NCERT 11.9.2 Q7

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Question: Find the sum of n terms of the A.P. whose kth term is 5k + 1.

TABLE 0
GIVEN DATA

Symbol	Value	Parameter
$x(0)$	1	First Term
$x(k)$	$5k+1$	kth Term
d	5	Common Difference
$S(n)$	?	Sum of N terms

Given:

$$\text{kth term of AP: } a_k = 5k + 1$$

Sequence Representation: The given arithmetic progression (AP) can be represented as:

$$x(n) = (5n + 1)u(n) \tag{1}$$

where $u(n)$ is the unit step function.

Z-transform: Apply the Z-transform to $x(n)$:

$$X(z) = \frac{5z^{-1}}{(1 - z^{-1})^2} + \frac{1}{(1 - z^{-1})} \tag{2}$$

Region of Convergence or R.O.C :

$$|z| > 1 \tag{3}$$

Sum of First $n+1$ Terms: Express the sum of the first $n + 1$ terms ($y(n)$) in terms of $x(n)$ using the convolution:

$$y(n) = x(n) * u(n) \tag{4}$$

Applying Z transform on both sides

$$Y(z) = X(z)U(z) \tag{5}$$

$$= \frac{1}{(1 - z^{-1})^2} + \frac{5z^{-1}}{(1 - z^{-1})^3} \tag{6}$$

Using contour integration to find inverse Z transform:

$$y(n) = \frac{1}{2\pi j}\oint_C Y(z)z^{n-1}dz \tag{7}$$

$$= \frac{1}{2\pi j}\oint_C \left[\frac{1}{(1 - z^{-1})^2} + \frac{5z^{-1}}{(1 - z^{-1})^3}\right] z^{n-1}\,dz \tag{8}$$

The sum of the terms of the sequence is computed using the residue theorem, expressed as R_i, which represents the residue of the Z-transform at $z = 1$ for the expression $Y(z)$.

$$R_i = R_1 + R_2 \tag{9}$$

R_1 and R_2 are residues calculated at the poles of the Z-transform.

$$R_1 = \frac{1}{(2-1)!}\left.\frac{d(z^{n+1})}{dz}\right|_{z=1} \tag{10}$$

$$= (n + 1) \tag{11}$$

$$R_2 = \frac{1}{(3-1)!}\left.\frac{d^2(5z^{n+1})}{dz^2}\right|_{z=1} \tag{12}$$

$$= \frac{5}{2}(n + 1)(n) \tag{13}$$

$$S(n) = R_1 + R_2 \tag{14}$$

$$= (n + 1) + \frac{5}{2}(n + 1)(n) \tag{15}$$