NCERT 11.9.2 Q7

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Question: Find the sum of n terms of the A.P. whose kth term is $5k + 1$.

Solution:

TABLE 0
GIVEN DATA

Symbol	Value	Parameter
$x(0)$	1	First Term
$x(n)$	$(5n+1)u(n)$	kth Term
d	5	Common Difference
$y(n)$	?	Sum of N terms

Apply the Z-transform to $x(n)$:

$$X(z) = \frac{5z^{-1}}{(1-z^{-1})^2} + \frac{1}{(1-z^{-1})} \quad |z| > 1 \tag{1}$$

Sum of First $n+1$ Terms: Express the sum of the first $n+1$ terms ($y(n)$) in terms of $x(n)$ using convolution:

$$y(n) = x(n) * u(n) \tag{2}$$

Applying Z transform on both sides:

$$Y(z) = X(z)U(z) \tag{3}$$

$$= \frac{1}{(1-z^{-1})^2} + \frac{5z^{-1}}{(1-z^{-1})^3} \tag{4}$$

Given expressions:

$$X_1(z) = \frac{1}{(1-z^{-1})^2} \tag{5}$$

$$X_2(z) = \frac{5z^{-1}}{(1-z^{-1})^3} \tag{6}$$

Using z-transform pairs:
1.$\mathcal{Z}\{nu[n]\} = \frac{z}{(z-1)^2}$ with ROC $|z| > 1$

2.$\mathcal{Z}\{n(n-1)u[n]\} = \frac{2z}{(z-1)^3}$ with ROC $|z| > 1$

Rewriting the expression:

$$\frac{1}{(1-z^{-1})^2} + \frac{5}{2} \cdot \frac{2z^{-1}}{(1-z^{-1})^3} \tag{7}$$

$$= \mathcal{Z}\{nu[n]\} + \frac{5}{2}\mathcal{Z}\{n(n-1)u[n]\} \tag{8}$$

Therefore, the inverse z-transform of the sum is:

$$y[n] = nu[n] + \frac{5}{2}n(n-1)u[n] \tag{9}$$

The stem plot is given as