Department of
Computer Engineering

Data Structure & Algorithms

Graph Search

Searching a graph

- Given: a graph G = (V, E), directed or undirected
- Goal: *methodically* explore every vertex (and every edge)
- Different methods of exploration =>
  - Different order of vertex discovery
  - Different shape of the exploration trace subgraph (which can be a spanning tree of the graph)
- Methods of exploration:
  - Breadth-First Search
  - Depth-First Search

Breadth-First Search

Breadth first search

- Given
  - a graph G=(V,E) – set of vertices and edges
  - a distinguished source vertex s
- Breadth first search systematically explores the edges of G to discover every vertex that is reachable from s.
- It also produces a ‘breadth first tree’ with roots that contains all the vertices reachable from s.
- For any vertex v reachable from s, the path in the breadth first tree corresponds to the shortest path in graph G from s to v.
- It works on both directed and undirected graphs.

Breadth first search - concepts

- To keep track of progress, it colors each vertex - white, gray or black.
- All vertices start white.
- A vertex discovered first time during the search becomes nonwhite.
- All vertices adjacent to black ones are discovered. Whereas, gray ones may have some white adjacent vertices.
- Gray represent the frontier between discovered and undiscovered vertices.

BFS – How it produces a Breadth first tree

- The tree initially contains only root: s
- Whenever a vertex v is discovered in scanning adjacency list of vertex u
  - Vertex v and edge (u,v) are added to the tree.

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BFS - algorithm

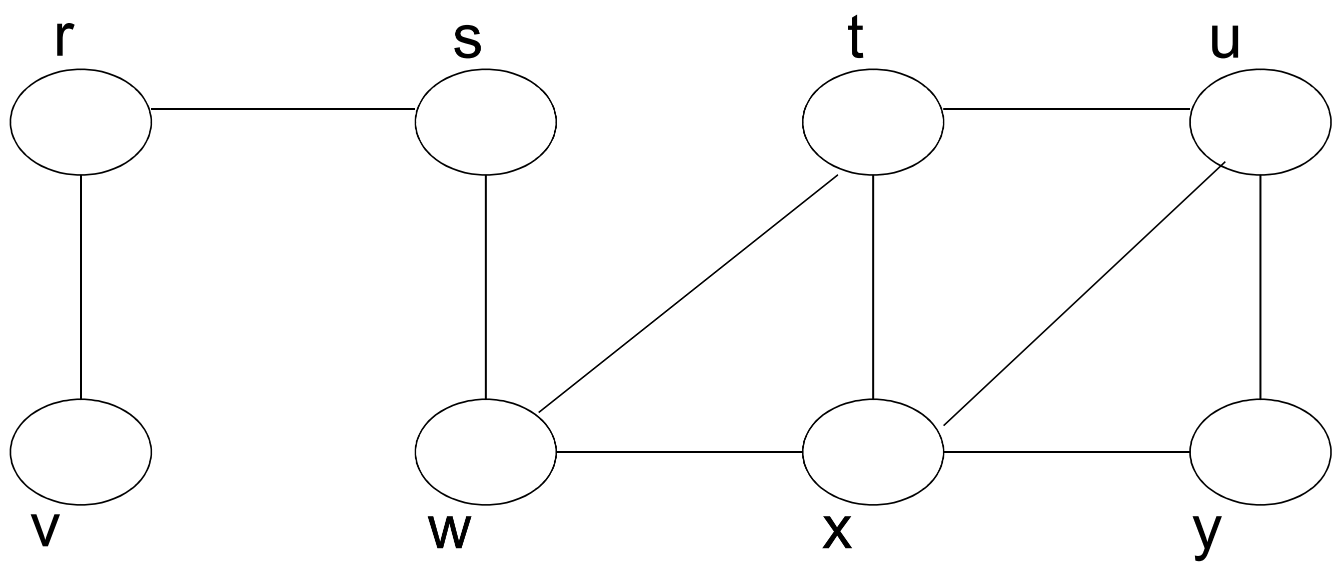
```
BFS(G, s)                    // G is the graph and s is the starting node
 1  for each vertex u ∈ V [G] - {s}
 2       do color[u] ← WHITE       // color of vertex u
 3          d[u] ← ∞               // distance from source s to vertex u
 4          π[u] ← NIL             // predecessor of u
 5  color[s] ← GRAY
 6  d[s] ← 0
 7  π[s] ← NIL
 8  Q ← Ø                          // Q is a FIFO - queue
 9  ENQUEUE(Q, s)
10  while Q ≠ Ø              // iterates as long as there are gray vertices. Lines 10-18
11      do u ← DEQUEUE(Q)
12         for each v ∈ Adj[u]
13             do if color[v] = WHITE       // discover the undiscovered adjacent
vertices
14                   then color[v] ← GRAY   // enqueued whenever painted gray
15                        d[v] ← d[u] + 1
16                        π[v] ← u
17                        ENQUEUE(Q, v)
18         color[u] ← BLACK         // painted black whenever dequeued
```

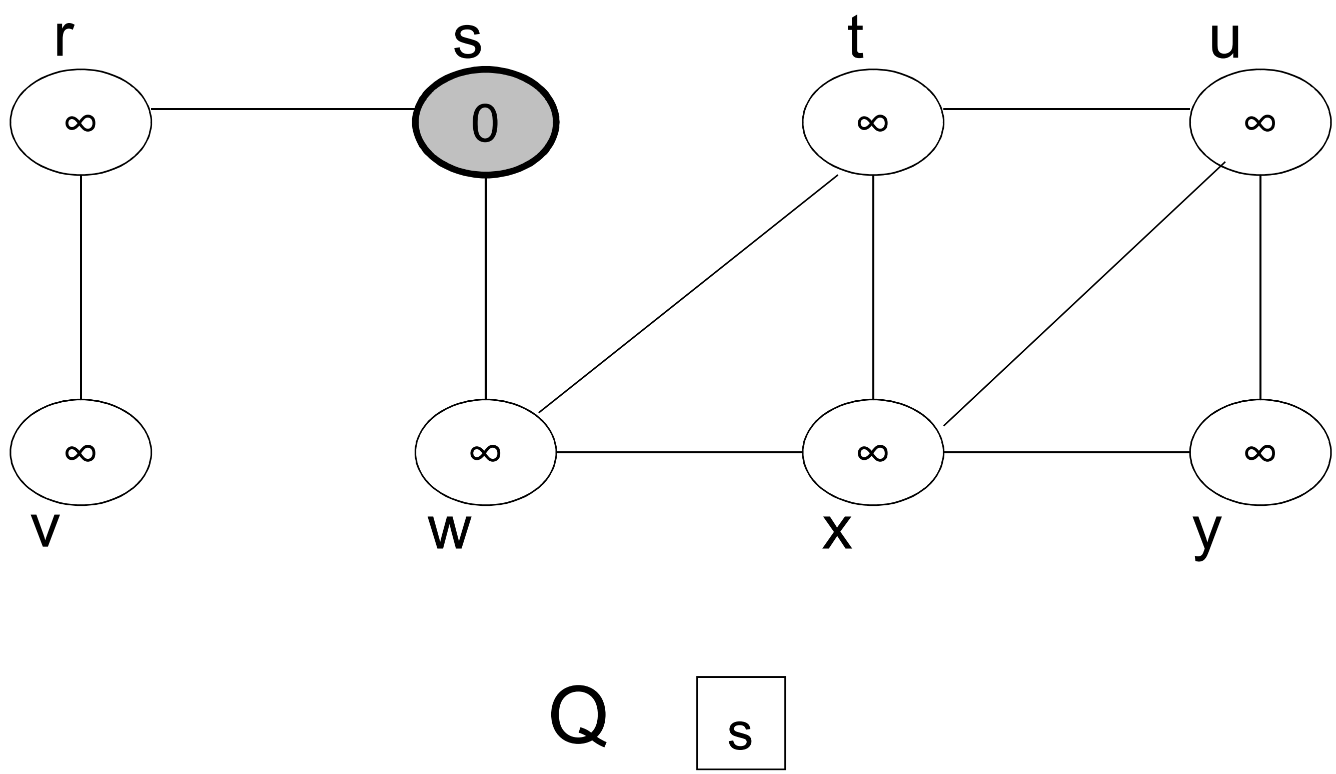
Breadth first search - analysis

- Enqueue and Dequeue happen only once for each node – O(V).
- Sum of the lengths of adjacency lists – θ(E) (for a directed graph)
- Initialization overhead O(V)

Total runtime O(V+E)

Example – Applying BFS





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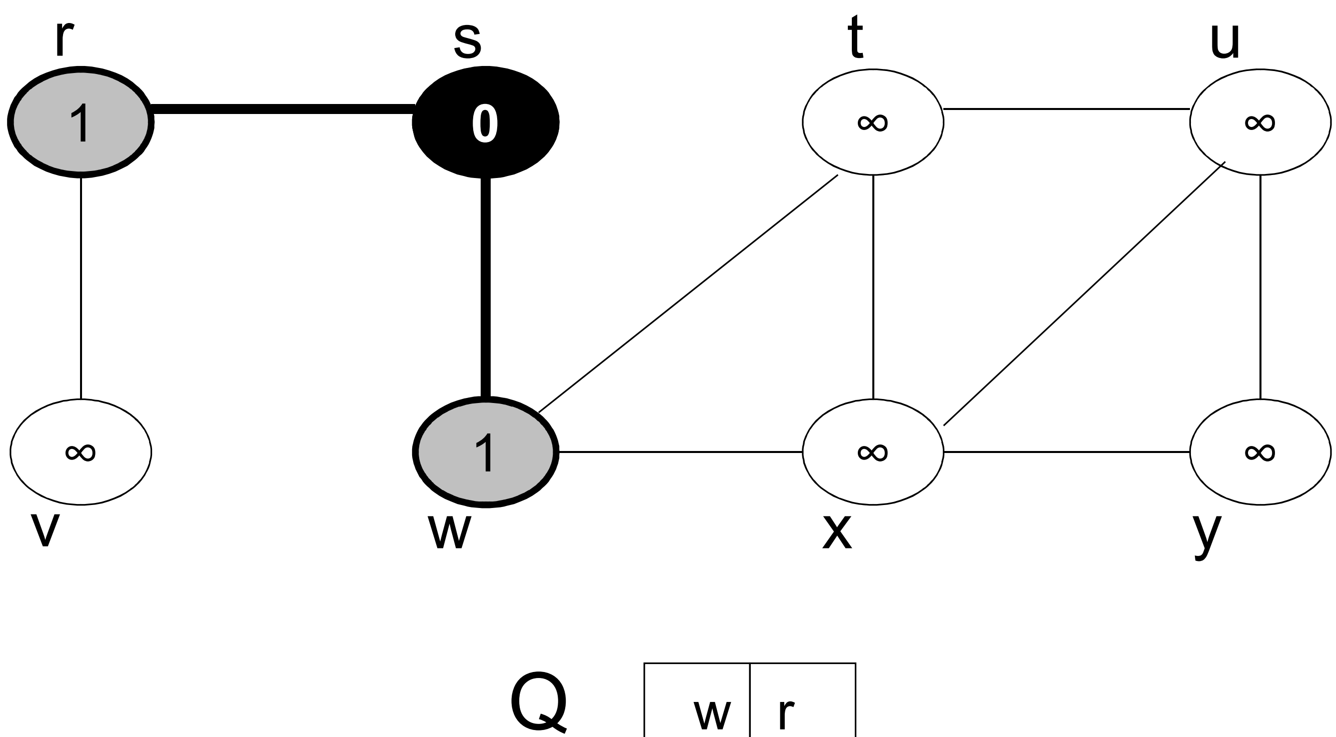
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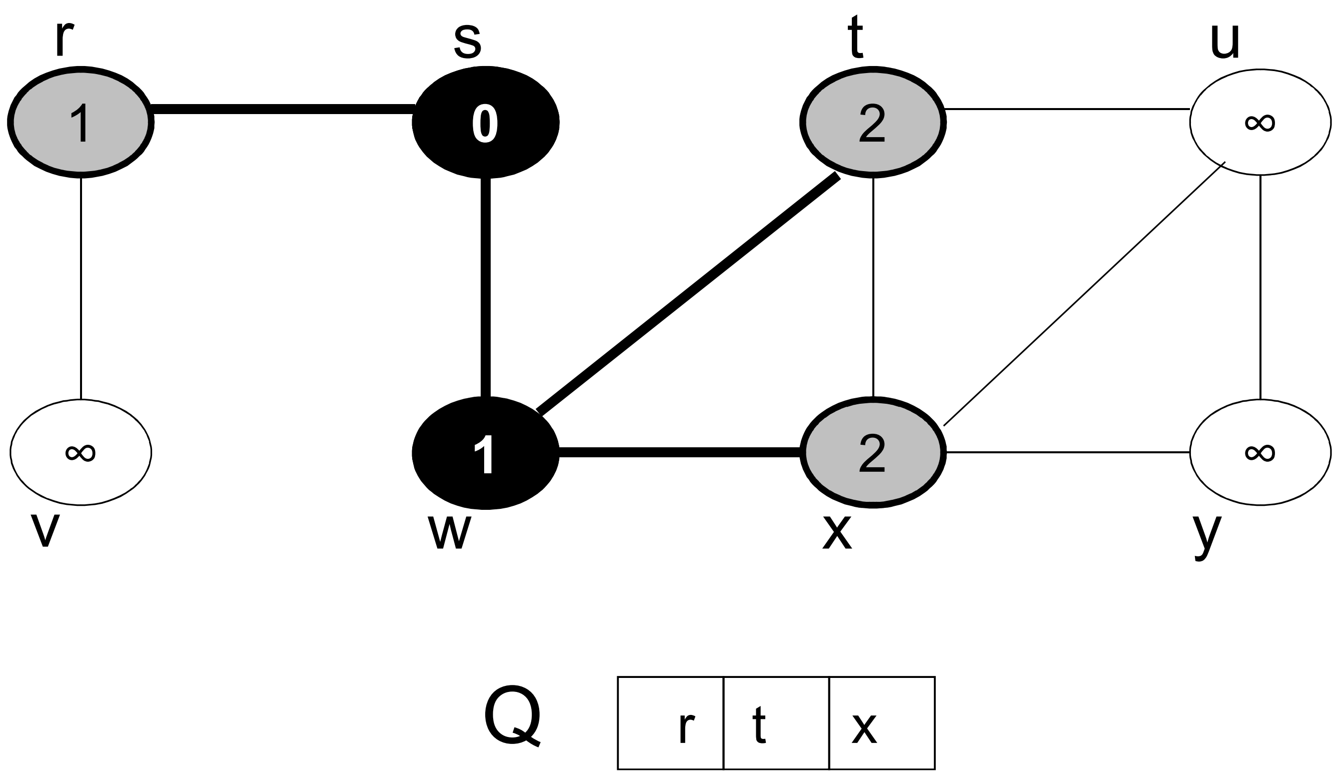
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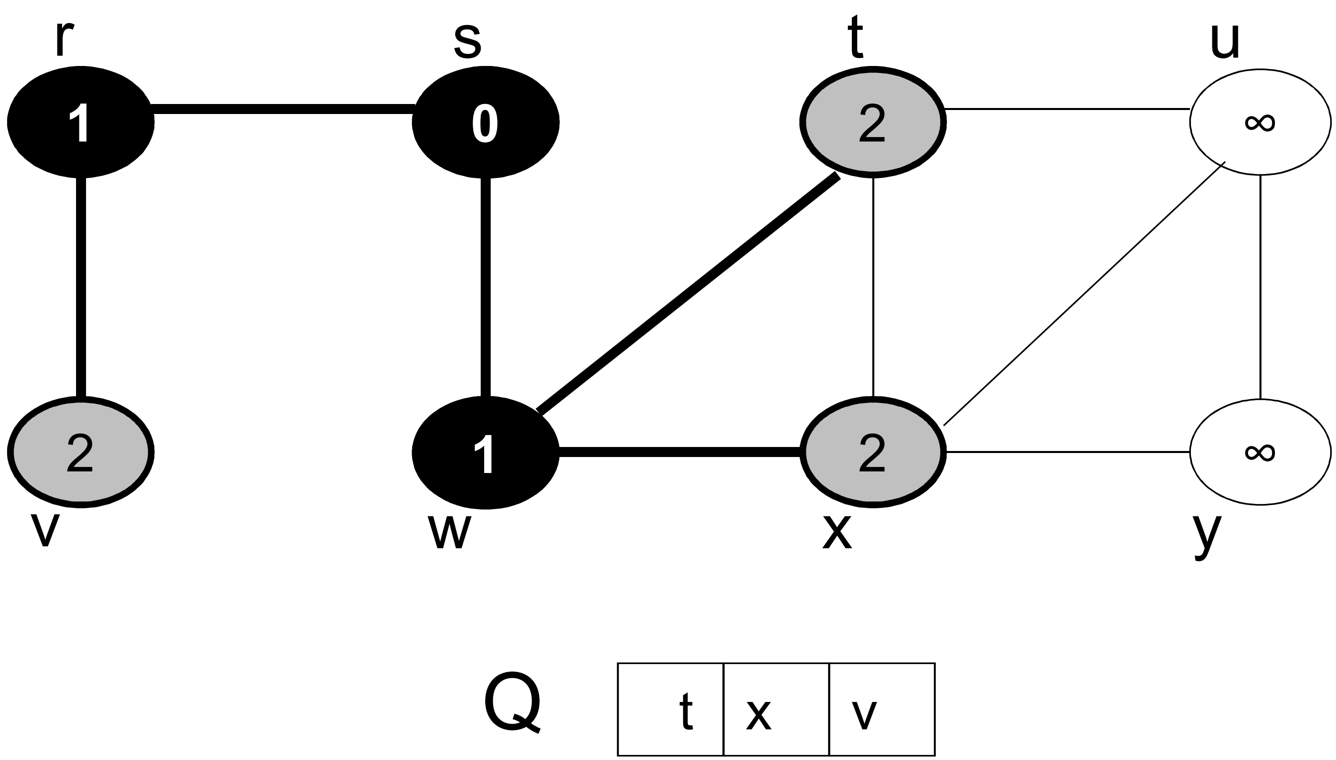
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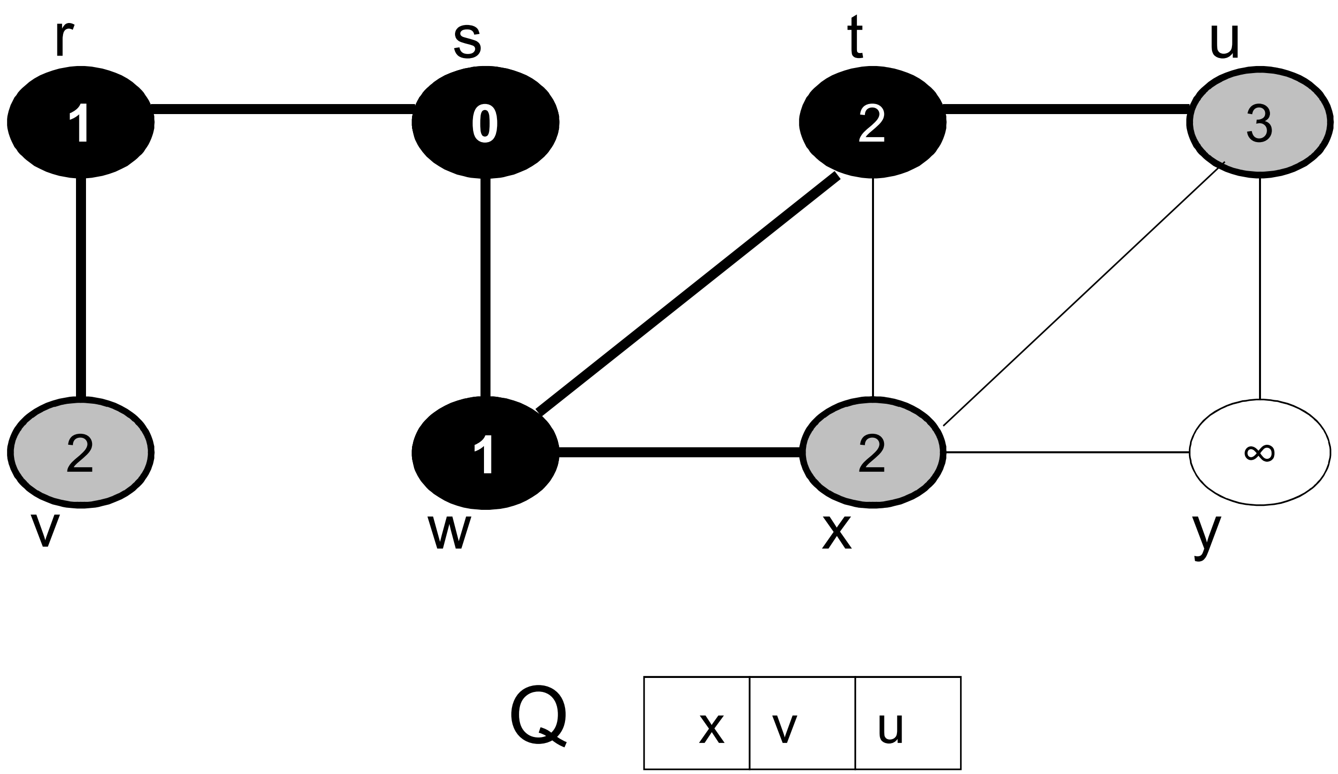

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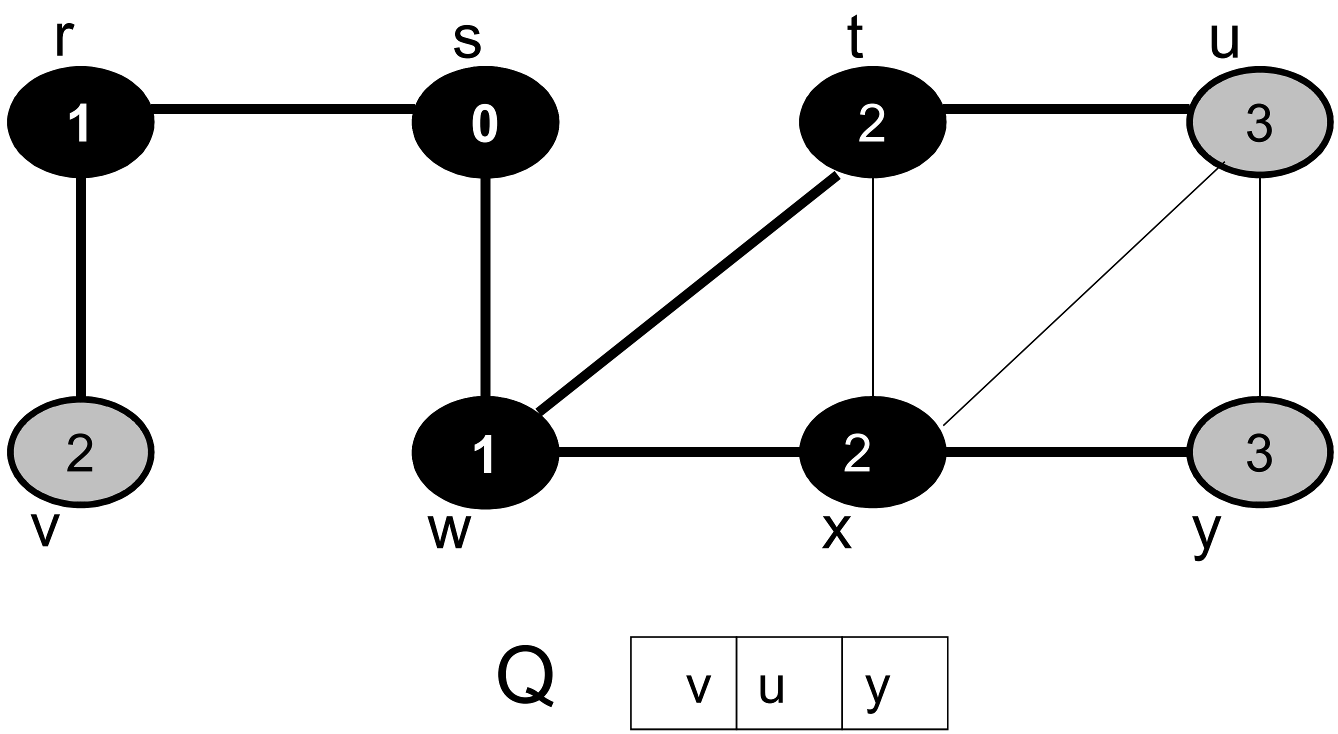
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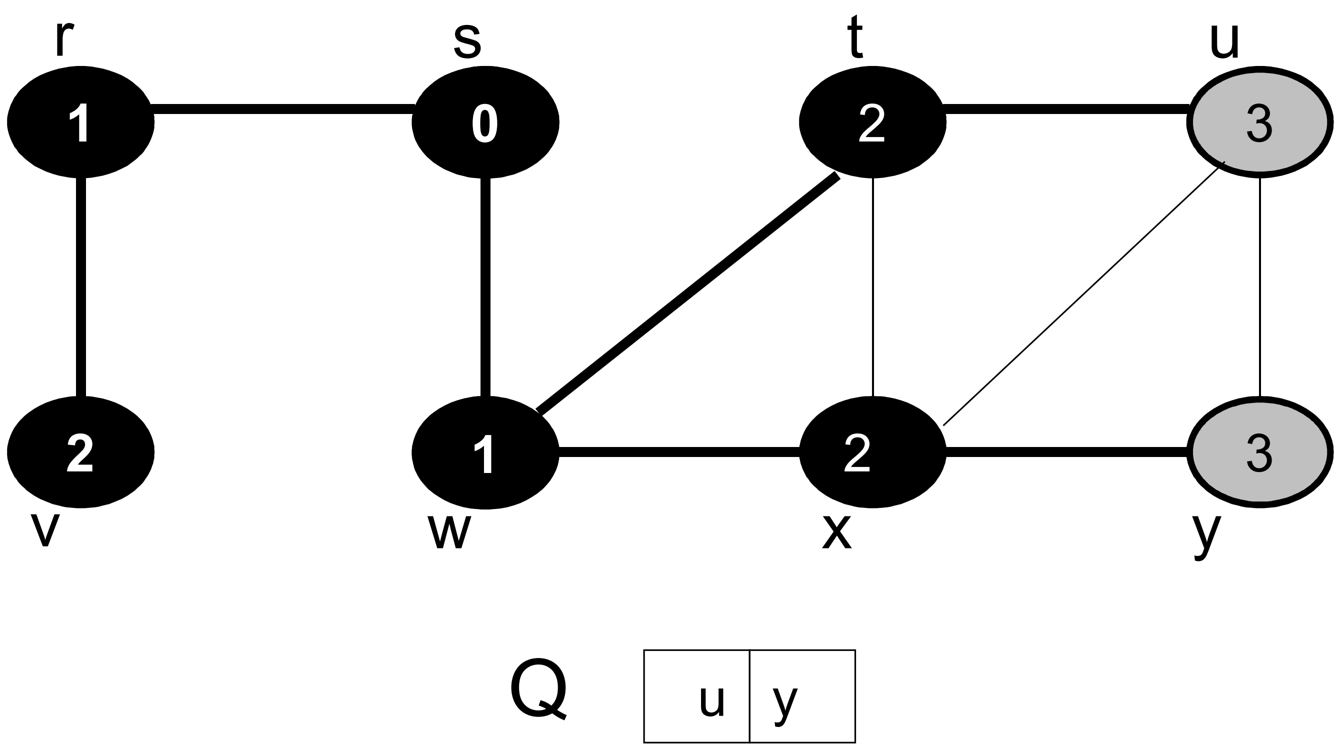
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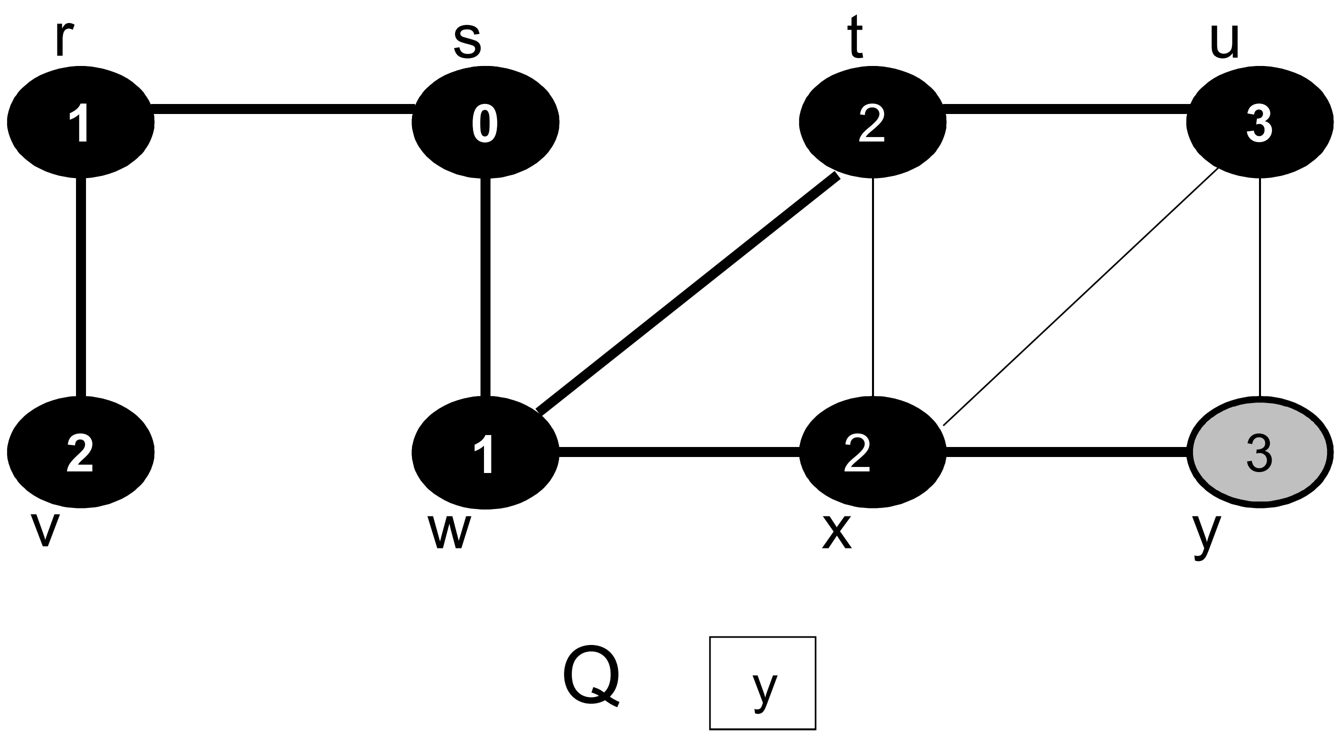
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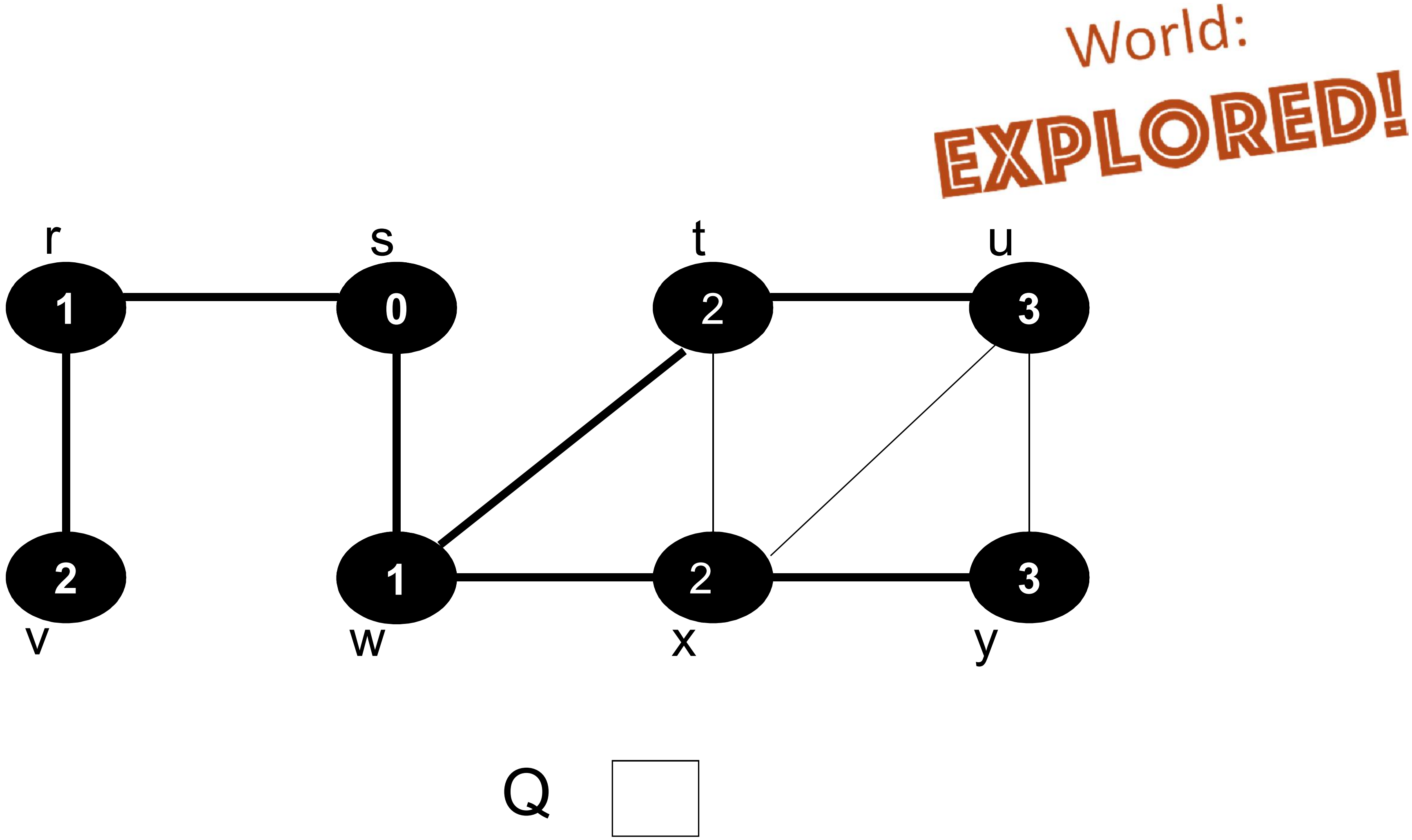
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Depth-First Search

Depth first search

- It searches ‘deeper’ the graph when possible.
- Starts at the selected node and explores as far as possible along each branch before backtracking.
- Vertices go through white, gray and black stages of color.
  - White – initially
  - Gray – when discovered first
  - Black – when finished i.e. the adjacency list of the vertex is completely examined.
- Also records timestamps for each vertex
  - d[v] when the vertex is first discovered
  - f[v] when the vertex is finished

Depth first search - algorithm

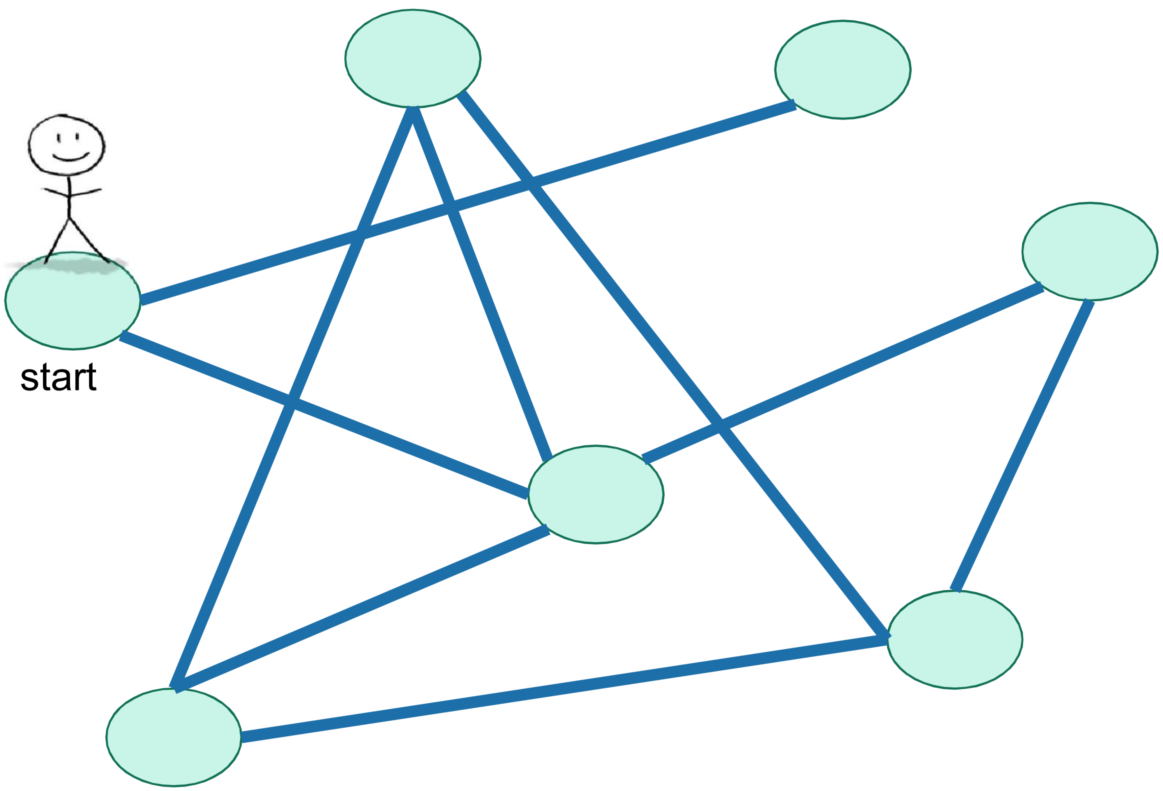
```
DFS(G)
for each vertex u ∈ V [G]
     do color[u] ← WHITE          // color all vertices white, set their parents NIL
        π[u] ← NIL
time ← 0                          // zero out time
for each vertex u ∈ V [G]                  // call only for unexplored vertices
     do if color[u] = WHITE                // this may result in multiple sources
           then DFS-VISIT(u)

DFS-VISIT(u)
color[u] ← GRAY      ▹White vertex u has just been discovered.
time ← time +1
d[u] time                 // record the discovery time
for each v ∈ Adj[u]       ▹Explore edge(u, v).
     do if color[v] = WHITE
           then π[v] ← u // set the parent value
                       DFS-VISIT(v)        // recursive call
color[u] BLACK       ▹ Blacken u; it is finished.
f [u] ▹ time ← time +1
```

Depth first search - analysis

- Lines 1-3, initialization take time $\Theta(V)$.
- Lines 5-7 take time $\Theta(V)$, excluding the time to call the DFS-VISIT.
- DFS-VISIT is called only once for each node (since it's called only for white nodes and the first step in it is to paint the node gray).
- Loop on line 4-7 is executed $|Adj(v)|$ times. Since, $\sum_{v \in V} |Adj(v)| = \Theta(E)$, the total cost of DFS-VISIT it $\theta(E)$

The total cost of DFS is $\theta(V+E)$

Depth First Search

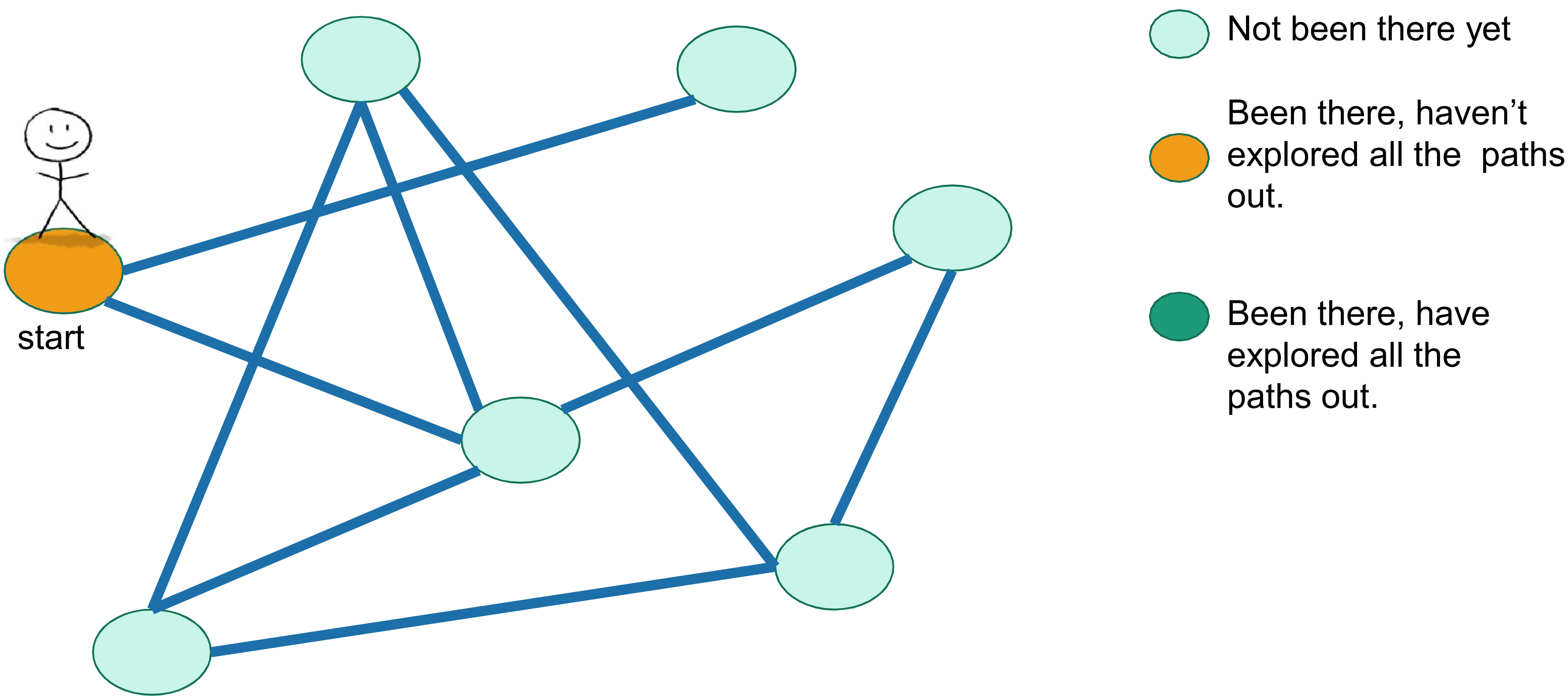


Not been there yet

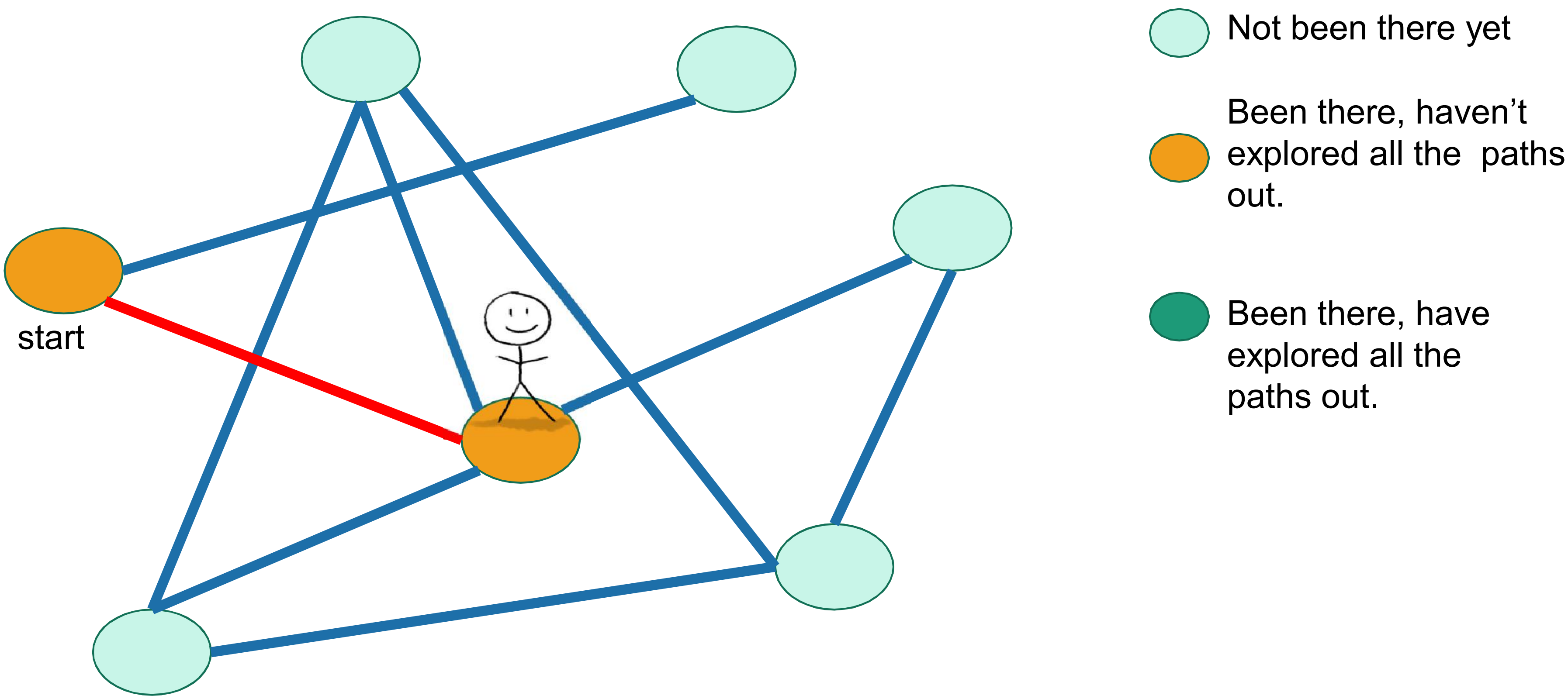
Been there, haven't explored all the paths out.

Been there, have explored all the paths out.

Depth First Search



Depth First Search



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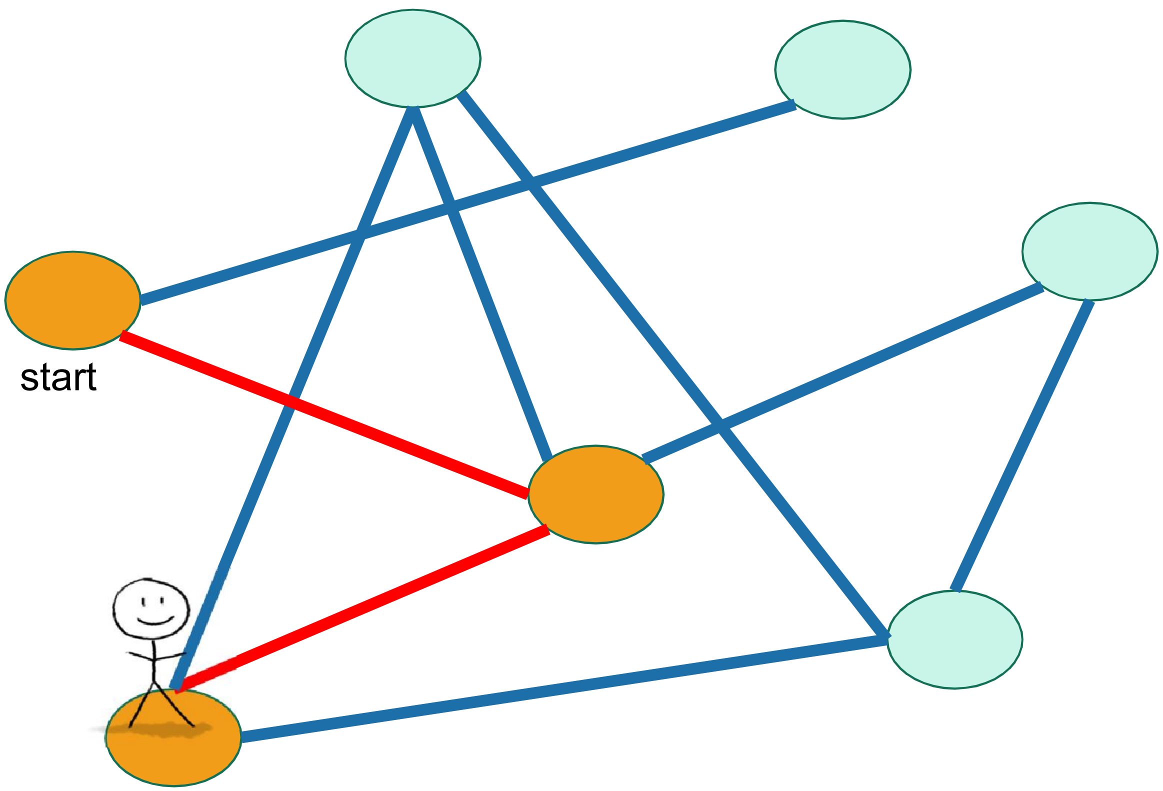

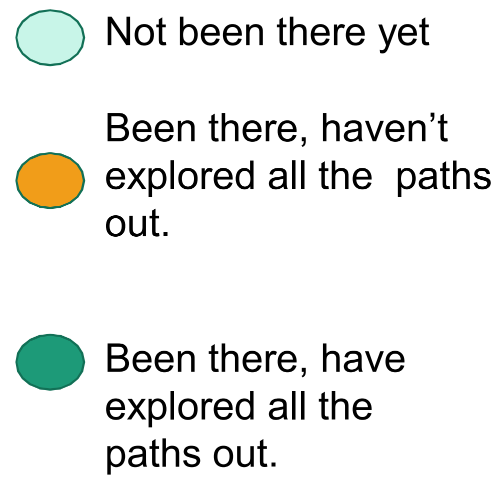

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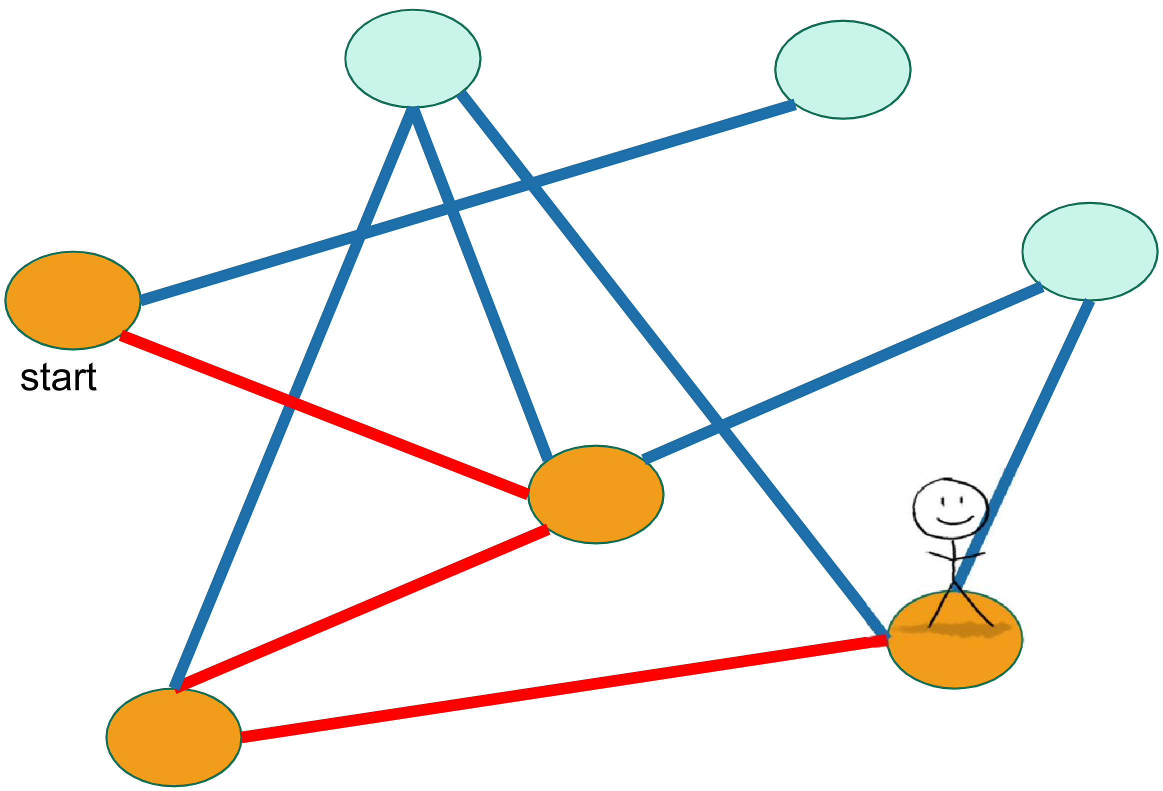

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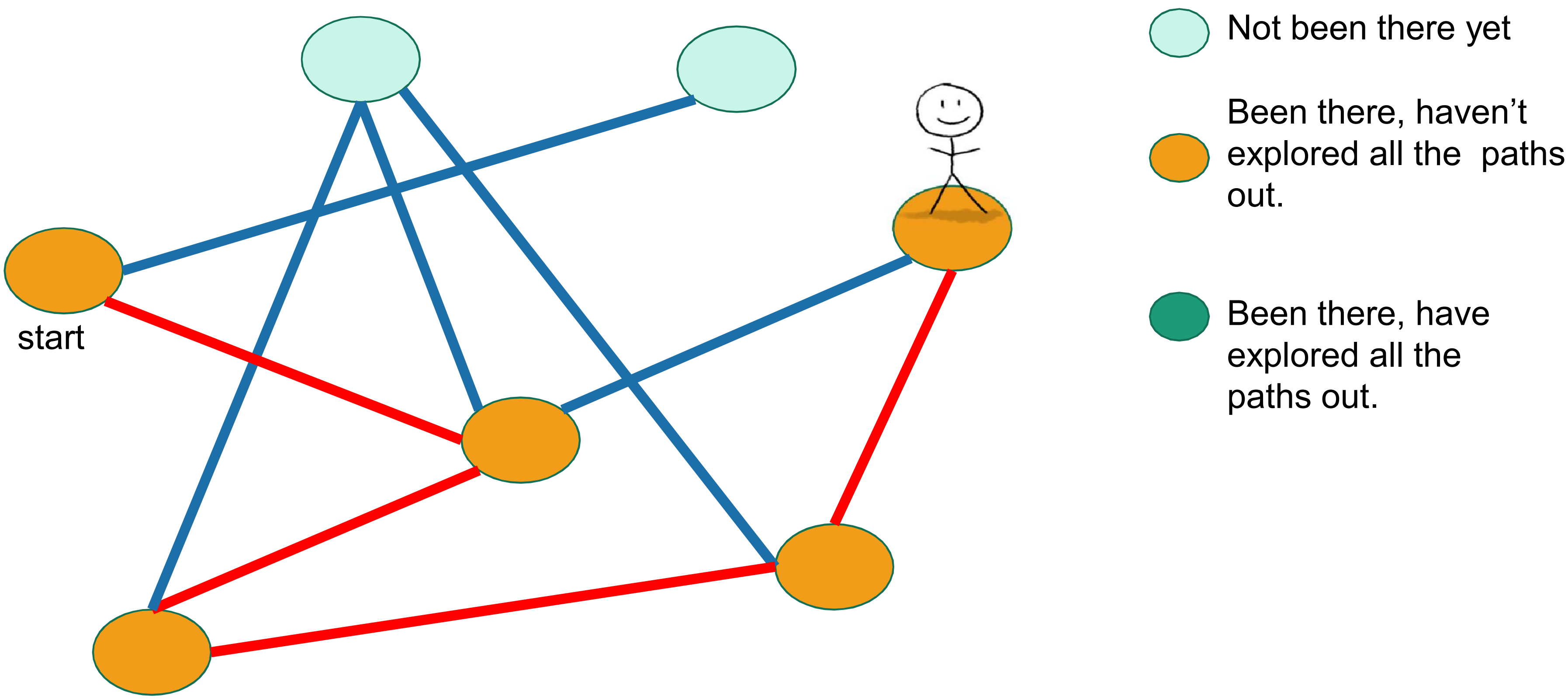

