ساختمان داده و الگوریتم ها (CE203)

جلسه شانزدهم:
جستجوی عمق اول (DFS)

سجاد شیرعلی شهرضا

پاییز 1401

شنبه، 19 آذر 1401

Based on Stanford CS161 Slides from Summer 2021 by Karey Shi

اطلاع رسانی

- بخش مرتبط کتاب برای این جلسه: 22

جستجوی عمق اول (DFS)

یک روش دیگر برای پیمایش گراف

BFS vs. DFS

Literally just BREADTH vs DEPTH:

While BFS first explores the nodes closest to the “source” and then moves outwards in layers, DFS goes as far down a path as it can before it comes back to explore other options.

DEPTH-FIRST SEARCH

An analogy:

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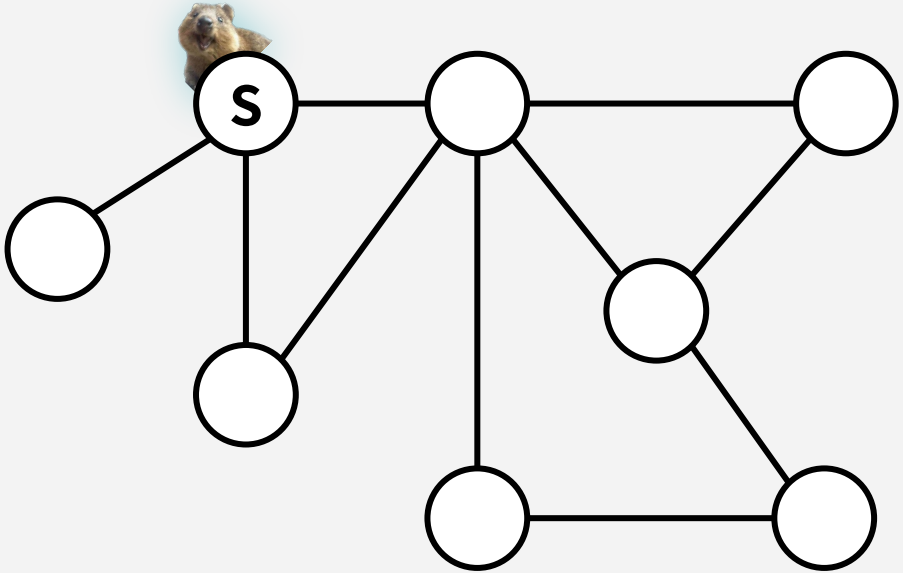

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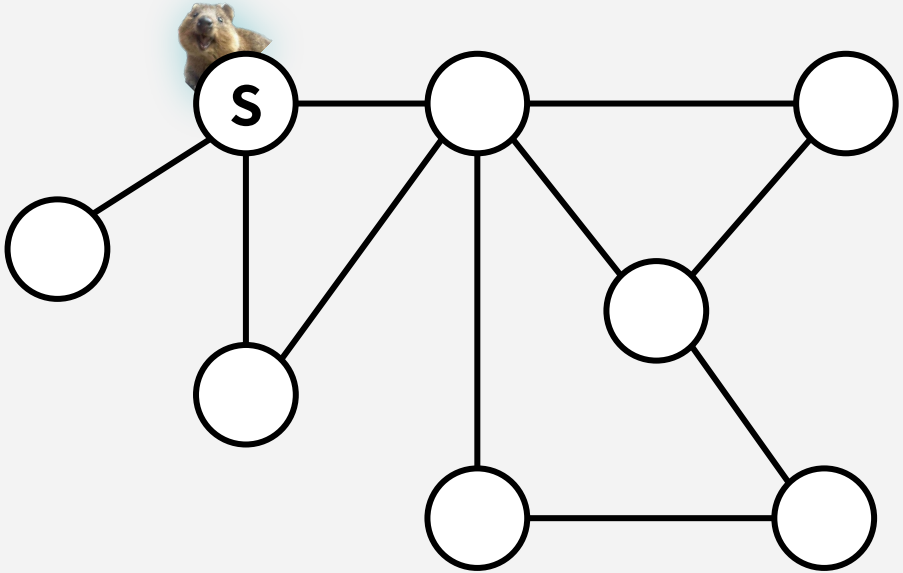

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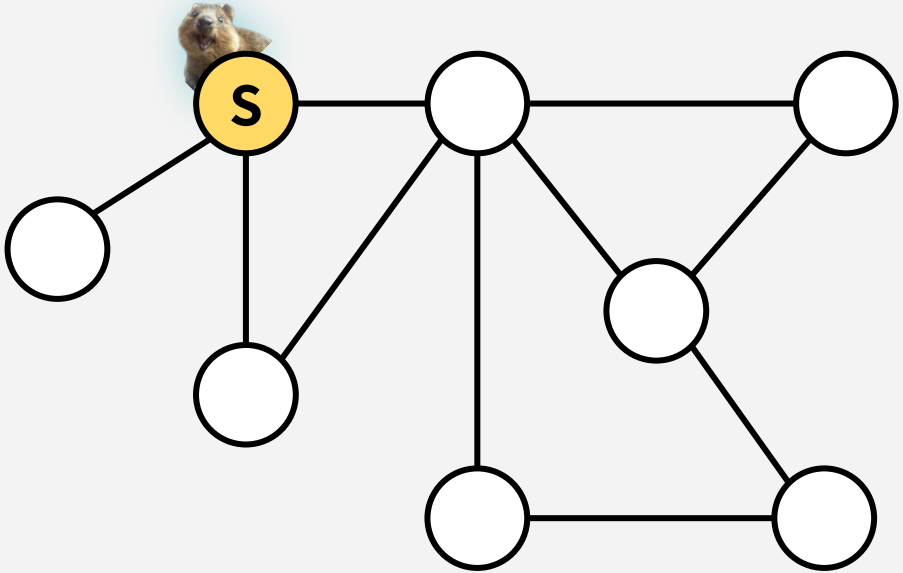

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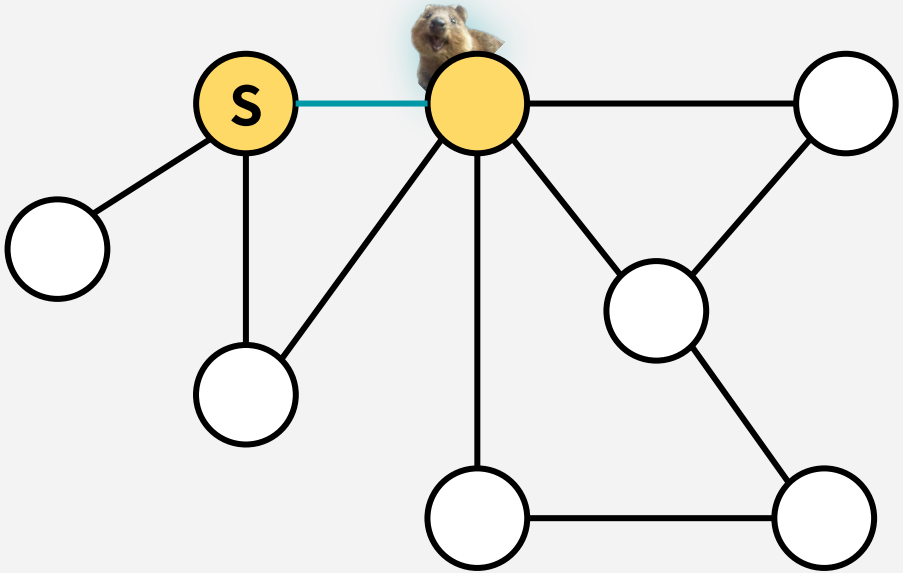

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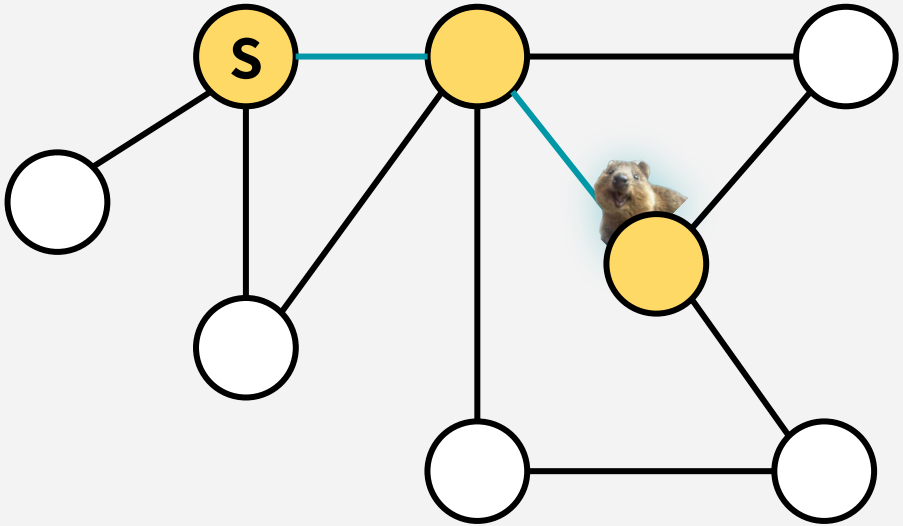

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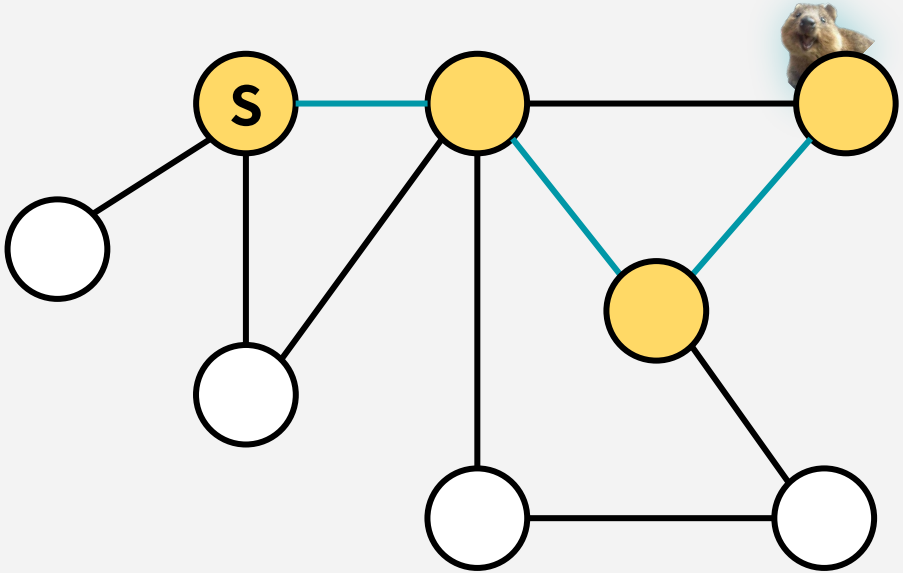

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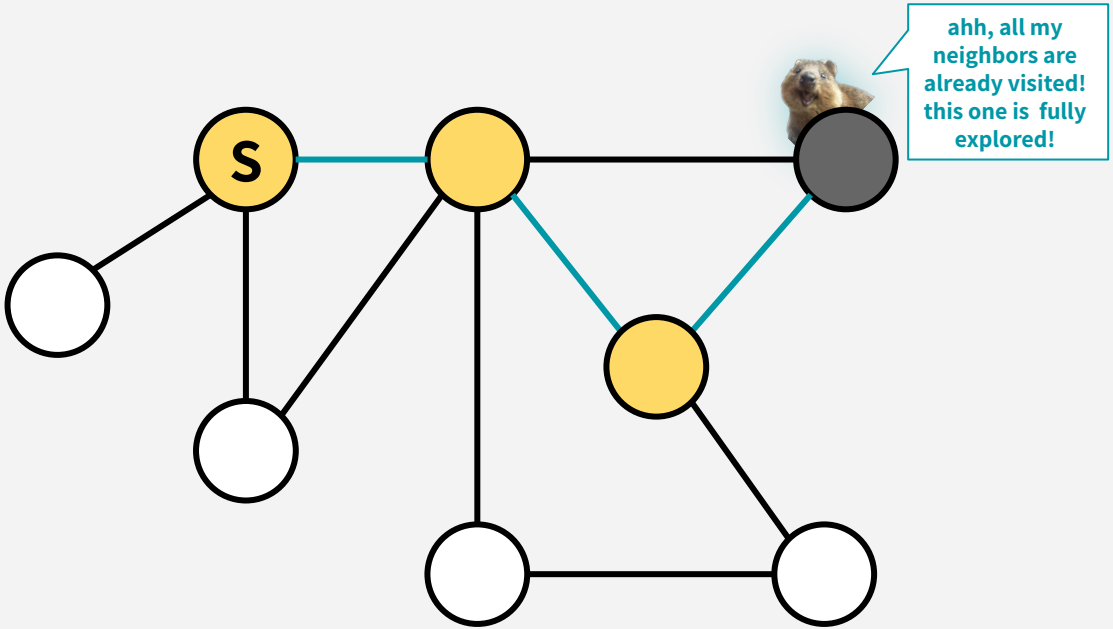

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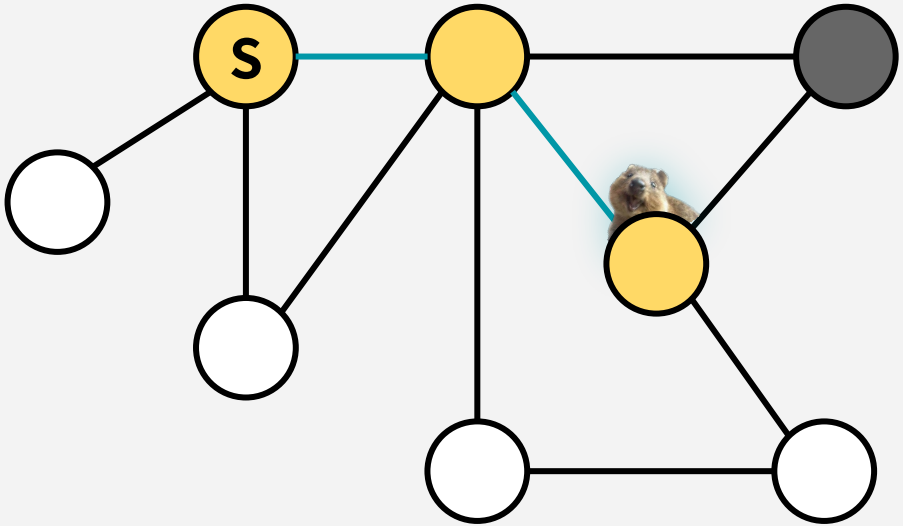

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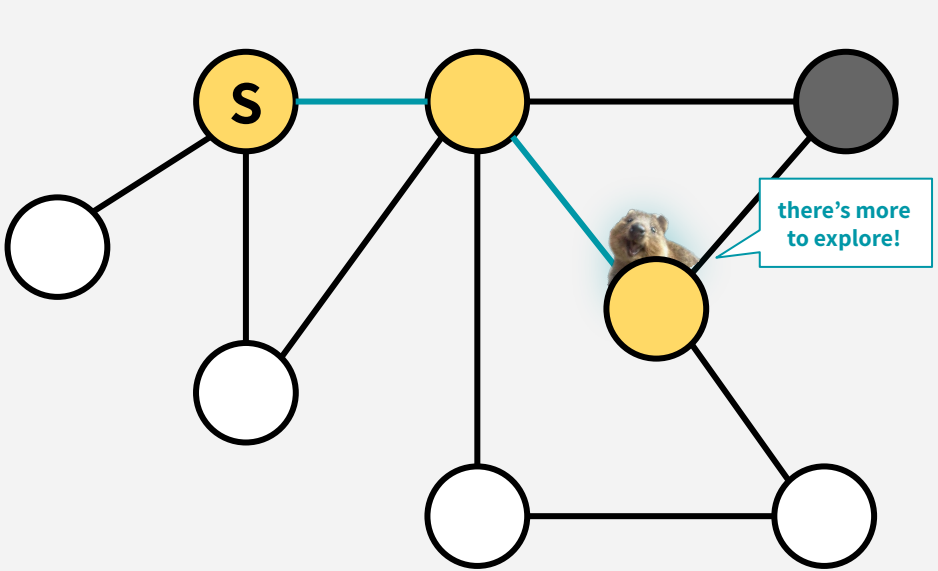

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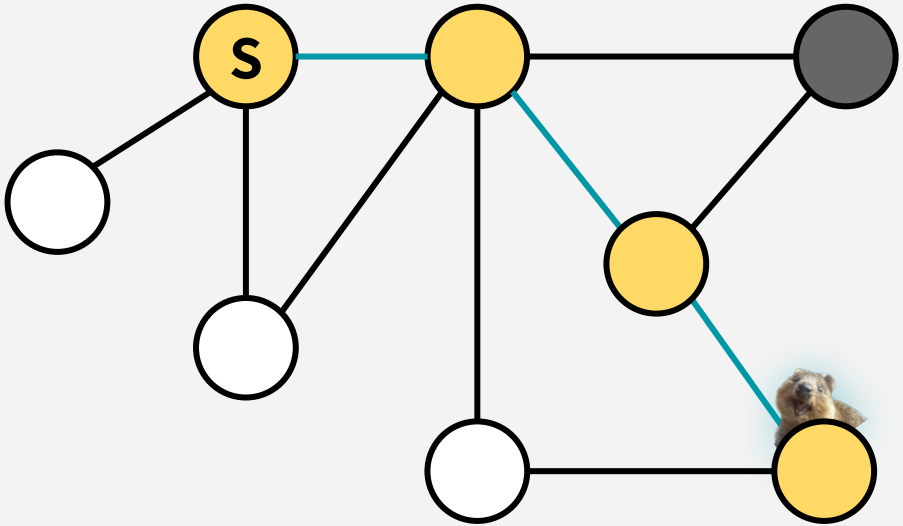

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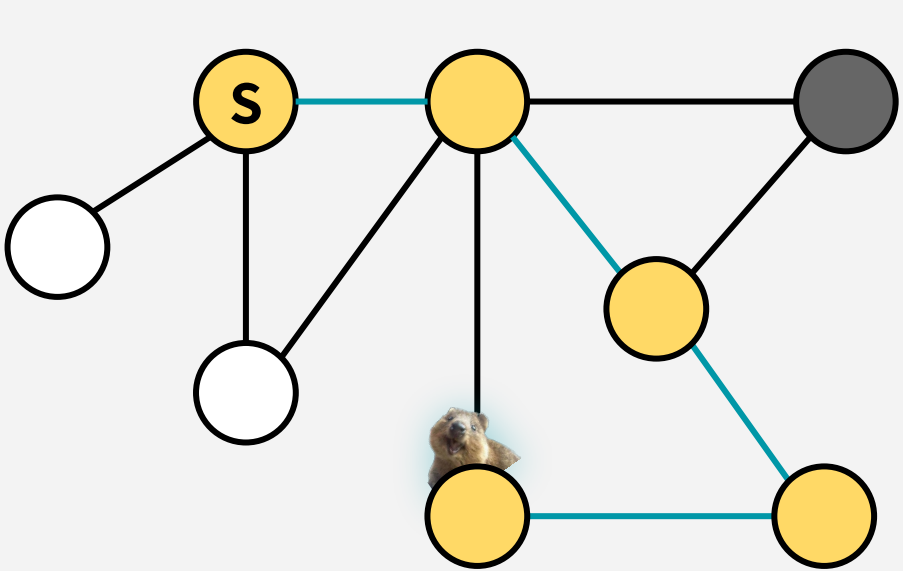

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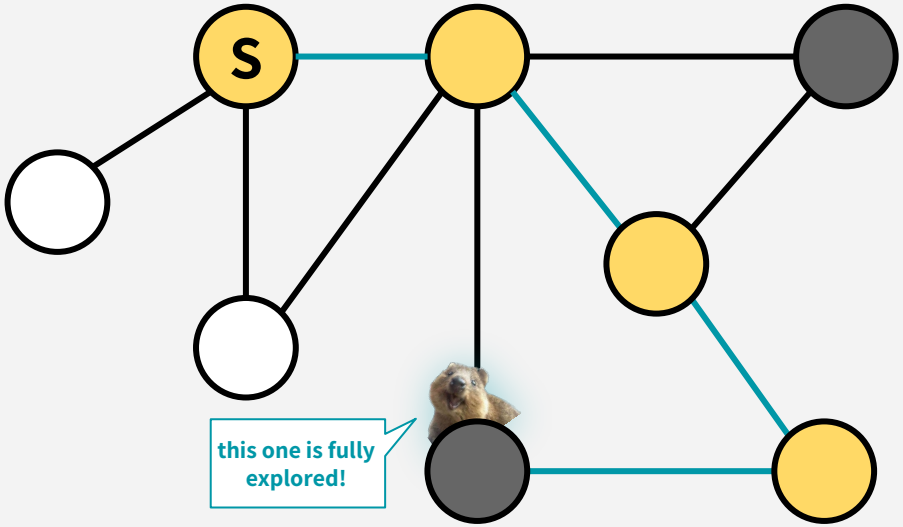

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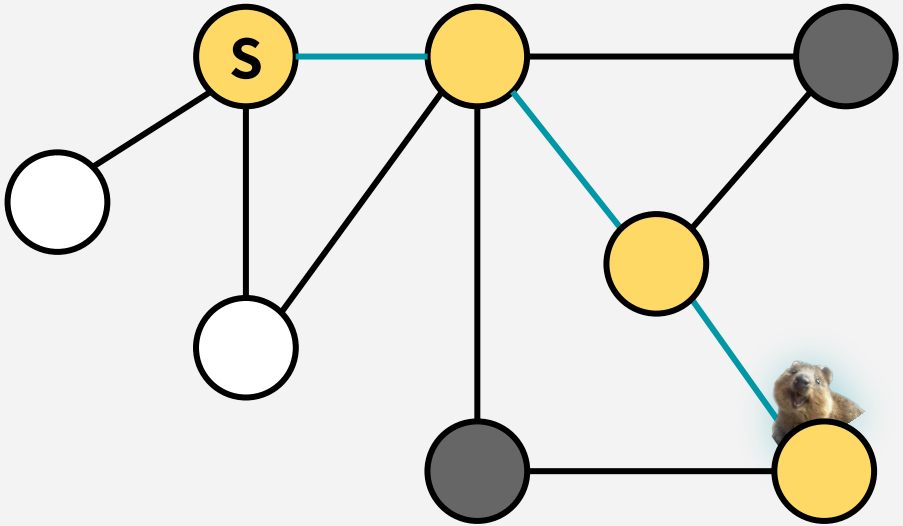

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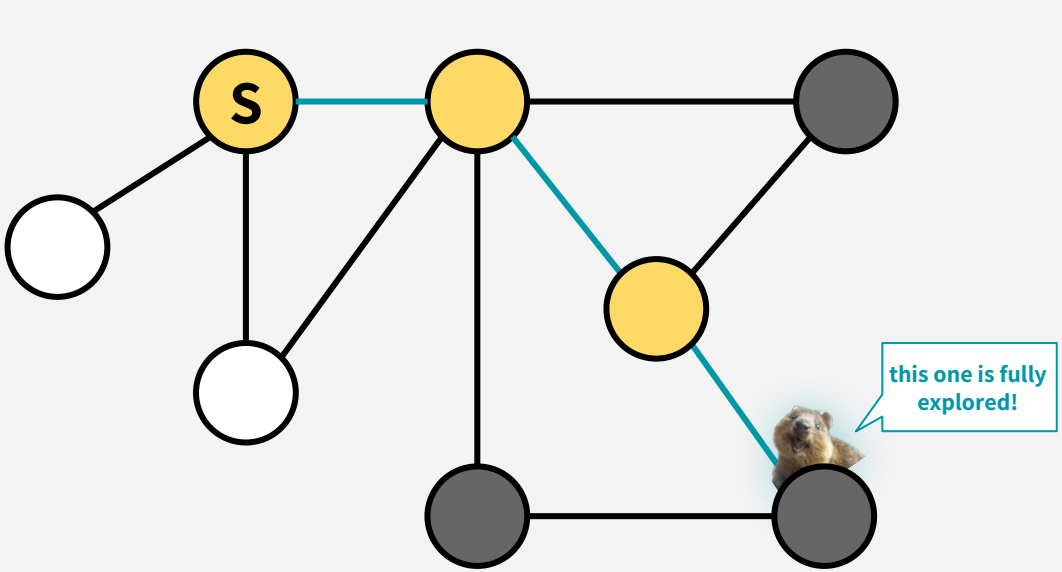

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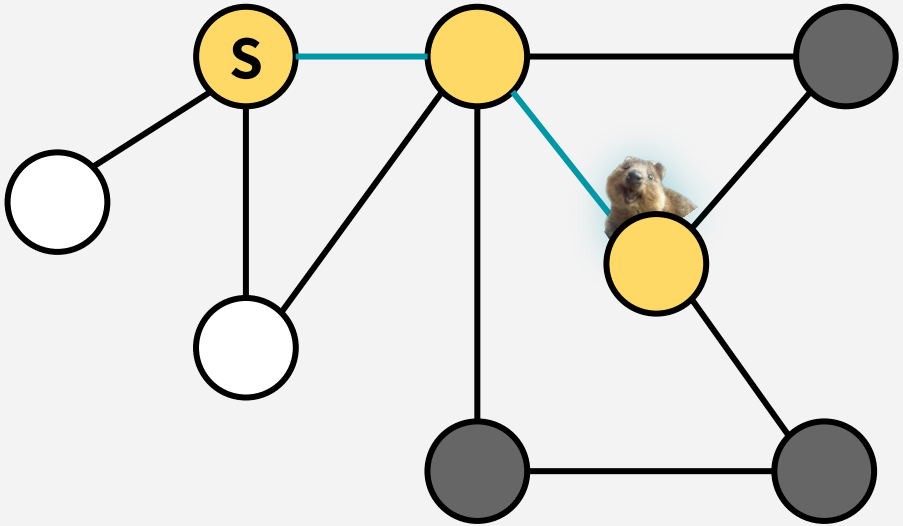

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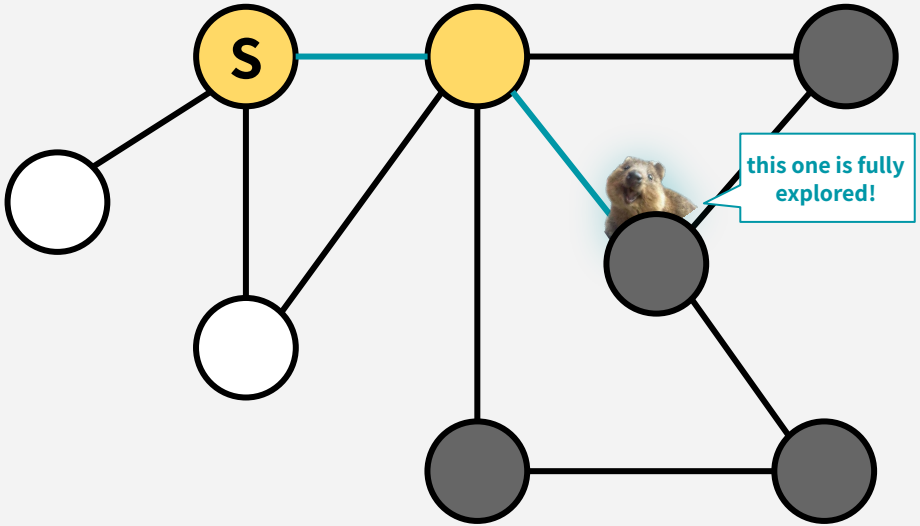

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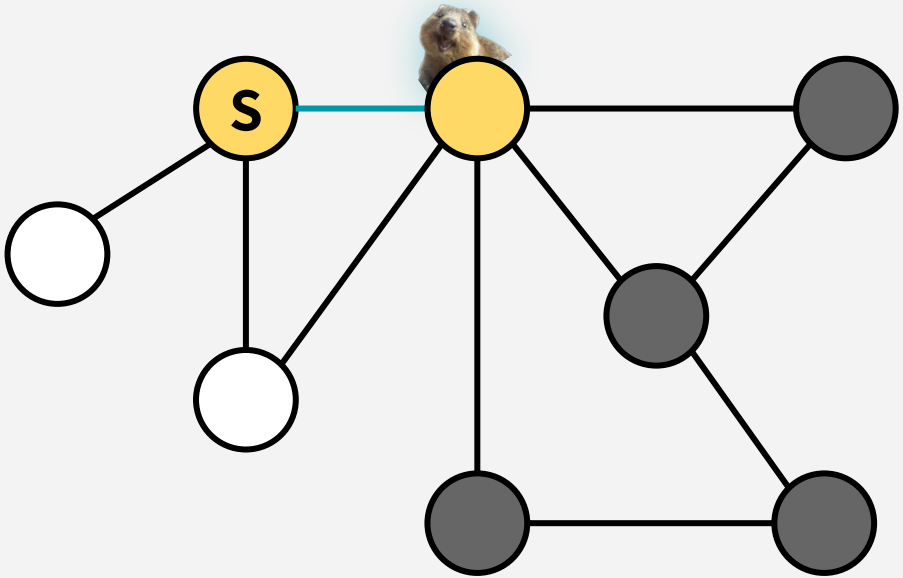

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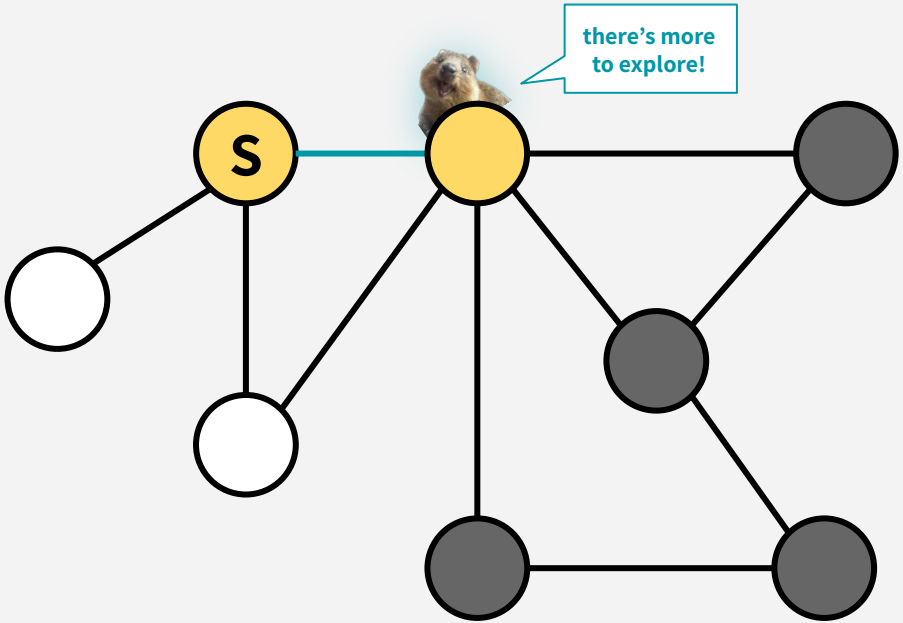

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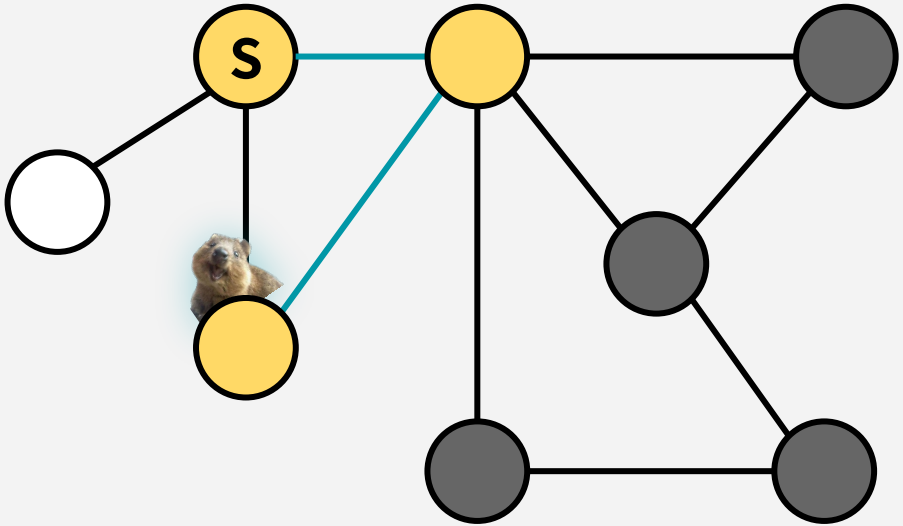

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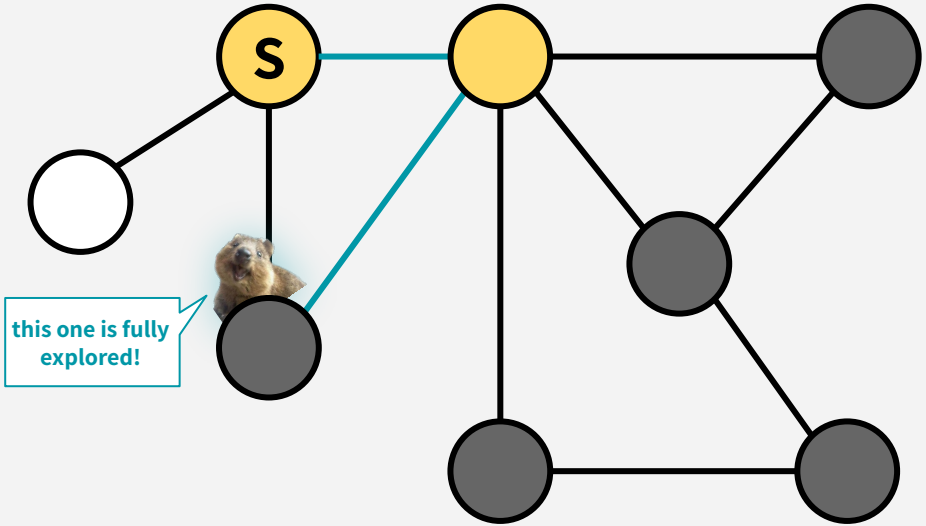

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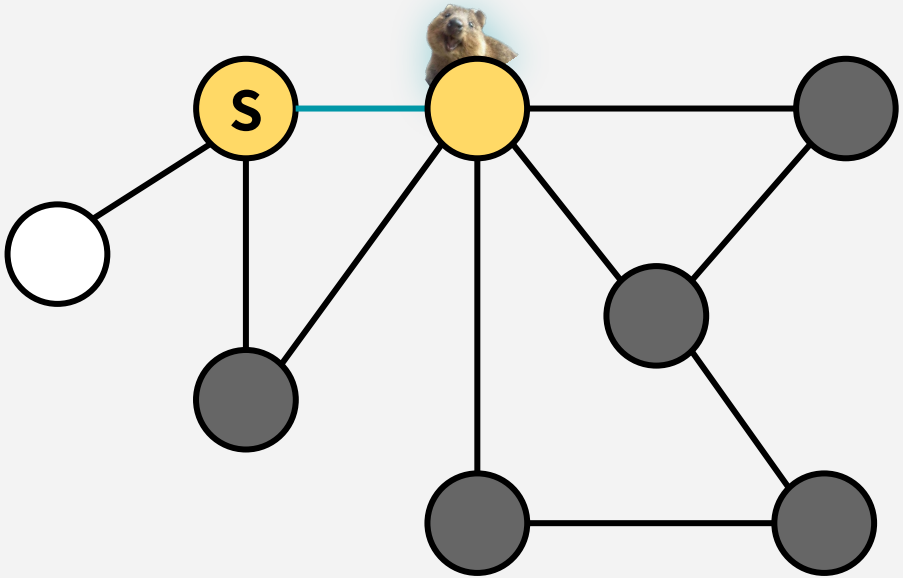

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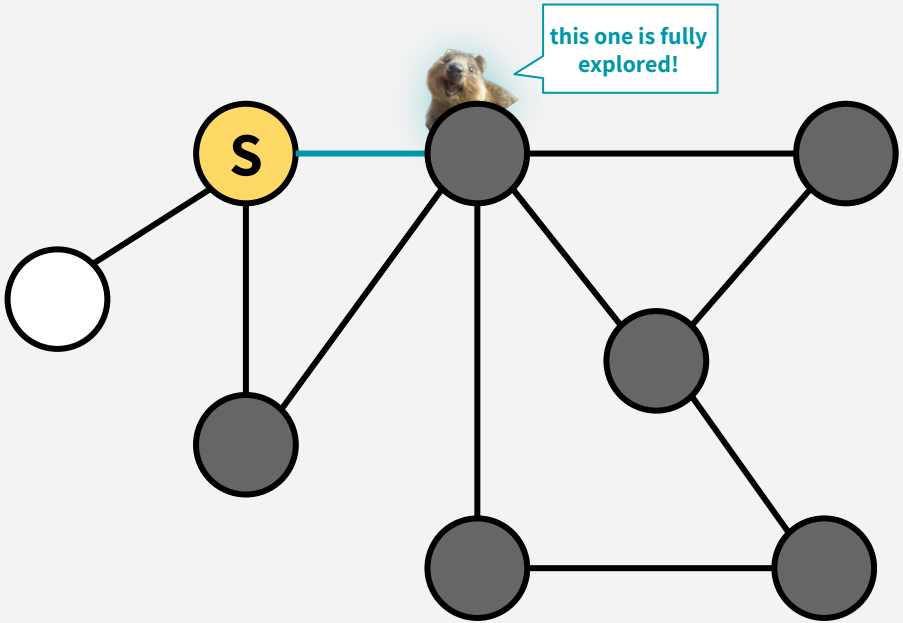

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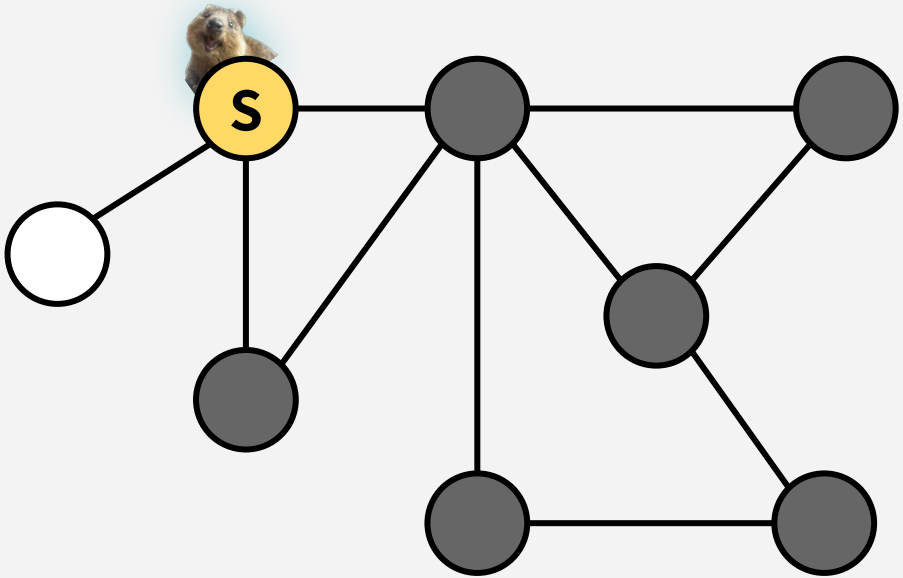

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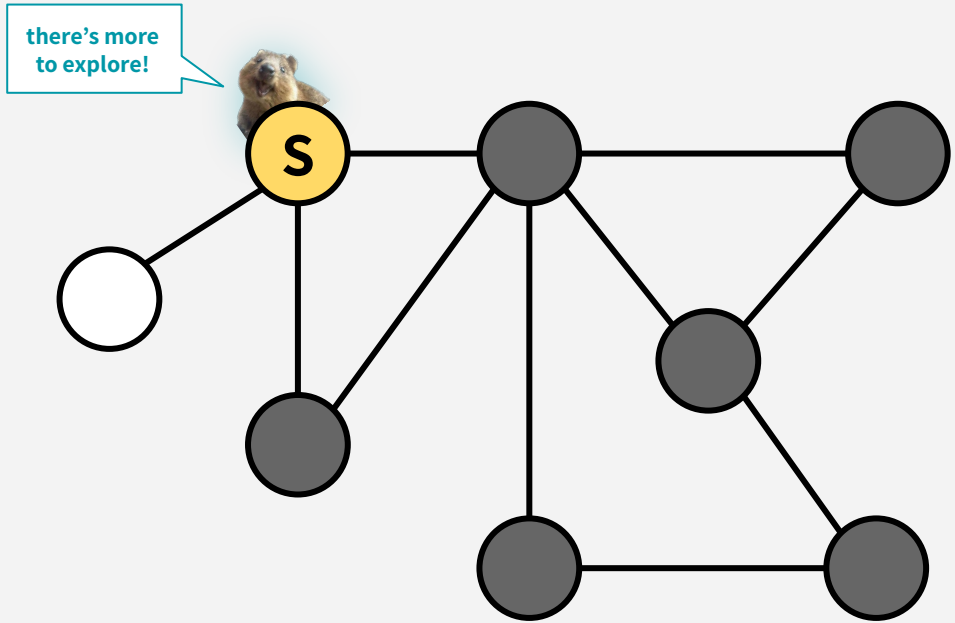

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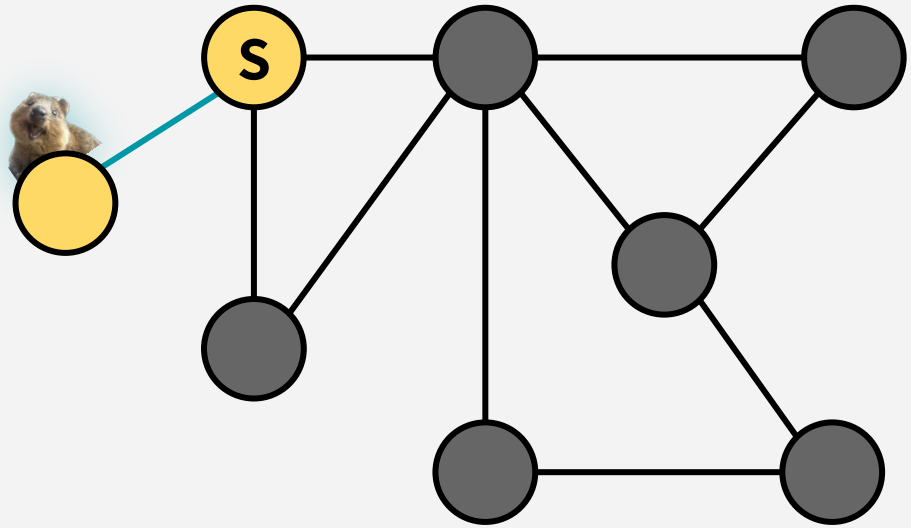

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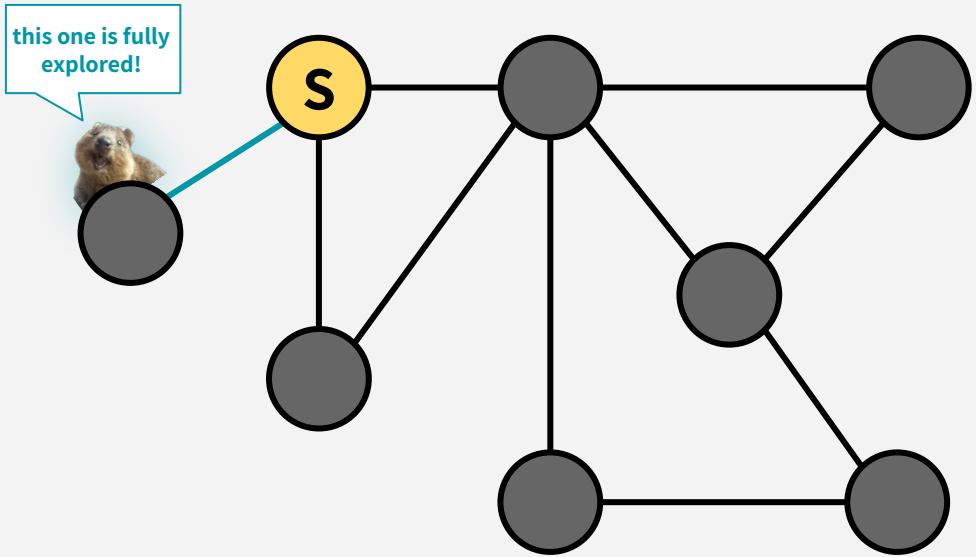

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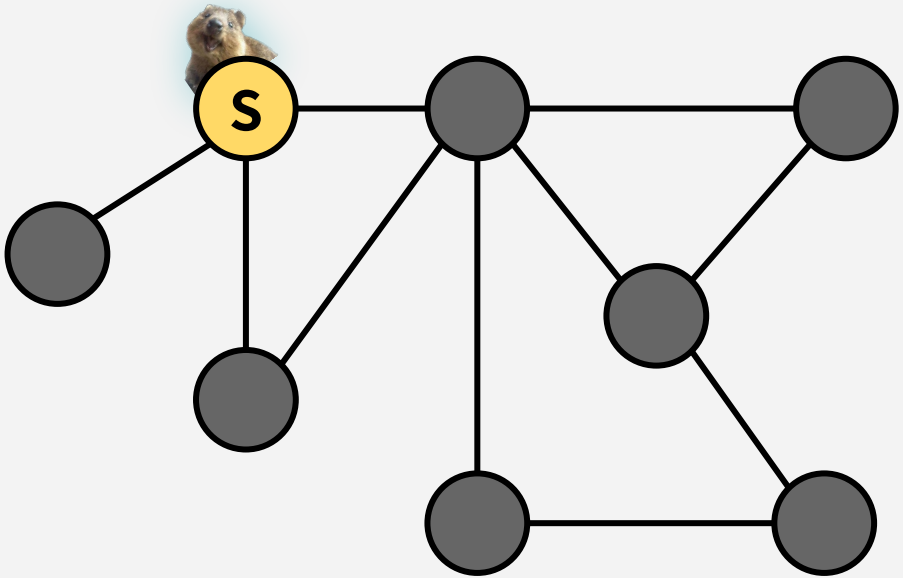

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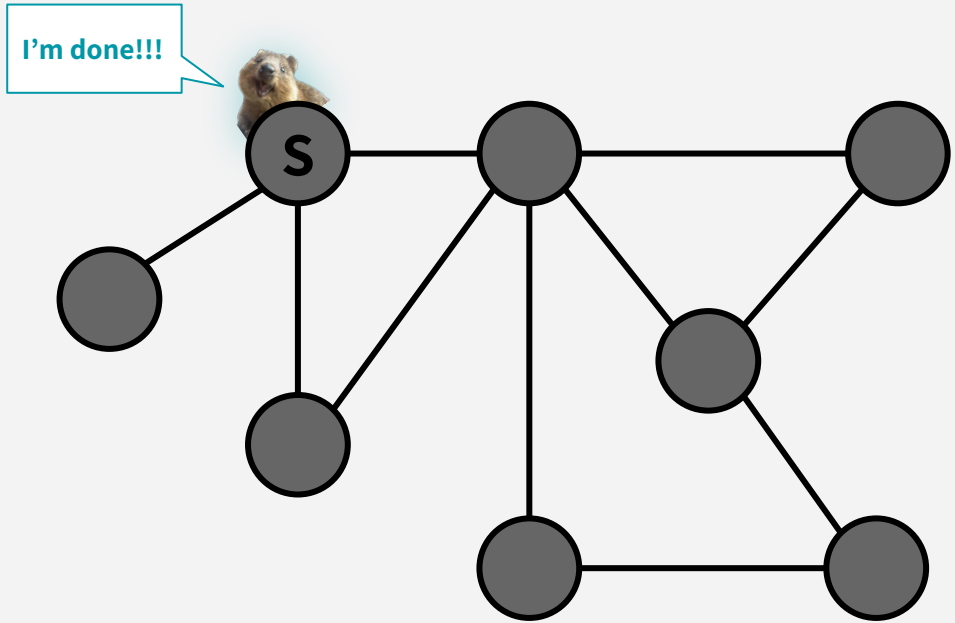

