Subject: اشکان شکیبا ۹۹۳۱۰۳۰ Year: Month: Date:

سوال اول

$$\alpha(t)=\left(\cos\left(\frac{\sqrt{2}}{2}t\right),\sin\left(\frac{\sqrt{2}}{2}t\right),\frac{\sqrt{2}}{2}t\right)$$

$$\Rightarrow \alpha'(t)=\left(\frac{-\sqrt{2}}{2}\sin\frac{\sqrt{2}}{2}t,\frac{\sqrt{2}}{2}\cos\frac{\sqrt{2}}{2}t,\frac{\sqrt{2}}{2}\right)$$

$$\Rightarrow |\alpha'(t)|=\sqrt{\frac{1}{2}\left(\sin^2\frac{\sqrt{2}}{2}t+\cos^2\frac{\sqrt{2}}{2}t\right)+\frac{1}{2}}=1$$

$$\vec{T}(t)=\frac{\alpha'(t)}{|\alpha'(t)|}=\left(\frac{-\sqrt{2}}{2}\sin\frac{\sqrt{2}}{2}t,\frac{\sqrt{2}}{2}\cos\frac{\sqrt{2}}{2}t,\frac{\sqrt{2}}{2}\right)$$

$$\alpha''(t)=\left(\frac{-1}{2}\cos\frac{\sqrt{2}}{2}t,\frac{-1}{2}\sin\frac{\sqrt{2}}{2}t,0\right)$$

$$\alpha'\times\alpha''=\begin{vmatrix} i & j & k \\ -\frac{\sqrt{2}}{2}\sin\frac{\sqrt{2}}{2}t & \frac{\sqrt{2}}{2}\cos\frac{\sqrt{2}}{2}t & \frac{\sqrt{2}}{2} \\ \frac{-1}{2}\cos\frac{\sqrt{2}}{2}t & \frac{-1}{2}\sin\frac{\sqrt{2}}{2}t & 0 \end{vmatrix}$$

$$=\left(\frac{\sqrt{2}}{4}\sin\frac{\sqrt{2}}{2}t,-\frac{\sqrt{2}}{4}\cos\frac{\sqrt{2}}{2}t,\frac{\sqrt{2}}{4}\right)$$

$$\Rightarrow |\alpha'\times\alpha''|=\sqrt{\frac{2}{16}\left(\sin^2\frac{\sqrt{2}}{2}t+\cos^2\frac{\sqrt{2}}{2}t\right)+\frac{2}{16}}=\frac{1}{2}$$

$$\rightarrow \vec{B}(t)=\frac{\alpha'\times\alpha''}{|\alpha'\times\alpha''|}=\left(\frac{\sqrt{2}}{2}\sin\frac{\sqrt{2}}{2}t,\frac{-\sqrt{2}}{2}\cos\frac{\sqrt{2}}{2}t,\frac{\sqrt{2}}{2}\right)$$

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$$\vec{N} = \vec{B} \times \vec{T} = \begin{vmatrix} i & j & k \\ \frac{\sqrt{2}}{2}\sin\frac{\sqrt{2}}{2}t & -\frac{\sqrt{2}}{2}\cos\frac{\sqrt{2}}{2}t & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2}\sin\frac{\sqrt{2}}{2}t & \frac{\sqrt{2}}{2}\cos\frac{\sqrt{2}}{2}t & \frac{\sqrt{2}}{2} \end{vmatrix}$$

$$= \left(-\cos\frac{\sqrt{2}}{2}t, -\sin\frac{\sqrt{2}}{2}t, 0\right)$$

$$K(t) = \frac{|\alpha' \times \alpha''|}{|\alpha'|^2} = \frac{\frac{1}{2}}{1^2} = \frac{1}{2}$$

$$\alpha'''(t) = \left(\frac{\sqrt{2}}{4}\sin\frac{\sqrt{2}}{2}t, \frac{-\sqrt{2}}{4}\cos\frac{\sqrt{2}}{2}t, 0\right)$$

