NCERT 11.9.2 Q7

EE23BTECH11204 - Ashley Ann Benoy*

Question: Find the sum of n terms of the A.P. whose kth term is 5k + 1.

Solution:

TABLE 0 GIVEN DATA

Symbol	Value	Parameter
<i>x</i> (0)	1	First Term
x(n)	(5n+1)u(n)	kth Term
d	5	Common Difference
y(n)	?	Sum of N terms

Apply the Z-transform to x(n):

$$X(z) = \frac{5z^{-1}}{(1 - z^{-1})^2} + \frac{1}{(1 - z^{-1})} \quad |z| > 1$$
 (1)

Sum of First n + 1 Terms: Express the sum of the first n + 1 terms (y(n)) in terms of x(n) using convolution:

$$y(n) = x(n) * u(n)$$
 (2)

Applying Z transform on both sides:

$$Y(z) = X(z)U(z)$$
 (3)

$$= \frac{1}{(1-z^{-1})^2} + \frac{5z^{-1}}{(1-z^{-1})^3} \tag{4}$$

TABLE 0
Z TRANSFORM PAIRS

Expression	Z-Transform	ROC
nu[n]	$\frac{z^{-1}}{(1-z^{-1})^2}$	z > 1
n(n-1)u[n]	$\frac{2z^{-1}}{(1-z^{-1})^3}$	z > 1

Rewriting the expression:

$$\frac{1}{(1-z^{-1})^2} + \frac{5}{2} \cdot \frac{2z^{-1}}{(1-z^{-1})^3} \tag{5}$$

On referring the above table we can obtain the Z trnasform inverse as follows:

$$y[n] = nu[n] + \frac{5}{2}n(n-1)u[n]$$
 (6)

The stem plot is given as

