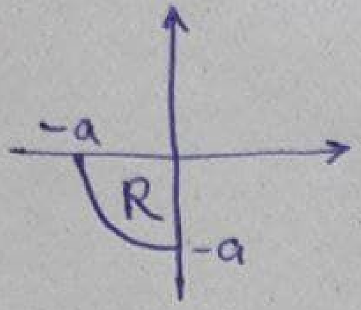
۹۹۳۱۰۳۰

اشکان شکیبا

۱۲) بنابر قضیه گرین می‌توان نوشت:




$$\oint M(x,y)dx + N(x,y)dy$$

$$= \int_R \int \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

$$M(x,y) = 2x^3 - 4y^3 \Rightarrow \frac{\partial M}{\partial y} = -12y^2$$

$$N(x,y) = 4x^3 + 8y^3 \Rightarrow \frac{\partial N}{\partial x} = 12x^2$$

$$\oint (2x^3 - 4y^3)dx + (4x^3 + 8y^3)dy = \int_R \int (12x^2 + 12y^2)\, dx dy$$

از مختصات قطبی استفاده می‌کنیم:

$$x = r\cos\theta, \quad y = r\sin\theta$$

$$J(r,\theta) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r$$

$$\Rightarrow dx dy = r\, dr d\theta$$

$$\int_R \int 12(x^2+y^2)\, dx dy = \int_{\pi}^{\frac{3\pi}{2}} \int_0^a 12r^2 . r . dr . d\theta = \int_{\pi}^{\frac{3\pi}{2}} 3r^4 \Big|_0^a d\theta$$

$$= 3a^4\theta \Big|_{\pi}^{\frac{3\pi}{2}} = \frac{3\pi}{2} a^4$$