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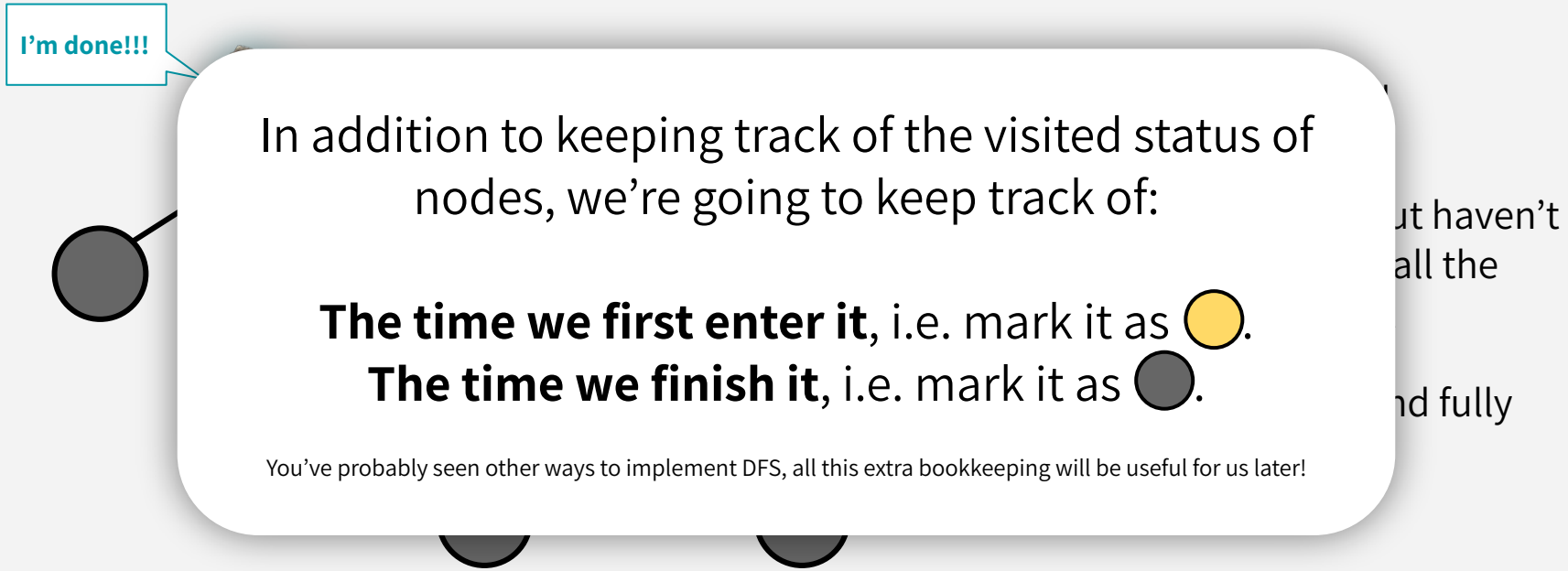

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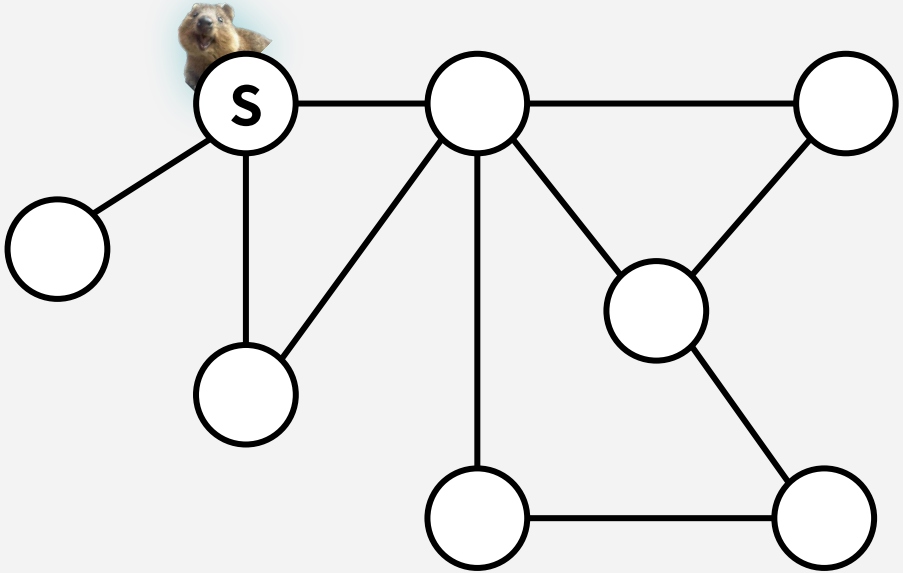


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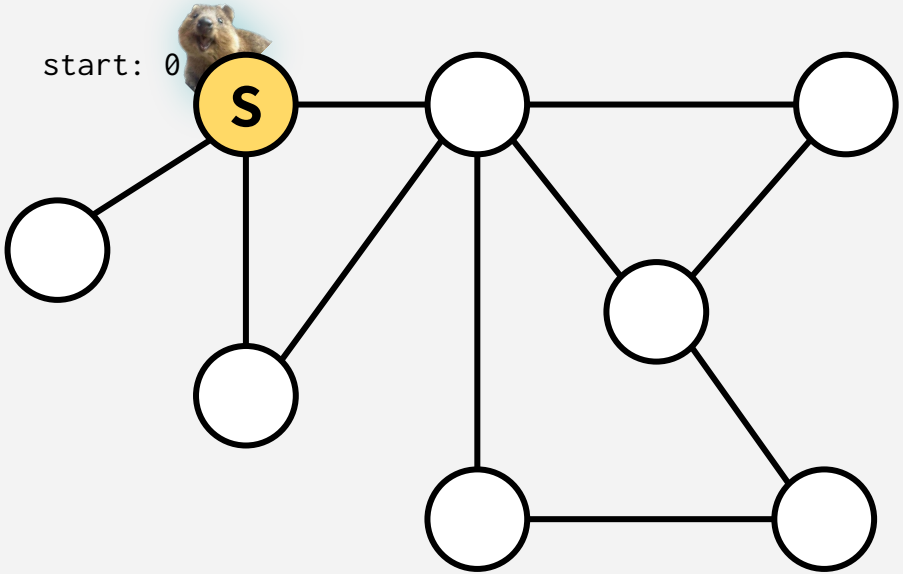


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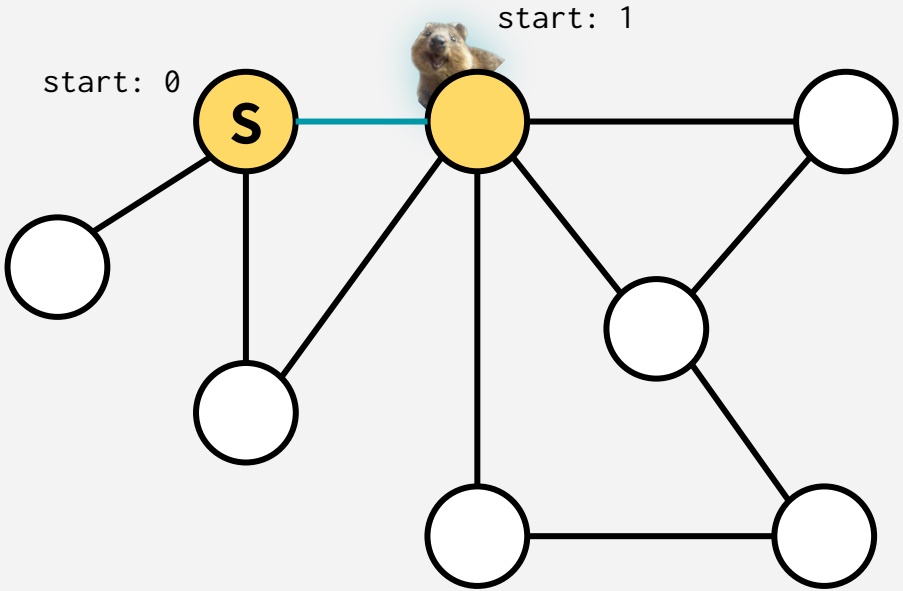


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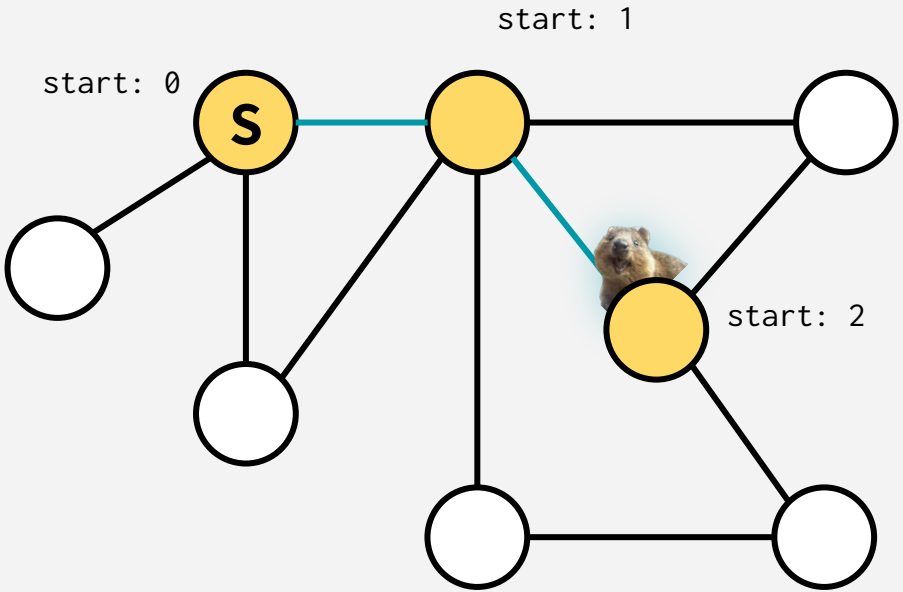


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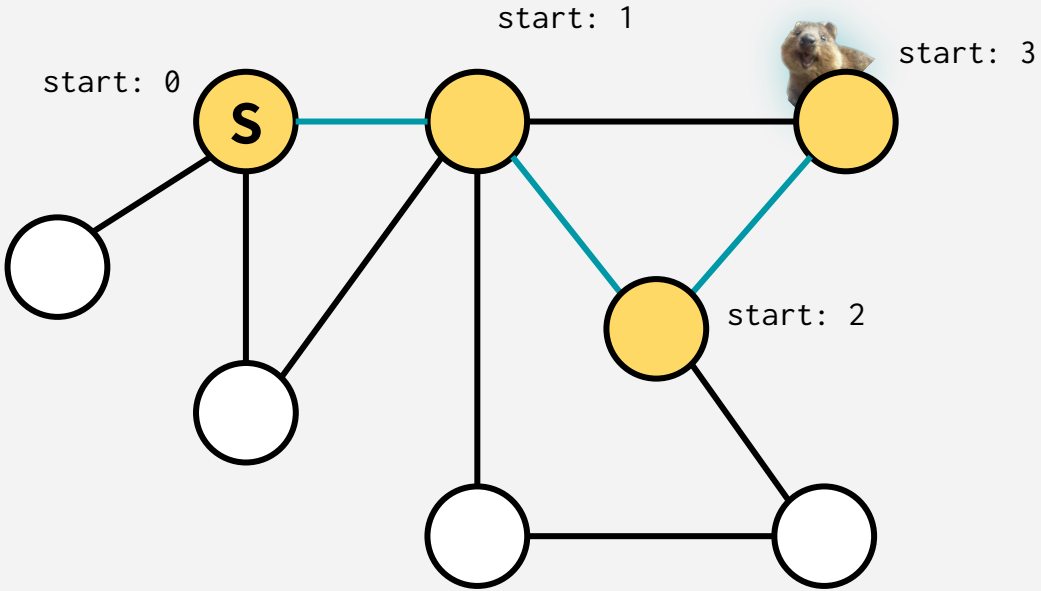


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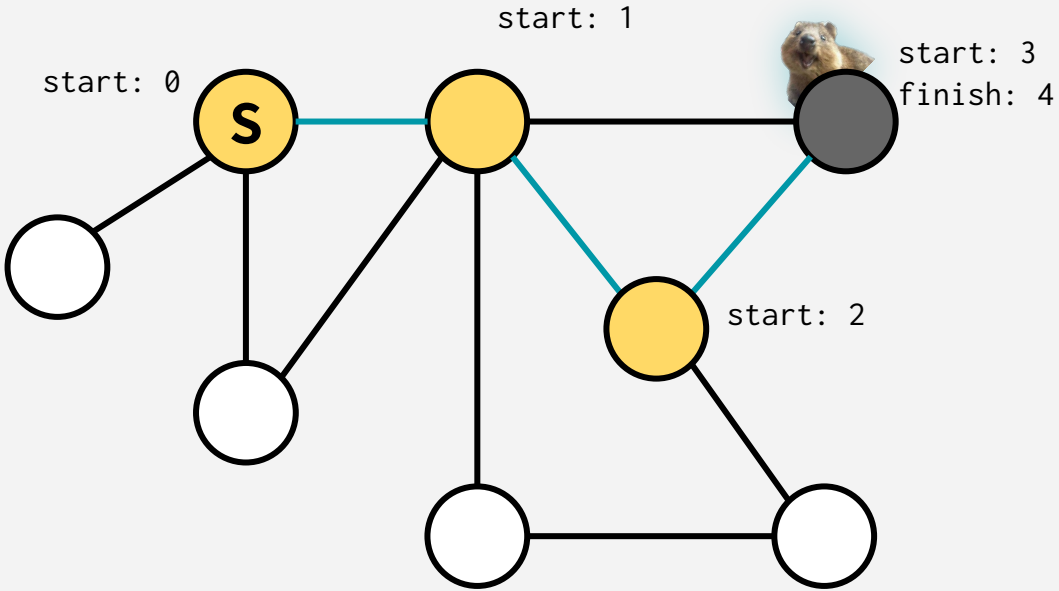


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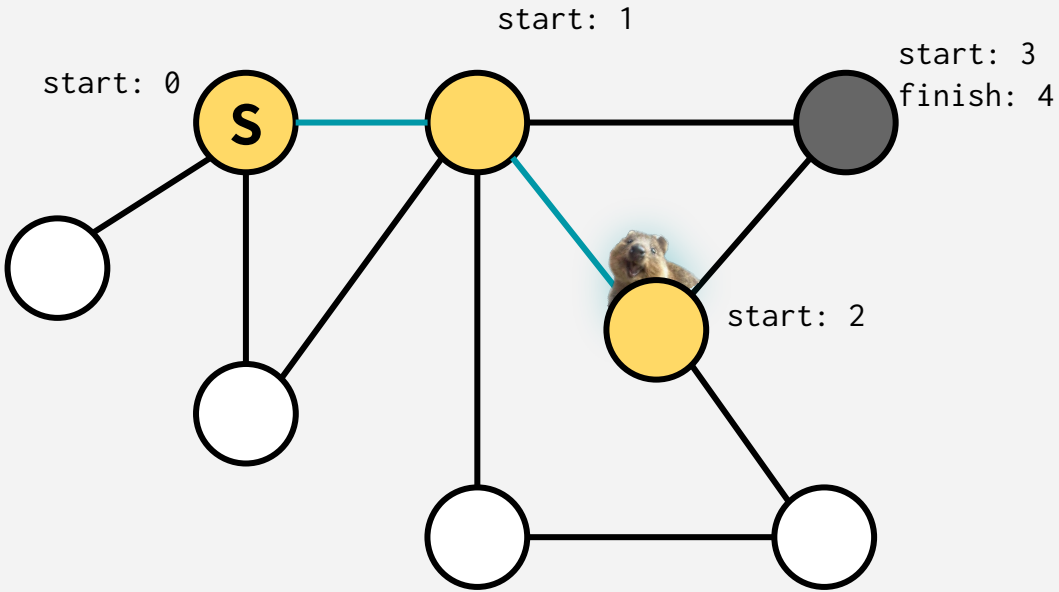


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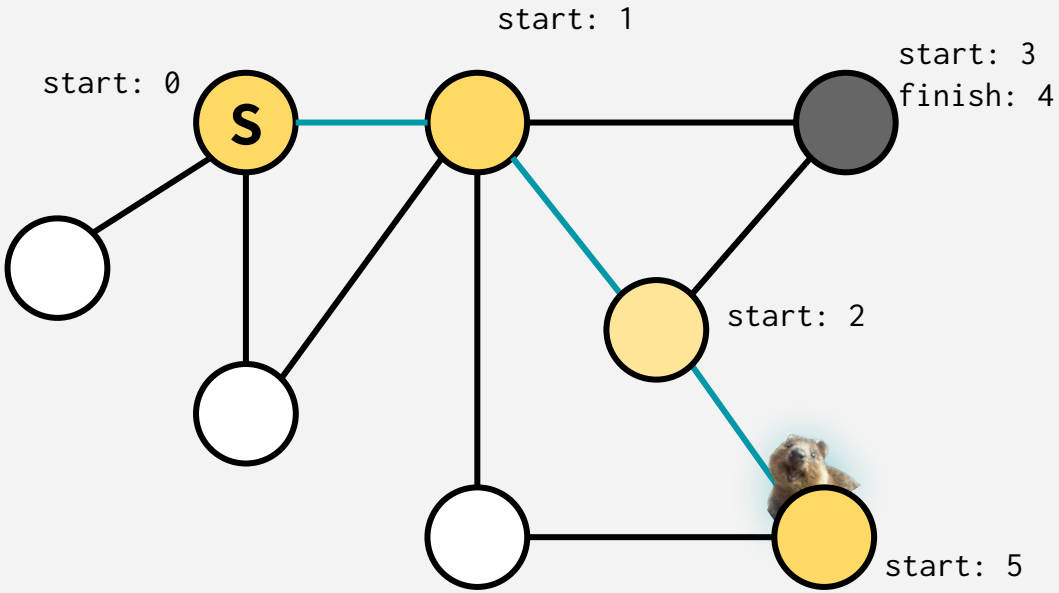


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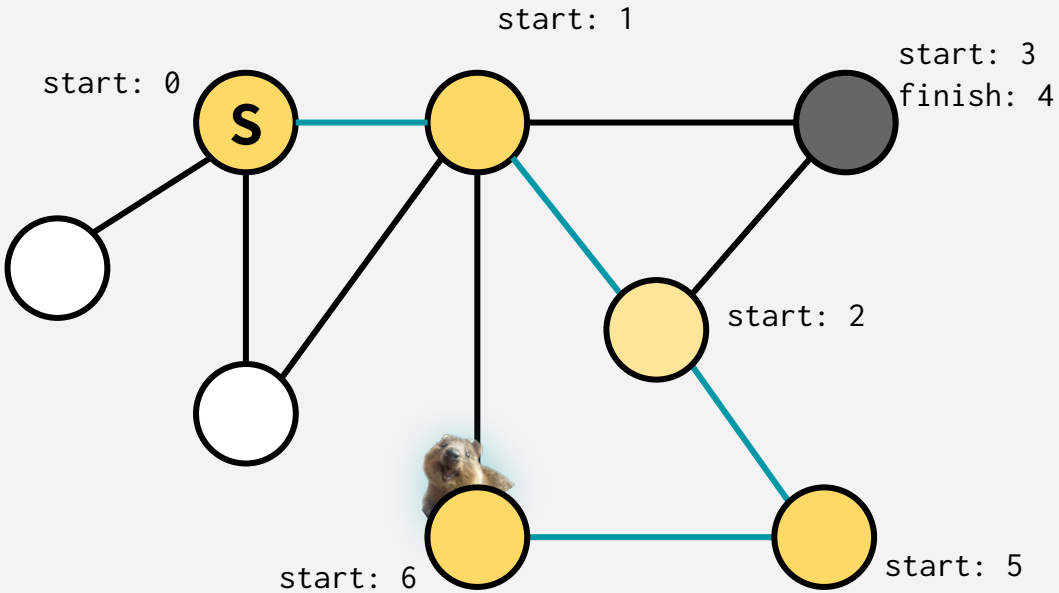


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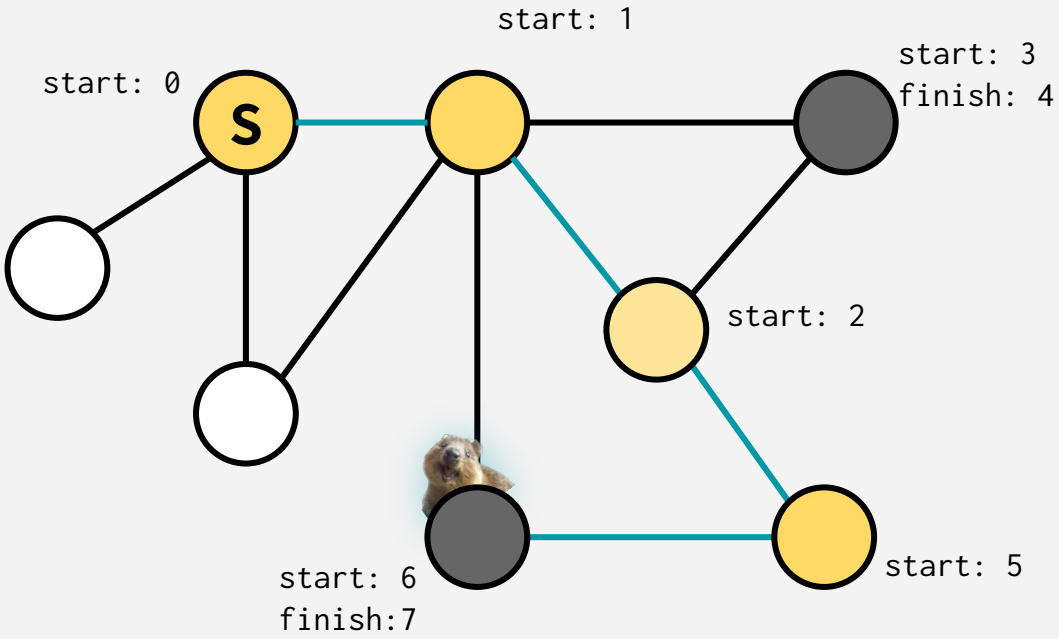


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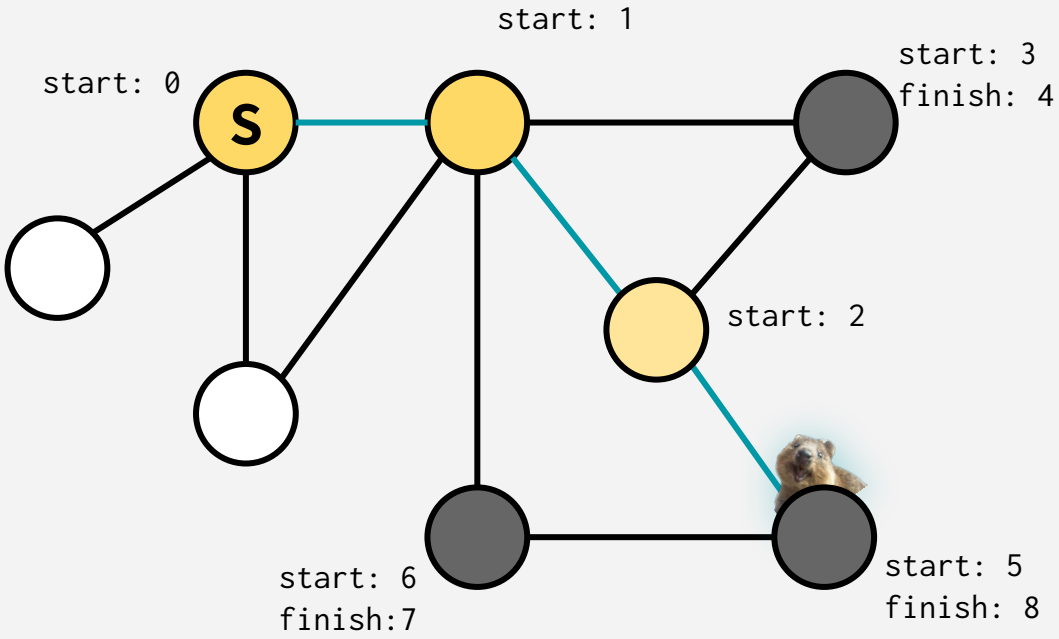


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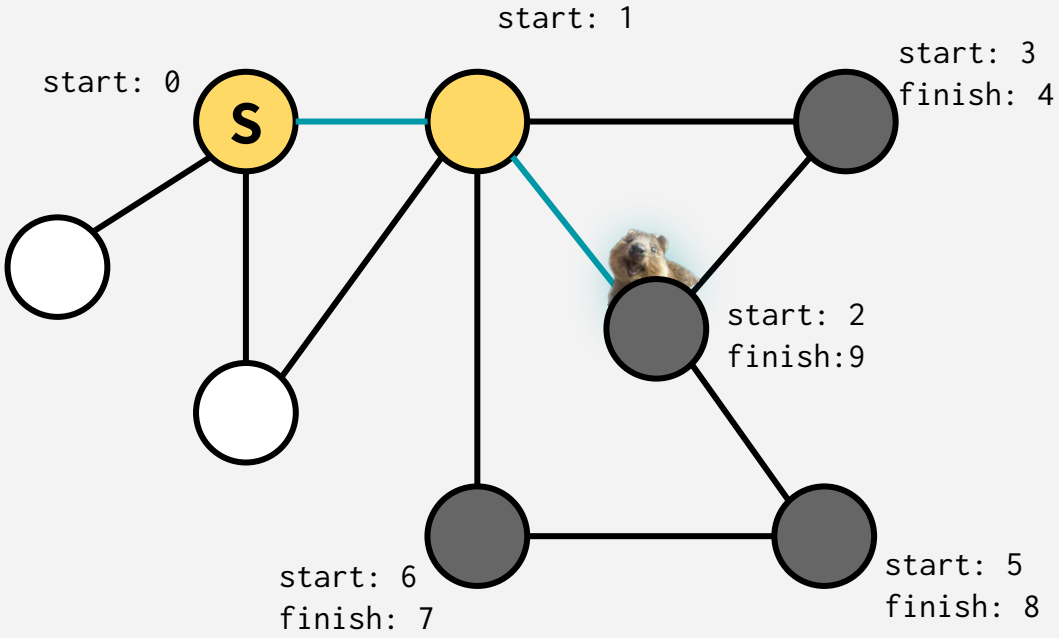


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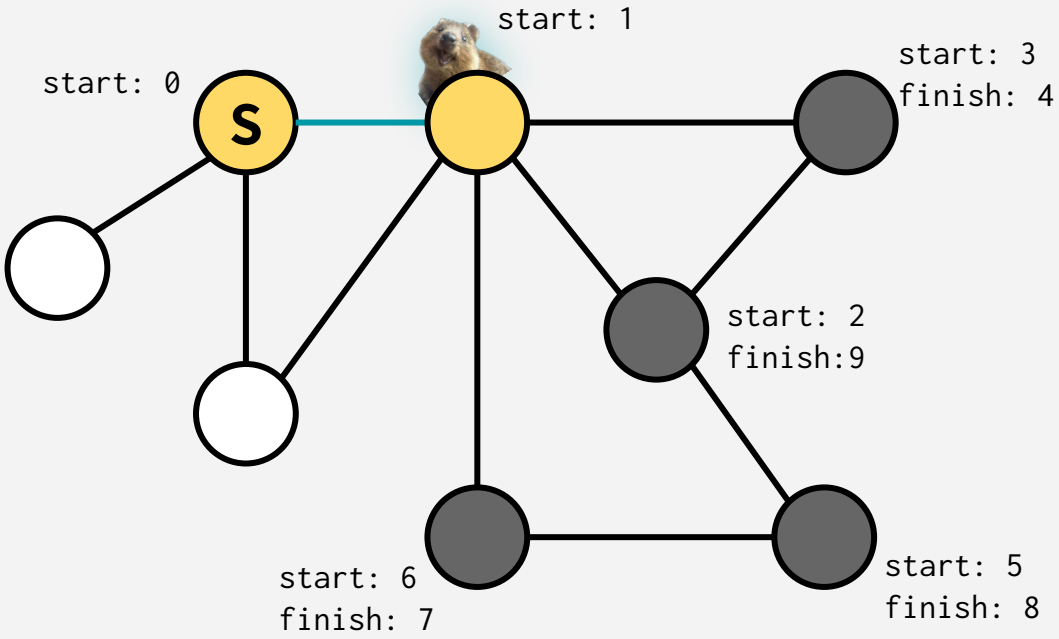


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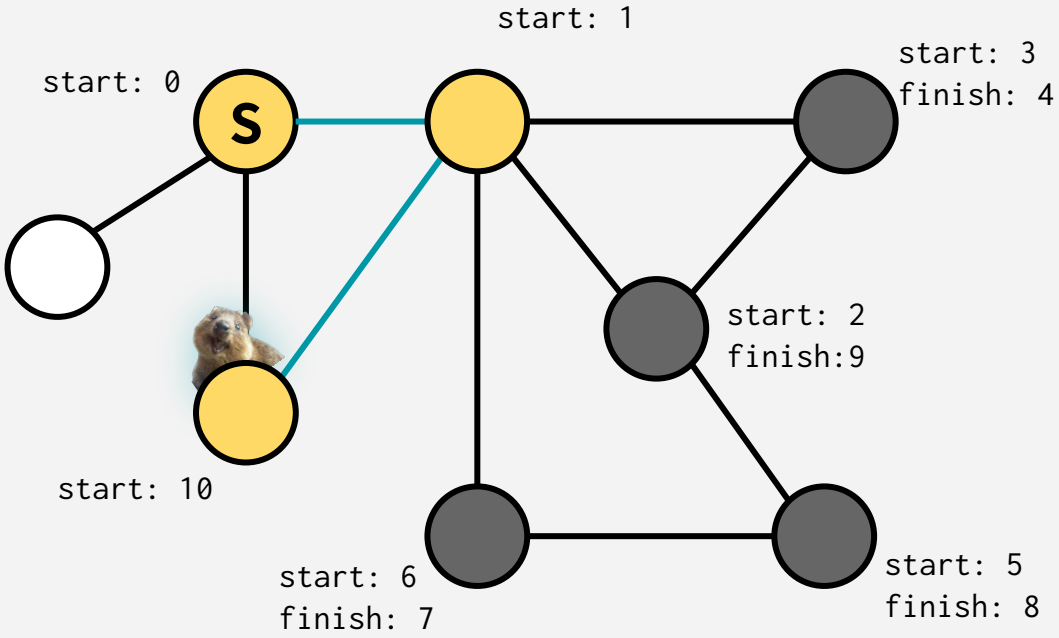


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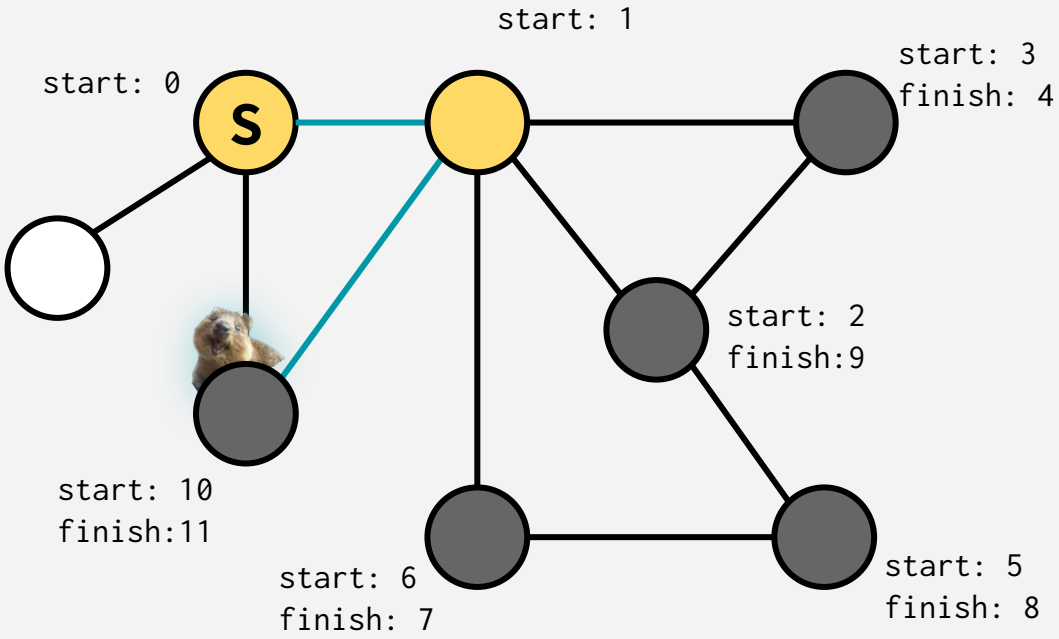


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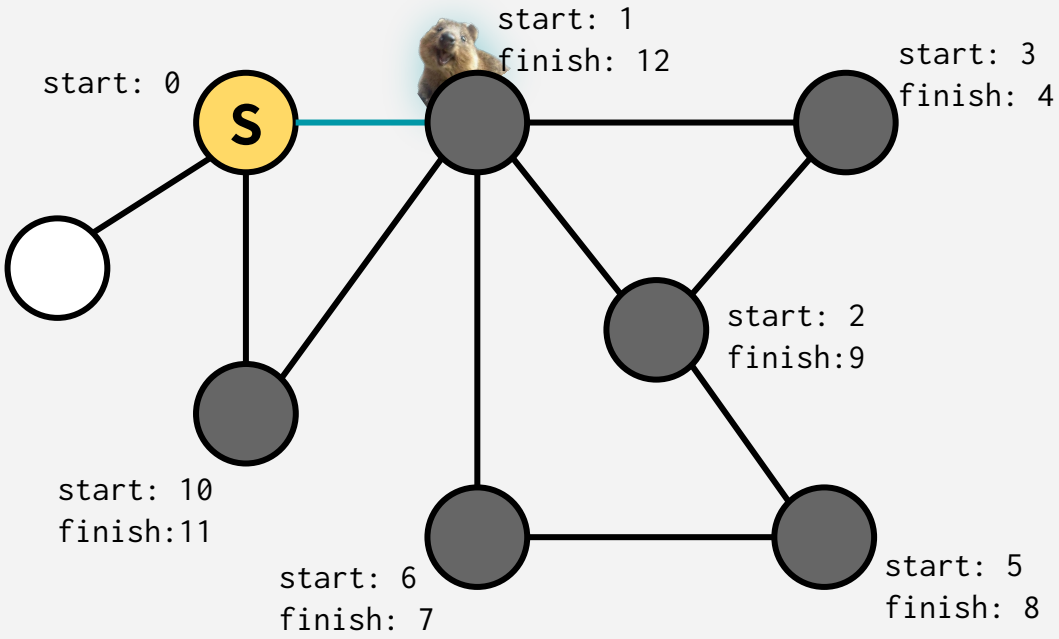


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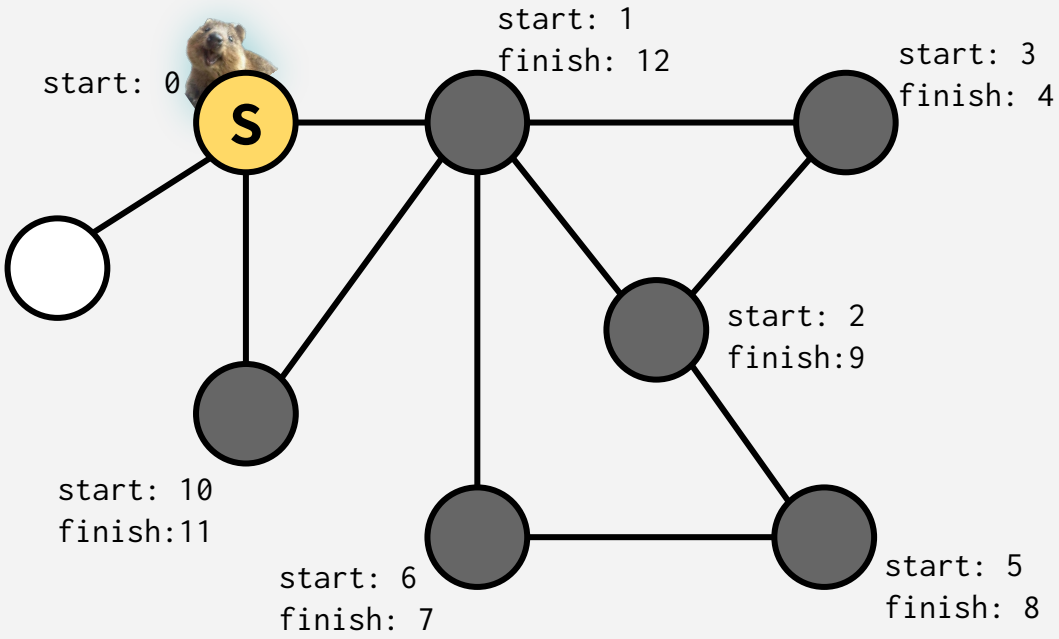


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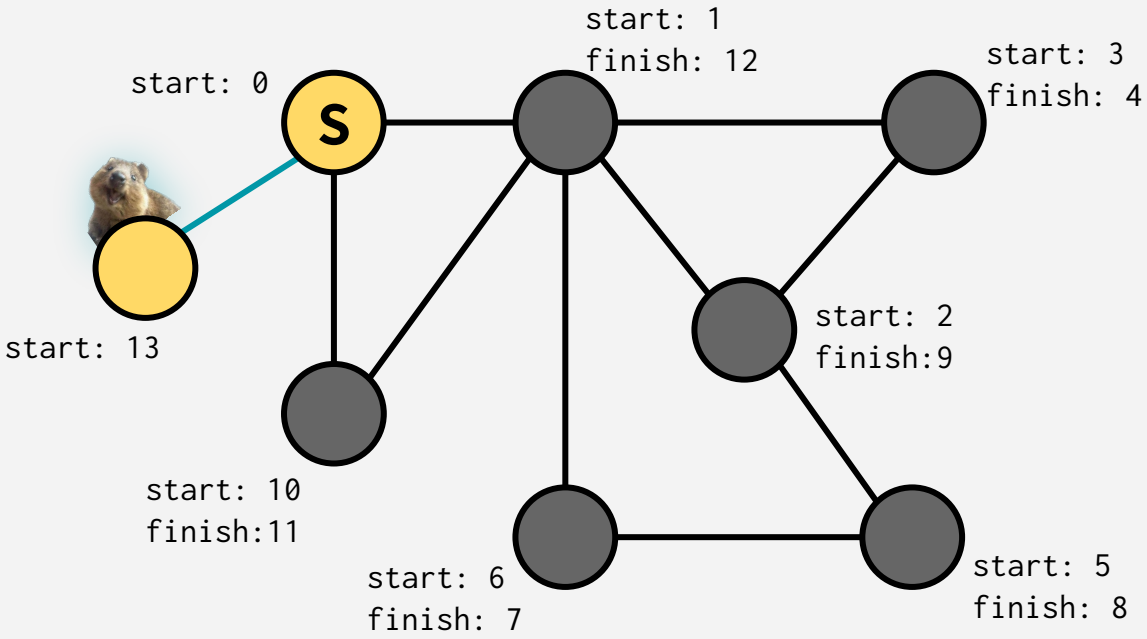


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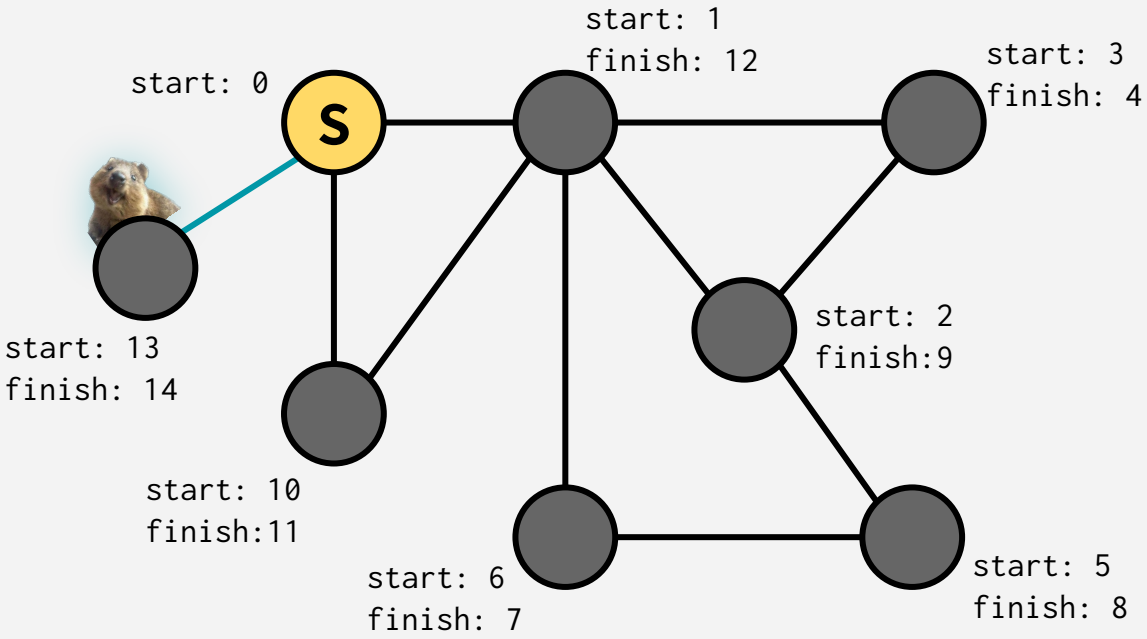


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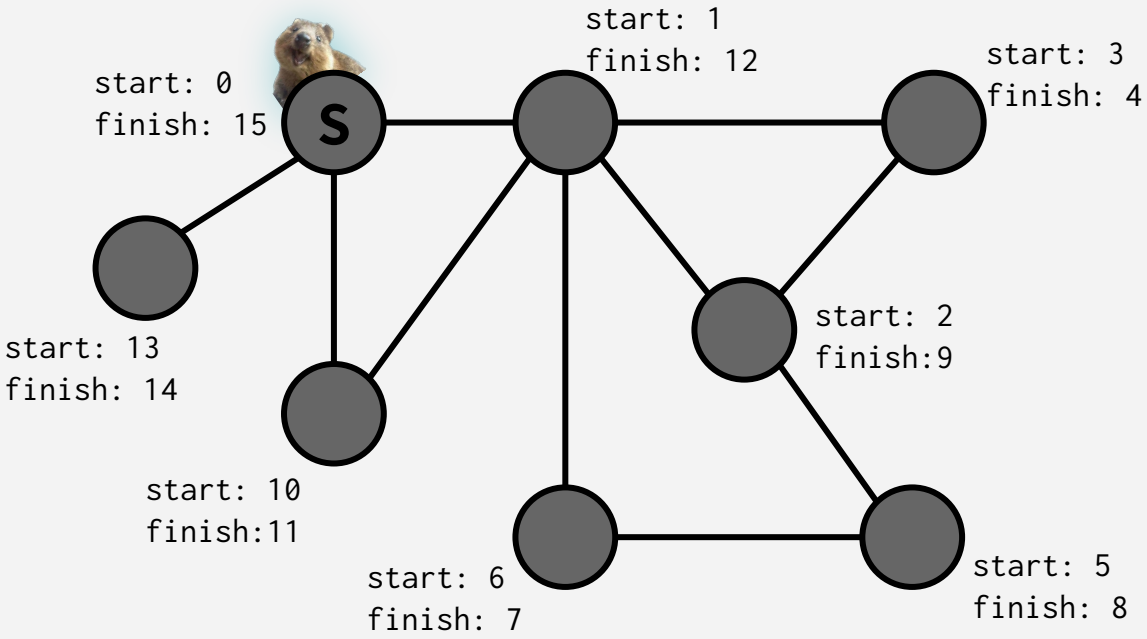


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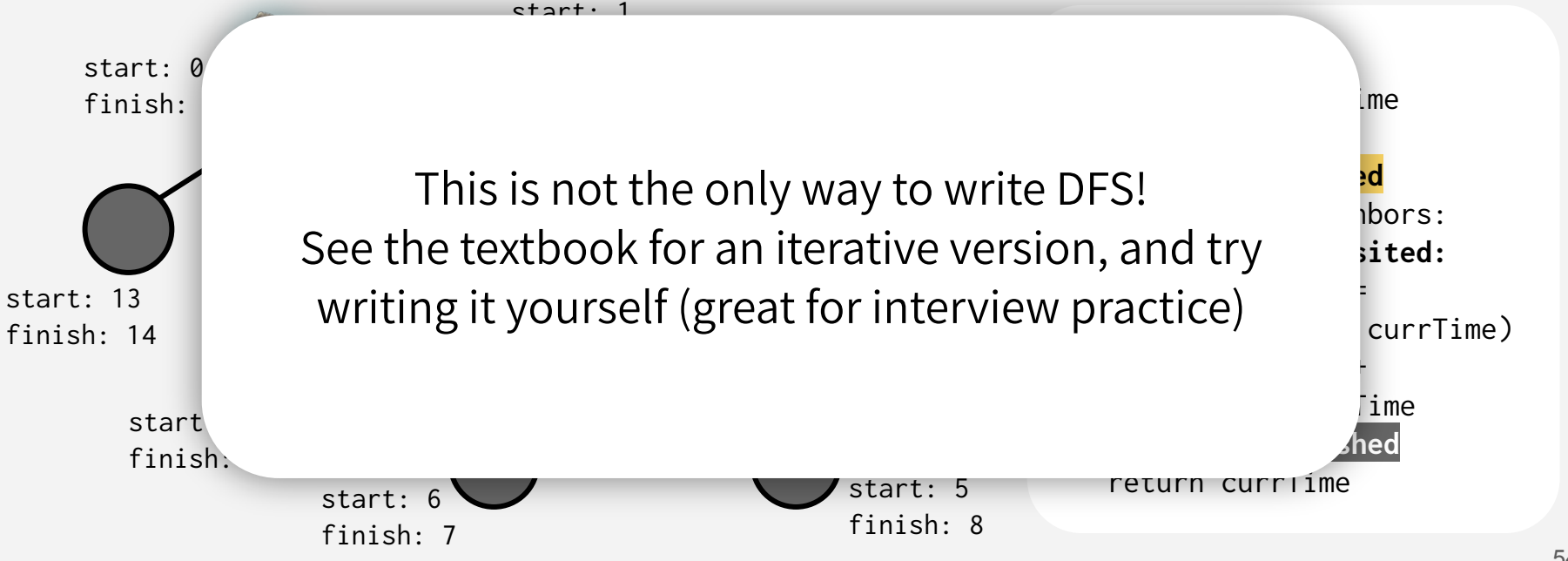
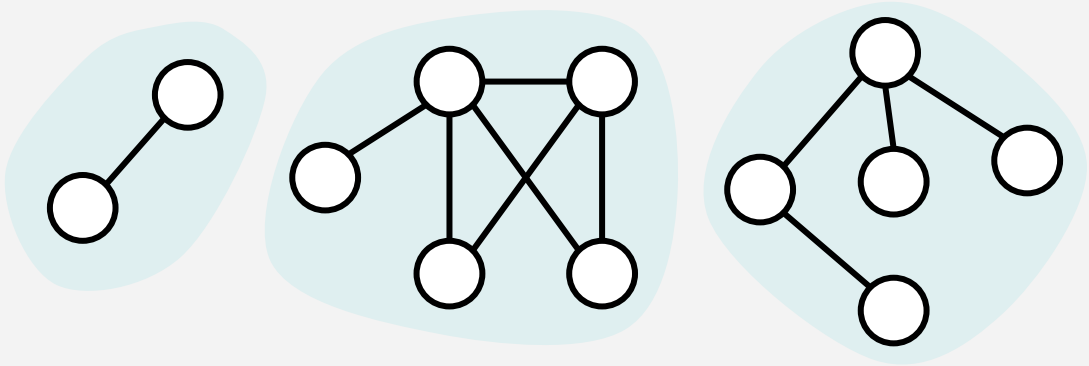


This is not the only way to write DFS!
See the textbook for an iterative version, and try writing it yourself (great for interview practice)

DEPTH-FIRST SEARCH

Like BFS, DFS finds all the nodes reachable from the starting point!

In undirected graphs, this is equivalent to finding a **connected component.**



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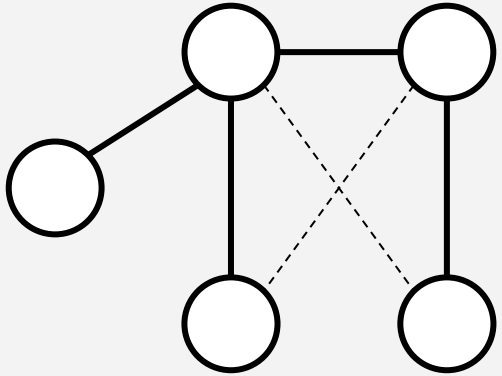
Why is it called depth-first?

We are implicitly building a **tree**!

(It's a tree because we never revisit a node)

We're going as "deep" as we can before "bubbling" back up.



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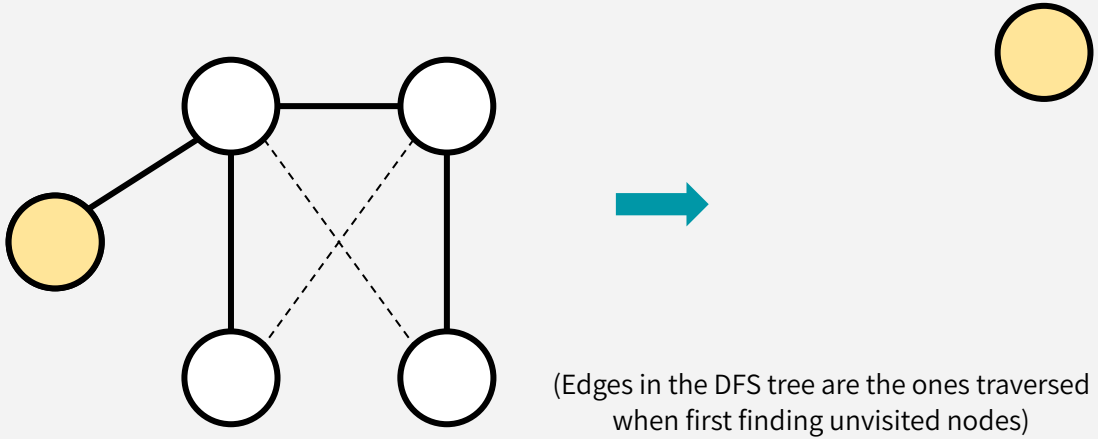

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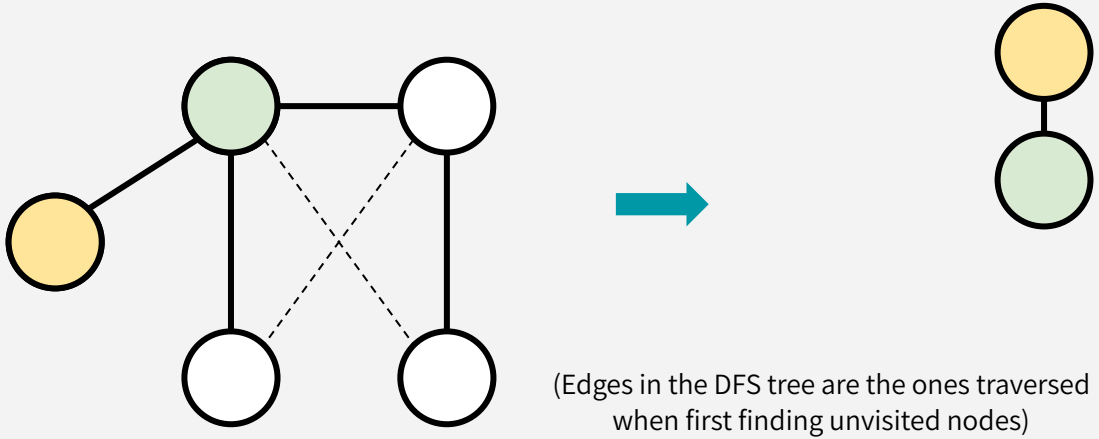

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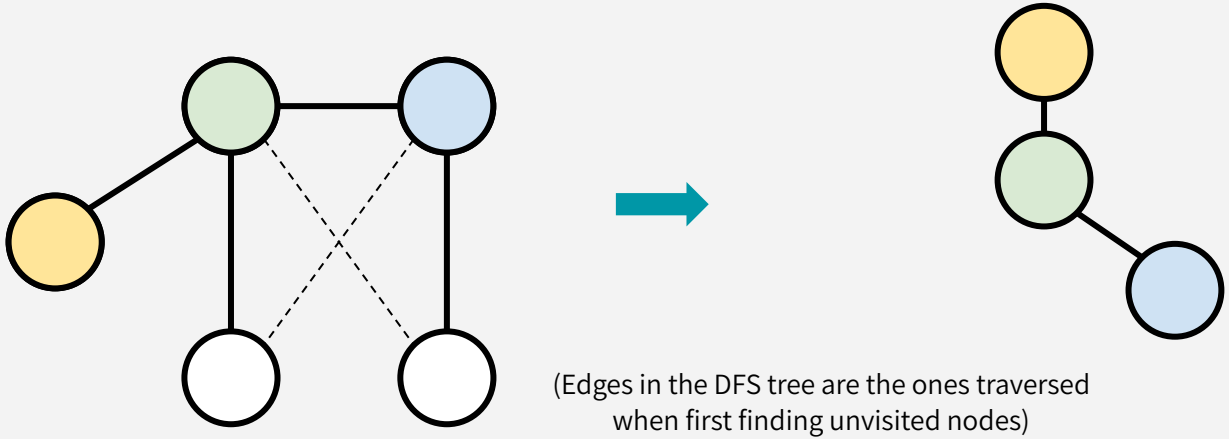

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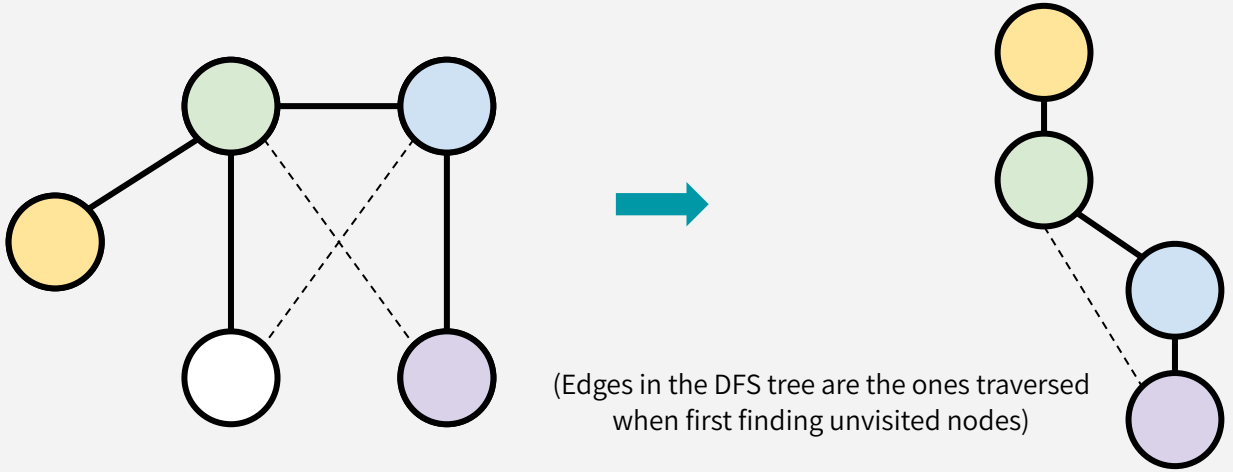

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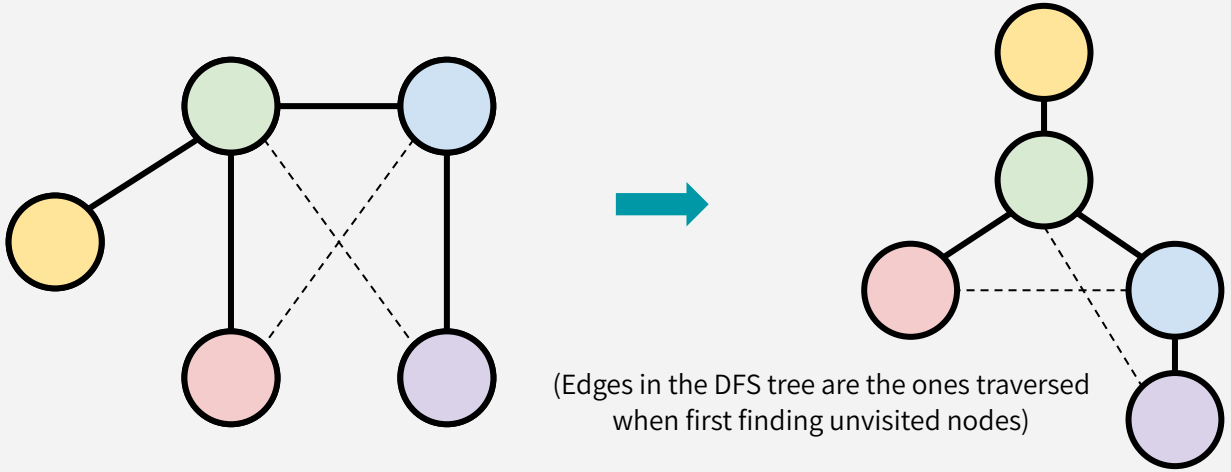

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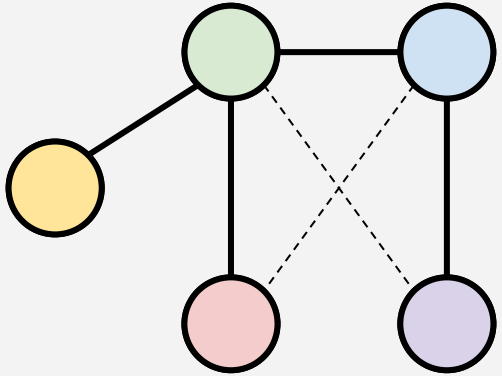

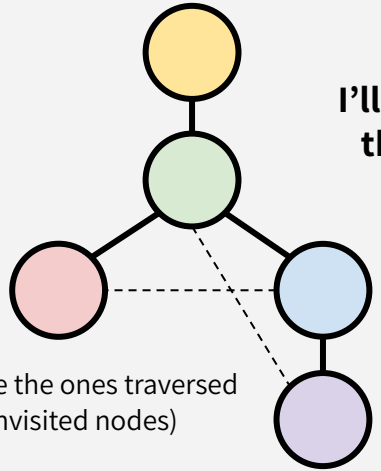

I'll refer to this as the "DFS tree"

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DEPTH-FIRST SEARCH: RUNTIME

To explore a graph's **i^{th} connected component** (n_i nodes, m_i edges):

We visit each vertex in the CC exactly once ("visit" = "call DFS on").
At each vertex v, we:

- Do some bookkeeping: **O(1)**
- Loop over v's neighbors & check if they are visited (& then potentially make a recursive call): O(1) per neighbor → **O(deg(v))** total.

Total: $\sum_v O(\deg(v)) + \sum_v O(1) = \mathbf{O(m_i + n_i)}$

DEPTH-FIRST SEARCH: RUNTIME

To explore **the entire graph** (n nodes, m edges):

A graph might have multiple connected components! To **explore the whole graph**, we would call our DFS routine once for each connected component (note that each vertex and each edge participates in exactly one connected component). The combined running time would be:

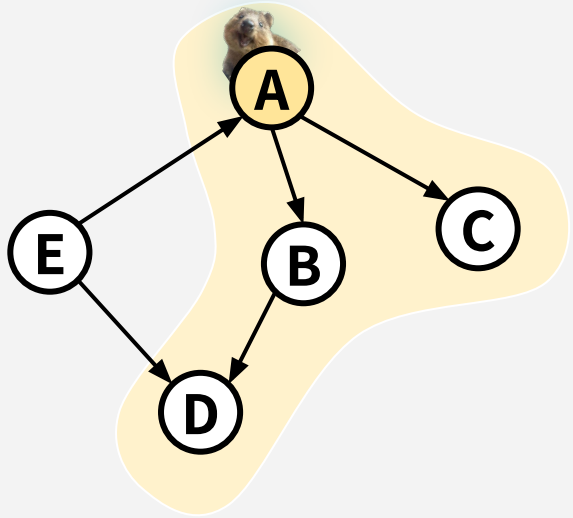
$$O(\sum_i m_i + \sum_i n_i) = \mathbf{O(m + n)}$$

DEPTH-FIRST SEARCH

DFS works fine on directed graphs too!

From a start node x, DFS would find all nodes ***reachable*** from x.

(In directed graphs, "connected component" isn't as well defined… more on that later!)



Verify this on your own: running DFS from A would still find all nodes reachable from A (E isn't reachable from A in this directed graph).

سوال؟

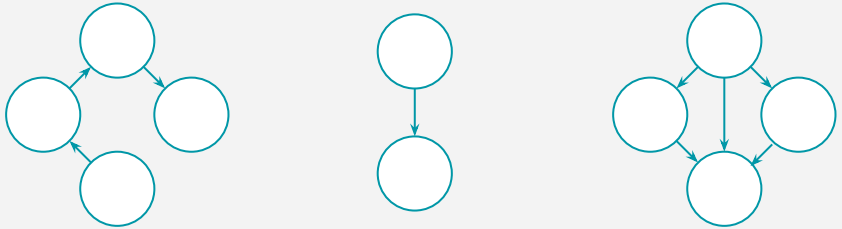
مرتب سازی توپولوژیکی

یک کاربرد از جستجوی عمق اول برای مسائل دارای پیش نیاز

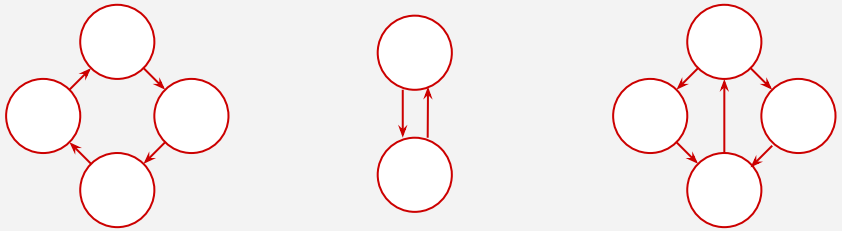
ASIDE: DIRECTED ACYCLIC GRAPHS

A **Directed Acyclic Graph (DAG)** is a directed graph with *no directed cycles*.

These are DAGs:



These are not DAGs:



TOPOLOGICAL SORTING: THE TASK

Given a DAG, find an ordering of vertices so that all of the dependency requirements are met

Example applications:

Given a package dependency graph, in what order should packages be installed?

Given a course prerequisites graph, in what order should we take classes?

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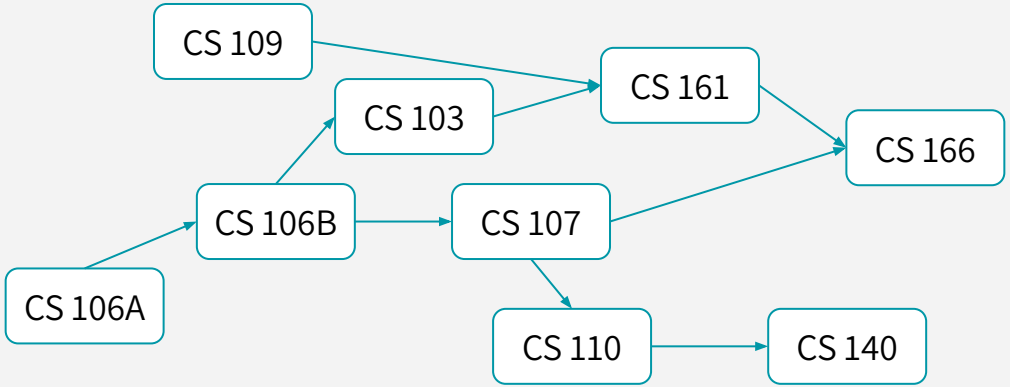

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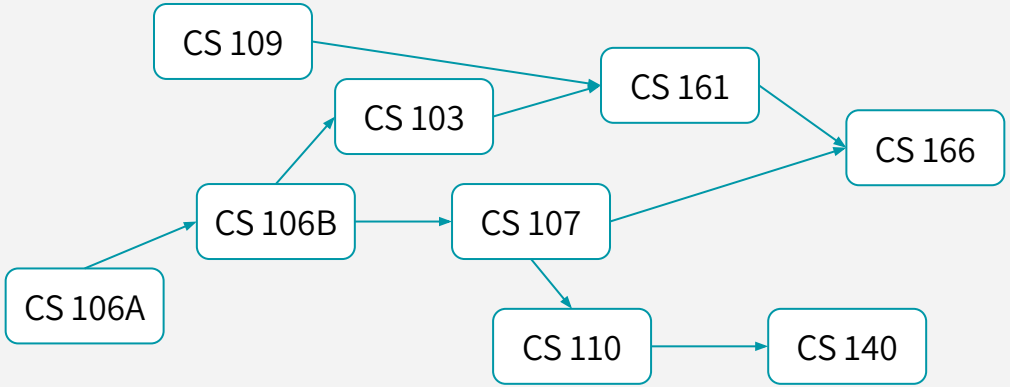

This prerequisite graph is a DAG!
(directed & acyclic)

TOPOLOGICAL SORTING: THE TASK

Given a DAG, find an ordering of vertices so that all of the dependency requirements are met

What does “meeting the dependency requirements” mean?

We want to produce an ordering such that:
for every edge **(v, w)** in E, **v** must appear before **w** in the ordering
(e.g. CS103 must come before CS161)

“d
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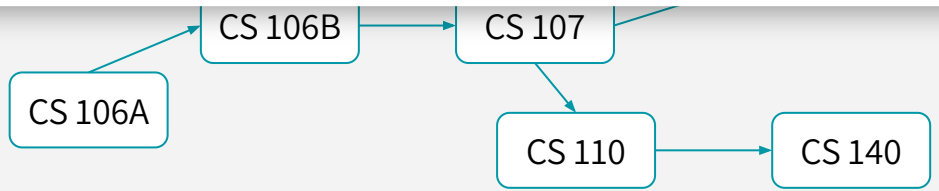

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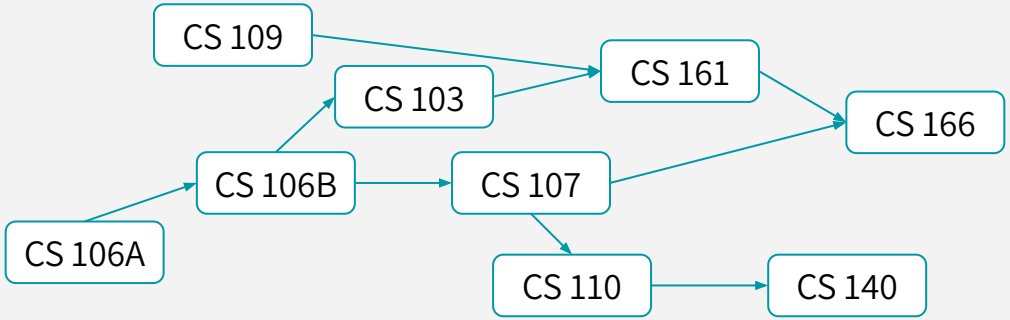

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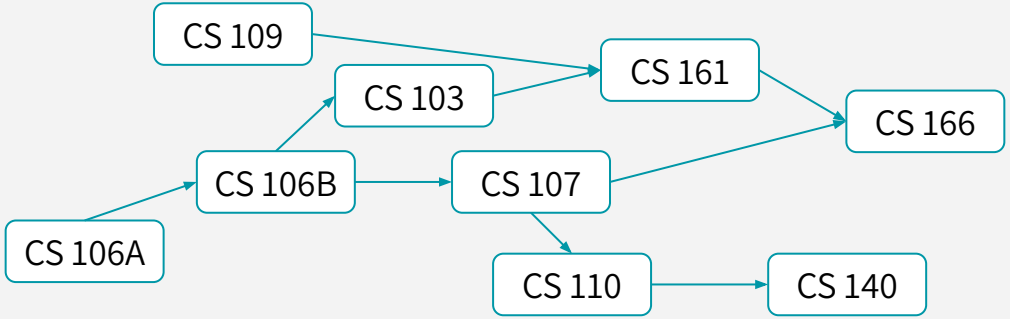

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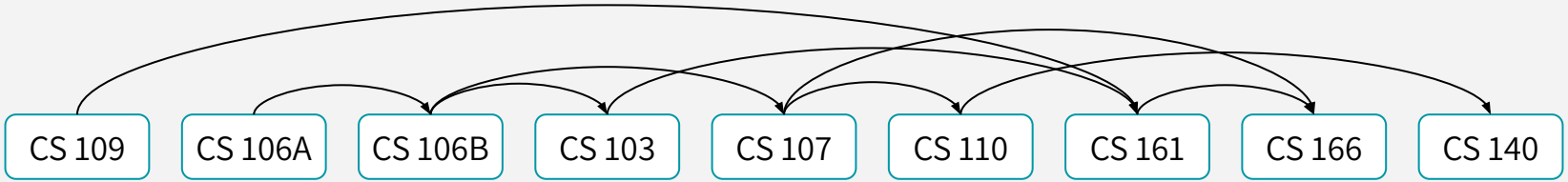

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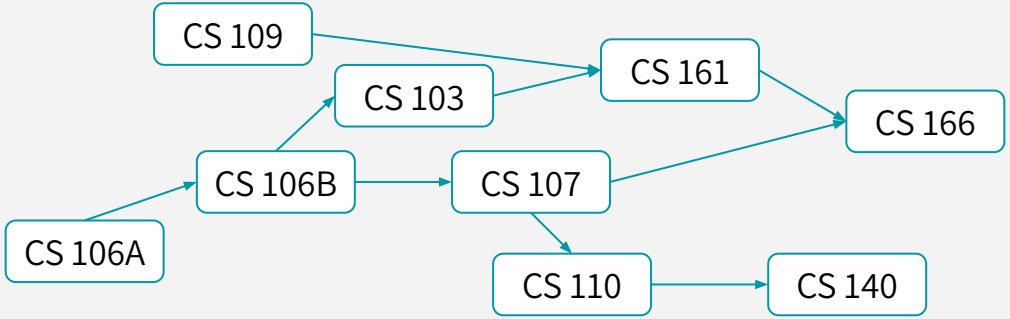
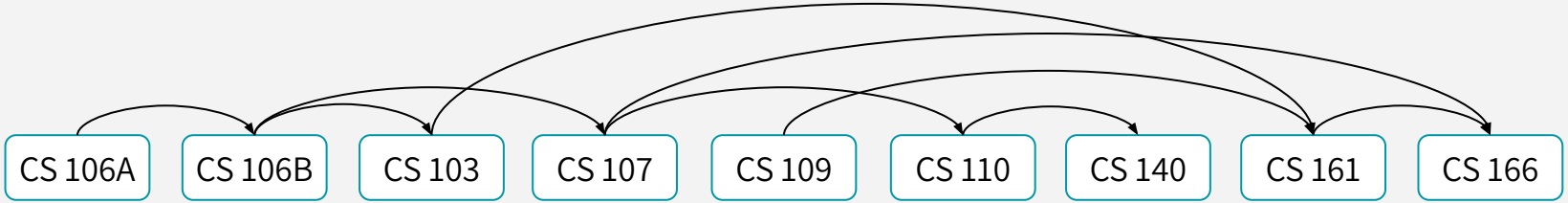


Also a correct toposort of this DAG:



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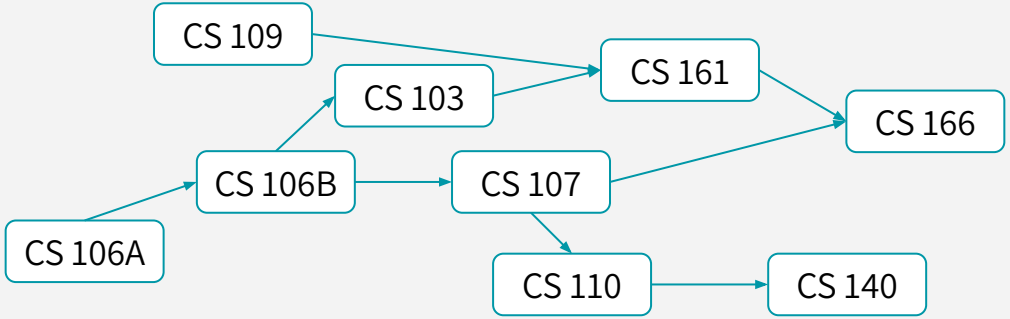
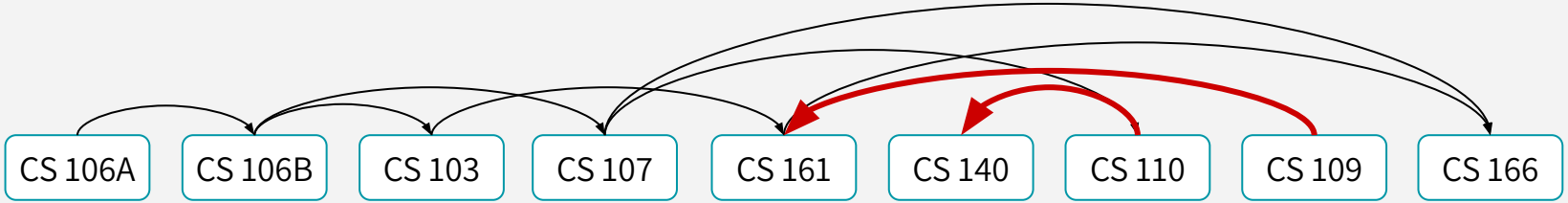


Not a correct toposort of this DAG:



TOPOSORT ON NON-DAGS?

