طراحی الگوریتم ها

مبحث یازدهم: گراف بدون وزن، BFS، و DFS

سجاد شیرعلی شهرضا

بهار 1402

سه شنبه، 23 اسفند 1401

Based on Stanford CS161 Slides from Summer 2021 by Karey Shi

اطلاع رسانی

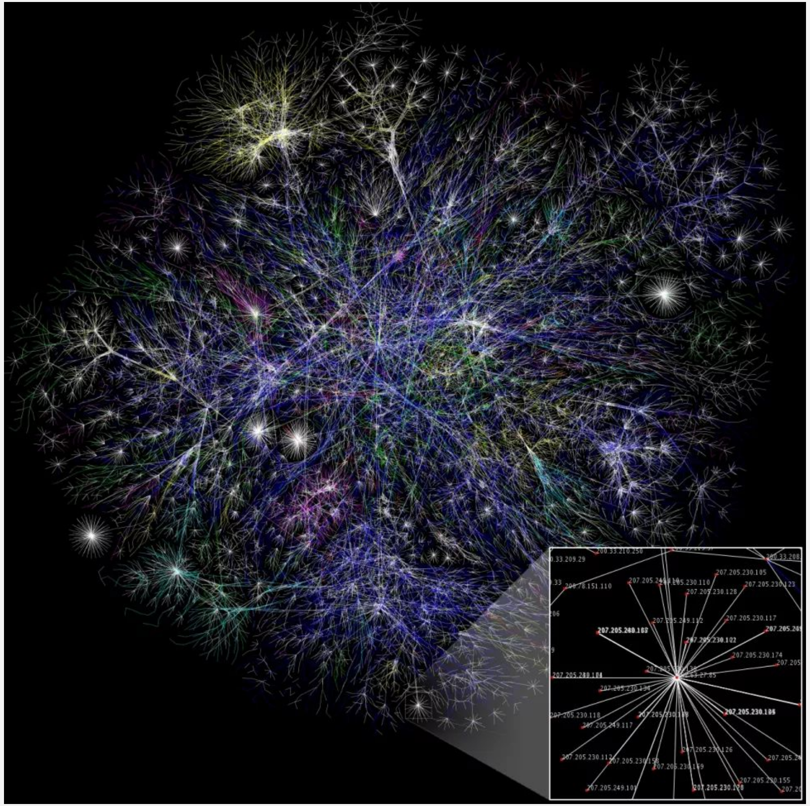
- بخش مرتبط کتاب برای این جلسه: 22

گراف

تعریف و نمونه

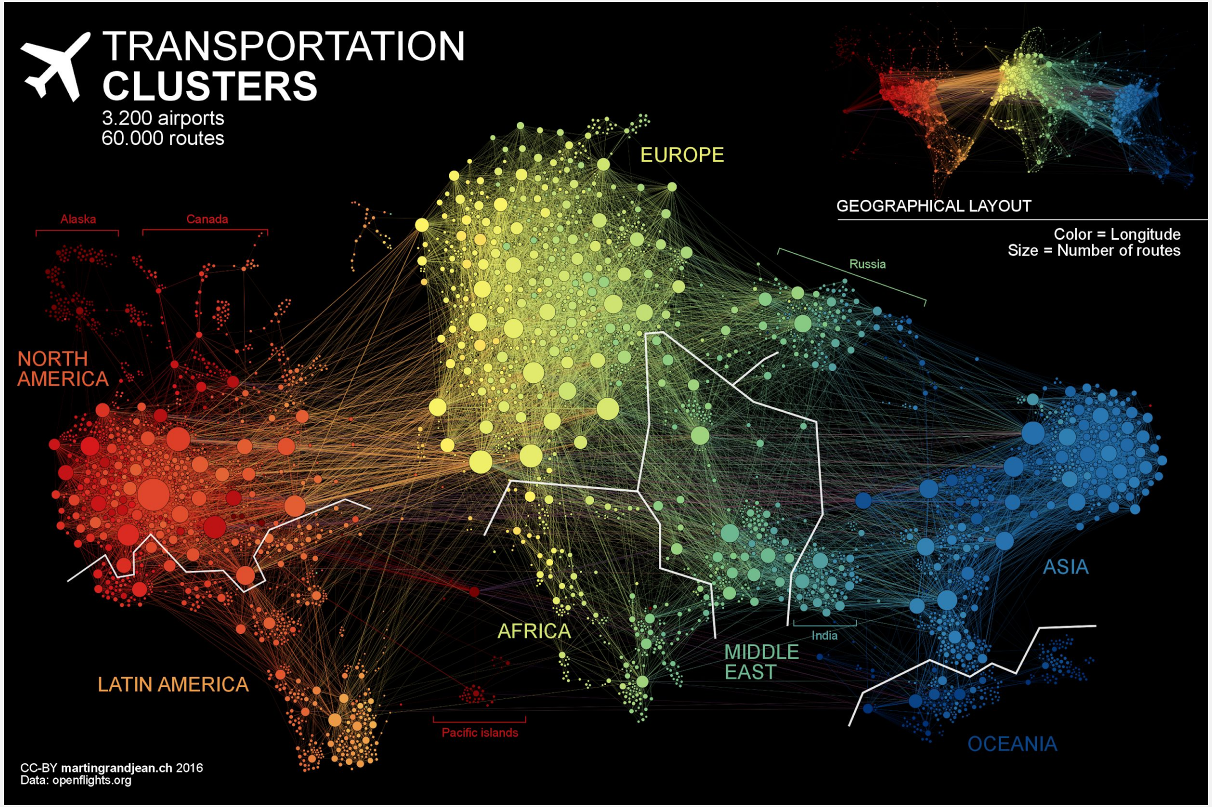
GRAPH EXAMPLES

Partial graph of the Internet (in 2005), where each “node” is an IP address, and the “edges” between them reveal connectivity delays (shorter lines = closer IP addresses)



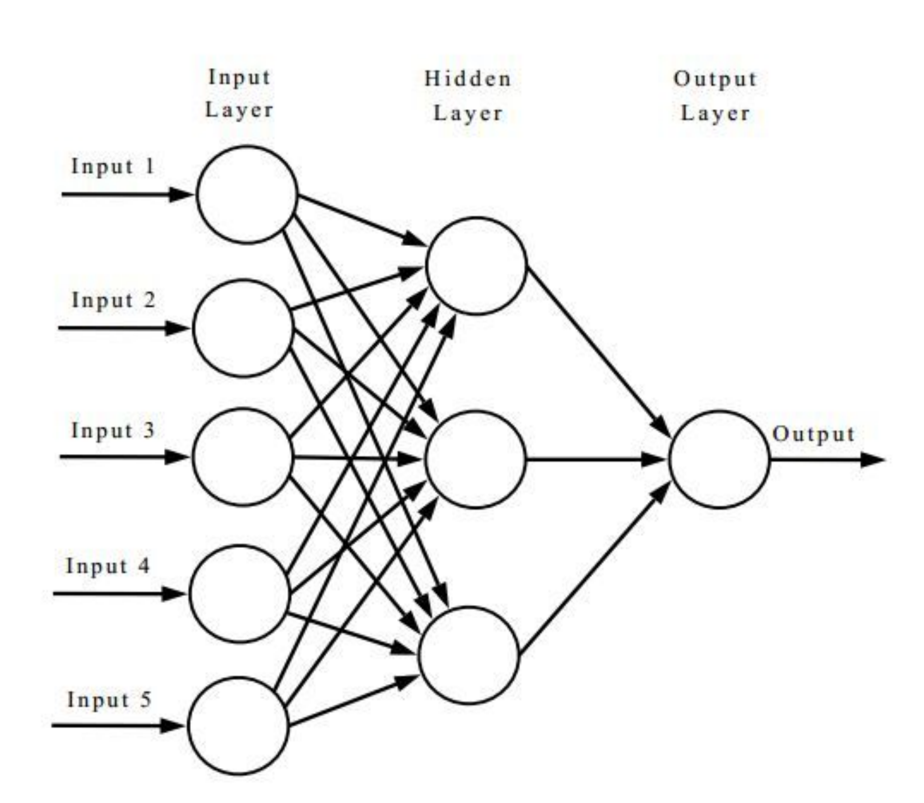
GRAPH EXAMPLES

Each “node” is an airport, and flight routes are represented by the “edge” in between them



GRAPH EXAMPLES

Neural networks! Each “node” represents a module of the neural network, and “edge” represent output/input relationships



GRAPH EXAMPLES

Graph of characters in the third book of Game of Thrones, where each “node” is a character, and “edge” reveal frequency of interaction (i.e. 2 names appearing within 15 words of one another).

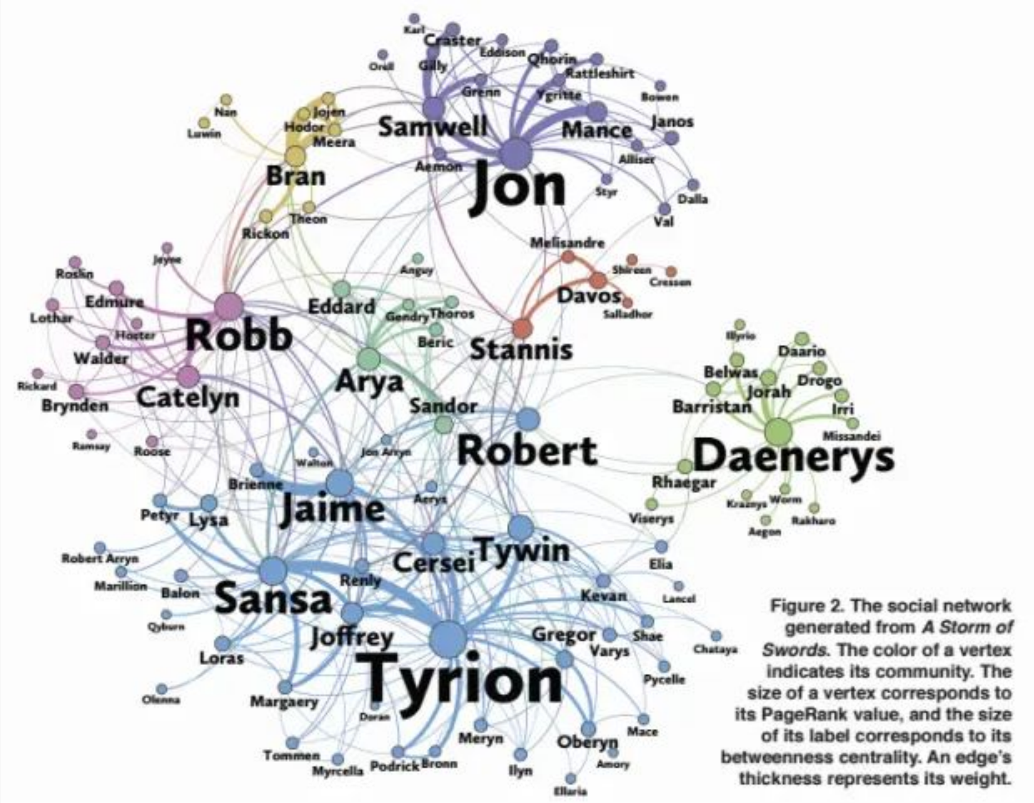
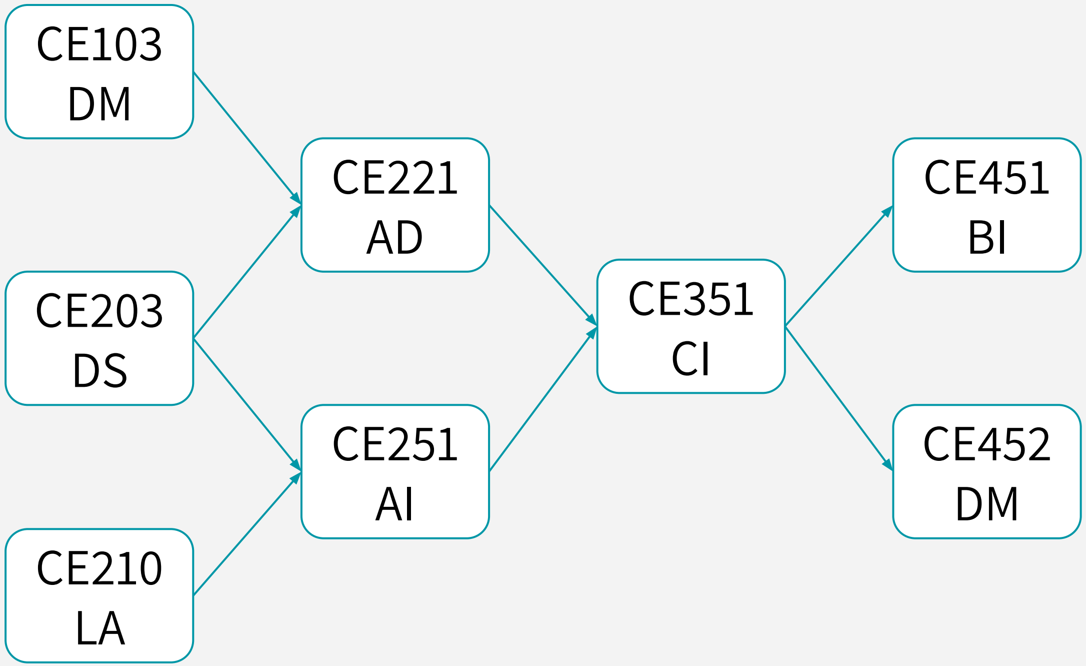


Figure 2. The social network generated from *A Storm of Swords*. The color of a vertex indicates its community. The size of a vertex corresponds to its PageRank value, and the size of its label corresponds to its betweenness centrality. An edge’s thickness represents its weight.

GRAPH EXAMPLES

CE prerequisites!
“nodes” are classes
and an “edge” from
class A to class B
means “class B
depends on class A”



WHAT ARE GRAPHS USED FOR?

- There are a lot of diverse problems that can be represented as graphs, and we want to answer questions about them
- For example:
  - How do we most efficiently route packets across the internet?
  - Are there natural “clusters” or “communities” in a graph?
  - Which character(s) are least related with ______?
  - How should I sign up for classes without violating pre-req constraints?

But first off, some terminology!

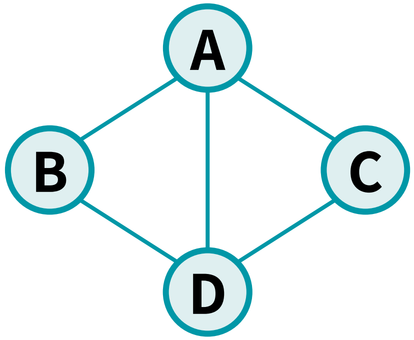
2 KINDS OF GRAPHS

We'll deal with both kinds of graphs in this class.

UNDIRECTED GRAPHS

An undirected graph has
a set of vertices (V) & a set of edges (E)

Formally,
G = (V, E)



V = {A, B, C, D}
E = { {A, B}, {A, C}, {A, D}, {B, D}, {C, D}}

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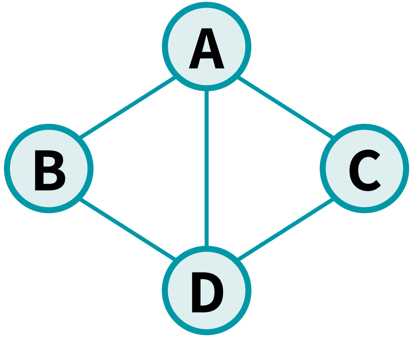

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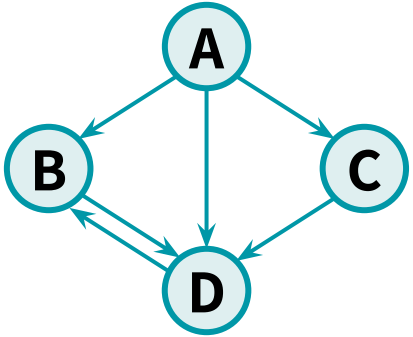
DIRECTED GRAPHS

A directed graph has
a set of vertices (V) & a set of **DIRECTED** edges (E)



Formally,
G = (V, E)

V = {A, B, C, D}
E = { [A, B], [A, C], [A, D], [B, D], [C, D], [D, B]}

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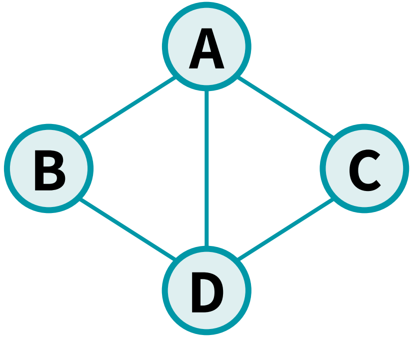

The **degree** of vertex D is 3
Vertex D's **neighbors** are A, B, and C

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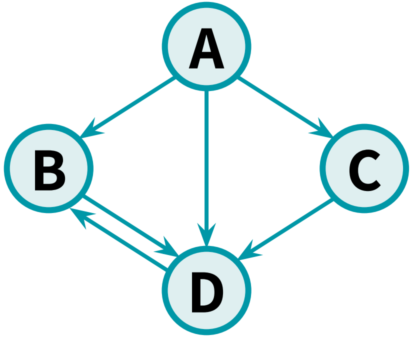


Formally,
G = (V, E)

The **in-degree** of vertex D is 3. The **out-degree** of vertex D is 1.
Vertex D's **incoming neighbors** are A, B, & C
Vertex D's **outgoing neighbor** is B

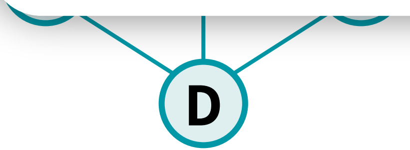
2 KINDS OF GRAPHS

We'll deal with both kinds of graphs in this class.

UNDIRECTED GRAPHS

a set

Formally,
G = (V, E)



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DIRECTED GRAPHS

edges (E)

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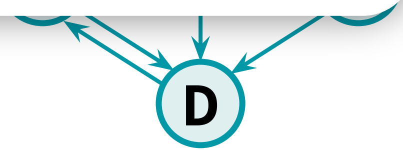

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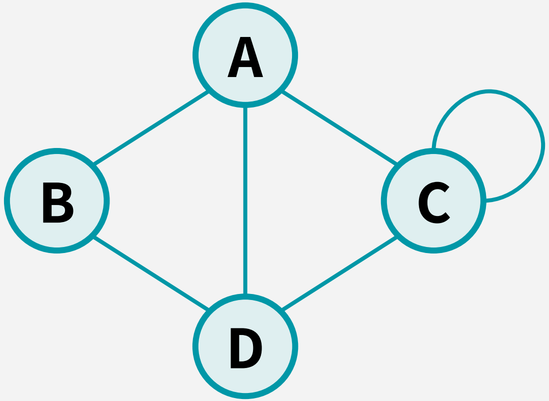
Vertex D's **incoming neighbors** are A, B, & C
Vertex D's **outgoing neighbor** is B

Today, we're only working with ***unweighted*** graphs.
These are graphs where edges aren't assigned weights, or all edges are assumed to have the same weight.

GRAPH REPRESENTATIONS

OPTION 1: **ADJACENCY MATRIX**



(An undirected graph)

(destination)

(source)

$$
\begin{array}{c|c}
 & \begin{matrix} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{matrix} \\
\begin{matrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}
\end{array}
$$

GRAPH REPRESENTATIONS

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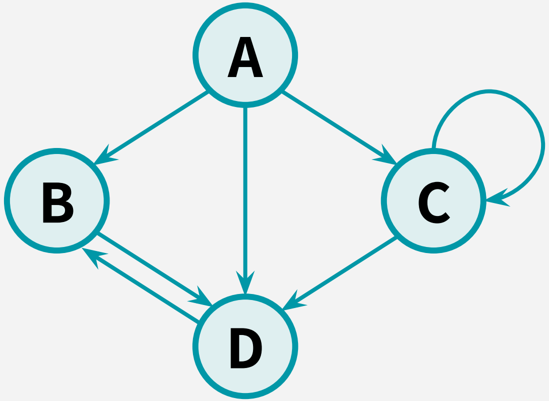


(A directed graph)

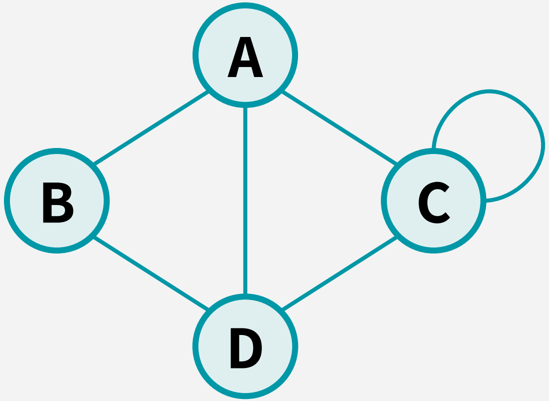
(destination)

(source)

$$
\begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\end{array}
$$

GRAPH REPRESENTATIONS

OPTION 2: **ADJACENCY LISTS**



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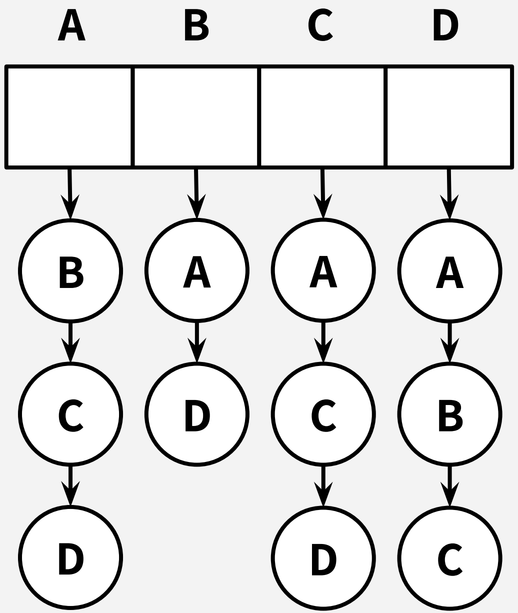
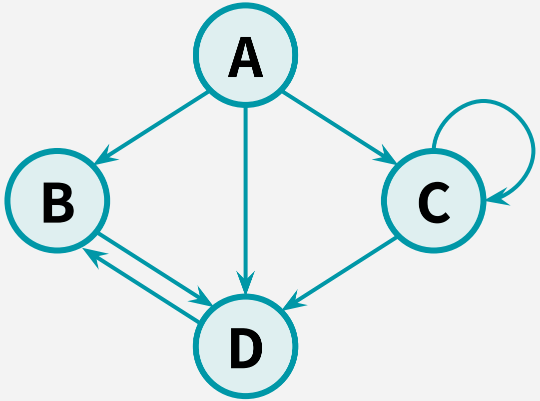


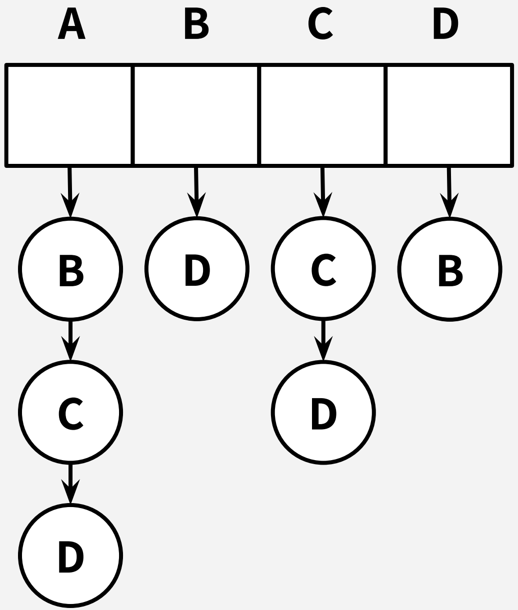
Each list stores a node's neighbors

GRAPH REPRESENTATIONS

OPTION 2: **ADJACENCY LISTS**



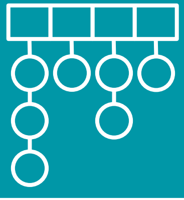
(A directed graph)



Tracks *outgoing neighbors.*

(You could also do the same for incoming neighbors as well)

GRAPH REPRESENTATIONS

For a graph G = (V, E) where $\|V\| = \mathbf{n}$, and $\|E\| = \mathbf{m}$	$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$	
EDGE MEMBERSHIP Is e = {v, w} in E?		
NEIGHBOR QUERY Give me v’s neighbors		
SPACE REQUIREMENTS		

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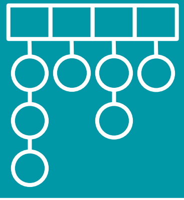
For a graph G = (V, E) where $\lvert V\rvert = \mathbf{n}$, and $\lvert E\rvert = \mathbf{m}$	$\begin{bmatrix}0&1&1&1\\1&0&0&1\\1&0&1&1\\1&1&1&0\end{bmatrix}$	
EDGE MEMBERSHIP Is e = {v, w} in E?	**O(1)**	
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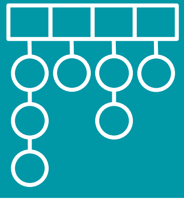
For a graph G = (V, E) where $\lvert V \rvert = \mathbf{n}$, and $\lvert E \rvert = \mathbf{m}$	$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$	
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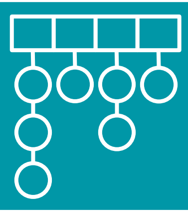
For a graph $G = (V, E)$ where $\lvert V \rvert = \mathbf{n}$, and $\lvert E \rvert = \mathbf{m}$	$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$	
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NEIGHBOR QUERY Give me v's neighbors	**O(n)**	
SPACE REQUIREMENTS		

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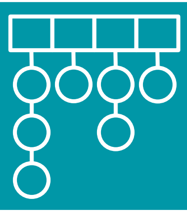
For a graph $G = (V, E)$ where $\lvert V \rvert = \mathbf{n}$, and $\lvert E \rvert = \mathbf{m}$	$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$	
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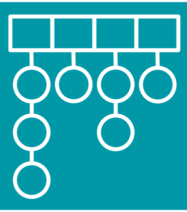
For a graph G = (V, E) where $\|V\| = \mathbf{n}$, and $\|E\| = \mathbf{m}$	$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$	
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SPACE REQUIREMENTS	**$O(n^2)$**	

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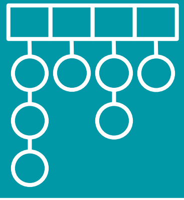
For a graph $G = (V, E)$ where $\|V\| = \mathbf{n}$, and $\|E\| = \mathbf{m}$	$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$	
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NEIGHBOR QUERY Give me v's neighbors	**O(n)**	**O(deg(v))**
SPACE REQUIREMENTS	**O(n^2)**	**O(n + m)**

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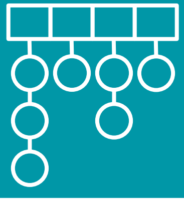
For a graph $G = (V, E)$ where $\lvert V \rvert = \mathbf{n}$, and $\lvert E \rvert = \mathbf{m}$	$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$	
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SPACE REQUIREMENTS	**$O(n^2)$**	**O(n + m)**

Generally, better for sparse graphs (where $m \ll n^2$).

We'll assume this representation, unless otherwise stated.

سوال؟

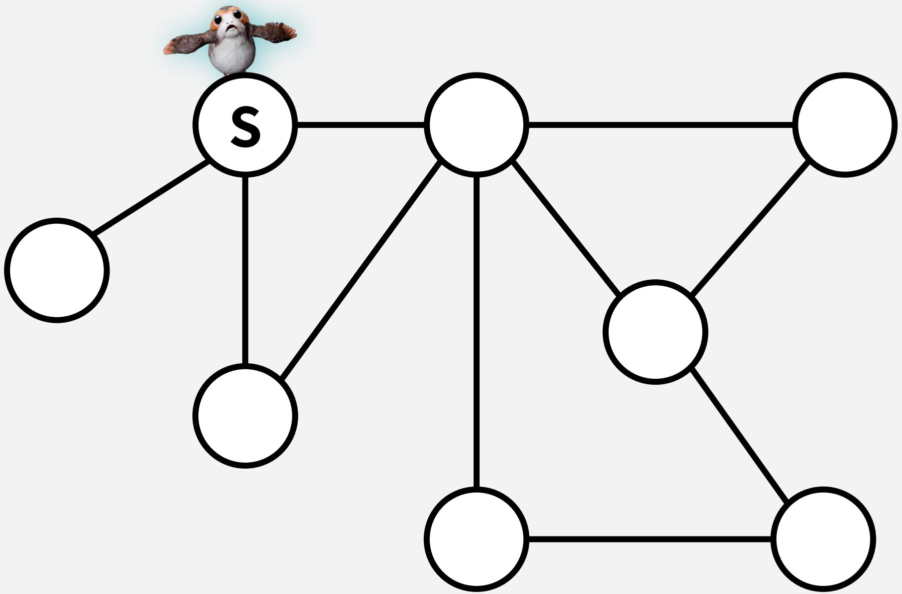
جستجوی سطح اول (BFS)

یک روش پیمایش گراف

BREADTH-FIRST SEARCH

An analogy:

A bird is exploring a labyrinth from above (with a bird's eye view)



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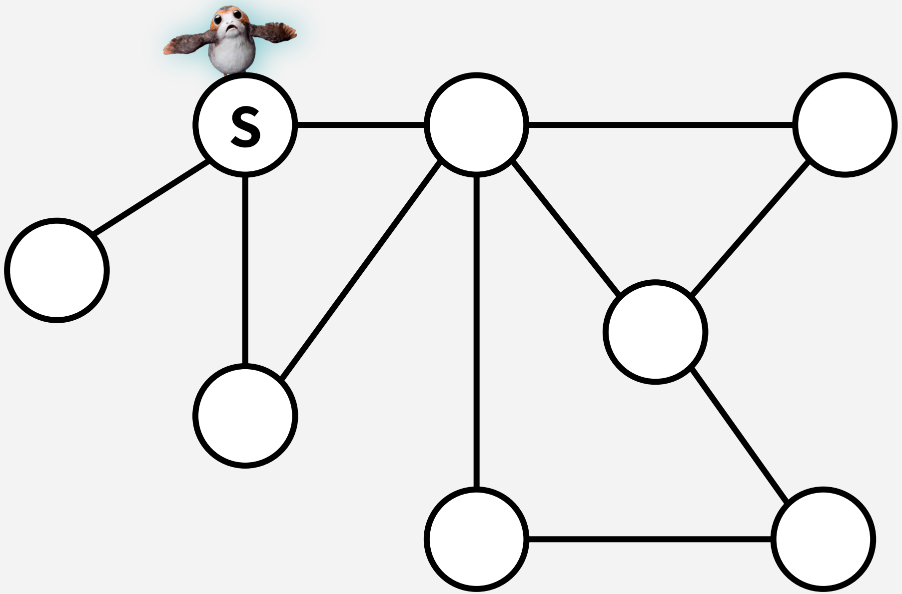

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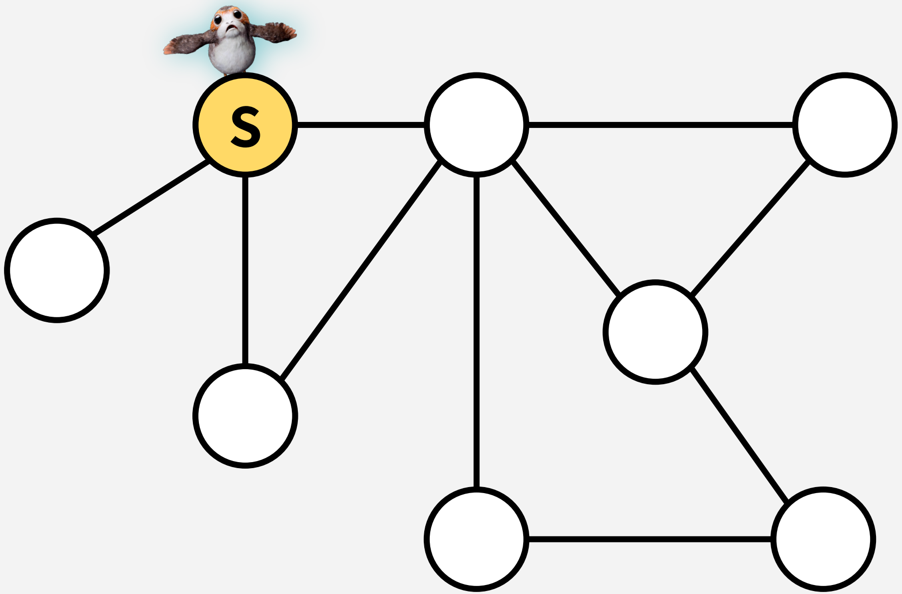

unvisited

Layer 0: reachable in 0 steps

Layer 1: reachable in 1 step

Layer 2: reachable in 2 steps

Layer 3: reachable in 3 steps

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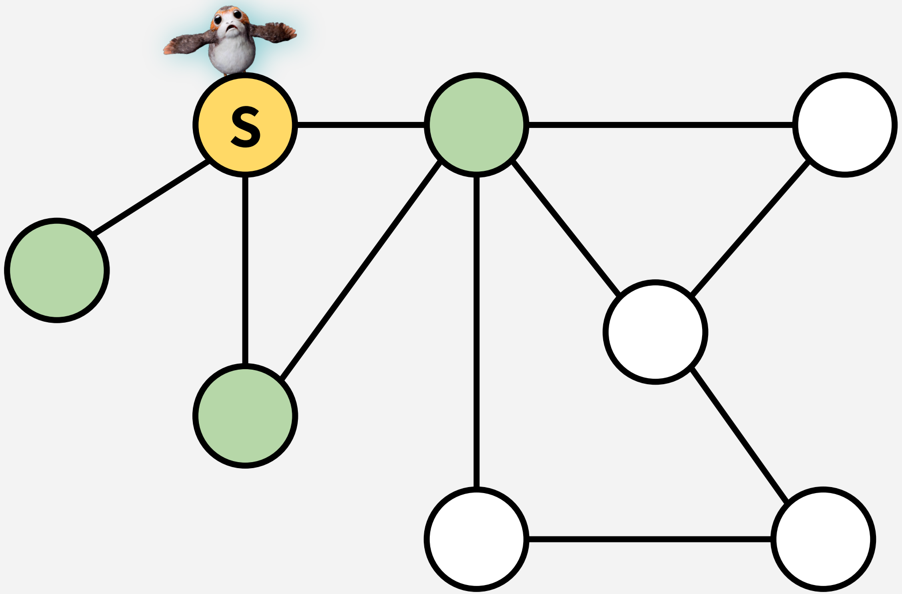

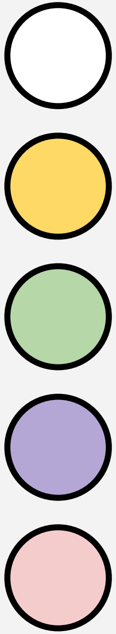

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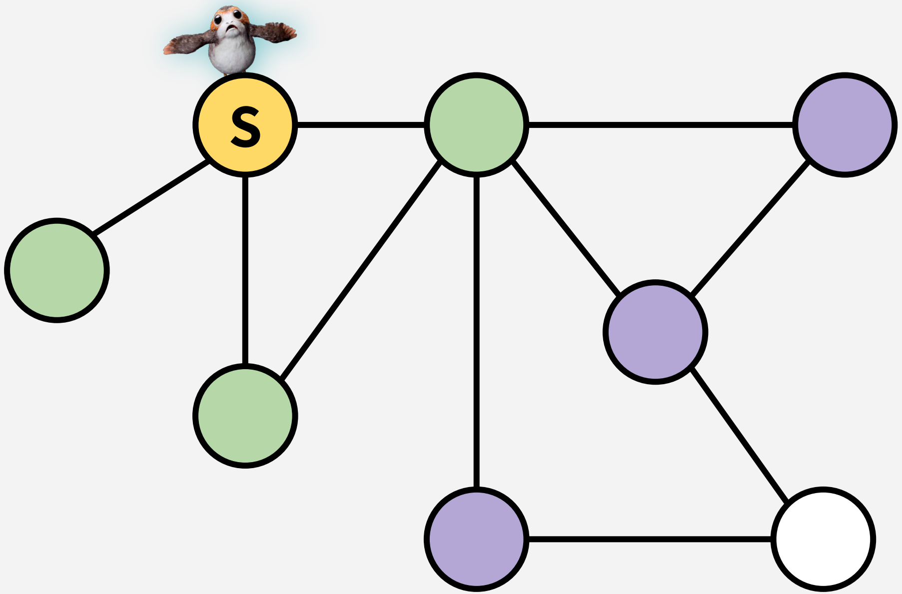

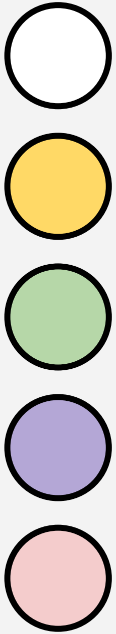

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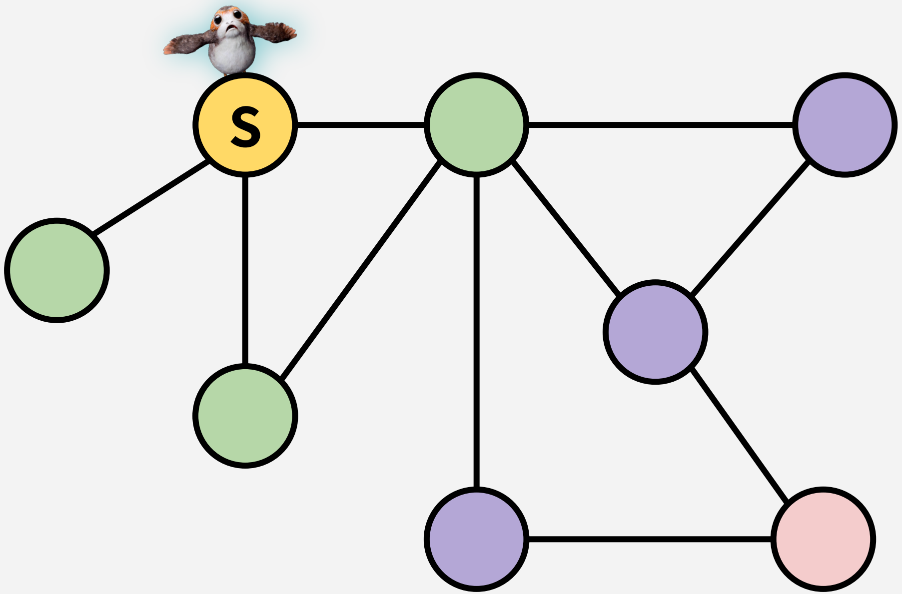

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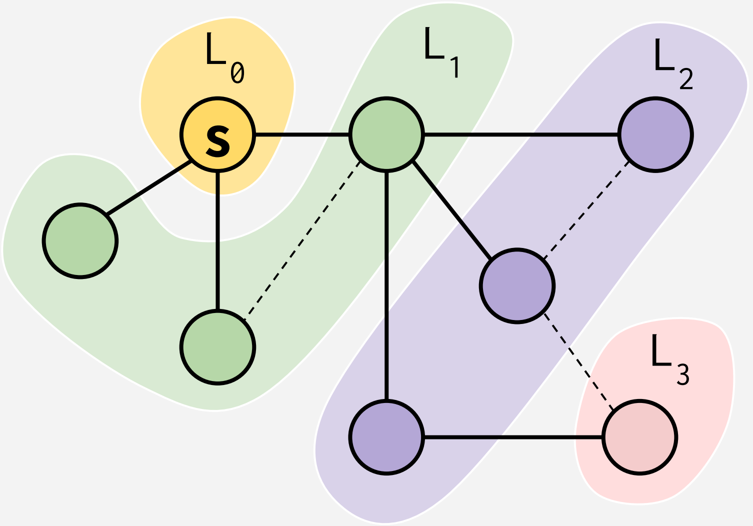
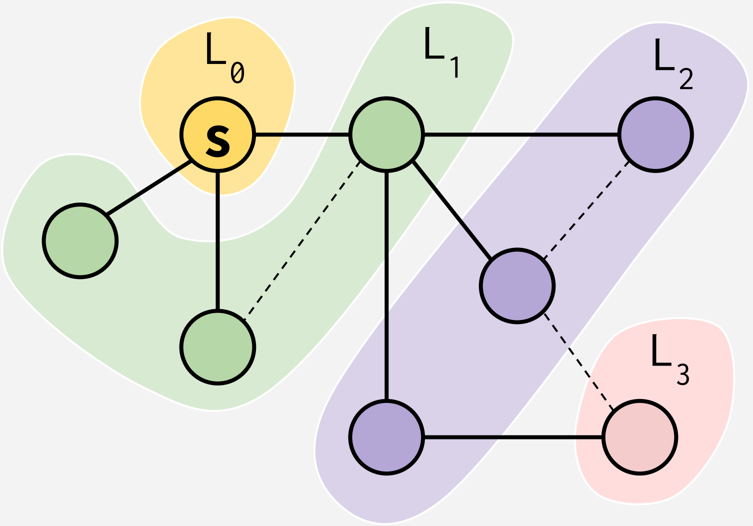



```
BFS(s):
    Set L_i = [] for i = 0, ..., n-1
    L_0 = s
    for i = 0, ..., n-1:
        for u in L_i:
            for v in u.neighbors:
                if v not yet visited:
                    mark v as visited
                    add v to L_{i+1}
```

L_i = The set of nodes we can reach in i steps from s

BREADTH-FIRST SEARCH



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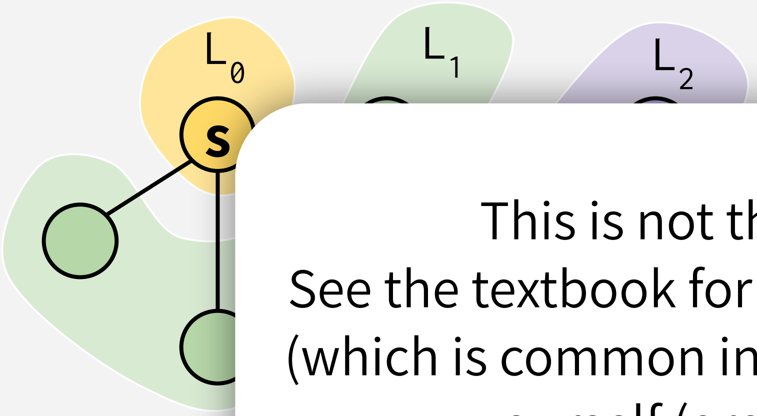
Go through all nodes in L_i and add their unvisited neighbors to L_{i+1}

BREADTH-FIRST SEARCH




```
BFS(s):
              ..., n-1
          rs:
          sited:
          sited
```

This is not the only way to write BFS!
See the textbook for a version that uses a FIFO **queue** (which is common in the real world), and try writing it yourself (great for interview practice)

L_i = The set of nodes we can reach in i steps from s

Go through all nodes in L_i and add their unvisited neighbors to L_{i+1}

BREADTH-FIRST SEARCH

BFS finds all the nodes reachable from the starting point!

