مشتقات $\sec x$, $\csc x$:

$$* \quad \sec x = \frac{1}{\cos x}$$

$$\Rightarrow \quad \frac{d(\sec x)}{dx} = \sec x \; tg\, x$$

$$* \quad \csc x = \frac{1}{\sin x}$$

$$\Rightarrow \quad \frac{d(\csc x)}{dx} = -\csc x \; cotg\, x$$

وارون توابع مثلثاتی :

① $y = \sin x \;;\; -\pi/2 \leq x \leq \pi/2$ (یک به یک)

$$\Rightarrow \sin^{-1} x : [-1, 1] \longrightarrow [-\pi/2, \pi/2]$$

$$\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

اثبات :

$$y = \sin^{-1} x \Longleftrightarrow x = \sin y$$

$$\Rightarrow 1 = \frac{dx}{dx} = \cos y \left(\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

(2) $y = \cos x \; ; \; 0 \leq x \leq \pi$ (یک به یک)

$\Rightarrow \cos^{-1} x : [-1, 1] \longrightarrow [0, \pi]$

$$\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-dx}{\sqrt{1-x^2}} = \cos^{-1} x + C$$

③ $y = tg\,x$ و $-\frac{\pi}{2} < x < \frac{\pi}{2}$ (یک به یک)

$$\Rightarrow tg^{-1}x : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\frac{d(tg^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\int \frac{dx}{1+x^2} = tg^{-1}x + C$$

$$\Rightarrow \int \frac{dx}{x^2+a^2} = \frac{1}{a}\, tg^{-1}\left(\frac{x}{a}\right) + C.$$

(4) $y = \cot x \; ; \; 0 < x < \pi$ (یک به یک)

$\Rightarrow \quad \cot^{-1} x : \mathbb{R} \longrightarrow (0, \pi)$

$$\frac{d(\cot^{-1} x)}{dx} = \frac{-1}{1+x^2}$$

$$\int \frac{-dx}{1+x^2} = \cot^{-1} x + C.$$

5) $y = \sec x$; یک به یک $[0,\pi]-\{\frac{\pi}{2}\}$ بر

$\Rightarrow \sec^{-1} x : (-\infty, -1] \cup [1, +\infty) \to [0,\pi]-\{\frac{\pi}{2}\}$

$$\frac{d(\sec^{-1} x)}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1} x + C$$

⑥ $y = \csc x$; بر $[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$ یک به یک

$$\Rightarrow \csc^{-1} x : (-\infty,-1] \cup [1,\infty) \longrightarrow [-\tfrac{\pi}{2},\tfrac{\pi}{2}]-\{0\}$$

$$\frac{d(\csc^{-1} x)}{dx} = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\int \frac{-dx}{|x|\sqrt{x^2-1}} = \csc^{-1} x + c$$

توابع هذلولوی

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{1}{\tanh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

قضیه: $\cosh^2 x - \sinh^2 x = 1$

* به‌ازای هر t، نقطه $(\cosh t, \sinh t)$ روی هذلولی $x^2-y^2=1$ قرار دارد.

* $\cosh x + \sinh x = e^x$

$\sinh(0)=0 \quad , \quad \cosh(0)=1$

$$\begin{cases} \cosh(-x) = \cosh x \\ \sinh(-x) = -\sinh x \end{cases}$$

$$\begin{cases} \cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y \\ \sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y \end{cases}$$

