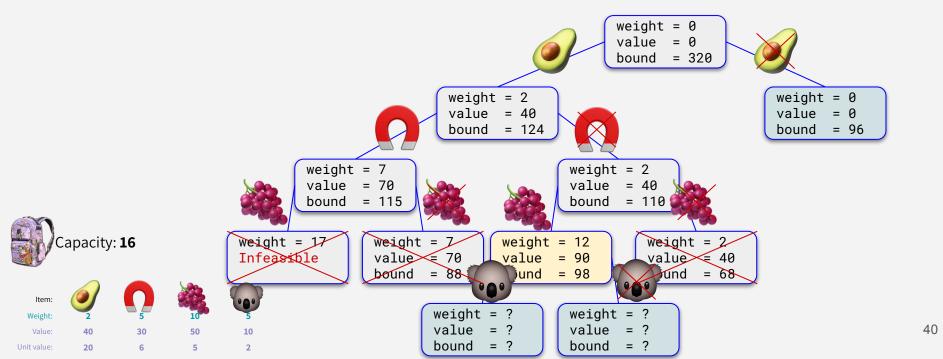
Weight:

Bound (68) < best known answer (90) weight = 0Do not continue value bound = 320 weight = 2weight = 0value = 40 value bound = 124bound = 96 weight = 7weight = 2value value = 40= 115 bound bound = 110Capacity: 16 weight = 17weight = 7weight = 2weight = 12 Infeasible = 90 value value value bound

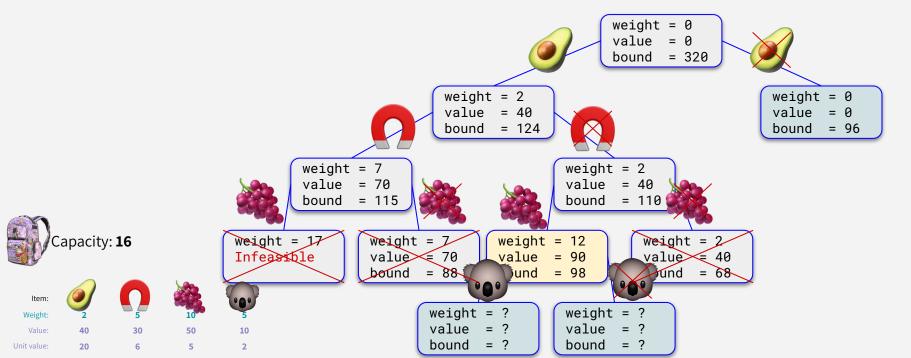
Expand the node with bound 98



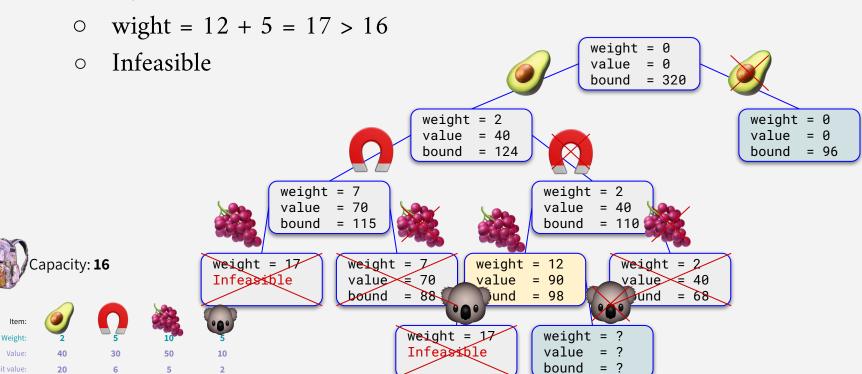
Expand the node with bound 98



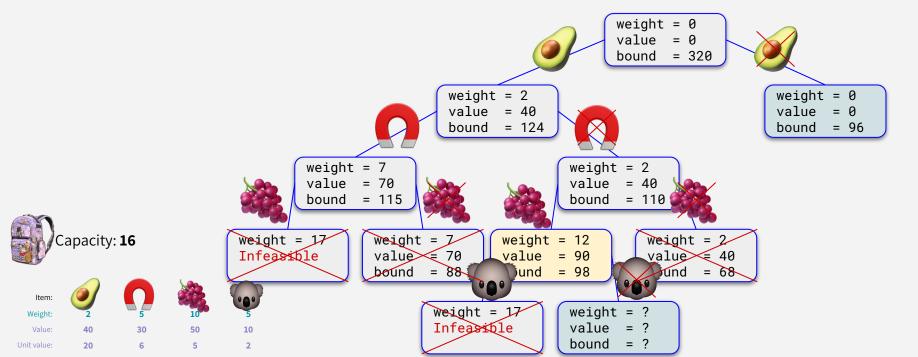
Pick Quala



• Pick Quala

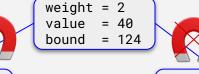


Do not pick Quala



- Do not pick Quala
 - Found a final answer with value 90 + 0 = 90
 - wight = 12 + 0 = 12
 - bound = 90 + (16-12) * 0 = 90

weight = 0value = 0bound = 320



weight = 0value bound = 96



weight = 7= 70 value = 115 bound



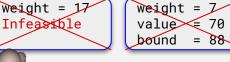
weight = 2value = 40bound = 110



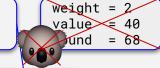
Item:	2	

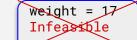


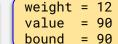




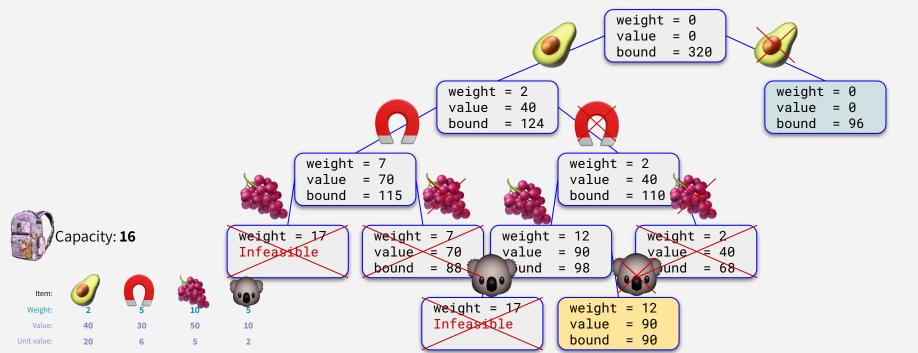




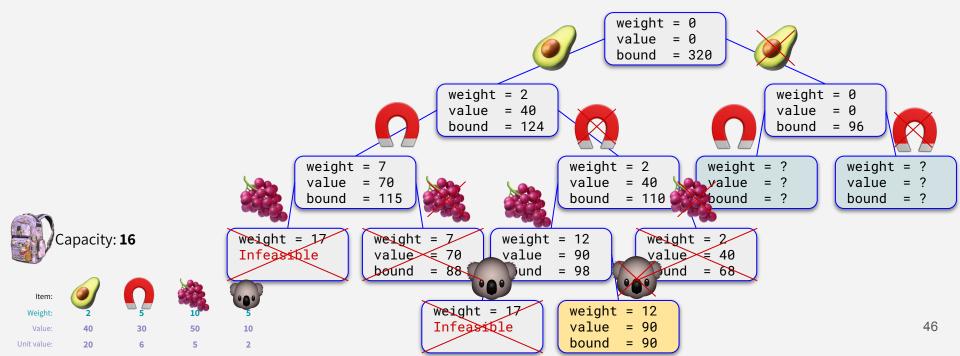




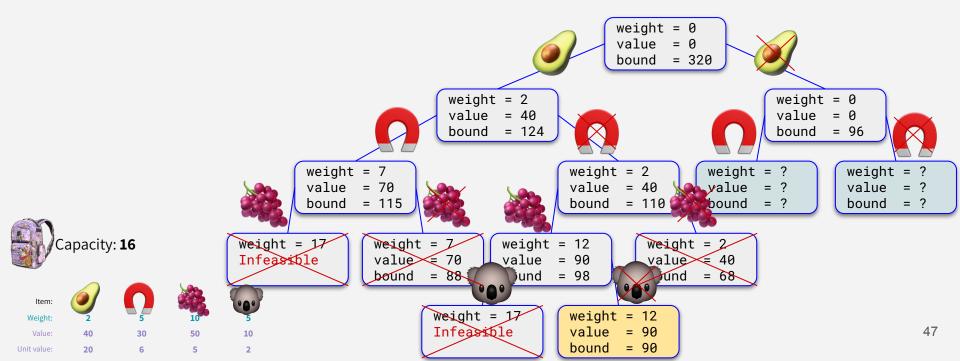
Expand node with bound 96



• Expand node with bound 96

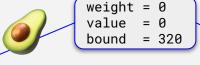


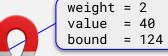
Pick Magnet

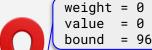


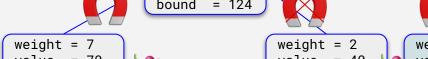
Pick Magnet

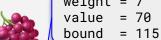
- value = 0 + 30 = 30
- weight = 0 + 5 = 5
- bound = 30 + (16-5) * 5 = 85

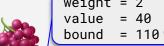












weight = 5**y**alue = 30 bound = 85 weight value bound

Capacity: **16** weight = 17 Infeasible

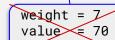
Item: eight:	2	\bigcap_{5}	
/alue:	40	30	