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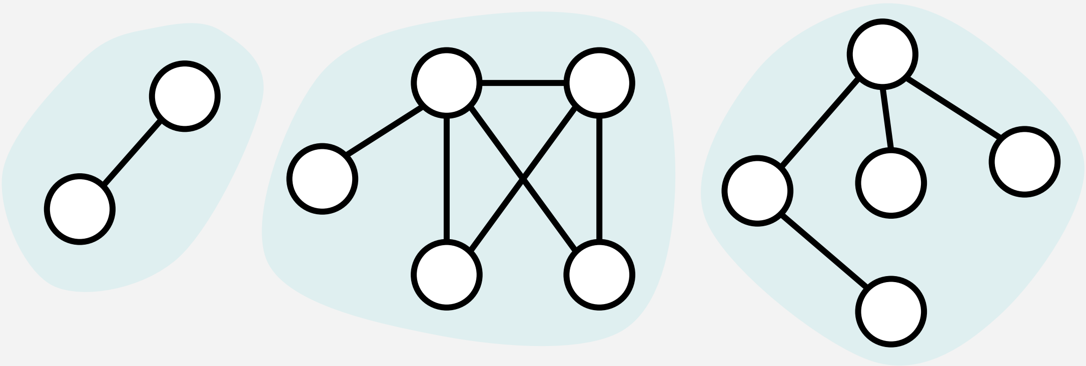
In undirected graphs, this is equivalent to finding the node's **connected component.**



BREADTH-FIRST SEARCH: RUNTIME

To explore a graph's **i^th^ connected component** (n_i nodes, m_i edges):

We visit each vertex in the CC exactly once ("visit" = grab from its L_i).

BREADTH-FIRST SEARCH: RUNTIME

To explore a graph's **i^{th} connected component** (n_i nodes, m_i edges):

We visit each vertex in the CC exactly once ("visit" = grab from its L_i).
At each vertex v, we:

- Do some bookkeeping: **O(1)**
- Loop over v's neighbors & check if they are visited (& then potentially mark the neighbor & place in L_{i+1}): O(1) per neighbor → **O(deg(v))** total.

BREADTH-FIRST SEARCH: RUNTIME

To explore a graph's i^{th} **connected component** (n_i nodes, m_i edges):

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Total: $\sum_v O(\deg(v)) + \sum_v O(1) = \mathbf{O(m_i + n_i)}$

BREADTH-FIRST SEARCH: RUNTIME

To explore **the entire graph** (n nodes, m edges):

A graph might have multiple connected components! To **explore the whole graph**, we would call our BFS routine once for each connected component (note that each vertex and each edge participates in exactly one connected component). The combined running time would be:

$$O(\sum_i m_i + \sum_i n_i) = \mathbf{O(m + n)}$$

BREADTH-FIRST SEARCH

Why is it called breadth-first?

We are implicitly building a **tree**!

(It's a tree because we never revisit a node)

We go as "broadly" as we can when building each layer of the tree

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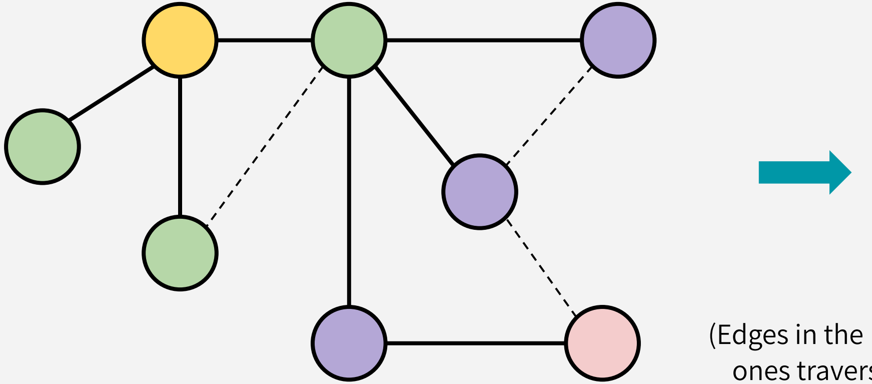
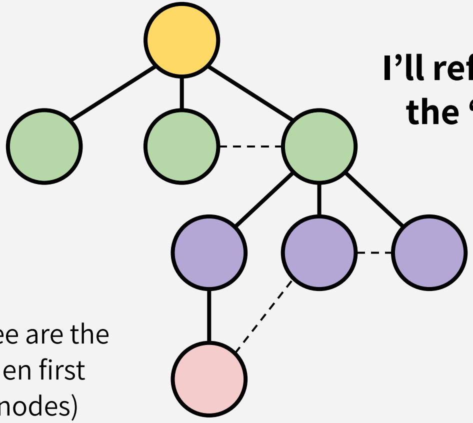




I'll refer to this as the "BFS tree"

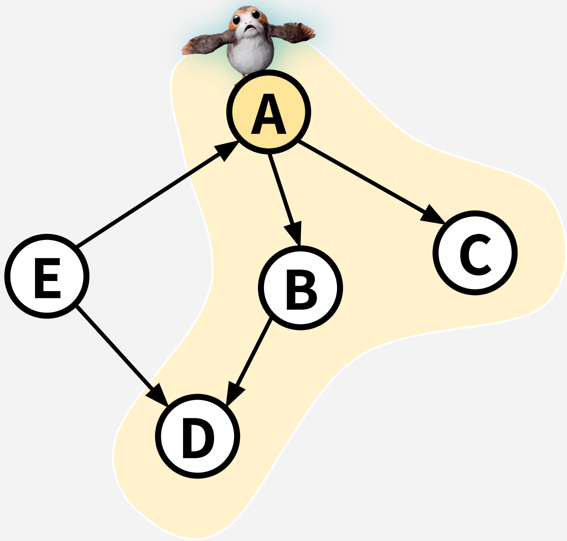
(Edges in the BFS tree are the ones traversed when first finding unvisited nodes)

BREADTH-FIRST SEARCH

BFS works fine on directed graphs too!

From a start node x, BFS would find all nodes ***reachable*** from x.

(In directed graphs, "connected component" isn't as well defined… more on that later!)



Verify this on your own:
running BFS from A would still find all nodes reachable from A (E isn't reachable from A in this directed graph).

BREADTH-FIRST SEARCH

What are some applications of BFS?

Finding a node's connected component (just run BFS)!
(or in directed graphs, finding reachable nodes from a starting node)

Single-source shortest paths

Testing bipartiteness

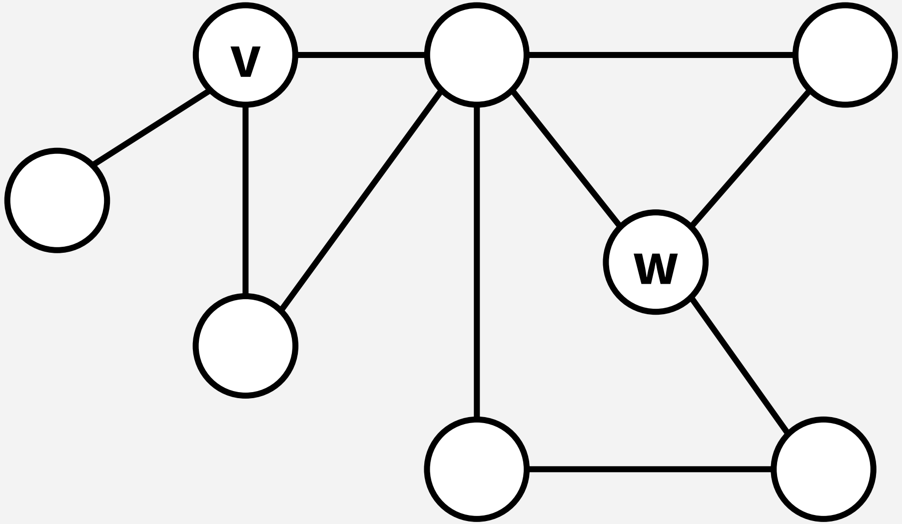
And more…

سوال؟

پیدا کردن کوتاه ترین مسیر با جستجوی سطح اول

SHORTEST PATH: THE TASK

How long is the shortest path between vertices v and w?



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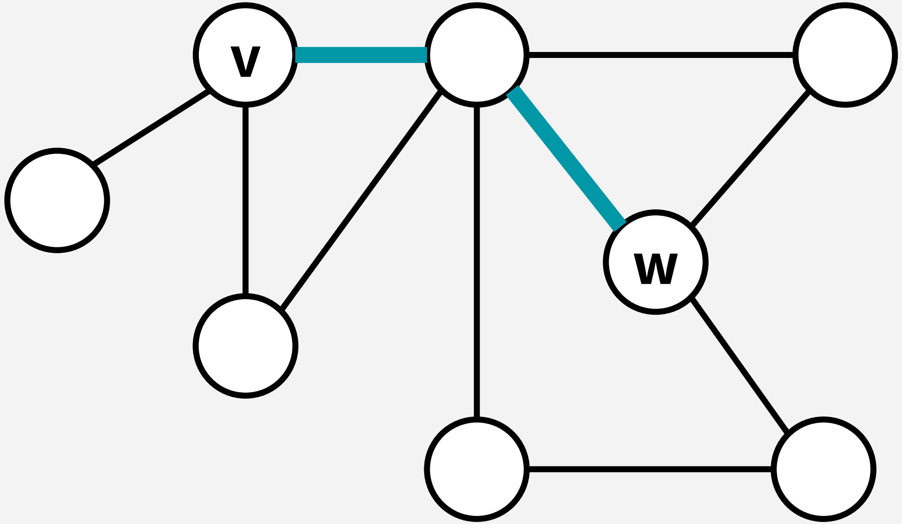

From visually inspecting the graph, we can see that the shortest path from **v** to **w** is 2 (there are 2 edges on that path)!

There are paths of length 3, 4, or 5 as well, but we can't do any better than 2.

SHORTEST PATH: THE TASK

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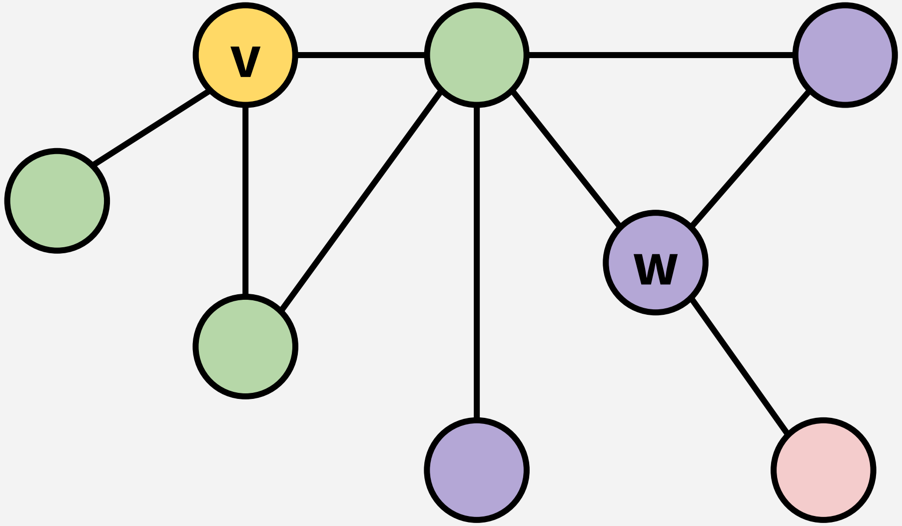

unvisited

Layer 0: reachable in 0 steps

Layer 1: reachable in 1 step

Layer 2: reachable in 2 steps

Layer 3: reachable in 3 steps

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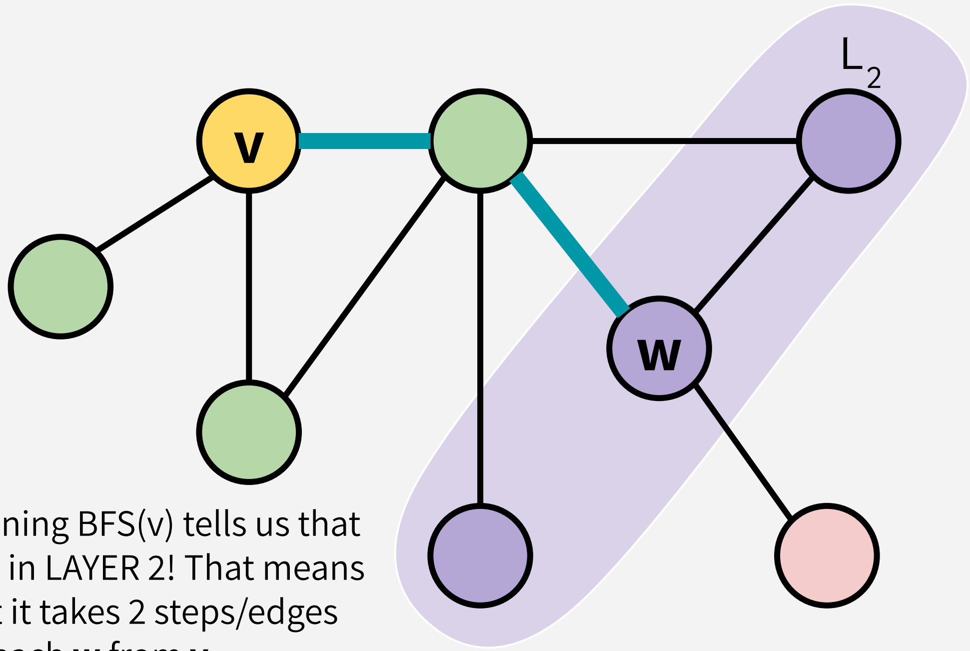
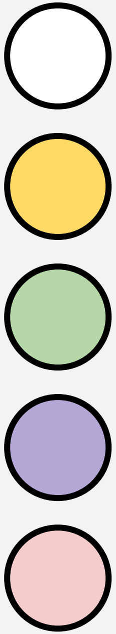


Running BFS(v) tells us that **w** is in LAYER 2! That means that it takes 2 steps/edges to reach **w** from **v**.



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Layer 0: reachable in 0 steps

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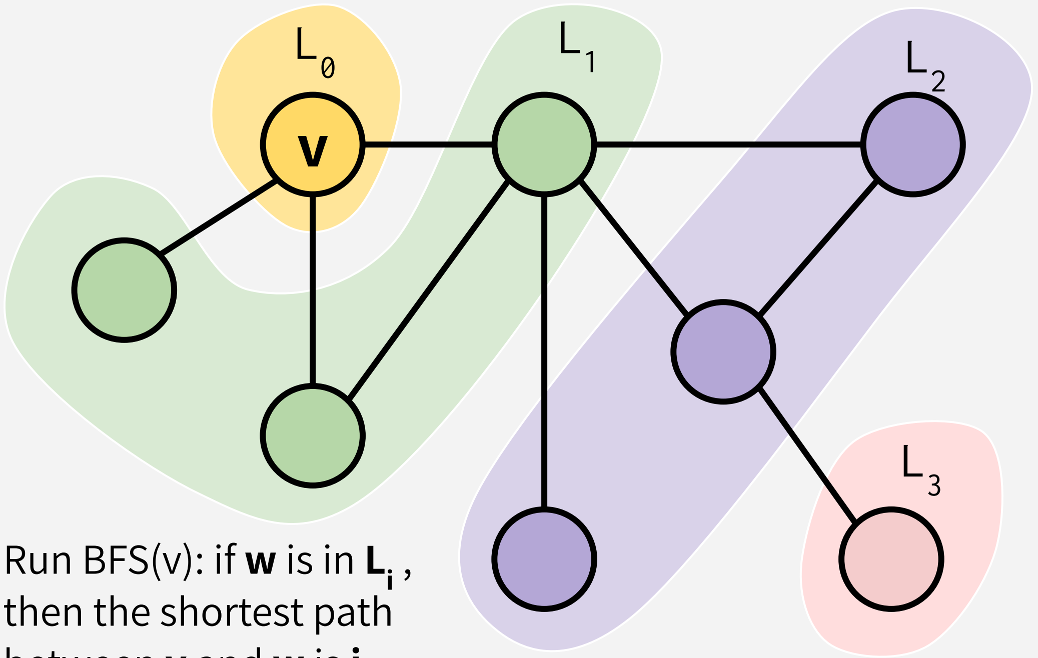
SINGLE-SOURCE SHORTEST PATH

How long is the shortest path between vertices v & *all other vertices* w?



- unvisited
- **Layer 0**: reachable in 0 steps
- **Layer 1**: reachable in 1 step
- **Layer 2**: reachable in 2 steps
- **Layer 3**: reachable in 3 steps

Run BFS(v): if **w** is in $\mathbf{L_i}$, then the shortest path between **v** and **w** is **i**

SINGLE-SOURCE SHORTEST PATH

How long is the shortest path between vertices v & *all other vertices* w?

```
findAllDistances(v):
    perform BFS(v) → gives us all L_i
    for all w in V:
        d[w] = ∞
    for each L_i:
        for all w in L_i:
            d[w] = i
```

SINGLE-SOURCE SHORTEST PATH

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Runtime: O(m+n)

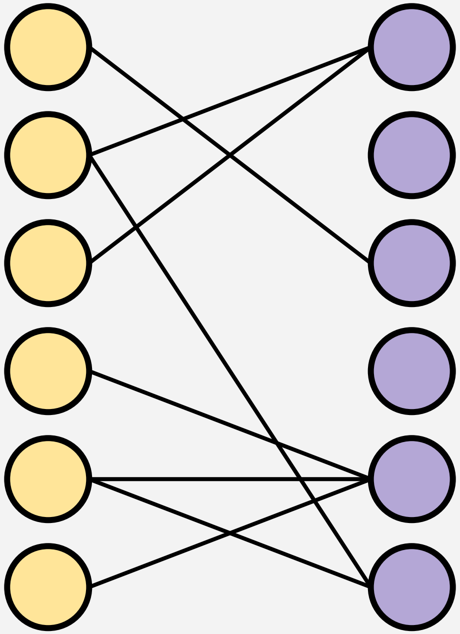
سوال؟

آزمایش دوبخشی بودن گراف

استفاده از جستجوی سطح اول برای آزمایش دوبخشی بودن گراف

BIPARTITE GRAPHS

A graph is **bipartite** iff there exists a 2-coloring such that there are no edges between same-colored vertices



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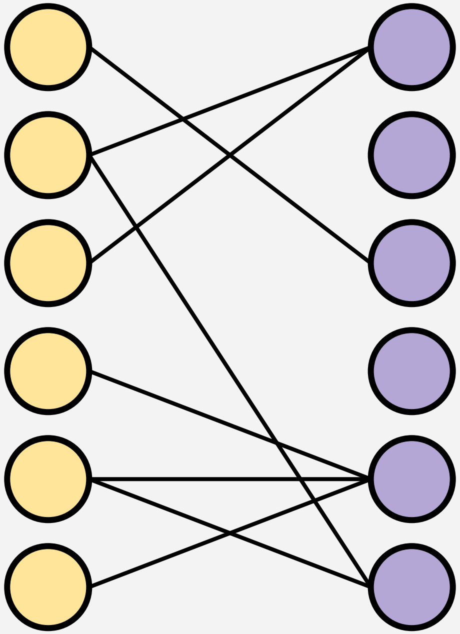
Example 1:

You're planning a cross-team exercise match between two school tennis players, and you polled everyone's preferences for their opponent. Can you verify that no students were listing someone from their school as one of their preferred opponents?



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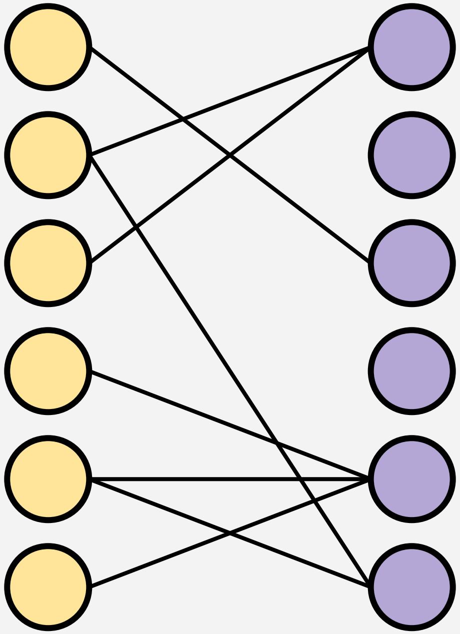
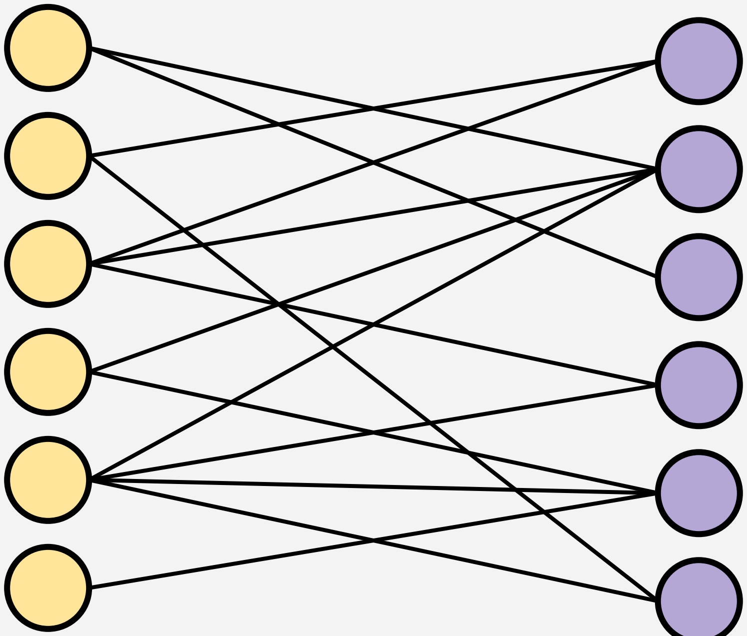


Example 2:

You have a bunch of fish and two fish tanks; some pairs of fish will fight if they're in the same tank. Can you separate the fish so that there's no fighting?

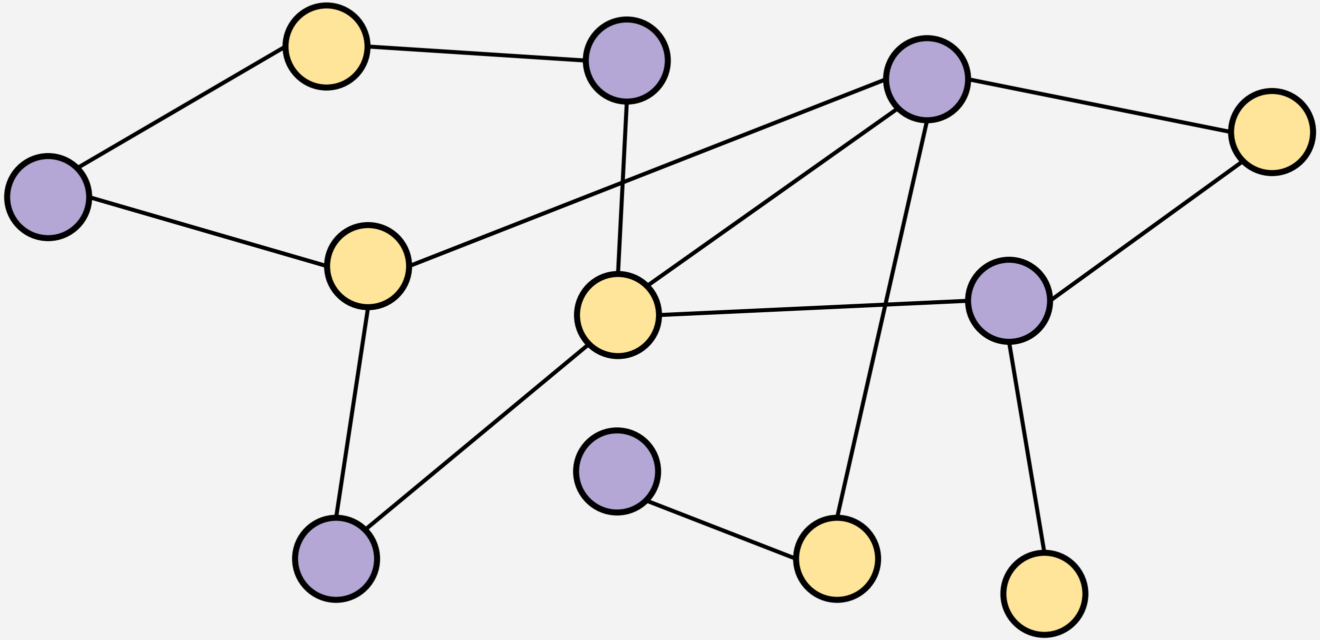
BIPARTITE GRAPHS

Is this graph bipartite?



BIPARTITE GRAPHS

How about this one?



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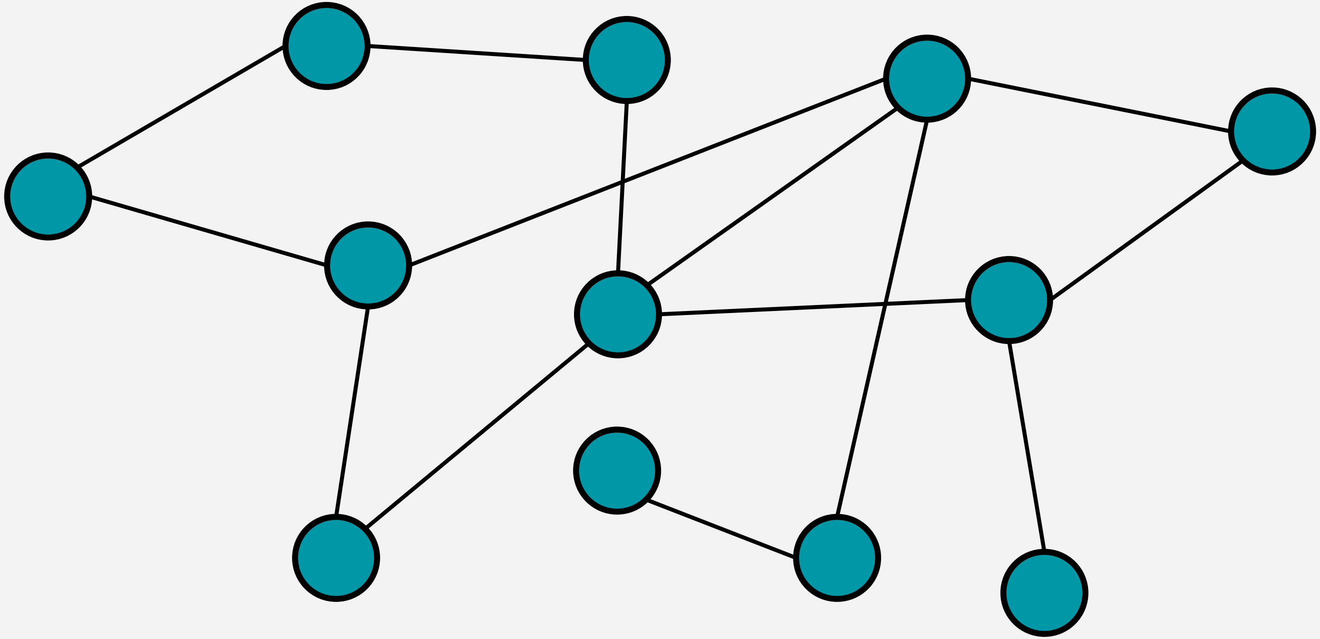

BIPARTITE GRAPHS

Application of BFS:

- Color the levels of the BFS tree in alternating colors (i.e. run BFS from any vertex, and alternate colors for each layer)
- If you attempt to color the same vertex different colors (i.e. revisit a node that's a different color than what you would have colored it), then the graph isn't bipartite!
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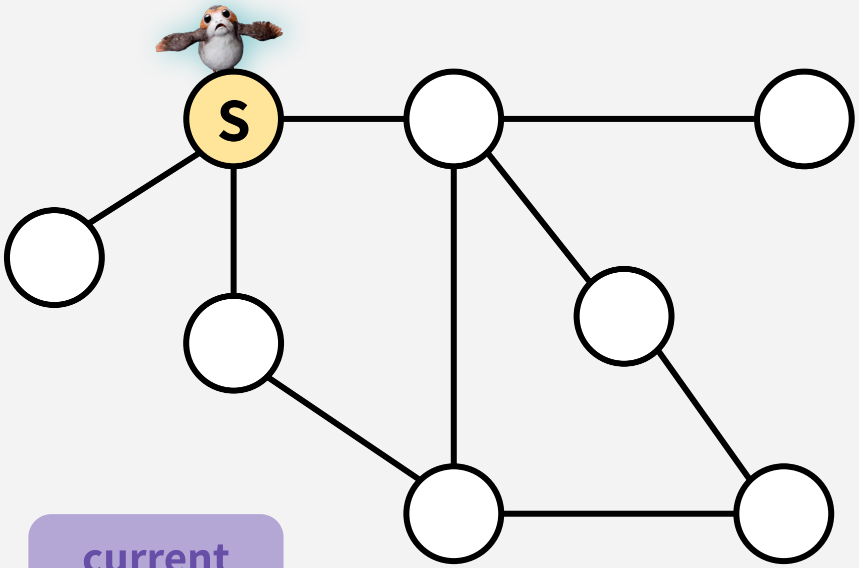

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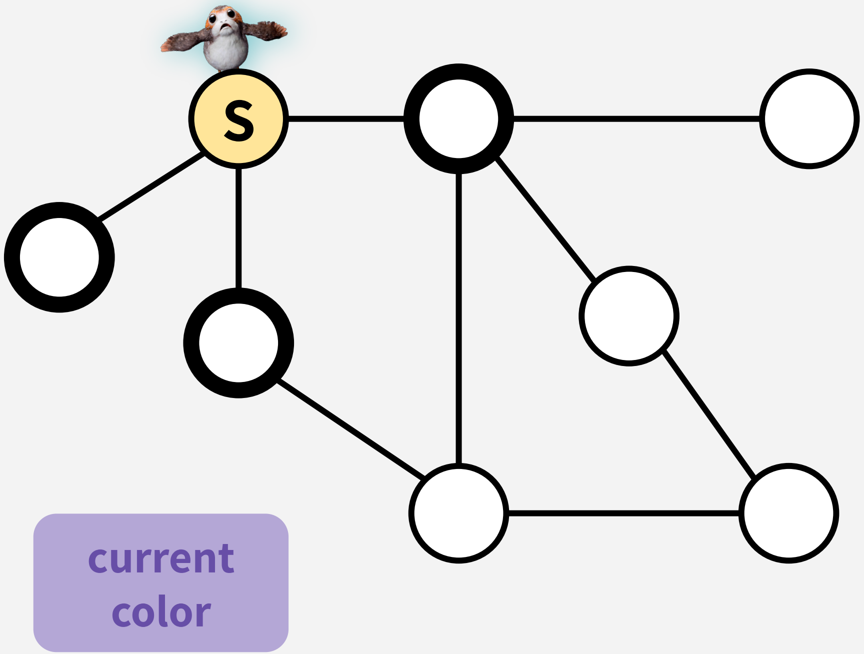

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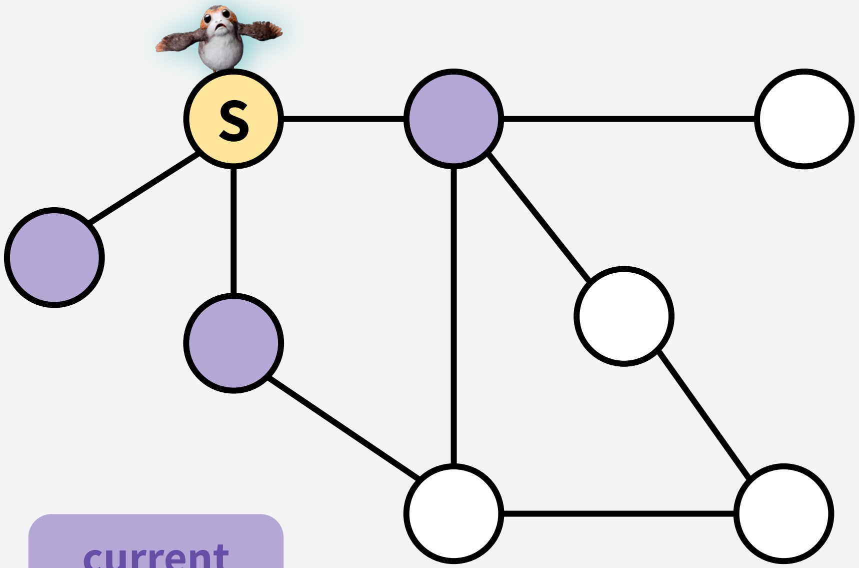

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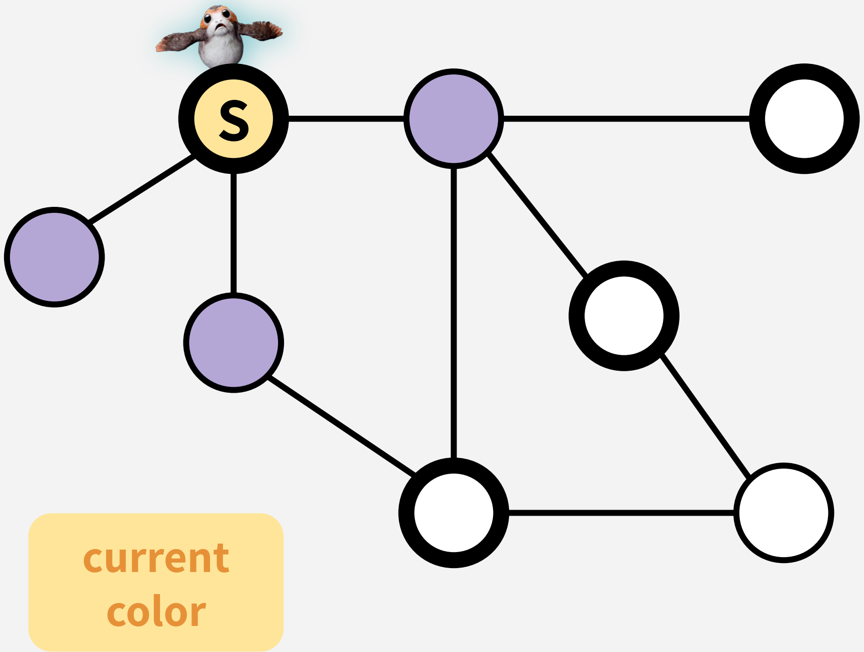

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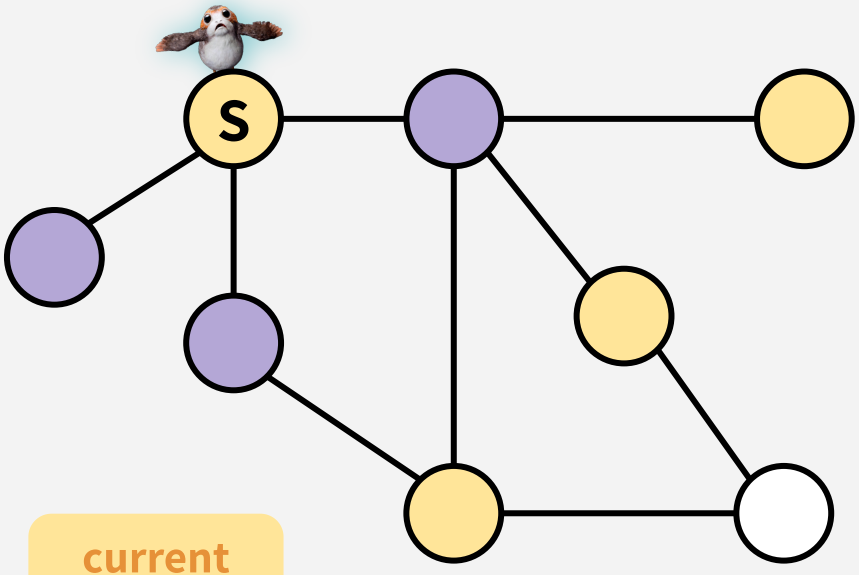

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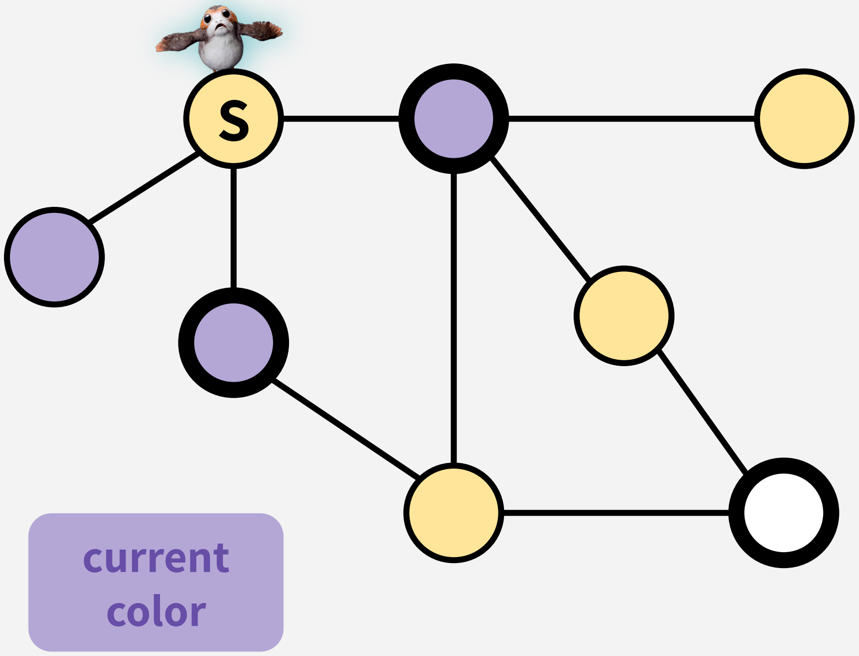

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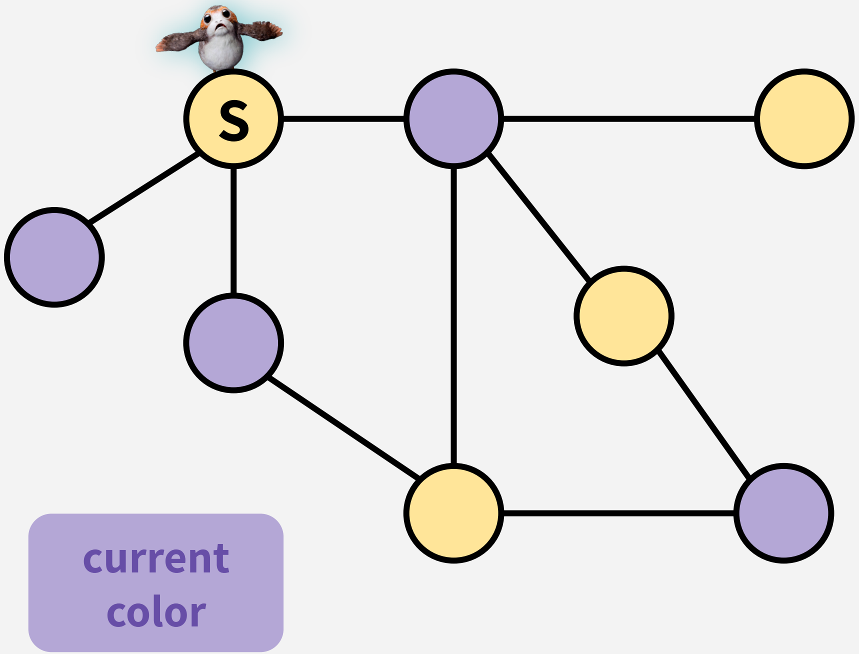

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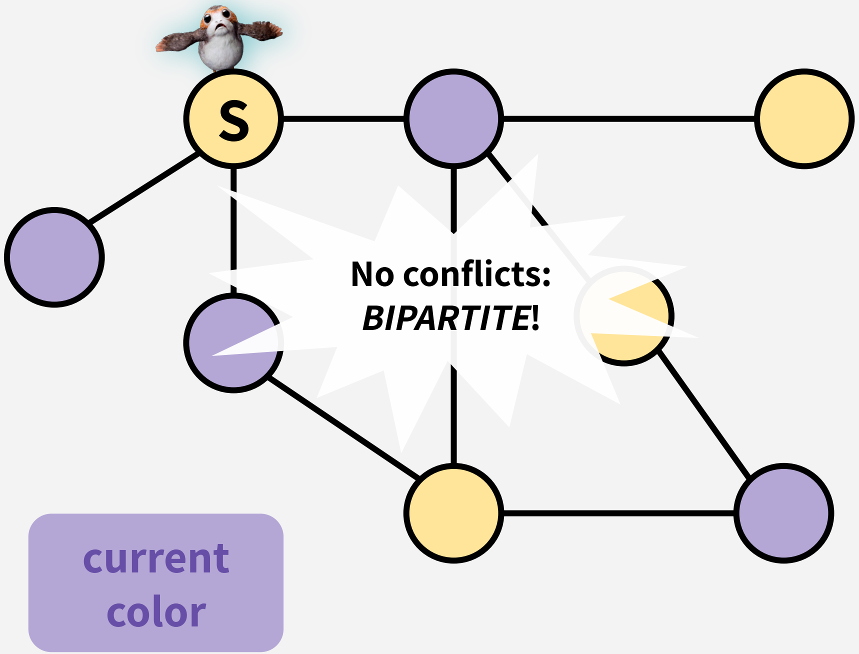

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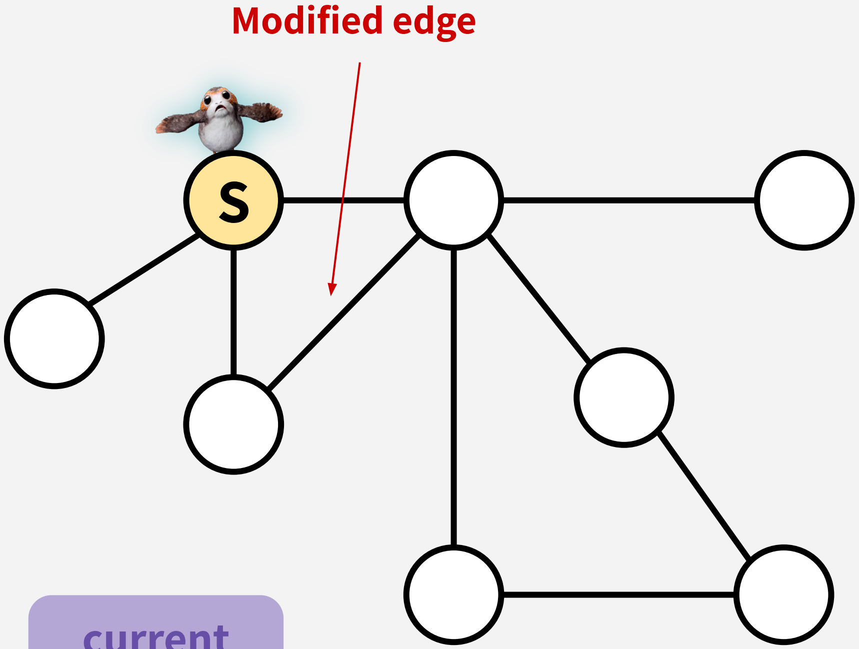

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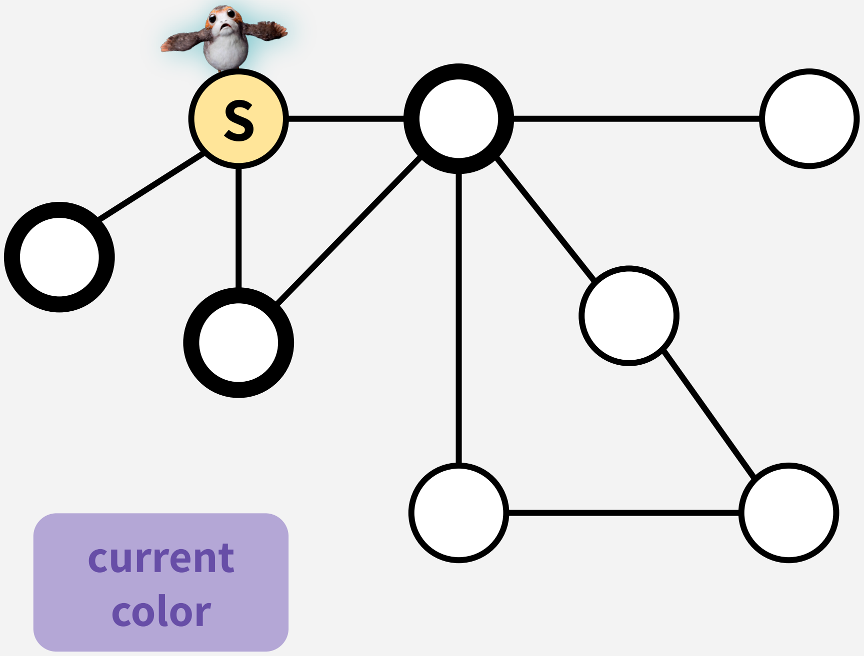

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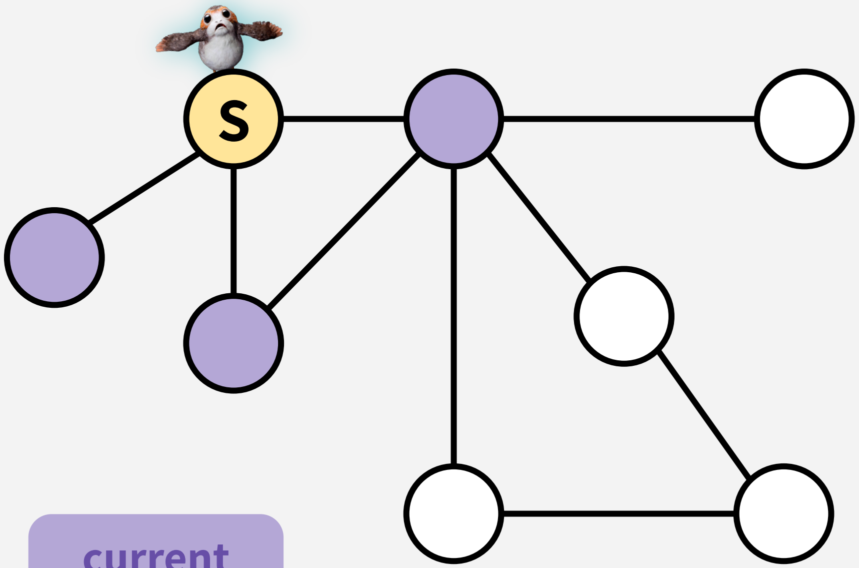

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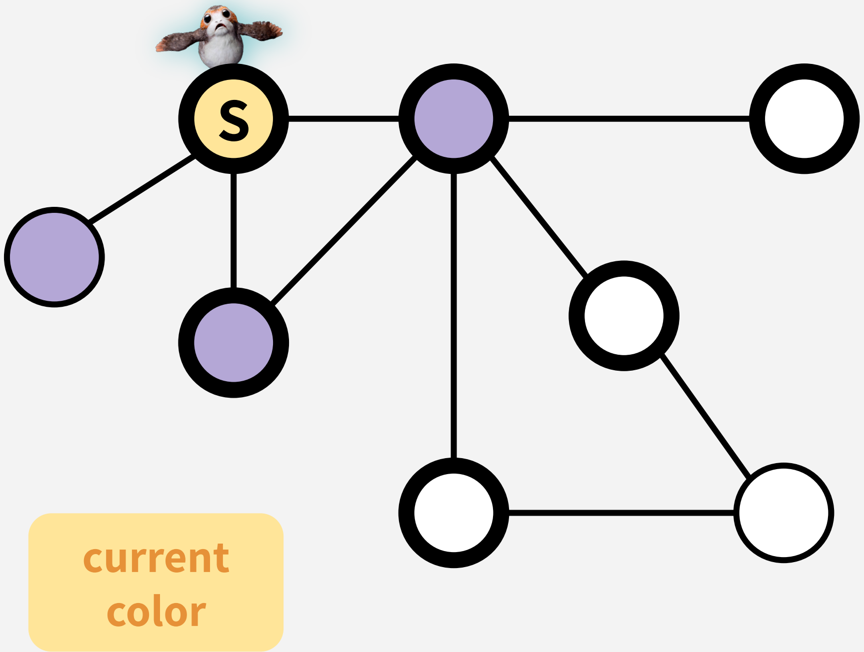

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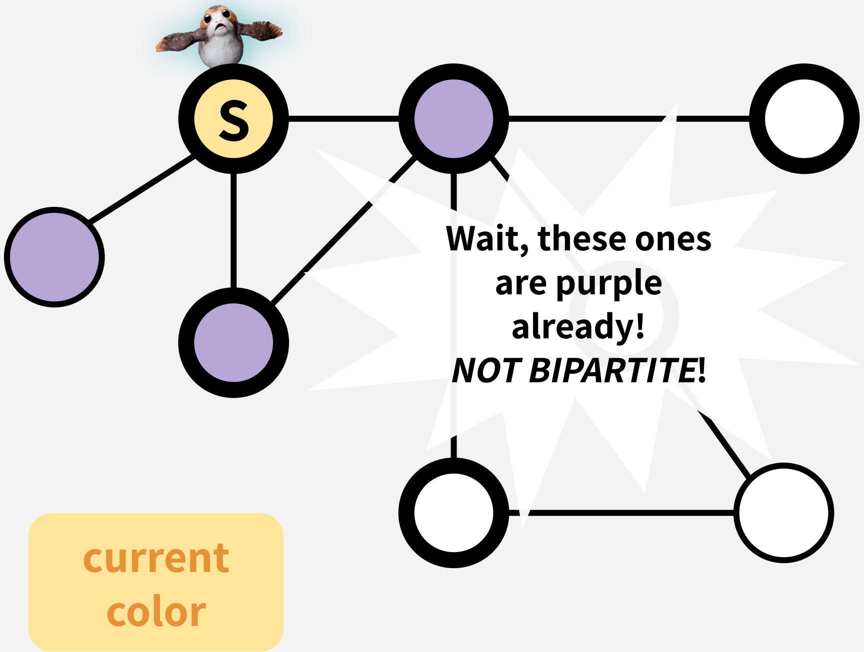
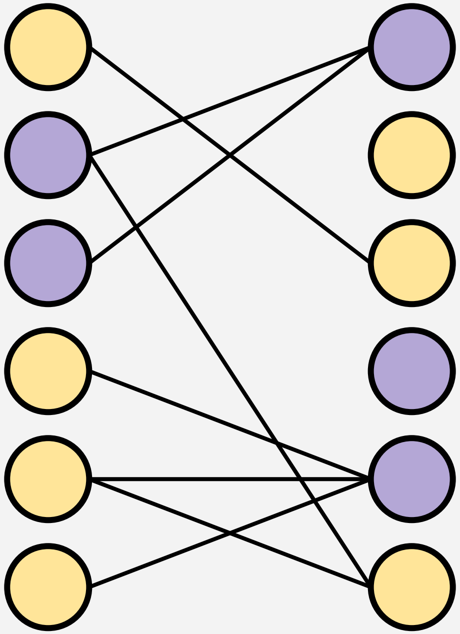


BIPARTITE GRAPHS

But wait… there exists many poor colorings on legitimate bipartite graphs.

Just because **the BFS coloring technique** doesn't work, why do we just throw up our hands and say **no** coloring works?



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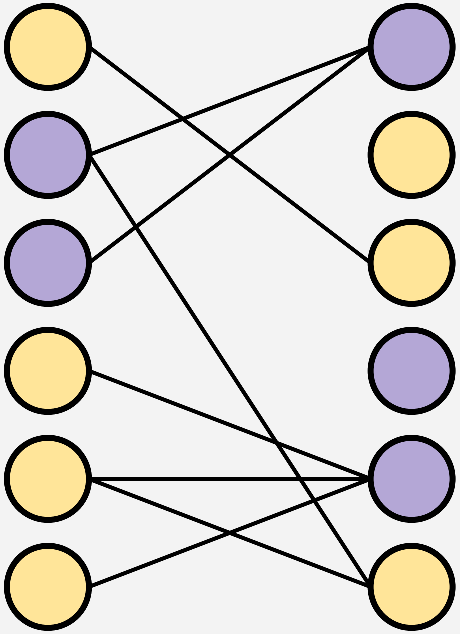
We can come up with plenty of bad coloring on this legitimate bipartite graph...



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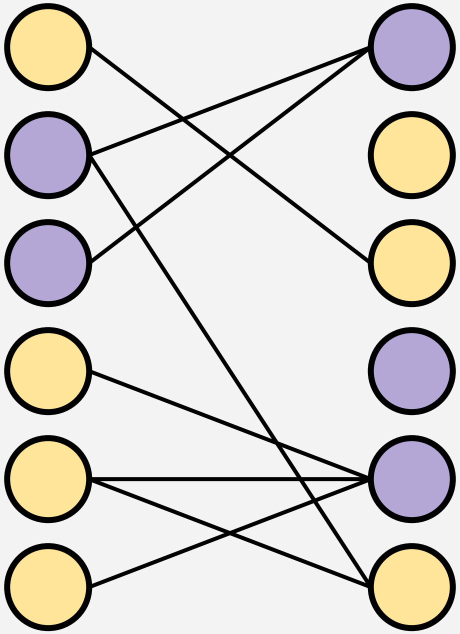
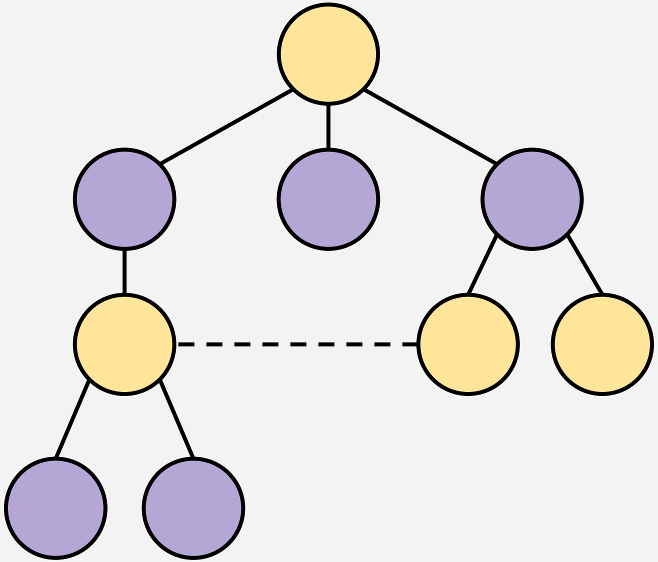


We need to prove that if BFS encounters a conflict (tries to color two neighbors the same color!), then there's no way the graph could be bipartite.

BIPARTITE GRAPHS

If BFS tries to color two neighbors the same color, then it's found a **cycle of odd length** in the graph

This is the BFS tree. Each level in this tree corresponds to each "BFS level". Our BFS coloring technique basically tries to alternate colors across levels.



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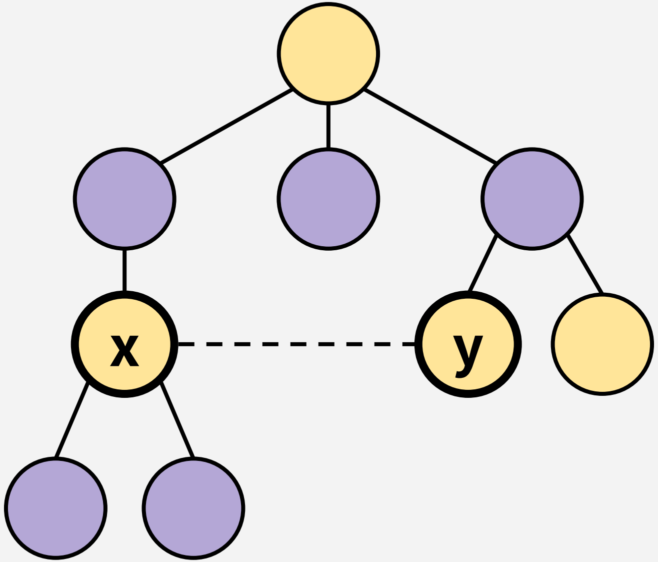
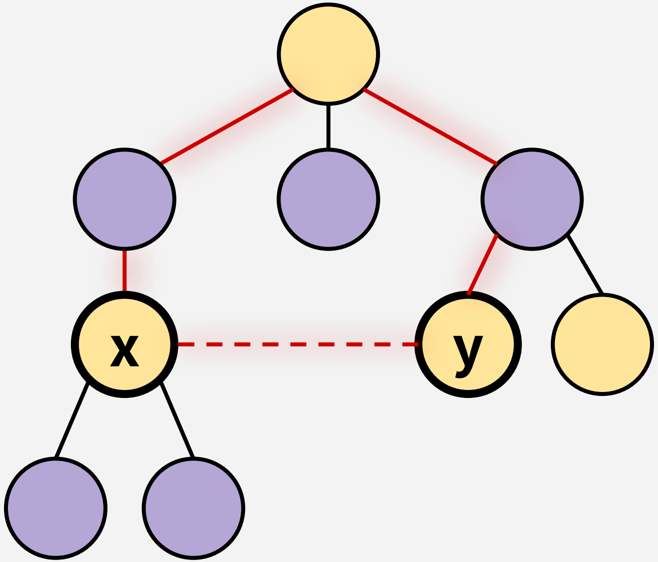


These neighbors are the conflict! BFS will try to color one of **x** or **y** purple, but it's already been colored yellow.

BIPARTITE GRAPHS

If BFS tries to color two neighbors the same color, then it's found a **cycle of odd length** in the graph

If **x** and **y** are the same color & are neighbors, then they are on the same level.



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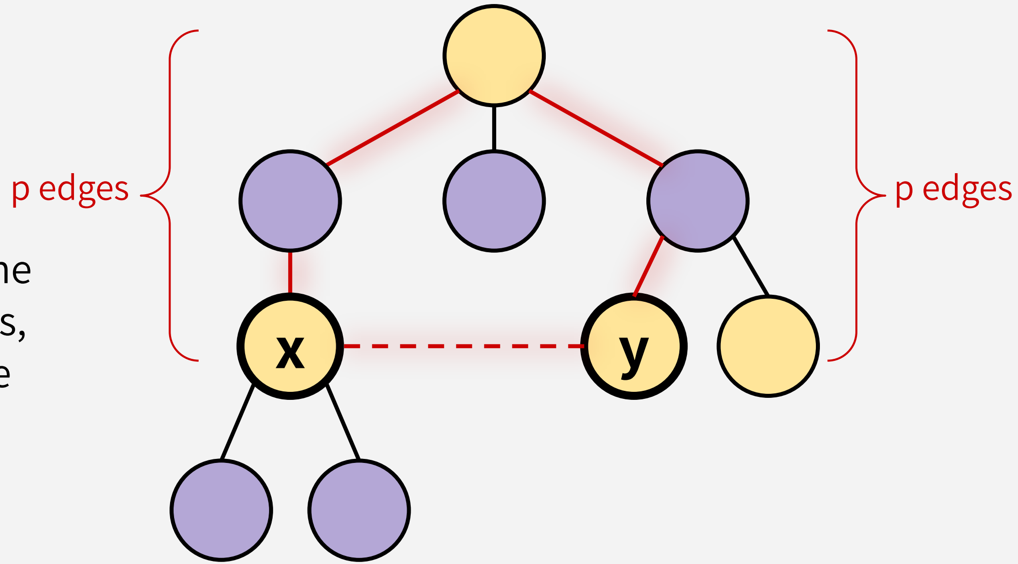

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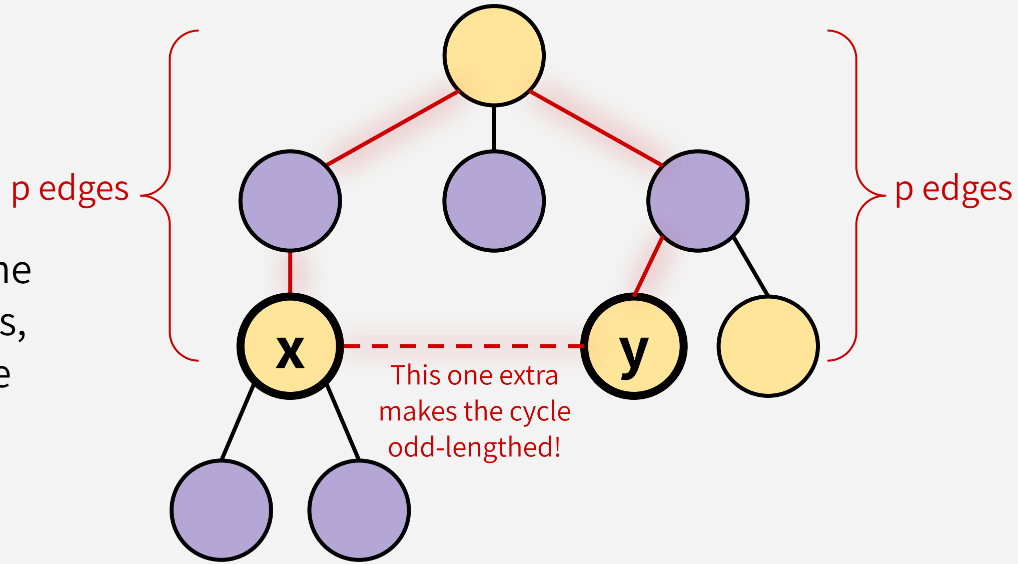

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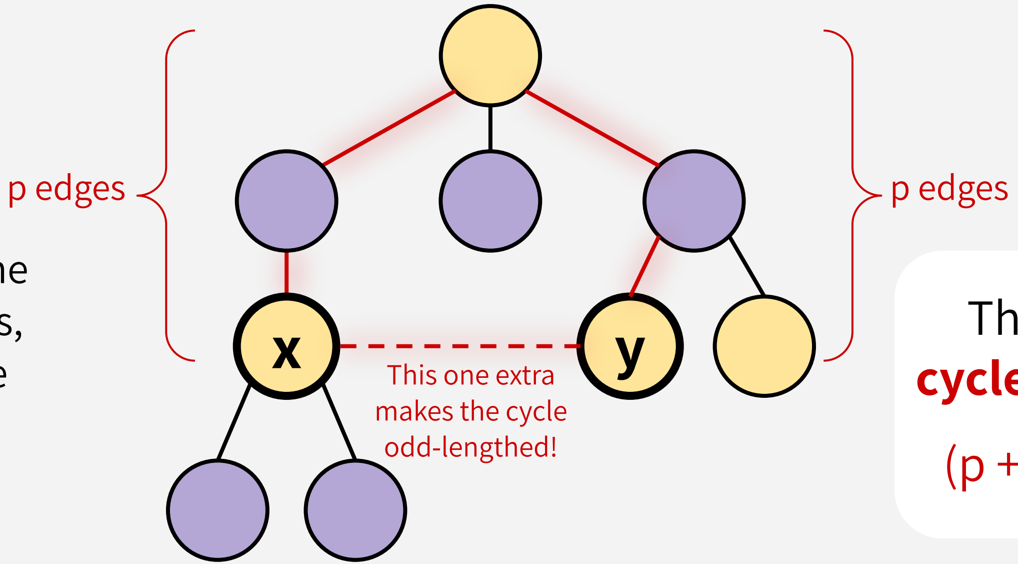

If **x** and **y** are the same color & are neighbors, then they are on the same level.

Thus, there is a **cycle of odd length**

(p + p + 1) = odd #

BIPARTITE GRAPHS

If BFS tries to color two neighbors the same color,

It’s impossible to color a cycle of odd length with two colors such that no two neighbors have the same color. Therefore, it’s impossible to two-color the graph such that no adjacent vertices are colored the same.

So, BFS colors two neighbors the same color iff the graph is not bipartite.

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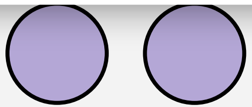


BFS & BIPARTITE GRAPHS RECAP

BFS can be used to detect bipartite-ness of a graph in time O(n + m), since all that coloring business is just O(1) extra work per node or edge.

This is one example of how you can take advantage of the “layers” that BFS constructs to reason about how to accomplish a task that might not seem like a “classic” BFS-shortest-path task (which you might be more familiar with).

سوال؟

جستجوی عمق اول (DFS)

یک روش دیگر برای پیمایش گراف

BFS vs. DFS

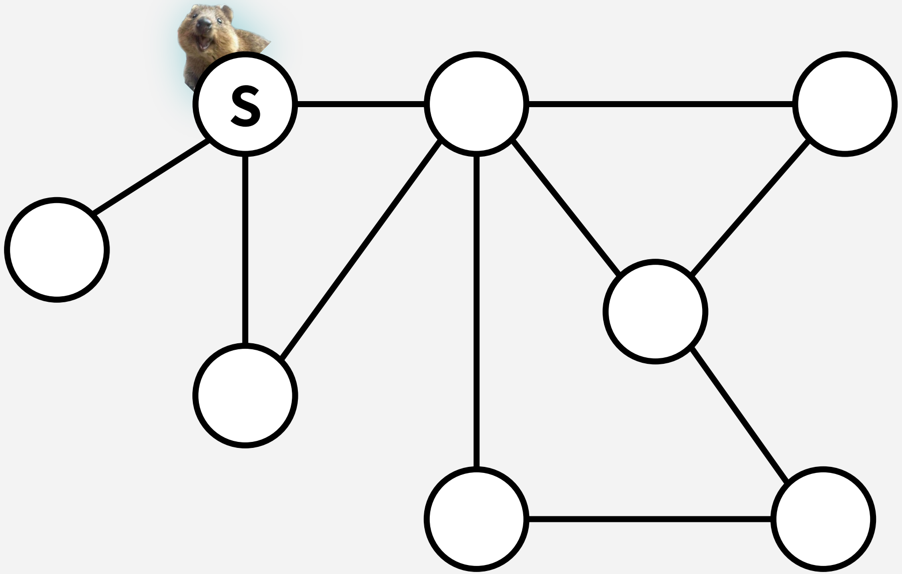
Literally just BREADTH vs DEPTH:

While BFS first explores the nodes closest to the “source” and then moves outwards in layers, DFS goes as far down a path as it can before it comes back to explore other options.

DEPTH-FIRST SEARCH

An analogy:

A smart quokka is exploring a labyrinth with chalk (to mark visited destinations) & thread (to retrace steps)



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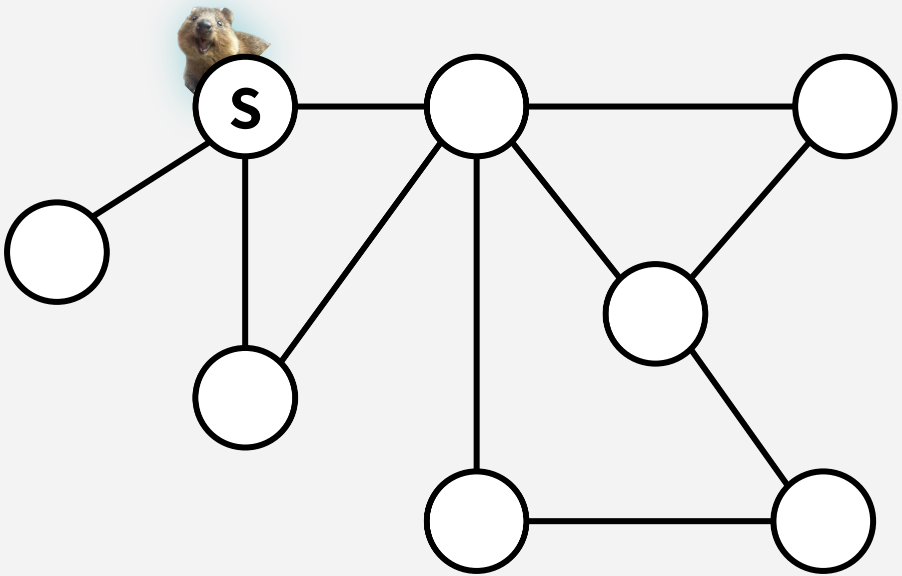

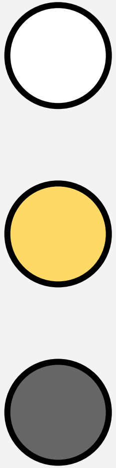

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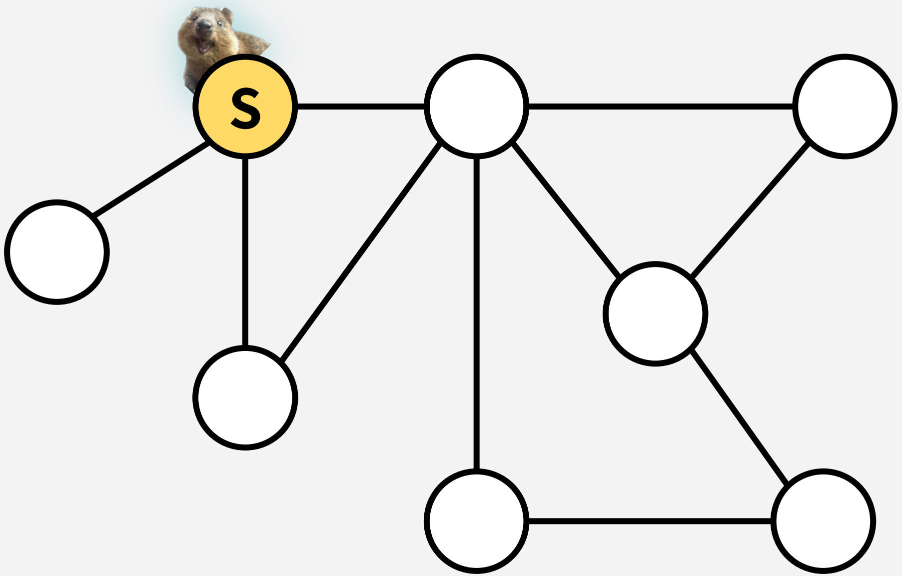

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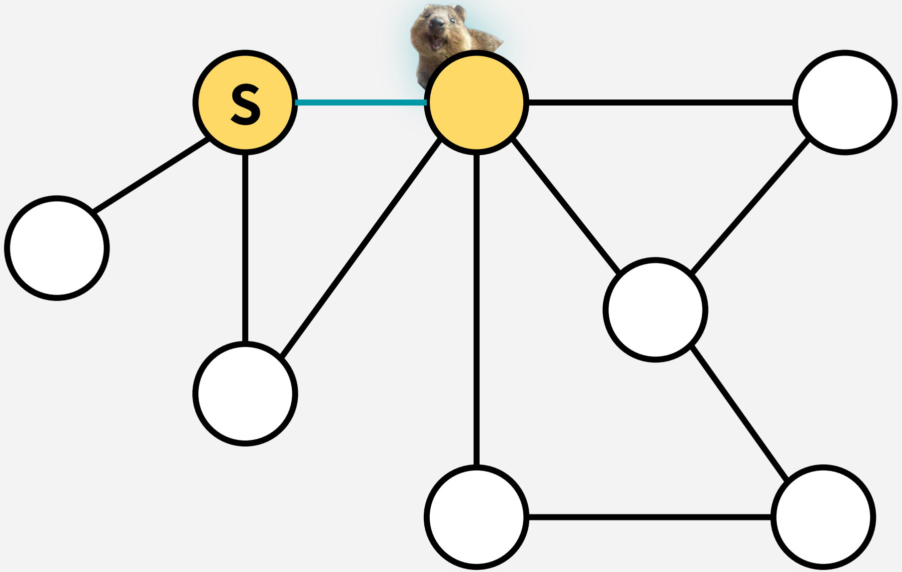

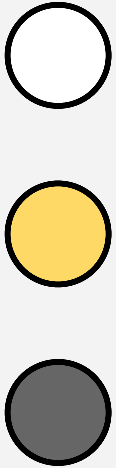

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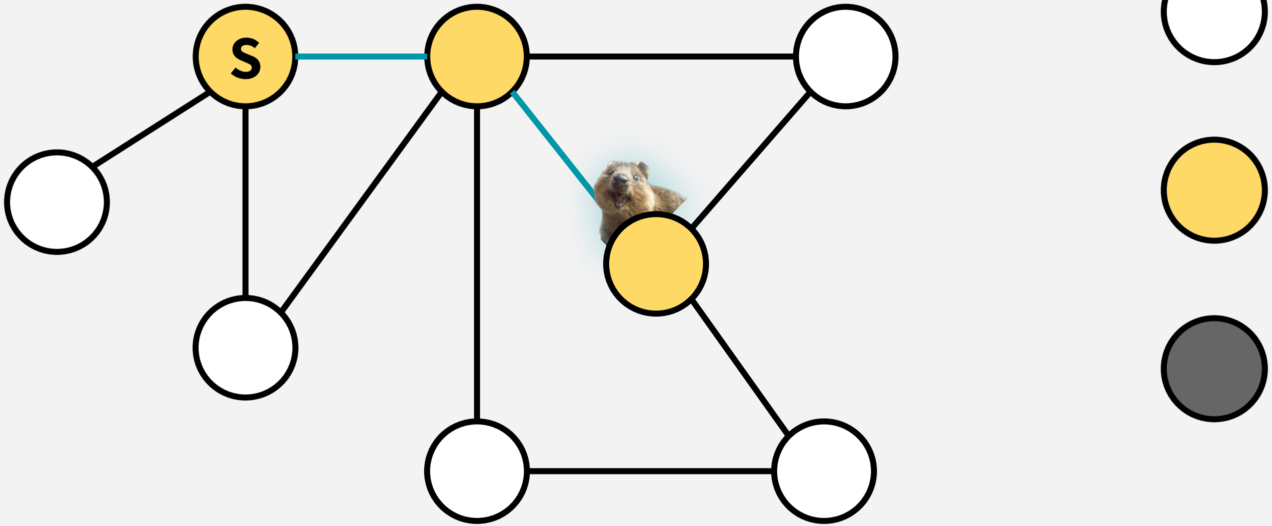

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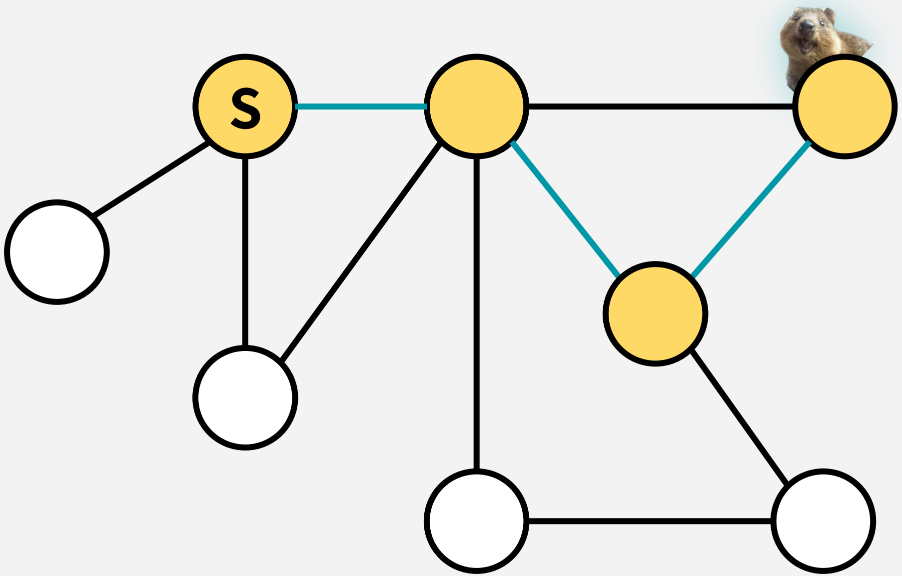

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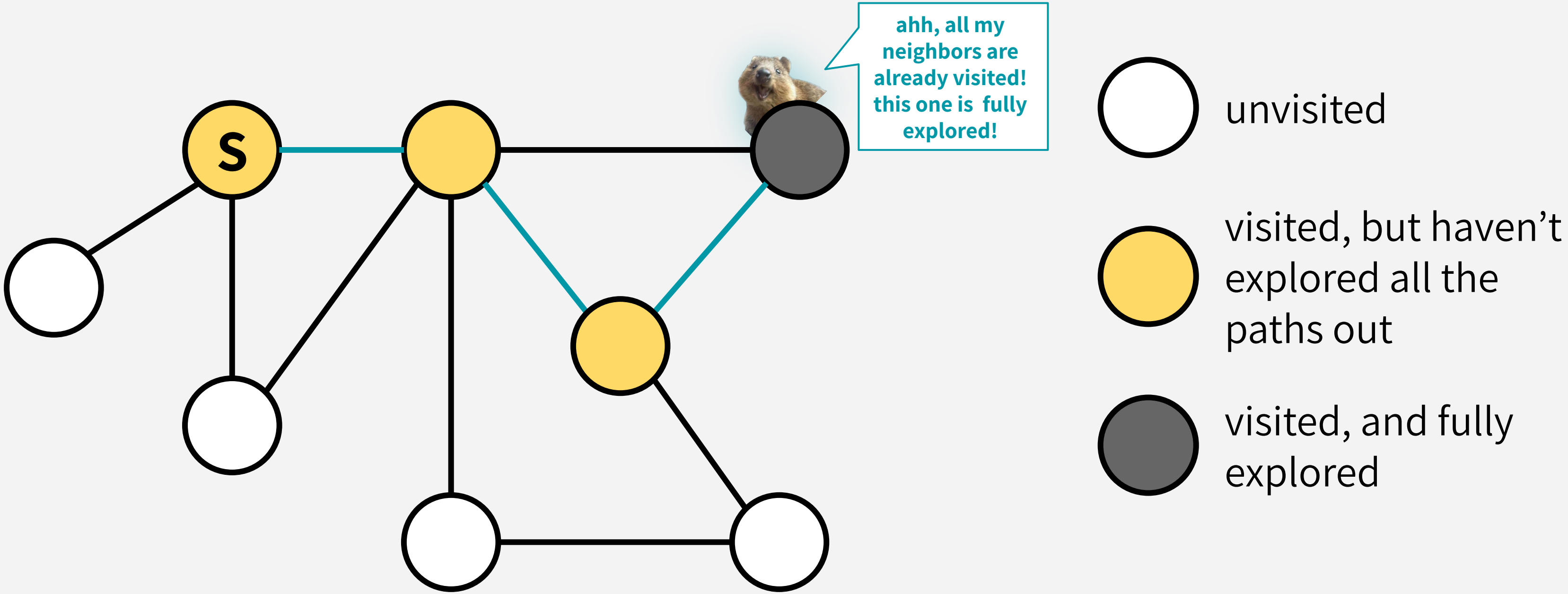

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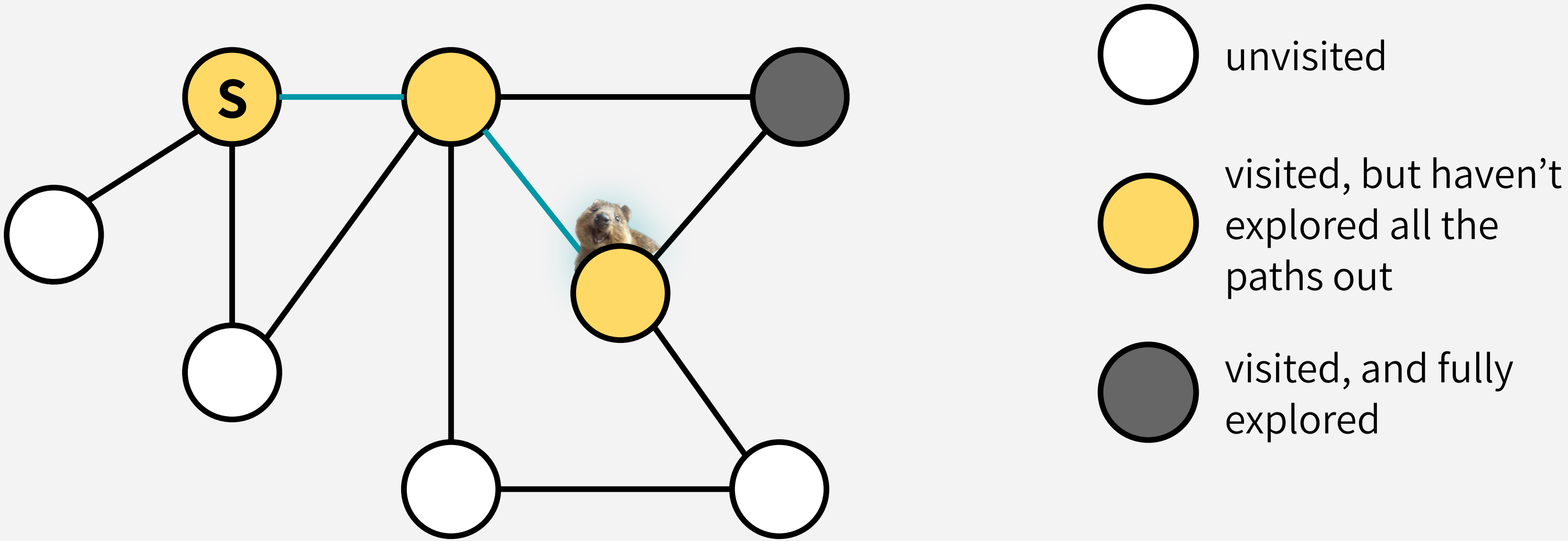

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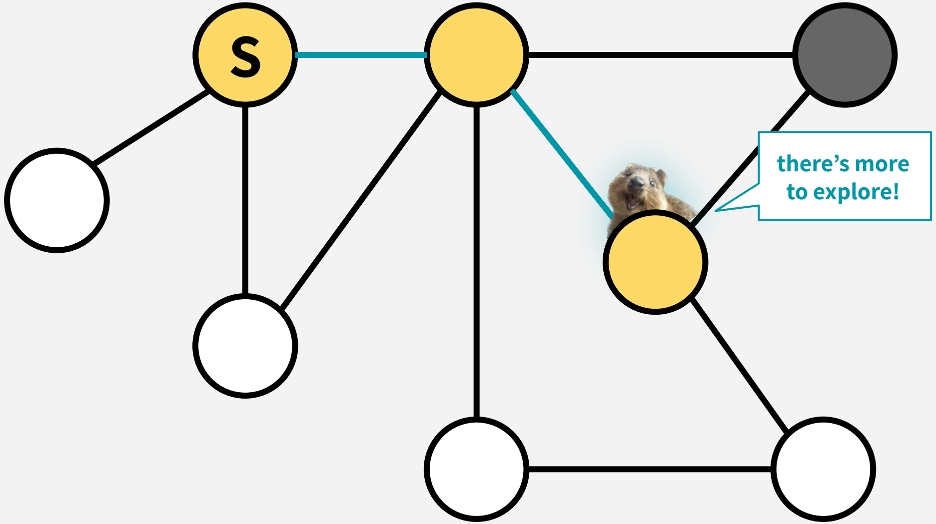

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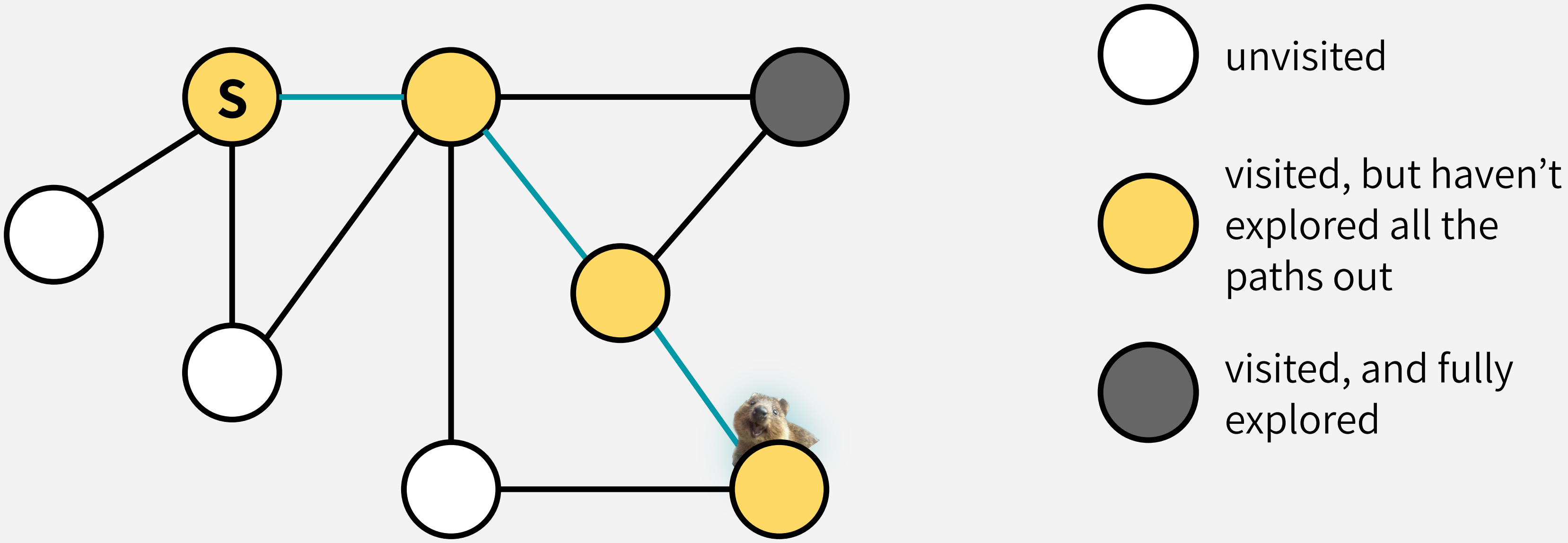

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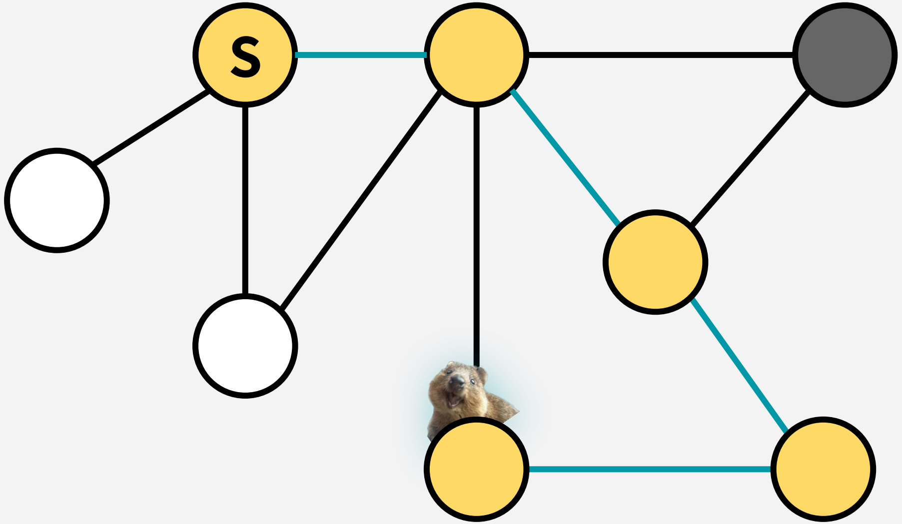

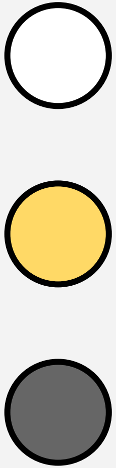

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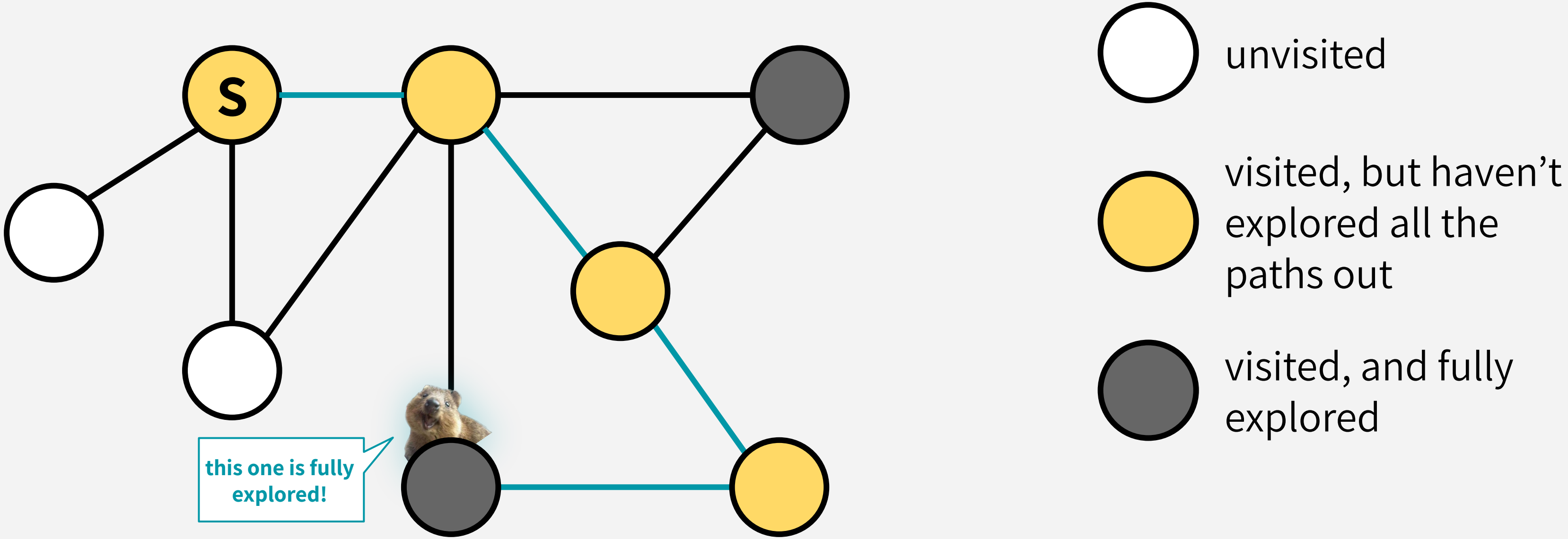

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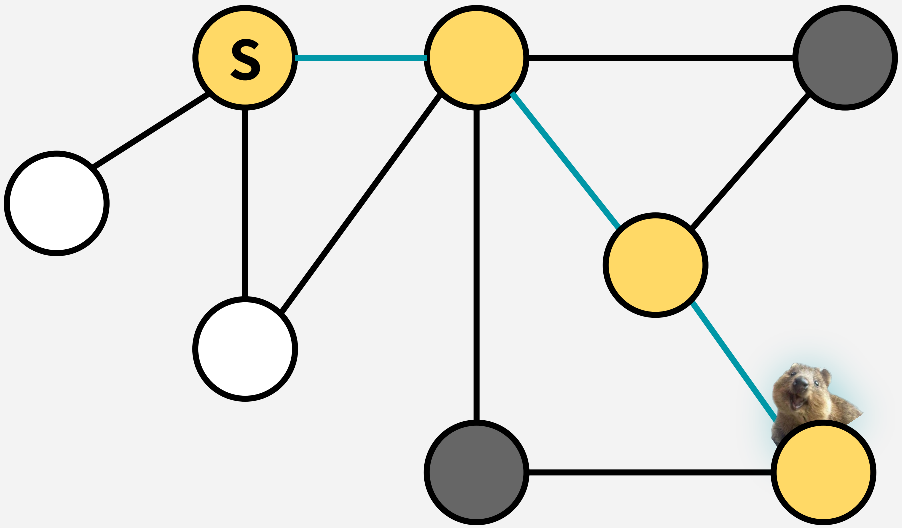

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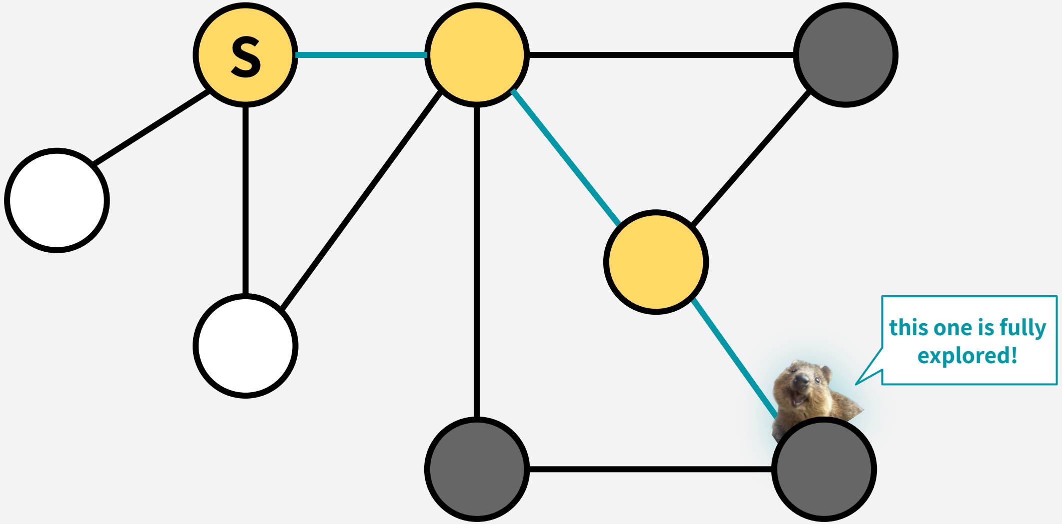

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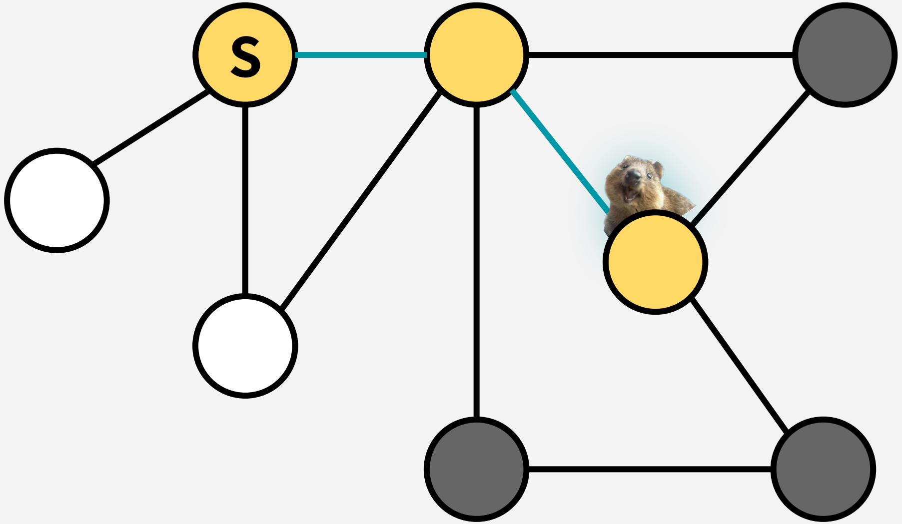

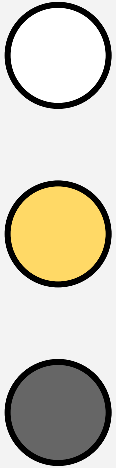

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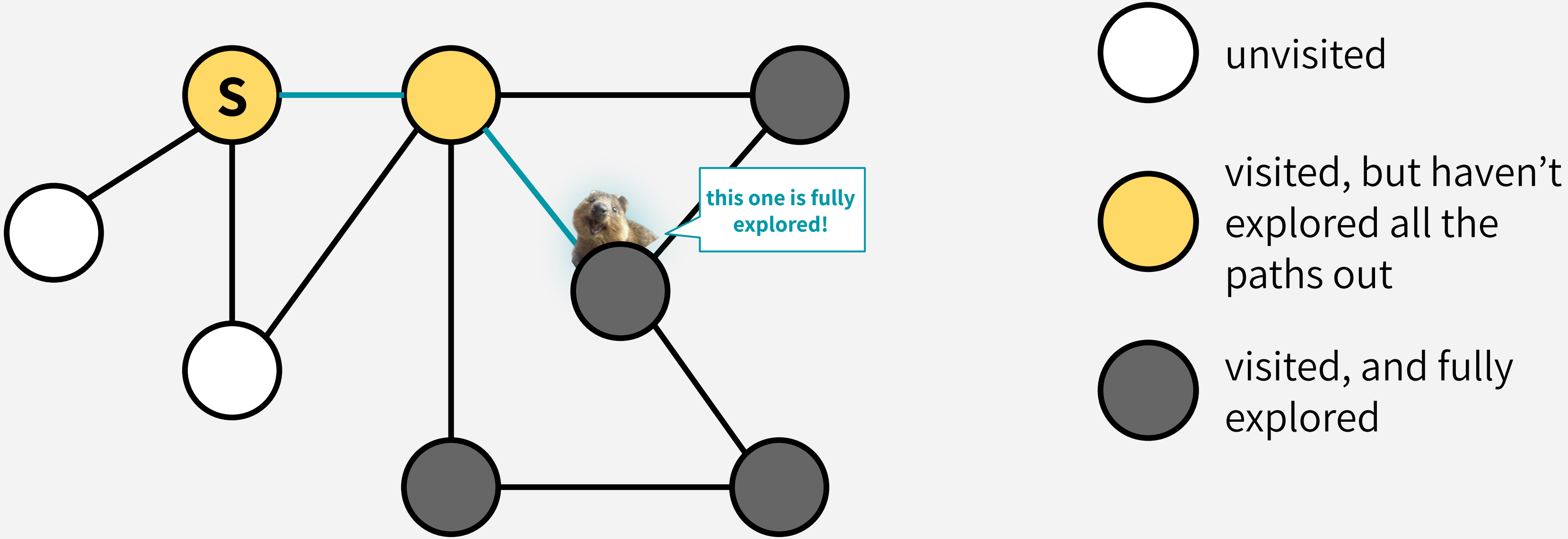

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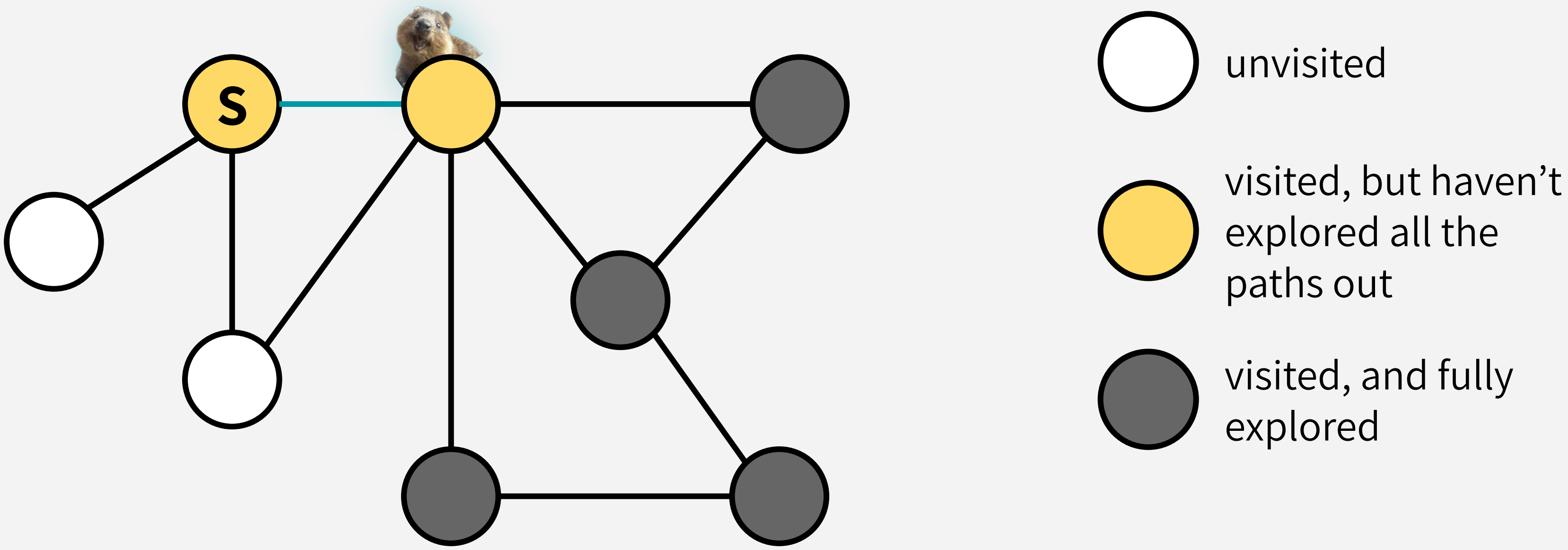

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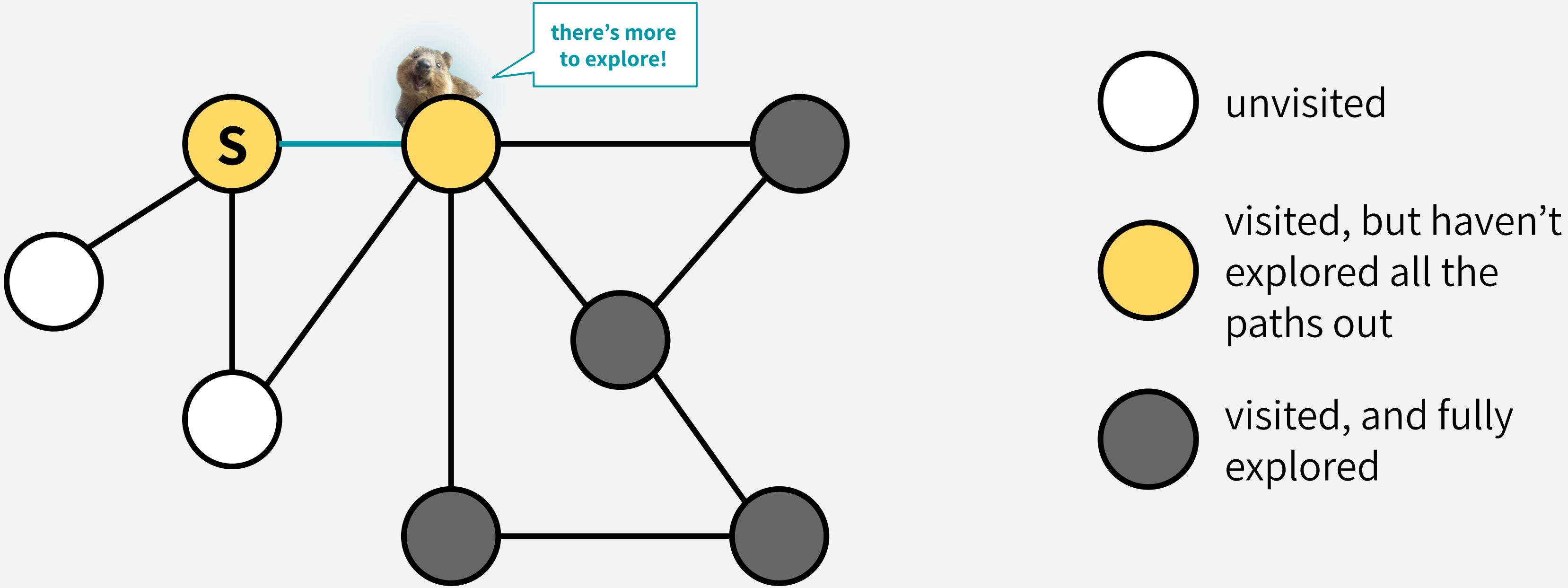

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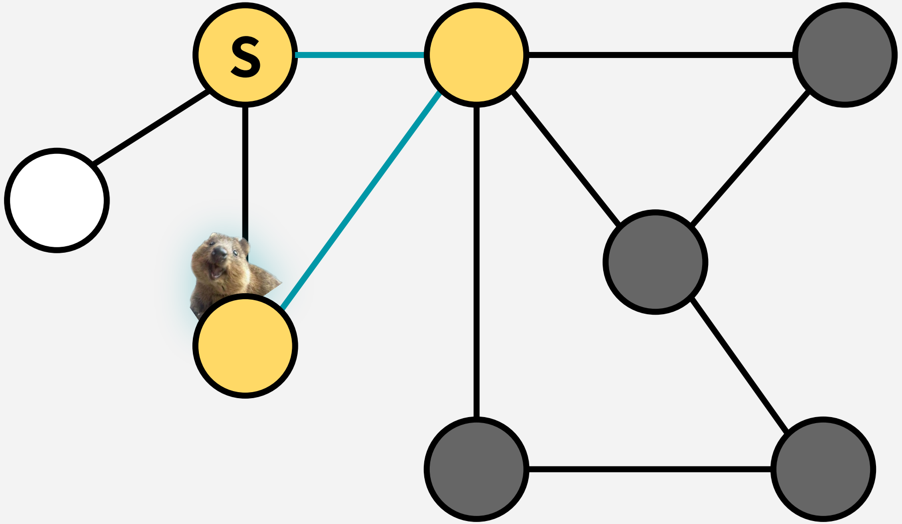

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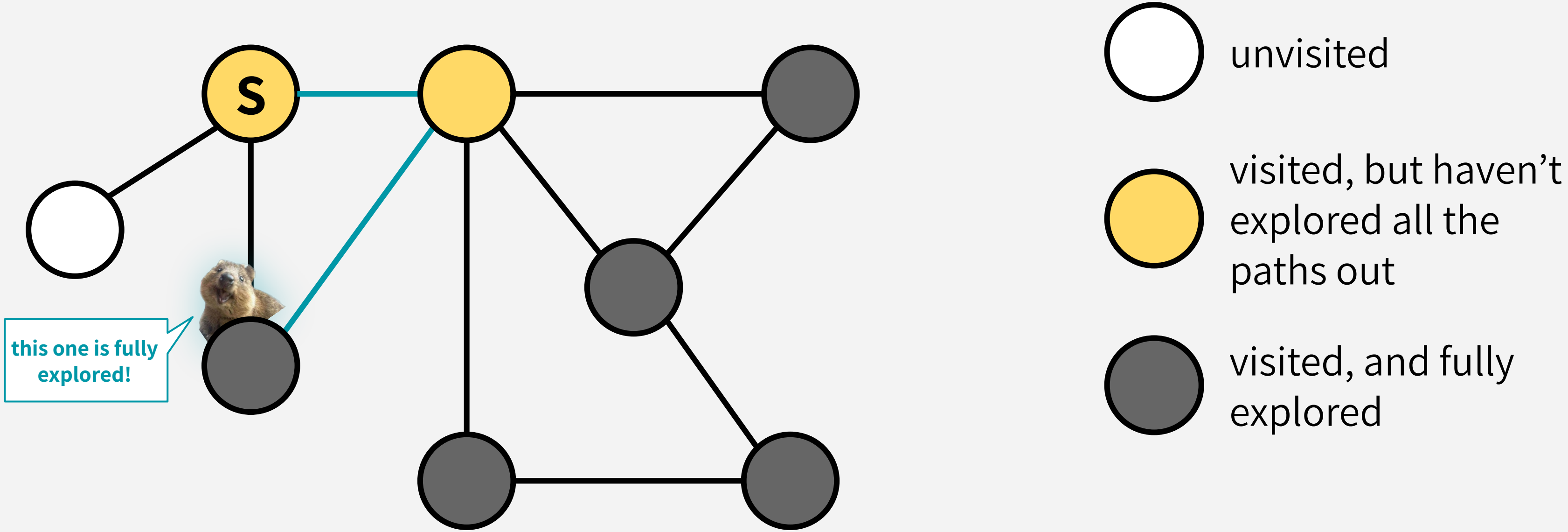

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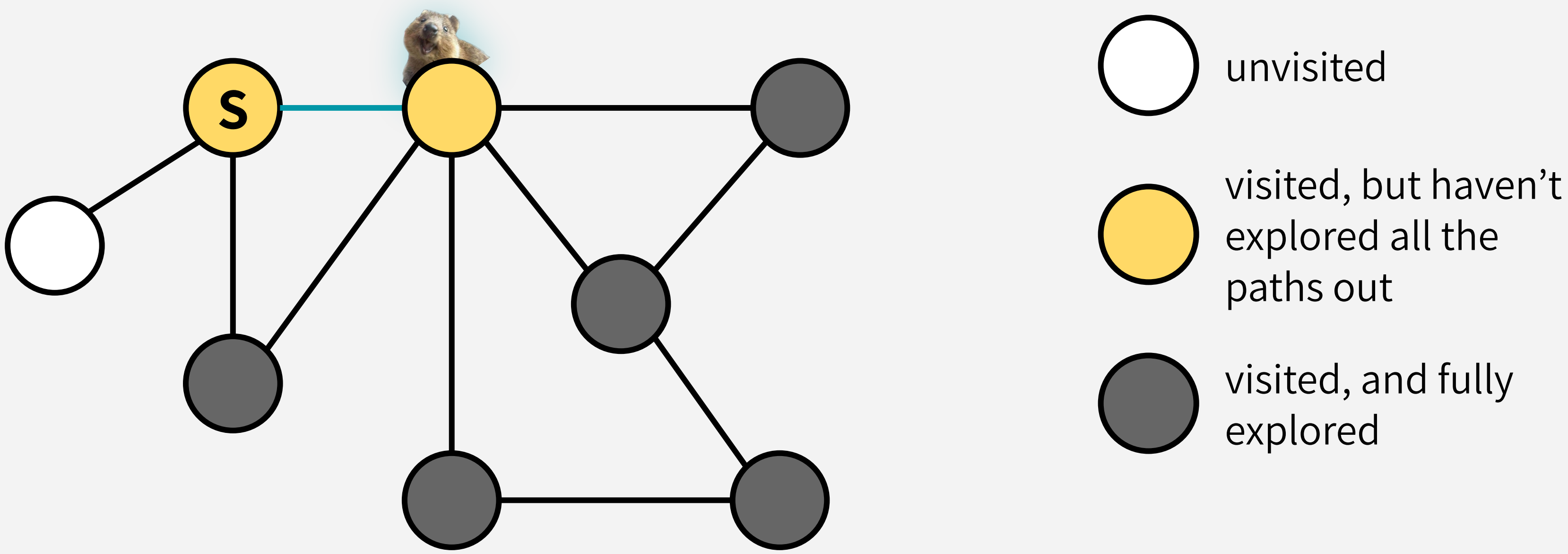

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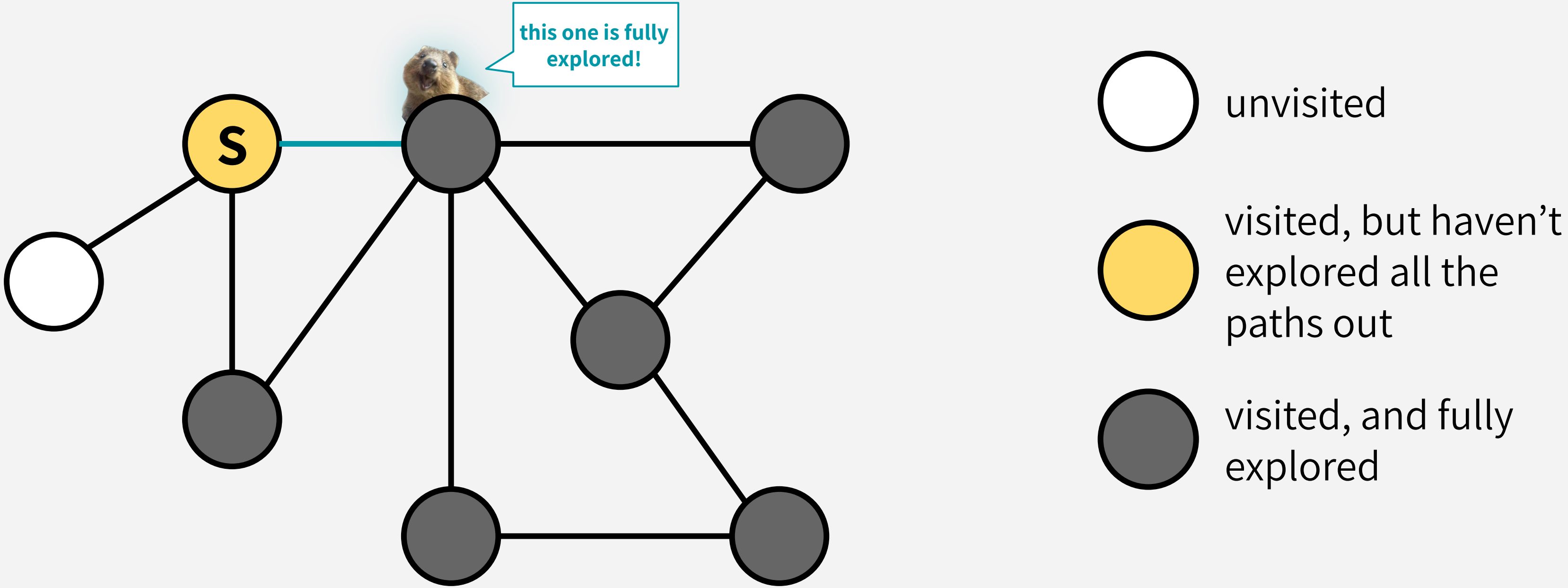

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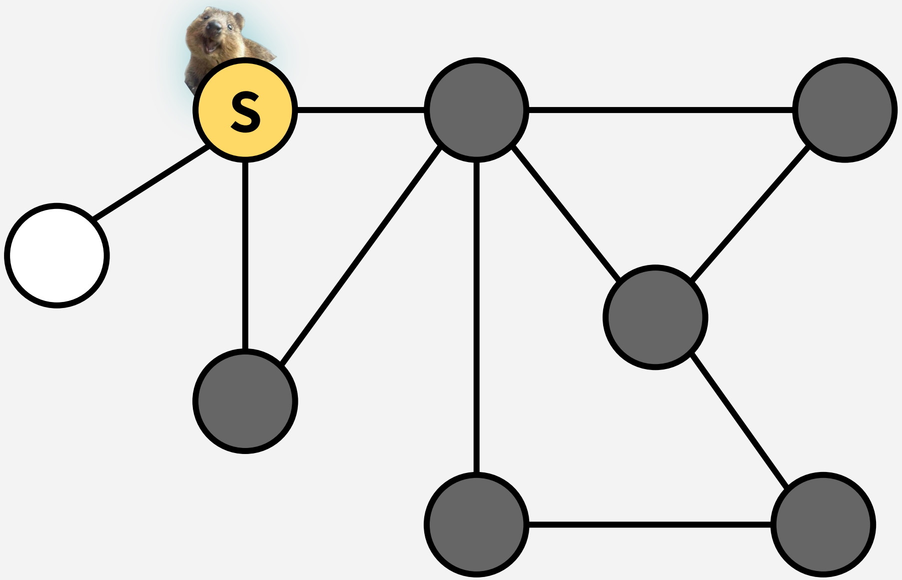

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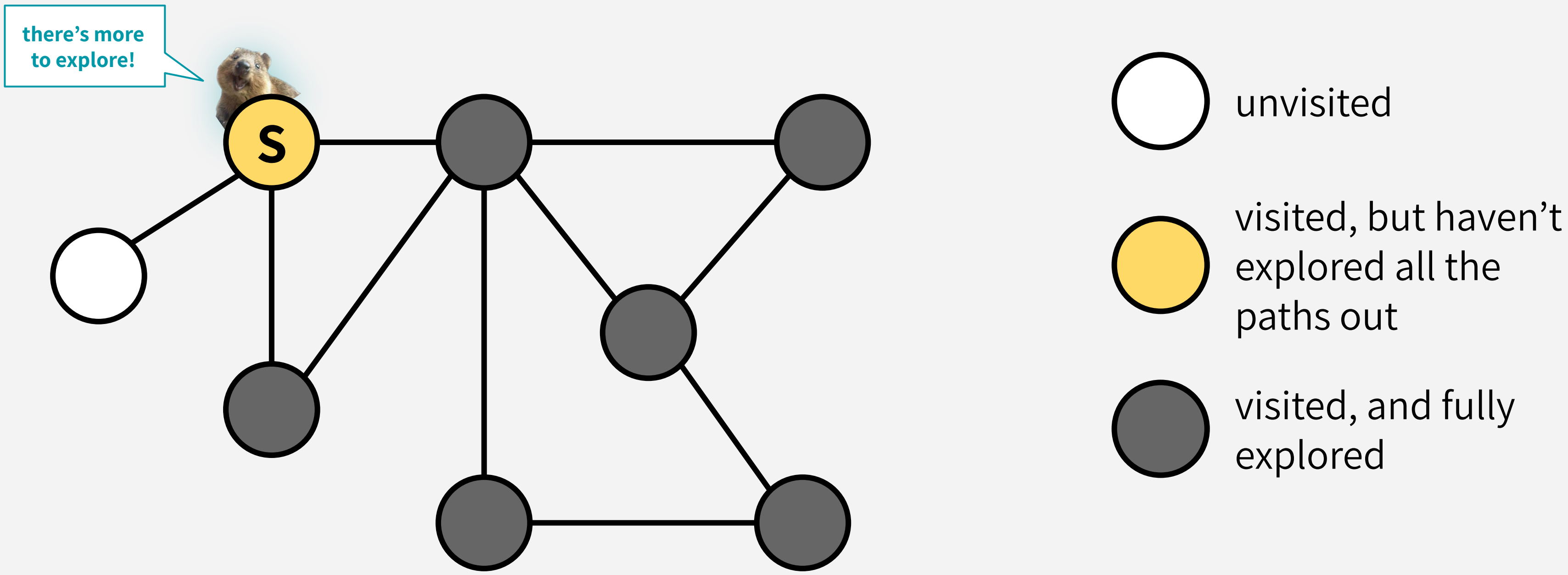

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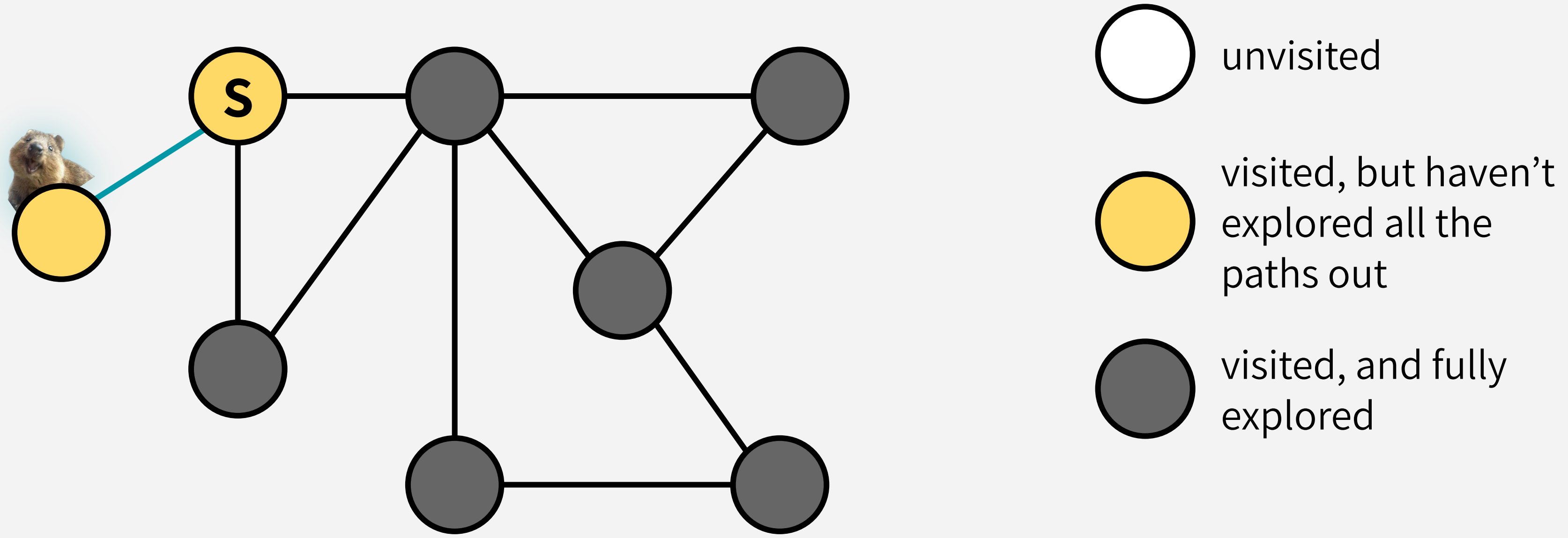

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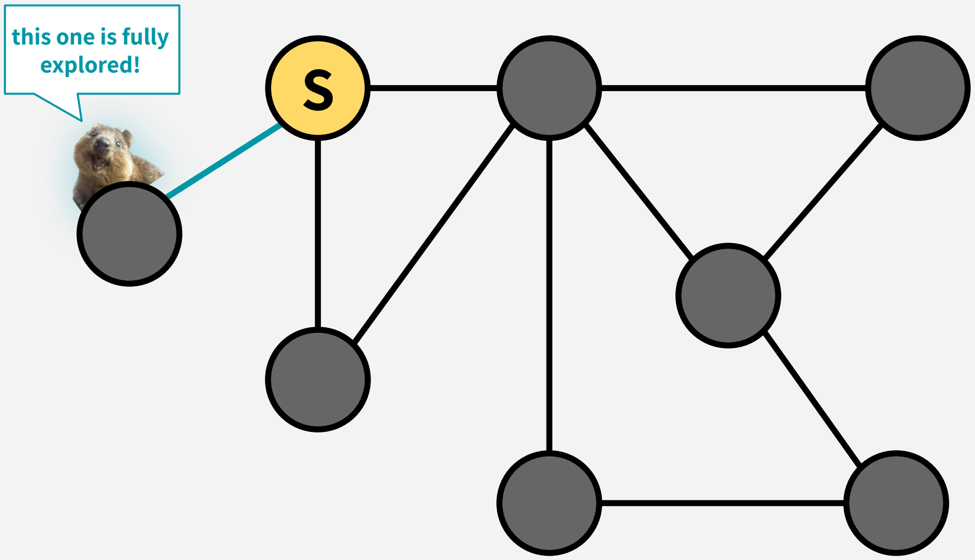

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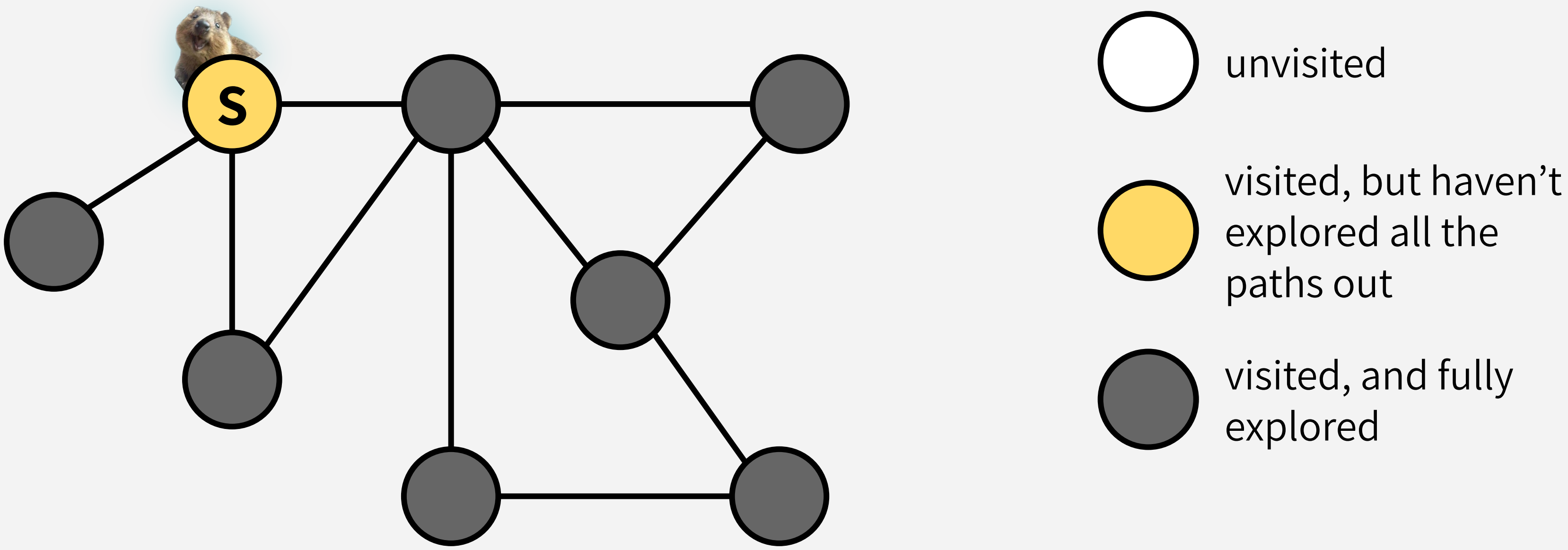

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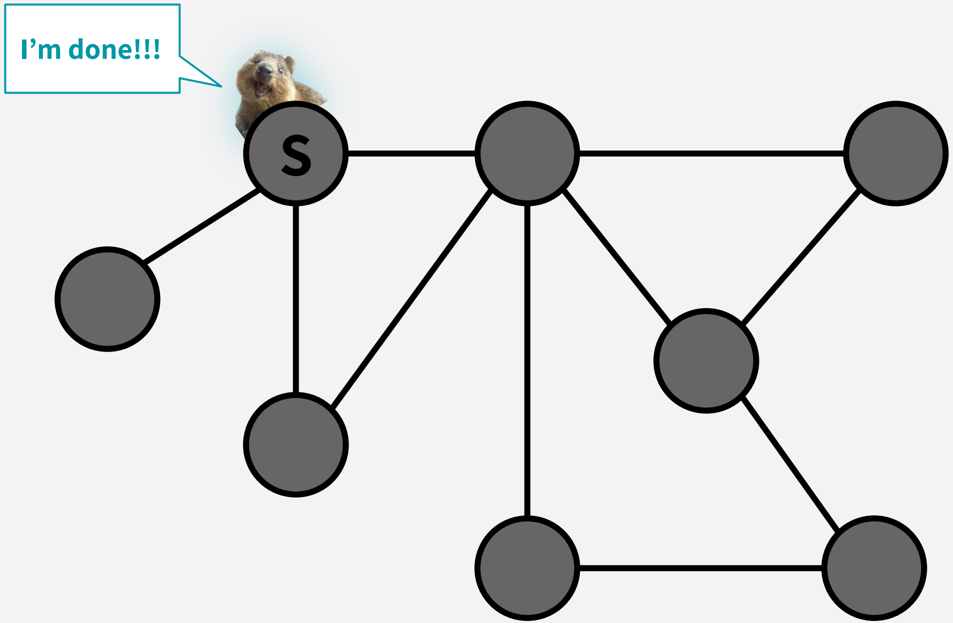

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I'm done!!!

In addition to keeping track of the visited status of nodes, we're going to keep track of:

The time we first enter it, i.e. mark it as ○.

The time we finish it, i.e. mark it as ●.

You've probably seen other ways to implement DFS, all this extra bookkeeping will be useful for us later!

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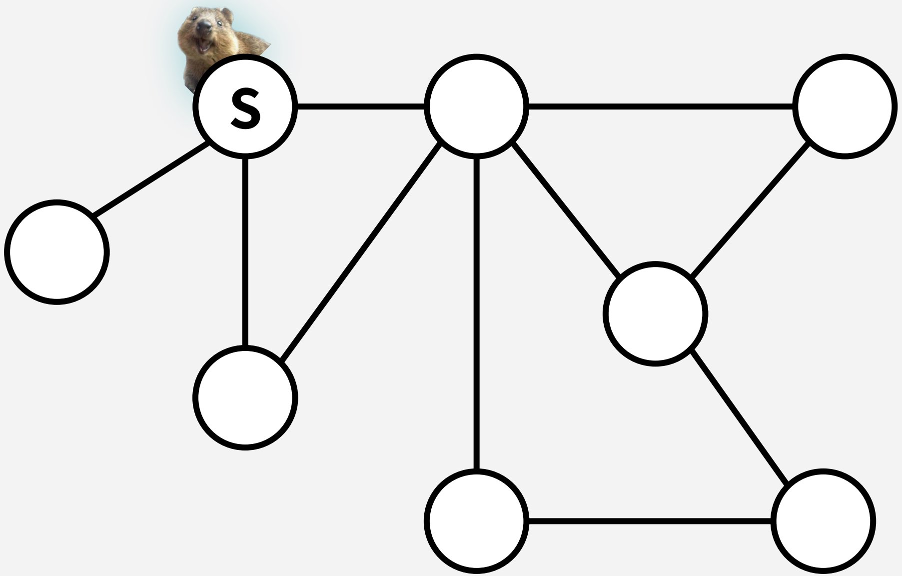


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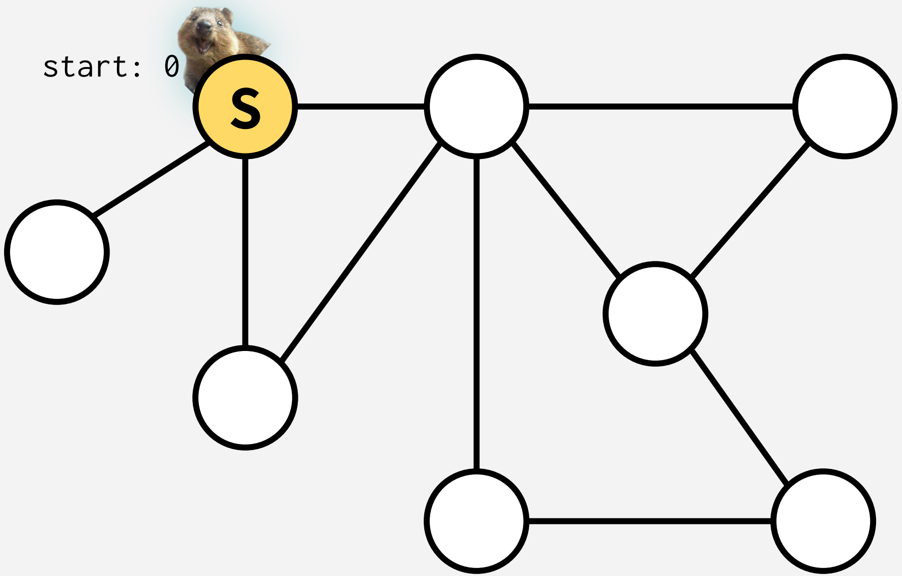


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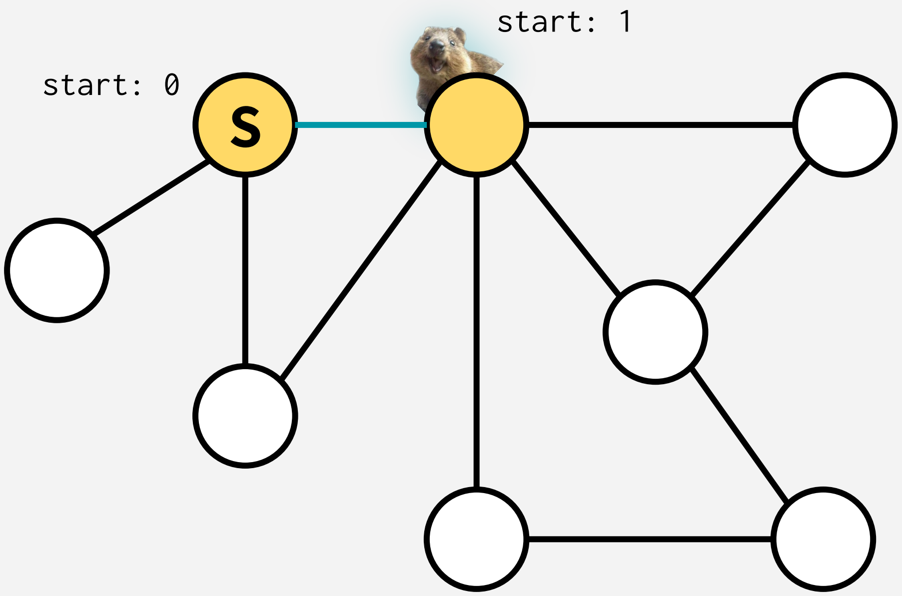


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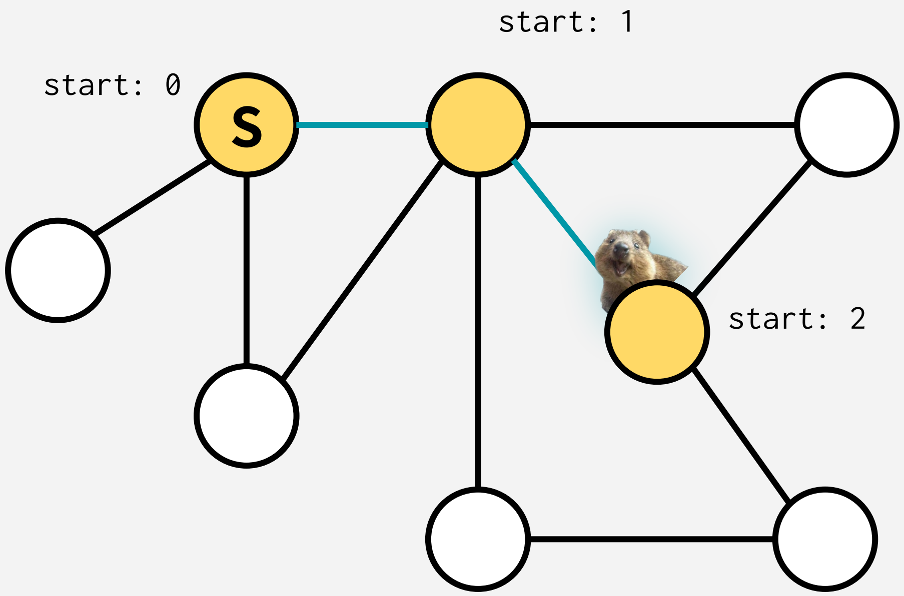


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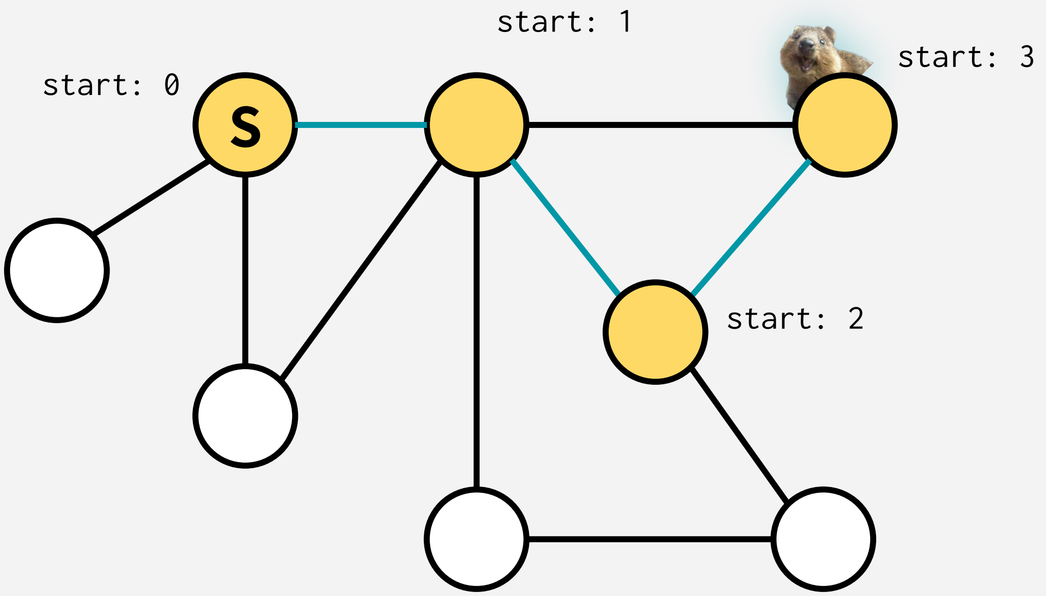


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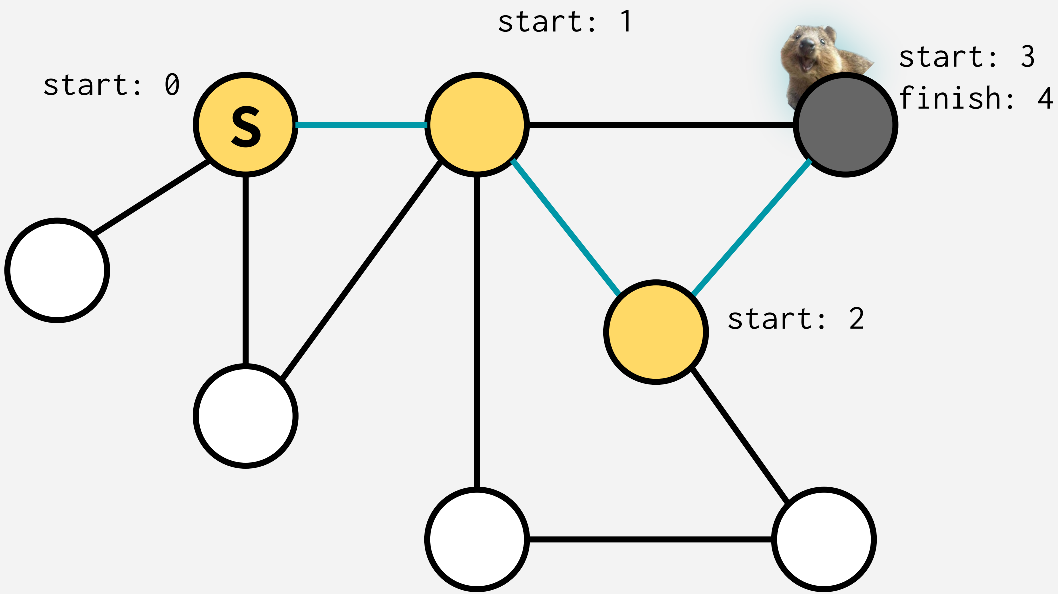


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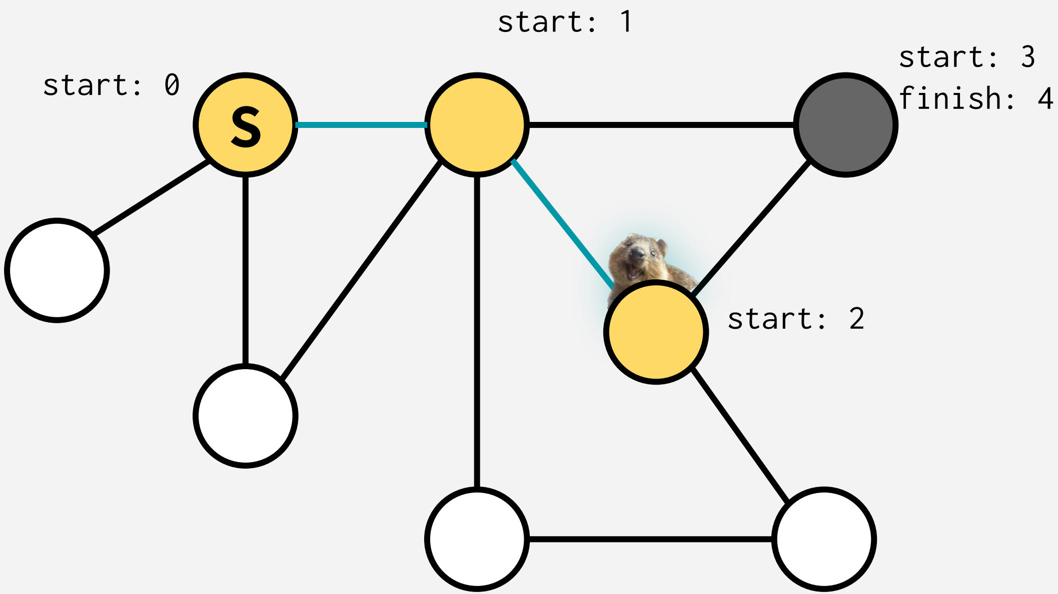


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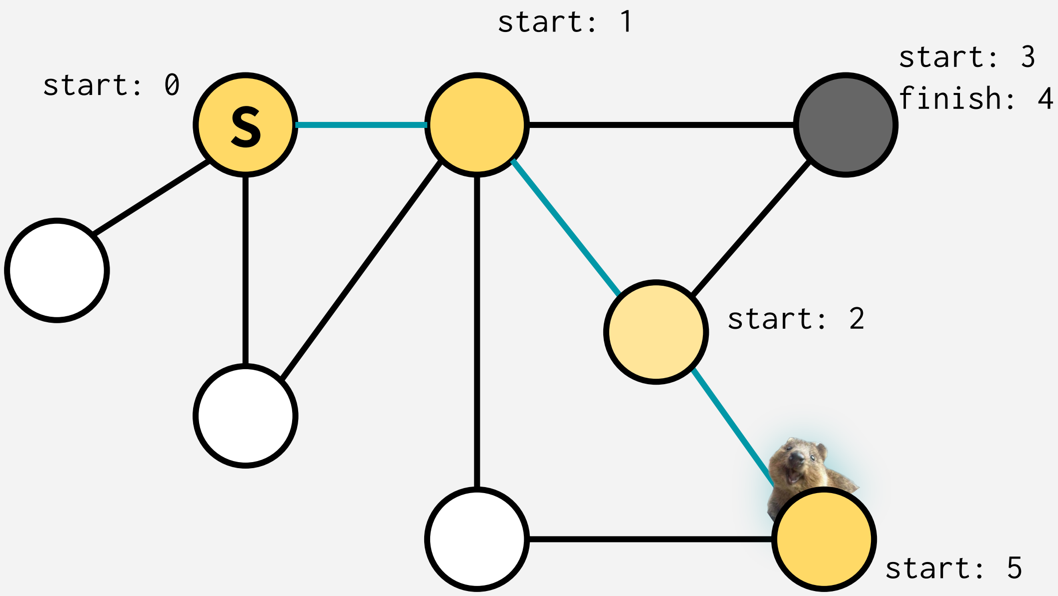


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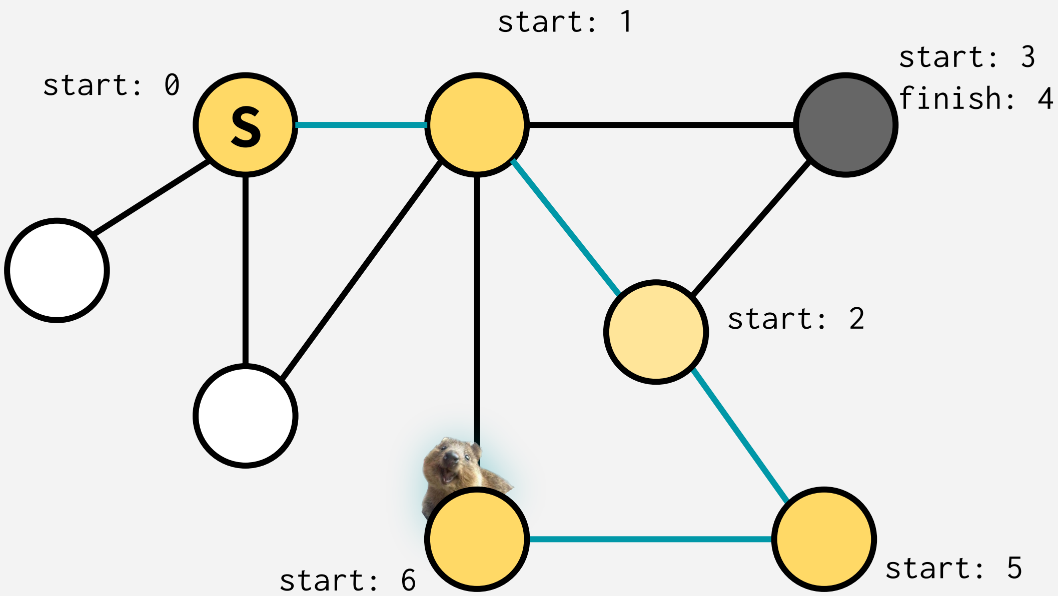


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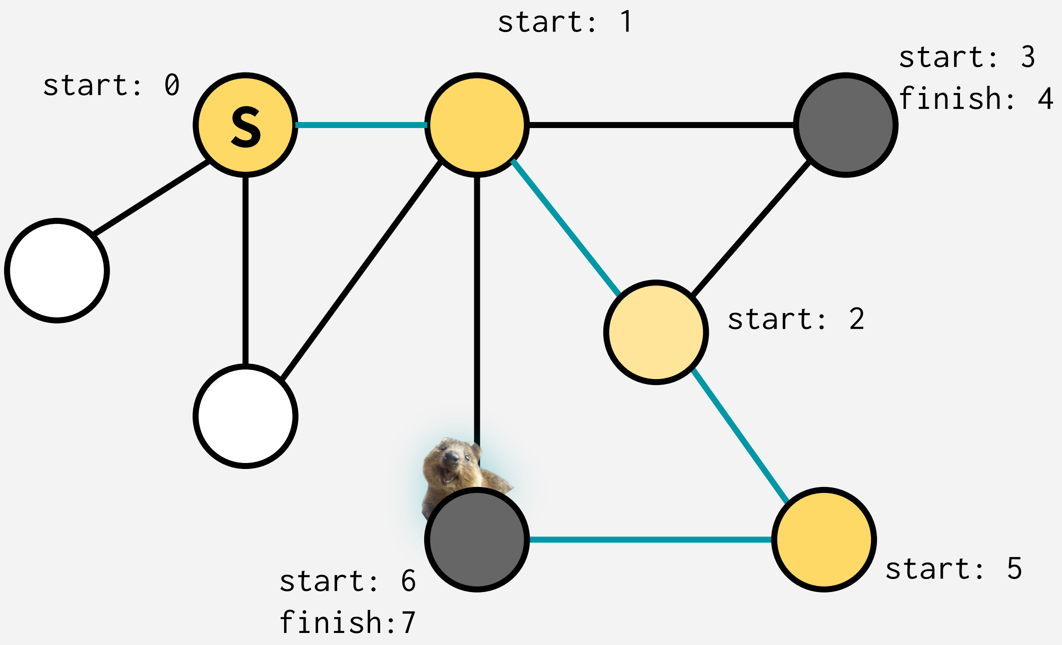


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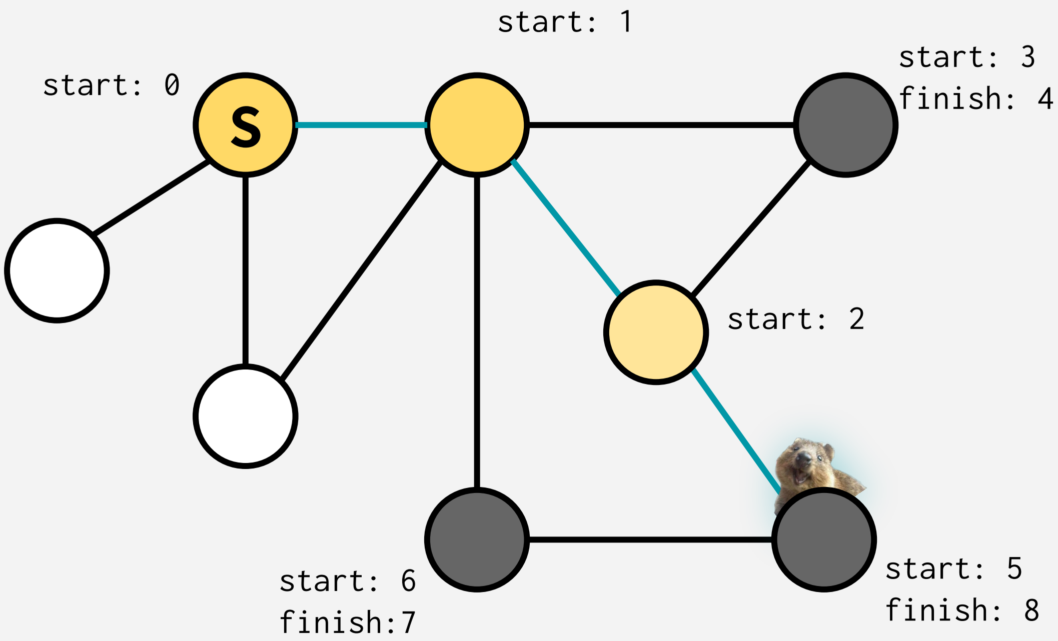


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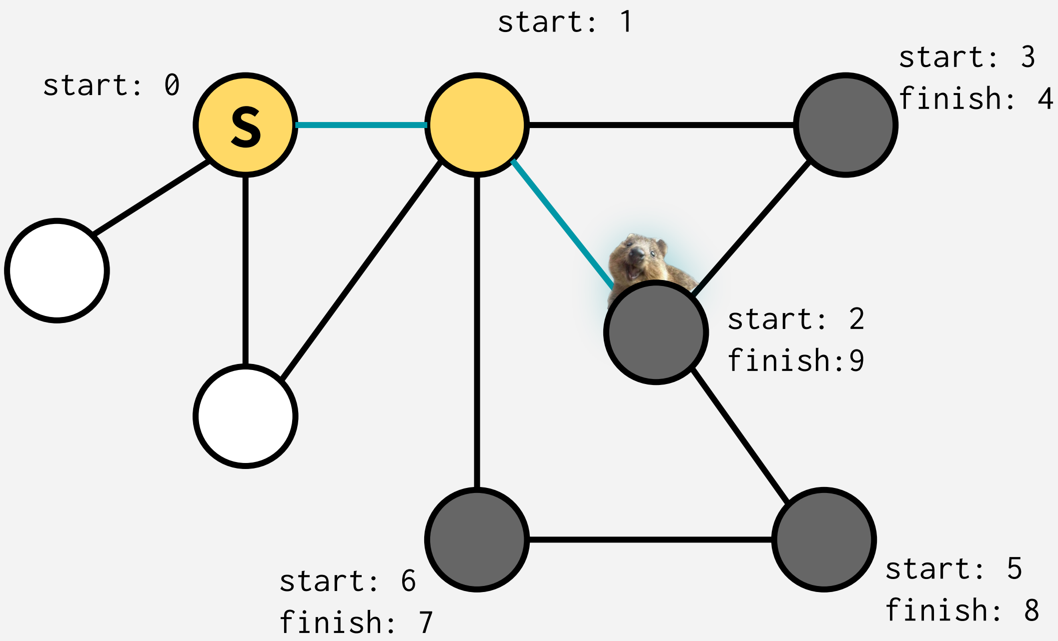


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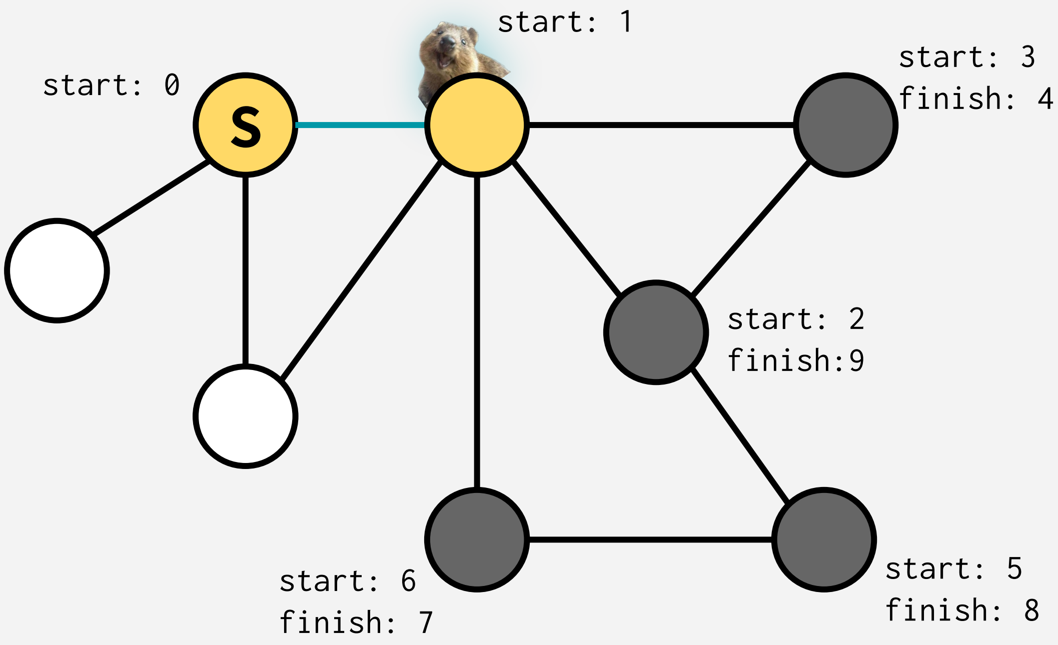


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DEPTH-FIRST SEARCH

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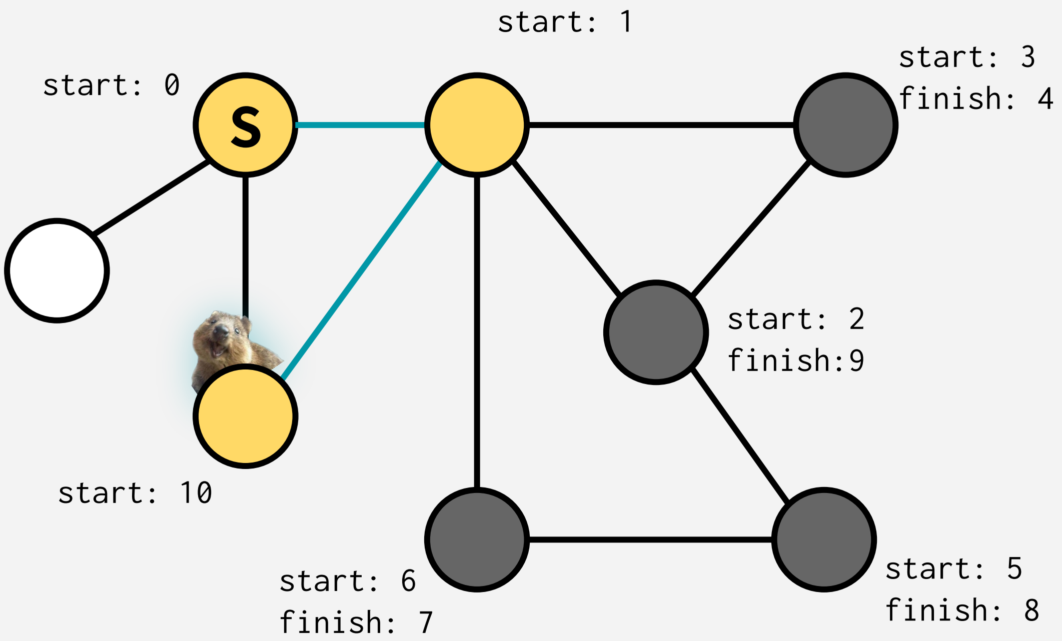
A smart quokka is exploring a labyrinth with chalk (to mark visited destinations) & thread (to retrace steps)

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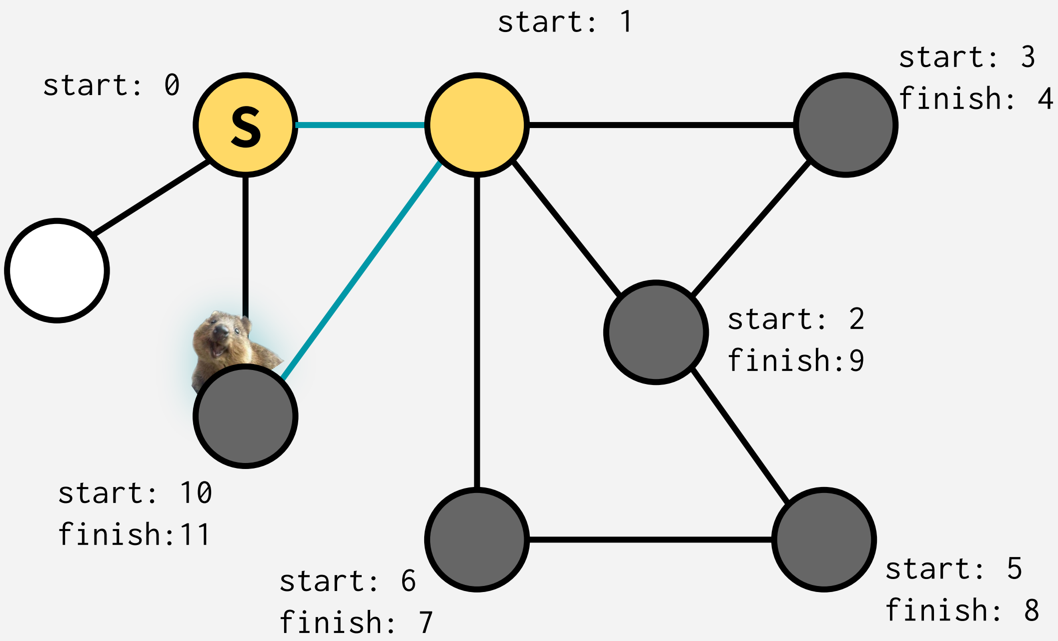


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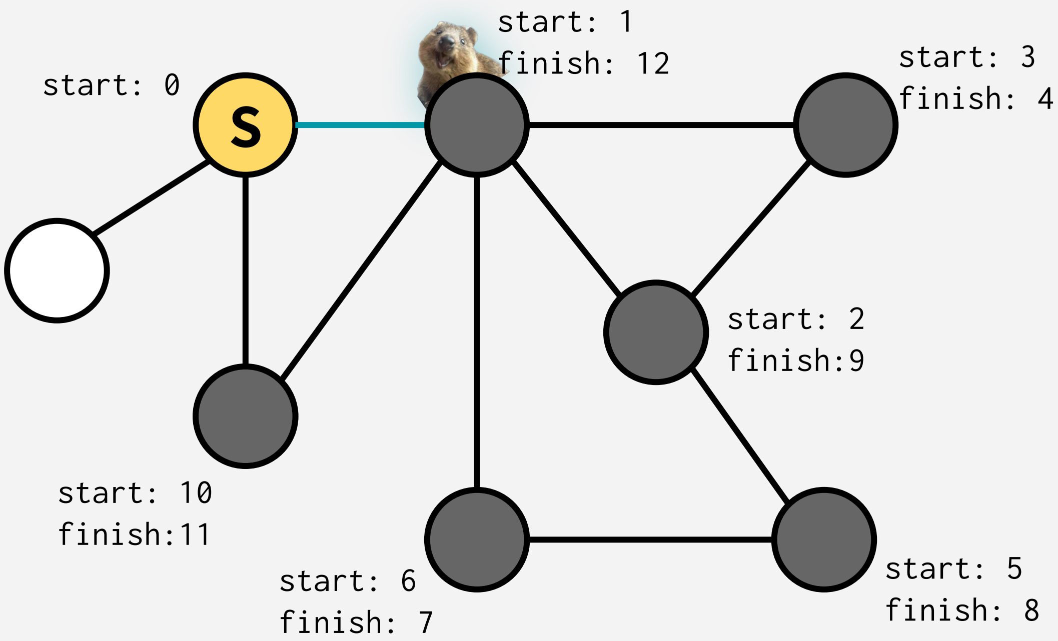


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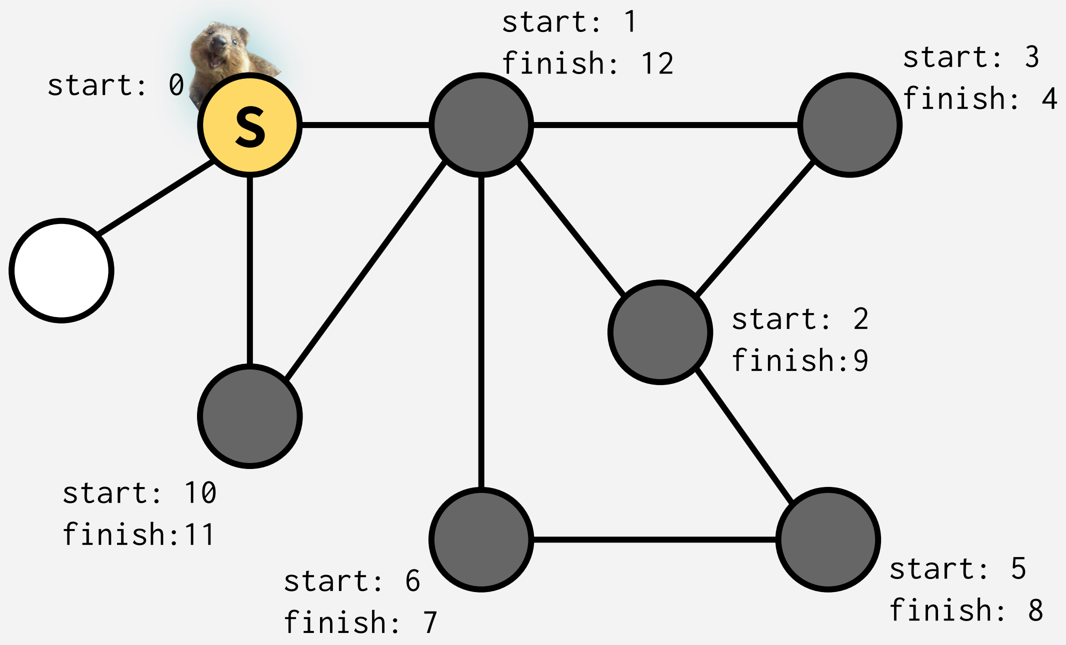


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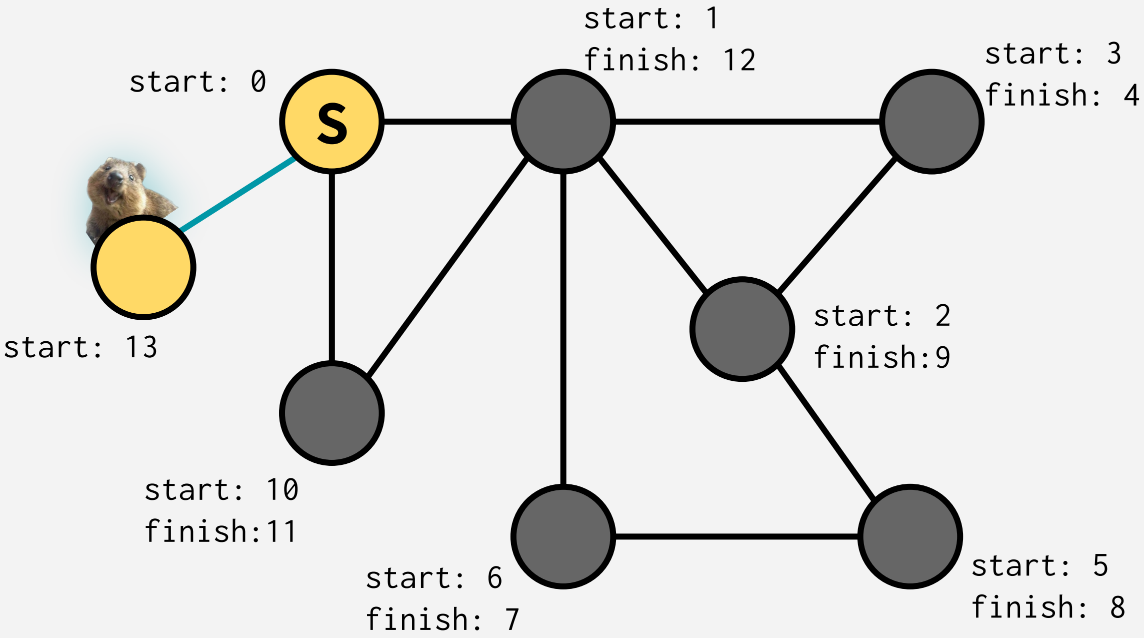


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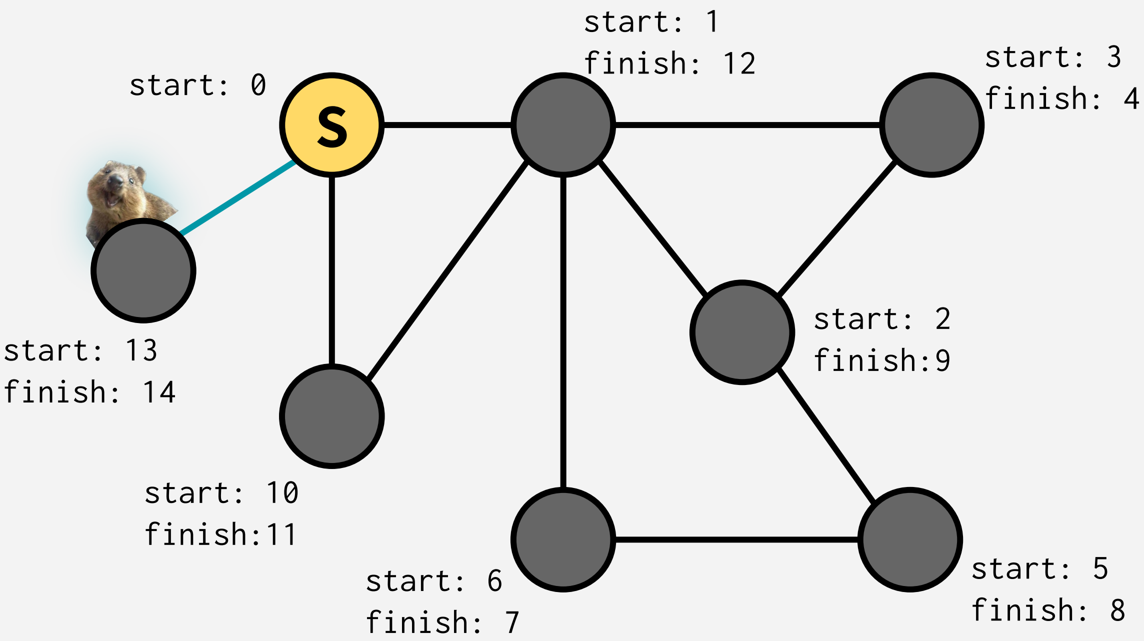


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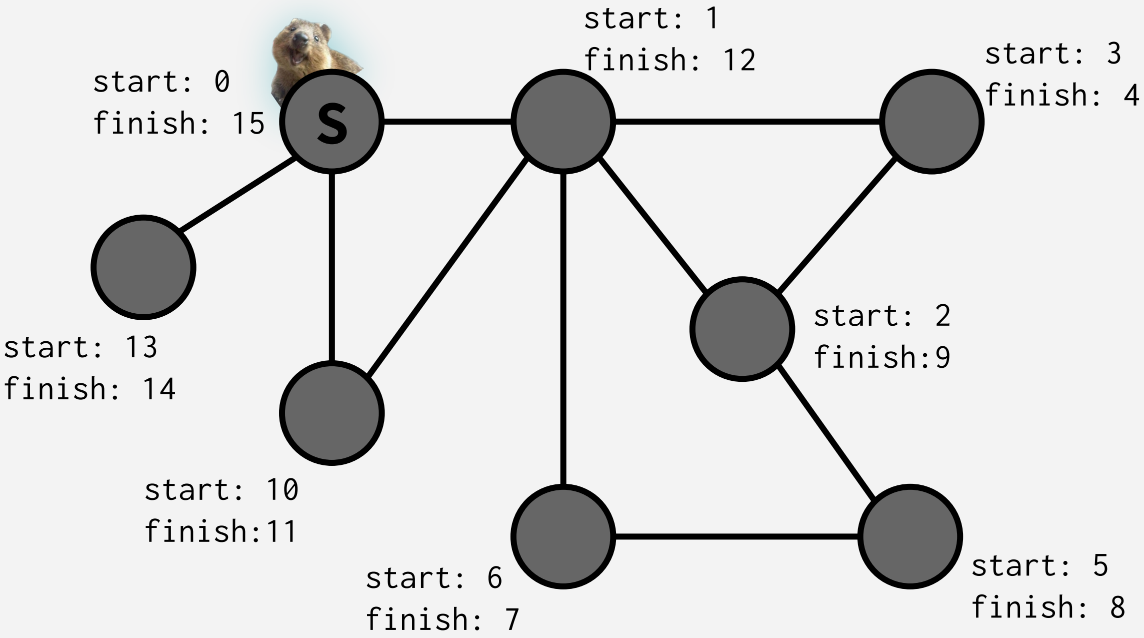


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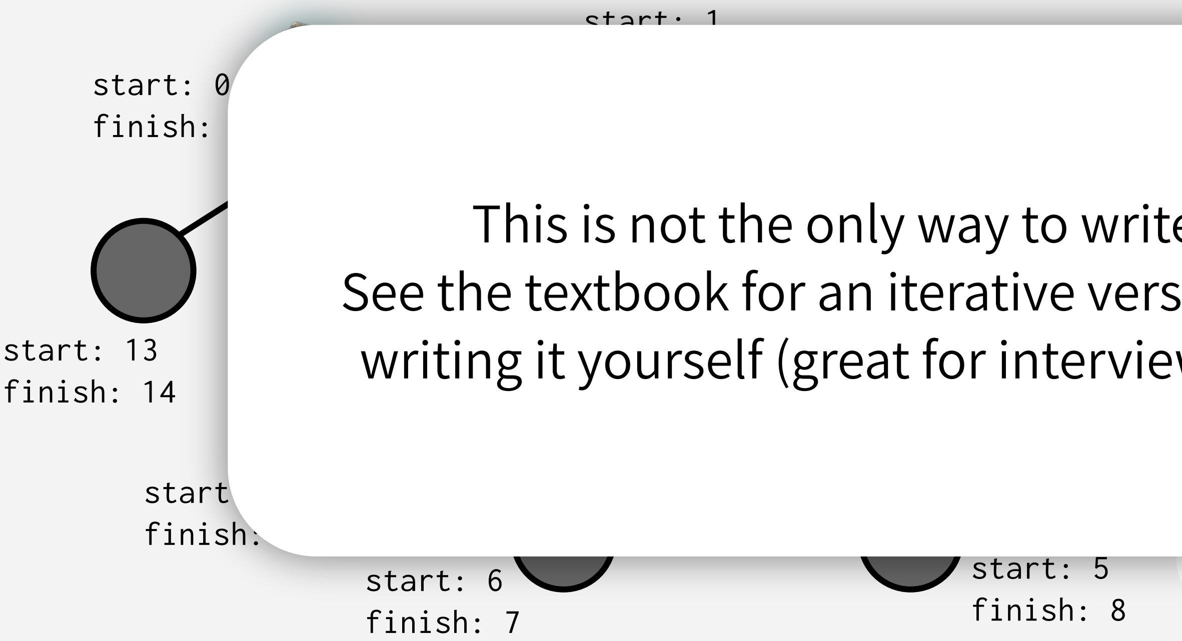


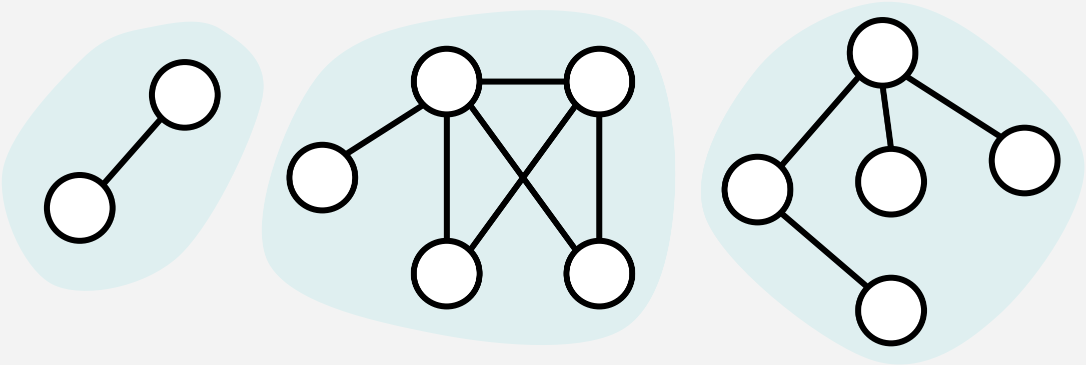
This is not the only way to write DFS!
See the textbook for an iterative version, and try writing it yourself (great for interview practice)

```
return currTime
```

DEPTH-FIRST SEARCH

Like BFS, DFS finds all the nodes reachable from the starting point!

In undirected graphs, this is equivalent to finding a **connected component.**



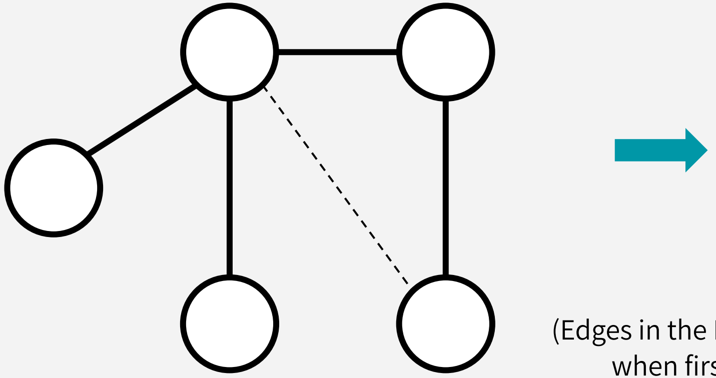
DEPTH-FIRST SEARCH

Why is it called depth-first?

We are implicitly building a **tree**!

(It's a tree because we never revisit a node)

We're going as "deep" as we can before "bubbling" back up.



(Edges in the DFS tree are the ones traversed when first finding unvisited nodes)

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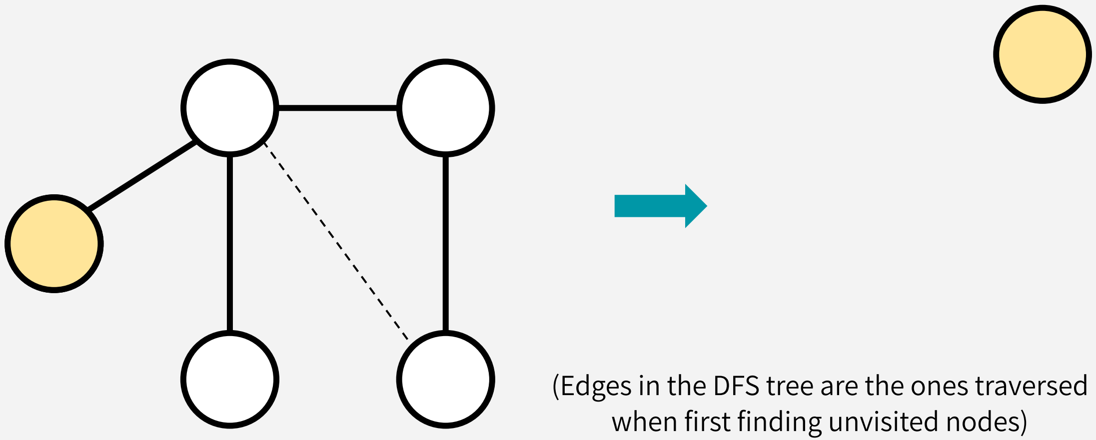

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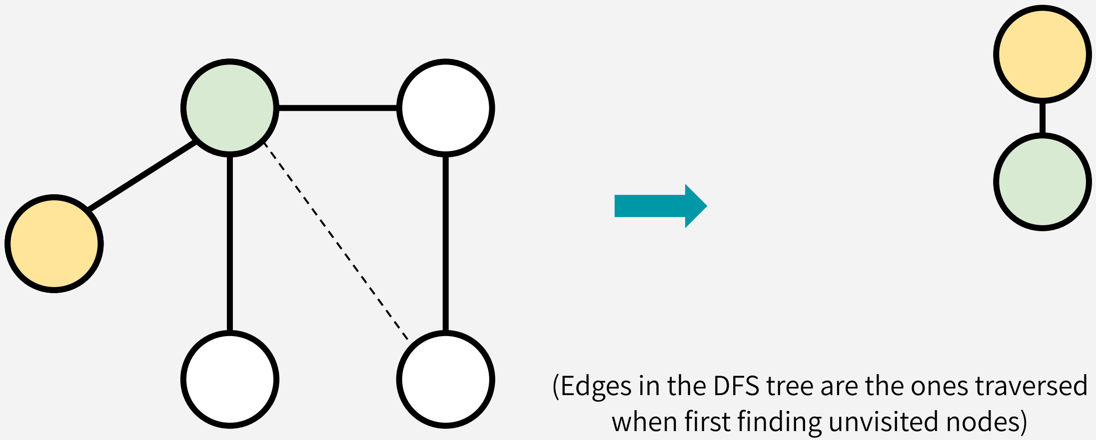

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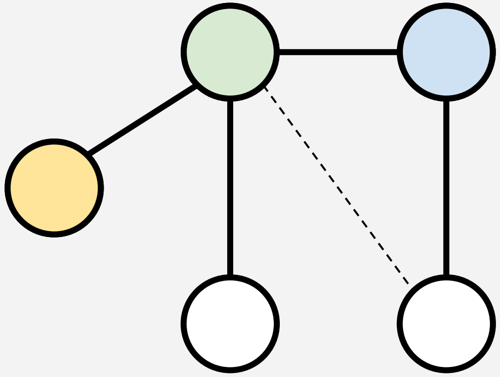

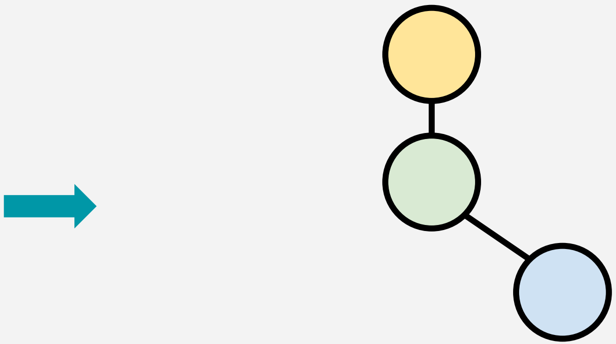

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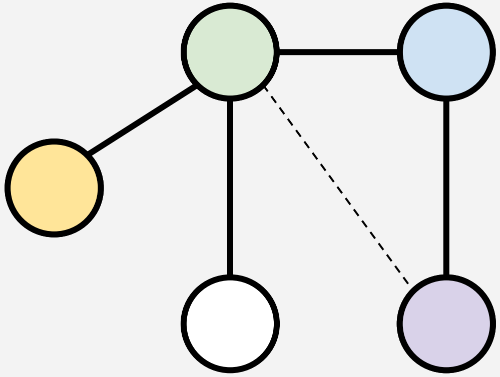

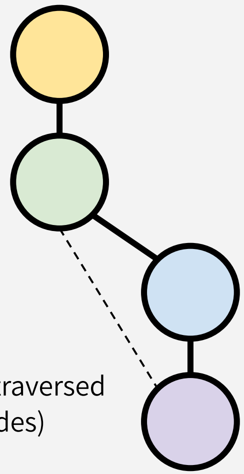

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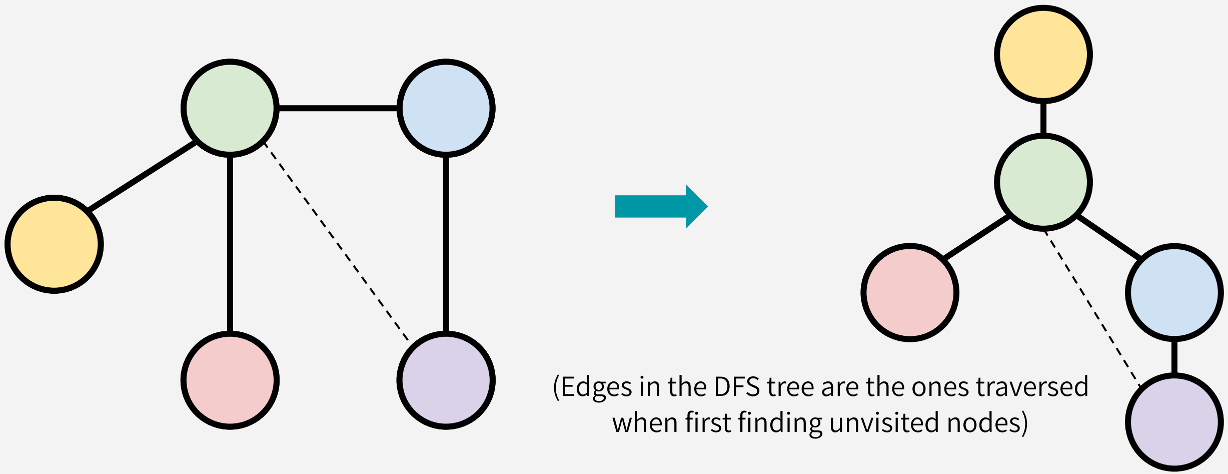

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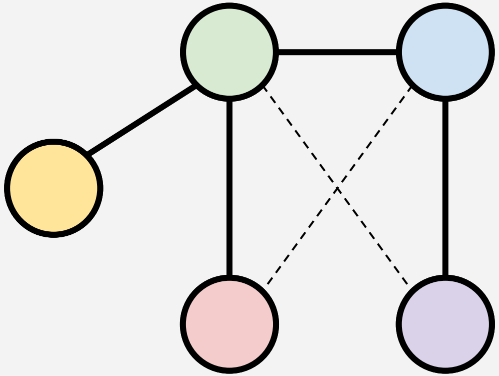
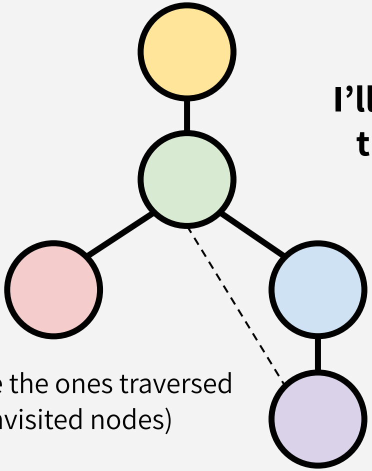




I'll refer to this as the "DFS tree"

(Edges in the DFS tree are the ones traversed when first finding unvisited nodes)

DEPTH-FIRST SEARCH: RUNTIME

To explore a graph's i^{th} **connected component** (n_i nodes, m_i edges):

We visit each vertex in the CC exactly once ("visit" = "call DFS on").
At each vertex v, we:

- Do some bookkeeping: **O(1)**
- Loop over v's neighbors & check if they are visited (& then potentially make a recursive call): O(1) per neighbor → **O(deg(v))** total.

Total: $\sum_v O(\deg(v)) + \sum_v O(1) = \mathbf{O(m_i + n_i)}$

DEPTH-FIRST SEARCH: RUNTIME

To explore **the entire graph** (n nodes, m edges):

A graph might have multiple connected components! To **explore the whole graph**, we would call our DFS routine once for each connected component (note that each vertex and each edge participates in exactly one connected component). The combined running time would be:

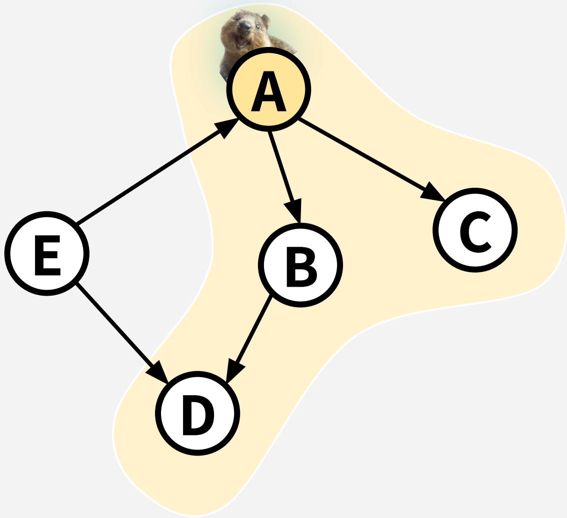
$$O(\textstyle\sum_i m_i + \sum_i n_i) = \mathbf{O(m + n)}$$

DEPTH-FIRST SEARCH

DFS works fine on directed graphs too!

From a start node x, DFS would find all nodes ***reachable*** from x.

(In directed graphs, “connected component” isn’t as well defined… more on that later!)



Verify this on your own:
running DFS from A would still find all nodes reachable from A (E isn’t reachable from A in this directed graph).

سوال؟

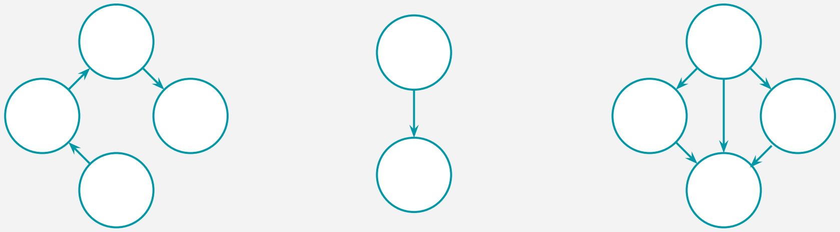
مرتب سازی توپولوژیکی

یک کاربرد از جستجوی عمق اول برای مسائل دارای پیش نیاز

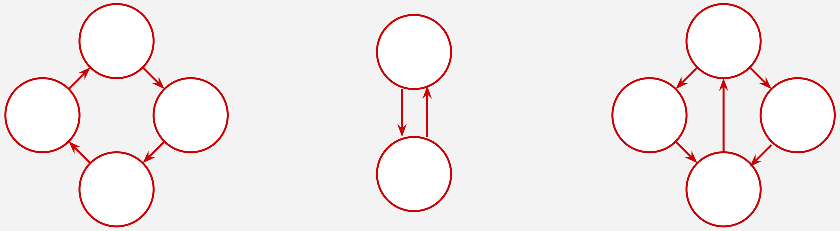
ASIDE: DIRECTED ACYCLIC GRAPHS

A **Directed Acyclic Graph (DAG)** is a directed graph with *no directed cycles*.

These are DAGs:



These are not DAGs:



TOPOLOGICAL SORTING: THE TASK

Given a DAG, find an ordering of vertices so that all of the dependency requirements are met

Example applications:

Given a package dependency graph, in what order should packages be installed?
Given a course prerequisites graph, in what order should we take classes?

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Given a package dependency graph, in what order should packages be installed?
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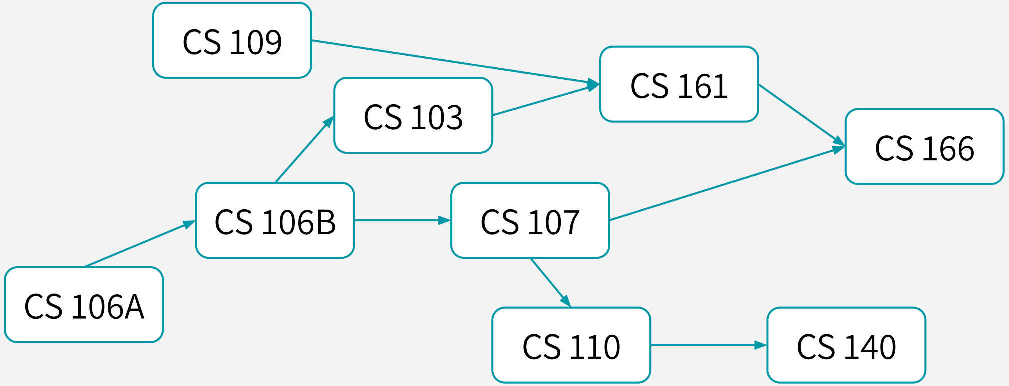
means B "depends" on A
(i.e. take class B after A)



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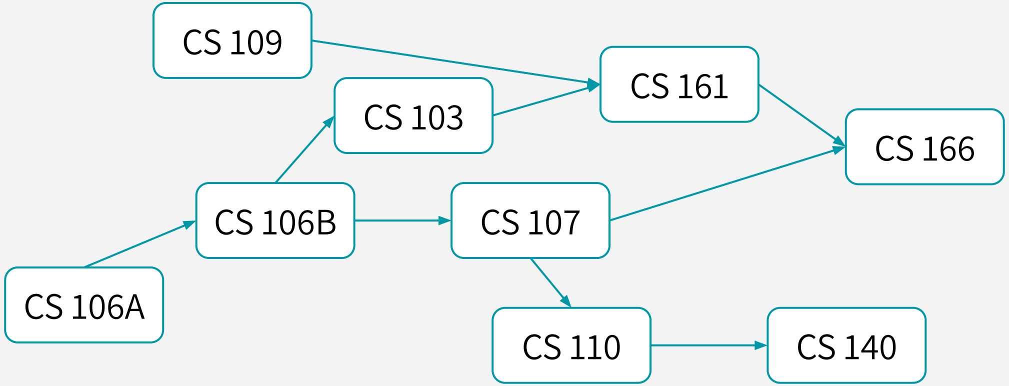


This prerequisite graph is a DAG!
(directed & acyclic)

TOPOLOGICAL SORTING: THE TASK

Given a DAG, find an ordering of vertices so that all of the dependency requirements are met

What does "meeting the dependency requirements" mean?

We want to produce an ordering such that:

for every edge **(v, w)** in E, **v** must appear before **w** in the ordering

(e.g. CS103 must come before CS161)

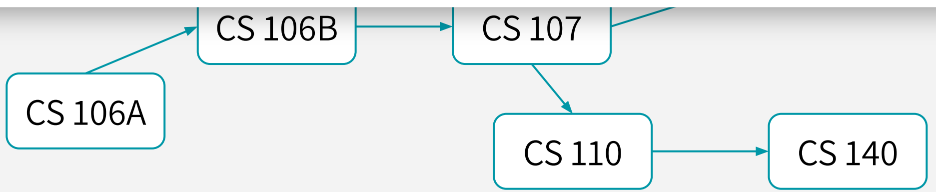
"d

(i.e. take class B after A)

te

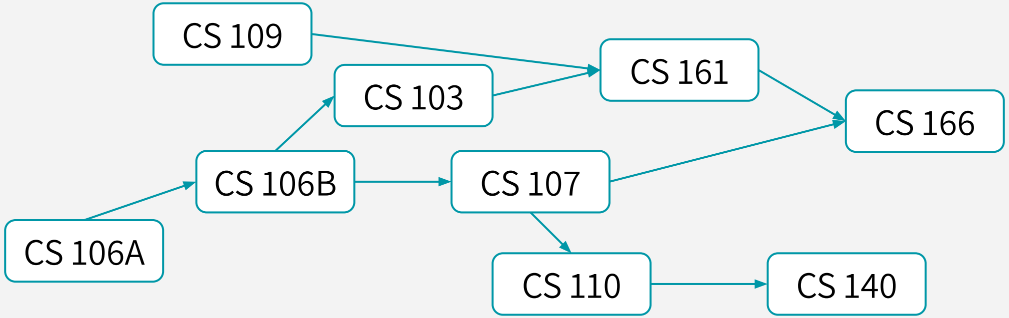
G!