$$\Rightarrow \cosh(2x) = \cosh^2 x + \sinh^2 x$$

$$= 1 + 2\sinh^2 x = 2\cosh^2 x - 1$$

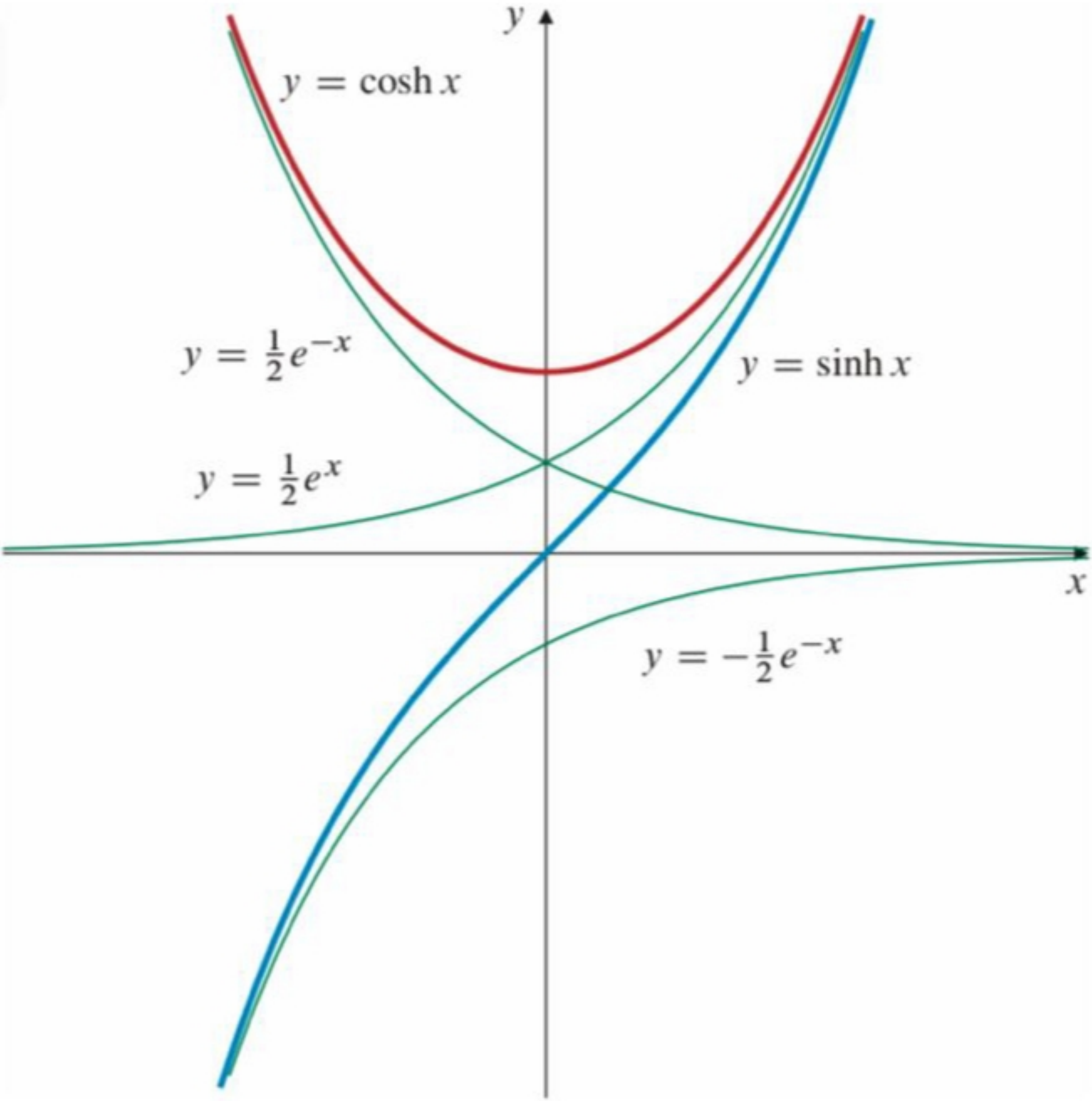
* $\sinh(2x) = 2\sinh x \cosh x$

* $\lim\limits_{x \to \pm\infty} \cosh x = +\infty$

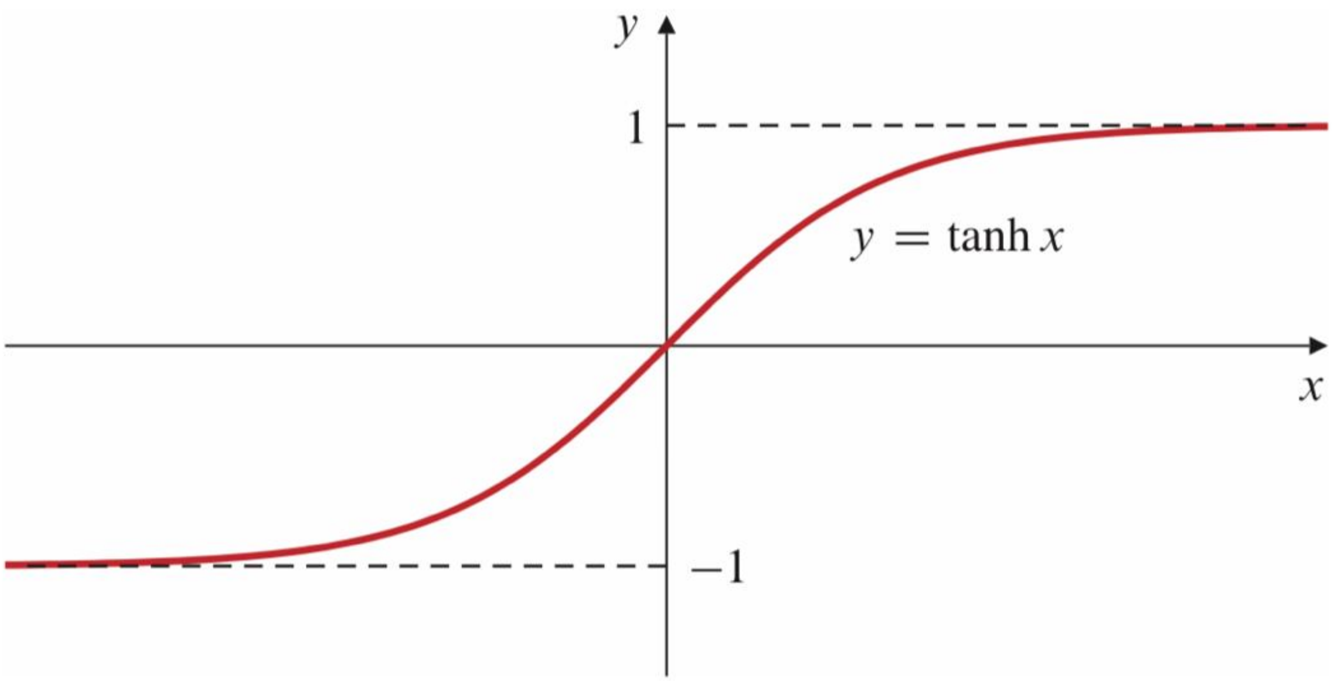
* $\begin{cases} \lim\limits_{x \to +\infty} \sinh x = +\infty \\ \lim\limits_{x \to -\infty} \sinh x = -\infty \end{cases}$

