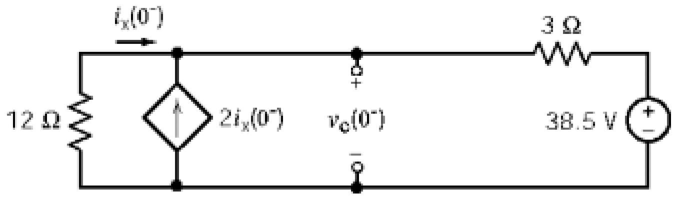
جواب تمرینات سری سوم

۲-

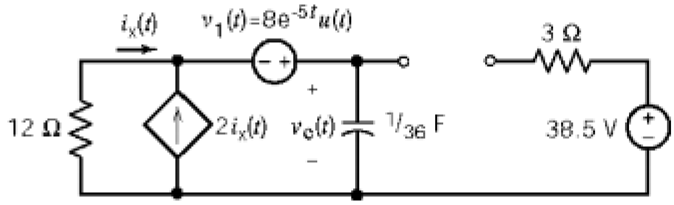
Solution: Assume that the circuit is at steady state before $t = 0$:



$$\text{KVL}: \ 12i_x + 3(3\,i_x) + 38.5 = 0 \ \Rightarrow \ i_x = -1.8\bar{3} \text{ A}$$

$$\text{Then } v_c(0^-) = -12\,i_x = 22 \text{ V} = v_c(0^+)$$

After $t = 0$:



$$\text{KVL}: \ 12i_x(t) - 8e^{-5t} + v_c(t) = 0$$

$$\text{KCL}: -ix(t) - 2ix(t) + (1/36)\frac{dv_c(t)}{dt} = 0 \ \Rightarrow \ ix(t) = \frac{1}{108}\frac{dv_c(t)}{dt}$$

$$\therefore \ 12\left[\frac{1}{108}\frac{dv_c(t)}{dt}\right] - 8e^{-5t} + v_c(t) = 0$$

$$\frac{dv_c(t)}{dt} + 9v_c(t) = 72e^{-5t} \ \Rightarrow \ v_{cn}(t) = Ae^{-9t}$$

Try $v_{cf}(t) = Be^{-5t}$ & substitute into the differential equation $\Rightarrow B = 18$

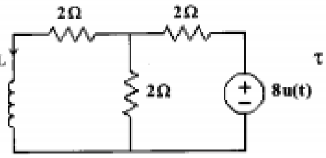
$$\therefore \ v_c(t) = Ae^{-9t} + 18\,e^{-5t}$$

$$v_c(0) = 22 = A + 18 \ \Rightarrow \ A = 4$$

$$\therefore v_c(t) = 4e^{-9t} + 18e^{-5t} \text{ V}$$

ابتدا مدار را در بازه زمانی $۰ < t < \frac{۲}{۳}$ تحلیل می‌کنیم. در این حالت کلید باز بوده و مدار به صورت زیر است.

$$I_L(۰^-) = I_L(۰^+) = ۰ \quad , \quad I_L(\infty) = \left(\frac{۸}{۲+۲\|۲}\right)\times\frac{۱}{۲} = \frac{۴}{۳}A$$



$$\tau = \frac{L}{R_{th}} \quad , \quad R_{th} = ۲+۲\|۲ = ۳\Omega \;\Rightarrow \tau = \frac{۲}{۳}\sec \Rightarrow I_L(t) = I_L(\infty) + [I_L(۰^+) - I_L(\infty)]e^{-\frac{t}{\tau}}$$

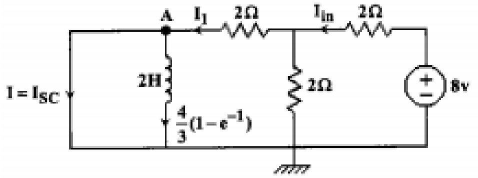
$$\Rightarrow I_L(t) = \frac{۴}{۳} + [۰ - \frac{۴}{۳}]e^{-\frac{۳}{۲}t} = \frac{۴}{۳} - \frac{۴}{۳}e^{-\frac{۳}{۲}t}$$

