NCERT 11.9.2 Q7

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Question: Find the sum of n terms of the A.P. whose kth term is 5k + 1.

TABLE 0
GIVEN DATA

Symbol	Value	Parameter
x_0	1	First Term
x_k	$5k+1$	kth Term
d	5	Common Difference
S_n	?	Sum of N terms

Given:

$$\text{kth term of AP: } a_k = 5k + 1$$

Sequence Representation: The given arithmetic progression (AP) can be represented as:

$$x(n) = (5n + 1)u(n) \tag{1}$$

where $u(n)$ is the unit step function.

Z-transform: Apply the Z-transform to $x(n)$:

$$x(z) = \frac{5 + z^{-1}}{(1 - z^{-1})^2} \tag{2}$$

Region of Convergence or R.O.C :

$$|z| > 1 \tag{3}$$

Sum of First $n+1$ Terms: Express the sum of the first $n + 1$ terms ($y(n)$) in terms of $x(n)$ using the convolution:

$$y(n) = x(n) * u(n) \tag{4}$$

Inverse Z-transform: Apply the Z-transform on both sides to get $Y(z) = X(z)U(z)$, where $U(z)$ is the Z-transform of the unit step function.

$$Y(z) = \frac{5 + z^{-1}}{(1 - z^{-1})^2} \tag{5}$$

Contour Integration for Inverse Z-transform: Use contour integration to find the inverse Z-transform ($y(n)$):

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z)z^{n-1} \, dz \tag{6}$$

Residue Calculations: Calculate the residues R_1 and R_2 at the poles of the Z-transform.

$$R_1 = 5(n + 1) \tag{7}$$

$$R_2 = -n \tag{8}$$

$$S_n = r_1 + r_2 = 4n + 5 \tag{9}$$

Stem Plot of x(n) = (5n + 1)u(n)