GATE 23 EE Q38

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Question: Consider a lead compensator of the form

$$K(s) = \frac{1+\frac{s}{a}}{1+\frac{s}{\beta a}}, \quad \beta > 1, \quad a > 0$$

The frequency at which this compensator produces maximum phase lead is 4 rad/s. At this frequency, the gain amplification provided by the controller, assuming an asymptotic Bode-magnitude plot of $K(s)$, is 6 dB. The values of a and β, respectively, are

(A) 1, 16 (B) 2, 4 (C) 3, 5 (D) 2.66, 2.25

Solution:

Parameter	Value
Transfer Function	$K(s) = \frac{1+\frac{s}{a}}{1+\frac{s}{\beta a}}$
Maximum Phase Lead Frequency	$\omega_m = 4\,\text{rad/s}$
Gain Amplification at ω_m	$20\log_{10}\lvert K(j\omega_m)\rvert = 6\,\text{dB}$
Conditions	$\beta > 1$, $a > 0$

TABLE I
GIVEN PARAMETERS

$$K(s) = \frac{1+\frac{s}{a}}{1+\frac{s}{a\beta}}$$

$$K(s) = \frac{s+a}{a} \cdot \frac{a\beta}{s+a\beta} \tag{1}$$

$$= \beta\frac{s+a}{s+a\beta} \tag{2}$$

1. The max phase lead is: $\omega_m = \sqrt{a \cdot a \cdot \beta}$

2. If $G(s) = \frac{k(s+a)}{s(s+a\beta)}$ has to act as a lead compensator, then $a\beta$ must be greater than a, i.e., $a\beta > a$.

Using the above properties we have:

$$\omega_m = \sqrt{a \cdot a\beta} = 4 \tag{3}$$

$$\beta > 1 \tag{4}$$

$$K(j\omega) = \frac{1+\frac{j\omega}{a}}{1+\frac{j\omega}{a\beta}} = \beta\frac{j\omega + a}{j\omega + a\beta} \tag{5}$$

$$|K(j\omega)| = \frac{\beta\sqrt{\omega^2+a^2}}{\sqrt{\omega^2+(a\beta)^2}} \tag{6}$$

Using Gain Amplification:

$$20\log_{10}|K(j\omega_m)| = 6 \tag{7}$$

$$|K(j\omega_m)| \approx 2 \tag{8}$$

$$|K(j\omega_m)| = \frac{\beta\sqrt{(\omega_m)^2+a^2}}{\sqrt{(\omega_m)^2+(a\beta)^2}} = 2 \tag{9}$$

$$\frac{\beta\sqrt{16+a^2}}{\sqrt{16+(a\beta)^2}} = 2 \tag{10}$$

Solving the above equation, we get $a \approx 2$ and $\beta = 4$. Therefore, the correct answer is **(B) 2, 4**.
The Bode plots for the same are as follows: