Filter Design

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1 Introduction

We are supposed to design the equivalent FIR and IIR filter realizations for given filter number. This is a bandpass filter whose specifications are available below.

2 Filter Specifications

2.1 The Digital Filter

1. Passband: The passband is from {4 + 0.6(j)}kHz to {4 + 0.6(j+2)}kHz.
   where

   $$j = (r - 11000) \quad \mod \sigma \tag{1}$$

   where σ is sum of digits of roll number and r is roll number.

   $$r = 11204 \tag{2}$$
   $$\sigma = 8 \tag{3}$$
   $$j = 4 \tag{4}$$

   substituting $j = 4$ gives the passband range for our bandpass filter as 6.4 kHz - 7.6 kHz. Hence, the un-normalized discrete time filter passband frequencies are F_{p1} = 6.4 kHz and F_{p2} = 7.6 kHz.
   The corresponding normalized digital filter passband frequencies are

   $$\omega_{p1} = 2\pi \frac{F_{p1}}{F_s} = 0.27\pi \tag{5}$$
   $$\omega_{p2} = 2\pi \frac{F_{p2}}{F_s} = 0.32\pi \tag{6}$$

2. Tolerances: The passband (δ_1) and stopband (δ_2) tolerances are given to be equal, so we let $\delta_1 = \delta_2 = \delta = 0.15$.

3. Stopband: The transition band for bandpass filters is $\Delta F = 0.3$ kHz on either side of the passband.

$$F_{s1} = 6.4 - 0.3 = 6.1\text{KHz} \tag{7}$$
$$F_{s2} = 7.6 + 0.3 = 7.9\text{KHz} \tag{8}$$

$$\omega_{s1} = 2\pi\frac{F_{s1}}{F_s} = 0.254\pi \tag{9}$$
$$\omega_{s2} = 2\pi\frac{F_{s2}}{F_s} = 0.329\pi \tag{10}$$
$$\tag{11}$$

2.2 The Analog filter

In the bilinear transform, the analog filter frequency (Ω) is related to the corresponding digital filter frequency(ω) :

$$\Omega = \tan\frac{\omega}{2} \tag{12}$$

Using this relation, we obtain the analog passband and stopband frequencies as: $\Omega_{p1} = 0.4515$, $\Omega_{p2} = 0.5497$ and $\Omega_{s1} = 0.4216$, $\Omega_{s2} = 0.5683$ respectively.

3 The IIR Filter Design

We are supposed to design filters whose stopband is monotonic and passband equiripple. Hence, we use the Chebyschev approximation to design our bandpass IIR filter.

3.1 The Analog Filter

1. Low Pass Filter Specifications: Let $H_{a,BP}(j\Omega)$ be the desired analog bandpass filter, with the specifications provided in Section 2.2, and $H_{a,LP}(j\Omega_L)$ be the equivalent low pass filter, then
$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega} \tag{13}$$
where $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.4982$ and $B = \Omega_{p2} - \Omega_{p1} = 0.0982$.
Substituting Ω_{s1} and Ω_{s2} in (13) we obtain the stopband edges of lowpass filter
$$\Omega_{Ls1} = \frac{\Omega_{s1}^2 - \Omega_0^2}{B\Omega_{s1}} = -1.7 \tag{14}$$
$$\Omega_{Ls2} = \frac{\Omega_{s2}^2 - \Omega_0^2}{B\Omega_{s2}} = 1.34 \tag{15}$$
And we choose the minimum of these two stopband edges
$$\Omega_{Ls} = \min(|\Omega_{Ls_1}|, |\Omega_{Ls_2}|) = 1.. \tag{16}$$

2. The Low Pass Chebyschev Filter Paramters: The magnitude of frequency response of the low pass filter is given by

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1+\epsilon^2 c_N^2(\Omega_L/\Omega_{Lp})} \tag{17}$$

The passband edge of the low pass filter is chosen as $\Omega_{Lp} = 1$. Therfore ,

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1+\epsilon^2 c_N^2(\Omega_L)} \tag{18}$$

Here c_N denote the chebyshev polynomials for a particular order N of the filter.

$$c_N(x) = \cosh(N\cosh^{-1} x), x = \Omega_L \tag{19}$$

$$c_0(x) = 1 \tag{20}$$

$$c_1(x) = x \tag{21}$$

There exists a recurssive relation from which all the polynomials can be found out.

$$c_{N+2} = 2xc_{N+1} - c_N \tag{22}$$

Imposing the band restrictions on (17)

$$|H_{a,LP}(j\Omega_L)|^2 < \delta_2 \;\text{ for }\; \Omega_L = \Omega_{Ls} \tag{23}$$

$$1-\delta_1 < |H_{a,LP}(j\Omega_L)|^2 < 1 \;\text{ for }\; \Omega_L = \Omega_{Lp} \tag{24}$$

$$\tag{25}$$

we obtain :

$$\frac{\sqrt{D_2}}{c_N(\Omega_{Ls})} \le \epsilon \le \sqrt{D_1},$$

$$N \ge \left\lceil \frac{\cosh^{-1}\sqrt{D_2/D_1}}{\cosh^{-1}\Omega_{Ls}} \right\rceil, \tag{26}$$

where $D_1 = \frac{1}{(1-\delta)^2} - 1$ and $D_2 = \frac{1}{\delta^2} - 1$ and $\lceil . \rceil$ is known as the ceiling operator .

Parameter	Value
D_1	0.384
D_2	43.44
N	4
$c_4(x)$	$8x^4 + 8x^2 + 1$

Table 1: Parameter Table

we get $N \ge 4$ and $0.16 \le \epsilon \le 0.62$

The below code plots (17) for different values of ϵ .

```
https://github.com/dhanushnayakh03/EE1205/tree/main/Audio_%20Filter/
    codes/plot1.py
```

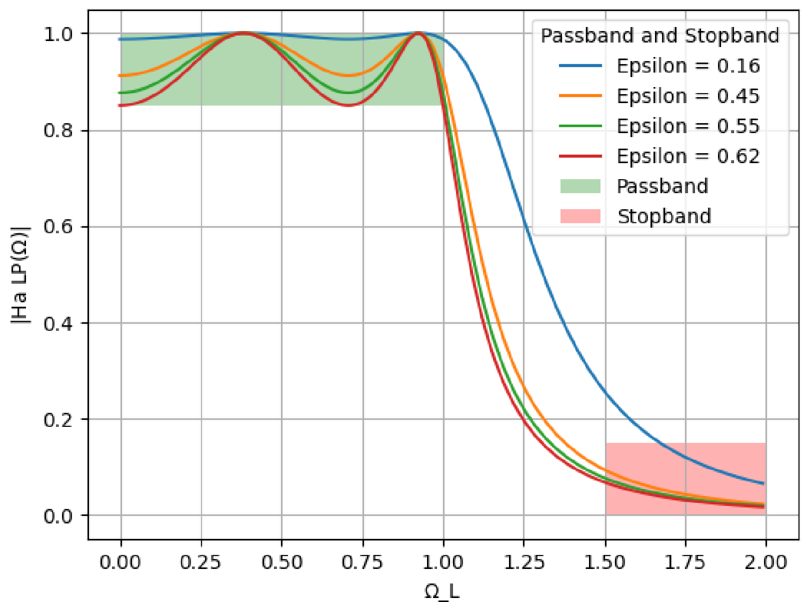

Figure 1: The Analog Low-Pass Frequency Response for $0.16 \leq \epsilon \leq 0.62$

In Fig. 1 we can observe the equiripple behaviour in passband and monotonic behaviour in stopband. As the value of ϵ increases the value of $|H_{a,LP}(j\Omega_L)|$ decreases.

3. The Low Pass Chebyschev Filter: The next step in design is to find an expression for magnitude response in s domain.

   Using $s = j\Omega$ or in this case $s_L = j\Omega_L$ we obtain:

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1 + \epsilon^2 c_N^2(\frac{s_L}{j})} \tag{27}$$

   To find poles equate the denominator to zero:

$$1 + \epsilon^2 c_N^2\left(\frac{s_L}{j}\right) = 0 \quad \text{where} \quad c_N(x) = cos\left(Ncos^{-1}(x)\right) \tag{28}$$

On solving (28) we obtain poles :

$$s_k = -\Omega_{Lp} \sin(A_k) \sinh(B_k) - j\Omega_{Lp} \cos(A_k) \cosh(B_k) \tag{29}$$

where k is the index of the pole and

$$A_k = (2k+1)\frac{\pi}{2N} \tag{30}$$

$$B_k = \frac{1}{N} \sinh^{-1}\left(\frac{1}{\epsilon}\right) \tag{31}$$

The below code computes the values of s_k and stores it in a text file.

https://github.com/dhanushnayakh03/EE1205/blob/main/Filter_Design/codes/sk_gen.c

The poles obtained are formulated in the table below.

Pole	*Value*
s_1	-0.3913 - j0.4156
s_2	-0.3913 + j0.4156
s_3	-0.1621 + j1.0033
s_4	0.1621 + j1.0033
s_5	0.3913 + j0.4156
s_6	0.3913 - j0.4156
s_7	0.1621 - j1.0033
s_8	-0.1621 - j1.0033

Table 2: Values of s_k

The below code plots the pole-zero plot.

https://github.com/dhanushnayakh03/EE1205/blob/main/Filter_Design/codes/plot1.py

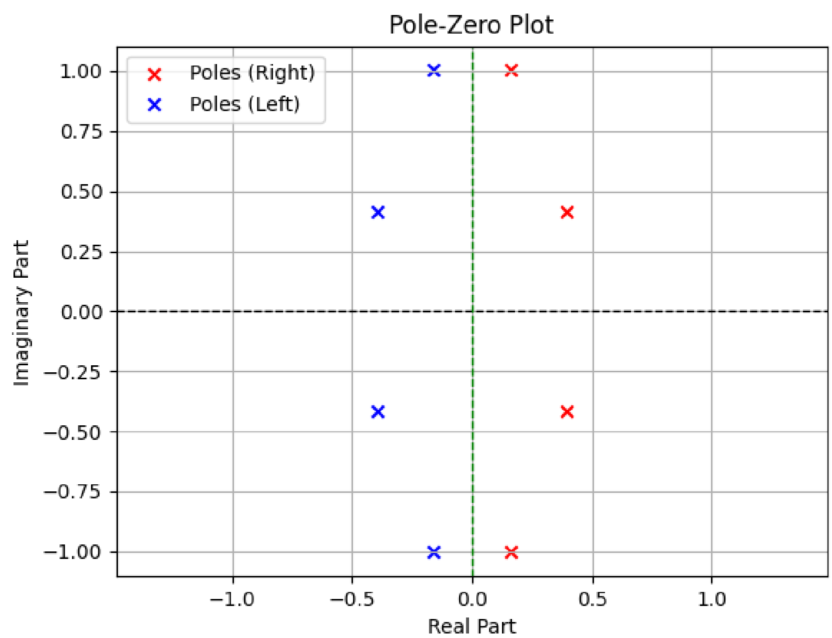

Figure 2: The Pole zero plot and all the poles lie on an ellipse. The left and right poles have been identified as shown.

The poles in the left half of the plane are considered in the design as we intend to design a stable system.
Therefore the magnitude response is written as :-

$$H_{a,LP}(s_L) = \frac{G_{LP}}{(s_L - s_5)(s_L - s_6)(s_L - s_7)(s_L - s_8)} \tag{32}$$

where G_{LP} is the gain of the Low pass filter. Refer to Table 2 for s_k values.

We know that from (17):-

$$\left|H_{a,LP}(s_L)\right| = \frac{1}{\sqrt{1+\epsilon^2}} \text{at } \Omega_L = 1 \implies s_L = j \tag{33}$$

Substituting respective values in (33) we get $G_{LP} = 0.4166$

$$H_{a,LP}(s_L) = \frac{0.4166}{(s_L - s_5)(s_L - s_6)(s_L - s_7)(s_L - s_8)} \tag{34}$$

$$= \frac{0.4166}{s_L^4 + 1.3022 s_L^3 + 1.84781 s_L^2 + 1.16512 s_L + 0.435003} \tag{35}$$