(directed & acyclic)



TOPOLOGICAL SORTING: THE TASK

It's helpful to think of this as **"linearizing"** the graph, where all edges point to the **right**



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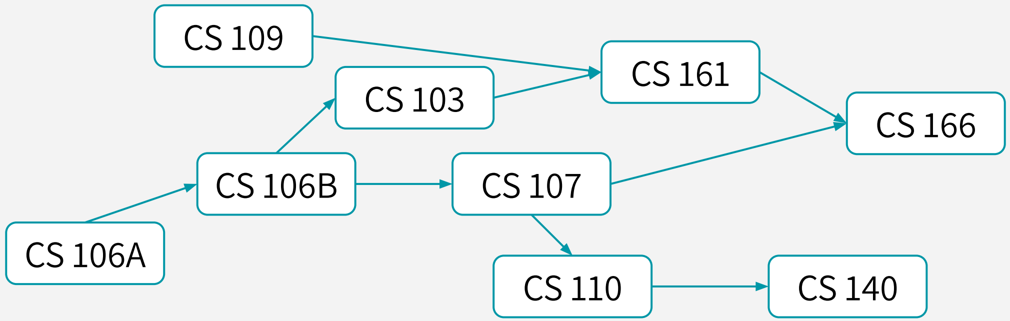

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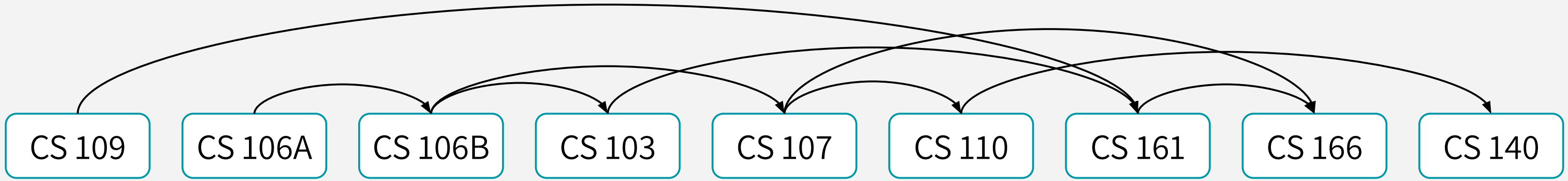

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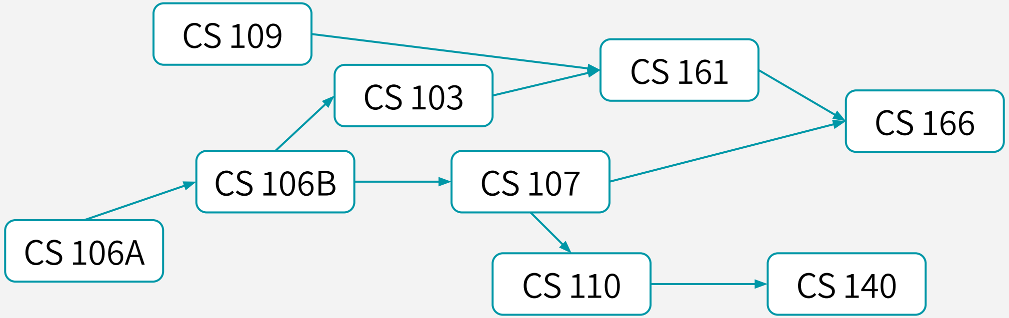

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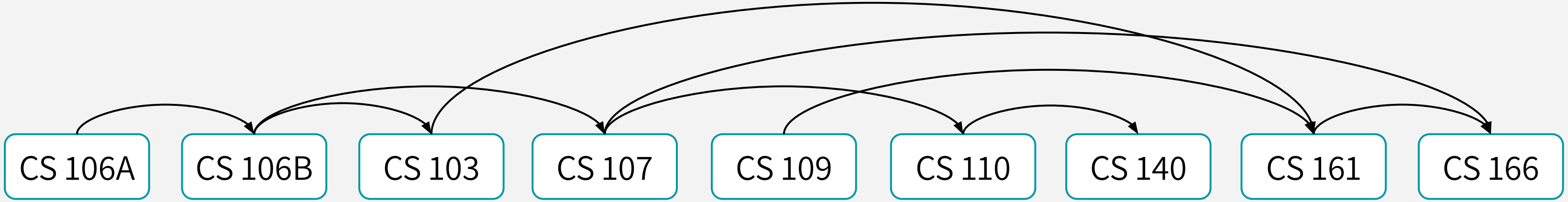

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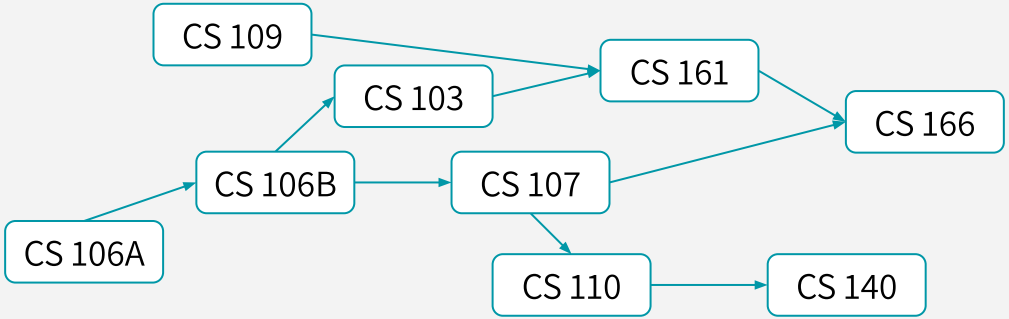

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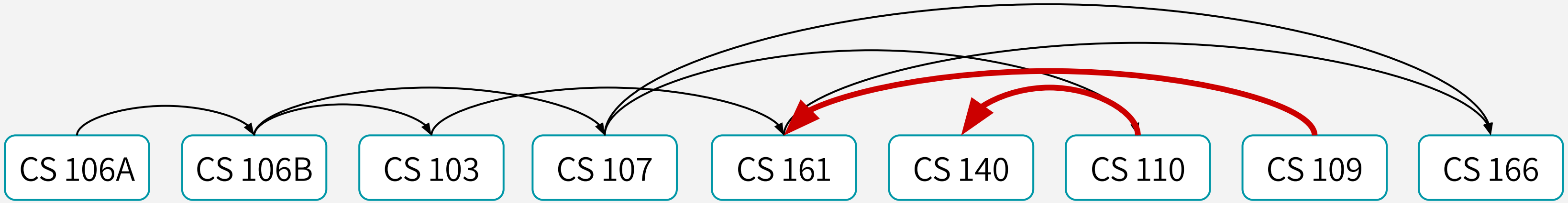


TOPOSORT ON NON-DAGS?

We assume these "dependency" graphs are all DAGs!

What about other graphs? Undirected graphs? Directed graphs with cycles?

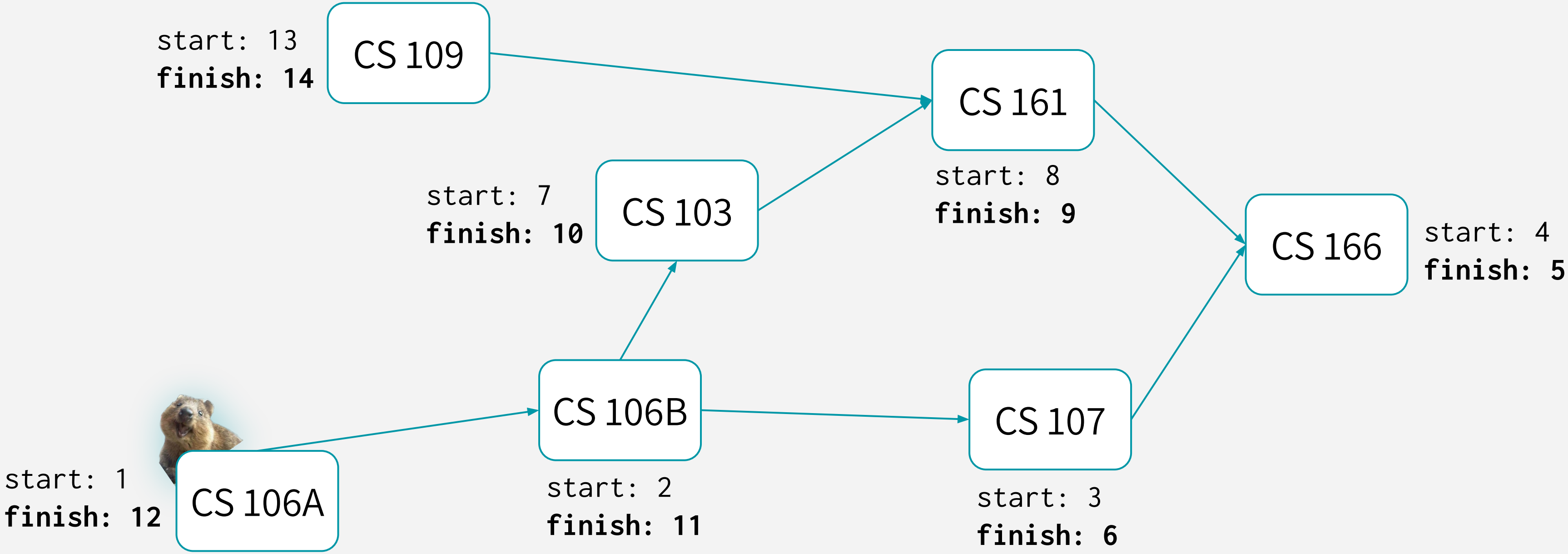
Toposort gives us a priority ordering of nodes (e.g. more intro classes are "higher priority" than more advanced classes). Edges in DAGs clearly illustrate priority: edge from **x** to **y** means **x** has priority over **y**.

In an undirected graph, if there's an **x-y** edge, which node has "priority"?

In a graph with cycles, if **x** and **y** are part of a cycle, then **x** can reach **y** and **y** can also reach **x**… so which node has "priority"?

DFS WILL GET US A TOPOSORT

Let’s run DFS. What do you notice about the finish times? What does it have to do with toposort?



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CLAIM: In general, if there’s an edge from **v** → **w**, **v**’s finish time will be *larger* than **w**’s finish time

Let’s consider two cases: (1) DFS visits **v** first, or (2) DFS visits **w** first.

start: 1
finish: 12

CS 106A

start: 2
finish: 11

start: 3
finish: 6

DFS WILL GET US A TOPOSORT

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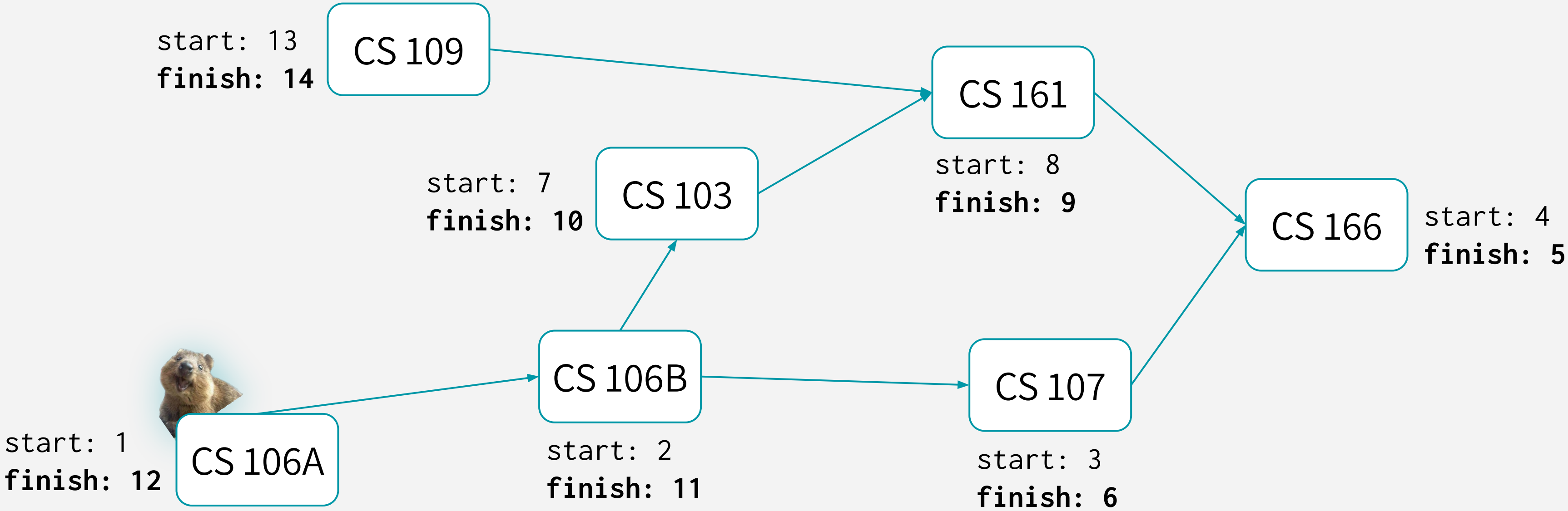
<u>CASE 1</u>: **v** → **w**, and **v is discovered first** by DFS



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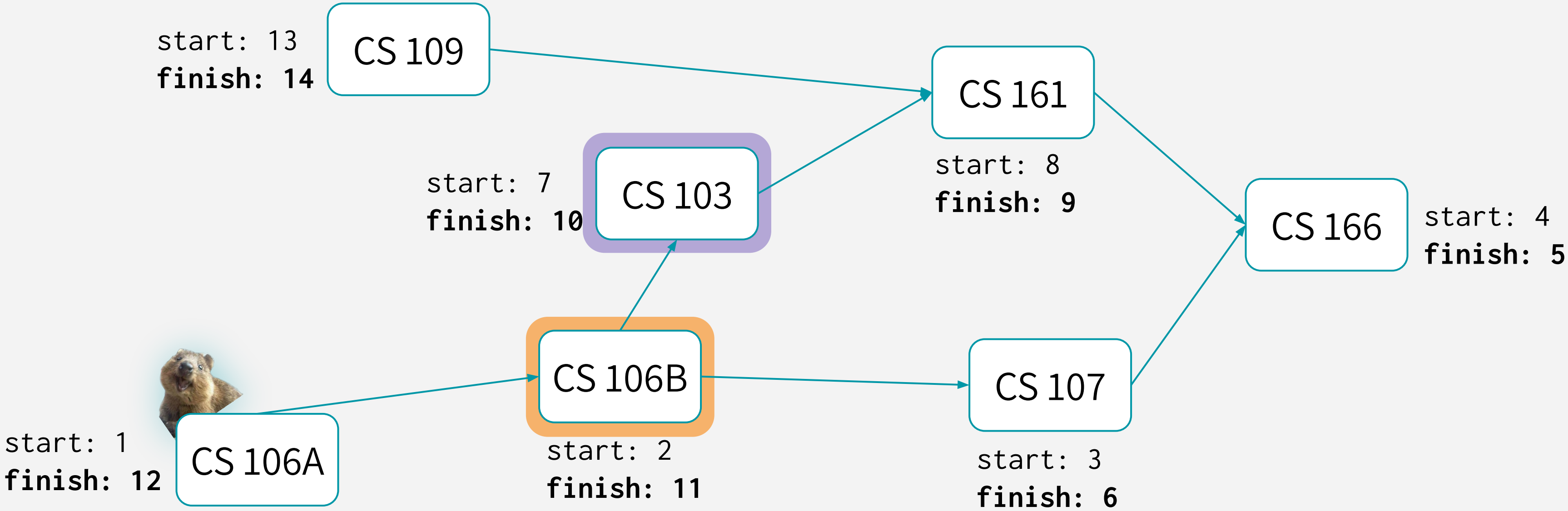

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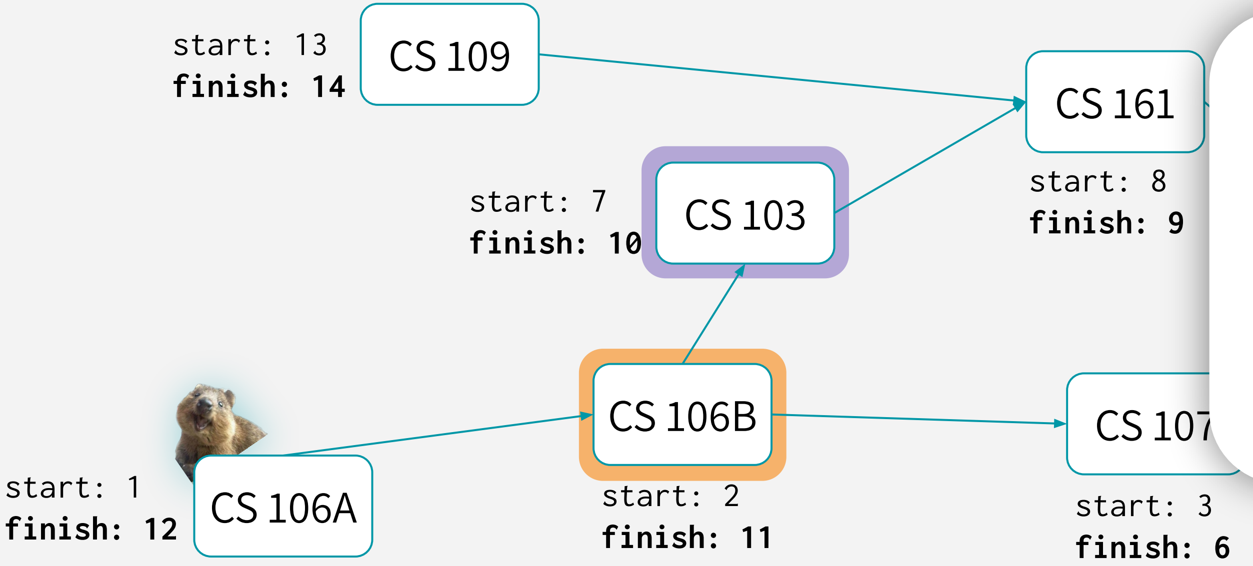

When v is discovered by DFS, this call will eventually discover w & recursively call DFS on w. Then, w will get its finish time before v gets its finish time, so v.finish > w.finish!

DFS WILL GET US A TOPOSORT

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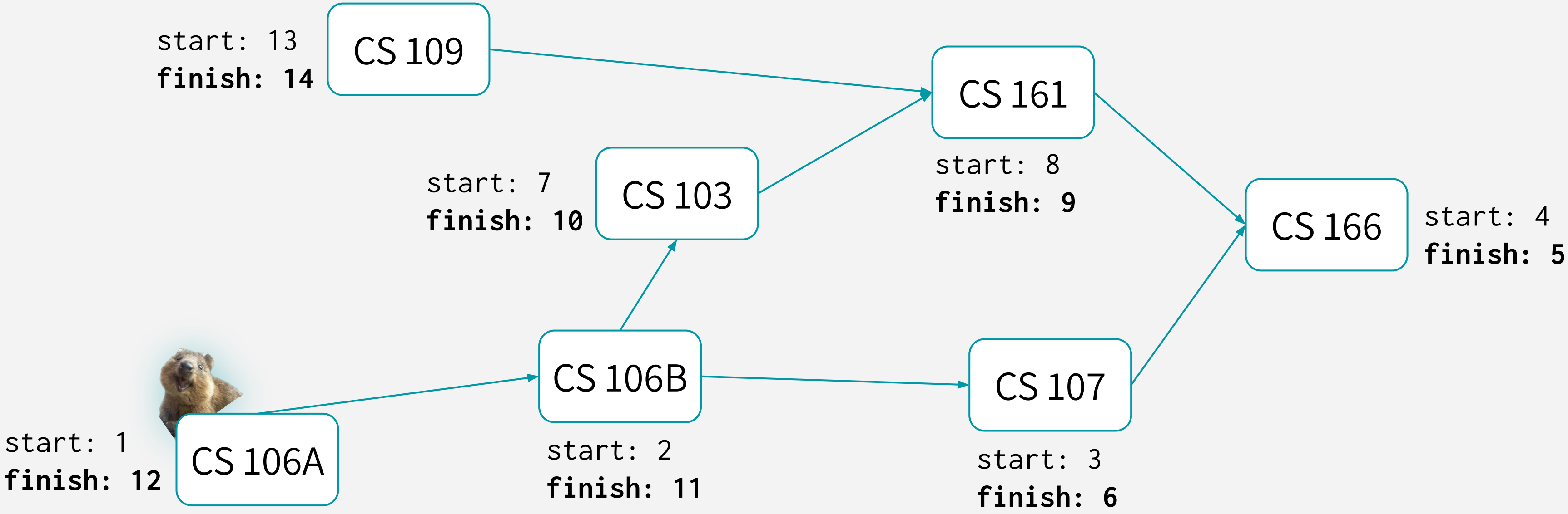
CASE 2: **v** → **w**, and **w is discovered first** by DFS



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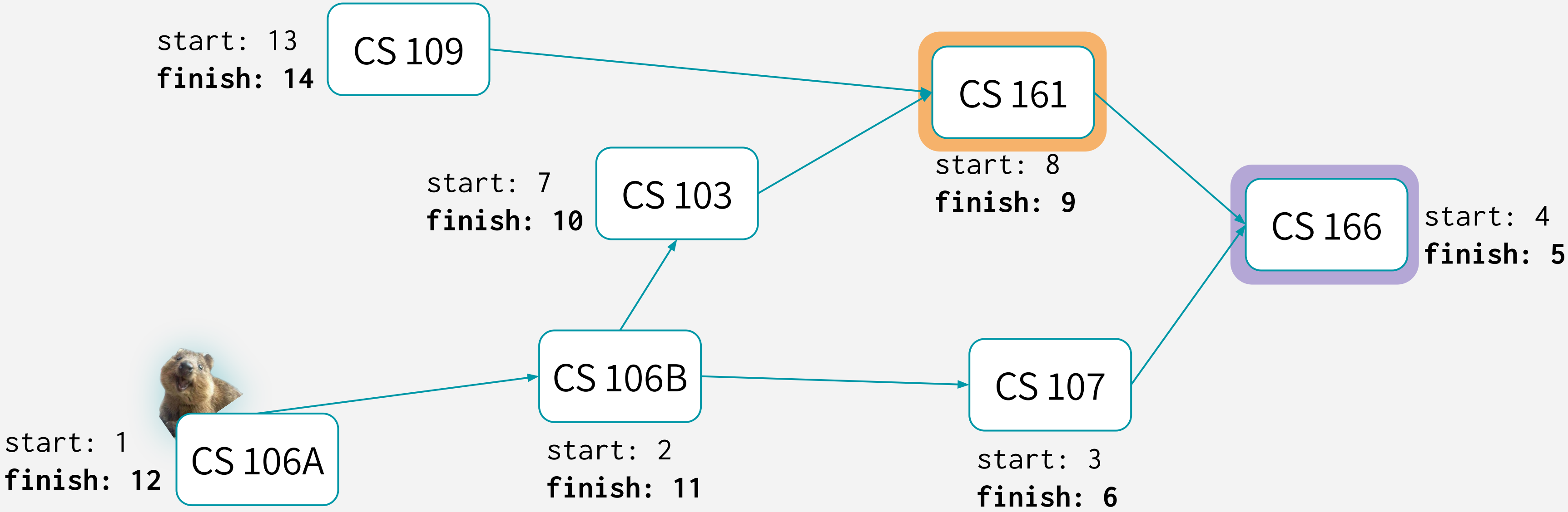

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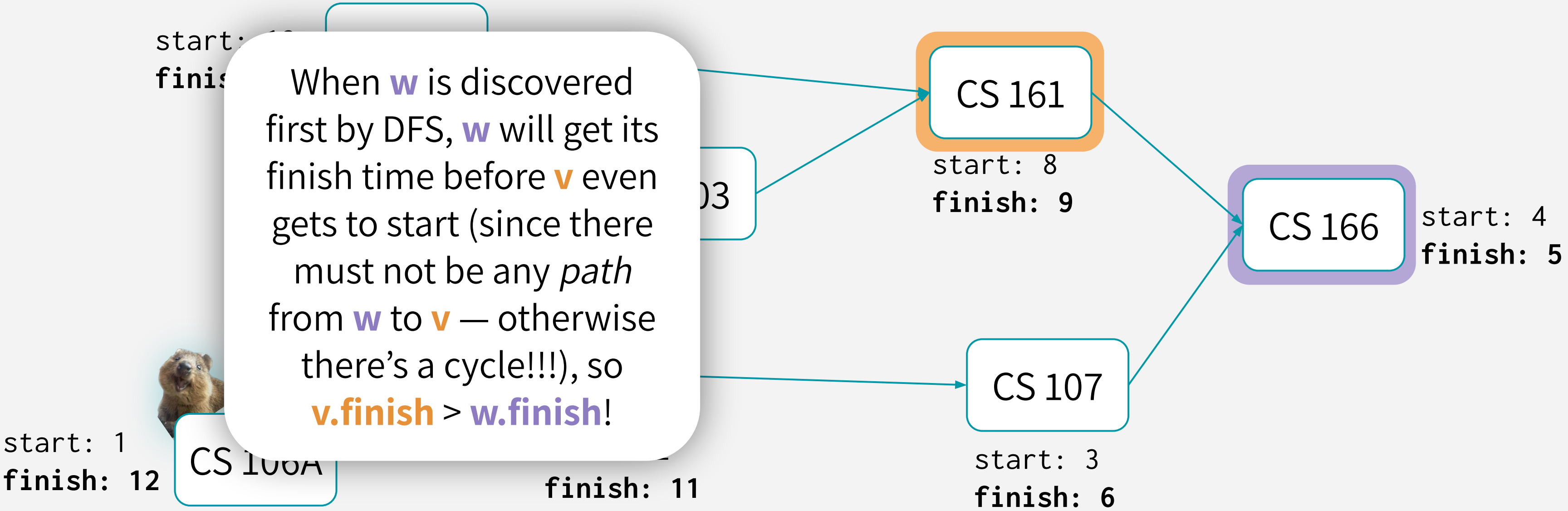
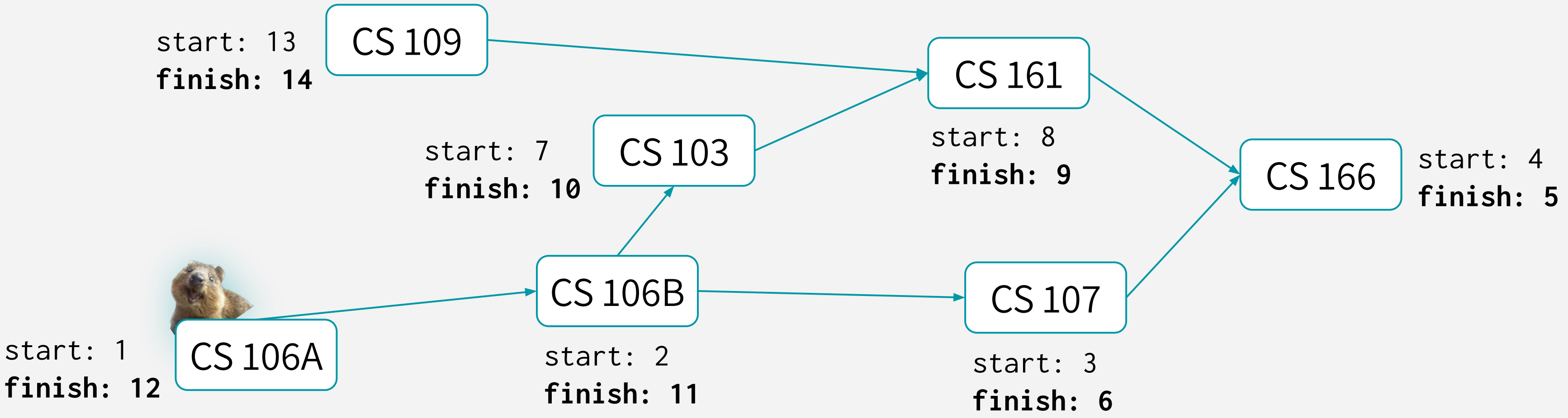


When **w** is discovered first by DFS, **w** will get its finish time before **v** even gets to start (since there must not be any *path* from **w** to **v** — otherwise there's a cycle!!!), so **v.finish** > **w.finish**!

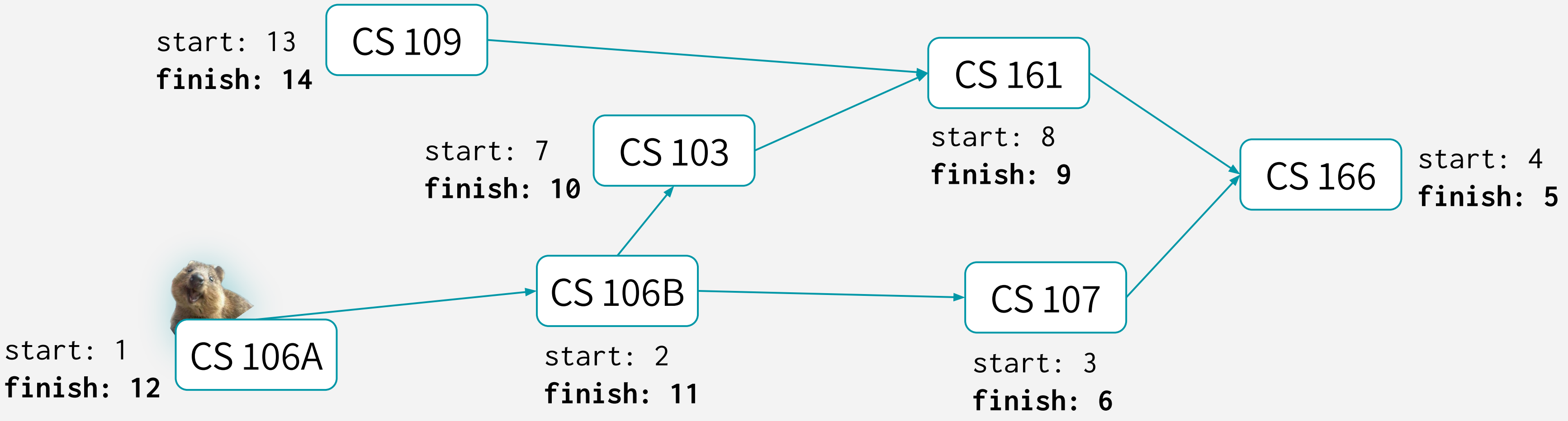
DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



DFS WILL GET US A TOPOSORT

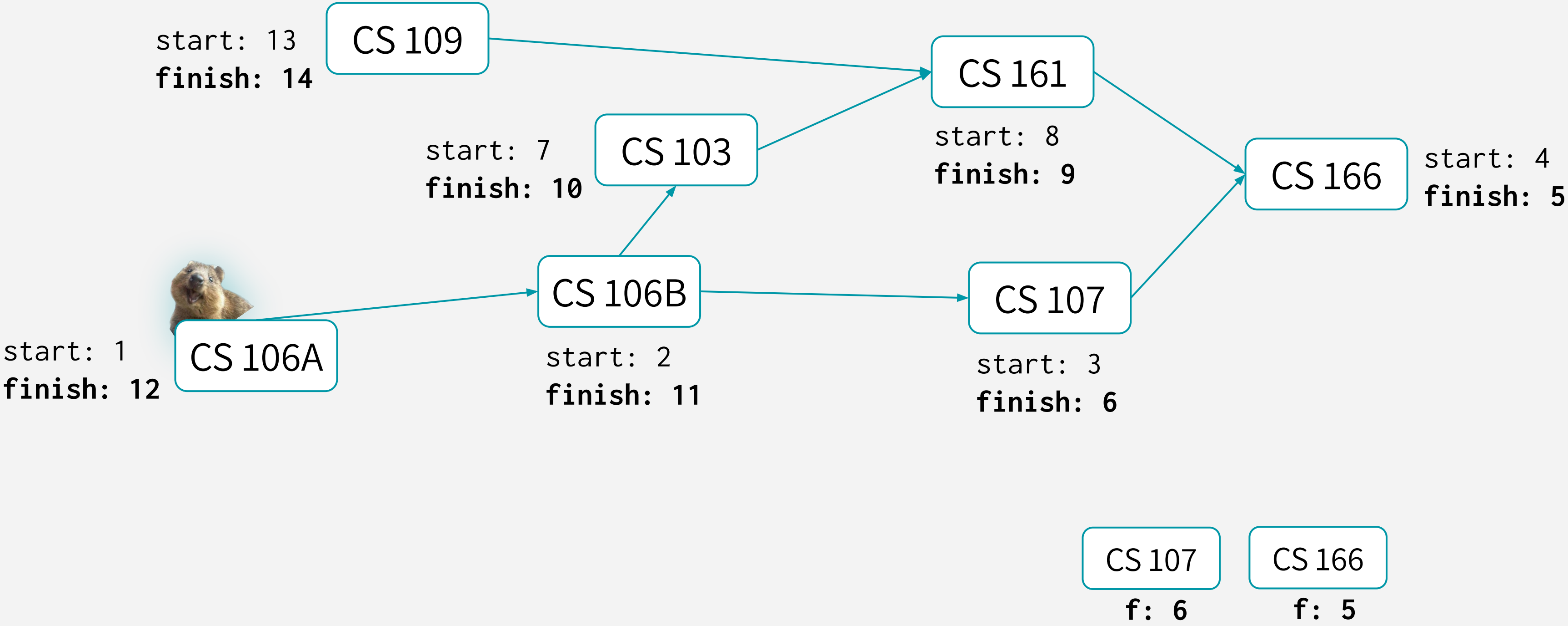
TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



CS 166
f: 5

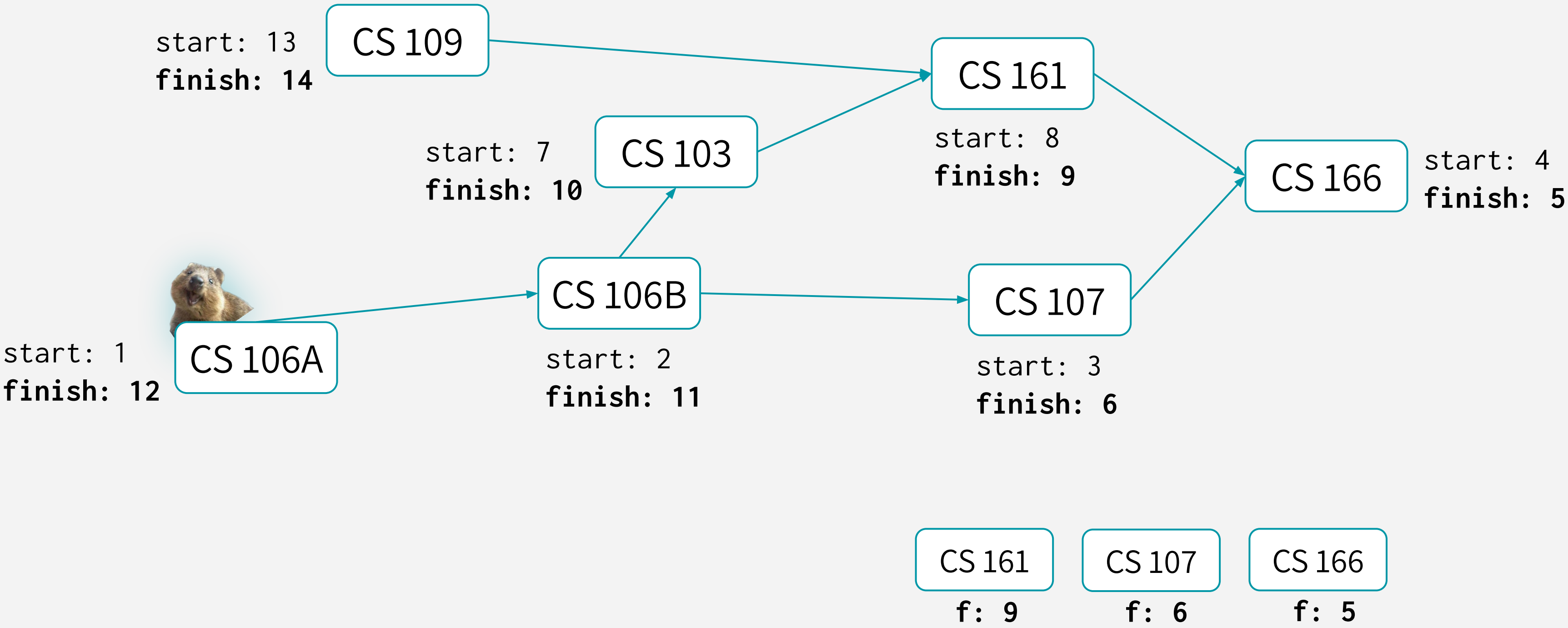
DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



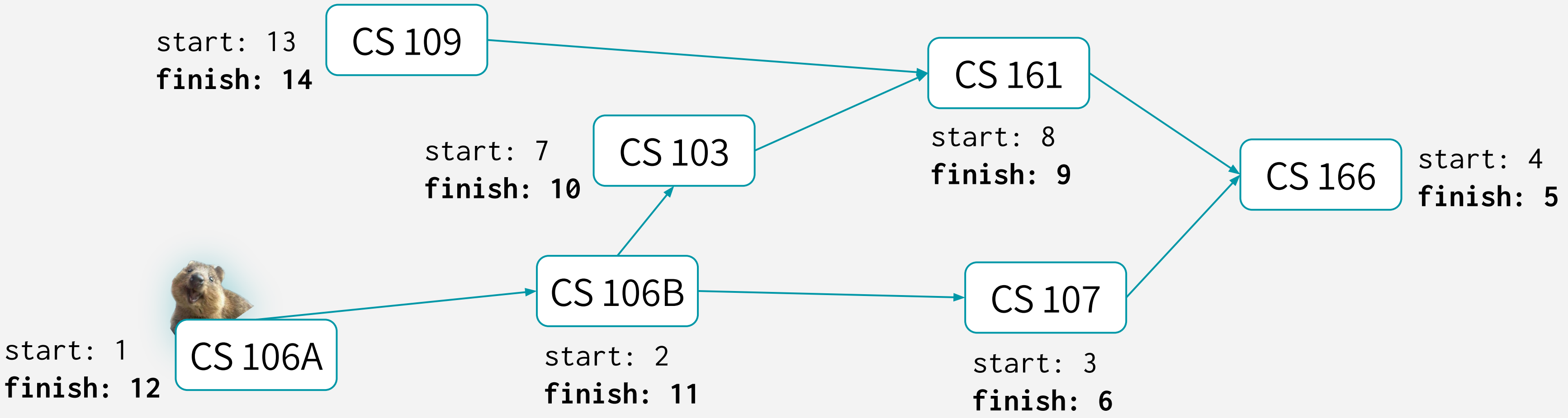
DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



DFS WILL GET US A TOPOSORT

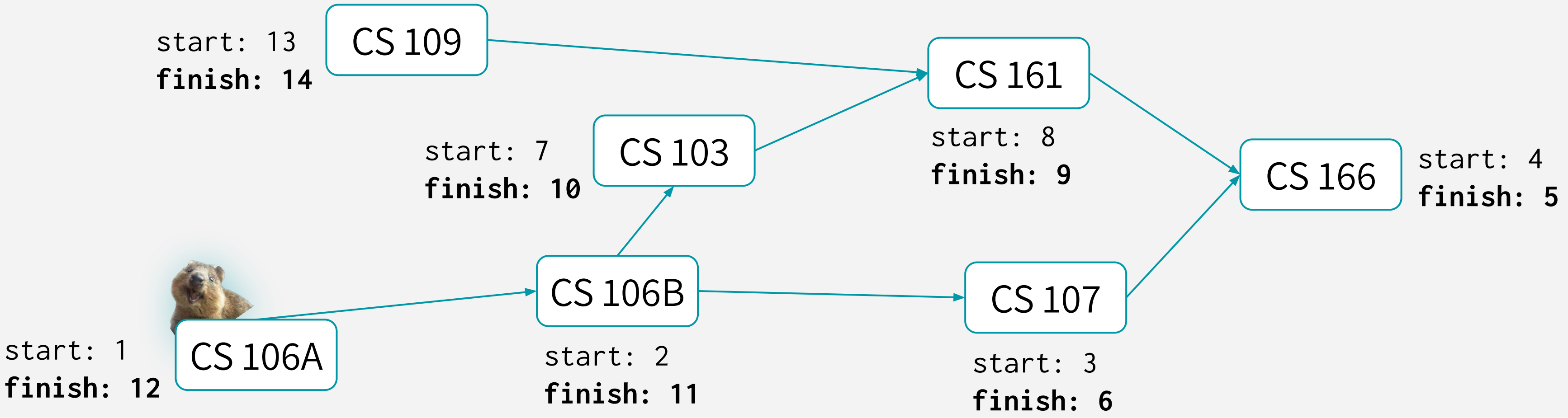
TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

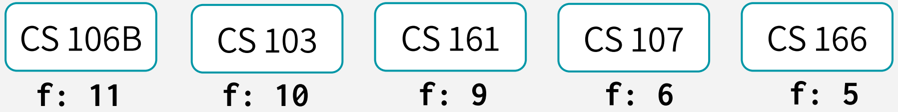


CS 103	CS 161	CS 107	CS 166
f: 10	f: 9	f: 6	f: 5

DFS WILL GET US A TOPOSORT

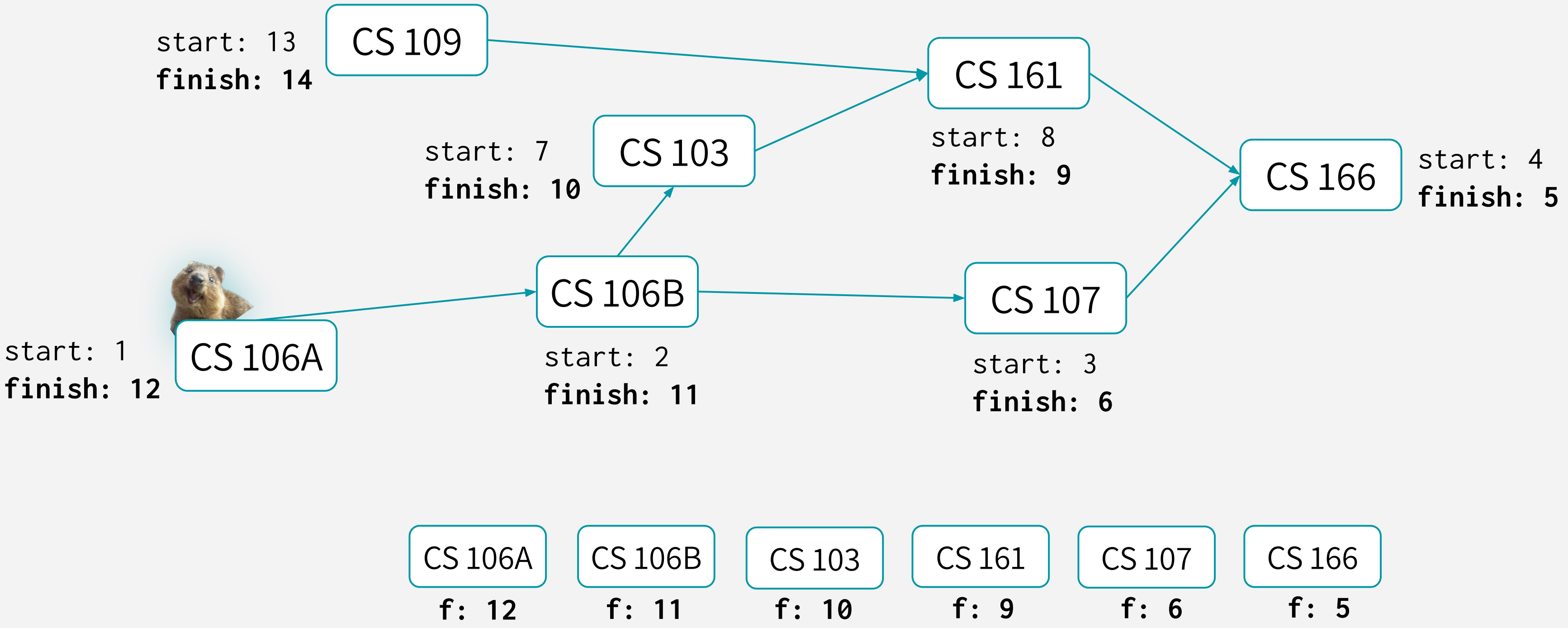
TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.