$$\tau(t) = \frac{(\alpha' \times \alpha'') \cdot \alpha'''}{|\alpha' \times \alpha''|^2}$$

$$= \frac{\left(\frac{\sqrt{2}}{4}\sin\frac{\sqrt{2}}{2}t, \frac{-\sqrt{2}}{4}\cos\frac{\sqrt{2}}{2}t, \frac{\sqrt{2}}{4}\right) \cdot \left(\frac{\sqrt{2}}{4}\sin\frac{\sqrt{2}}{2}t, \right.}{\left(\frac{1}{2}\right)^2} \cdots$$

$$\cdots \frac{\left. \frac{-\sqrt{2}}{4}\cos\frac{\sqrt{2}}{2}t, 0\right)}{} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}$$

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$a=0, b=2, c=0 \quad (99/10/25)$

$\Rightarrow \beta(t) = \sqrt{2}\, T(t) + \frac{V}{2} N(t) + \frac{\sqrt{2}}{2} B(t)$

$= \left(-\sin\frac{\sqrt{2}}{2}t, \cos\frac{\sqrt{2}}{2}t, 1\right) + \left(\frac{-V}{2}\cos\frac{\sqrt{2}}{2}t, \frac{-V}{2}\sin\frac{\sqrt{2}}{2}t, 0\right)$

$+\left(\frac{1}{2}\sin\frac{\sqrt{2}}{2}t, \frac{-1}{2}\cos\frac{\sqrt{2}}{2}t, \frac{1}{2}\right)$

$= \left(\frac{-1}{2}\sin\frac{\sqrt{2}}{2}t - \frac{V}{2}\cos\frac{\sqrt{2}}{2}t, \frac{1}{2}\cos\frac{\sqrt{2}}{2}t - \frac{V}{2}\sin\frac{\sqrt{2}}{2}t, \frac{3}{2}\right)$

$\beta'(t) = \left(\frac{V\sqrt{2}}{4}\sin\frac{\sqrt{2}}{2}t - \frac{\sqrt{2}}{4}\cos\frac{\sqrt{2}}{2}t, \frac{-\sqrt{2}}{4}\sin\frac{\sqrt{2}}{2}t\right.$

$\left. - \frac{V\sqrt{2}}{4}\cos\frac{\sqrt{2}}{2}t, 0\right) \Rightarrow |\beta'(t)| = \frac{5}{2}$

$\beta''(t) = \left(\frac{1}{4}\sin\frac{\sqrt{2}}{2}t + \frac{V}{4}\cos\frac{\sqrt{2}}{2}t, \frac{V}{4}\sin\frac{\sqrt{2}}{2}t - \frac{1}{4}\cos\frac{\sqrt{2}}{2}t, 0\right)$

$k = \frac{|\beta' \times \beta''|}{|\beta'|^3} = \frac{\frac{25}{25\sqrt{2}}}{\frac{125}{8}} = \frac{\sqrt{2}}{5}$

$\beta' \times \beta'' = \left(0, 0, \frac{25}{25\sqrt{2}}\right) \Rightarrow |\beta' \times \beta''| = \frac{25}{25\sqrt{2}}$

$$\frac{\sqrt{2}}{8}\sin\frac{\sqrt{2}}{2}t+\frac{7\sqrt{2}}{8}\cos\frac{\sqrt{2}}{2}t$$

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$$\beta'''(t)=\left(\frac{-7\sqrt{2}}{8}\sin\frac{\sqrt{2}}{2}t-\frac{\sqrt{2}}{8}\cos\frac{\sqrt{2}}{2}t,\ 0\right)$$

$$\tau=\frac{(\beta'\times\beta'').\beta'''}{|\beta'\times\beta''|^2}=0$$

تاب (صفر) و انحنای آن ثابت است؛ بنابراین مسیر (منحنی) β نیز دایره‌ای به شعاع $\frac{1}{K}$ خواهد بود.