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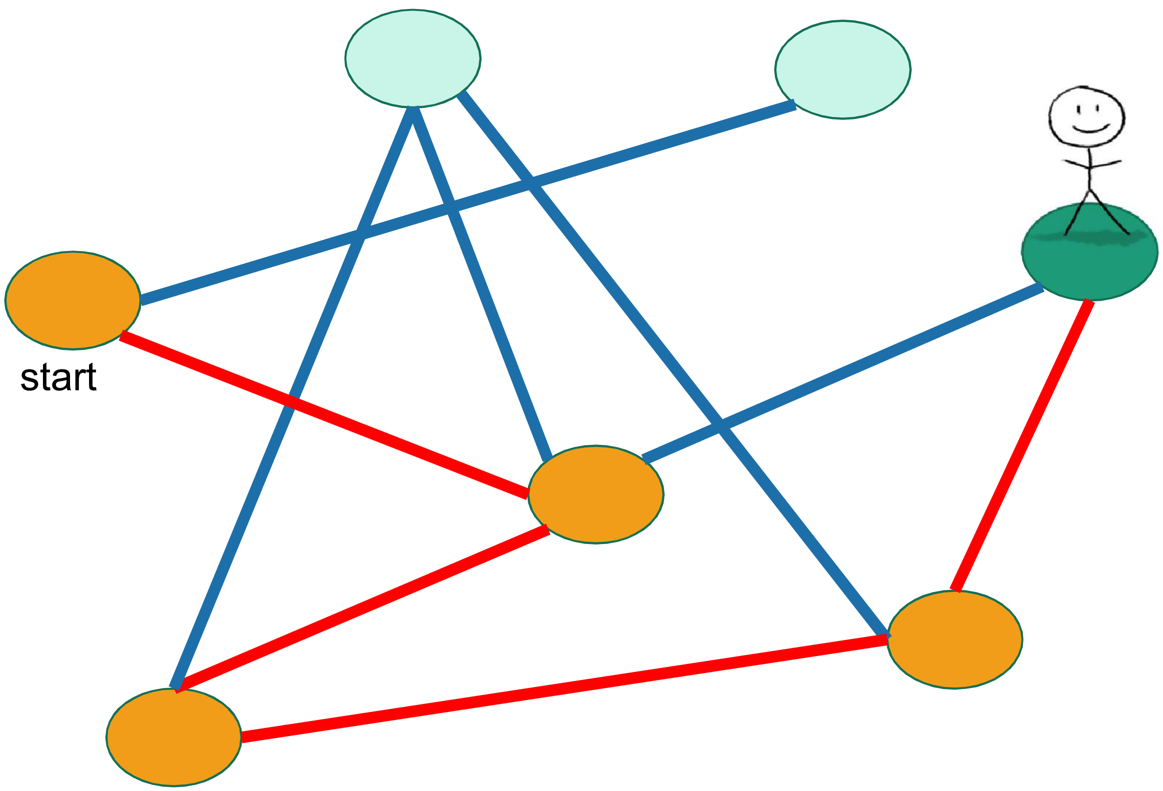

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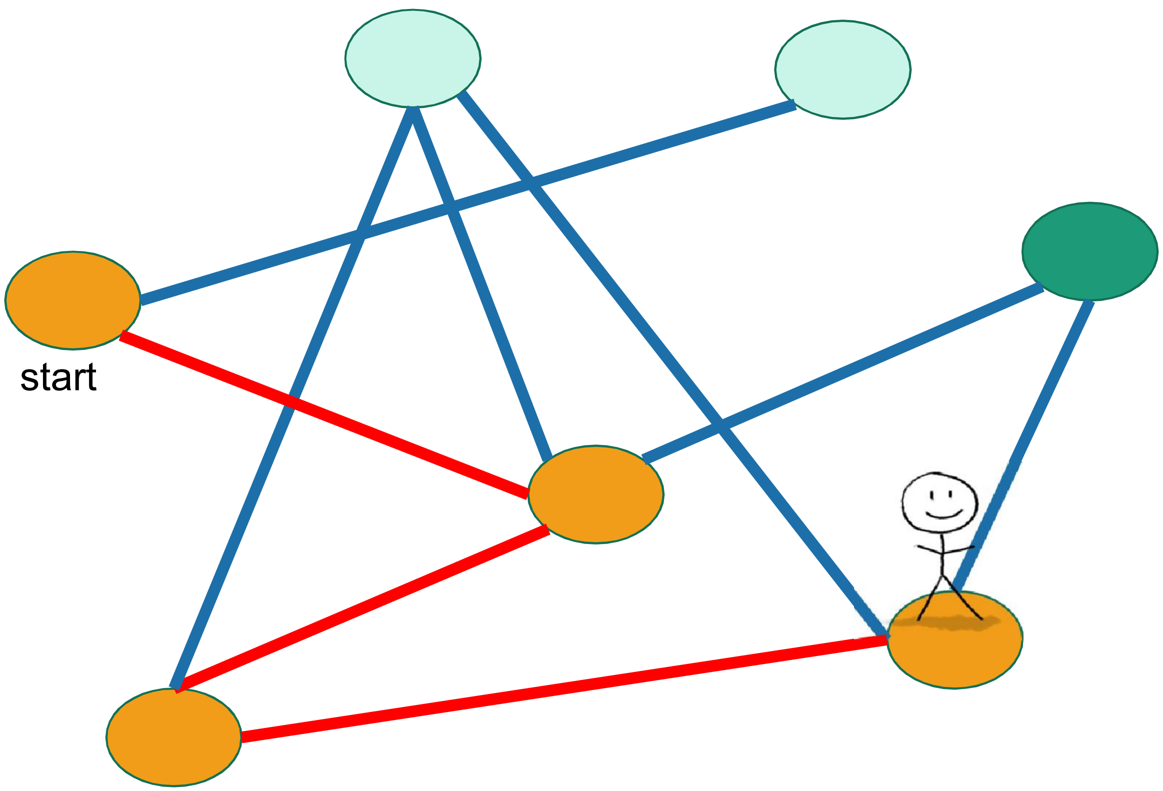

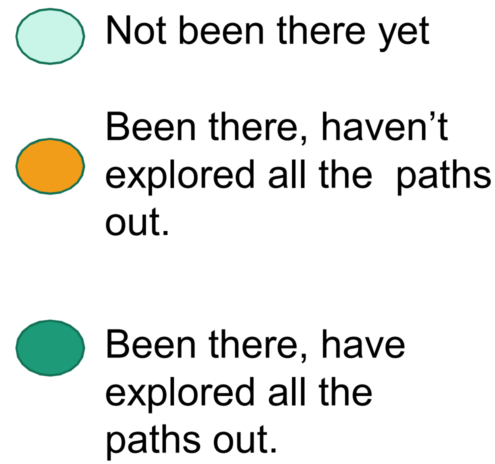

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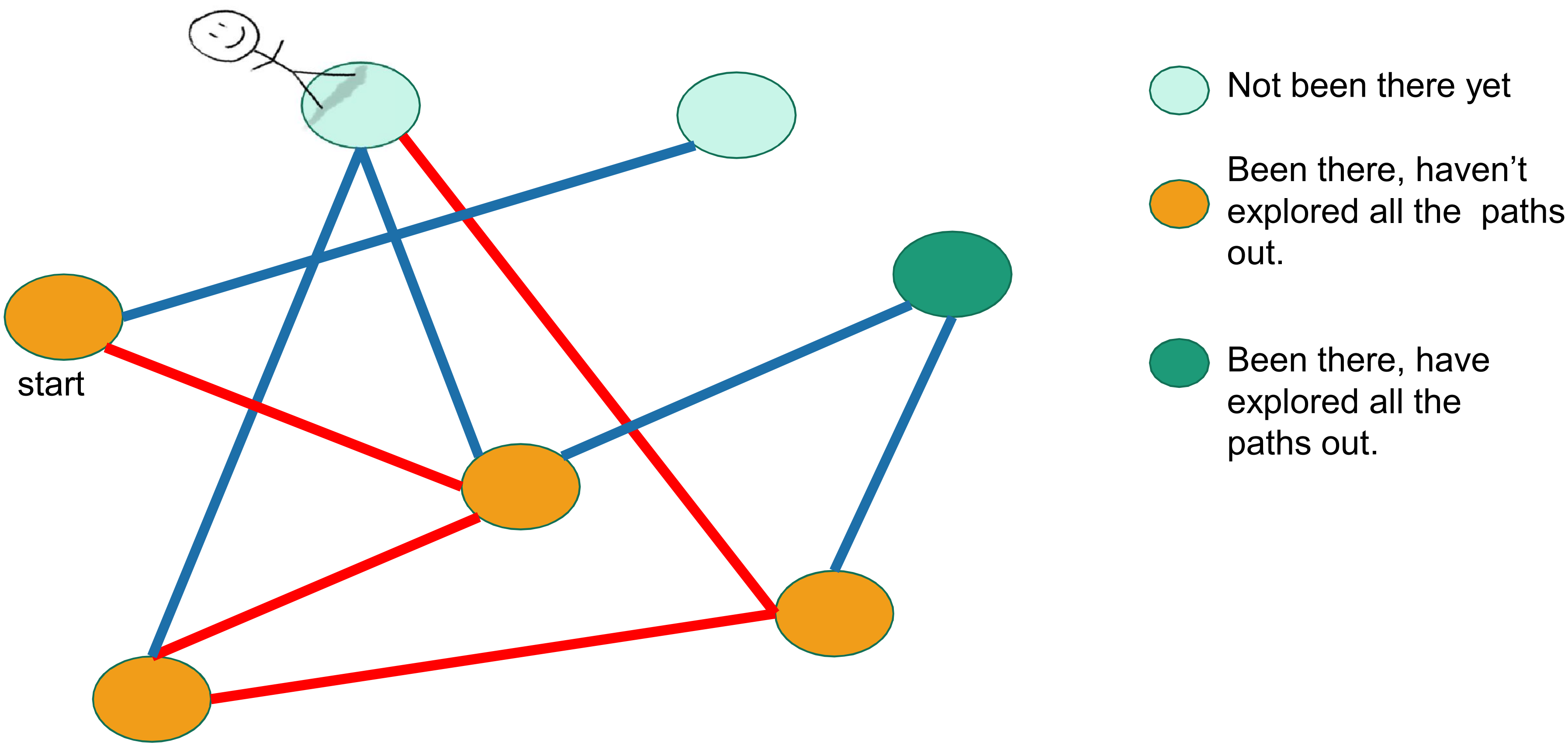

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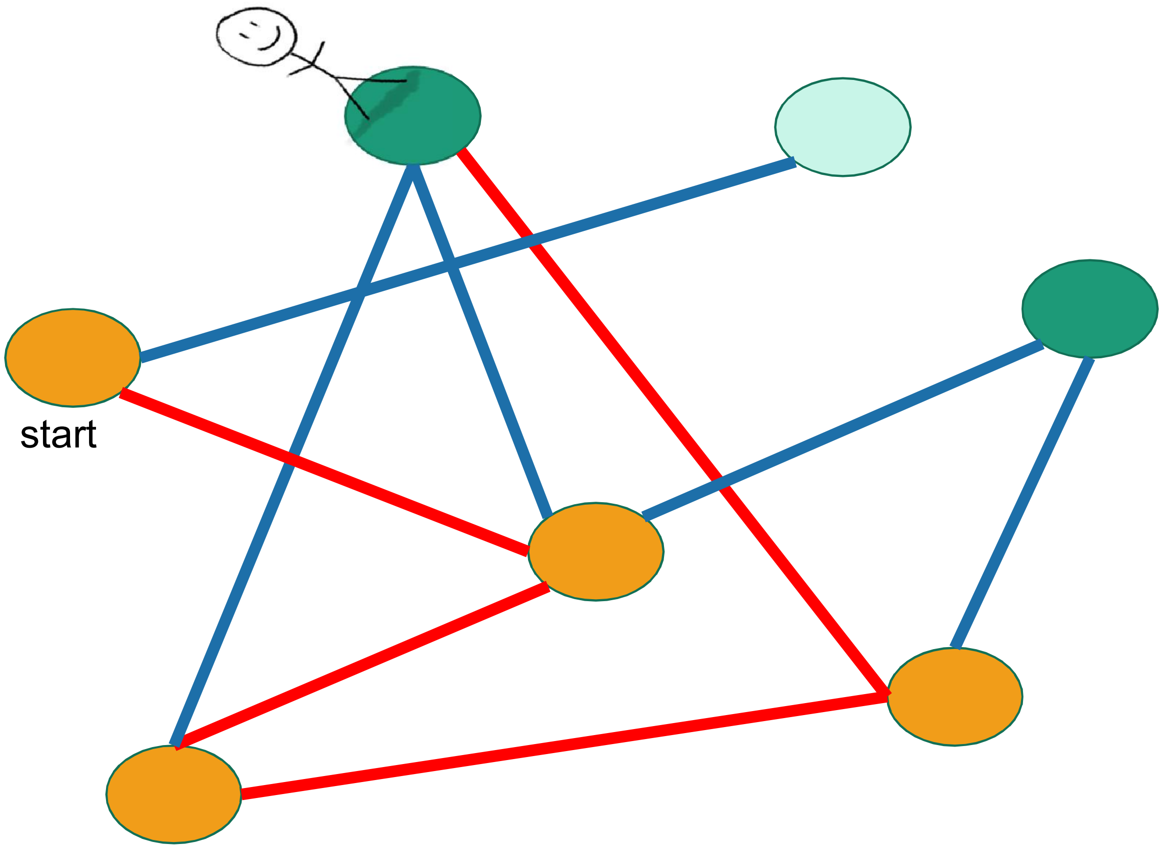

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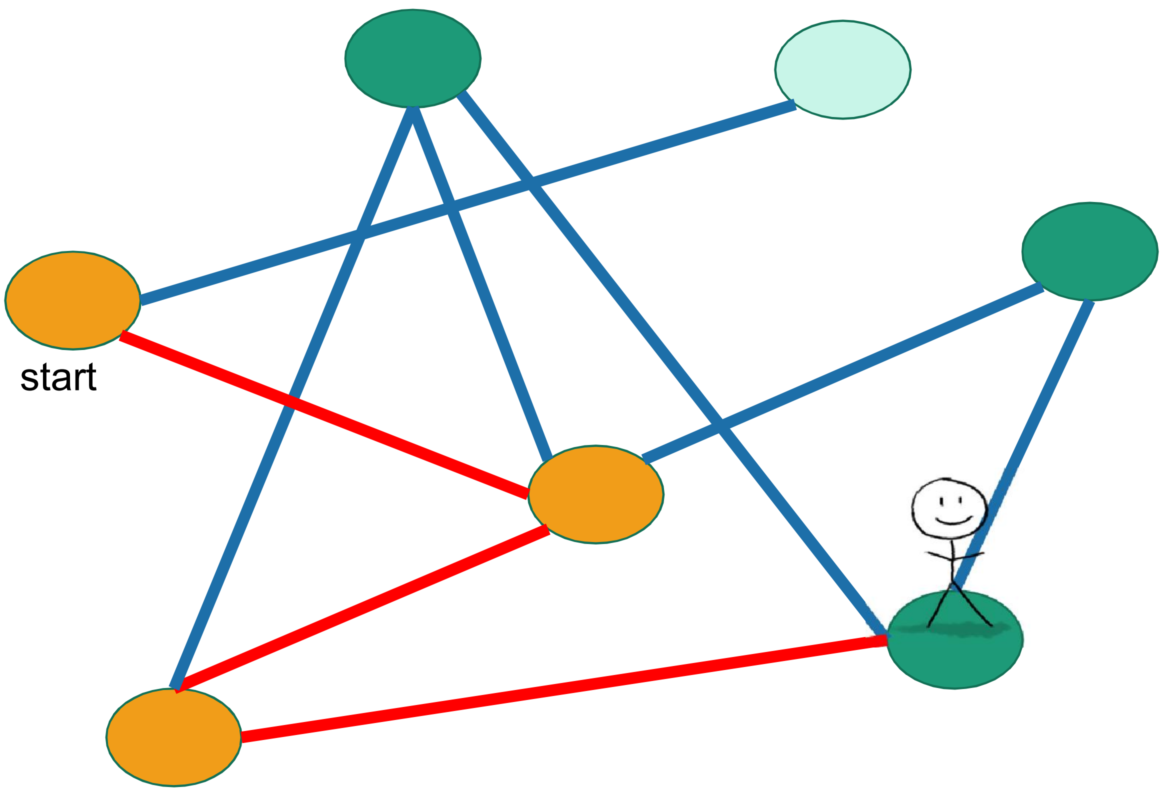

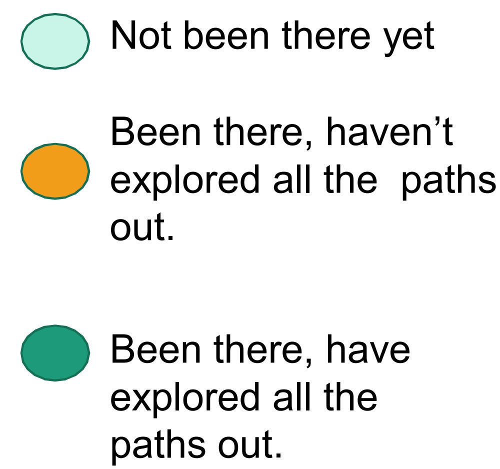

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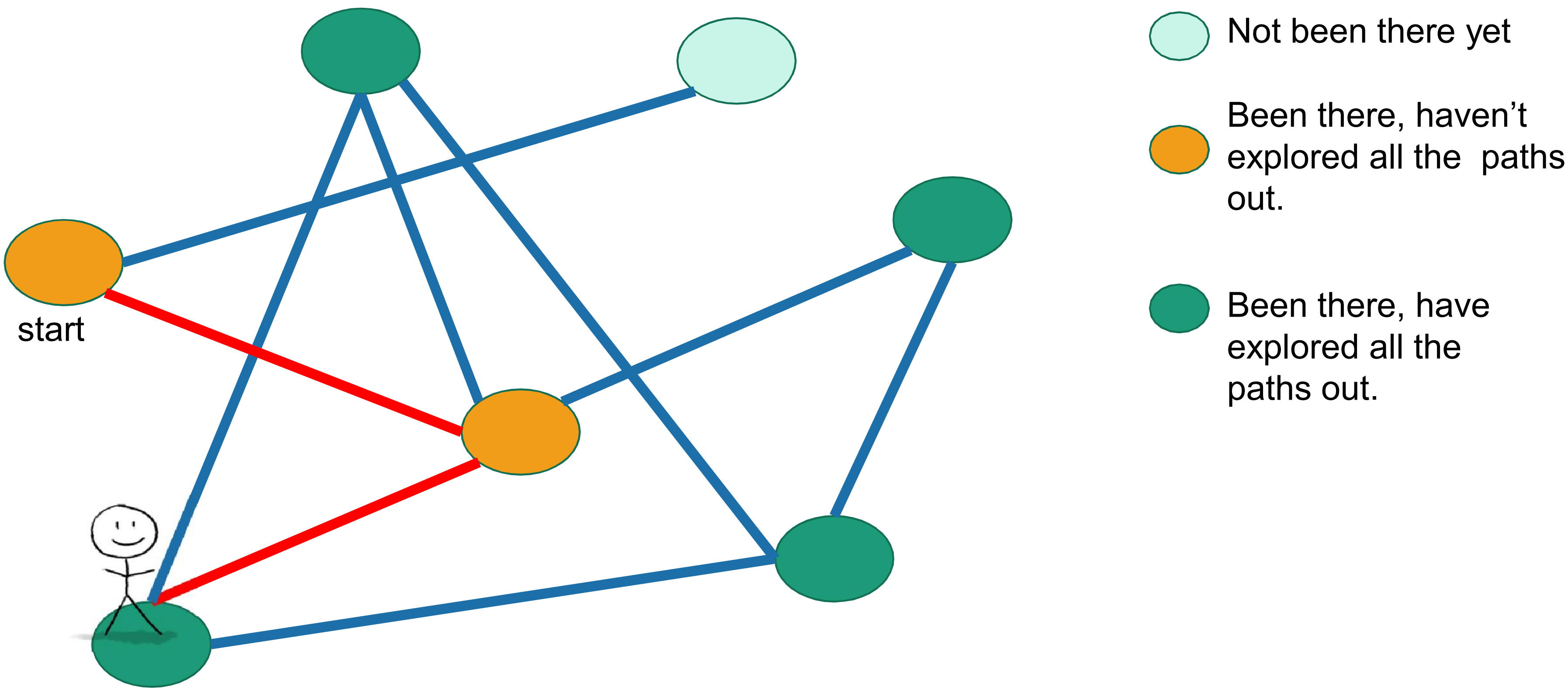

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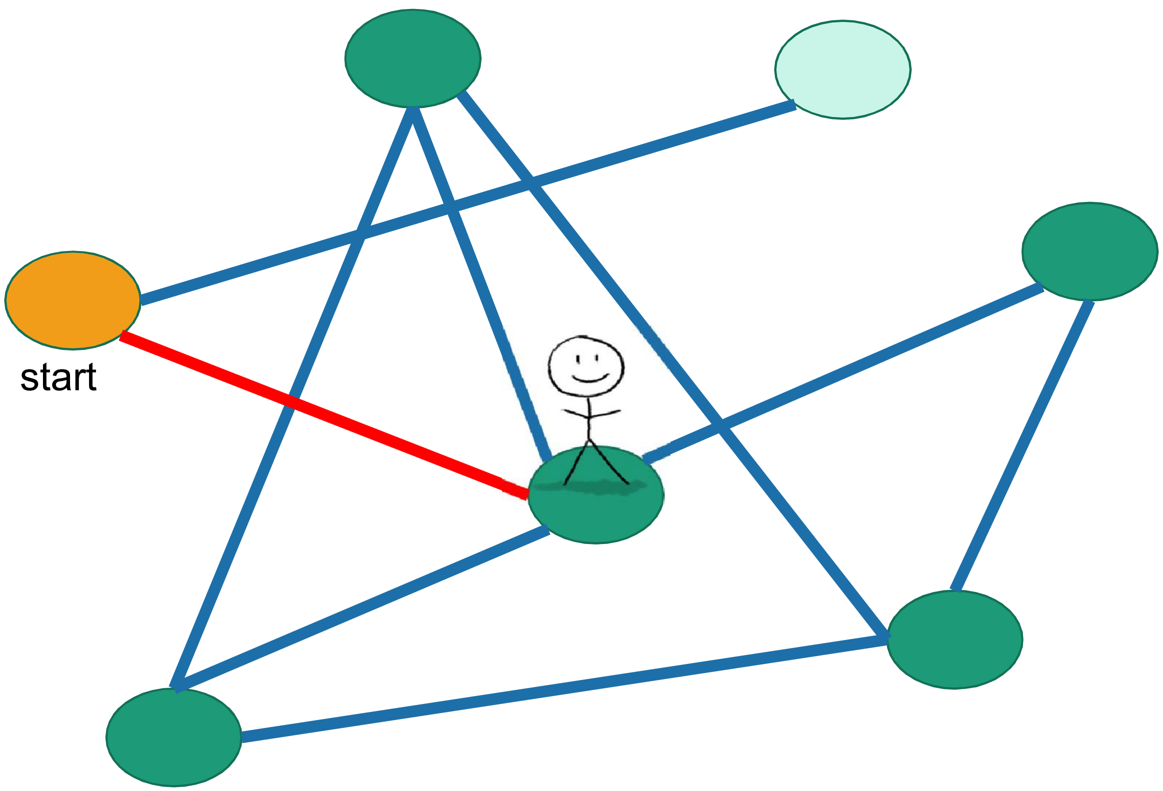