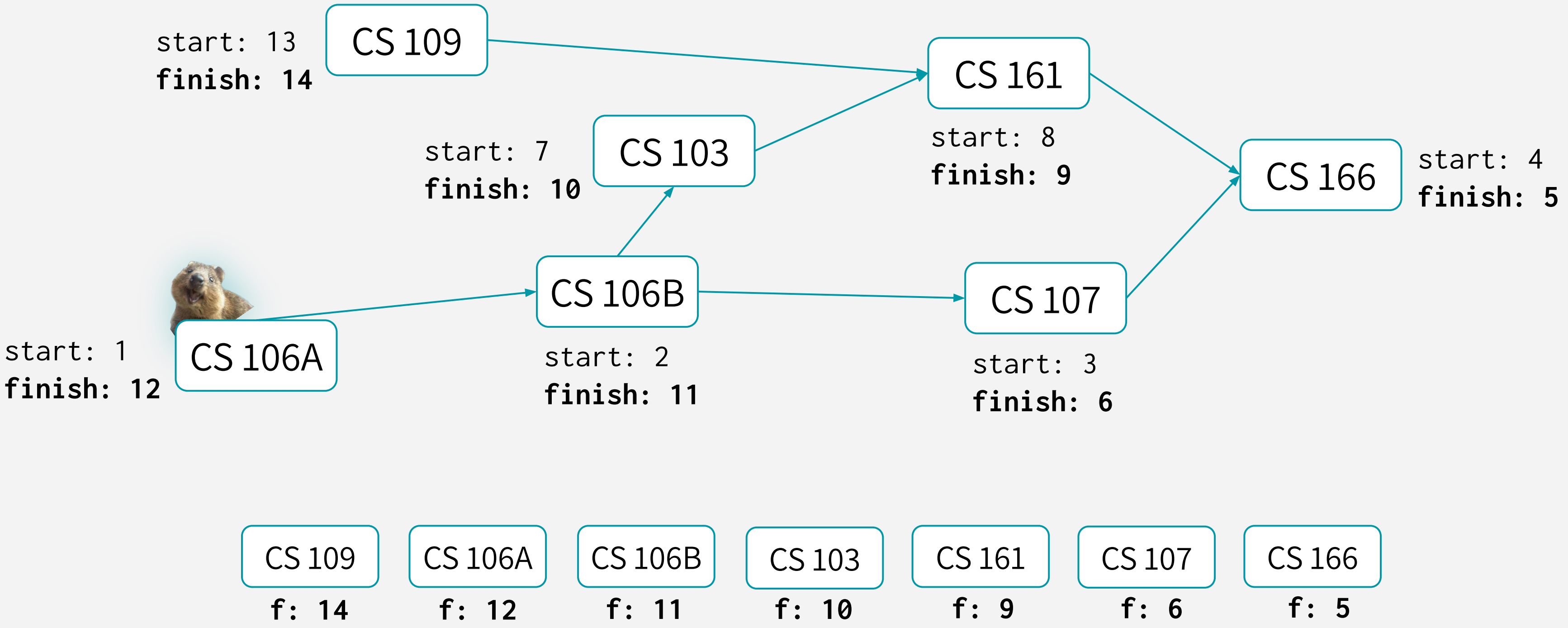
DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



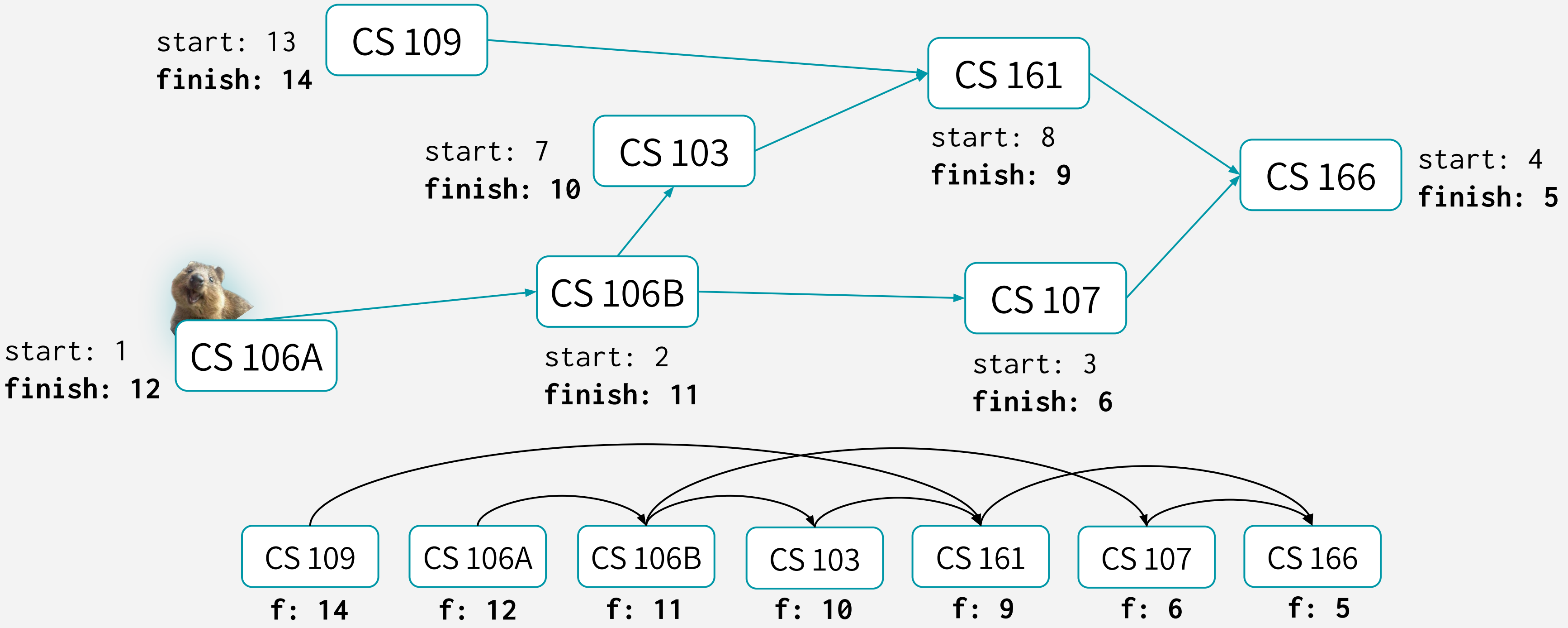
DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



DFS WILL GET US A TOPOSORT

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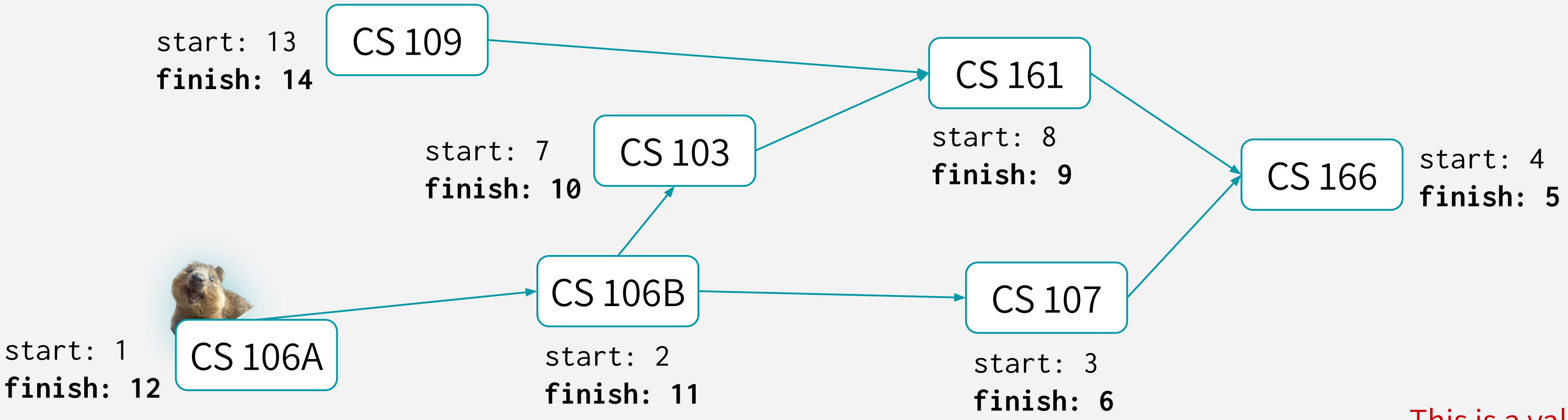
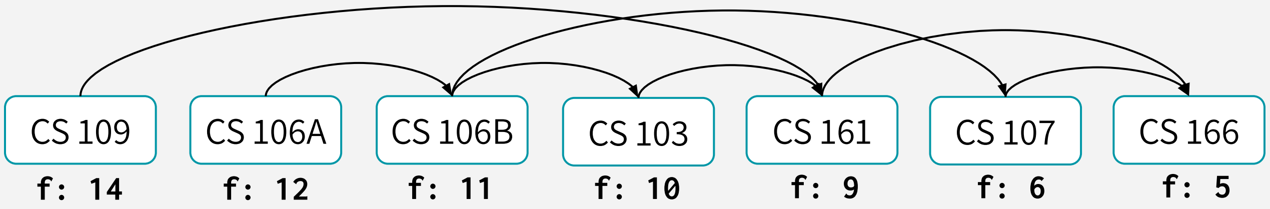




This is a valid toposort ordering!

DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

start: 13
finish: 14
CS 109

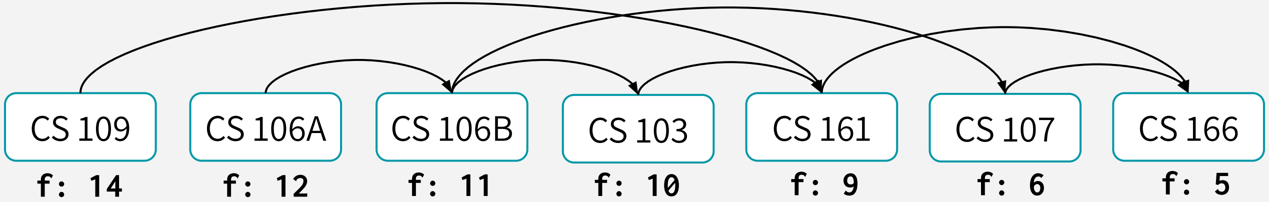
CS 161

Regardless of which vertex your DFS starts, it'll get you a correct Toposort ordering of your DAG

start: 1
finish: 12
CS 106A

start: 2
finish: 11

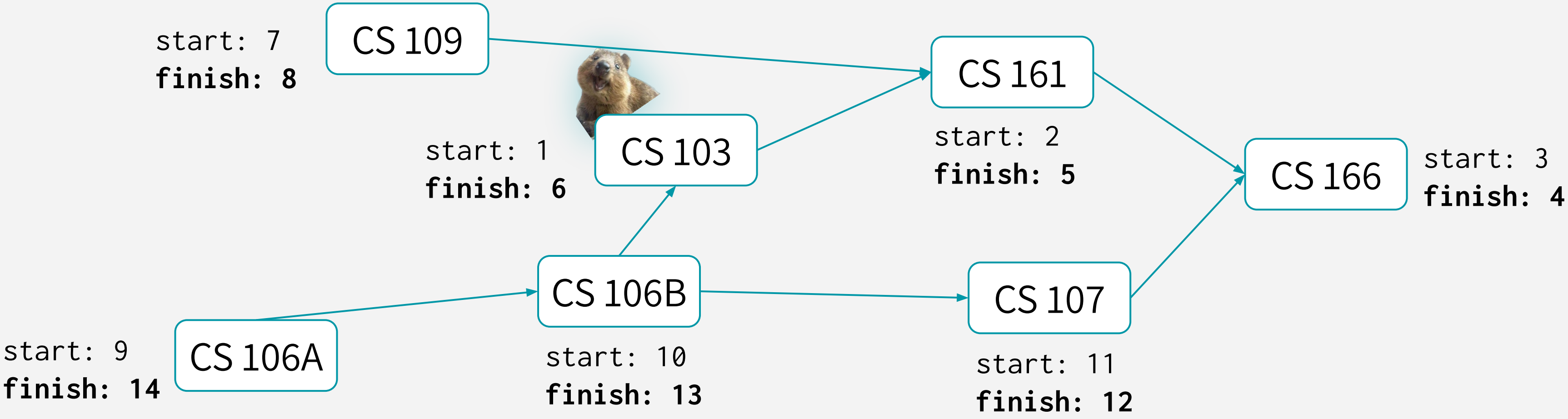
start: 3
finish: 6



DFS WILL GET US A TOPOSORT

Different order of running

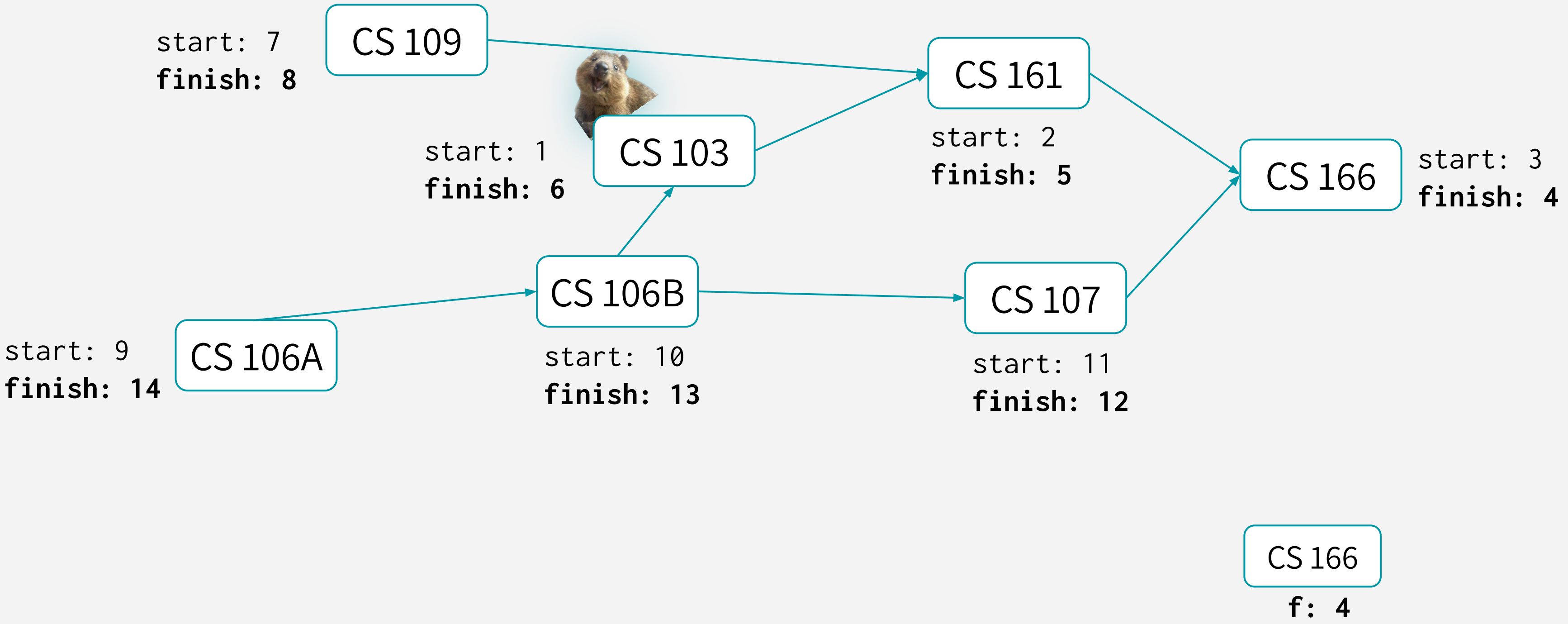
TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



DFS WILL GET US A TOPOSORT

Different order of running

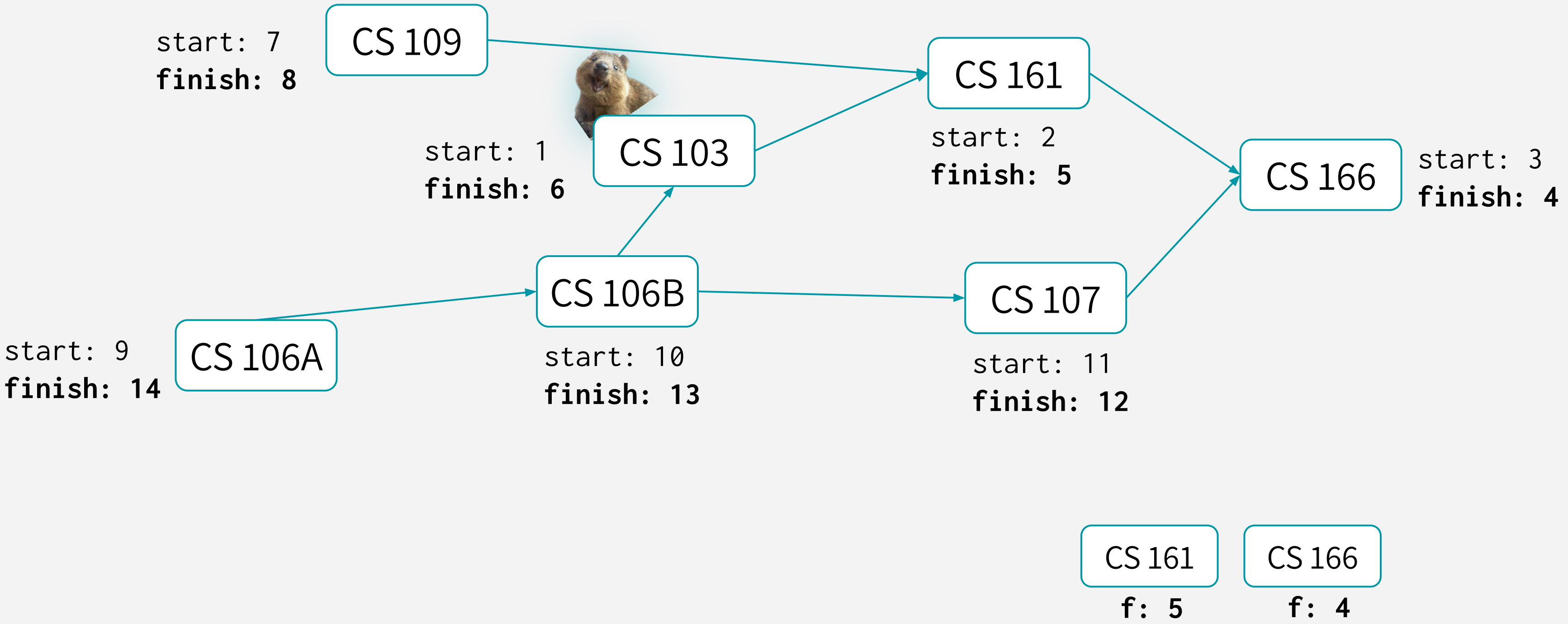
TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



DFS WILL GET US A TOPOSORT

Different order of running

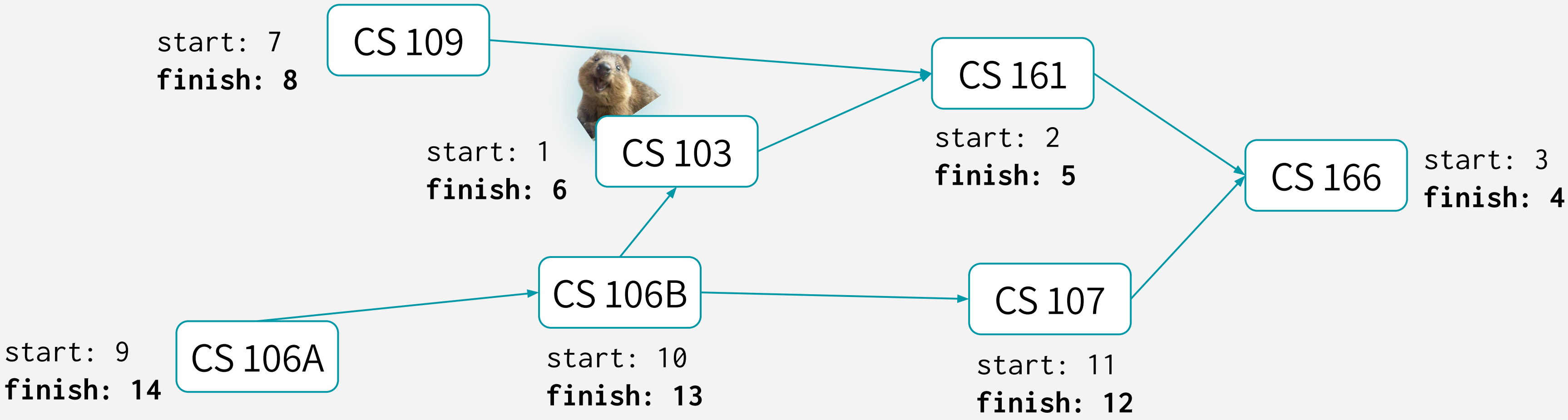
TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

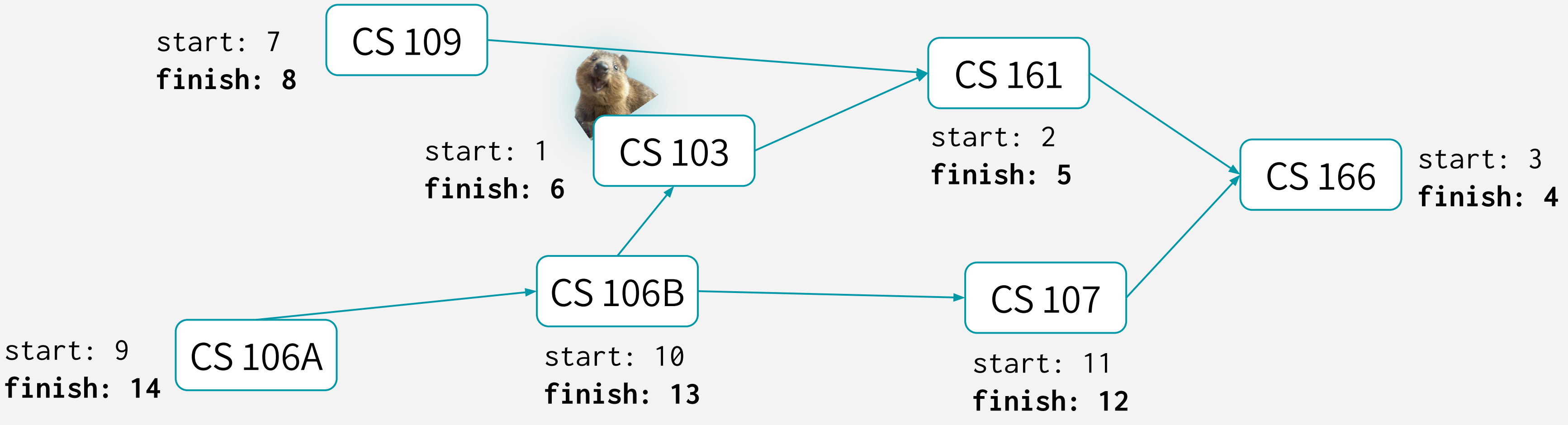


CS 103	CS 161	CS 166
f: 6	f: 5	f: 4

DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

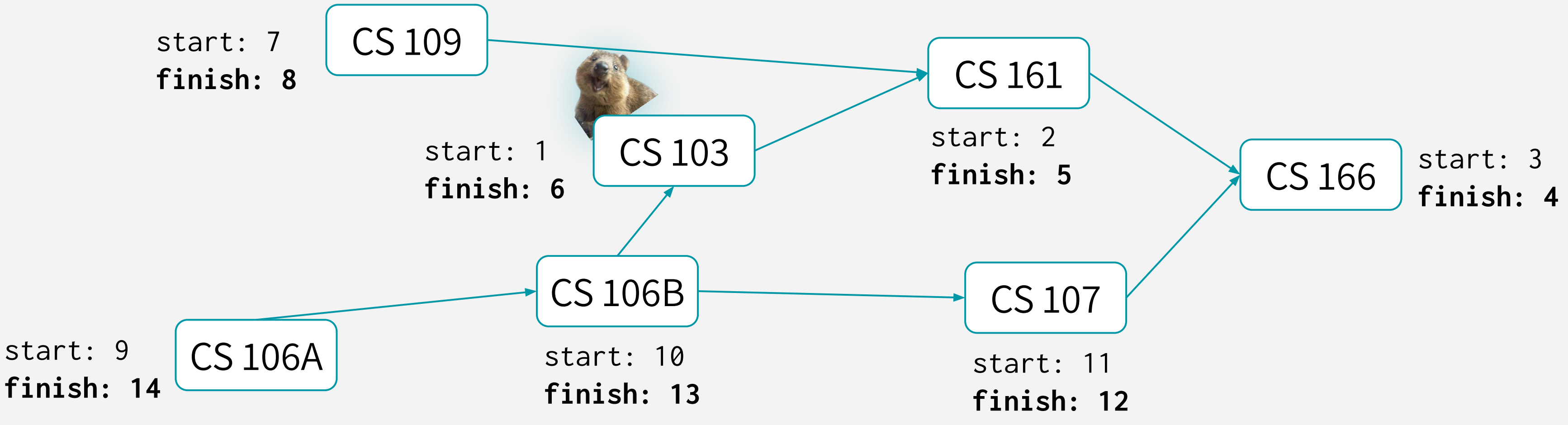


CS 109	CS 103	CS 161	CS 166
f: 8	f: 6	f: 5	f: 4

DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

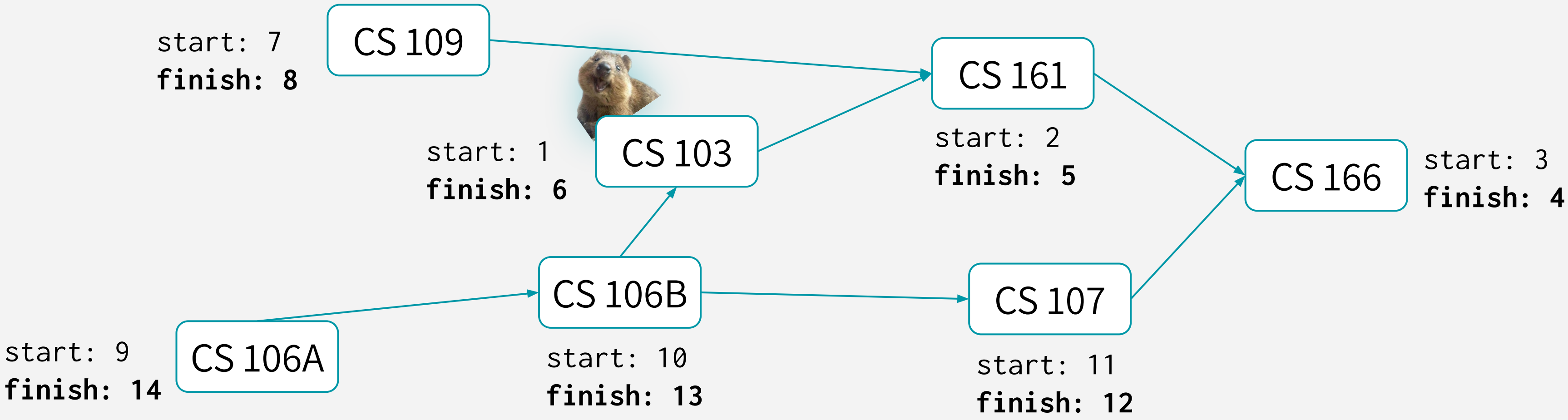


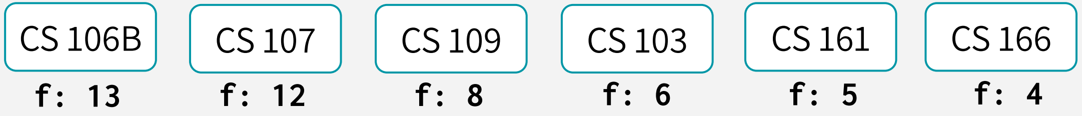
CS 107	CS 109	CS 103	CS 161	CS 166
f: 12	f: 8	f: 6	f: 5	f: 4

DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

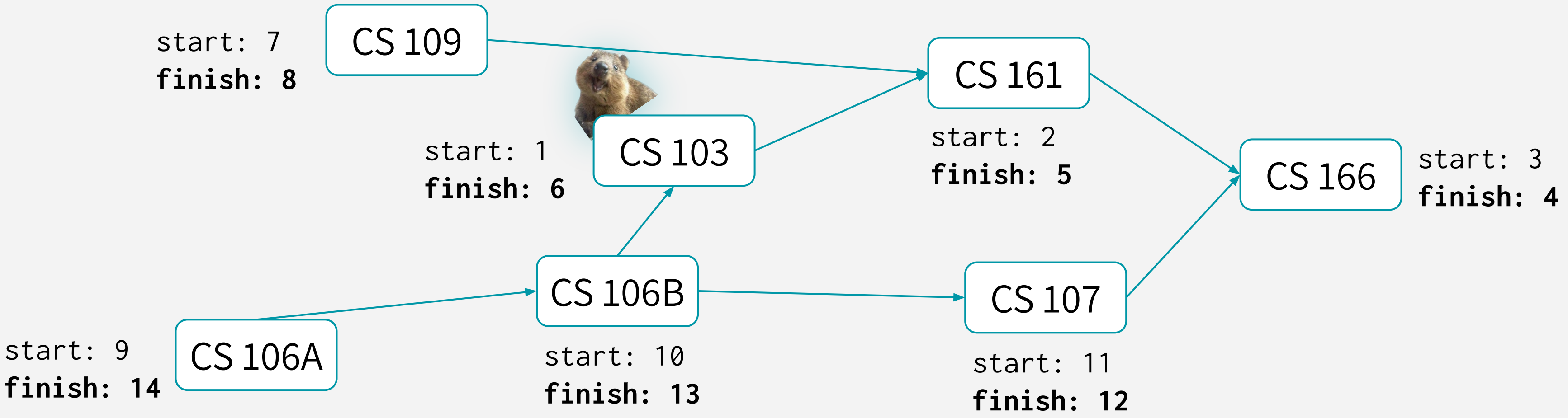




DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

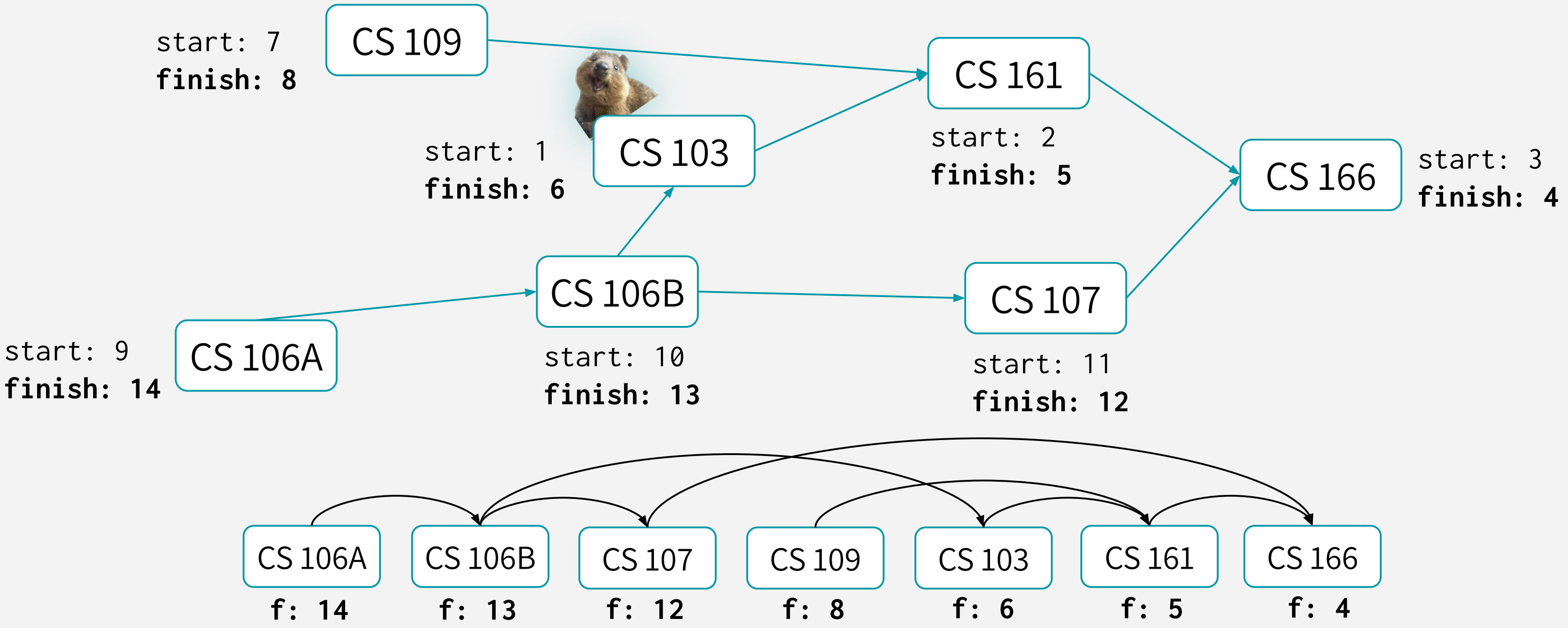


CS 106A	CS 106B	CS 107	CS 109	CS 103	CS 161	CS 166
f: 14	f: 13	f: 12	f: 8	f: 6	f: 5	f: 4

DFS WILL GET US A TOPOSORT

Different order of running

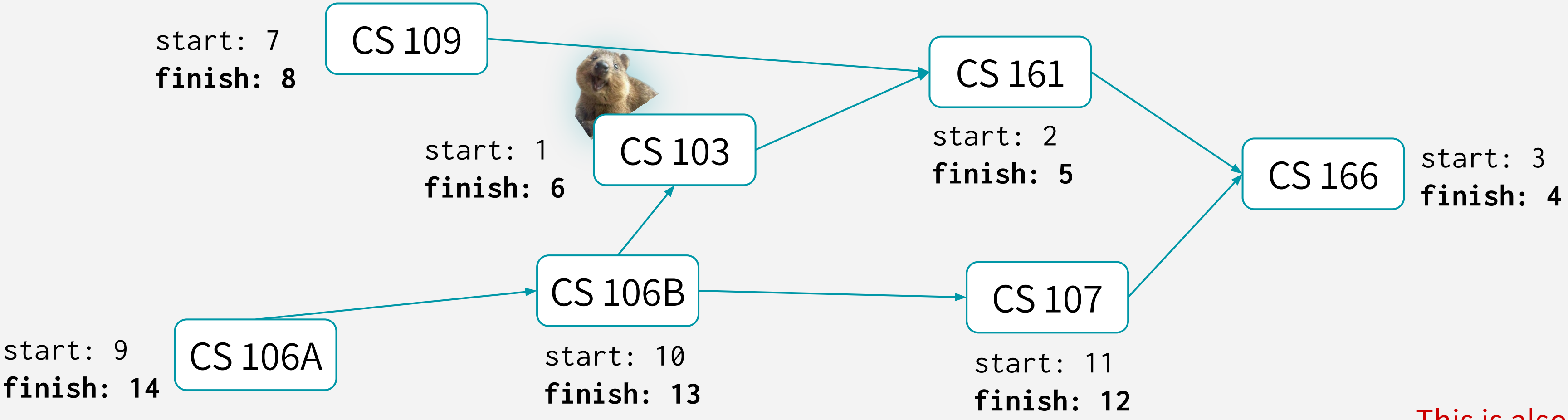
TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

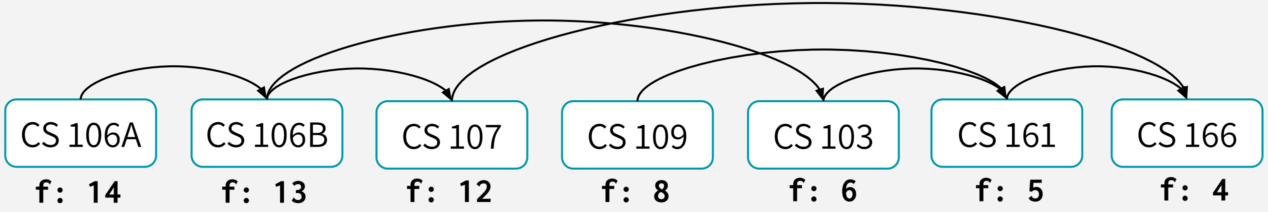


DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.





This is also a valid toposort ordering!

DFS & TOPOSORT RECAP

DFS can help you solve the Topological Sorting Problem.

That's just the fancy name for the problem of finding an ordering of the vertices which respect all the dependencies.

سوال؟