* $\begin{cases} \lim\limits_{x \to +\infty} \operatorname{tgh} x = 1 \\ \lim\limits_{x \to -\infty} \operatorname{tgh} x = -1 \end{cases}$

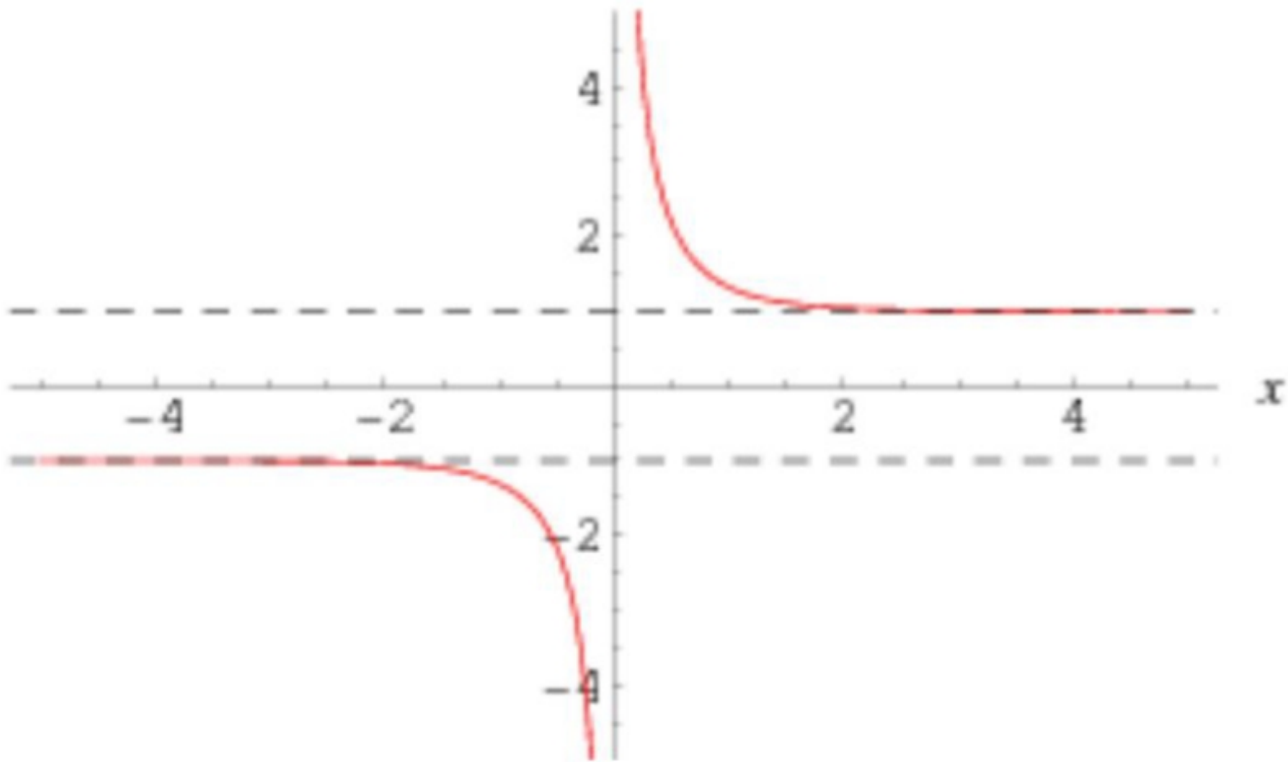
* $\begin{cases} \lim\limits_{x \to -\infty} \operatorname{cotgh} x = -1 \\ \lim\limits_{x \to +\infty} \operatorname{cotgh} x = 1 \end{cases}$



