

# NCERT 11.9.2 Q7

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**Question: Find the sum of n terms of the A.P. whose kth term is  $5k + 1$ .**

The sum of the terms of the sequence is computed using the residue theorem, expressed as  $R_i$ , which represents the residue of the Z-transform at  $z = 1$  for the expression  $Y(z)$ .

TABLE 0  
GIVEN DATA

Symbol	Value	Parameter
$x(0)$	1	First Term
$x(k)$	$5k + 1$	kth Term
$d$	5	Common Difference
$S(n)$	?	Sum of N terms

Given:

kth term of AP:  $a_k = 5k + 1$

Sequence Representation: The given arithmetic progression (AP) can be represented as:

$$x(n) = (5n + 1)u(n) \quad (1)$$

where  $u(n)$  is the unit step function.

Z-transform: Apply the Z-transform to  $x(n)$ :

$$X(z) = \frac{5z^{-1}}{(1 - z^{-1})^2} + \frac{1}{(1 - z^{-1})} \quad |z| > 1 \quad (2)$$

Sum of First  $n + 1$  Terms: Express the sum of the first  $n + 1$  terms ( $y(n)$ ) in terms of  $x(n)$  using the convolution:

$$y(n) = x(n) * u(n) \quad (3)$$

Applying Z transform on both sides

$$Y(z) = X(z)U(z) \quad (4)$$

$$= \frac{1}{(1 - z^{-1})^2} + \frac{5z^{-1}}{(1 - z^{-1})^3} \quad (5)$$

Using contour integration to find inverse Z transform:

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z)z^{n-1} dz \quad (6)$$

$$= \frac{1}{2\pi j} \oint_C \left[ \frac{1}{(1 - z^{-1})^2} + \frac{5z^{-1}}{(1 - z^{-1})^3} \right] z^{n-1} dz \quad (7)$$

$$R_i = R_1 + R_2 \quad (8)$$

$R_1$  and  $R_2$  are residues calculated at the poles of the Z-transform.

$$R_1 = \frac{1}{(2 - 1)!} \left. \frac{d(z^{n+1})}{dz} \right|_{z=1} \quad (9)$$

$$= (n + 1) \quad (10)$$

$$R_2 = \frac{1}{(3 - 1)!} \left. \frac{d^2(5z^{n+1})}{dz^2} \right|_{z=1} \quad (11)$$

$$= \frac{5}{2}(n + 1)(n) \quad (12)$$

$$S(n) = R_1 + R_2 \quad (13)$$

$$= (n + 1) + \frac{5}{2}(n + 1)(n) \quad (14)$$

