

GATE 23 EE Q38

EE23BTECH11204 - Ashley Ann Benoy*

Question: Consider a lead compensator of the form

$$K(s) = \frac{1 + \frac{s}{a}}{1 + \frac{s}{\beta a}}, \quad \beta > 1, \quad a > 0$$

The frequency at which this compensator produces maximum phase lead is 4 rad/s. At this frequency, the gain amplification provided by the controller, assuming an asymptotic Bode-magnitude plot of $K(s)$, is 6 dB. The values of a and β , respectively, are

- (A) 1, 16 (B) 2, 4 (C) 3, 5 (D) 2.66, 2.25

Solution:

Parameter	Value
Transfer Function	$K(s) = \frac{1 + \frac{s}{a}}{1 + \frac{s}{\beta a}}$
Maximum Phase Lead Frequency	$\omega_m = 4 \text{ rad/s}$
Gain Amplification at ω_m	$20 \log_{10} K(j\omega_m) = 6 \text{ dB}$
Conditions	$\beta > 1, a > 0$

TABLE I
GIVEN PARAMETERS

$$K(s) = \frac{1 + \frac{s}{a}}{1 + \frac{s}{a\beta}}$$

$$K(s) = \frac{s + a}{a} \cdot \frac{a\beta}{s + a\beta} \quad (1)$$

$$= \beta \frac{s + a}{s + a\beta} \quad (2)$$

1. The max phase lead is: $\omega_m = \sqrt{a \cdot a\beta}$

2. If $G(s) = \frac{k(s+a)}{s(s+a\beta)}$ has to act as a lead compensator, then $a\beta$ must be greater than a , i.e., $a\beta > a$.

Using the above properties we have:

$$\omega_m = \sqrt{a \cdot a\beta} = 4 \quad (3)$$

$$\beta > 1 \quad (4)$$

$$K(j\omega) = \frac{1 + \frac{j\omega}{a}}{1 + \frac{j\omega}{a\beta}} = \beta \frac{j\omega + a}{j\omega + a\beta} \quad (5)$$

$$|K(j\omega)| = \frac{\beta \sqrt{\omega^2 + a^2}}{\sqrt{\omega^2 + (a\beta)^2}} \quad (6)$$

Using Gain Amplification:

$$20 \log_{10} |K(j\omega_m)| = 6 \quad (7)$$

$$|K(j\omega_m)| \approx 2 \quad (8)$$

$$|K(j\omega_m)| = \frac{\beta \sqrt{(\omega_m)^2 + a^2}}{\sqrt{(\omega_m)^2 + (a\beta)^2}} = 2 \quad (9)$$

$$\frac{\beta \sqrt{16 + a^2}}{\sqrt{16 + (a\beta)^2}} = 2 \quad (10)$$

Solving the above equation, we get $a \approx 2$ and $\beta = 4$. Therefore, the correct answer is **(B) 2, 4**.

The Bode plots for the same are as follows:

