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اشکان شکیبا

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$x = r\cos\theta ,\ y = r\sin\theta ,\ z = z$

$$\Rightarrow J(r,\theta,z) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

$$\Rightarrow dV = dx\,dy\,dz = r\,dr\,d\theta\,dz$$

$y = \frac{\sqrt{3}}{3}x \rightarrow \theta = \frac{\pi}{6} ,\quad y = \sqrt{3}x \rightarrow \theta = \frac{\pi}{3}$

$x^2+y^2 = 1 \Rightarrow r = 1 ,\quad x^2+y^2 = 4 \Rightarrow r = 2$

$$z = \frac{(x^2+y^2)^3}{x^2} = \frac{r^6}{r^2\cos^2\theta} = r^4\sec^2\theta$$

$$V = \iiint dV = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \int_1^2 \int_0^{r^4\sec^2\theta} r\,dr\,d\theta\,dz = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \int_1^2 z\Big|_0^{r^4\sec^2\theta} \cdot r\,dr\,d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \int_1^2 r^5\sec^2\theta\,dr\,d\theta = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{r^6}{6}\Big|_1^2 \cdot \sec^2\theta\,d\theta$$

$$= \frac{63}{6} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2\theta\,d\theta = \frac{21}{2}\tan\theta\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$\Rightarrow V = \frac{21}{2}\left(\sqrt{3} - \frac{\sqrt{3}}{3}\right) = 7\sqrt{3}$$

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اشکان شکیبا

۱۰) داریم $2x-3y+z=6$ ؛ بنابراین:

$$(x,z)=(0,0) \Rightarrow -3y=6 \Rightarrow y=-2$$

$$(y,z)=(0,0) \Rightarrow 2x=6 \Rightarrow x=3$$

$$z=0 \Rightarrow 2x-3y=6 \Rightarrow y=-2+\frac{2}{3}x$$

$$\iiint y\,dV=\int_0^3\int_{-2+\frac{2}{3}x}^{0}\int_0^{6-2x+3y} y\,dz\,dy\,dx$$

$$=\int_0^3\int_{-2+\frac{2}{3}x}^{0} z\Big|_0^{6-2x+3y} y\,dy\,dx=\int_0^3\int_{-2+\frac{2}{3}x}^{0} y(6-2x+3y)\,dy\,dx$$

$$=\int_0^3 \frac{y^2}{2}(6-2x)+y^3\Big|_{-2+\frac{2}{3}x}^{0}dx$$

$$=\int_0^3 -\left(\left(-2+\frac{2}{3}x\right)^2\frac{6-2x}{2}+\left(-2+\frac{2}{3}x\right)^3\right)dx$$

$$=\int_0^3 -\left(-\frac{3}{2}+1\right)\left(-2+\frac{2}{3}x\right)^3dx$$

u را به شکل روبرو تعریف می‌کنیم:

$$u=-2+\frac{2}{3}x \Rightarrow du=\frac{2}{3}dx \Rightarrow dx=\frac{3}{2}du$$

$$\Rightarrow x=0\to u=2\ ,\ x=3\to u=0$$

$$\Rightarrow \int_0^3\frac{1}{2}\left(-2+\frac{2}{3}x\right)^3dx=\frac{1}{2}\int_2^0 u^3\times\frac{3}{2}du=\frac{3u^4}{16}\Big|_2^0=-3$$