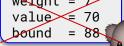




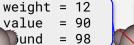




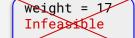


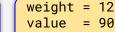




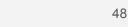




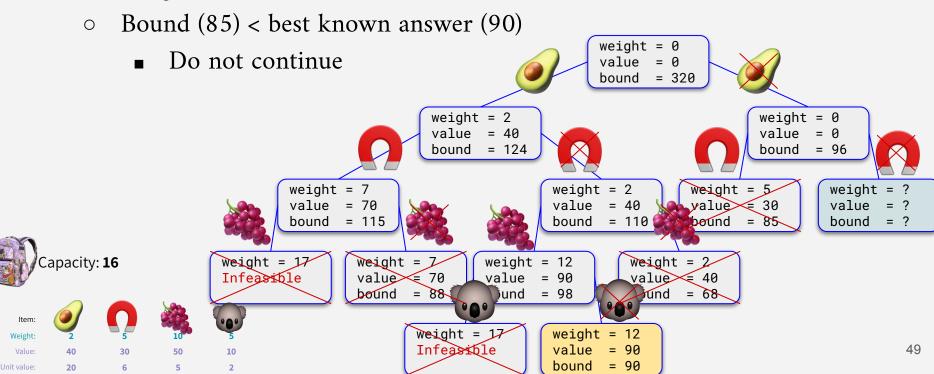




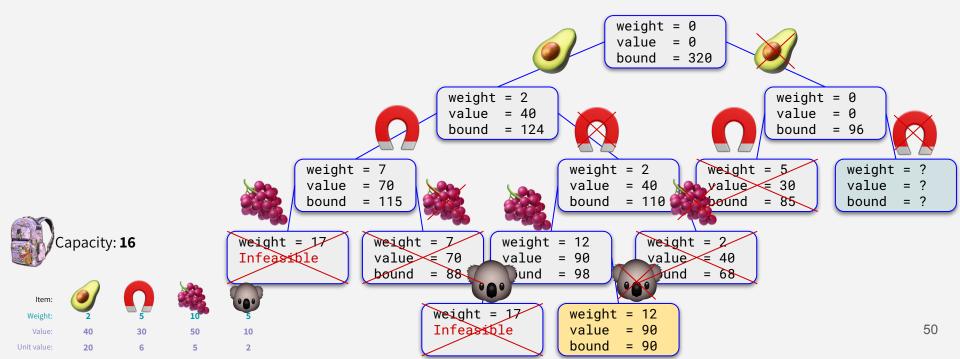
bound



Pick Magnet

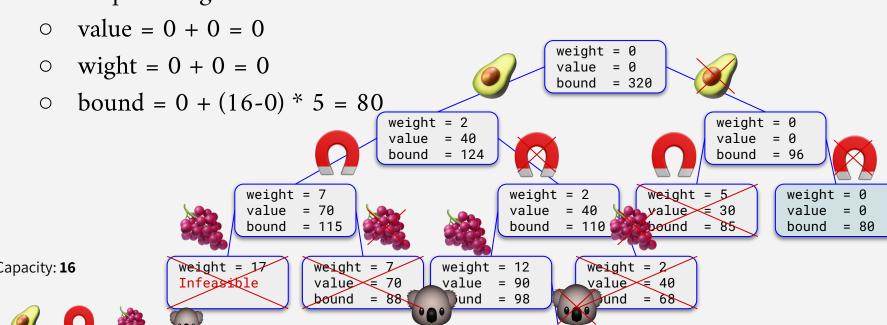


• Do not pick Magnet



• Do not pick Magnet

Weight:



weight = 17

Infeasible

weight

= 90

value

bound

51

• Do not pick Magnet

Bound (80) < best known answer (90) weight = 0Do not continue value bound = 320 weight = 2weight = 0value = 40 value bound = 124bound = 96weight = 7weight = 2weight = 5weight = 0 value = 70 value yalue √≡ 30 value = 115 = 110 bound = 85 bound bound bound Capacity: 16 weight = 17weight = 7weight = weight = 12 Infeasible value 70 90 value value bound = 88= 98 bund weight = 17weight 52 Infeasible value = 90

bound

No more partial answer to expand

Current best answer is the best answer weight = 0value bound = 320 weight = 2weight = 0value = 40 value bound = 124bound = 96 weight = 7weight = 2weight = 5weight = 0value = 70 value yalue √≡ 30 value = 110 bound = 85 bound = 115 bound bound Capacity: 16 weight = 17weight = 7weight = weight = 12 Infeasible value 70 value 90 value bound = 88= 98 bund weight = 17weight Weight:

Infeasible

value

bound

53

No more partial answer to expand

Current best answer is the best answer weight = 0• Best Final Answer: 90 value bound = 320 Avocado and Grape weight = 2weight = 0value = 40 value bound = 124bound = 96 weight = 7weight = 2weight = 5weight = 0 = 70 value value yalue √≡ 30 value = 110 bound = 85 = 115 bound bound bound Capacity: 16 weight = 17weight = 7weight = 2weight 12 = value 70 90 Infeasible value value bound = 88= 98 weight = 17weight

Infeasible

value

bound

= 90



سوال؟