NCERT 11.9.2 Q7

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Question: Find the sum of n terms of the A.P. whose kth term is $5k+1$.

Symbol	Value	Parameter
$x(0)$	1	First Term
$x(n)$	$(5n+1)u(n)$	kth Term
d	5	Common Difference

TABLE I
GIVEN PARAMETERS

Apply the Z-transform to $x(n)$:

$$X(z) = \frac{5z^{-1}}{(1-z^{-1})^2} + \frac{1}{(1-z^{-1})} \quad |z| > 1 \tag{1}$$

Sum of First n Terms:

$$y(n) = x(n) * u(n) \tag{2}$$

Applying Z transform on both sides:

$$Y(z) = X(z)\,U(z) \tag{3}$$

$$= \frac{1}{(1-z^{-1})^2} + \frac{5}{2} \cdot \frac{2z^{-1}}{(1-z^{-1})^3} \tag{4}$$

Now we can compare the above pairs as;

$$nu(n) \overset{\text{Z}}{\leftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \tag{5}$$

$$u(n) \overset{\text{Z}}{\leftrightarrow} \frac{1}{(1-z^{-1})} \tag{6}$$

$$n(n-1)\,u(n) \overset{\text{Z}}{\leftrightarrow} \frac{2z^{-1}}{(1-z^{-1})^3} \tag{7}$$

On referring the above equations and comparing, we can obtain the Z transform inverse as follows:

$$y(n) = (n+1)\,u(n) + \frac{5}{2}n(n-1)\,u(n) \tag{8}$$

$$= \left(n+1+\frac{5}{2}n(n-1)\right)u(n) \tag{9}$$

Since we are taking n starting from 0 we replace n with n+1 to make our simulation match with the theory
Therefore, we have got the sum of n terms as:

$$y(n) = \left(n+2+\frac{5}{2}n(n+1)\right)u(n+1) \tag{10}$$

The stem plot is given as