We assume these "dependency" graphs are all DAGs!

What about other graphs? Undirected graphs? Directed graphs with cycles?

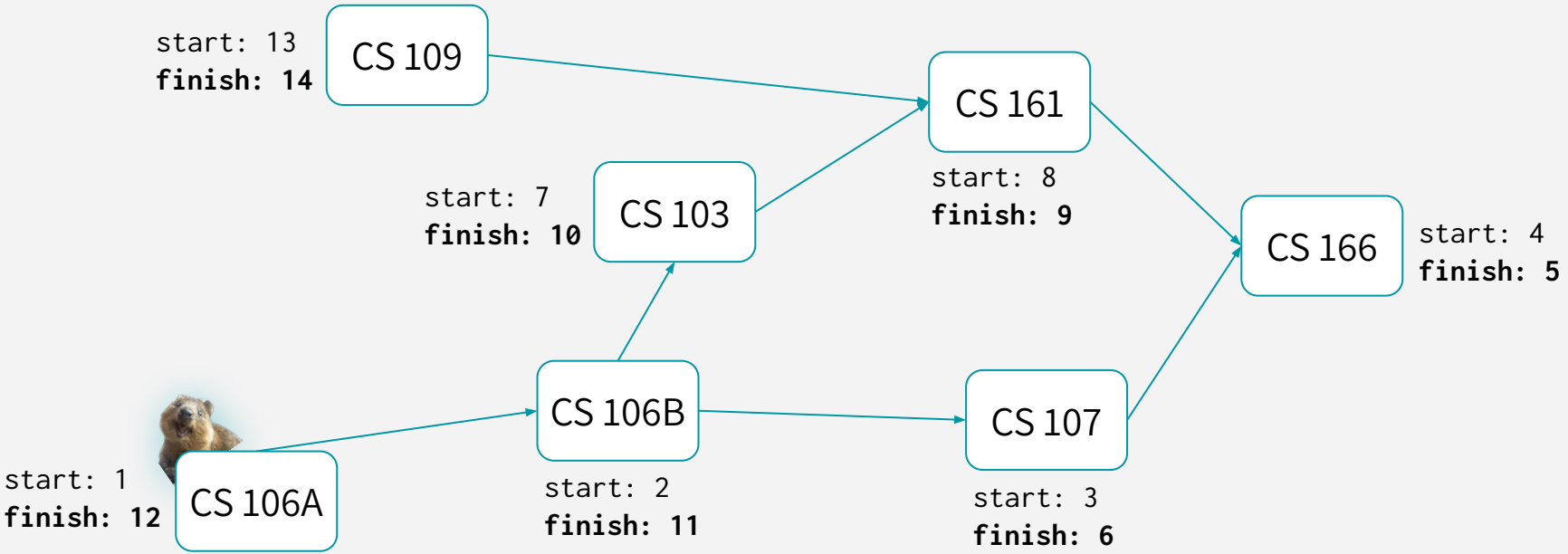
Toposort gives us a priority ordering of nodes (e.g. more intro classes are "higher priority" than more advanced classes). Edges in DAGs clearly illustrate priority: edge from **x** to **y** means **x** has priority over **y**.

In an undirected graph, if there's an **x-y** edge, which node has "priority"?

In a graph with cycles, if **x** and **y** are part of a cycle, then **x** can reach **y** and **y** can also reach **x**… so which node has "priority"?

DFS WILL GET US A TOPOSORT

Let's run DFS. What do you notice about the finish times? What does it have to do with toposort?



DFS WILL GET US A TOPOSORT

Let's run DFS. What do you notice about the finish times? What does it have to do with toposort?

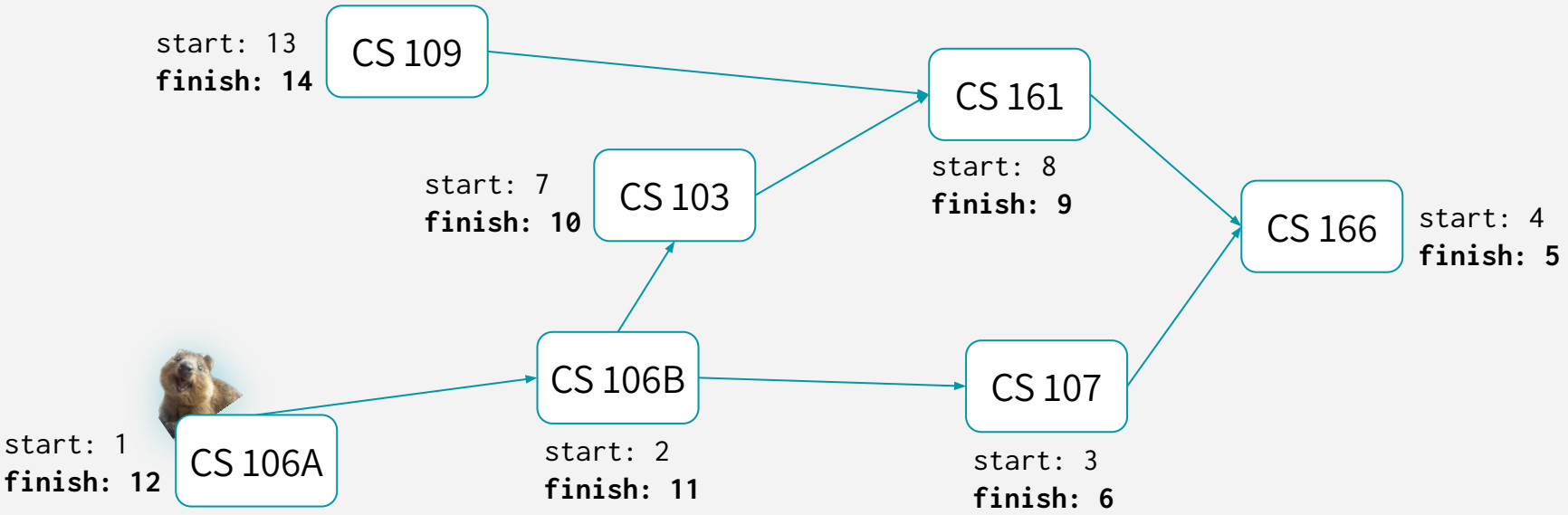
CLAIM: In general, if there's an edge from **v** → **w**, **v**'s finish time will be *larger* than **w**'s finish time

Let's consider two cases: (1) DFS visits **v** first, or (2) DFS visits **w** first.

DFS WILL GET US A TOPOSORT

CLAIM: In general, if there's an edge from **v** → **w**, **v**'s finish time will be *larger* than **w**'s finish time

CASE 1: **v** → **w**, and **v** **is discovered first** by DFS



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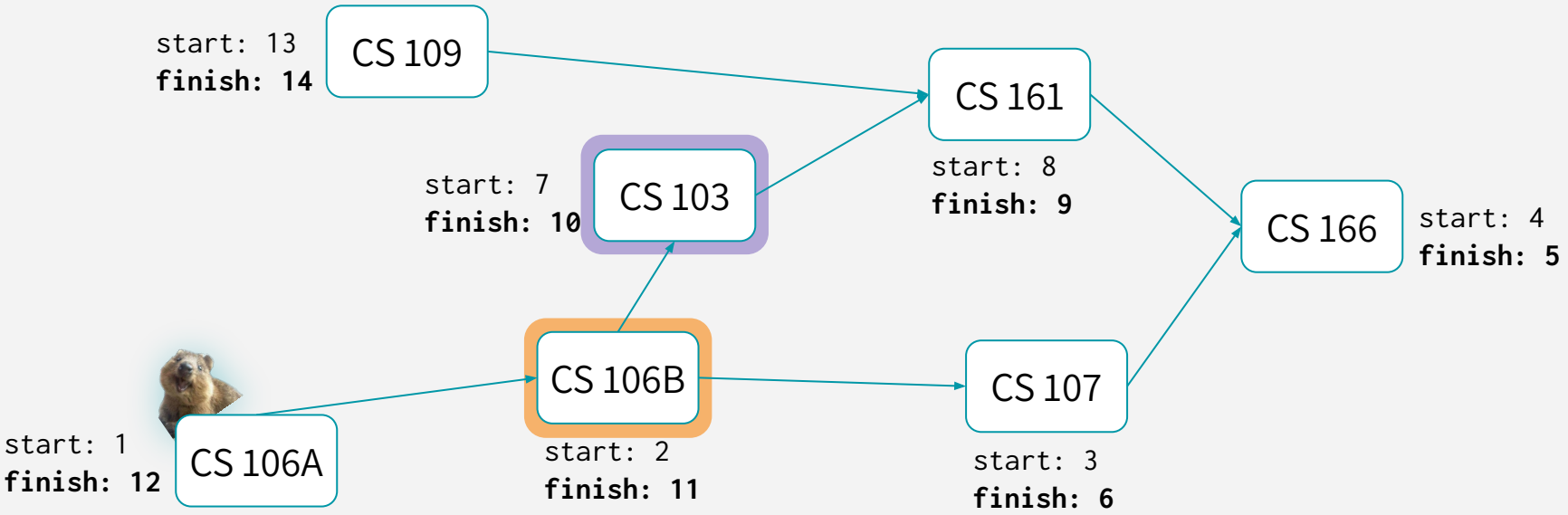

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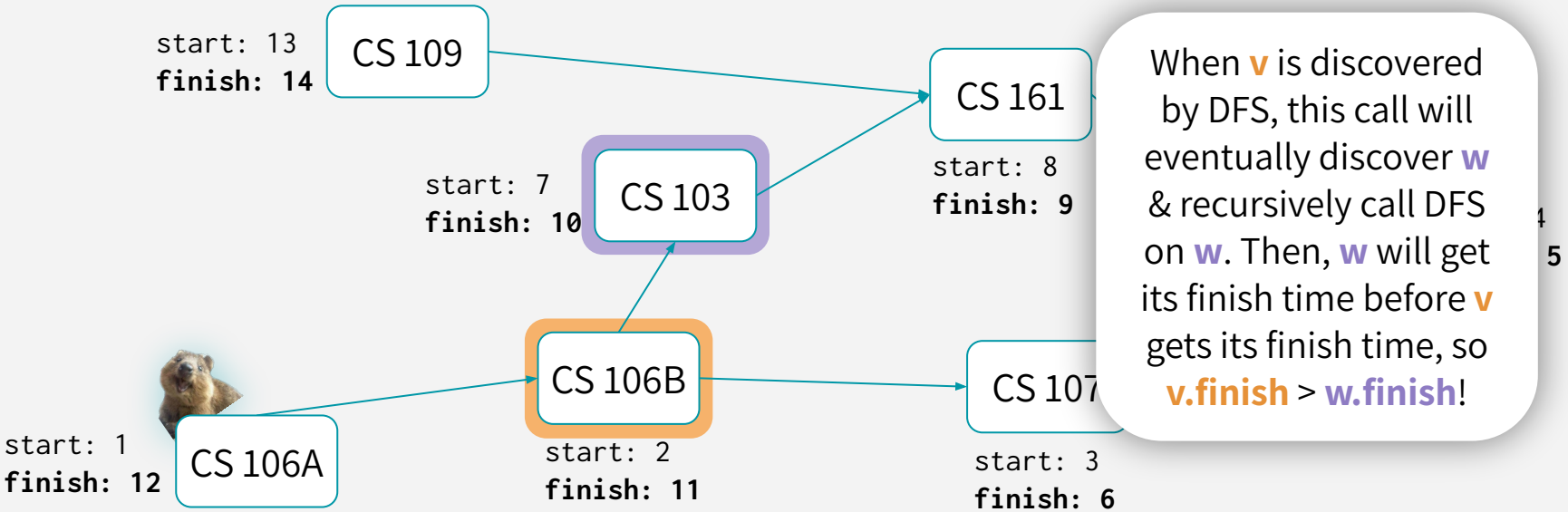

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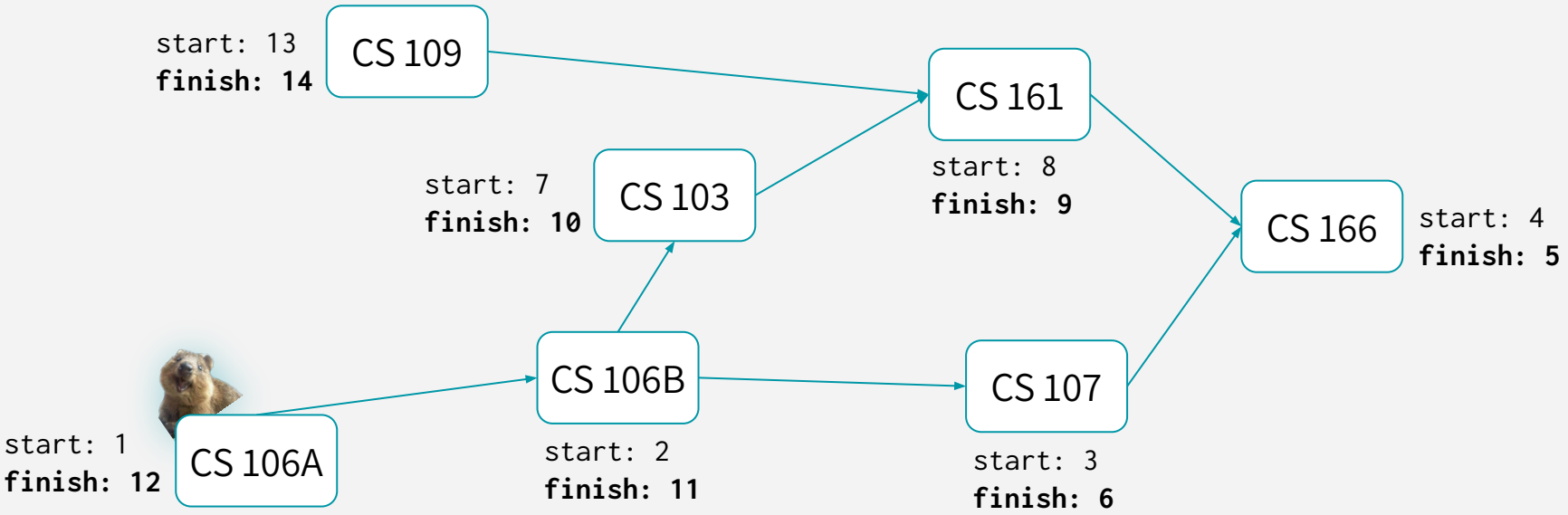
CASE 2: **v** → **w**, and **w** **is discovered first** by DFS



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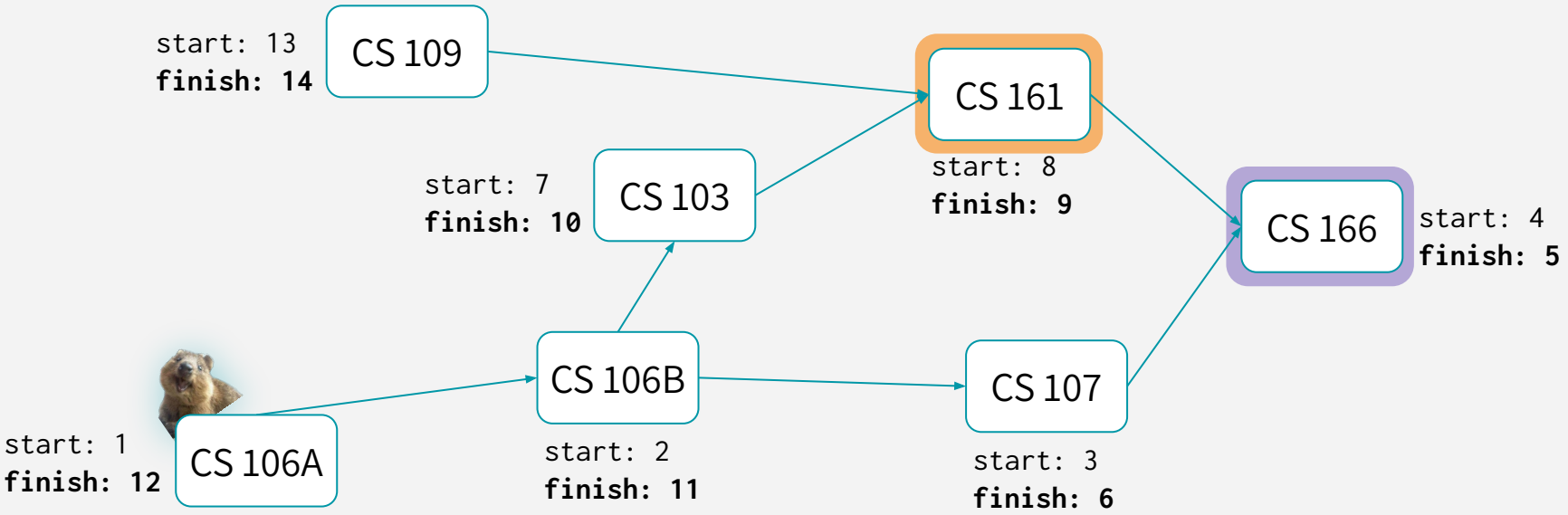
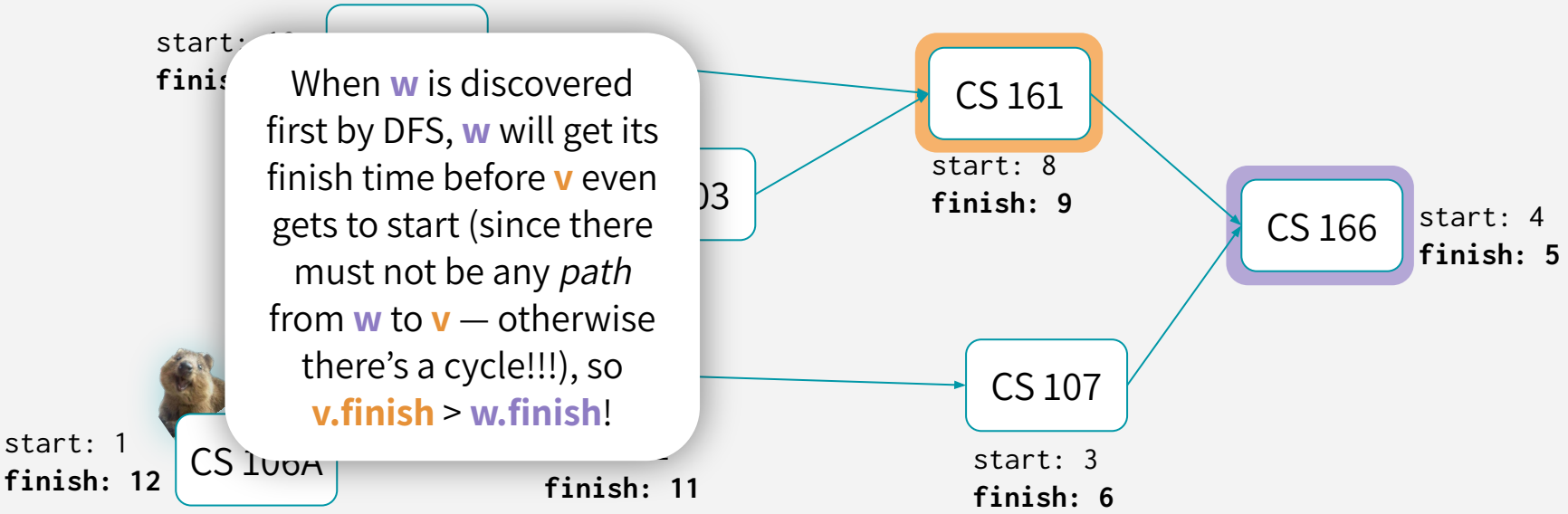


DFS WILL GET US A TOPOSORT

CLAIM: In general, if there's an edge from **v** → **w**, **v**'s finish time will be *larger* than **w**'s finish time

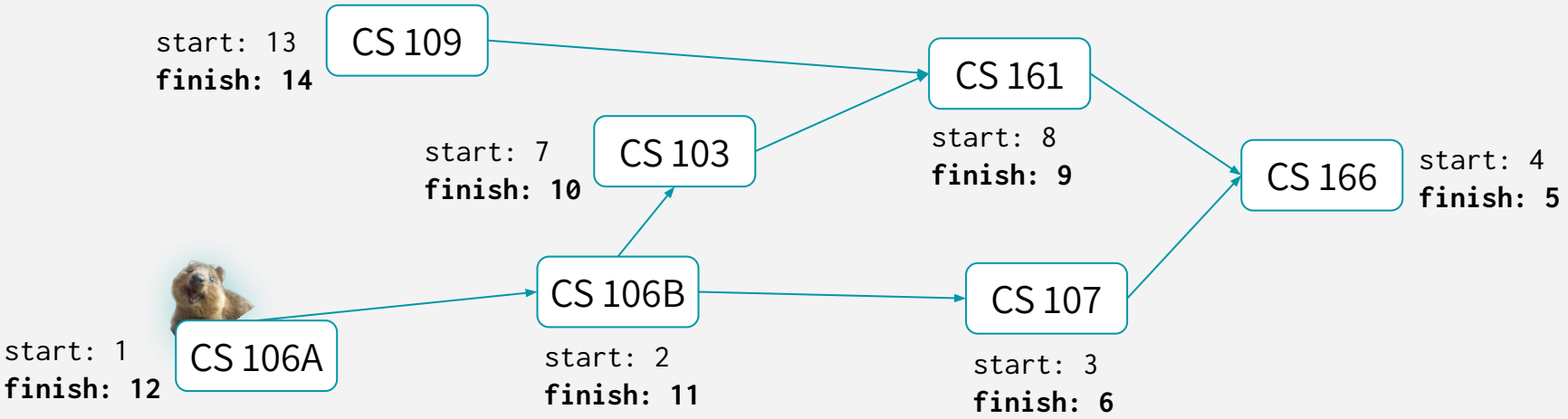
CASE 2: **v** → **w**, and **w** **is discovered first** by DFS

When **w** is discovered first by DFS, **w** will get its finish time before **v** even gets to start (since there must not be any *path* from **w** to **v** — otherwise there's a cycle!!!), so **v.finish** > **w.finish**!



DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



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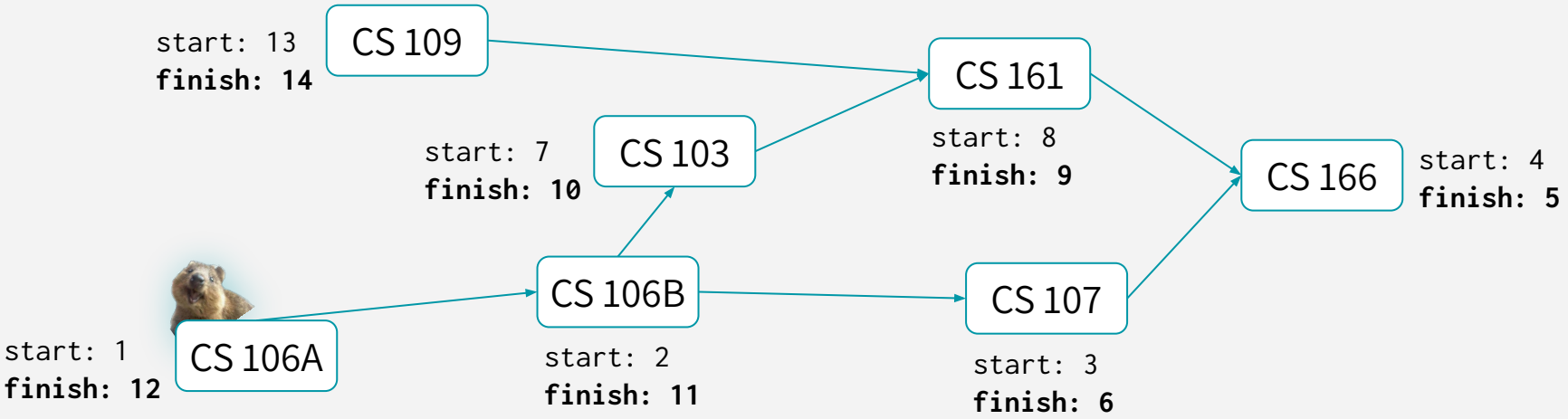

CS 166
f: 5

DFS WILL GET US A TOPOSORT

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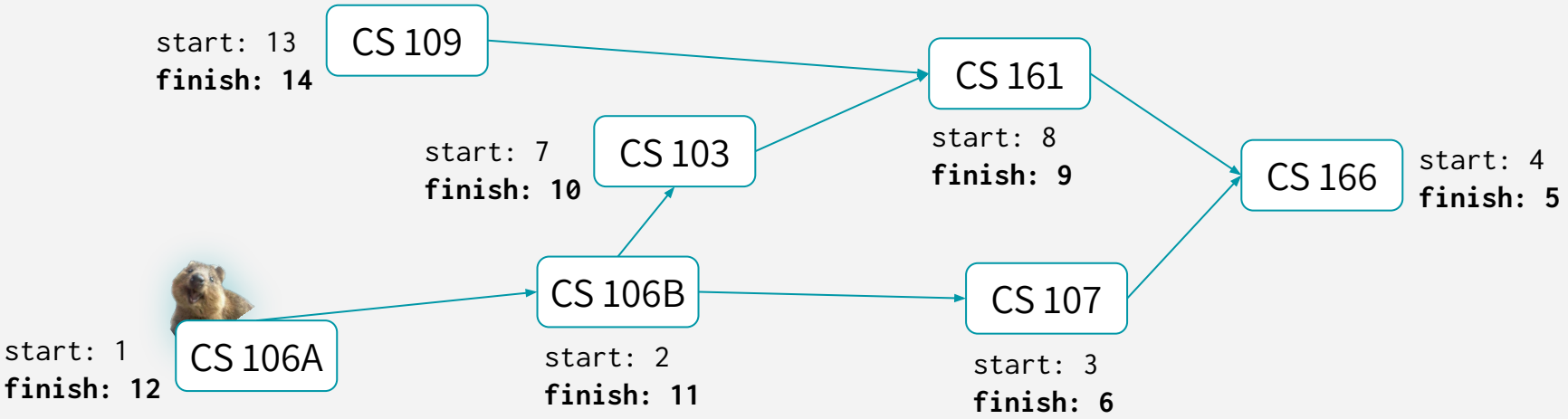

CS 107 **f: 6**

CS 166 **f: 5**

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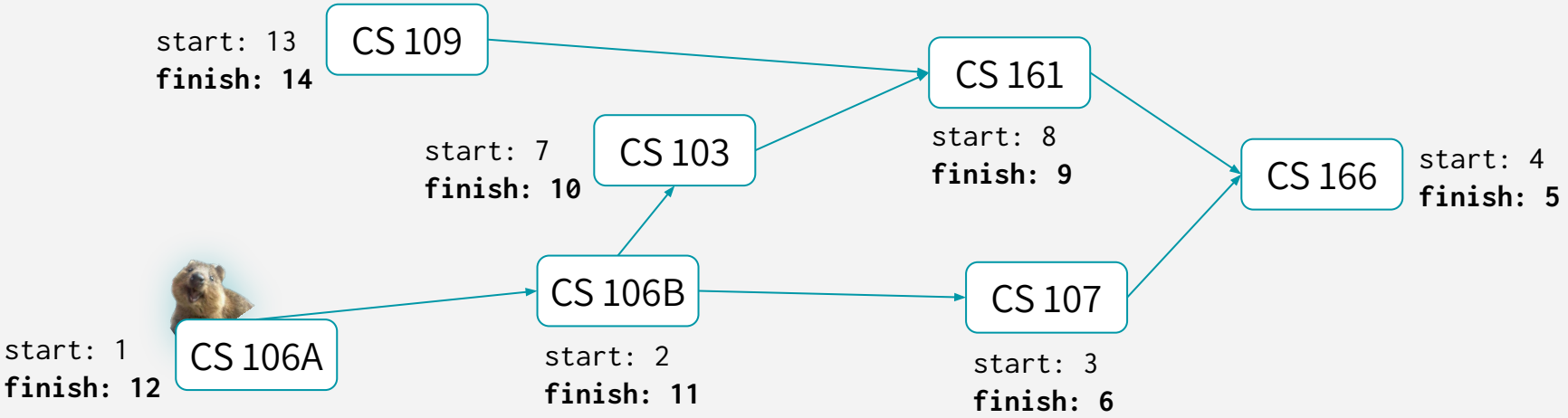
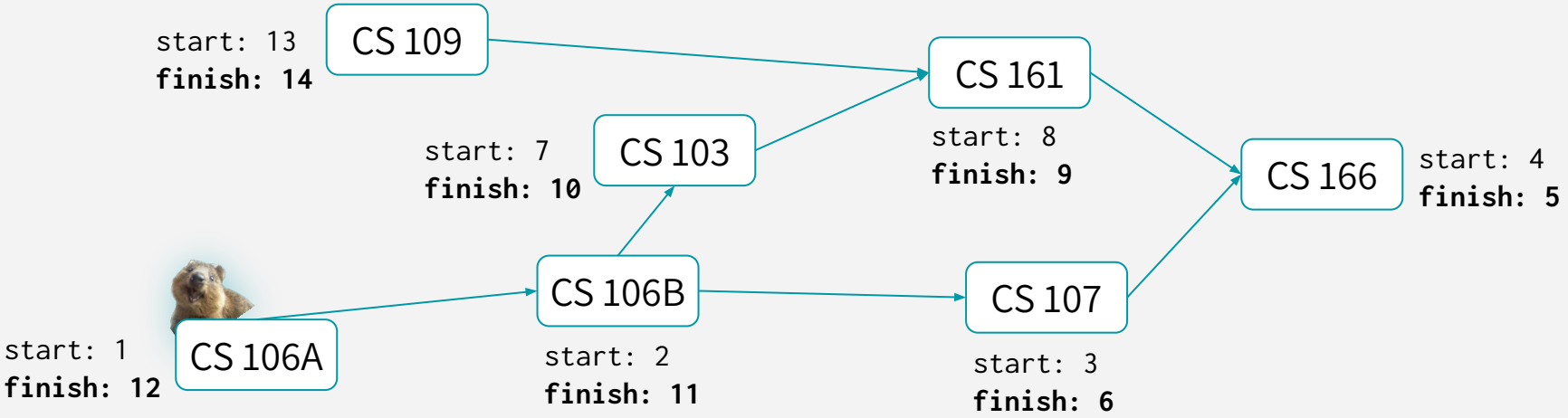


CS 161 f: 9 | CS 107 f: 6 | CS 166 f: 5

DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



CS 103 **f: 10** | CS 161 **f: 9** | CS 107 **f: 6** | CS 166 **f: 5**

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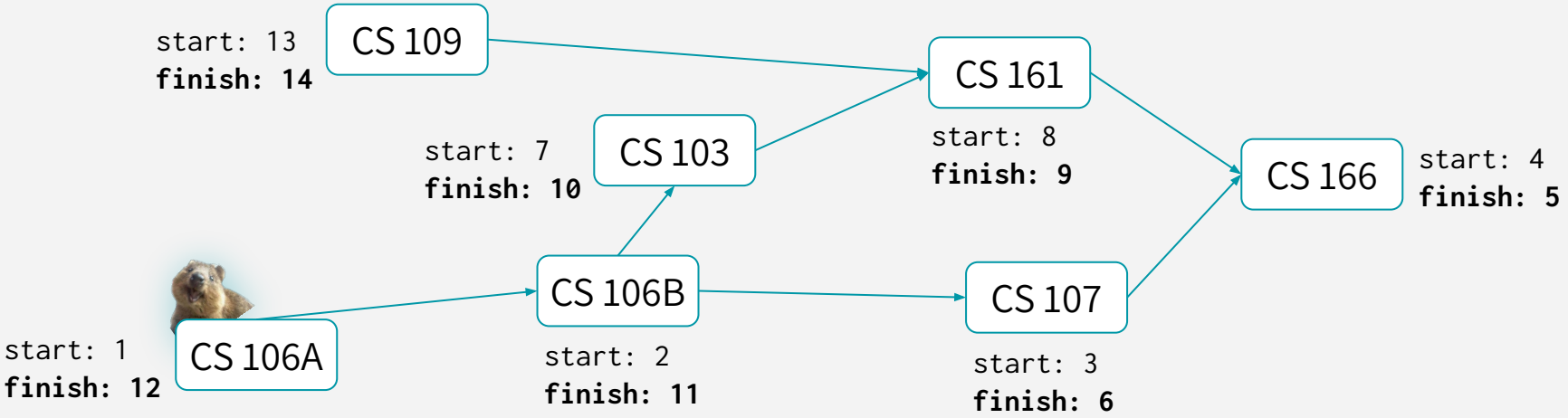

CS 106B **f: 11** | CS 103 **f: 10** | CS 161 **f: 9** | CS 107 **f: 6** | CS 166 **f: 5**

DFS WILL GET US A TOPOSORT

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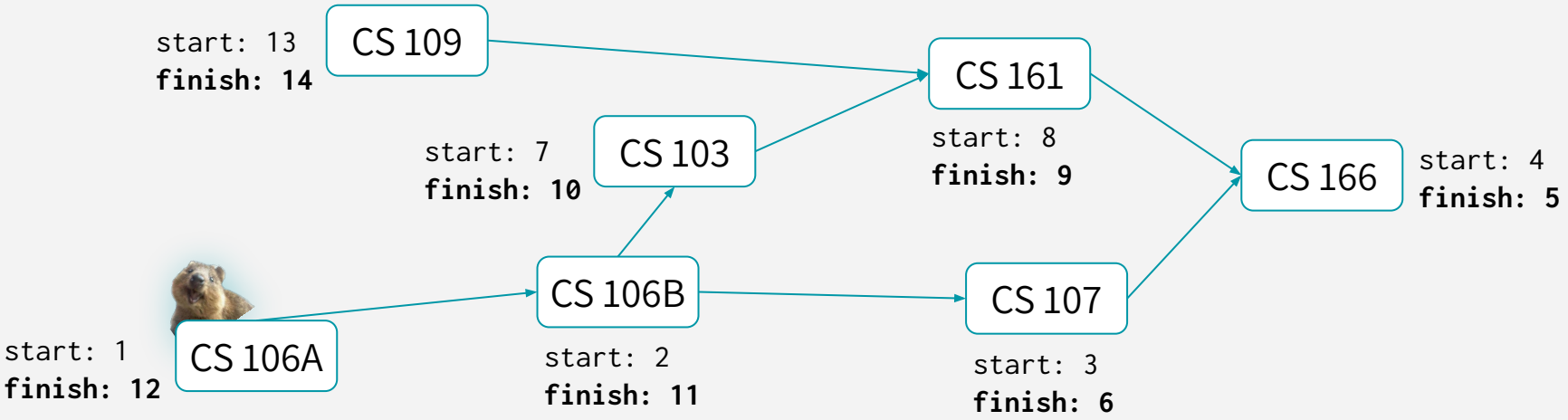

CS 106A **f: 12** | CS 106B **f: 11** | CS 103 **f: 10** | CS 161 **f: 9** | CS 107 **f: 6** | CS 166 **f: 5**

DFS WILL GET US A TOPOSORT

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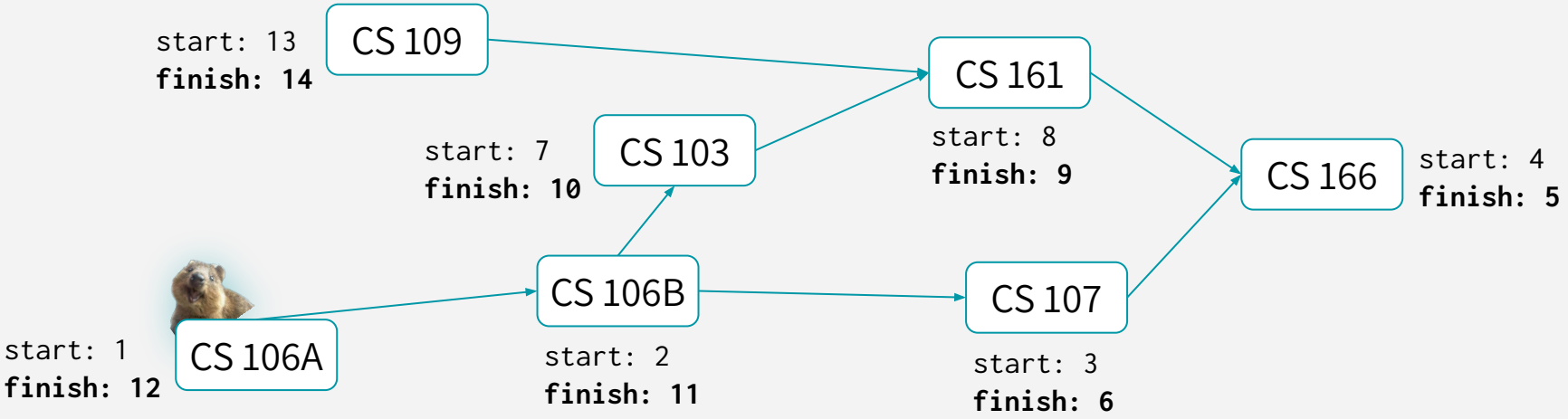

CS 109 **f: 14** | CS 106A **f: 12** | CS 106B **f: 11** | CS 103 **f: 10** | CS 161 **f: 9** | CS 107 **f: 6** | CS 166 **f: 5**

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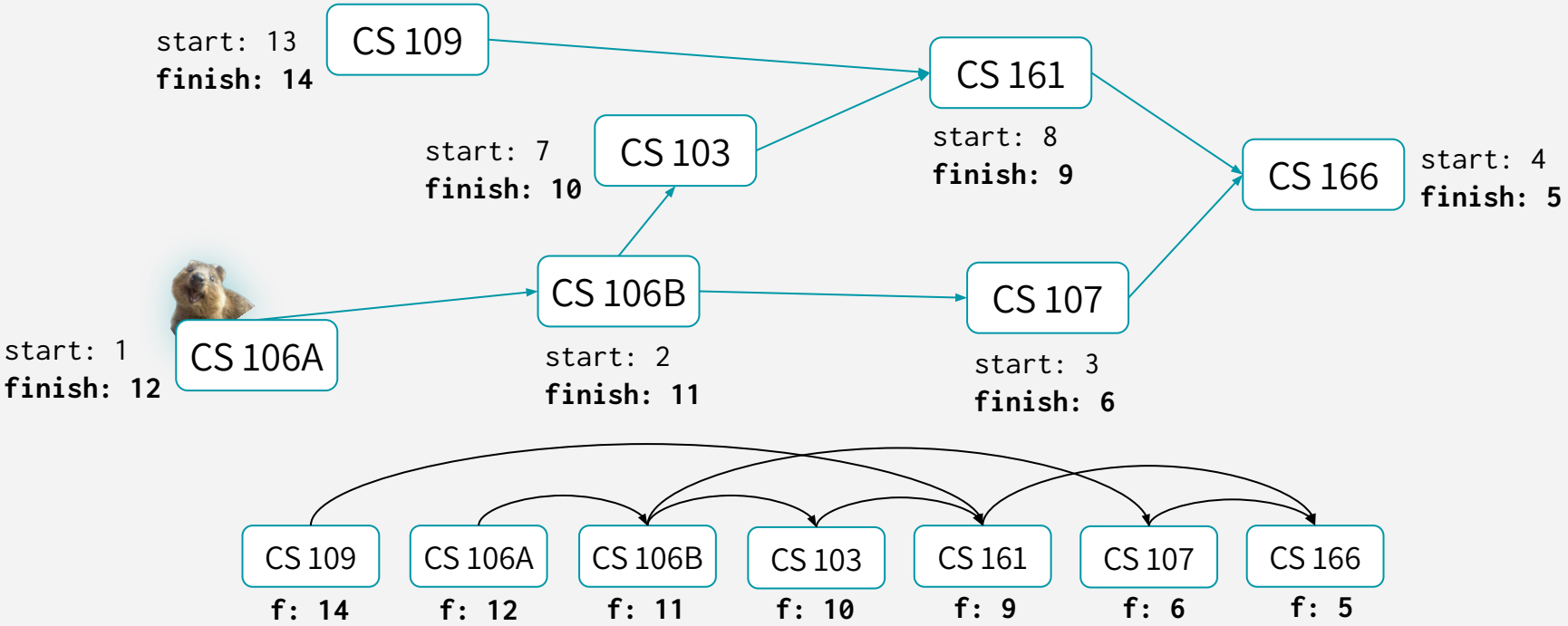

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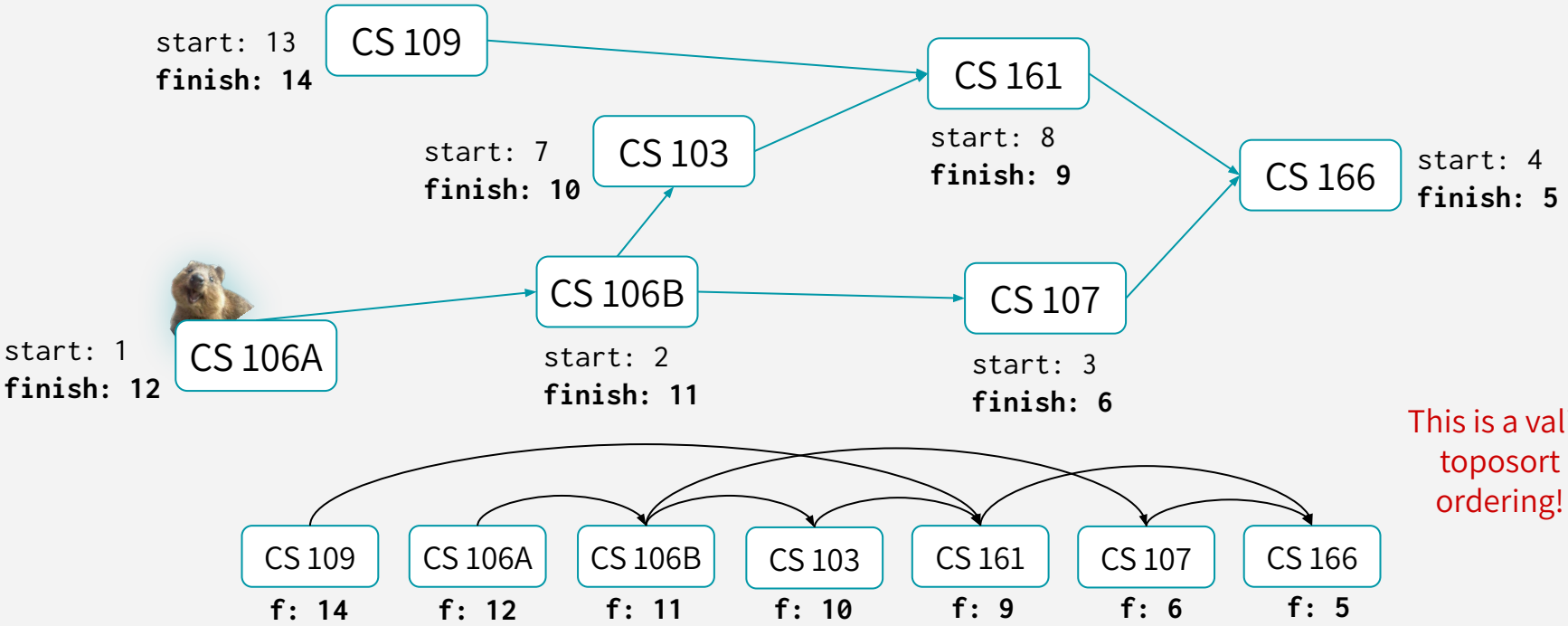
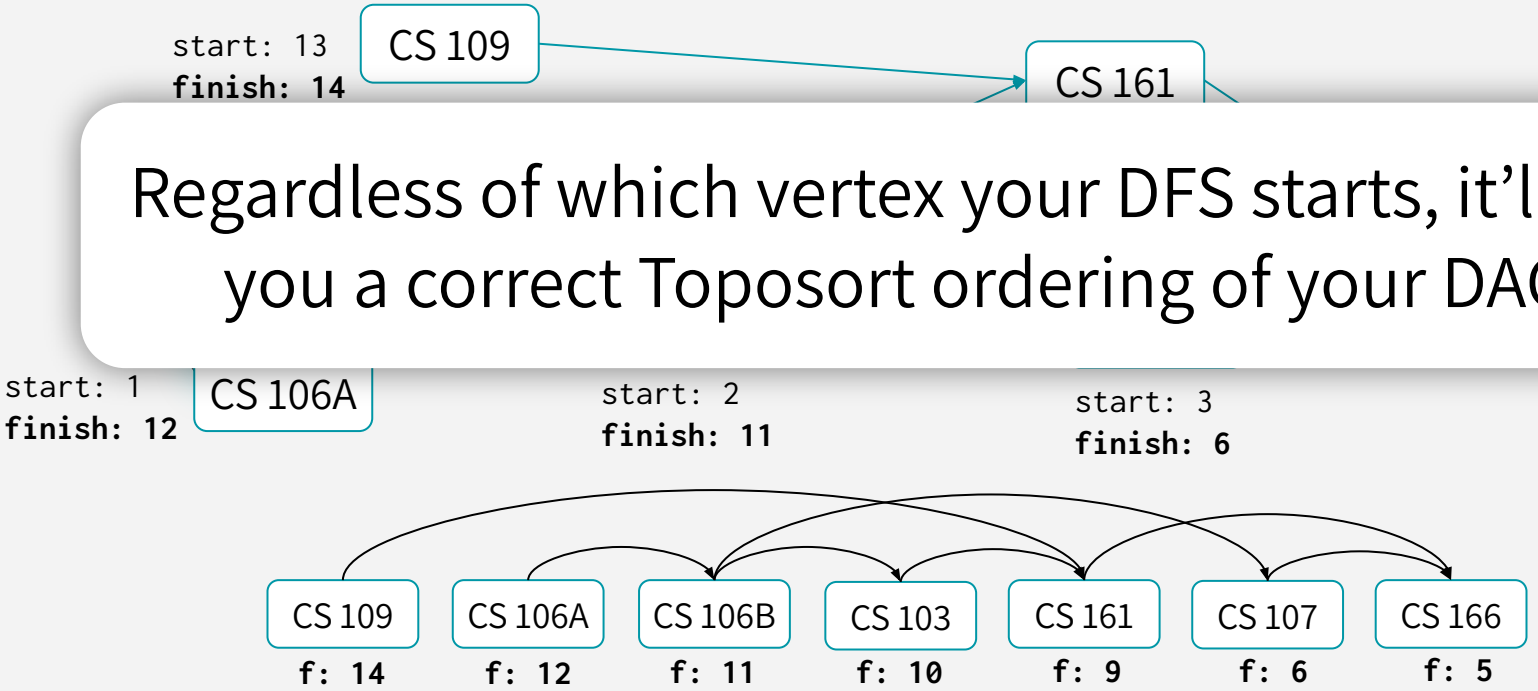


This is a valid toposort ordering!