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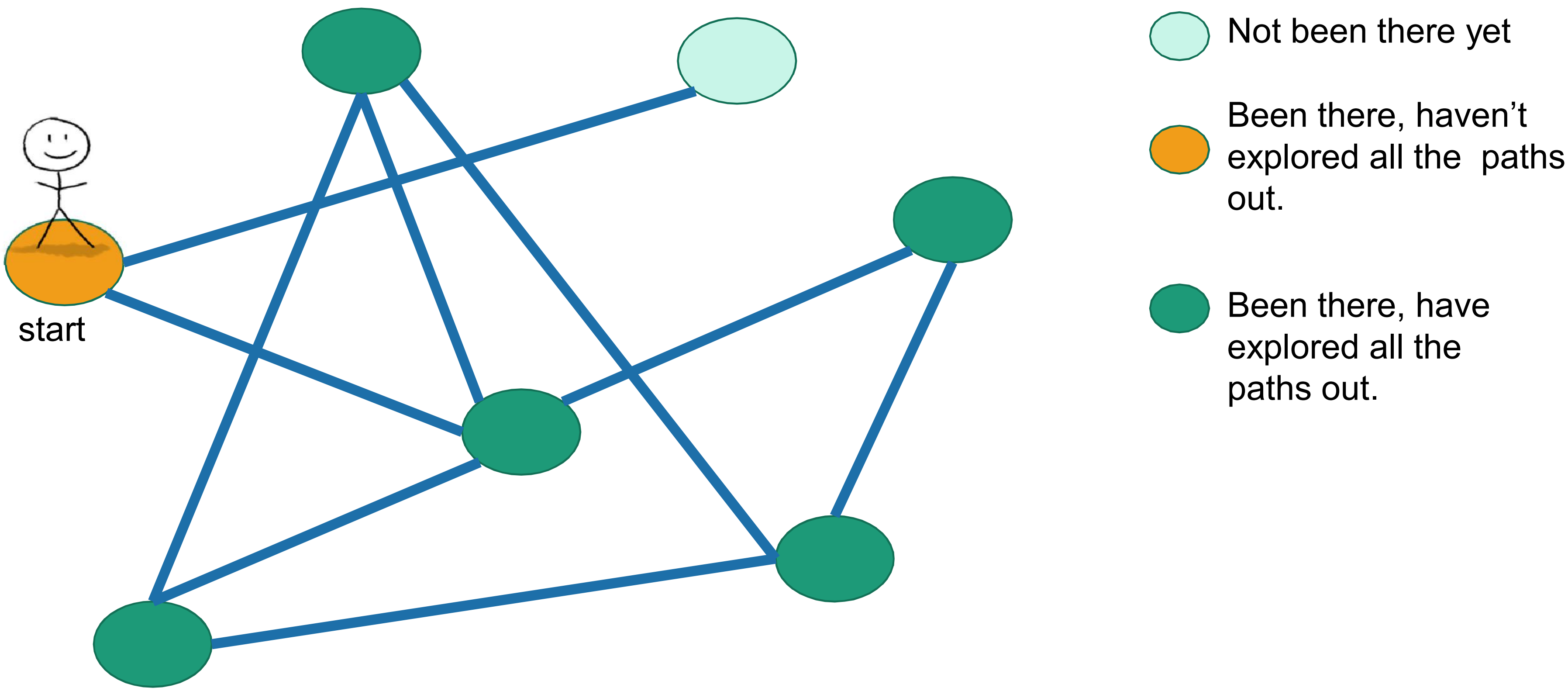

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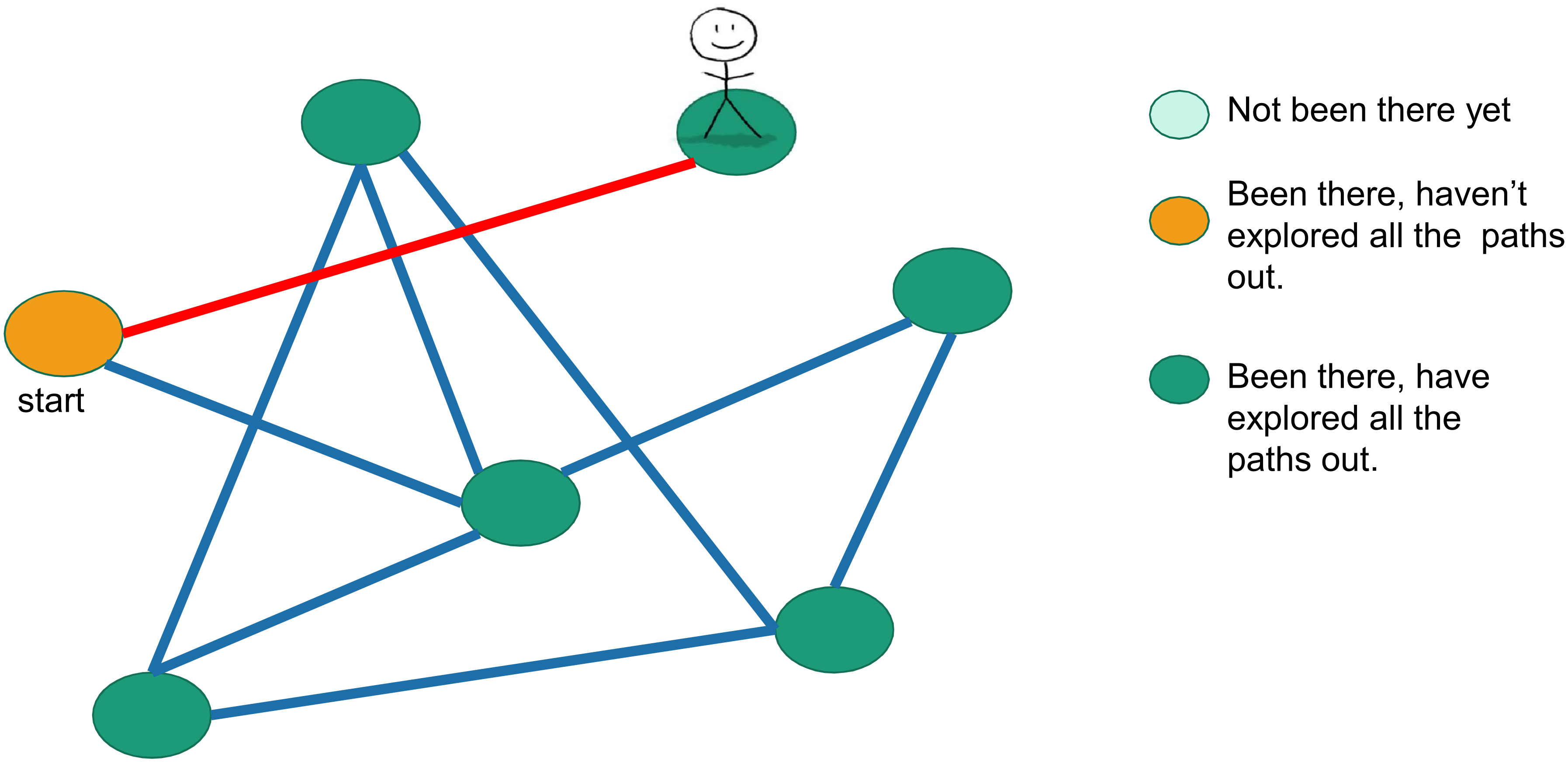

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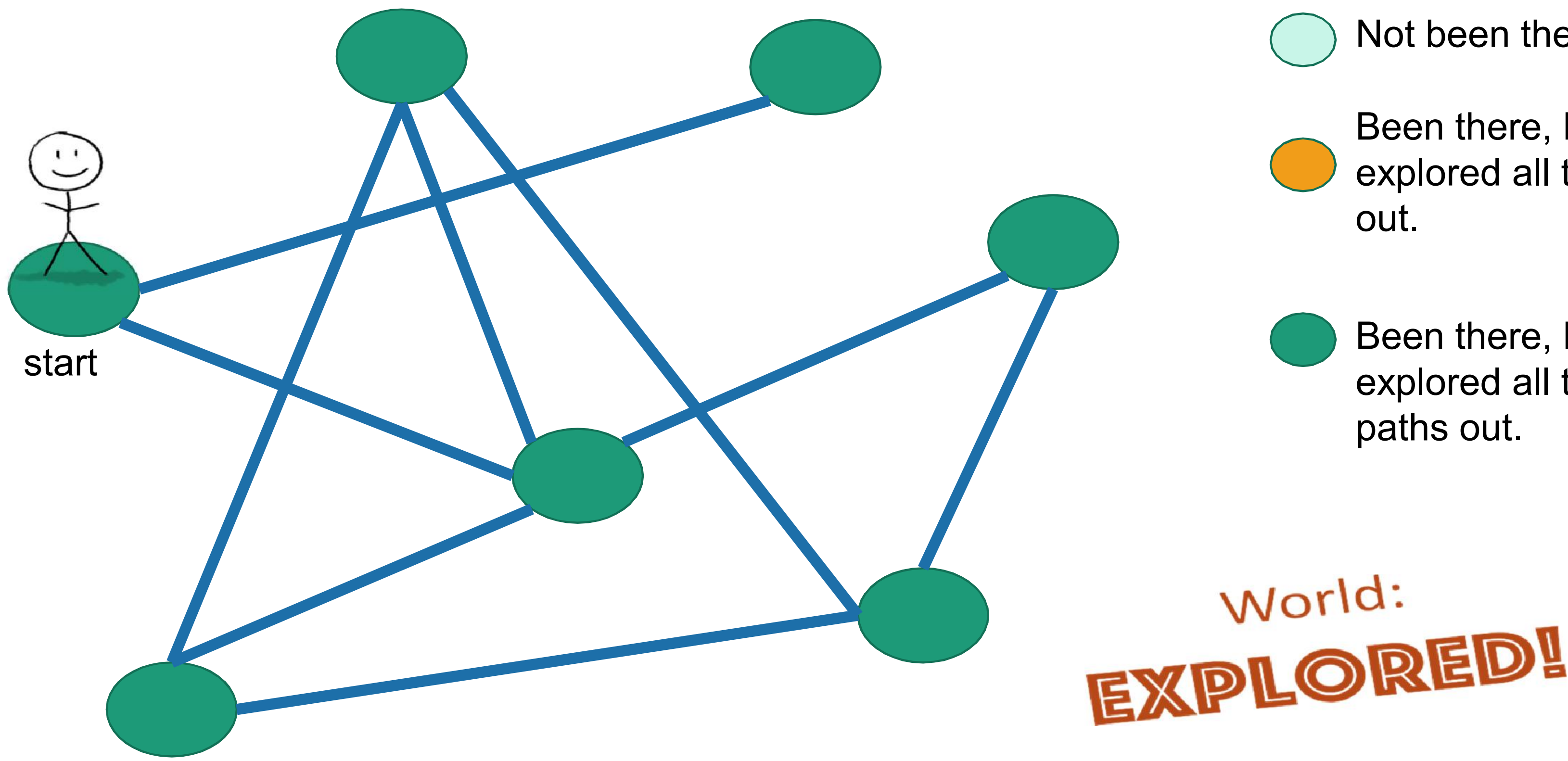

BFS and DFS - comparison

- Space complexity of DFS is lower than that of BFS.
- Time complexity of both is same – $O(|V|+|E|)$.
- The behavior differs for graphs where not all the vertices can be reached from the given vertex s.
- Predecessor subgraphs produced by DFS may be different than those produced by BFS. The BFS product is just one tree whereas the DFS product may be multiple trees.