y
$y = \cosh x$
$y = \frac{1}{2}e^{-x}$
$y = \sinh x$
$y = \frac{1}{2}e^{x}$
x
$y = -\frac{1}{2}e^{-x}$


y
1
y = tanh x
x
−1

coth x



مشتق توابع هذلولوی

$$(\sinh)'(x) = \cosh x$$

$$(\cosh)'(x) = \sinh x$$

$$(\tanh)'(x) = \operatorname{sech}^2 x$$

$$(\coth)'(x) = -\operatorname{csch}^2 x$$

$$(\operatorname{sech})'x = -\operatorname{sech} x \ \tanh x$$

$$(\operatorname{csch})'x = -\operatorname{csch} x \ \coth x$$

قضیه:

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

* توجه کنید که $y=\sinh x$ اکیداً صعودی است،

پس یک به یک است و داریم:

$$y=\sinh^{-1}x \iff x=\sinh y.$$

$$\rightarrow x=\frac{e^y-e^{-y}}{2} \overset{\times \frac{e^y}{e^y}}{=\!=\!=} \frac{e^{2y}-1}{2e^y}$$

$$\rightarrow e^{2y}-2xe^{2y}-1=0$$

$$\Rightarrow (e^y)^2-2x(e^y)-1=0$$

$$\Rightarrow e^y=x \underset{-}{\overset{\text{قابل قبول}}{+}} \sqrt{x^2+1} \qquad (e^y>0)$$

$$\Rightarrow y=\operatorname{Ln}(x+\sqrt{x^2+1})$$

$$\Rightarrow \boxed{\sinh^{-1}x=\operatorname{Ln}(x+\sqrt{x^2+1})}$$

$$\Rightarrow \frac{d(\sinh^{-1} x)}{dx} = \frac{1}{\sqrt{x^2+1}}$$

$$\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1} x + C$$

* $\cosh^{-1} x = \mathrm{Ln}\left(x + \sqrt{x^2 - 1}\right)$; $x \geqslant 1$

* $\mathrm{tgh}^{-1} x = \frac{1}{2} \mathrm{Ln}\left(\frac{1+x}{1-x}\right)$; $-1 < x < 1$

* $\mathrm{Cotgh}^{-1} x = \frac{1}{2} \mathrm{Ln}\left(\frac{x+1}{x-1}\right)$; $|x| > 1$

مشتق وارون توابع هذلولوی

$f(x) = \sinh^{-1} x$ $\quad D_f = R_f = \mathbb{R}$

$$f'(x) = \frac{1}{\sinh'(\sinh^{-1}(x))} = \frac{1}{\cosh(\sinh^{-1}(x))} = \frac{1}{\sqrt{1+\sinh^2(\sinh^{-1}(x))}} = \frac{1}{\sqrt{1+x^2}}$$

$f(x) = \cosh^{-1} x$ $\quad D_f = [1, +\infty) \quad R_f = \mathbb{R}$ $\quad f'(x) = \dfrac{1}{\sqrt{x^2-1}}$

$f(x) = \tanh^{-1} x$ $\quad D_f = (-1, 1) \quad R_f = \mathbb{R}$ $\quad f'(x) = \dfrac{1}{1-x^2}$

$$\frac{d(\operatorname{Coth}^{-1} x)}{dx} = \frac{1}{1-x^2} \; ; \; |x| > 1$$