DFS WILL GET US A TOPOSORT

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

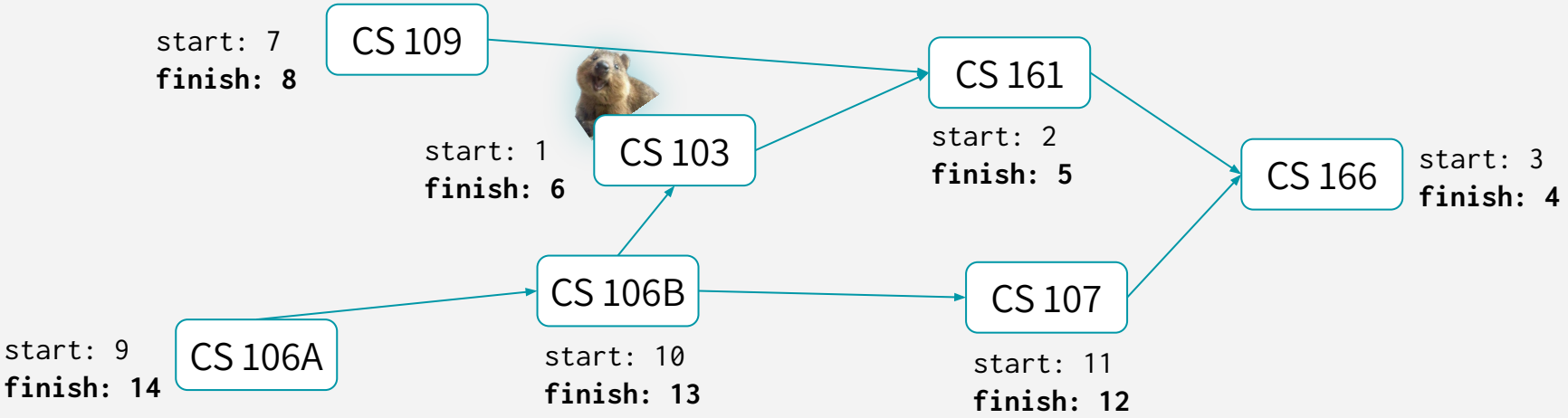


Regardless of which vertex your DFS starts, it'll get you a correct Toposort ordering of your DAG

DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



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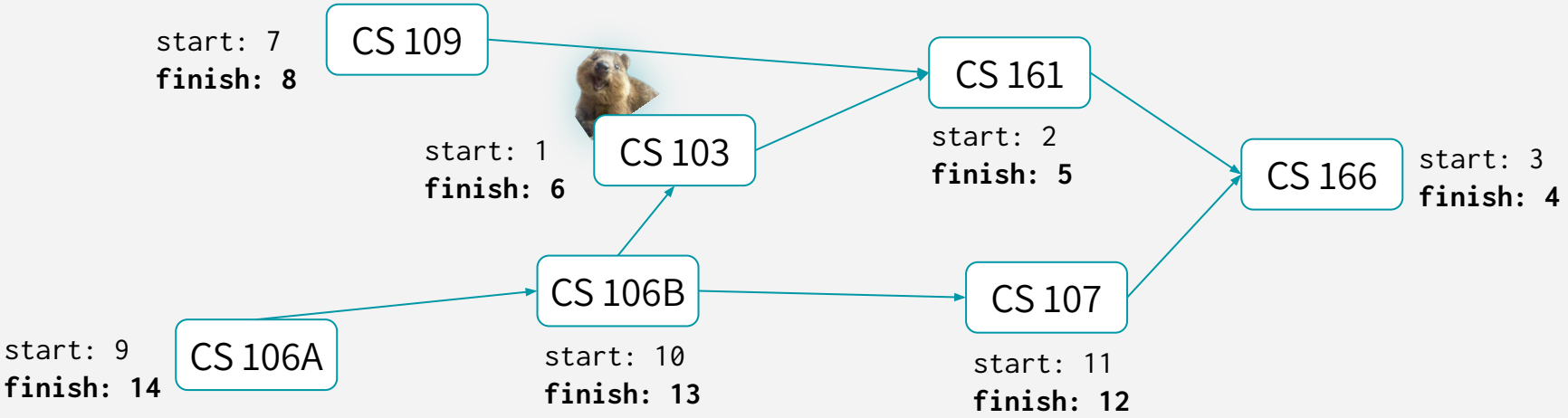
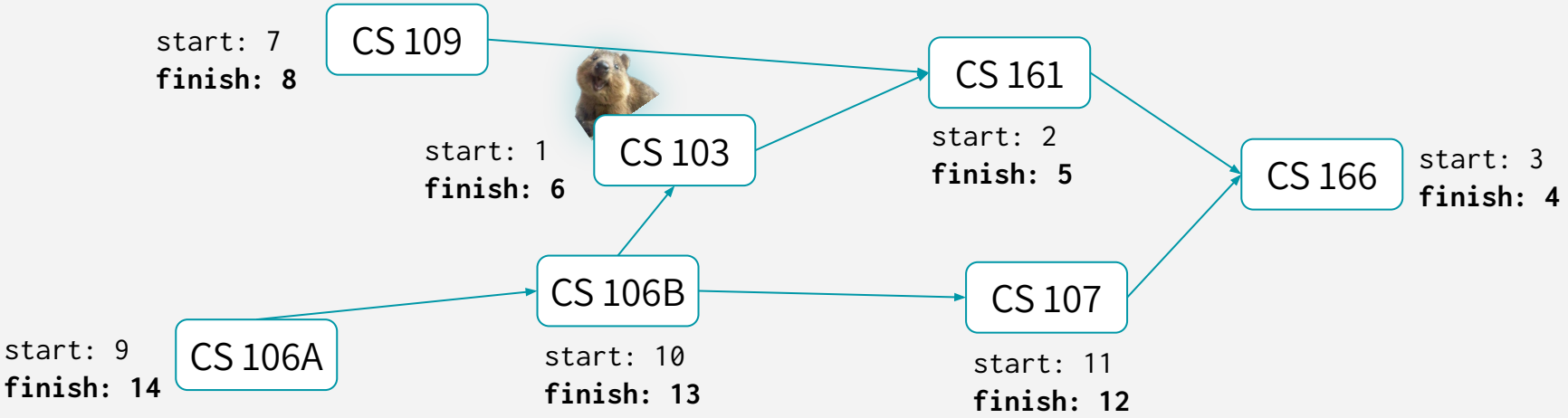


CS 166
f: 4

DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



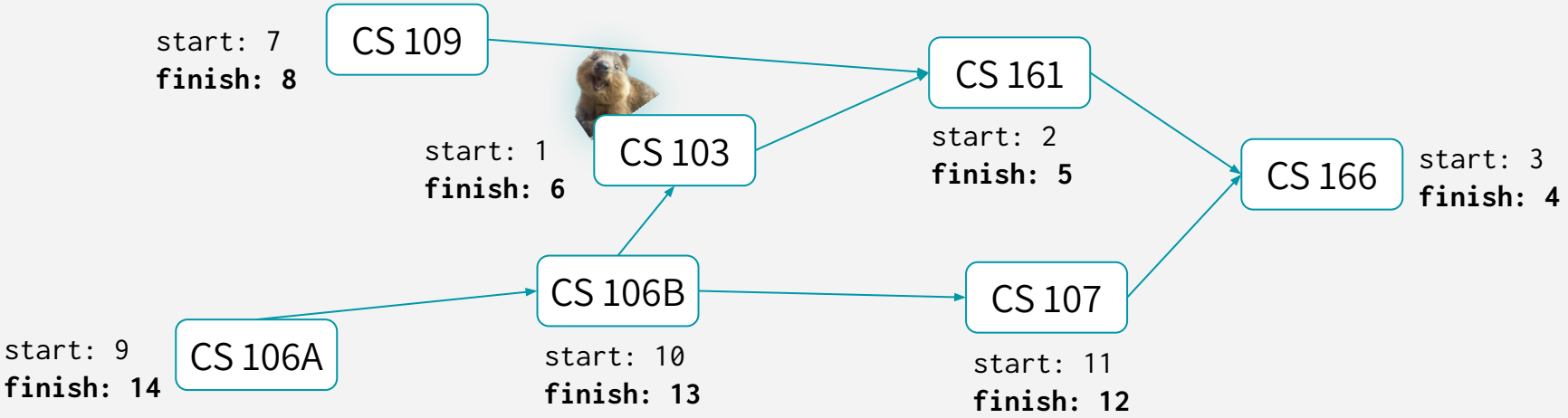
CS 161 **f: 5**

CS 166 **f: 4**

DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.

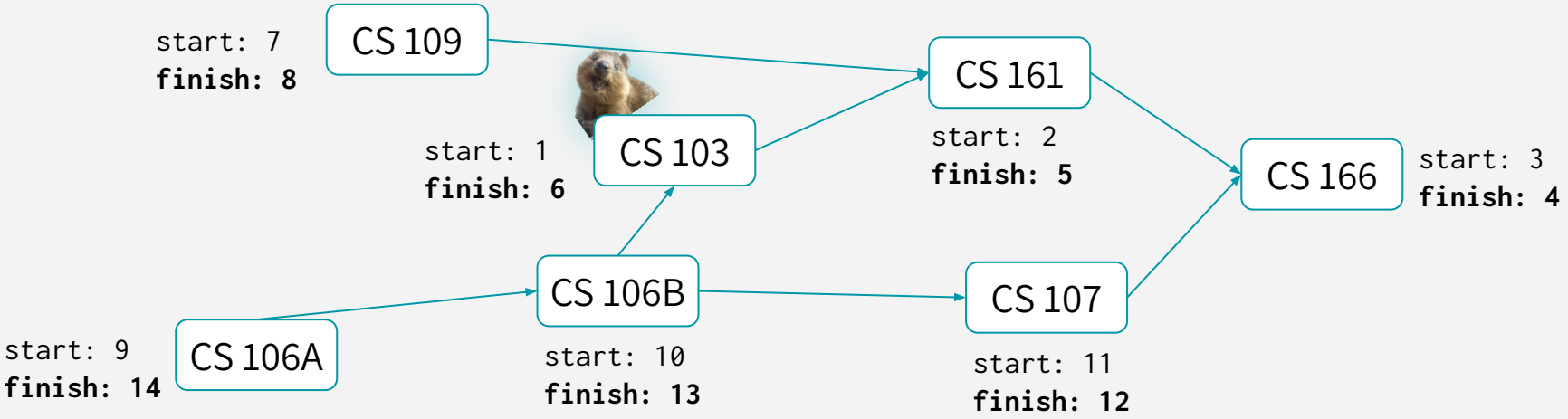


CS 103 **f: 6** CS 161 **f: 5** CS 166 **f: 4**

DFS WILL GET US A TOPOSORT

Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



CS 109 **f: 8** | CS 103 **f: 6** | CS 161 **f: 5** | CS 166 **f: 4**

DFS WILL GET US A TOPOSORT

Different order of running

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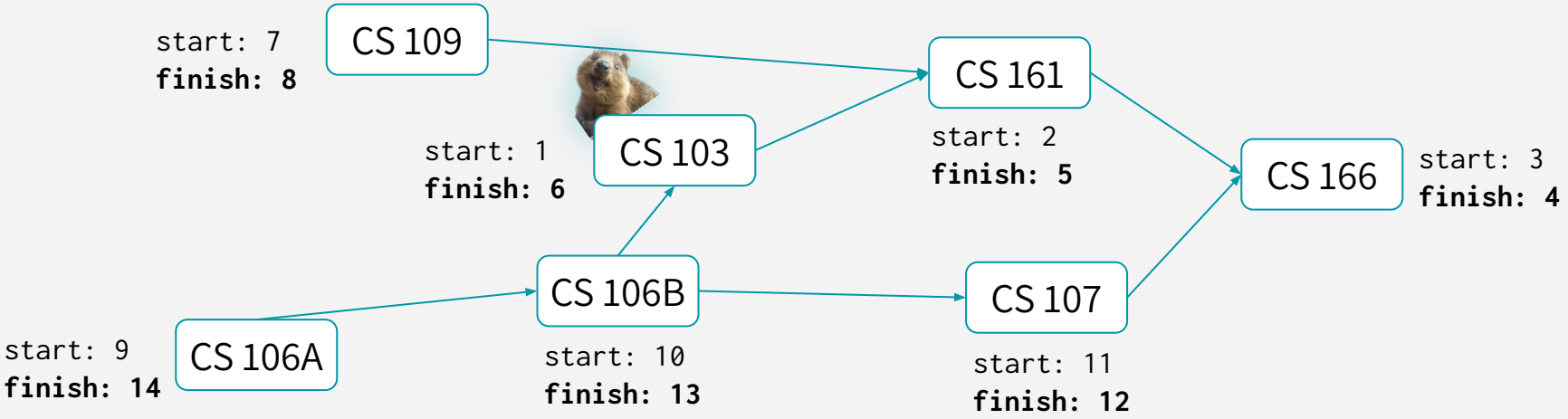

CS 107 **f: 12** | CS 109 **f: 8** | CS 103 **f: 6** | CS 161 **f: 5** | CS 166 **f: 4**

DFS WILL GET US A TOPOSORT

Different order of running

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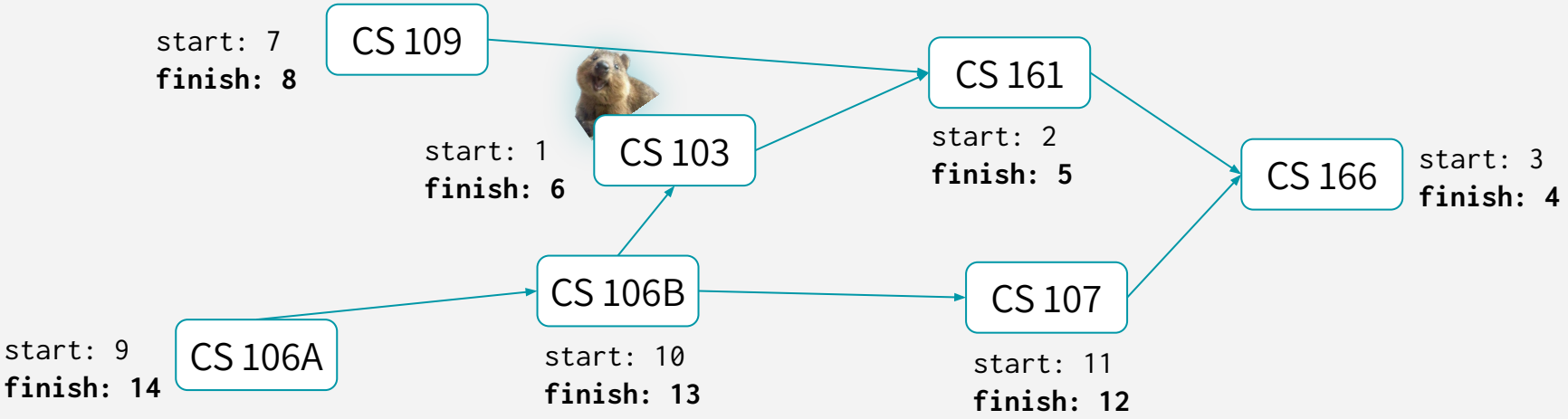

CS 106B	CS 107	CS 109	CS 103	CS 161	CS 166
f: 13	f: 12	f: 8	f: 6	f: 5	f: 4

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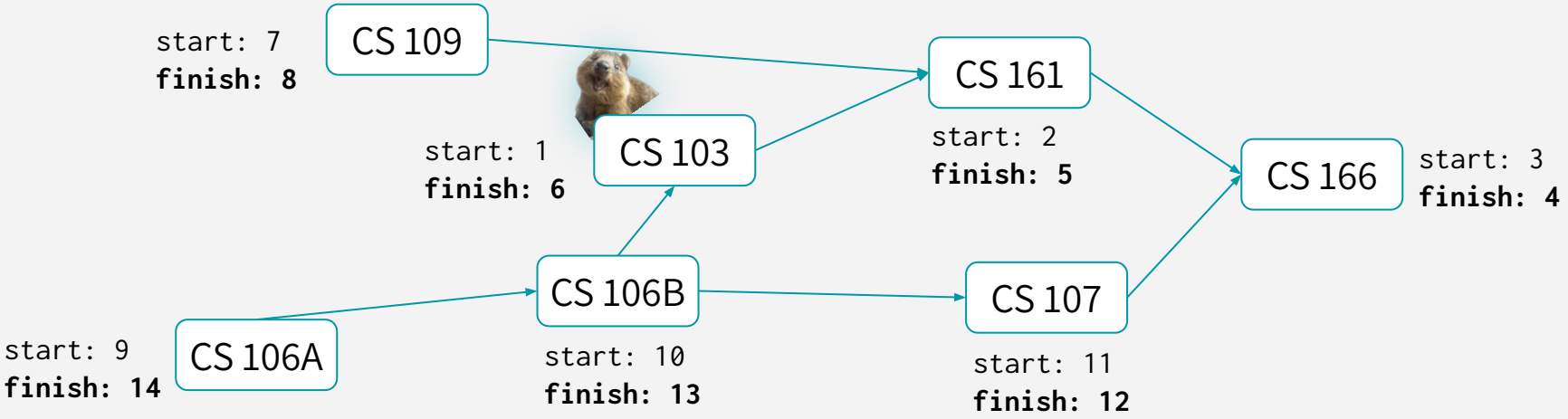
Different order of running

TOPOSORT: Perform DFS. When a vertex gets its finish time, insert it at the start of the list.



CS 106A **f: 14** | CS 106B **f: 13** | CS 107 **f: 12** | CS 109 **f: 8** | CS 103 **f: 6** | CS 161 **f: 5** | CS 166 **f: 4**

DFS WILL GET US A TOPOSORT

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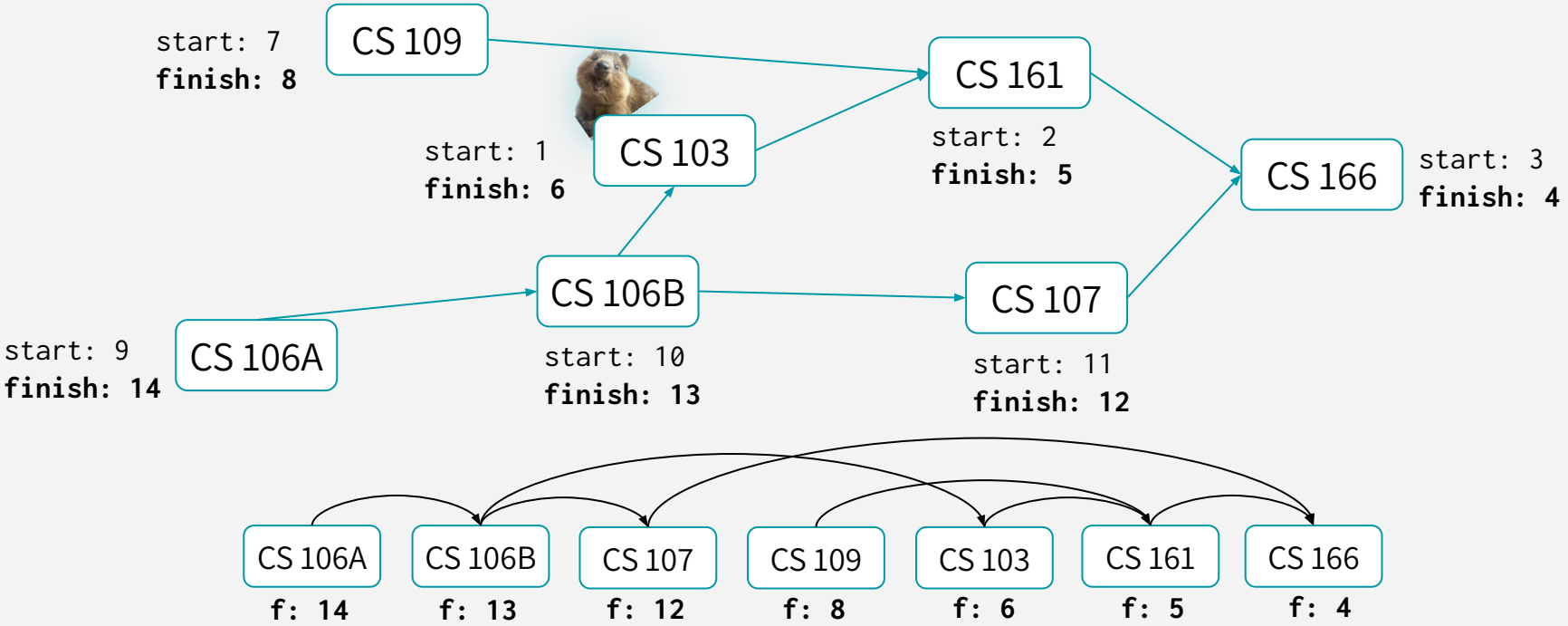

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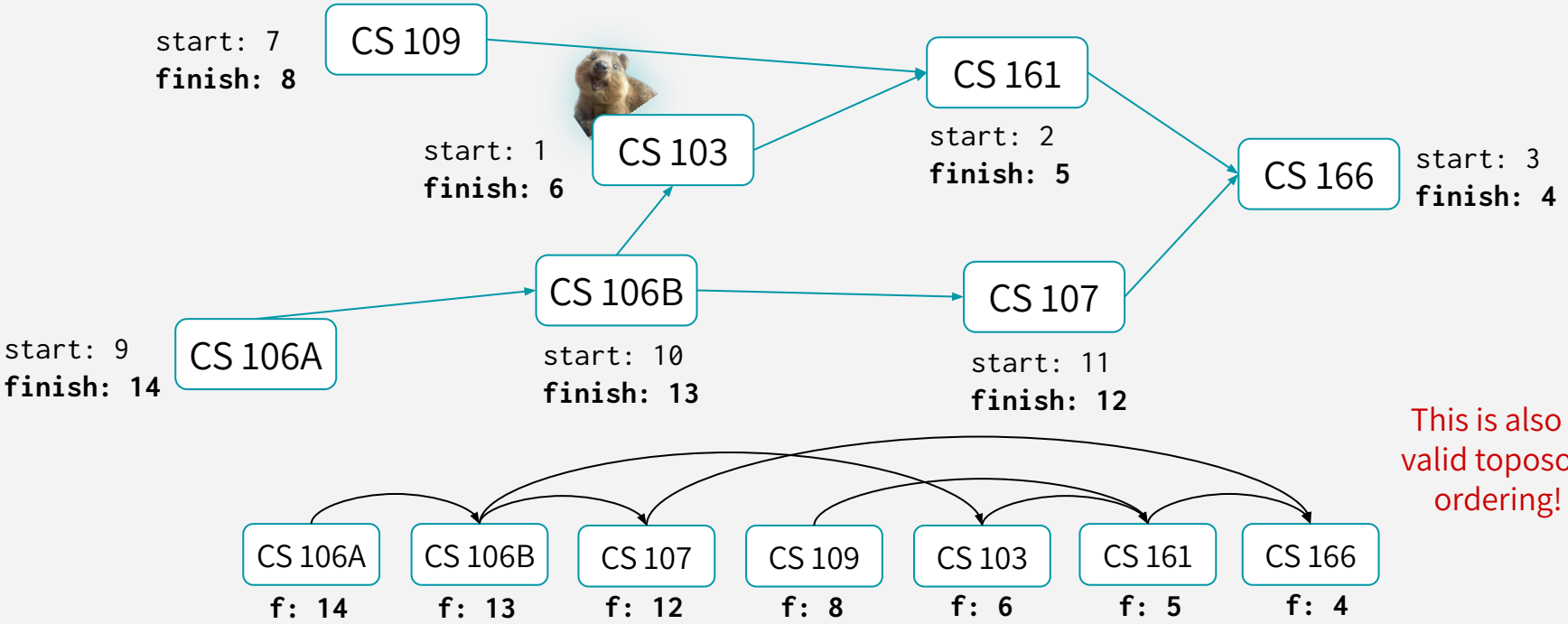

This is also a valid toposort ordering!

DFS & TOPOSORT RECAP

DFS can help you solve the Topological Sorting Problem.

That's just the fancy name for the problem of finding an ordering of the vertices which respect all the dependencies.

سوال؟