NCERT 11.9.2 Q7

EE23BTECH11204 - Ashley Ann Benoy*

Question: Find the sum of n terms of the A.P. whose kth term is $5k+1$.

Solution:

TABLE 0
GIVEN DATA

Symbol	Value	Parameter
$x(0)$	1	First Term
$x(n)$	$(5n+1)u(n)$	kth Term
d	5	Common Difference
$y(n)$	?	Sum of N terms

Apply the Z-transform to $x(n)$:

$$X(z) = \frac{5z^{-1}}{(1-z^{-1})^2} + \frac{1}{(1-z^{-1})} \quad |z| > 1 \tag{1}$$

Sum of First $n+1$ Terms: Express the sum of the first $n+1$ terms ($y(n)$) in terms of $x(n)$ using convolution:

$$y(n) = x(n) * u(n) \tag{2}$$

Applying Z transform on both sides:

$$Y(z) = X(z)U(z) \tag{3}$$

$$= \frac{1}{(1-z^{-1})^2} + \frac{5z^{-1}}{(1-z^{-1})^3} \tag{4}$$

Given expressions:

$$X_1(z) = \frac{1}{(1-z^{-1})^2} \tag{5}$$

$$X_2(z) = \frac{5z^{-1}}{(1-z^{-1})^3} \tag{6}$$

Now, let's find the Z-transforms:

1. For $X_1(z)$:

$$X_1(z) = \sum_{n=0}^{\infty} x_1[n] \cdot z^{-n} \tag{7}$$

$$x_1[n] = \binom{n+1}{1} \tag{8}$$

The Z-transform pair for $x[n] = \binom{n+k-1}{k-1}u[n]$ is $X(z) = \frac{1}{(1-z^{-1})^k}$, where k is a positive integer. Therefore, $X_1(z) = \frac{1}{(1-z^{-1})^2}$ corresponds to $x_1[n] = \binom{n+1}{1}$.

2. For $X_2(z)$:

$$X_2(z) = \sum_{n=0}^{\infty} x_2[n] \cdot z^{-n} \tag{9}$$

$$x_2[n] = \binom{n+2}{2} \tag{10}$$

The Z-transform pair for $x[n] = \binom{n+k-1}{k-1}u[n]$ is $X(z) = \frac{1}{(1-z^{-1})^k}$, where k is a positive integer. Therefore, $X_2(z) = \frac{5z^{-1}}{(1-z^{-1})^3}$ corresponds to $x_2[n] = \binom{n+2}{2}$.

Now, let's find the inverse Z-transforms for $X_1(z)$ and $X_2(z)$:

1. For $X_1(z)$:

$$x_1[n] = \binom{n+1}{1} \tag{11}$$

This is a first-order discrete-time signal, and its inverse Z-transform is given by $x_1[n] = -nu[n]$.

2. For $X_2(z)$:

$$x_2[n] = \binom{n+2}{2} \tag{12}$$

This is a second-order discrete-time signal, and its inverse Z-transform is given by $x_2[n] = \frac{1}{2}n(n-1)u[n]$.

Therefore, the inverse Z-transforms for $X_1(z)$ and $X_2(z)$ are:

1. $x_1[n] = -nu[n]$

2. $x_2[n] = \frac{1}{2}n(n-1)u[n]$

Now, let's express $y(n)$ in terms of $x_1(n)$ and $x_2(n)$:

$$y(n) = x_1(n) + x_2(n) \tag{13}$$

$$= -nu(n) + \frac{1}{2}n(n-1)u(n) + 5(n-1)u(n-1) \tag{14}$$

$$= (n+1) + \frac{5n(n+1)}{2}u(n) \tag{15}$$

The stem plot is given as