حال جریان سلف را در $t = \frac{۲}{۳}\sec$ به دست می‌آوریم.

$$I_L(t = \frac{۲}{۳}) = \frac{۴}{۳} - \frac{۴}{۳}e^{-\frac{۳}{۲}\times\frac{۲}{۳}} = \frac{۴}{۳} - \frac{۴}{۳}\times e^{-۱} = \frac{۴}{۳}[۱ - e^{-۱}]$$

بعد از بسته شدن کلید، با توجه به اتصال کوتاه شدن سلف، مقدار R_{th} از دو سر آن صفر شده و لذا ثابت زمانی مدار بی‌نهایت می‌شود.

بنابراین جریان سلف برای همه زمان‌ها بعد از $t = \frac{۲}{۳}$ ثابت می‌ماند و به اندازه $I_L(t = \frac{۲}{۳}) = \frac{۴}{۳}(۱ - e^{-۱})$ خواهد شد.



$$I_{in} = \frac{۸}{۲+۲\|۲} = \frac{۸}{۳}A \qquad I_۱ = \frac{۱}{۲}I_{in} = \frac{۸}{۳}\times\frac{۱}{۲} = \frac{۴}{۳}A$$

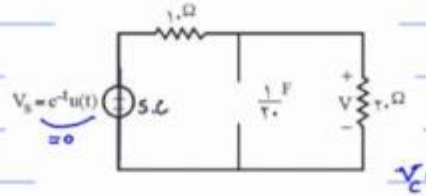
با نوشتن KCL در گره A داریم:

$$I_{sc} + I_L = I_۱ \;\Rightarrow I_{sc} + \frac{۴}{۳}(۱ - e^{-۱}) = \frac{۴}{۳} \;\Rightarrow I_{sc} = I = \frac{۴}{۳}e^{-۱}$$

۴-

$v = v_c$

تحلیل 0^-

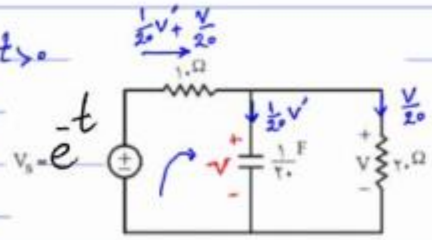


چون منبع مستقل در مدار در $t=0^-$ نداریم

تمام متغیرها صفر هستند. یعنی: $v_c(0^-) = 0$

چون مدار خاص نیست متغیر حالت پیوسته خواهد بود $v_c(0^+) = v_c(0^-) = 0$

$t > 0$



KVL $\quad -e^{-t} + 10\left(\frac{1}{20}v' + \frac{v}{20}\right) + v = 0 \;\; ① \xrightarrow{t=0^+}$

$: \quad -1 + \frac{1}{2}v'(0^+) + \frac{1}{2}v(0^+) + v(0^+) = 0$

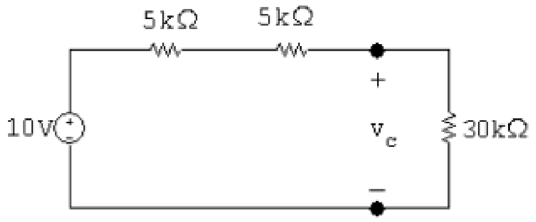
$v'(0^+) = 2\left(1 - \frac{3}{2} \times 0\right) = 2$

مشتق معادله ① : $\quad e^{-t} + \frac{1}{2}v'' + \frac{1}{2}v' + v' = 0 \xrightarrow{t=0^+} 1 + \frac{1}{2}v''(0^+) + \frac{3}{2}\underbrace{v'(0^+)}_{2} = 0$

$\Rightarrow v''(0^+) = -8 \;\frac{v}{sec^2}$

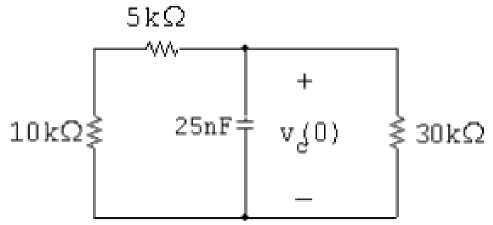
Note that for $t > 0$, $v_o = (10/15)v_c$, where v_c is the voltage across the 25 nF capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{30}{40}(10) = 7.5\,\text{V}$$

$0 \le t \le 0.2\,\text{ms}$:



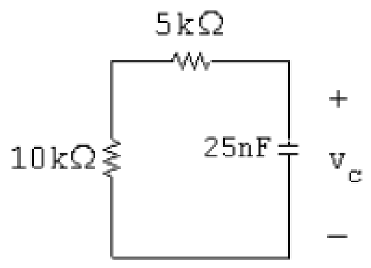
$$\tau = R_eC, \qquad R_e = 15{,}000 \| 30{,}000 = 10\,\text{k}\Omega$$

$$\tau = (10 \times 10^3)(25 \times 10^{-9}) = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$$

$$v_c = 7.5e^{-4000t}\,\text{V}, \qquad t \ge 0$$

$$v_c(0.2\,\text{ms}) = 7.5e^{-0.8} = 3.37\,\text{V}$$

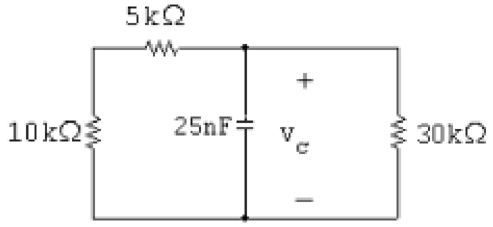
$0.2\,\text{ms} \le t \le 0.8\,\text{ms}$:



$\tau = (15 \times 10^3)(2.5 \times 10^{-9}) = 375\,\mu\text{s}, \qquad \frac{1}{\tau} = 2666.67$

$v_c = 3.37e^{-2666.67(t-200\times10^{-6})}\,\text{V}$

$0.8\,\text{ms} \le t <$:



$\tau = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$

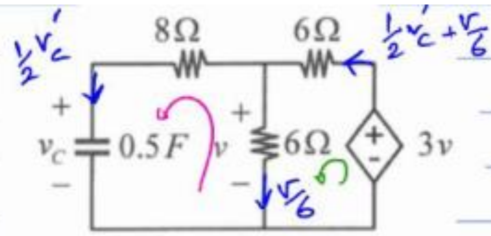
$v_c(0.8\,\text{ms}) = 3.37e^{-2666.67(800-200)10^{-6}} = 3.37e^{-1.6} = 0.68\,\text{V}$

$v_c = 0.68e^{-4000(t-0.8\times10^{-3})}\,\text{V}$

$v_c(1\,\text{ms}) = 0.68e^{-4000(1-0.8)10^{-3}} = 0.68e^{-0.8} = 0.306\,\text{V}$

$v_o = (10/15)(0.306) = 0.204\,\text{V}$

۶-



می‌توان لحظه $t_0 = 0$ فرض کنیم

$$v_c(0^-) = v_c(0^+) = 2$$

$$v_c(t_1) = \frac{1}{2} \times 2 = 1$$

$$\text{KVL}: -3v + 6\left(\frac{1}{2}v_c' + \frac{v}{6}\right) + v = 0$$

$$-v + 8\left(\frac{1}{2}v_c'\right) + v_c = 0$$

$$-4v_c' - v_c + 3v_c' = 0 \longrightarrow v_c' + v_c = 0$$

$$v_c(t) = 2e^{-t} = 1 \longrightarrow e^{-t} = \frac{1}{2} \xrightarrow{\ln} -t = -\ln 2$$

$$\boxed{t_1 = \ln 2 \ (s)}$$